An iterative approach for distribution chain design in agile virtual environment

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Abstract

Purpose – The study sets out to explore the use of an iterative approach for designing distribution chain in an agile virtual environment; in an agile virtual environment, quick adaptation to changing market situation and automation of supply chain management processes are essential.

Design/methodology/approach – The iterative approach consists of two parts: the strategic model and the tactical model. First, the strategic model (including number of distributors, location of distributors) is determined. Then, based on the output of the strategic model, the tactical model (i.e. the inventory planning at each node, and vehicle routeing between different nodes of the chain) is determined. After determining the tactical model, the operation-related parameters from the tactical model are input into the strategic model again, and the configuration of distribution chain is re-optimized. Such iterations proceed until the design result converges.

Findings – The proposed iterative design process provides many advantages such as reuse of knowledge, adaptive to changing market conditions, modular design, and optimal results guaranteed by mixed mathematical usage. The proposed approach is also realizable as a supply chain management software tool.

Originality/value – An important contribution of this study is the iterative process that uses three different types of mathematics. For determination of the strategic model mixed integer programming is used. Determination of the tactical model is effected using genetic algorithm and probability theory.

Keywords - distribution management, organizational structures, inventory control, supply chain management

1. Introduction

In this paper, an iterative approach is proposed for designing the distribution chain. The proposed iterative process is most suitable for agile virtual environment in which quick adaptation to changing market situation and automation of supply chain management processes are essential.

The iterative approach consists of two parts: the first part is to determine the strategic model and the second part is to determine the tactical model. The strategic model gets the initial values for the configuration of the distribution chain (e.g. number of distributors, location of distributors, etc.) from the ongoing or past collaborations of the virtual enterprise. Mixed integer programming (MIP) is used for determining the strategic model. In the tactical model, two issues are mainly considered: identifying routes for vehicles by genetic algorithm, and determining parameters for inventory control by probability theory.
After determining tactical model, the newly calculated operation-related parameters (e.g. the unit product delivering cost, average inventory level, etc.) are input again into the strategic model in order to re-optimize the configuration of the distribution chain. Such iterative process proceeds until the design result converges meaning the difference between successive iterations is not remarkable.

1.1 Distribution chain design
A supply chain generally centered with the nucleus enterprise may be divided into three parts: supply, production and distribution part (Davidrajuh, 2003a, b). To satisfy customer demand, maximize its profit, and win the competition in the increasingly globalized economy, first, the nucleus enterprise must analyze the market and understand the customer demand, then, it may begin to plan its production process and organize its supply chain. As the distribution part plays crucial role in the management of a supply chain, separation of it from supply chain and dealing with it independently is recommended (Ma and Deng, 2002). When designing a distribution chain, following questions must be answered:

- Where to locate the retailers and wholesalers so as to deliver products to customers efficiently, and maximize profit for the nucleus enterprise effectively?
- What kind of transportation channels and modes will be used?
- How many products need to be held at different locations?

The ultimate objectives for supply chain management are: to maximize profit for the nucleus enterprise and its collaborators; and to satisfy customers’ requirements. Distribution chain design is crucial to achieving these objectives. First, the revenue of a supply chain is largely related to the places where the retailers are located; it is the distribution chain design that decides where to locate these retailers. Second, logistics cost takes an important part in the total cost of a supply chain. In the proposed design approach, a set of optimization methods is applied to minimize the logistics cost; by maximizing revenue and by minimizing the cost, profit for the supply chain is maximized. Finally, distribution chain is directly faced with customers. Both inventory control and product delivery planning are related to customer service. These two issues are addressed in the tactical model of the distribution chain design.

There are three levels of distribution chain design that can be distinguished depending on the time horizon. The three levels are strategic, tactical and operational design (Vidal and Goetschalckx, 1997). The strategic design considers time horizons of more than one year; operational design involves short-term decision, often less than an hour or a day; tactical design falls in between those two extremes with respect to the time horizon. As strategic design has more significant effect on the management of distribution chain, it is the main concern of this paper. At the same time, to determine the operation related parameters more precisely, the tactical model of a distribution chain is also considered here.

1.2 Organization of this paper
In the next section (section 2), a literature review on mathematical approaches for designing distribution chain is presented. In section 3, the iterative approach is presented. In sections 4 and 5, design of the strategic model and the tactical model are
presented respectively. Finally, in section 6, the managerial implication of this design approach is discussed.

2. Mathematics for distribution chain design

Probably, Geoffrion and Graves (1974) was the first to use MIP to design a multi-commodity single-period production-distribution system. Since then, mathematical programming, heuristics, simulation and artificial intelligence have largely been used to design a distribution chain (or production-distribution network as called in those papers). Cohen and Lee (1985), and Dogan and Goetschalckx (1999) considered multi-commodity single-product production-distribution systems, and used mathematical programming methods to determine the locations of facilities by minimizing cost or maximizing profit. To make the design more valuable in practice, Jayaraman (1998) extended the model into a multi-product situation, and different production methods were considered in his design model.

To model the stochastic environment of a distribution chain, Escudero and Galindo (1999) and MirHassani (2000) took customer demand as random variable, and used Integer stochastic programming (ISP) to design their production-distribution systems.

Besides mathematical programming, other optimization methods were also used to design a production-distribution system. For example, Berry et al. (1998) used genetic algorithm to optimize the topology of distribution network. Anthony (2000) used simulated annealing method to determine the configuration of a production-distribution system, etc.

In these existing design models, following general optimization form was explicitly or implicitly used:

(1) **Objective function:**
   - minimize cost = production cost + inventory holding cost + transportation cost + ...; or
   - maximize profit = all revenues – total cost.

(2) **Subject to constraints:**
   - production capacity constraints;
   - inventory capacity constraints; and
   - transportation constraints.

Two types of notations are used in these formulae: parameters (e.g. demand at retailers, unit product delivering cost, average inventory level, etc.) and decision variables (e.g. binary variables to indicate the selection/rejection of a distributor or a transportation channel, etc.).

3. The design process

The proposed design process is shown in Figure 1. In the initialization block, all parameters (including operation-related parameters) are assigned initial values from the ongoing (and/or from the past) collaboration. Then, for the strategic model, the number and location of wholesalers and retailers are determined. Given the output of strategic model, the tactical model is determined. The tactical model is used to determine the transportation routes and inventory control parameters.
After determining the tactical model, the iterative process inputs the operation-related parameters into the strategic model, and the configuration of the distribution chain is re-optimized. At the end of each iteration the successive design results are compared. If there is no remarkable difference between the successive design results means the design process converges, and that the design process is accomplished. Hence, the latest configuration of the distribution chain, inventory control parameters, and product delivery routes are output as the final design.

When the market conditions change, there may be changes in the values of some of the parameters such as demand, transportation costs, etc. These new values can be input into the system and the design process is invoked again. First, based on these new values, the tactical model is adjusted (operation-related parameters are re-calculated) to adapt to the new market situation. When there is significant change at the operation-related parameters, the enterprise may reconsider its distribution chain configuration to adapt to this new market.

4. The strategic model
Ma et al. (2004) developed an on-line approach for benchmarking potential distributors. In this approach, distributors were evaluated individually; hence, the distributors selected in that approach can only be viewed as potential distributors for further selection. However, it is the structure of the distribution chain (Caputo et al., 2004; Childerhouse et al., 2003) and the relationship between different distributors (Rahman, 2004; Wu et al., 2004)
that determine the performance of a distribution chain. In this paper, the potential distributors will be used for the configuration of the distribution chain (strategic model), i.e. the optimal number and location of distributors including wholesalers and retailers. Among several possible optimization methods, MIP offers the following advantages:

- it is capable of solving large-scale problems; and
- it is easy to be implemented as computer applications. Hence, MIP is chosen to optimize the strategic model.

A MIP model is composed of two types of formulae: objective function and constraints. The objective function in this strategic model is to maximize profit, while the constraints here mainly contain flexibility constraint, material flow balance constraint,

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<td>(v_j)</td>
<td>1 if the wholesaler (j) is selected, 0 otherwise</td>
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<td>(w_{ij})</td>
<td>1 if retailer (i) is assigned to wholesaler (j), 0 otherwise</td>
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Table I. Explanation for notations in the strategic model
etc. The formulae for objective function and constraints are shown as follows, and the corresponding notation explanation is shown in Table I:

Maximise profit = \[ \sum_{i=1}^{n} u_i P_i D_i - \sum_{j=1}^{m} \sum_{i=1}^{n} W_{ij} C_{ji} D_i - \sum_{j=1}^{m} v_j (C_{openj}(I_{kj}) + C_j I_j T) \]

\[ - \sum_{j=1}^{m} v_j C_0 D_{wj} - C_0 I_0 T \] (1)

subject to:

\[ \frac{C_{p_{\text{max}}} - \sum_{i=1}^{n} u_i D_i}{C_{p_{\text{max}}}} \geq \varepsilon_p \] (2)

\[ \sum_{i=1}^{n} u_i D_i \geq C_{p_{\text{min}}} \] (3)

\[ \frac{C_{t_{\text{max}j}} - \sum_{i=1}^{n} W_{ij} D_i}{C_{t_{\text{max}j}}} \geq \varepsilon_{ij} \forall j \] (4)

\[ \sum_{i=1}^{n} W_{ij} D_i \geq C_{t_{\text{min}j}} \forall j \] (5)

\[ d_{wj} = \sum_{i=1}^{n} W_{ij} D_i \] (6)

\[ P_p = \sum_{j=1}^{m} d_{wj} \] (7)

\[ \sum_{j=1}^{m} W_{ij} = 1 \forall i \] (8)

\[ W_{ij} \leq u_i \; \text{and} \; W_{ij} \leq v_j \forall i, j \] (9)

\[ d_{wj} \geq 0 \geq P_p \geq 0 \forall j \] (10)

\[ u_i, v_j, w_{ij} = 0 \; \text{or} \; 1 \forall i, j \] (11)

As indicated in Table I, the unit product delivering cost (\( C_{ji} \) and \( C_0j \)) and average inventory level (\( I_0 \) and \( I_j \)) are identified as operation-related parameters.
In the objective function, profit equals total revenue minus total cost. Here, only four types of costs are considered:

1. product delivery cost from wholesalers to retailers (the second item);
2. opening and inventory holding cost at wholesalers (the third item);
3. product delivery cost from distribution center to wholesalers (the fourth item); and
4. inventory holding cost at the distribution center (the fifth item).

Here, it is assumed that the retailers do not belong to the nucleus enterprise, so the inventory holding cost at retailers is not considered.

Formulae (2) to (11) represent different types of constraints, which are explained as follows:

- **Formula (2)** specifies the production volume flexibility requirement. Production volume flexibility for an enterprise is measured by the percentage of its slack production capacity (Slack, 1987).
- **Formula (3)** ensures that sum of demands must exceed the minimal production capability to guarantee enough resource utilization rate.
- **Formula (4)** specifies the delivery volume flexibility requirement. Similar to production volume flexibility, delivery volume flexibility for a warehouse is defined as the percentage of its slack through-put.
- **Formula (5)** ensures the sum of demands at a wholesaler exceeds its minimal throughput capacity.
- **Formula (6)** specifies the material flow balance at wholesalers, and **Formula (7)** specifies the material flow balance at the distribution center.
- **Formula (8)** ensures that a retailer can only be assigned to one wholesaler.
- **Formula (9)** ensures that only when both retailer $i$ and wholesaler $j$ are selected, retailer $i$ can possibly be assigned to wholesaler $j$.
- **Formula (10)** ensures continuous variable $d_{wj}$ and $p_p$ are positive, while **Formula (11)** ensures decision variables $u_i$, $v_j$ and $w_{ij}$ are binary.

5. The tactical model
The tactical model is determined based on the output of the strategic model. The demands at retailers, which were represented by average value in the strategic model, will be viewed as random variable in the tactical model due to its short time horizon. The tactical model has the following sub-models:

1. Inventory control model at retailers;
2. Transportation model from wholesalers to retailers;
3. Inventory control model at wholesalers and at distribution center; and
4. Transportation model from distribution center to wholesalers.

5.1 Inventory control model at a retailer
According to Tijms (1994), there are mainly two types of inventory control models: periodic review $(R, S)$ model and continuous review $(s, Q)$ model. For $(R, S)$ model, an order is placed every
$R$ unit time to raise the inventory level to $S$. For $(s, Q)$ model, an order $Q$ is placed when at hand inventory is less than or equal to $s$. Due to the random customer demand at a retailer, $(R, S)$ model may cause following problems: the order may come when it is unnecessary (the at hand inventory level is high enough to meet the demand), or may not come when it is necessary (the at hand inventory level is less than safety stock). Both situations may cause extra cost for the retailer. Hence, $(s, Q)$ model is selected for the inventory control policy.

For $(s, Q)$ model, following formula was normally used to determine parameter $s$ (Silver and Peterson, 1985; Johnson et al., 1996):

$$s = L + n\sigma$$  \hspace{1cm} (12)

where:

$s$ = the reorder point (or safety stock).
$L$ = the total demand during lead time.
$n$ = the safety factor. It is determined by subjective judgment.
$\sigma$ = the standard deviation of demand during lead time.

Parameter $Q$ was determined by minimizing inventory holding cost which is shown as follows:

$$TC = \frac{TD}{Q} \times c_o + \left( \frac{Q}{2} + n\sigma \right) \times c_h$$  \hspace{1cm} (13)

where:

$TC$ = the total inventory holding cost.
$TD$ = the total demand during successive replenishments.
$Q$ = the ordering quantity.
$c_o$, $c_h$ = the unit ordering and holding cost respectively.

In these formulae, when determining $Q$, the demand was implicitly assumed to be linear with respect to time, so the average inventory level at the retailer was expressed as: $ns + Q/2$. In practice, such assumption may not hold, and it may cause error when determining the inventory model at a retailer. To determine the inventory control parameters more precisely, the working process at a retailer can be described as the retailer facing a random demand. When the at hand inventory level is less than or equal to $s$, an order $Q$ is placed. This order will come after a lead time. The objective here is to minimize the inventory holding cost per unit time (e.g. per day). This approach assumes that demand is a normal distribution random process in which mean is $\mu$ and standard deviation is $\sigma$. 

The inventory holding cost at a retailer includes carrying cost and shortage cost. The cost function is expressed as:

\[
\varphi(N_t, U_t) = \begin{cases} 
C_h N_t U_t & N_t \geq 0 \\
C_s N_t U_t & N_t < 0 
\end{cases} \quad (14)
\]

where:

- \(U_t\) = the inter-arrival time for successive demands.
- \(N_t\) = net inventory, which equals to on hand inventory minus backlogged orders. Both \(U_t\) and \(N_t\) are random variables.
- \(C_h\) = carrying cost per unit product per unit time.
- \(C_s\) = shortage cost per unit product per unit time.

According to Reuven (1998), if a process can be split into several replicas, and for every replica, they have the same initiate state and independent, identical distribution, then this process can be called regenerative process. The inventory holding process at a node can approximately be viewed as a regenerative process. Every time when the absolute value of net inventory is less than a small number, a new regenerative cycle begins. Assume that the inventory holding process at a node is simulated within \(N\) regenerative cycles, then the inventory holding cost per unit time can be estimated by following formula:

\[
C(s, Q) = \frac{\sum_{i=1}^{N} \sum_{j=1}^{M_i} \sum_{t=0}^{\tau_{i}-1} \varphi(s + Q - \sum_{t} d_{ijt}, U_{ijt} + K)}{\sum_{i=1}^{N} \sum_{j=1}^{M_i} \sum_{t=0}^{\tau_{i}-1} U_{ijt}} \quad (15)
\]

where:

- \(C(s, Q)\) = the inventory holding cost per unit time. It is a function with respect to \(s\) and \(Q\).
- \(N\) = the number of regenerative cycles.
- \(M_i\) = order times during regenerative cycle \(i\).
- \(t_{ij}\) = the number of demands in a replenishment cycle.
- \(\varphi(s + Q - \Sigma d_{ijt}, U_{ijt})\) = the cost function defined in formula (16). Its value is obtained by simulation.

The first argument in this function represents the net inventory. \(K\) is the fixed ordering cost.

Following procedure is used to realize this simulation model and search for the optimal parameter pair \((s, Q)\):

1. Generate random samples: inter-arrival time \((u_{1,1}, \ldots, u_{t1,1}, \ldots, u_{1,N}, \ldots, u_{tN,N})\) and demand \((d_{1,1}, \ldots, d_{r1,1}, \ldots, d_{1,N}, \ldots, d_{rN,N})\) from
corresponding PDFs (Probability Density Functions) based on \( N \) regenerative cycles.

- Simulate the working process at a retailer, and calculate the corresponding inventory holding cost by function \( \phi(l(s + Q_2 \sum d_{ijt}), U_{ijt}) \).
- Determine range for parameter \( s \) and \( Q \) from the ongoing collaboration, and evenly take values from their corresponding ranges to form parameter pairs \((s_k, Q_l)\).
- Estimate inventory holding cost \( C(s_k, Q_l) \) by formula (15), find the minimal cost. The parameter pair corresponding to this minimal cost is the optimal one \((s^*, Q^*)\).

5.2 Transportation model from a wholesaler to its retailers

After determining optimal \((s, Q)\) for each retailer, the transportation system between wholesalers and retailers can be planned. In practice, the order quantity by a retailer is normally small, so it is possible for a vehicle to serve several retailers in one journey. In such situation, following questions are raised:

- How to cluster retailers?
- How to determine routes for vehicles?

Vehicle routing algorithm can answer these questions. At present, there are mainly three types of routing algorithms: heuristics (e.g. Christofides, 1985; Viswanathan and Mathur, 1997; Chao, 2002), mathematical programming (e.g. Popken, 1994; Wendy et al., 1999), and genetic algorithm (e.g. Gabbert et al., 1991; Chen and Gen, 1996, Chen et al., 1998). For its easy implementation, emphasis on global as well as local search, use of randomization in search process, genetic algorithm has been proven to be a versatile and effective approach for solving vehicle routing problems. Hence, this paper will use genetic algorithm to optimize the transportation system between wholesalers and retailers.

In the existing genetic algorithm based routing approaches, normally the transportation aspect was emphasized, but inventory aspect was ignored. For example, when calculating cost in their objective function, the inventory holding cost was not considered. Such ignorance may cause severe mistakes when determining routes for vehicles in a distribution chain. To overcome this drawback, this paper proposes a genetic algorithm based approach that deals with both transportation aspect and inventory aspect simultaneously.

The problem to be solved is depicted in Figure 2. The wholesaler (denoted by 0) that possesses a set of identical vehicles will serve a set of retailers (denoted by 1, \( \ldots, p, \ldots, q, \ldots, n \)). Retailer \( i \ (i = 1 \sim n) \) will order \( Q_i \) units product when its inventory level is less than or equal to \( s_i \). \( Q_i \) and \( s_i \) are determined in the previous subsection. Thus, the problem here is to route vehicles to different retailers by minimizing the sum of transportation cost and inventory holding cost.

![Figure 2. Problem illustration](image-url)
For optimizing the routes by genetic algorithm, the chromosome and fitness function must be specified first, in addition to the mutation (or crossover) strategy to guide the optimization process.

### 5.2.1 Chromosome and fitness function

#### Chromosome

The chromosome is represented by two strings: The first string includes all retailers except wholesaler. The second string has the same number of fields as the first one, and it denotes the vehicle numbers assigned to the retailers at the corresponding fields in the first string. For example, the following chromosome represents a routing solution, which has three vehicles (denoted by 1 ~ 3) and 15 retailers (denoted by 1 ~ 15):

1. **String 1**: 2, 4, 6, 8, 9, 3, 12, 15, 14, 11, 13, 7, 1, 5, 10; and
2. **String 2**: 1, 2, 1, 3, 3, 2, 1, 2, 1, 3, 2, 1, 1, 3, 2.

This example represents three routes: 0-2-6-12-14-7-1-0 (served by vehicle 1), 0-4-3-15-13-10-0 (served by vehicle 2), and 0-8-9-11-5-0 (served by vehicle 3), where 0 denotes the wholesaler.

#### Fitness function

Fitness function is determined based on the following two objectives: maximizing diversity and minimizing cost. As stated in (Patrick, 1993), largely scattered chromosomes can prevent the optimization process from premature selection (or local optimum). To avoid local optimum, the design approach introduces diversity (used to indicate the scatteration of chromosomes) into the fitness function. Cost is composed of time related cost, distance related cost, and vehicle renting cost.

#### Time related cost

For retailer \( i \), there is an ideal refill point \( R_i \) (\( R_i \geq 0 \)) which is determined by decision makers of this retailer. Any early or late arrival of the serving vehicle may cause extra inventory holding cost for this retailer. This extra cost is defined as time related cost. Time related cost is composed of three parts (as shown in Figure 3): early arrival cost \( \alpha \), late arrival cost \( \beta \), and stock out cost \( \gamma \) (\( \alpha, \beta, \gamma \) also represent the slopes of corresponding lines).

The inventory level of retailer \( i \) (\( I_i \)) when vehicle arrives at it can be calculated by:

\[
I_i = S_i - (T_j + L_i)d_i
\]  \hspace{1cm} (16)

where:

- \( S_i \) = the order up to level for retailer \( i \).
- \( T_j \) = the inter-service time for route \( j \), which is defined as the period between successive replenishments at this route;

![Figure 3. Time-related cost](image-url)
\( L_i = \) the lead time for retailer \( i \).

\( d_i = \) the random demand at retailer \( i \). As assumed before, \( d_i \) is a normal distribution random variable with mean \( m_i \) and standard deviation \( s_i \).

By \( l_i \), we can use following formula to estimate the time related cost for retailer \( i \):

\[
C_{ti} = \int_{\alpha L_i}^{+\infty} \alpha (l_i - R_i) f(l_i) dl_i + \int_0^{R_i} \beta (R_i - l_i) f(l_i) dl_i + \int_{-\infty}^{0} (\gamma |l_i| + \beta R_i) f(l_i) dl_i
\]

(17)

Where \( f(l_i) \) is the PDF of \( l_i \) which can be calculated from PDF of demand \( d_i \) based on formula (16).

Distance-related cost. It is proportional to the sum of distances traveled by all vehicles in a routing solution, shown as follows:

\[
C_d = c_d \sum_j L_j
\]

(18)

where:

\( L_j \) = the traveling distance for vehicle \( j \).

\( c_d \) = the unit traveling cost.

\( C_d \) = the total distance related cost for a routing solution.

Vehicle renting cost. It is proportional to the number of vehicles used in a routing solution, shown as:

\[
C_r = c_r n
\]

(19)

where:

\( n \) = the number of vehicles used in a routing solution.

\( c_r \) = the unit renting cost.

Diversity. Diversity for a chromosome is calculated by the sum of inverse squared distances between that chromosome and all others in this gene pool. The distance between chromosomes \( i \) and \( j \) is defined as:

\[
d_{ij}^2 = \sum_{k=1}^{m} (V_{ik} - V_{jk})^2
\]

(20)

where:

\( V_{ik} \) = the vehicle number for retailer \( k \) in chromosome \( i \), \( k = 1 \sim m \), \( m \) is the number of retailers in a chromosome.

\( V_{jk} \) = the vehicle number for retailer \( k \) in chromosome \( j \).
So, the diversity for a routing solution is:

\[ D_d = \sum_{i} \sum_{j \in N_i} \frac{1}{d_{ij}^2} \]  

(21)

where: \( N \) is the collection of all chromosomes in the gene pool. This implies that maximizing diversity results in minimizing \( D_d \).

Summing up the four parts mentioned above produces the fitness function:

\[ f = w_c(C_t + C_d + C_r) + w_d D_d \]  

(22)

where: \( w_c \) and \( w_d \) are weights for cost and diversity, and both are determined by subjective judgment. The objective for this optimization process is to minimize \( f \).

5.2.2 Optimizing the transportation model. Normally, genetic algorithm uses random mutation and crossover to find the optimal solution. Because of the limited capacity of vehicles and other routing constraints, such random process may cause infeasible solution. To avoid this problem, this paper applies guided mutation in this optimization process. The flow chart for this optimization process is shown in Figure 4.

Generation of initial gene pool. A gene pool is composed of a set of chromosomes. Because of the capacity constraint for vehicles, it is impossible to generate chromosomes randomly. This paper uses the following guided procedure to generate a chromosome. The first string of a chromosome is generated randomly, and it must include all retailers in it. The corresponding second string is constructed by following way: assign vehicle 1 to the rightmost retailer in the first string, then find the geographically closest retailer to latest assigned one, and assign vehicle 1 to it. Such process will continue until the capacity of vehicle 1 is full. Then assign vehicle 2 to the rightmost unassigned retailer, repeat the assigning process as for vehicle 1 to finish the assignment of vehicle 2, and so on. When all retailers in the first string are assigned, this chromosome is constructed.

Optimization of routes in a chromosome. In the previous step, all retailers are clustered in each chromosome. The following procedure can be used to optimize routes inside a chromosome. According to combinatorial optimization algorithm in Kreyzig (2000), the shortest path for the vehicle in a route can be found by following procedure:

- Step 1: Label wholesaler with 0;
- Step 2: Set \( i = 0 \);
- Step 3: Among all unlabeled retailers, find the retailer closest to wholesaler (or retailer) \( i \), label it with \( i + 1 \), and update \( i (i = i + 1) \); and
- Step 4: Go to Step 3 until all retailers are labeled.

After all retailers are labeled, the shortest path is found, and the serving sequence (or the route for the vehicle) is: 0-1-2-...-k-0, \( k \) is the number of retailers in this route.

Mutation to reduce time-related cost. For each retailer, there is an ideal inter-service time, which minimizes the time related cost for it. For all retailers in a route, as they are served by one vehicle, the actual inter-service time is the same, and most of them will not be served according to their own ideal inter-service time. If the ideal inter-service times of retailers in one route can be made as close as possible, the
The mutation to reduce time related cost is designed according to this idea, and it is completed by the following steps:

- Step 1. Select the route with largest time related cost in a chromosome; and
- Step 2. In this route select the retailer $i$ that satisfies: 

$$\max_{i \in N_j} (T_i - T_j)^2$$  \hspace{1cm} (23)

where:

$T_j$ = the inter-service time for route $j$.
$N_j$ = the collection of all retailers in route $j$.
$T_i$ = the ideal inter-service time for retailer $i$. 

Figure 4. Optimizing a gene pool
Step 3. Change the route number for this retailer to another route that satisfies:

$$\min_{j \in M} (T_i - T_j)^2$$

where:

- $T_i$ = the ideal inter-service time for retailer $i$ that is the one selected in previous step.
- $M$ = collection of all routes in this chromosome that have enough free vehicle capacity to accept retailer $i$. If $M$ is empty, give up this mutation.

Mutation to reduce distance-related cost. For a route that has $k$ retailers, the combinatorial optimization algorithm mentioned previously can be used to determine the service sequence: 0-1-2-...-k-0. The traveling distance for circle 0-1-...-k-0 is called total traveling distance for this route, and it is used to calculate the distance related cost.

The traveling distance from 1 to $k$ is called service traveling distance. This mutation is used to reduce service traveling distance, and it is achieved by grouping retailers which are close to one another into a route. The mutation is completed by following steps:

1. Step 1. Select the route (e.g. route $j$) with largest service traveling distance; and
2. Step 2. Select the retailer $k$ (in route $j$) that satisfies:

$$\max_{l \in N_j} \sum_{k \in N_j} d_{kl}^2$$

where:

- $d_{kl} = $ the distance between retailer $k$ and $l$.
3. Step 3. Change its route number to another route that is closest to it. If it is impossible to find any route to accept this retailer, just give up this mutation.

Mutation to reduce vehicle-renting cost. This mutation is used to reduce the number of vehicles used in a chromosome. It is accomplished according to the following steps:

1. Step 1. Select the route with largest slack vehicle capacity; and
2. Step 2. Distribute all retailers in this route into other routes that have enough free vehicle capacity to accept one or two of them. After distributing all retailers in this route, the number of vehicles needed for this chromosome is reduced by one. If such distribution is impossible, give up this mutation.

As indicated in Figure 4, after one generation, the fitness value for each chromosome is calculated, and the best $m$ chromosomes are selected to form the new gene pool for next generation. The optimization process is complete if the average fitness value for newly formed gene pool converges, i.e. there is no significant change at it. After this optimization, the best chromosome is selected as the routing solution.

5.2.3 Allocating distance-related cost to retailers in a route. After determining the routes for vehicles, the unit product delivering cost can be calculated for the retailers.
Figure 5 shows a route with $n$ retailers ($1 \sim n$) and one wholesaler (0). The direct traveling distance from wholesaler to retailer $i$ is denoted as $d_i$. Similar to Berman and Larson (2001), we provide the following formulae to allocate the travel distance $A_i$ to retailer $i$:

$$A_i = 2d_i - R_i + \frac{1}{n}d_1R_n = 2 \times \frac{n-1}{n}d_nR_{n-1}$$

$$= R_n + 2 \times \frac{n-2}{n-1}(d_{n-1} - d_n) \ldots R_2 = R_3 + 2 \times \frac{1}{2}(d_2 - d_3)R_1 = R_2$$

Finally, the unit product delivering cost from wholesaler 0 to retailer $i$ is:

$$c_{0i} = \frac{1}{Q_i} \sum_i A_i C_{dj}$$

where:

$Q_i$ = the order quantity for retailer $i$,

$C_{dj}$ = the distance related cost for route $j$.

### 5.3 Inventory control model at wholesalers and at distribution center

A wholesaler faces several retailers. The demand process at the wholesaler is also a random process, and the characteristics of this random process (i.e., mean and standard deviation of the demand) can be generated by a simulation model. After getting these characteristics, the formulae provided in the subsection 5.1 “Inventory control model at a retailer” can be used to determine the optimal inventory control parameter pair ($s, Q$). For the distribution center, the parameter determining process is the same as the one for wholesalers. For brevity, these details are illustrated here.

### 5.4 Transportation model from distribution center to wholesalers

Normally, the amount of product demanded by a wholesaler is large and hence, a vehicle can only serve one wholesaler in its journey. Thus, it is assumed that there is no routing problem in this transportation model.

Figure 5.

A route with $n$ retailers and a wholesaler
The unit product delivering cost from distribution center to wholesaler \( j \) can be calculated by:

\[
C_j = C_{l0} + C_{0j} + C_{uj} 
\]  

(28)

where:

- \( C_{l0} \) = the unit loading cost at distribution center.
- \( C_{0j} \) = the unit transportation cost from distribution center 0 to wholesaler \( j \).
- \( C_{uj} \) = the unit unloading cost at wholesaler \( j \).

If there is routing problem in this transportation model, models provided in subsection 5.2 “Transportation model from a wholesaler to its retailers” can be used again to determine routes for vehicles, and formula (27) can be used to calculate the unit product delivering cost from distribution center to wholesalers.

### 6. Conclusion and managerial implications

This paper proposes an iterative approach for developing a distribution chain in which optimization of the strategic model and the tactical model is done successively. The design approach proposed in this paper turn out:

- optimal configuration of the distribution chain, including the number and locations of retailers and wholesalers, and the assignment of retailers to wholesalers;
- optimal inventory control policy and parameters at each node; and
- optimal routing of vehicles to deliver products between different nodes (i.e. from distribution center to wholesalers, and from wholesalers to retailers).

The contribution of this paper is as follows:

1. **Reuse of knowledge**: In this paper, we focus on an iterative process that uses knowledge from the past and ongoing collaborations to design a new or better distribution chain.

2. **Adaptive system**: The iterative process proposed in this paper accommodates input of latest market details at the start of each iteration.

3. **Modular design**: The iterative approach consists of two parts: the strategic model and the tactical model. The tactical model is further divided into four distinct sub-models such as: Inventory control model at the retailers; Inventory control model at the wholesalers and at the distribution center; Transportation model from the wholesaler to retailers, and Transportation model from the distribution center to the wholesalers (see Figure 6).

4. **Integrated approach**: We believe that there is no single type of mathematics that is most useful for modeling and design of distribution chain. Thus, in this paper, we use three different types of mathematics. For determination of the strategic model of a distribution chain (including number of distributors, location of distributors) Mixed integer programming
(MIP) is used. Based on the output of the strategic model, determination of the tactical model (i.e. the inventory planning at each node, and vehicle routing between different nodes of the chain) is done using genetic algorithm and probability theory.

(5) Realizable system: All models, formulae and algorithms in this design approach can be implemented as a software package for designing distribution chain. This means, the design approach proposed in this paper is not only is theoretically elegant but also applicable in practice.
References


**Further reading**
