Moral Hazard and Adverse Selection in Delegated Portfolio Management

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Abstract

We investigate the investor-manager relationship in a partial equilibrium principal agent model. The industry for the delegated portfolio management is characterized by information asymmetry on many different levels. Investors will have inferior knowledge on the alpha generating ability of a specific manager and the work he puts into his ex-ante analyses or in some cases the gross return generated. Existing literature confirms that there is a large variation mechanism design can be explained through agency frictions caused by uncertainty related to expectations of active management. Our approach internalizes both frictions caused by unobservability of the output and uncertainty related to the quality of the asset manager. We find that screening is more likely to occur when allowing good managers to differentiate themselves by short selling or use of leverage. However, if truth telling is to be insured in all states of nature, a regulator must be careful when restricting the size of short selling or leverage, since this will only influence the lower bound of the wage for the low type in the separation state and not the high bound for the high type. Moreover, we also demonstrate that bad managers can potentially play a role of financial advisors if they have additional information on distribution of managers.

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**Keywords:** Delegated Asset Management Industry, Moral Hazard, Adverse Selection, Agency Cost, Portfolio Management, Short Selling, Leverage
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Introduction

Delegation of wealth management is an essential and an evolving feature of financial markets. History has been witness to the fact that the asset management industry has undergone major changes over the years. The period from 1990s to 2008 has contributed heavily towards changing the face of the industry. Technological change, deregulation and institutional changes (formation of hedge funds and private equity firms) paved the way for easy access to credit and ever increasing asset prices. As a result, the assets under management (AuM) escalated partially due to this rise in liquidity. It was not before 2008 that we saw a drastic decline in these figures. This happened around the time when credit crisis occurred and significantly affected the global financial industry. Many asset managers saw a 30-40% reduction in their AuM not only because of the crash in asset prices but also due to lack in confidence among investors who pulled their money out. Among other things, this brought into light the failures due to market liberalization and incentives that the industry gives to the managers to take more risks (Rajan 2005 and Michael Pinedo 2010).

Following the credit crisis, the industry witnessed major structural changes and transformations. We see de-liberalization in many markets, investors setting new norms and standards for managers, more importance put on risk management and managers finding redemption in up-and-coming Middle east and Asian markets.

In 2010, the total assets under management (AuM) were estimated to be $121.1 trillion globally. This was the highest ever annual increase in global wealth level. North America and Europe together held nearly 60% of the total wealth managed in world, with North America having the highest proportion ($38 trillion). While rest of the world shared the rest, the Asia-pacific (ex Japan) region experienced the fastest growth in wealth amounting to 17.2%. Emerging countries together shared around $ 29.7 trillion in assets under management.

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2 Shaping a New Tomorrow: How to Capitalise on the Momentum of Change - BCG annual global wealth report 2011
Due to the size of the industry and increased centralization of money management, crisis such that we have recently witnessed can have large unpredictable effects on the economy. Hence, gaining deeper understanding of this industry is equally important for the academics and policy makers (Stracca 2005).

The asset management industry houses various types of funds (investment vehicles). Each fund has its own significance and fulfills needs of different investors that exist in the market. Moreover, they differ in their cost features, risk profiles and productivity characteristics. The industry is broadly classified into two categories; institutional asset management and the retail market. Institutional asset management works directly with large institutions and high net-worth individuals. Hedge funds and private equity funds can be classified under this category. Retail includes all the mutual and some pension funds. It pools the investments made by smaller individual investors (Michael Pinedo 2010) and is more regulated in contrast to its counterpart. Other funds not categorized above but are part of asset management industry are sovereign-wealth funds (state-owned fund), insurance funds and exchange-traded funds (follows an index or a commodity, or a basket of assets like index funds but trades like a stock or an exchange - Investopedia).

The focus of this paper is primary not to acquaint the reader with historic or prevalent industry structure, trends or figures. The discussion above is only to motivate systematic understanding of the industry. The main purpose of the thesis is to shed light on the problem on possible incentive problems in relation to delegation of investment management by investors to the asset managers and also formally model it.

Asset management revolves around the idea that asset managers specialize in obtaining information from the market, obtaining certain skill which helps them generate returns on the wealth they manage or generate excess risk-adjusted returns resulting from the economies of scale. Since investors lack this specialization ability, they entrust that managers are better to manage their wealth than doing it themselves and manager is paid in return for his services.
This creates the classic case of information asymmetry where, when an agent is better informed than principal, it may result in an outcome characterized by dead weight loss.

The problem that investor faces is two-fold; the first problem arises when agent indulges in perverse behavior such as shirking, taking too much risk, not making optimal portfolio choices, misreporting etc (moral hazard). The second problem arises when due to asymmetric information, investor struggles in choosing a good manager that maximizes the return on his wealth (adverse selection). Our paper studies the general theoretical work on these problems and their application to this and other industries. This preliminary work complements our original contribution towards modeling these problems in a principal agent model where an investor hires a manager to manage his money which helps us derive some meaningful conclusions and results at the end.

The paper evolves as follows. Section 1 illustrates the prevalent compensation structures in the industry and how some of them contribute towards exacerbating the agency problem. Section 2 discusses the prevalent theoretical literature under moral hazard and adverse selection with many possible extensions that have been studied. We extend our discussion as we show the application of the standard theoretical frameworks to asset management industry and other sectors in section 3. The purpose of the section 2 and 3 is not only to discuss the agency problem, but also present relevant contributions that provide us with intuitive solutions and optimal contracts that mitigate the problem. Section 4 presents a principal agent model where we analyze the inefficiencies related to different situations of asymmetric information and characterize optimal contracts in presence of the moral hazard and adverse selection together and lastly section 5 concludes our main findings and proposes possible extensions.
1. **Common Fee Structures in asset management industry**

The fee structures vary across the industry. Different investment vehicles available to the investors differ in their characteristics such as the regulatory environment, transparency and ability to generate returns. Therefore, the fee structures employed, usually depends on the type of investment vehicle the investor chooses to invest his money in.

Conventionally, the industry has used the asset-based fees structure where remuneration is simply a fixed percentage proportional to the value of assets held by the investor. It is also known as management fee.

Another class of compensation schemes often used is the performance-based fee structures, which have become an integral part of this industry. The supporters of these structures believe that performance-based remuneration is desirable, since they align the interests of the manager and the investor. But other people have criticized them to be biased in the favor of the manager and creating conflicts of interest. They claim that a performance fee, which originally provides the manager with the incentives to boost returns, also increases his risk appetite due to his limited liability\(^3\). When chips are low, this structure encourages manager to take more risk since the downside is very limited compared to potential upside. The performance-based fee structures came under scrutiny in the financial crisis of 2008, and following this the absolute fees levels for some investment vehicles were reduced. In the following part of this section, we will discuss the various performance fees structures prevalent in the industry and how they differ among investment vehicles.

Figure 1 provides a graphical illustration of some of the most commonly found fee structures in the industry.

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\(^3\) [http://www.statpro.com/blog/performance-fees-good-or-bad/](http://www.statpro.com/blog/performance-fees-good-or-bad/)
There are essentially two types of performance fees structures: symmetric and asymmetric.

In an asymmetric fee structure, fees are only paid over an increase in net asset value (NAV) of the management fund. In other words, when the fund is earning profits for its investors, the managers are entitled to the part of this increase in the value of the fund which is their performance fee. However, managers earn zero when investors are losing money.

Different asymmetric fee structures are the following:

- **High-water mark (HWM)** - Once performance fees are paid, the fund establishes a high-water mark with their highest periodical closing NAV. When the value of fund goes below HWM, no performance fee is earned until the NAV goes beyond the current HWM level. This structure ensures that manager receives performance fee only when he generates profits for the investors and not recovering losses. They are widely used in hedge funds. Black (2004) quotes a study by Van Hedge Fund Advisors that
states that 89 percent of hedge funds (at the end of the first quarter of 2003) had fee structures that contain a HWM provision.

\[ \text{Per Share Performance Fee} = \max (K \cdot (\text{End NAV} - \text{Beginning HWM NAV}), 0) \]

where \( K \) is the performance fee.

- **Hurdle Rate** - Fees are only paid if fund performs over a certain benchmark eg. LIBOR. It is usually linear over this benchmark. It is also most commonly used in hedge funds. Black (2004) also found out that that only 18 percent of funds have a hurdle rate.

- **Fulcrum fees** - Managers charge performance fee when they generate a return over a specified predetermined benchmark. However, if they are unable to do so their base (management) fee will be reduced. Also, only institutional or high end investors can be charged fulcrum fee. This performance fee structure is used in some mutual funds and is the only incentive system available in this particular industry, enforced by law (1970 amendment to the Investment Company Act of 1940). However, it is not widely applied. In 1999 only 1.7% of the all the mutual funds in the market used fulcrum fee (Elton, Gruber and Blake 2002).

Hedge funds usually employ a combination of a management fee and a performance fee. The ratio ranges from 1.5%/15% to 3%/30%. But majority of them use 2%/20%. The management fee in mutual funds ranges between 1%-3%, and performance fees differs from fund to fund. The mutual funds that do employ incentive fees have a fixed and a variable component which should be symmetrical to the benchmark.

Academics have questioned the common belief that an asymmetric fee structures necessarily induce managers to take more risk. Empirical and theoretical results show that individuals are risk averse when facing gains and risk seeking when faced with losses (Kahneman and Tversky 1979, 1992; Coval and Shumway 2005; Haigh and List 2005). In the given context, it means that
when over a benchmark, managers try to moderate the risk, hence securing themselves in a profitable position. However, when below the benchmark they will tend to increase the risk (Ross 2004; Coleman and Siegel 1999). In contrast, Kouwenberg and Ziemba (2007) find that risk seeking among loss-averse managers increases with higher performance fee.

Moreover, in a symmetric fee structure, a manager typically receives an asset-based fee and a bonus for outperformance while he has to pay a penalty if he underperforms. Hence, it relaxes the limited liability condition for a downside that is inherent in asymmetric fee structures. This very attribute has made it a quite popular research topic among academics. Theoretical results show that it dominates asymmetric fees in aligning the interests of managers and investors since it will encourage manager to take the risk desired by investor rather than motivating risky behavior, but it does not completely removes the agency problem (Stark 1987; Golec and Starks 2004). However, it is a very rare possibility that one would see this fee structure often employed in reality.

2. Literature Review

2.1. Efficiency and private information

In 1960 Ronald Coase presented his famous theorem stating that with clearly defined property rights, market participants would simply bargain and reach an efficient outcome. This proved to be a very important result because it did not rest on the perfect competition assumption of the first welfare theorem. According to Coase, no such assumption is needed, only the absence of transaction costs, designated property rights and voluntarily private bargaining. In later years, many researches have focused on the possible breakdown of this theorem due to asymmetric information of the market participants. Farrell (1987), models this by assuming asymmetrical information with respect to the preferences in the market. He shows that in the presence of private information, private contracting is only weakly efficient⁴. Moreover, Schmitz (2001) shows

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⁴ That is, it will depend on the parameters
that in the case with information asymmetry and where property rights may be limited⁵, efficiency can always be achieved.

It is not the purpose of this thesis to discuss outcomes under different bargaining powers. Hence, we restrict ourselves to the case when there are only two parties bargaining, and one party has all the bargaining power.

### 2.2 Contract theory and private information

In economics we are often concerned with how information asymmetries will affect the outcome of some fiduciary relationship between two or more parties. If the objects of all parties were the same, there would be no inefficiencies and no formalization of the agreement will be needed. However, this is often not the case. A worker will have objectives that do not correspond to the objectives of the company, a manager of a mutual fund will want to maximize his own payment or an insured agent may take action he would not otherwise take if not protected. It is not difficult to see that examples like this could be found in all aspects of the economy. If the objectives between the players differ, the principal could propose a contract which would protect his interest such that the agent is given the incentives to take actions the principal would have done in the absence of the relationship (Laffont and Martimort 2001). Contract theory is a field of economics which is concerned with the design of such incentive compatible contract under various information structures (Bolton and Dewatripont 2005). A contract is an agreement between parties (individuals, businesses, organizations or government agencies) which may or may not be legally binding⁶ and involves a promise to do something in return for a valuable benefit (exchange of promises). The legal nature of the ‘Contract’ (‘Contract law’) is derived from the principle expressed in the Latin phrase *pacta sunt servanda*, which literally means "pacts must be kept". Moreover, a contract specifies the terms and conditions under each parties commitment.

In contract theory two important cases of information asymmetry are described. The first case is referred as ‘Adverse Selection’, where because some parties

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⁵ He model this by assuming if no property rights exist, the decision variable will be stochastically determined.

⁶ Reflecting the power of enforcement
have more information than others regarding the state of the world, results in market inefficiency and possibly a full market breakdown. In an influential paper by Akerlof (1970), he shows that, using the market for used cars as an example, car owners with good quality cars will never put their car up for sale since the buyers are only willing to pay an amount equal to an estimate based on the distribution of quality (private value). Thus reducing the amount the buyers are willing to pay for a car given that they only know the distribution. The idea is that this looped mechanism may then, in the worst case, lead to a market breakdown, or possibly, “a market for lemons”. This will be covered in more detail, as well as possible solutions to the adverse selection problem under section 4.

The second case is known as ‘Moral Hazard’ which occurs when one party has private information and the actions taken would influence the fiduciary relationship. Examples of this may be a worker, which is costly to monitor for the employer, could work hard or shirk, or a manager of mutual fund could do thorough analytical work prior to his investments or he could more or less arbitrarily pick a portfolio matching his own risk profile. These cases are often explained and analyzed in the literature using a Principal-Agent model, in which a principal that hires an agent to pursue his interests’ faces difficulties arising from asymmetric information and this problem is called ‘agency problem’ or ‘principal agent problem’. There exists a large literature that discusses analytical properties of principal agent framework and provides us with an enlightening understanding of how optimal contracts are designed when faced with agency problem.

2.2.1 Adverse Selection

In his famous article “market for lemons”, George A. Akerlof introduced the idea that information asymmetry regarding the quality of a good could lead to a complete market breakdown. Akerlof showed, under the assumption of linear utility functions and uniformly distributed quality, there will not be any trade at the market clearing price, even if there exist sellers and buyers willing to trade. The main intuition is that only the average quality of goods matter to the uninformed buyer. Hence, the informed seller will not put up goods whose
quality exceeds the reservation price of the buyer (Akerlof assumes this to be equal to the quality) and the distribution gets augmented until the point where the price is equal to 0 and no trade is possible. This is a direct consequence of the continuous quality distribution.

Similarly, if quality is defined discrete, Akerlof showed the important result that the bad cars drive the good out of the market. The reason for this is that, due to information asymmetry between buyers and sellers, buyers can only infer the true quality of the good based on the distribution. This implies that all cars will sell for the same price. Thus, any seller knowing his car is of high quality would never trade since he would be implicitly subsidizing the sales of lower quality cars. This, in turn, reduces the average quality of cars in the market and lowers the price even further.

Akerlof uses these adverse selection problems to explain a variety of economic observations. In addition to his well-known example of the used cars market, he applies the theory to insurance markets, discrimination in the labor market and credit market in underdeveloped countries. We will discuss some of these and other important applications in section 3.2.

Much focus has been put on possible solutions to the adverse selection problem. Akerlof himself suggests brand names as a way to indicate quality and to give the buyer leverage when the quality is below expectations, thus guaranties to ensure the consumer some minimum expected quality and licensing to counteract uncertainty.

Laffont and Martimort (2001) point out that in the case of adverse selection where an agent may get access to information which is not available to the principal when a task is delegated, in order to achieve efficiency, an incentive compatible contract must be prepared such that it entices the agent to share the private information. In the literature, this is referred as the “revelation principle”. The term first appeared in Baron and Myerson (1982), where the authors considered the problem of regulating a monopolist with cost unknown to the regulator. In their proposition they define the revelation principle as:
Without any loss of generality, the regulator may be restricted to regulatory policies which require the firm to report its cost parameter $\vartheta$ and which give the firm no incentive to lie.

However, the most common way of modeling solutions to the problem is signaling (Spence 1973) and screening (Rotchield and Stiglitz 1976). The next subsections will briefly go through the theoretical background for these ideas.

**Signaling**

The main idea in this approach is that the informed party can take actions prior to signing the contract which signals their ability. In Spence’s model, workers with privately known productivity select an education level that maximizes expected wage minus cost of education given their productivity. Moreover, the labor market is perfectly competitive and firms are risk neutral. This implies that all workers are paid their expected productivity. Thus, by observing a specific education level, firms will offer contracts contingent on this specific level. This in turn leads to two special cases.

If, all types selects the same education (e.g. no education at all), we have what’s called a pooling equilibrium. In this case, all workers are hired and offered a wage equal to the expected productivity. It follows from this that there are an infinite pooling equilibria, but only one is socially optimal, and that’s when no agent get educated. This is because, in contrast to human capital theory, in these models education has no effect on the productivity of the workers.

Another possible equilibrium is a separating one. In this case all types choose a different education level. Consequently, each are paid their true productivity (and not just the expected productivity of a randomly selected worker), and firms are able to perfectly verify the type of the agent based on their education decision. Moreover, as long as productivity of the high types is larger than the low types, and cost of education is decreasing in type, there will always be a possible separating equilibrium.

A key property in separating the workers is the so called Spence-Mirrlees or single crossing property (Spence 1973 and Mirrlees 1971). This condition means
benefit from deviating from the equilibrium path and this deviation is unique for this type, the firm should be able to infer the type deviating. Bank and Sobell (1987) puts even more restrictions on stable equilibrium’s. Their divinity equilibrium condition states that in addition to the intuitive criteria, if there is a profitable deviation by one type which is also profitable by another type then no weight should be put on that type.

**Screening**

Conversely to the case with signaling, screening models assume the uninformed part moves first. The idea, first proposed by Rotchild and Stiglitz (1976) was that the uninformed party could offer a menu of contracts inducing the informed party to self-select and reveal their type.

An important result which has huge impact on the solution to screening models is the so called revelation principle (Myerson 1981). With respect to mechanism design, it means the uninformed party can restrict himself to self to direct revelation mechanisms and he can never do better than this. This result is quite intuitive. Consider an indirect mechanism with some Bayesian Nash equilibrium strategies for the principal and agents. According to the revelation principle, this can be replaced with a direct revelation mechanism. If this was not true then, an agent of type $\theta_j$ would report some other type $\hat{\theta}$ and receive strictly larger expected utility. But this contradicts the original equilibrium condition, hence in equilibrium the agents must report truthfully. Since this can be thought of as reporting to a neutral third party who then implements the optimal strategy based on the reported information, the revelation principle implies that centralization dominates decentralization (Flrckinger 2010).

Another principle, related to the revelation principle is the so called taxation principle Guesnerie (1981, 1995) and Rochet(1986). This means that any equilibrium price schedule can be written as a nonlinear pricing scheme. Moreover, the principal can focus on more realistic compensation structures when designing the contract (Caillaud and Hermalín 2000).
2.2.2 Moral hazard

Pauly (1968) was the first to introduce the idea of moral hazard in an economic context. He considered the problem in the medical insurance industry and defined moral hazard as *intangible loss-producing propensities of the individual assured*. Moreover, he showed that individuals opting for more medical care with insurance than in its absence is due to *rational economic behavior* and has little to do with morality. Arrow (1970) broadened the implications of the intuition made by Pauly. He said that *optimality of complete insurance is no longer valid when method of insurance influences the demand for the services provided by the insurance policy*. They both agreed that if the agent is risk averse, it is not socially optimal to provide him with full insurance. Hence, due to moral hazard, the market allocations will always be constrained Pareto optimal\(^7\). But neither of them showed how to formally model this aspect. It was in the paper by Zeckhauser (1970) that moral hazard was formally modeled for the first time. In the same institutional setting as Pauly, he presented the optimization problem of the individual. The results were determined using first order conditions. Spence and Zeckhauser (1971) and Mirrlees (1972) later applied these principal-agent problems in a general setting of expected utility subject to *deliberately chosen contingent contracts*.

The first order approach was applied to other particular applications such as share cropping (Stiglitz 1974), capital markets, and incentive and pay structures (Stiglitz 1975, Haris and Raviv 1979, Mirrlees 1976). But, Mirrlees (1975, 1999) in a critical paper pointed out the common problem of assuming that the first order approach is valid even when certain assumptions established in the model does not hold. He shows that this technique will be invalid unless at the optimum, the outcome to the agent’s problem is unique. If there is no uniqueness, the first order conditions are not even necessary conditions for optimality. In other words, the procedure ignores that the solution of the problem should be a global maximum not a local maxima. Mirrlees claimed that in a situation where, the density of output is conditioned on the action of the agent, it needs to fulfill two conditions; monotone likelihood function (MLR) which implies that that payment

\(^7\) We extend the Pauly’s and Arrow’s discussion later in section 3.2.2
schedule of an agent needs to be increasing in output and an agent’s action be convex at each level of output (CDF). He showed that inclusion of these two conditions makes the first order approach valid. Rogerson (1985) concurred with Mirrlees and proved the conditions above to be true using a simpler proof. Grossman and Hart (1983) demonstrated the same, but were criticized for its applicability when principal is risk averse (Rogerson 1985).

Holmström (1979) found the first order approach to be valid for the case where the distribution function of outcomes is a combination of two convex fixed distributions and the agent’s action evaluates the weights of this combination. Jewitt (1988) criticized Mirrless-Rogerson approach of first order condition validation and prophesized that it lacks applicability in situations where the CDF property does not hold. He presented a procedure for validity; the first step is to ensure that the multiplier on the incentive constraint is positive. Secondly, ensure that the utility of the agent falls within a known class, and finally, make sure that the utility function of the agent is concave.

Hart and Holmström (1987) discuss two widely used methodologies in moral hazard problems. First is known as a state-space formulation introduced by Spence and Zeckhauser (1971) and Ross (1973). Here the verifiable outcome is determined together by the set of actions the agent chooses and possible the state of nature. The payoff function which also depends on the actions and outcomes is controlled by the principal. The principal’s problem is to construct an incentive structure that maps the outcomes into the payments of the agent. It is assumed in the model that exerting effort is costly to the agent. The agent’s utility function is separable. That is his utility function consists of a function of the payment he receives minus a cost function of effort. The principal’s utility function is the profit earned minus the transfers given to the agent. The distribution of the state of nature, the outcome function, and cost and utility functions are agreed upon together by principal and agent.

An alternative and rather economically insightful formulation of the problem was initiated by Mirrlees (1974, 1976) and later explored by Holmström (1979). In this formulation, the agent’s choice of action ‘a’ determines the distribution over
output ‘x’ and profit ‘π’, which is derived from the distribution of state of nature. As Hart and Holmström explain, they suppose that the derived distribution takes a form of $F(\pi, x, a)$ with a density function $f(\pi, x, a)$. The principal chooses the least cost reward scheme based on the action he wants agent to take. Moreover, the principal knows the preferences of the agent and thus he is able to infer the action the agent will take despite of the fact that he cannot directly observe it. Participation is ensured by promising the agent some minimum expected utility and that incentive scheme provided to him is homogenous to the set of actions he will choose. This approach is called parameterized distribution formulation.

Hart and Holmström (1987) discuss the formulations above in a basic model, where the actions can take on two values H (Work hard) or L (being lazy) and in a general setting where actions are defined continuously. They infer that the optimal incentive scheme is characteristically similar in both cases. But an important distinction is that in a two-action model one has to compare the results of being hard working with being lazy, meaning that since there is no continuous trade-off, it is hard to conclude anything about the choice of action. In contrast, given that first order approach\(^8\) is valid, the continuous effort case implies that it can be proved that effort satisfies first order stochastic dominance. Furthermore, they criticize solutions obtained in the static moral hazard problems by being highly sensitive to the information technology, meaning that by manipulating the structure one can formulate any reward system. They explain that incentive schemes prevalent in reality are far simpler than what is used theoretically and fail to be consistent across wide range of circumstances.\(^9\)

In the literature, the moral hazard problem can be extended to a case where a potential risk neutral agent has limited liability (LL). This case was first formally modeled by David Sappington (1983) in an adverse selection model\(^10\), which discussed that the principal in some cases has to satisfy some LL constraints in

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\(^8\) Incentive constraint is relaxed in case with continuous effort. This very phenomenon is referred as first order approach.

\(^9\) Aggregation and linearity in the provision of intertemporal incentives justifies the optimality of using linear scheme in principal agent contracts

\(^10\) Please refer to the section 2.2.1
addition to the participation and incentive compatibility constraints. The paper discussed two situations; first referred as LL resulting from limited wealth/no wealth and second referred as LL on utility associated to a minimum level of utility. In this paper, he concludes that when solving the adverse selection problem in light of these two LL constraints separately it results in a similar optimal contract. But Lawarrée and Audenrode (1999) successfully demonstrated that the same result would not hold in a moral hazard model. They show that constraints on transfers Pareto dominate the constraints on utility in a moral hazard setting. They argue that the difference in the results is due to adverse selection models have a one to one relationship between the choice variable of agent (For ex-effort) and output. But in the moral hazard models the output’s value is uncertain when agent chooses the effort level.

Prior to Lawarrée and Audenrode (1999), Innes (1990) introduced limited liability, where a risk neutral entrepreneur makes an unobservable ex-ante effort choice while employing the investment funds of a risk-neutral investor. He imposes LL constraints on the transfers, which require transfers to be above a minimum level. Laffont and Martimort (2002) showed the same. They explained the limited liability case where the optimal contract is deduced to the point that the delegation will no longer be costless (which is actually the case in first best\(^{11}\)) to the principal, because the agent is now only liable to some extent and has to be given transfers greater than some exogenous level. If this exogenous given level is smaller than the optimal transfers subject to some realization of nature, the agent must be provided with positive limited liability rent. This means that cost of requesting effort is second best cost to the principal (higher than first best cost). They generalize the same result to the case when the agent is risk averse. Here, due to the agent’s risk preferences, the optimal transfers would entail that the agent has to be paid a risk premium to induce participation. Therefore the delegation would again be costlier than first best cost, which will increase with the degree of risk aversion.

\(^{11}\) Laffont and Martimort (2002) refer to this as the case when the effort is observable and both agent and principal as risk neutral. Here principal is able to extract all the rent from the agent since he only has to pay him transfers that equal his reservation utility.
Another important extension to the moral hazard case is when the principal hires many agents together to do a task. The common problem discussed in the literature is an issue of cooperation. Itoh (1991) compares the optimal contracts in a situation when the principal who can’t observe the action choices made by the agents, can either design a task structure which makes the agent only specialize on their own task and does not require helping others in the team or designing a non-specialized task structure which encourages agents to help others. He finds that teamwork will be an optimal choice if each agent is able to increase his own actions in response to rise in help from other agents in the team. It is also demonstrated that when doing a task and the agents receive marginal disutility, if free riding occurs, teamwork would still be optimal since it would cause the agent to cut down his (her) cost that he had to induce on the task. Mookherjee (1984) discusses that when working in teams, if agents are able to learn about each other’s’ actions, it motivates collusive behavior. Therefore agents can together choose an outcome which is Pareto inferior and likely to be different than the Nash equilibrium outcome that principal prefers. Ma (1988) showed an approach to deal with this problem. He suggested that, by providing agents with an extensive set of strategies such that the principal can employ second best Bayes-Nash equilibrium as a unique outcome to the problem.

Non-cooperative behavior in the team will make efficiency unsustainable if joint output is the only informative signal of the collective effort exerted. Each agent has to induce full cost of effort while only sharing part of outcome. Hence, it leads to a potential free riding problem. The solution suggested involves providing group incentives such that agents are punished for resulting output level below optimal level and given bonuses when it is above. This would induce agents to take productive actions together and hence imply efficiency. However, a third party (principal)\(^{12}\) is needed not only to monitor but also to impose the punishments or to endorse the bonuses. He (she) will not contribute towards output, but will have a share in it (Holmström 1982). In the similar setting, Ma (1988) shows that when actions of the agents are observable to each other, it induces perfect information and thereby removing the agency cost involved.

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\(^{12}\) Principal cannot be a team member to make this incentive scheme credible.
Lazear and Rosen (1981) analyze the linear compensation schemes relative to the payment based on ranking in multi-agent organizations. They show that when agents are risk neutral, both schemes result in same efficient allocation of resources, but the same does not hold when agents are risk-averse. In this case, the payment based on ranking is preferred by the agents since it provides them with insurance against uncertain events.

In another setting Holmström and Milgrom (1991) extended the single task models and investigated moral hazard in a multi-tasks principal agent model. They allow the principal to either choose to have various tasks for the agent or allot a task that has many different dimensions to work on. The problem highlighted in this setting is to design an incentive scheme that will not only fulfill its fundamental job of making agents work hard or to distribute risk, but also serve the purpose of getting agents to focus their time and attention to various tasks or dimensions. They emphasize that task(s) design is an essential tool for an optimal incentive scheme. In addition to this, one can use variety of other instruments to control the agent’s actions such as combining similar tasks into one, manipulate the bounds on incentives that one receives on task completion, and alter the restrictions on different approaches the agents take to do a job. All these suggestions are minutely explored, but under a set of very strong assumptions which made this analysis simpler and opened up avenues for research in multi-task setting.

An interesting development was later made by Dewatripont, Jewitt and Tirole (1999 II) where they studied incentives of government officials in a multi-task career concern model. Their paper aimed to support the sociological findings in a survey on US government agencies.\(^{13}\) The technology is adopted from the influential paper of Holmström (1982) where output is a function of the agent’s talent (static), continuous effort, and a stochastic noise term. Unlike Holmström\(^{14}\), they investigate the impact of effort over different tasks, such that the different talents suggest strengths in different tasks. The basic premise of career concern models is that in absence of explicit incentives, the agents exert

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\(^{13}\) James Q. Wilson survey of US governmental agencies 1989

\(^{14}\) Single-task two period model with static talent variable.
effort in order to make the market realize their true potential, something that is subject to uncertainty. Dewatripont, Jewitt and Tirole paper is able to successfully extend implications of this basic premise when the moral hazard problem is modeled dynamically over tasks than time. They show that uncertainty in the market about the agent true type is positively correlated with the agent’s effort. In other words, with higher uncertainty, the agent exerts more effort to let the market know about his (her) true type. Therefore it pays off for the organization to specialize their agents in certain tasks. However, this may weaken the overall performance of the organization since some value driving tasks may be neglected. This result confirms one of the findings in the US survey, as they conclude the result above as an essential tradeoff in government agencies something which is found to be reverse in explicit incentive private firms. Furthermore, this demonstrates the idea that when modeling moral hazard problems dynamically, implicit incentives play a significant role to deter agent to involve in perverse behavior.

Prior to work from Dewatripont, Jewitt and Tirole (1999 I &II) and Holmström (1982), introductory work in dynamic moral hazard models came into picture when economists pointed out the importance of the potential dynamic nature of principal agent relationship in removing the agency problem. First to do so, was Radner (1981) who found that in a principal agent setting over a repeated game, a contractual setting can sustain since the principal will generally find a way to prevent an agent to indulge in undesirable behavior. He finds that in a finite repeated game this leads to the existence of an epsilon-equilibrium (almost Nash equilibrium). Rubinstein (1977) finds the same to be close to Pareto optimal in presence of a Nash equilibria using law of iterated logarithm.

Later Fama (1980) concluded that since reputation of the manager plays a significant role as a career aspect, the market will alone eliminate the agency problem. This would imply that implicit incentives will be sufficient to relax the incentive problems, and one would not need formal contacts. Holmström (1982) found that this efficiency can only be reached under some restrictive assumptions. If the agent is risk averse and positively discounts future profits, efficiency will not be met since implicit incentives alone will not compensate him
for the risk. In addition he points out that empirical evidence has shown the importance of implicit and explicit incentives in reality.

In the literature of moral hazard, one significant contribution comes from Fudenberg and Tirole (1990). They relax the assumption of commitment and assume effort exerted by the agent today has long term effect over the periods that follow. Therefore, present effort level influences the probability distributions of outcomes for all subsequent periods. The principal will throughout the renegotiating stages not have any private information on the probability distributions of the outcomes in the future. Thus, a contract can be renegotiated after the agent has chosen his (her) action and before the realization of outcome. The game progresses as follows; after the agent has exerted effort, the contract can be renegotiated and principal can offer a set of contracts for the each level of effort the agent may have taken before the renegotiation stage. Basically, the problem transforms into an adverse selection problem with equilibrium being a set of mixed strategies and different effort levels correspond to the agents’ type. If payment is independent of outcomes, an agent can choose the lowest possible effort. So in equilibrium it requires that the principal induces agent to choose higher effort by promising higher transfers for good outcome, but also make him (her) choose lower effort levels with sufficiently high probability that contract would not by renegotiated.

Another set of models developed under dynamic moral hazard comes from the recent work of Biasis (2009a) and Laffont and Martimort (2001). They present a similar, discrete time, two-period models. In these models, incentive compatibility implies an increase in expected utility of the agent after success and decrease after failure. Here the principal can take advantage of the dynamic nature of the relationship in order to relax the incentive constraints. For instance - the transfer promised at the second period after two successes can be used to entice effort at both first and second period. Moreover, Biasis allows for scaling of the project between the periods, and finds that closing the firm at the second period if production is low in the first period can help avoiding a full market breakdown. The intuition is that the agent rationally expect the project to be liquidated (or the investor exercising his walkaway option in the case of money
management) when the project performs poorly. While such liquidation is good to provide the agent with incentives, it hurts efficiency. Therefore, by introducing this extension to the model it still reflects the rent/efficiency tradeoff. Moreover, if the manager can be replaced, Biasis postulates that the principal needs to weigh this alternative in relation to their current state and cost of finding a new manager.

Under the discrete infinite time horizon case, the continuation utility of the agent and the value function of the principal are evaluated given the information available at the beginning of period $n$. All the other assumptions are same as in one and two period case. Here, the continuation utility of the agent evolves as a function of the cash flow realizations. The idea is that following a negative (positive) realization of output, the continuation utility of the agent must be reduced (increased), such that the incentive compatibility holds. Moreover, Biais (2009b) discusses that due to the sensitive nature of the continuation utility to performance of the firm, incentive compatibility requires that this sensitivity be equal to the magnitude of the moral hazard problem (reflected by the agents outside option and his cost of effort).

The last dimension that we will discuss under the theoretical literature related to moral hazard is when output is unobservable to the principal. This topic is quite important part for our paper since we adopt it in section 4. The basic idea is that in some cases principal cannot observe and verify the output generated by the agent and must therefore rely on him (her) to report it truthfully. Due to the unobservability of cash flows (or any other measure of success) the agent can divert it to his advantage and can report lower profits. This feature was first initiated in economic models by Bolton and Sharfstein (1990). In their one-period model, they study the situation in a debt contract where agents can divert the funds. In the event when he defaults on his debt, he is then faced with a threat of losing his collateral or not getting financing in the future. De Marzo and Fishman (2007) employ similar assumptions to a long term financial contracting problem where agents use debt and equity as optimal tools to structure the capital. Due to the dynamic nature of the contract, the threat of termination at any time by the principal (investor) makes the agent truthfully share the cash
flow than divert it. In their model they define the benefit from diverting funds equal to \( \lambda \) which varies between 0 and 1. They suggest that an optimal contract entails that in order to induce agent to report truthfully he must be paid in terms of \( \lambda \) per dollar of reported cash flow. However when the agent decides to steal the cash \((1-\lambda)\) is lost by the agent, which is regarded as the cost of diversion. This is so because the stealing is considered as an inefficient activity. Finally, in case \( \lambda = 0 \) there is no moral hazard problem.

In contrast to the discrete-time study in this paper, DeMarzo and Sannikov (2006) conducted a similar analysis in a continuous-time infinite horizon. Our model in section 4 also illustrates the possibility of cash diversion by an agent which is unobservable to the principal in one-period adverse selection and moral hazard problem.

3. Theoretical perspective: Asset management industry and other institutional settings

3.1 Asset management industry, stylized facts and theoretical relevance

3.1.1 Stylized facts about active management

As institutional savings drastically increase both absolute and relative to the retail market, in a market characterized by fierce competition and huge economies of scale, different asset managers battle for the right to manage, and potentially collect large fees on these investments. Nevertheless, active managed funds generally perform poorly relative to their passive counterparts (Malkiel, 1995; Gruber, 1996; Fernandes et al 2009). But Shleifer and Vishny, 1997 interprets this as the investors, having inferior knowledge, evaluates the manager on wrong (or potentially misleading) criteria, leading to a mispricing in the market Fernandes et al 2009.

Furthermore, there exists little evidence of persistence in returns (Berk and Green, 2004). Berk and Green argue that this is because investors evaluate
managers based on their historical performance and will continue to pool funds into managers until the expected return reaches the competitive level\textsuperscript{15}.

\subsection{Incentive theory and delegates portfolio management}

Information asymmetry is prominent in the asset management industry. An investor will not have complete information with respect to the true skill of the managers in the market. Neither will he be able to observe the effort put into ex-ante analyses; and in some cases the portfolio selection and risk profile. Naturally, delegated portfolio management has seen much research into these topics and we now attempt to give a brief review on the rich literature that exists on these areas.

In general, investors delegating the management of their assets to an agent cause frictions due to the different object functions of the investor and the manager. The manager, will privately make decisions which may not be in the best interest of the investor. There are several factors which may help mitigate these frictions and (Ackermann, Mcenally, and Ravenscraft 1999) identify four such factors in the investor manager relationship.

1. The first factor is the \textbf{contract}. That is, the contract can be designed such that the manager is given the incentives to make effort and risk decisions that are optimal for the investor. The problem for the investor is that if he cannot observe the effort or risk choices of the manager and the agent is risk averse or protected by limited liability, he cannot directly dictate the effort or risk level in the contract. Thus, the first best cannot be reached. The investor will instead have to settle for the second best, implying a positive expected rent to the manager\textsuperscript{16}. If there is adverse selection, the contract can also be used to screen the managers\textsuperscript{17}.

2. \textbf{Ownership structure} may also help mitigate the problem of moral hazard. Often, especially in hedge funds, managers have substantial

\begin{footnotesize}
\begin{itemize}
\item[15] Implicitly assuming decreasing returns to scale
\item[16] If the manager could make effort and risk part of the contract he could extract all rent from the agent.
\item[17] See section 4
\end{itemize}
\end{footnotesize}
investments in their own fund. This joint ownership structure exposes the manager to the same risks as the investors and thus helps align the objectives of the investors and managers. However, as pointed out by Ackermann, Mcenally, and Ravenscraft (1999), given a risk averse manager, he may end up choosing a risk level below that is preferred by the investors. Moreover, Kouwenberg and Ziemba (2007) show that risk taking by the manager is greatly reduced if the manager has a substantial investment of his own wealth into his own fund (at least 30%). When managers differ in their ability to generate excess return above the risk free rate, ownership structure could also potentially be used as a signaling device (we will discuss this part in more detail in section 4.5.4).

iii. Thirdly, **government regulation** may put restrictions on the extent at which the managers may adopt certain risky strategies, lockup periods and investments criteria (such as allowing for smaller minimum investment). This is less relevant for the hedge funds than the more regulated mutual funds industry. One argument often used to rationalize the minimal regulation facing hedge funds, is that investors of such investment vehicles are assumed to be more rational and informed than the average investor (considering the high minimum investment and the large degree of institutional clients), and thus does not need the same protection as for instance investors in mutual funds. Nevertheless, there has been increasing concern regarding the level of regulation in the industry, magnified by the financial crisis, where short selling in many financial institutions was temporarily banned in order to reduce the negative impact short selling can have in an already vulnerable sector.

iv. Finally, **market forces** may also help reduce the principal agent problem. The idea is that well informed, rational investors, will exit funds where effort or risk does not match their optimal preferences and invest in higher performing funds with better matching risk profiles. Obviously, these selection mechanisms scale with the degree of fund transparency. Thus, for hedge funds, especially with a joint ownership where replacing
the manager may prove difficult, this effect is limited (Ackermann, Mcenally, and Ravenscraft 1999).

In his influential paper, Stoughton (1993) introduced an alternative formulation of the principal agent problem for delegated portfolio management. He assumed the existence of two assets in the market, one risk free and one risky. The risky asset is assumed to be normally distributed. Managers can, in light of their expertise observe a signal in the market that is correlated with the true value of the risky asset before selecting the portfolio. By exerting costly and unobservable (to the investor) effort, the manager can potentially increase the precision of this signal. Hence, the investor must consider two dimensions of the moral hazard problem. First, the unobservable level of effort and then the portfolio choice will affect the risk and return of the gross portfolio. With these conditions, Stoughton finds that a linear sharing rule leads to significant underinvestment, but by introducing a quadratic compensation structure this underinvestment becomes mitigated. An important consequence of this has been termed the irrelevance result (Admati and Pfleiderer 1997). Admati and Pfleiderer finds that the inefficient low investment caused by a linear compensation structure is due to the manager have the possibility to scale his response to the signal and undo the incentive part of the contract. But Gomez and Sharma (2003) finds that this is only true if there are no restrictions on the amount the manager can short-sell. Conversely, linear sharing rules can be optimal and Ozerturk (2004) finds that if the manager can affect the price of the risky asset by trading, these contracts do give the manager the incentives to gather information. Moreover, if the investor is sufficiently large, Bhattacharya and Pfleiderer (1984) show that a quadratic contract dominates linear sharing rules.

Palomino and Prat (2002) show that when the agent controls effort and risk, and has limited liability, the optimal contract can be characterized by a simple bonus scheme where he is paid a fixed fee above some benchmark18. Livdan and Tchistyiy (2010) also considers a two-dimensional dynamic problem with hidden

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18 However, this is a construct of the binominal effort technology
cash flow and risk choice and demonstrate that if the state of nature is non-verifiable, it may be optimal for the investor to gradually increase investment.

In the literature, many researchers have focused on one specific compensation structure commonly observed in the hedge-fund industry, i.e. a high-watermark loss recovery system. Ray and Chakraborty (2008) employ a two-period principal agent model where the agent is compensated by such a rule. They find that walkaway by the investor is more likely when the fund moves away from the watermark. Moreover, when the fund is below the watermark, the manager reduces effort and increases risk. They find this to be partially consistent with empirical observations. A fund below the watermark will generally increase variance in return, but following a one-period bad performance, managers will actually reduce risk in order to maintain the continuation value of the contract. Zhan (2010) finds similar results in a multi-period simulated model. Furthermore, he shows that if the fund is only a little under the watermark, the manager may actually increase his effort level, but once the cost to reach the watermark becomes sufficiently large, he reduces costly effort and the fund drops even further below the required value. In this case, Zhan proposes that it may be optimal for both the investor and the manager to reset the watermark for incentive purposes. Ray (2011) focuses on the downside of high-watermark contracts and he points out that funds below the watermark increases subsequent risk and have lower sharpe ratios.

In a model with adverse selection, Aragon and Qian (2006) analyses the use of high-watermarks in the compensation structure as a signaling device and demonstrate that information asymmetry is a requirement for justification of the mechanism. Moreover, they also find empirical support for their predications as high-watermarks are more common in funds with shorter track records or long lock-out periods. Brown, Goetzmann and Park (2001) focus on the prominent use of high-watermark contracts in the hedge fund industry relative to the mutual fund industry. They claim that high-watermarks can be used as a credible signal of diminishing returns in the industry (mutual fund managers rely more on accumulation of assets under management to increase compensation). In an
empirical paper Deuskar et al (2011) studies the change in fee structures and demonstrate that less established funds change their fee structure more frequently. They also find support that high skilled managers with short track record reduce the fixed proportion of the compensation and increase the incentive part to signal ability.

Using the formulation of Stoughton (1993), Stremme (2001) considers an adverse selection problem where one type of manager has superior market information. He finds that in a dynamic environment with learning, bonus contracts are optimal. Stoughton and Heinkel (1994) show, using a model with adverse selection and two periods, that an optimal pooling contract requires lower incentives than what is the case for a single period. Interestingly, they also find that if the fund outperforms a benchmark with sufficient amount, this indicates that the performance is not due to the deep skill parameter but rather pure luck. Das and Sundaram (1999) use a slightly more complex technology with two assets where the informed manager knows the true combined distribution. With a special focus on fulcrum fees (see section 1) and letting the manager propose the fee structure (have all the bargaining power) they find that investors can be made strictly better off by prohibiting fulcrum fees\(^{19}\).

In a principal agent model with moral hazard similar to the carrier concerns model of Holmström (1982), Farnsworth (2003) uses a dynamic model to analyze the gains from learning, but unlike Holmström the managers are assumed to have an informational advantage over the principal. He demonstrates that when beliefs are non-homogenous, the optimal contract constitutes a fixed proportion of the assets under management and an increased probability of receiving a bonus based on performance as the reputation of the manager increases. Focusing more on the effects of career concerns in a model with high-watermark provision, Brown, Goetzmann and Park (2001) find in their empirical study that high-watermarks have little effect on the risk profile of the manager. They interpret this as an indication that hedge fund managers put much weight on their reputation in the industry.

\(^{19}\) Arguably, they only compare fulcrum fees to an asymmetric fee structure
3.2 Theoretical work in other institutional settings

3.2.1 Credit rationing

This phenomenon refers to a situation where lenders limit the supply of additional credit to borrowers, even if the borrowers are willing to pay higher interest rates. Banks usually in reality are able to distinguish between borrowers (bank statements/wealth is an essential tool) to a certain extent. They charge higher interest to the high-risk borrowers or ask for more collateral requirements and thus credit rationing becomes less likely. But in spite of this, it is impossible to identify the types perfectly, so credit rationing occurs (Paschke notes)\(^{20}\)

Moral hazard

Tirole (2006) uses a principal agent model to explain credit rationing, with a focus on moral hazard. A common idea is that an increase in the interest rate has no effect on the borrower in the event of bankruptcy since he is protected by limited liability. A higher interest rate would imply a lower stake for the borrower, since he now has to pay more than the normal rate to get access to certain amount of money. This reduced stake motivates him to pursue projects with high private benefits, which in turn may reduce the probability of reimbursement.

Tirole studies the phenomena in a simple agency model and uses it to illustrate the role of net worth. In this model, an “entrepreneur”(borrower) does not have enough money to finance a fixed-size project and must therefore resort to outside funding (common lenders such as bank). The entrepreneur controls his effort level and may also chose between projects with different probability of success. Both players are assumed risk neutral, but due to the competitive nature of the market, the lenders make zero profits. This assumption will act as a constraint in the maximization problem of the lender. Finally, the borrower who is protected by limited liability is ensured non negative transfers from the relationship.

Moreover, the project will only be financed in absence of the moral hazard problem. Due the zero profit condition of the investors, when the project is funded, the borrower receives the entire surplus. Tirole shows that a necessary condition to receive financing is that the break even constraint holds. This constraint says that if the entrepreneur’s own stake in the project is less than some limit depending on the severity of the moral hazard problem, the probability of high return and the required lending, he cannot afford to give incentives the agent to exert high effort (which is assumed to be the only case where it is possible to get financing in the first place). Thus, he will not receive financing.

**Adverse Selection**

An adverse selection argument for this phenomenon is that higher interest rates tend to attract low quality borrowers and since these low-quality borrowers (high-risk takers) are more likely to default on their loan, they are less affected by a rise in the interest rate than high quality borrowers. Therefore to keep a good sample of borrowers, lenders may want to keep interest rates low (Tirole 2006).

Stiglitz and Weiss (1981) proved that since it is not possible to perfectly screen high and low risk borrowers, banks offer the same interest rate to both of them. They present a model of credit rationing in which potential borrowers who are denied loans will not be able to borrow even if they offer to pay high interest rates or accepts higher collateral requirements. Their analysis implies that raising interest rates or increasing the requirements of collateral could reduce the profitability of the bank since it encourages high risk types and discourage low types to borrow. Moreover, banks shouldn’t use either of them as instruments to balance the demand of the loans with more supply. It is rather suggested to offer quality of loans than quantity. They conclude that in light of this situation it will lead to excess demand equilibria.

However, they discuss the existence of excess supply equilibria in the market, where in a competitive environment banks must in order to attract good borrowers cut down interest rates. But a cut in an interest rate from one bank
would be met by an equal/lower cut by another bank. So the resulting equilibrium will be excess supply where no bank cuts its interest rate.

### 3.2.2 Insurance

Insurance is a type of risk management where one is partially or fully protected against an uncertain undesired outcome. It can be further defined as transferring risk from one party to another. The party that sells the insurance is called an insurer and the party that receives it is called insuree. Insurance can be bought for a fixed or infinite period of time. An insurer charges the insuree a risk premium for the protection. The risk premium can be a one-time payment or can be paid in installments until the insurance policy (insurance contract) comes to an end. If an undesirable outcome is realized when insured, the insuree receives a compensation (depends on the type of contract).

### Moral Hazard

The work of Pauly (1968) and Arrow (1970) brought us some economic intuition of moral hazard in the medical industry. They concluded that individuals are free to indulge in any action. Furthermore, with the guarantee that insurance company will pay, the outcome of this will be socially inefficient. However, the insurance company could allocate the resources to all concerned with the policy by rationing the quantity of medical services it will provide under the medical insurance. Pauly argues that insurance at times works as a subsidy\(^\text{21}\) for individuals, hence insurance should only be provided in cases where quantity demanded at zero price would equal the quantity demanded at positive price. This implies that complete insurance falls out if supply of insurance affects the demand for the services it represents. Newhouse and Taylor (1969) suggested that one can eliminate part of this problem by restricting the consumers’ choice with respect to quality of insurance, but letting them receive the subsidy corresponding to quantity.\(^\text{22}\)

In the first formalization of moral hazard in the industry, Spence and Zeckhauser (1971) show that if an insurance company can monitor the actions or observe

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\(^{21}\) Raises their income when sick

\(^{22}\) They referred to this as “Variable cost insurance” where insuree will pay higher premiums for expensive hospitals
the state of the nature (which the insured individual payoff is based on), then full risk spreading can be received and individuals will not indulge in perverse behavior. In absence of this monitoring facility, the optimal insurance payoff will always be second best.

Holmström (1979) applies the moral hazard problem to accident insurance. He is formally able to show the intuition of Arrow (1970) and Pauly (1968). In case of an accident (case insurance materializes), the optimal outcome would entail deductibles (i.e., no complete insurance). Laffont and Martimort (2002) also use accident insurance to showcase moral hazard. They define effort of the agent as level of safety care. Like others, their formal model also demonstrates that in the presence of moral hazard, the cost incurred by the insurance company will be higher than in absence of moral hazard.

**Adverse Selection**

Dionne, Doherty, and Fombaron (2000) study a model where the insurance company is not able to distinguish between the risk types. It is assumed that risk neutral insurees denoted by high and low risk takers are not able to influence the probabilities of accidents and damage, which again removes moral hazard in model. Moreover, high risk takers have greater probability to be in a state of accident than low risk takers. The paper is quite comprehensive and investigates adverse selection in insurance market across different levels. They use the work of many academics to compile this extensive literature.

In the case where there is only one insurance company in the market which has full information about the types, the optimal contract will be characterized by full insurance coverage, extraction of whole consumer surplus and no dead weight loss (no loss in efficiency). When this single period model is extended to a private information setting, the monopolist must offer self-selection contracts to the insurees which makes them reveal their true types. Here the low type will buy the insurance policy giving him limited protection at a low price while a high type will buy a full insurance. The same result was primarily shown by Stiglitz (1977).
Cooper and Hayes (1987) use experience ratings in addition to self-selection constraints when looking at the problem over two-periods with full commitment. They assume that each agent’s accidents’ history becomes known to the insurer after first period. This implies that the monopolist is able to raise his profits by offering contracts where premiums in the second period are function of history or damages in the first period. The resulting equilibrium contract is a separating one with high types getting full insurance and low risk individuals obtaining partial insurance. Experience rating feature of the model is very enticing for the potential insurers since it gives them a chance to pay lower premiums in the second period, given than accident does not occur.

In another multi-period setting, Dionne (1983) uses a Stackelberg game to induce risk revelation than using self-selection constraints. The individual chooses a starting premium based on his “risk announcement”. If the insuree reports himself (herself) as a low risk type then, he (she) pays a low risk premium taking into consideration that average damage is less than the anticipated damage given his (her) risk announcement. However if this does not hold he (she) has to pay a penalty premium.

It is discussed in the literature that if we augment the setting with full commitment to a possibility of renegotiating in a finite horizon case, the self-selection in the first period would imply inefficiency in the periods that follow given that the information types become public. Inefficiency would mean for instance that if high types were aware that renegotiation will occur, they would never reveal their type (Dionne, Doherty, and Fombaron 2000 and Dionne, Doherty 1994). Hosios and Peters (1989) discuss that accident reports available after the second period works as a tool to reveal information about the type and hence can be used in renegotiation. They show that when neither party can commit to the contract, a separating equilibrium does not exist; only pooling and semi-separating equilibria are possible.

Dionne, Doherty, and Fombaron (2000) also extend the adverse selection problem in insurance to competitive markets. In a benchmark case with symmetric competition they demonstrate that insurance companies make zero
profit with premiums set equal to their marginal cost, there is full insurance coverage and insurees keep their surplus. Hence the optimal contract is first best. However if information is not public, Rothschild and Stiglitz (1976) show that a separating equilibrium exists where high types buy full insurance while low types obtain only partial insurance. Moreover, the firm earns zero expected profit on each contract. The contract implies that high types obtain first best allocation, but low type receives less insurance compared to the full information case. Wilson (1977) suggests an alternative approach where a firm takes into account the strategy/behavior of others when deciding their own strategy. For instance, an equilibrium can exist where firms abandon their policies after anticipating the reactions of other firms, such that the remaining at least break even.

The literature analyzing the adverse selection problem in the insurance industry with competitive firms is quite vast, and since the main aim of this section of our paper is only to acquaint the reader with relevant topics, we will restrict our discussion here and move to adverse selection in multi-period case.

In a multi-period setting Cooper and Hayes (1987) studies two types of contracts; one where both parties commit to the contract referred as full commitment and the other where the insurees can costless switch in second period and buy the insurance at other available competitors referred as semi-commitment contract. It is shown that in the former, the optimal contract will be “qualitatively” similar to the monopoly case with full commitment, meaning that high types will buy full insurance and low type will obtain partial insurance. However, in the latter due to the fact that the insurees can move to other firms in the second period who offer single period contracts, it reduces the possibility of punishing for accidents in first period. They assume that entrant firms offer single period contracts without any information about the history of accidents or what type of contract was chosen by insurees. Moreover, these new firms offer contracts pertaining to Rothschild and Stiglitz one-period optimal contract with separating equilibrium.

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23 The problem is only constrained to find an optimal contract for low type.
The optimal contract for two-periods by competitive firms under semi-commitment involves maximizing the expected utility of low types under the constraints of *non-negative intertemporal expected profits* and *no-switching constraint*. The former eliminates the possibility of *cross-subsidizations between high and low types*. Hence under the assumption that there is a Nash equilibrium, the optimal contract entails; high types get full insurance and low type gets partial insurance. High types are then indifferent between low and high types’ contracts while low types prefer only the contract designed for them. Thus both types retain their consumer surplus, and the sequential profits from high types are flat while on the low types, the competitive firms earn positive expected profits from old insurees and expected losses on new insurees.

There exists extensive work on the situation when there is no-commitment on both sides. In this case both parties sign a one-period contract which will be followed by a *conditionally optimal* and *experience rated* second-period contract. It is seen in the literature that this sequential contracting with no commitment gives rise to “intertemporal welfare” lower than the case with full commitment but higher than in the situation with repetition of static contracts without memory (meaning for instance accident history remains unknown to the insurance company) (Kunreuther and Pauly 1985, Nilssen 1990, Fombaron 1997).

### 3.2.3 Sharecropping

Sharecropping is a system in agriculture where the landowner lets a tenant use the land in return for a part of production or is paid corresponding transfers for working on the land. Hence contracts are needed to decide the sharing attributes between the parties. Sharecropping has been a debatable subject where some have regarded it as an inefficient form of agriculture. Basu (1984) referred to it as *deplorable form of cultivation, the daughter of necessity and mother of misery*. It has been long believed that due to technological advancement, sharecropping would soon come to an end, but it still continues to be popular form of tenancy.
contract in developing countries and also developed countries\textsuperscript{25} (Chaudari and Maitra 2006).

**Moral Hazard**

Moral hazard in agrarian economies involves a landlord acting as a principal and tenant as an agent. Laffont and Martimort (2002), applied this problem to a setting with a monopolist and a risk neutral agent who has a limited liability. They compared a flexible second best contract\textsuperscript{26}(criticized for not corresponding to agrarian economies) to a linear contract where agent is offered a fixed share of the realized production. They show that with a linear sharing rule, the tenant always benefits from positive yield on production even in the worst case. Hence his transfers are more than what a second best (transfers correspond to high output and low output) entails in a worst case i.e. zero.

Prior to this, another interesting work was presented by Agarwal (1998) who uses the work of Eswaran and Kotwal (1985) in a double sided moral hazard problem where landlord and the tenant indulge in mutual monitoring of each other. They show that optimal contract entails sharing a rent that maximizes the risk-adjusted expected output net of their risk premium and agency cost. Moreover, they strengthen their result through a discussion of various empirical results which show that agency costs which are essentially monitoring costs can be removed.

**Adverse selection**

From an adverse selection perspective, it is often assumed that tenants have different unobservable ability to produce efficiently (privately known). The problem was first formally modeled by Hallagan (1978) who showed that the very existence of firms eliminates this problem i.e. the institutional setting where coexistence of wages, rents and a contracts shared by the two parties produces

\textsuperscript{25} Allen and Lueck (1992) discuss its existence in US mid-west. However the amount of theoretical and empirical work on developed nations in sharecropping reduced after 1980s.

\textsuperscript{26} Agent receives a wage that makes him exert high effort and principal has to give up part of his return generated to induce output
useful information such that it makes the landlord allocate resources efficiently. They show since the resulting wage is ratio of marginal disutility of labor to marginal utility of consumption/production (assumed that all consume and produce), this wage then can be used in order to make the tenants self-select themselves corresponding to their ability. This result was also shared by Newbery and Stiglitz (1979). The primary difference between the two papers is the way ability was defined. Hallagan defined it as entrepreneurial ability of an individual making it independent of the hours used on the land, while Newbery and Stiglitz defined it as the efficiency of an individual in terms of output units produced in one hour. Their models were succeeded by Allen (1982) who modeled two unobservable attributes as compared to one. His model not only included tenants with different abilities but also landlords with different quality of lands. Each of this attributes were assumed to be unobservable to the other party. Contracts are similar as in the Hallagan, Newbery and Stiglitz papers discussed above, but landlords are contracted upon the ratio of effective land to effective labor. If a landlord attempts to scam a tenant and claims to own a more effective land than he actually does, he will pay a penalty in the form of wage that corresponds to the type of worker that accepted the contract. Hence efficiency can be reached.

3.2.4 Environmental risks

Laffont (1995) investigates both the problems of moral hazard and adverse selection together in a case where the principal is a regulator and agent is a firm (efficient and inefficient) with a responsibility of a project which has a social value and a probability of a catastrophe both corresponding to an effort level. The effort level regarding catastrophe which has environmental risks can take two values 0 for no safety measures and 1 for taking safety measures. The incentive constraints are then put together such that it not only induces the firms to reveal their true types but also makes them to take safety measures. In the case where agents are risk neutral with no limited liability, the contract entails that safety measures can be induced by imposing a penalty if catastrophe occurs.  

27 It is assumed in the model that social cost of the catastrophe is large enough, so regulator wants to implement safety measures irrespective of the firm type.
The rents promised are the same as they would have been when safety measures were not needed, but at lower effort levels. When the limited liability constraints are removed, the optimal contracts suggest a pooling equilibrium where rents are promised even to the most inefficient firms such that so that safety measures are implemented. With risk averse agents, the effort distortion is even more significant with higher rents paid for risk insurance and solution suggested is a trade-off less favorable to efficiency.

4. The Model

4.1 Introduction

We consider an asset management problem which involves two types of players, an investor (the principal) and an investment manager (the agent). Different interactions between them are spread over only one period and the manager’s job is to manage the investor’s money by choosing a portfolio comprising of the assets available to him. The manager has access to two assets in the market, a risk-free and a risky asset. The return of the risky asset has distribution $F(R)$. Prior to selecting the portfolio, the manager may observe an informative signal $S$, giving him access to the conditional distribution $F(R | S)$. Only the manager can have access to $S$ and he may then base his investment decision on a more precise estimate of the true value of the risky asset. This type of situation was first modeled by Stoughton (1993), where the informative signal received by the agent is contingent on the amount of effort he exerts. Our model is primarily an adverse selection model, where this technology is used to differentiate the types of the managers in the market. The problem that we investigate in our paper deals with designing an optimal contract that makes an hired manager choose a portfolio that maximizes return for the investor and also bind manager to truthfully report the realized returns.

4.1.1 Information structure, Assets and Return Distribution

We borrow the technology used by Stremme (2001), where there are two assets available in the market, a risk-free and a risky asset. The gross return on the risky asset can take three values $R^+, R^- or R^0$, with $R^+ > R^0 > R^-$, whereas the
The gross return on the risk-free asset is $r^f = R^0 > 0$. The signal $S^j$, where $j \in \{+, −\}$ is assumed to take on two only values, either positive or negative. We further assume that following a positive signal, the return can only take the form of $R^+$ or $R^0$. Conversely, for a negative signal returns must be either $R^0$ or $R^-$. This in turn implies that after the market observes either $R^+$ or $R^-$, the signal would be reviled with certainty. It is also assumed that $P^l$, where $l \in \{+, −\}$ is the unconditional probably that $R$ will take the values $R^+$ or $R^-$ respectfully. The conditional probabilities, denoted $p^j_k$ for $k \in \{+, −\}$ are the probabilities of returns $(R^+, R^-\text{or } R^0)$ given the signal $S^j$.

**Assumption 1:** The unconditional probabilities of a high and low state of nature are equal to each other. Formally $P^+ = P^−$, where $p^+ p^+_k = P^+$ and $p^- p^-_k = P^−$

Assumption 1 will occasionally be relaxed when results are qualitatively dependent on this distributive property. Consequently, Figure 3 and 4 illustrates the distribution of the risky asset and (un)conditional probabilities of different state of natures

**Figure 3:** Two stage binomial tree illustrating distribution of the risky asset
4.1.2 Agents - Investment Managers

There are two kinds of managers, a good manager denoted by $\theta_G$ and a bad manager denoted by $\theta_B$. The good manager is what we refer as high skilled and the bad manager as low skilled. Both managers differ in their ability to obtain information from the market, on which they base their investment decision. Only high skilled manager has the ability to obtain such information in form of a signal $S^1$ with certainty. Since the low skilled manager does not have access to this information, he can be considered uninformed compared to the high skilled manager. Thus, he chooses his portfolio based only on the unconditional return probabilities.

**Assumption 2:** Good managers observe an informative signal prior to choosing their portfolio, while bad manager must base their decision on the posterior probabilities.
The assumption above implies that a high skilled manager will always generate return equal or above the risk-free rate, this however is not true for the bad agent as he runs the risk of a failed strategy.

However, unlike Stremme (2001) we also allow managers to use leverage at the risk-free rate and short sell at the market price of the risky asset. Thus, by adding this dimension to our model, it allows managers to have more diversity to their portfolio and investment decisions.

Managers are protected by limited liability i.e. they will have to be paid positive transfers and can’t be punished by levying negative transfers on them. However, later in the thesis we will discuss the implications of relaxing this assumption.

4.1.3 Investor - Principal
The investor has a certain amount of exogenous wealth and potentially invest 1$ with the hired agent.

We assume that the investor cannot observe the actual returns generated and relies on the manager to report him the true returns. Because of this unobservability assumption, the agent could potentially steal a fraction of realized returns and report lower returns to the investor. We also assume that stealing is inefficient and manager loses part of the stolen cash. This approach is similar to Demarzo and Fishman (2007), and like them, we argue that this can be justified due to inefficient consumption.

The principal maximizes his utility, a function of expected returns subject to the participation constraints, truth telling constraints, and limited liability constraints. This problem will be discussed in detail later in the paper.

4.1.4 Preferences

Assumption 3: Both managers and the investor are risk neutral with respect to wealth.

As far as managers are concerned, risk neutrality plays a crucial role to highlight the value of information. It gives us an option to assess value in monetary terms without having to assume specific shapes of the utility function. Consequently, this allows us to analyze the model in a more tractable way. Moreover, risk
neutrality allows us to arrive at more explicit solutions, which further facilitates the focus of our thesis.

In case of the principal, he only invests part of his wealth with a manager and therefore diversifies his money in various alternatives. Hence we view, risk neutrality on the part of the principal as a plausible assumption.

4.1.5 The Timing of Events

We model a one shot game. There is information asymmetry between the investor and the managers, with only managers being fully aware of their type $\theta_i$. Given these assumptions, the game evolves as follows:

1. At first, the investor proposes a take it or leave it contract to a manager. If not accepted the investor will repeat this procedure costless until the contract is accepted by an agent and participation is satisfied by at least one type and the investor.

2. If the contract is accepted and the hired manager is a good type then he receives information from the market and his portfolio choices are then established upon the conditional return probabilities. But if a bad manager is employed, he does not have access to such information and his portfolio choices are based purely on the unconditional return probabilities.

3. The returns $Y(\alpha)$, are realized

4. Manager reports $\tilde{Y}$

5. Manager is paid the transfers $W(\tilde{Y})$

Figure 5: Timing of the one shot contractual game between the investor and managers

![Figure 5: Timing of the one shot contractual game between the investor and managers](image-url)
4.2 Portfolio selection and strategies

In this section we state several conditions for optimal portfolio selection both for the investor and the manager. The manager will, since he is risk neutral, select the portfolio that maximizes his expected transfers. For the investor on the other hand, the optimal portfolio maximizes the expected net portfolio return. Since both players are risk neutral, this alignment problem is costless for the investor. Nevertheless, he must consider certain properties in the mechanism design such that this holds regardless of strategy. These properties and strategies will now be discussed in more detail.

First a brief recapture of the possible trading strategies and limitations available to the manager. For any manager \((G, B)\), there exists a risk free asset yielding the safe return \(R^0\) and a risky asset taking one of three values \((R^+, R^0, R^-)\). The precise distribution of the risky asset is described in detail in section (4.1.1). After receiving the ex-ante signal \(S^i\) (if the manager is good), the manager selects how much to invest in the risky asset. We allow for both short-selling and leverage (though under restrictions \(a \in (-a^*, a^*)\)) to increase the benefit of obtaining ex-ante additional information about the state of nature. Conversely, the bad manager’s uniformed strategies will simply be based with regard on assuming a realized value of the signal \(S^i\).

**Definition 1:** The bad manager has two viable strategies which we denote:

- **Leverage mimicking strategy:** The manager acts as he has received a positive signal when selecting his portfolio.

- **Short-sale mimicking strategy:** The manager acts as he has received a negative signal when selecting his portfolio.

For the most part of this thesis, we will only derive results for one of these strategies, unless it is qualitatively important to consider them both. This is because many of the results we derive will simply be mirrored in a converse strategy. As we show later, the marginal return with respect to ‘\(a\)’ following a successful implementation of a strategy will have significant influence on the implementation of an optimal compensation scheme. With this in mind, we now
turn our attention to two specific cases of the distribution. We will use graphical arguments to see the conditions at which each strategy constitutes the maximum portfolio return. These illustrations are given in Figure 6 and 7. Obviously, a good manager will never fall beneath $R^0$ in any case. However a bad manager could, given that his strategy turned out ex-post wrong, generate significant losses for the investor.

**Figure 6:** Portfolio return when maximum return is given by a short strategy

The wage function mapping these gross returns into transfers to the agent is denoted $W(Y)$. For now we don’t assume any particular structure on $W(Y)$, but...
we will discuss what conditions on this mapping function that leads to optimal portfolio selection for the investor. With this in mind we state our first proposition:

**Proposition 1** Let \( W(R) \) be the function mapping return to transfer and let it be increasing in all \( R > 1 \). Furthermore, let the truth telling constraint (1.1) be satisfied for all \( Y \)

1. Following a positive signal, the high skilled manager selects the portfolio \( (1 - a^*, a^*) \) and if he receives a negative signal, he selects the portfolio \( (1 + a^*, -a^*) \), i.e., he shorts the risky asset and invests all in the safe asset.

2. The bad manager will if \( P^+/P^- > W(\cdot)/W(\cdot) \) follow the leverage mimicking strategy and follow the short-sale mimicking strategy otherwise.

**Proof:** See appendix A1

Arguably, the manager will consider the possible gain from reporting a low return and stealing the difference when selecting his portfolio, but as we shall see in section (4.3), in equilibrium the wage function will satisfy truthful revelation of the gross return. Moreover, even if this was not the case, he will still be best off by maximizing the difference between the realized return and the lowest supported return. Hence he will still maximize expected gross return.

Moreover, if \( W'(R) \geq 0 \), such that \( W(R^I) = W(R^i) \) for some \( I \) and \( j \), and \( R^j \) or \( R^i \) constitutes the maximum of return, there may not be a unique solution to the problem. Suppose for instance, the manager will be paid a fixed fee with a bonus over some benchmark. Such a compensation structure can be described formally as:

\[
F(w, B, b) = w + B \cdot 1_{(Y > b)}
\]

\( 1_{(Y > b)} \) is an indicator function taking the value 1 when return exceeds the benchmark and 0 otherwise. If \( b < R^0 \) then any portfolio except the short selling strategy is optimal for both managers following a good signal. Following a bad
signal, any portfolio with $a \in [-a^*, 0)$ is optimal. We will however not consider such compensation schemes to a large extent.

An important feature of this model setup is the value of accessing additional information above what the market has priced in. The uninformed investor will in fact always generate less (or equal) return than the informed manager. This is stated formally in proposition 2.

Suppose now that the manager follows the optimal portfolio strategy for the investor. In this case, the expected conditional and unconditional returns are summarized in table 1, 2 and 3.

<table>
<thead>
<tr>
<th>Table 1: Expected return of a good manager</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Expected return</strong></td>
</tr>
<tr>
<td>$(1 - a^*)R^0 + a^*R^+$</td>
</tr>
<tr>
<td>$R^0$</td>
</tr>
<tr>
<td>$(1 + a^*)R^0 - a^*R^-$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2: Expected return of a bad manager following strategy $\mathcal{L}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Expected return</strong></td>
</tr>
<tr>
<td>$(1 - a^*)R^0 + a^*R^+$</td>
</tr>
<tr>
<td>$R^0$</td>
</tr>
<tr>
<td>$(1 - a^*)R^0 + a^*R^-$</td>
</tr>
</tbody>
</table>
Proposition 2: The expected gross return of a good manager exceeds that of a bad regardless of strategy and this difference increases in allowed leverage and short selling.

Proof: See appendix A2

It is clear the investor would like to design a contract which provides the incentives for unique optimal portfolio selection. Proposition 1, gives some structure to the wage function and we see that this corresponds well to usual fee structures in the industry such as such as asymmetrical bonus contracts with or without a hurdle rate, or contracts which are simply linear in return. However, although these conditions are sufficient for optimal portfolio selection, they are not necessary. In the next proposition, we attempt to put more structure on a general wage function yielding optimal selection for the investor.

Proposition 3: Let \( P_{l,s} = \max \left( (1 - a)R^0 + aR \right) \) be the maximum possible value of gross portfolio return. The wage at \( P_{l,s} \) must be strictly larger than all other returns to achieve optimal portfolio selection.

Proof: See appendix A 3

Proposition 3 implies that the investor is allowed to write a compensation scheme which need not be strictly increasing in R to achieve optimal selection, thus providing him with more flexibility when considering the agents participation constraints.
4.3 Unobservable cash-flow

In this model, the investor cannot observe the return generated and thus must rely on the return reported by the agent. To account for this we assume that given the true realized return $Y(a)$ the manager can report $\tilde{Y}(a)$ and steal the difference. However, in the process of doing this some of the funds are lost to both the agent, and he only gets away with a fraction. We denote the fraction lost by $\Sigma$ and treat it as exogenous (i.e. neither the principal nor the agent can influence the size of $\Sigma$).

Assume now that all conditions on $W(\tilde{Y})$ satisfy optimal portfolio selection. We then know that from our simple distribution, only four outcomes are possible. From this point on in the thesis we will assume this hold and then later check if the wage function satisfies the conditions for optimal selection. Table 4 summarizes these returns as well as introduces some simplifying notation.

<table>
<thead>
<tr>
<th>Return</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(1 - a^*)R^0 + a^*R^+$</td>
<td>$Y(\cdot)$</td>
</tr>
<tr>
<td>$R^0$</td>
<td>$Y(0)$</td>
</tr>
<tr>
<td>$(1 + a^*)R^0 - a^*R^-$</td>
<td>$Y(\cdot)$</td>
</tr>
<tr>
<td>$(1 - a^*)R^0 + a^*R^-$</td>
<td>$Y(\cdot)$</td>
</tr>
<tr>
<td>$(1 + a^*)R^0 - a^*R^+$</td>
<td>$Y(\cdot)$</td>
</tr>
</tbody>
</table>

In general, if the investor request truth telling, he must take into account the following maximization problem for the manager:

$$\tilde{Y} \in \arg\max W(\tilde{Y}) + (1 - \Sigma)\max(0, Y - \tilde{Y})$$

(1.1)

Here $\tilde{Y}$ is the reported return and $Y$ is the actual realized return. The last max operator is present due to the fact that the manager cannot use his own wealth
to boost the return in order to achieve higher wage\textsuperscript{28}. Although portfolio return is defined continuously, our simple distribution of the risky asset and assumption of optimal portfolio selection implies we can simplify the truth telling condition for computational purposes. Hence, condition (1.1), which implies an infinite number of truth telling constraints can be expressed as the only relevant cases (i.e. the only possible realized returns). We now attempt so solve explicitly for the conditions of optimal truth telling.

To solve for the conditions that ensure truth telling, we must first find what the lowest possible return is. The two candidates are:

1. The agent can choose maximum leverage and invest in the risky asset such that the portfolio return is 
\[ (1 - a^*)R^0 + a^*R^- \]

2. The agent can choose a maximum short strategy and get the return: 
\[ (1 + a^*)R^0 - a^*R^+ \]

The investor must consider these two cases when designing the contract as they will define a minimum payment he must promise the manager in different reported states. Hence, we have two cases to consider. To summarize, case 1 is the lowest possible return if the following condition holds. If not, then case 2 is the lowest possible return the manager can generate.

\[
\frac{R^+ + R^-}{2} < R^0
\]  

(1.2)

**Case 1: Lower bound of return is given by a failed leverage strategy**

Let’s now assume that condition (1.2) hold. We know that the maximum return possible is given by shorting the risky asset and invest in the risk-free asset. If this is indeed the case, it must be so that the manager truthfully report $\tilde{Y}(-a^*, R^-)$ and not $Y(a^*, R^-)$ if the following truth telling constraint is satisfied.

\[
W\left(Y(\tilde{\tau})\right) \geq W\left(Y(-\tau)\right) + (1 - \Sigma)(R^0 - R^-)2a^*  
\]  

(1.3)

\textsuperscript{28} If, for instance, the manager faces a convex compensation scheme such as described in the portfolio selection section, the manager have both the incentive and opportunity to boost the return with his own saving when he is marginally below the bonus benchmark rate.
With this condition we can now show that the investor will always request truth. The least cost condition for truth implementation is to bind (1.3). Hence, what the investor is left with after compensating the manager is:

\[
V(\bar{Y}(\bar{\bar{\mathcal{Y}}})) = (1 + \alpha^*)R^0 - a^*R^- - W(\bar{Y}(-)) \\
= (1 - \Sigma)(R^0 - R^-)2a^*
\] (1.4)

While if the agent lies and reports low return, the value of the fund after fees is:

\[
V(\bar{Y}(-)) = (1 - a^*)R^0 + a^*R^- - W(\bar{Y}(-))
\] (1.5)

Subtracting (1.5) from (1.4) we get:

\[
2\alpha(R^0 - R^-) \geq 2\alpha(R^0 - R^-)(1 - \Sigma)
\]

This is always true as long as \( R^0 > R^- \). Moreover, since this holds when the agents can steal the largest return in case 1, we conclude that it must be so that the investor will always prefer truth telling, regardless of realized return. This is intuitive as stealing is inefficient (part of the profit is “lost” by the agent). Definition 2 summarizes the four relevant truth-telling conditions for case 1.

**Definition 2:** For \( \frac{R^+ + R^-}{2} < R^0 \), the following four truth-telling conditions must be satisfied in equilibrium.

- \( W(\bar{Y}(\bar{\bar{\mathcal{Y}}})) \geq W(\bar{Y}(-)) + (1 - \Sigma)(R^0 - R^-)2a^* \)
- \( W(0) \geq W(\bar{Y}(-)) + (1 - \Sigma)(R^0 - R^-)a^* \)
- \( W(\bar{Y}(\bar{\bar{\mathcal{Y}}})) \geq W(\bar{Y}(-)) + (1 - \Sigma)(R^+ - R^-)a^* \)
- \( W(\bar{Y}(\bar{\mathcal{Y}})\bar{\mathcal{Y}}) \geq W(\bar{Y}(-)) + (1 - \Sigma)(2R^0 - R^- - R^+)a^* \)

These conditions constitute a floor at which compensation cannot fall beneath. How, and if this implies additional costs for the investor, will be addressed later in the thesis under section (4.5).

**Case 2: Lower bound of return is given by short selling strategy**

In case two, the highest possible return is defined by a max leverage strategy and the realization of \( R^+ \) while the lowest return is given by a short sale strategy and the realization of \( R^- \). Truth telling is always desired if \( 2\alpha(R^+ - R^0) \geq \)
2\alpha (R^+ - R^0)(1 - \Sigma). This is, for the same reasons as above, always true. Thus, we summarize the conditions for truth-telling in definition 3.

**Definition 3:** For \( \frac{R^+ + R^-}{2} > R^0 \), the following four truth-telling conditions must be satisfied in equilibrium.

- \( W(+) \geq W(Y(-)) + (1 - \Sigma)(R^+ - R^0)2a^* \)
- \( W(0) \geq W(Y(-)) + (1 - \Sigma)(R^+ - R^0)a^* \)
- \( W(\bar{\tau}) \geq W(Y(-)) + (1 - \Sigma)(R^+ - R^-)a^* \)
- \( W(-) \geq W(Y(-)) + (1 - \Sigma)(R^- - 2R^0 + R^+)a^* \)

For a good manager, only return \( \geq R^0 \) is relevant, while for the bad manager, these conditions must hold for all the support of \( Y \). One may think that this rationalizes the use of a linear sharing rule above a benchmark, but this would be wrong. In fact, this is simply a construct by the distribution. One thing worth considering however is the case where the distribution of returns is unbounded (e.g. if the risky asset \( R \sim N(\mu, \sigma) \)). In this case we can think of the relevant bound being such that the value of the fund is 0. We believe a more natural and intuitive explanation would be that the non-verifiable output assumption must be put into a context where the reporting of extreme portfolio returns would imply some sort of reputational effects on the market’s perception of the manager, and this would hurt his continuation utility. However, this is beyond the scope of this thesis and we only consider it as a possible extension in section 5.

### 4.4 Participation

Any contract proposed by the investor and accepted by the agent must provide expected utility to each of the players greater than their reservation utility. Let \( \Gamma_g \) and \( \Gamma_b \) be the reservation utilities of the good and bad manager respectfully, where \( \Gamma_g \geq \Gamma_b > 0 \). The investor has the outside option to invest on his own behalf in the risk-free asset, yielding the safe return \( R^0 \). Hence, the investor will never propose a contract which does not satisfy his participation constraint:

\[
E[V(\bar{\tau}) - W(\bar{\tau})] \geq R_0
\]  
(1.6)
Since the agents learn their type before signing the contract we have the following two participation constraints:

\[ E[W(\tilde{\theta} | \theta = G)] \geq \Gamma_g \]  
(1.7)

and

\[ E[W(\tilde{\theta} | \theta = B)] \geq \Gamma_b \]  
(1.8)

We see from these expressions that we implicitly assume that the manager rationally assumes truth-telling. The positive reservation utility of the bad manager deserves some explanation. We can think of this as managers having specific market knowledge and good managers are the selection of managers with specialized knowledge of the market under consideration, while bad managers are the homogenized pool of managers with expertise in different areas.

Moreover, these conditions imply some possible difficulties for the investor when designing the contract. First, if the difference in reservation utilities is large, a shutdown contract for the inefficient type may not be feasible. That is, all contracts satisfying the investor’s participation constraint (1.6) and the participation constraint of the efficient manager may also satisfy the participation constraint of the inefficient type. It may also be such that, due to the information asymmetry with respect to skill, the investor cannot promise the efficient type expected utility that satisfy his participation constraint. This is the classical adverse selection argument where the bad drive out the good. Each of these cases will be discussed in the next section.

4.5 Equilibrium outcomes

4.5.1 First best: Observable skill and observable cash flow

**Assumption 4:** The investor prefers to hire the efficient manager. That is we assume the surplus under efficient management exceeds that of inefficient given by the condition: \( E(Y_g | a = a_{inv}) - E(Y_b | a = a_{inv}) \geq \Gamma_g - \Gamma_b \) where \( a_{inv} \) is the optimal portfolio for the investor.
We first consider the benchmark case of full information. That is, the investor can observe both the return generated and the type of the manager. In this case there is no longer a tradeoff between rent extraction and efficiency. Only good managers are hired and bad managers are left with their reservation utility $\Gamma_g$. Hence, the investor can disregard the truth telling constraint (or set of constraints) and the participation constraint for the low type. He is then left with the following maximization problem:

$$
\max_{W(+), W(-), W(0)} E[Y - W(Y)]
$$

s.t.

$$
E[W(\tilde{Y} | \theta = G)] \geq \Gamma_g
$$

$$
a \in \arg\max_{a \in (-a^*, a^*)} E[W(Y, a)]
$$

and

$$
W(Y) \geq 0 \forall Y
$$

(1.9)

We assume that if the manager in optimum has a range of portfolios satisfying his maximization problem, he will just simply invest in the safest one. For instance, a compensation scheme where the investor pays the manager a fixed wage of $\Gamma_g$ binds the participation constraint, but it is neither optimal nor feasible as the expected return of the portfolio falls to $R^0$ and the investor is left with $R^0 - \Gamma_g < R^0$. In fact, the manager must receive a strictly positive return following both a successful short selling and leverage strategy. One possible optimal contract is summarized in proposition (4).

**Proposition 4:** Let the maximization problem of the investor be defined by (1.9), the following contract then efficiently implements first best for the investor:

- $W(Y) = \alpha \Gamma_g \ \forall Y \in \{Y(+), Y(\pm)\}$

$$W(+) = \alpha \Gamma_g + (1 - \alpha) \frac{\delta \Gamma_g}{p^+}, \ W(\pm) = \alpha \Gamma_g + (1 - \alpha) \frac{(1-\delta) \Gamma_g}{p^-} \text{ for } \alpha \in (0,1]$$

and

$$\delta \in \left\{ \left[ \frac{p^+}{p^-+p^+}, 1 \right] \text{ if } \max Y = (1 - a^*) R^0 + a^* R^+, \right. \left[ \frac{p^+}{p^-+p^+} \right] \text{ if } \max Y = (1 + a^*) R^0 - a^* R^- \right\}

**Proof:** See appendix A 4
An important assumption in proposition 4 is that the manager and investor can fully commit to the contract. Suppose this is not the case and setting $\alpha = 0$, then following a positive signal, the manager must be promised: $W(+) = \delta \Gamma_g / p^+ \Gamma_t$ and $W(\pm) = \delta \Gamma_g / p^- \Gamma_t$ with a bad signal. Suppose now that we are indeed in the situation where $\max Y = (1 - a^*)R^0 + a^*R^+$. For $\delta > p^+_t$, the investor who has all the bargaining power, reduces his promise to the agent, again leaving him with 0 rent. Conversely, if $\delta < p^+_t$, he must increase his promise to ensure participation by the agent. To describe the complete set of contracts implementing first best we use graphical argumentation

**Figure 8**: Feasible contracts implementing first best

First, notice that this graph assumes the case where maximum return is defined by positive leverage strategy. But the contracting problem where the maximum is given by a successful shorting strategy completely mirrors this, so it is sufficient to discuss the problem under one case. The only thing the investor needs to worry about in this first best scenario is to provide the manager with expected utility equal to his reservation utility and to give him incentives to choose the portfolio which maximizes expected return. Now, assume the manager has received a negative signal, according to proposition 3, the manager will chose the optimal portfolio for the investor if $W(Y(\pm)) > W(Y(j)) \forall j < \pm$. Thus, as we can see from figure 8, point C constitutes a conditional maximum.
Now consider the case where the manager has received a positive signal. The requirement for optimal portfolio choice is the same as for the negative signal (i.e. the maximum payment must be promised in the maximum return case). We immediately see that this gives the investor some flexibility when mapping the returns from point B -> A in the figure, as the only requirement is that they are strictly less than $\alpha \Gamma_g + (1 - \alpha) \delta \Gamma_g / P^+$. Also notice that this only constitutes a roof on the payments in this interval. In fact, the investor could reduce them to as low as $\alpha \Gamma_g$.

The payment given at point A is the roof of all transfers (we do not allow negative transfers). Hence, if the manager reduces this payment by increasing $\delta$, he must make sure this will not increase the payment in a successful short strategy by so much it becomes the unconditional maximum. The reason for this is that if $Y(+)\text{ is indeed the maximum following a leverage strategy, then the return} Y(\pm) \text{ must be feasible in an interior leverage strategy which is not optimal for the investor as shown in proposition 3.}$

Finally, the straight (curved) lines going from $R^0 \rightarrow Y(\pm)$ is an example of a linear and quadratic compensation scheme over the hurdle rate $R^0$, indicating that it should be possible for the investor to achieve first best with such a compensation structure.

### 4.5.2 Second best: Observable type, non-observable cash flow

As in the first best, only good managers are hired, but now the investor must also consider the truth telling condition (1.1) when designing the contract. Thus, the problem for the investor is:

\[
\max_{W(\pm), W(-\pm), W(0)} \mathbb{E}[Y - W(\bar{Y})]
\]

s.t. \( \mathbb{E}[W(\bar{Y}|\theta = G)] \geq \Gamma_g \)

\( a \in \arg\max_{a \in (-a^*, a^*)} \mathbb{E}[W(Y, a)] \) \hspace{1cm} (1.10)

\( \bar{Y} \in \arg\max \mathbb{E}[W(\bar{Y}) + (1 - \Sigma) \max\{0, Y - \bar{Y}\}] \)

and \( W(\bar{Y}) \geq 0 \ \forall \bar{Y} \)
In the first best case, the investor was able to extract all rent from the manager, but now this may no longer be the case. This is because, as discussed above, the truth telling condition implies a floor for the payments in all positive states (including the $R^0$ state).

**Proposition 5:** In the case where the investor can observe types but not cash flow, the following contract describe the equilibrium outcome

- **Case 1:** $R^+ + R^- / 2 < R^0 \quad W(\bar{Y}) = \Delta Q \forall \bar{Y} \in \{Y(+) , Y(\pm) , Y(0)\}$

  \[
  Y(\pm) = \Delta Q + (1 - \Sigma)(R^0 - R^-)2a^* \quad Y(+) = \Delta Q + (1 - \Sigma)(R^+ - R^-)a^* \quad Y(0) = (1 - \Sigma)(R^0 - R^-)a^* \quad \text{where } \Delta Q = \max\{0, \Gamma_g - E(W|Y_1)\}
  \]

- **Case 2:** $R^+ + R^- / 2 > R^0 \quad W(\bar{\tilde{Y}}) = \Delta Q \forall \bar{\tilde{Y}} \in \{Y(+) , Y(\pm) , Y(0)\}$

  \[
  Y(\pm) = \Delta Q + (1 - \Sigma)(R^+ - R^-)a^* \quad Y(+) = \Delta Q + (1 - \Sigma)(R^+ - R^0)2a^* \quad Y(0) = \Delta Q + (1 - \Sigma)(R^+ - R^0)a^* \quad \text{where } \Delta Q = \max\{0, \Gamma_g - E(W|Y_1)\}
  \]

**Proof:** See appendix A 5

For a more general discussion about the set of feasible optimal contracts, we again use graphical arguments. Figure 9 represents an illustration of contracts satisfying optimality.

**Figure 9: Feasible contracts when cash flow is unobservable**
As we can see from figure 9 implementing a second best contract is more difficult for the investor. He now faces floor payments on three cases instead of two as where the case before. Still, the requirements for optimal portfolio selection remain the same. As long as transfers in point A is larger than all other transfers and transfers in point B is larger than all transfers for reported Y less than $(1 + a^*)R^0 - a^*R^-$. In this figure, participation is ensured by varying $\Delta Q$, which can be interpreted as a fixed fee. Alternatively, the investor could just add a term to the three relevant states. Suppose he adds $a, b, c$ to the $+, \pm, 0$ states. He can then ensure participation if $P^+a + P^+c + P^-b = \Delta Q$. However, if the manager were risk averse, a fixed fee would dominate the latter strategy.

Since this is the implementation of a second best contract the more interesting case with respect to agency cost is that when $\Delta Q$ is equal to zero. The size of the agency cost will depend on the outside option for the high skilled manager relative to the fraction he can steal upon realizing return $Y$.

4.5.3 Second best: Hidden type, observable cash flow

Since the investor can no longer simply select the high type to manage his assets, but depending on the outside options of the managers, he may be able to screen the good manager by offering a contract that will be accepted by the productive high type, but rejected by the low type. However, if this is not the case, he must generally pay a rent to the low type to ensure participation of the high type as shown below. The participation constraint is binding in equilibrium, hence we can use proposition 4 to derive the conditions at which the investor is able screen the good managers.

**Theorem 1:** There exists a contract which is accepted by the high type and declined by the low type if the following condition is satisfied.

- When the optimal strategy for the bad type is given by a leverage strategy, the investor can reach first best when:
  
  \[
  \Gamma_g < \Gamma_b \left(1 + \frac{p^-}{p^+}\right) \quad \text{for} \quad \frac{p^-}{p^+} < 1 \\
  \Gamma_g < 2\Gamma_b \quad \text{otherwise}
  \]
• *When the optimal strategy for the bad type is given by a short-sale strategy, the investor can reach first best when:*

\[
\Gamma_g < \Gamma_b \left(1 + \frac{p^+}{p^-}\right) \quad \text{for} \quad \frac{p^+}{p^-} < 1 \\
\Gamma_g < 2\Gamma_b \quad \text{otherwise}
\]

**Proof:** See appendix A 6

Comparing these results to the proposition given in Stremme (2001), we see that allowing for short selling and leverage strengthen the investors position in screening the good agents. The reason is, he can now make some of his promises to the good agent contingent on the state impossible for the bad agent to reach. We call this state the separation state for further reference. That is, in these states he must only consider the roof of payments given by the optimal portfolio conditions.

The intuition behind Theorem 1 is that maximum utility the investor can promise the good agent and still satisfy optimal portfolio selection in any case is twice that of a bad manager. If, however the distribution is such that the probability of reaching the desired state for the bad manager (for example the $R^+$ state with a leverage strategy) is large relative to the undesired state, the screening condition tightens as he is forced to reduce payment in the attractive state (and thus also in the separation state).

To further explain the accepted range on the good agents reservation that allows for separation we refer to figure 10\(^{29}\). We see that if $\Gamma_b$ lies in the triangle A, the investor can potentially construct a contract that is accepted by the good manager and rejected by the bad. Once, $P^+$ approaches $P^-$ however, the condition reaches its maximum and stays there for all $P^+ > P^-$.  

\(^{29}\) This is only for when it is optimal for the bad manager to employ the leverage strategy. Since the case when he selects the short sale strategy completely mirrors this, we only included it in the appendix
Theorem 1 implies that since the condition is either regulated by the optimal portfolio condition or the strategy constraint for the bad guy, there is always a possibility of shutdown of the least efficient type as long as \( \Gamma_g < 2\Gamma_b \). The reason is simply that if the distribution and wage function is defined such that this is not true, then it is possible to change the wage function in a way so it is optimal for the bad manager to change his strategy (or potentially shutting him down) without compromising the participation of the efficient type.

4.5.4 **Unobservable types and cash-flow when managers can delegate active management.**

In this part we present a version of the model which includes both unobservable cash flow and hidden type. There is a continuum of agents normalized to 1. The probability of randomly selecting a good manager is given by \( P(\theta = G) = \Pi \) and for the low guy \( (\theta = B) = 1 - \Pi \). As before the high guy possesses an informative advantage relative to the low skilled manager in for of the signal \( S_i \), but all managers, regardless of type is more informed than the investor. To clarify, all managers observe both their own type and all other types in the market, as well as returns generated by other managers. This means that a potentially viable strategy for the bad agent may be to mimic the behavior of the investor and simply reinvest with a good manager. We assume that the minimum cost to ensure truth telling also implies that the participation constraint for the high guy is slack (i.e. the investor must pay a rent to the high type).
**Proposition 6:** When the investor cannot observe cash-flow, there exist a contract that will be actively managed by the high guy, but not by the low guy if the following conditions hold:

\[ \Gamma_g < (1 + 1/P^+) \Gamma_b + M(a^*) \]

where \( M(a^*) = \frac{a^*(1-\Sigma)}{P^+} ((R_0 - R^-)(P^+ - P^-) - (R^+ - R^0)) \)

**Proof:** See appendix A7

We see that \( M(a^*) \) is generally decreasing in \( a^* \). The reason is that with hidden cash-flow, the bad agent will be promised positive transfers in states \( Y(0) \) and \( Y(-) \). Thus, the wage at \( W(+) \) must be reduced accordingly to avoid the bad type signing the contract. Obviously now the requirements are no longer just on the difference in reservation utilities between the managers, but also on the reservation utility of the bad manager relative to the fraction he can steal. That is, binding all truth telling conditions may automatically imply participation.

Furthermore, this result is given as a proposition and not a theorem as in section 4.5.3. The reason is that the investor could propose a contract where the wage for the low guy won’t satisfy truth-telling in the separation state (i.e., he could set \( W(j) = 0 \ \forall j \leq - \)). We now see that the only condition for screening is the wage to the efficient type in the separation state. But this is the same problem as in section 4.5.3 with the solution \( \Gamma_B < 2 \Gamma_G \). We can consider out of equilibrium behavior by assuming the good manager reports the largest return \( Y(-) \). He is then paid 0, but inefficiently steals \( (1 - \Sigma)(Y(+) - Y(-)) \). But, by truthfully
reporting \( Y(+) \) he receives at least \( (1 - \Sigma)(Y(+) - Y(-)) \) which is always larger. Hence, if \( \Gamma_B < 2\Gamma_G \), there will exist a contract accepted only by efficient managers and it will satisfy truth telling for all realistic reported return).

But what if we relax the assumption that \( P^+ = P^- \)? In this case we could be in a situation where \( P^+/P^- > Y(+) \), but \( Y(+) > Y(+) \). This is not unreasonable and can be true for many situations where extreme losses are huge in absolute value relative to extreme gains, but less likely to happen\(^{30}\). Now, to ensure that the good types don’t select an interior portfolio (see proposition 3), the investor must promise higher transfers following a reported \( Y(+) \) than \( Y(+) \). But now, the wage \( W(+) \) is no longer constrained by the participation condition of the bad manager. The bad manager will accept all contracts satisfying truth-telling if the following condition holds:

\[
P^+(R^+ - R^-) + (1 - P^-)(R^0 - R^-) > \frac{\Gamma_B}{a^*(1 - \Sigma)} \tag{1.11}
\]

Assume now that (1.11) does not hold. The investor could then offer a contract where he rewards the good manager for employing a successful shorting strategy, and this bonus will not be bounded upwards by optimal portfolio selection criteria, but rather the strategy implementation criteria of the bad manager (see proposition 1). Hence, if the probability of the catastrophe case is sufficiently small relative to the probability of positive excessive return over the risk free rate, the investor could take advantage of this rare state when designing the contract. This shows the dilemma for a potential regulator with respect to the restriction on short selling \( a^* \). He cannot increase \( a^* \) such that (1.1) hold, but by increasing it marginally below, he also increases the potential extra utility the investor can promise the good agent relative to the bad agent.

Our next step is to consider the value of mimicking the investor for the bad agent. If he chooses to do so, he can take advantage of the information advantage he has over the uniformed investor to reduce the rent given to the good manager. For the investor on the other hand he will pay for the participation of the good manager and truth telling for the bad one.

\(^{30}\) For instance, if \( Y(+) \) represent a chatostrophy state.
**Proposition 7:** Both the investor and the bad manager prefers delegation when the following conditions holds:

\[
E[Y^{FB} - Y^{SB}] > \Gamma_g \\
E[Y^{FB} - \Gamma_g - Y^{SB}] \Sigma > 0
\]

**Proof:** See appendix A8

From assumption 4, Proposition 7 indirectly implies that delegation is always preferable. This is also an efficient outcome as the capital is now managed by the more productive high type. But will this state always be reached? We first use proposition 3 to obtain explicit results for both a leverage and a short-sale strategy for the bad manager.

\[
\Gamma_b < \begin{cases} 
    P^+\left(Y(+) - Y(-)ight) & \text{For a short - sale strategy} \\
    P^-\left(Y(\bar{y}) - Y(-)ight) & \text{For a leverage strategy} 
\end{cases} 
\] (1.12)

Until now we have assumed that the bad managers have perfect information about the asset management industry. We now relax this assumption and state that he believes the probability of selecting a good manager is \( \Pi^B > \Pi \). In real terms we can view this as the bad managers have through his experience gained knowledge of some other bad managers and are able to screen them out. We also relax the assumption that bad managers can observe returns. Hence, he must also consider optimal truth telling when designing the contract. So for a bad manager, the expected wage he receives when managing the portfolio himself is, as before:

\[
E(W|\theta = b) = E\left[(Y^{SB} - \bar{Y})(1 - \Sigma)\right]
\]

Before proceeding we need to define some important informational restrictions in the model.

**Assumption 5** Bad manager are totally homogenous with respect to the information they have about the market. That is, all bad managers believe the probability of randomly selecting a good manager from the pool of managers is equal to \( \Pi^B \)
Although assumption 5 is a strict assumption it is important to the tractability of the model and it does not compromise the validity or reliability of our results. The important thing is that each step of delegation is costly to the investor.

If the bad manager delegates the portfolio, there is a probability that he will get unlucky and select a bad manager, if he does, the selected manager will simply delegate the management to another manager and this keeps going until a good manager has been selected. For each new delegation, the new manager must be promised utility such that he has the incentive to tell the truth. The next theorem describes the conditions for when delegation is optimal for a bad manager:

**Theorem 2:** Let $W^+$ be the wage that ensures optimal truth telling from a good manager. Then, a bad manager will delegate the management of the portfolio if the following condition holds:

$$K > E[Y^{SB}] \text{ where } K = y^{FB} - W^+ - \frac{(1 - \Pi)\Sigma W^+}{(1 - (1 - \Pi)\Sigma)^2}$$

**Proof:** See Appendix A9

Theorem 2 states that the manager will delegate the active management as long as expected return from a good manager is sufficiently large such that it at least covers the expected additional cost resulting from potential several steps of delegation. To obtain further insight into the role of $K$, we employ some comparative statics.

**Comparative statics**

$$\frac{\partial K}{\partial \Pi^B} = \frac{\Pi^B \Sigma W^+}{(1 - (1 - \Pi^B)\Sigma)^2} + \frac{2(1 - \Pi^B)\Sigma^2 W^+}{(1 - (1 - \Pi^B)\Sigma)^3} > 0$$

$$\frac{\partial K}{\partial \Sigma} = -\frac{(1 - \Pi^B)W^+}{(1 - (1 - \Pi^B)\Sigma)^2} + \frac{2\Sigma(1 - \Pi^B)W^+(\Pi^B - 1)}{(1 - (1 - \Pi^B)\Sigma)^3} < 0 \quad (1.13)$$

We see that $K$ is increasing in the probability of finding a good manager and decreasing in the potential stolen share of the profit. This is intuitive since a greater probability of finding a good manager reduces the number of payments being made to different managers, and an increase in the potential stolen
amount reduces \( K \) as for a given probability \( \Pi^B \) investors (both the true and mimicking investors) must promise their agents a larger amount of the return.

\[
\lim_{\Pi^B \to 1} K = E(Y^{FB}) - W^+
\]

\[
\lim_{\Pi^B \to 0} K = E(Y^{FB}) - W^+ - \frac{\Sigma W^+}{(1 - \Sigma)^2}
\]

(1.14)

From (1.14) we can confirm that as the searching skill of a bad manager approaches perfection, the return generated by him approaches second best. Moreover, as the probability of finding a good manager approaches zero, the bad manager must pay an expected fee which is the sum of an infinite payments to other delegated manager. To see this, consider the payments being made in delegation \( n \). This must be:

\[
\sum_{j=1}^{\infty} \Sigma^j W^+ + \sum_{j=2}^{\infty} \Sigma^j W^+ + \cdots + \sum_{j=\infty}^{\infty} \Sigma^j W^+ = \frac{\Sigma W^+}{1 - \Sigma} + \frac{\Sigma^2 W^+}{1 - \Sigma} + \cdots + \frac{\Sigma^\infty W^+}{1 - \Sigma}
\]

\[
= \frac{\Sigma W^+}{(1 - \Sigma)^2}
\]

\[
\lim_{\Sigma \to 1} K = Y^{FB}
\]

\[
\lim_{\Sigma \to 0} K = Y
\]

(1.15)

Naturally, if the fraction of potential stolen return goes to zero, return goes to first best. But, it is important to see that we have implicitly assumed that satisfying truth-telling also ensures participation. We see that in this case, this is a very unrealistic assumption. More likely, the limit will rather be a function of the reservation utilities of the bad managers. The last condition is more intuitive. It says that when manager can steal almost all the return, gross return will be driven to the lowest possible reported return.

**Ownership structure**

Suppose now that theorem 2 holds. In this case a bad manager can do better by delegating the management of the assets under his control than to actively manage them himself. If we now relax the assumption of zero initial wealth of the manager and assume he will seek active management on a fraction of this
initial wealth throughout the period (the rest is left for continuous consumption etc). A good manager will then always manage his own money. If he did not, then there must be some manager in the market with greater alpha generating ability. A bad manager can potentially invest in his own fund to mimic the good manager. Thus, for ownership to be a credible signal, the following condition must hold:

$$E[(1 - \xi)(K - Y^{SB})] \geq E(W|\theta_B)$$

$\xi$ is the fraction of wealth put away for consumption when total initial wealth is normalized to 1. Thus, when the difference in skill is sufficiently large or the bad managers are highly skilled in selecting good managers, ownership can be used as a credible signal. In this case only good managers will be left in the market, but the bad managers still play an important role. They will ensure that joint ownership is maintained in equilibrium as if this is relaxed, it will once again be profitable for the bad managers to enter the market.

A potential problem is if the bad managers are heterogeneous in their ability to screen the market for bad manager (i.e $\Pi \in (\Pi, \bar{\Pi})$). Now delegation may be optimal for some agents, while self-active management for others. However, this is beyond the scope of this thesis.

5. Conclusions: Implications and limitations

Our analysis is consistent with the observed industry practice where active managers potentially can receive large fees for their services despite the stylized fact that they generally underperform their passive benchmark. We argue that one potential reason for this could be due to agency costs resulting from asymmetric information between the investor and the managers. Moreover, we show that it may not always be feasible for the investor to construct a shutdown contract of the inefficient type if difference in outside options between types is too great. This could be argued is due to sector specific knowledge dividing the managers.

Our analysis provides some rationale for the existence of financial advisers in the asset management market. We show that when manager have additional
knowledge about the distribution of types, low skilled manager may have an incentive of simply mimicking the investor and delegate the active management to another manager. Moreover, in social terms this would be efficient as the capital available for management is scarce and high skilled managers generate in expectations larger return than a poor skilled manager.

We also make an argument for the allowance of trading strategies such as short selling or use of leverage in the case where there is a small probability for catastrophic losses. Bad managers will to a less degree be able to receive ex-ante information about the asset(s) currently trading in the market and hence they will benefit less from employing these conditional strategies. Thus, by increasing the bound of short-selling/leverage, the condition for screening managers relaxes. However, in general the effects from varying short-selling restrictions are somewhat ambiguous and very dependent on the distribution. If for instance, a leverage strategy is dominating for the bad manager both in terms of expected utility and in potential maximum payout, increasing short-selling makes the screening problem more difficult. The reason is the upward bound in the state where the good manager is able to differentiate himself is restricted by the participation condition of the bad guy while the lower bound of the transfers in the “bad” state (e.g. $Y(-)$) will be increasing in the fraction the manager can steal. Since only the difference in these two wages matter when comparing the reservation utilities of the managers with respect to shutdown of the inefficient type, short-selling actually makes a first best implementation more difficult.

Nevertheless, our simple technology and assumptions leads to several points where the model could potentially be enriched to achieve more realistic predictions. Adding more assets available for investment could potentially increase the value of the signal received by the manager. In this case, he can then employ a more sophisticated strategy conditioned on several signals.

Another possibility is to make the managers (and potentially also the principal) risk averse. This would add to the realism of the model, but make it significantly less tractable. It will no longer be optimal for the manager to take on an extreme

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31 First best with respect to only efficient managers are employed
position in his portfolio selection, but rather an interior solution conditioned on the signal he receives (or assumes), his degree of risk aversion and on the mechanism design. The problem for the investor is that it is no longer simply a question of efficiency, but also optimal risk sharing. That is, he must give up rent to the risk averse manager to make sure he makes the efficient choices. A natural consequence of this would be to relax the assumption of hidden cash-flow and introduce a more traditional moral hazard interpretation (see Stoughton 2003).

In our model, we only have two types of managers in the market, and each type is homogenous with respect to market knowledge. One could potentially increase the number of types or make them continuous over some interval. If this was the case then unique knowledge about the distribution of types would also be implied. Another possible implication of adding more types is the possibility of an equilibrium where some of the bad managers works as a financial advisor and the rest actively manages both investments and own wealth.

We could also transform the model from a static to a dynamic model and include learning. In this case the investor could take advantage of the dynamic nature of return and make the bonus conditional on the history of return. In this respect, this would also allow us to analyze potential career concerns and reputational effects in the market. One possibility is to include decreasing returns to scale and in a market where investors have limited information, attempt to find an equilibrium which is comparable to the stylized fact that returns is generally inconsistent.
6. References


7. Appendices

A1 Proof of proposition 1

Suppose a high skilled manager has observed a positive signal, the expected return given his portfolio weights is then:

\[ E_{S^+}[Y(a)] = p^+ a(R^+ - R^0) + R^0 \]  \hspace{1cm} (1.16)

Where \( E_{S^+} \) is the expectations operator, conditioned on observing the signal \( S^+ \).

The manager has the following maximization problem

\[ \max_a E_{S^+}[W(Y,a)] \]

s.t.

\[ a^* \geq a \geq -a^* \]  \hspace{1cm} (1.17)

Let \( Y^*(a) \) and \( W^*(a) \) be the maximum expected value of \( Y \) parameterized by the portfolio weight \( a \). We see from (1.17) that:

\[ \frac{\partial E_{S^+}[Y(a)]}{\partial a} > 0 \]  \hspace{1cm} (1.18)

From this we see, by employing the envelope theorem that:

\[ \frac{dE_{S^+}W^*(a)}{da} = \frac{\partial E_{S^+}W(Y,a)}{\partial a} \bigg|_{Y=Y^*(a)} > 0 \]  \hspace{1cm} (1.19)

Hence \( E_{S^+}W^*(a) \) is maximized at \( a^* \). Conversely, if he observes a bad signal the expected conditional return is:

\[ E_{S^-}[Y(a)] = p^- a(R^- - R^0) + R^0 \]  \hspace{1cm} (1.20)

And we have:

\[ \max_a E_{S^-}[W(Y,a)] \]

s.t.

\[ a^* \geq a \geq -a^* \]  \hspace{1cm} (1.21)

Using the same notation, we have that:
And
\[
\frac{\partial E_s - Y(a)}{\partial a} < 0 \quad (1.22)
\]

And
\[
\frac{dE_s - W^*(a)}{da} = \frac{\partial E_s - W(Y, a)}{\partial a} \bigg|_{y = y^*(a)} < 0 \quad (1.23)
\]

We see that the manager reduces his exposure to the risky asset and invests all funds in the risk-free asset without leverage, thus proving the first part of the proposition.

For the second part, consider the bad agent. He does not receive any signal and must thus only rely on the prior probabilities. We first denote the unconditional probabilities of \( R^+, R^- \) and \( R^0 \) as \( P^+, P^- \) and \( P^0 \) respectfully.

The bad manager has two possible optimal strategies:

**Strategy 1, mimic the leverage strategy**: In this strategy the bad manager assumes the realization of \( S^+ \) and act according to this. Thus, he will implement the portfolio \((1 - a^*, a^*)\).

**Strategy 2, mimic the short-sale strategy**: In this strategy the bad manager assumes the realization of \( S^- \) and act according to this, implementing the portfolio \((1 + a^*, -a^*)\).

We now attempt to find the conditions at which these strategies are optimal. Without loss of generality, we can assume that the negative state at each strategy gives the wage 0 (this only means that the lowest possible return is not yielding a bonus for the agent). Thus, we can write the expected utility following each of these strategies as

\[
E(W) = P^+W(+) + P^0W(0) \quad \text{For the leverage strategy}
\]

\[
E(W) = P^0W(0) + P^-W(-) \quad \text{For the shorting strategy}
\]

Calculating the difference between these two expectations gives the condition for optimality and we see that it is optimal to employ a leverage strategy if:
From assumption 1, we see that this implies that he will employ the strategy which gives the highest wage in the high return state.

**A2 Proof of proposition 2**

We subtract $E_{q=a} [Y]$ from $E_{q=b} [Y]$ and get:

$$p^+ p^- ( (1 + a^*) R^0 - a^* R^- - (1 - a^*) R^0 - a^* R^- )$$

Simplifying and we get the condition showing this must be positive and increasing in $a^*$:

$$2 a^* P^- ( R^0 - R^- ) > 0$$

(1.25)

Moreover, if the bad manager selects the short sale strategy, we get a similar result:

$$2 a^* P^+ ( R^+ - R^0 ) > 0$$

(1.26)

**A3 Proof of proposition 3**

We want to show that if this condition does not hold, the manager will select an interior solution to the portfolio problem and not maximize expected return.

Suppose a skilled manager is hired and maximum return is given by a positive leverage strategy. That is, $Y(+) > Y(\bar{+}) > R^0$. Now let transfers in these three states be defined by $W(-) > W(+) > W(0) > 0$. Furthermore, suppose that the manager has observed a positive signal. The portfolio weight that maximizes expected return is then $a^*$. We must have the following expression for a successful implementation of a leverage strategy:

$$Y(+) = (1 - a^*) + a^* R^+$$

And then return under shorting is
We must show that there is an interior solution to \( a \) which gives the portfolio return \( Y^- \) under the assumption of a realization of \( R^+ \). Hence we set:

\[
(1 - a)R^0 + aR^+ = (1 + a^*)R^0 - a^*R^-
\]

Rearranging

\[
a = \frac{R^0 - R^-}{R^+ - R^0} a^*
\]

Since the condition that a maximum leverage strategy gives the potential highest return is:

\[
R^0 < \frac{R^+ + R^-}{2}
\]

This condition implies that

\[
\frac{R^+ - R^0}{R^0 - R^-} < 1
\]

Thus showing duplication requires an interior solution.

Finally, this is sufficient since varying \( a^* \) will not influence probability of receiving \( W(0) \)

---

**A4 Proof of proposition 4**

Let \( p^+p^+_r = P^+ \) and \( p^-p^-_r = P^- \). Since the optimal response to any signal for the investor is to adopt an extreme short or leverage strategy, the optimal contract must provide the agent with the incentives of choosing this over any other weights. The investor must try to ensure that the agent never chooses to invest in the safe asset without shorting the risky asset, so without loss of generality we assume he rationally sets \( W(0) = 0 \). This, along with the result that all interior solutions are suboptimal and binding the participation constraint implies the following expression for expected utility for the manager.

\[
P^+W(+) + P^-W(+) = \Gamma_g
\]
To satisfy the incentive constraint w.r.t portfolio selection, it is sufficient to promise a positive utility in the desired states. Hence, it is satisfied for \( W(+) > 0 \) and \( W(-) > 0 \). For any value of \( W \), they must satisfy the following limits:

\[
\lim_{W(-) \to 0} W(+) = \frac{\Gamma_g}{p^+}
\]

\[
\lim_{W(+) \to 0} W(+) = \frac{\Gamma_g}{p^-}
\]

To find all values of these transfers that are optimal for the investor, we introduce the parameter \( \delta \) to represent the relative weight on the leverage strategy and see this satisfy the participation constraint for \( \delta \in [0,1] \).

\[
\frac{p^+ \Gamma_g \delta}{p^+} + \frac{p^- \Gamma_g (1 - \delta)}{p^-} = \Gamma_g
\]

This shows that the investor can vary the payments in the desired states to satisfy both the participation and the portfolio selection constraint. But, the only requirement ensuring optimal portfolio selection is that the desired states yield strictly more transfers than any other state. Hence it can also be optimal to pay a small bonus when reaching the desired states and vary the fixed fee such that participation is ensured. If we let \( \alpha \) be the weight put on the fixed part of the compensation we end up with the expression in the proposition. We also see that the participation constraint is binding:

\[
\alpha \Gamma_g + (1 - \alpha) \left[ \frac{p^+ \Gamma_g \delta}{p^+} + \frac{p^- \Gamma_g (1 - \delta)}{p^-} \right] = \Gamma_g
\]

Finally, we attempt to identify restrictions on \( \delta \). Assume now that \( (1 - a^+) R^0 + a^+ R^+ \) represent the upper bound of portfolio return. According to proposition 3 this implies that optimal portfolio selection requires that transfers at this node is strictly larger than any other possible realization. Since this represents a successful leverage strategy, we can write:

\[
\frac{\delta \Gamma_g}{p^+} > \frac{(1 - \delta) \Gamma_g}{p^-}
\]

Rewriting this gives:
\[ \delta > \frac{p^+}{p^- + p^+} \]

Since the opposite must be true if short-selling gives the potential highest return, this condition will simply reverse.

A5 Proof of proposition 5

We can no longer assume a binding participation constraint, but optimality requires that we set the payment in the lowest possible return to 0, hence

\[ W(Y(-)) = 0 \]

With this condition we can check that binding the truth telling constraints solves the max problem for the investor. First they must be increasing in return as was assumed under optimal portfolio. The condition is that

\[ W(Y(\bar{y})) > W(Y(+)) > W(0) > W(Y(-)) \]

It is easily verifiable that this must hold considering the payments are made as a markup on the potential stolen returns for the agent. If the truth telling conditions are to bind will depend on the size of the manager’s reservation utility. Let the compensation scheme under binding truth telling conditions be \( Y \). If we define the difference in expected utilities gained for the agent under \( Y \) and \( \Gamma_g \) as: \( \Delta Q = \Gamma_g - E(W|Y) \). We see that, as in the first best case, a fixed transfer can be added to the contract to ensure participation.

Proposition 4 implies that if \( \Delta Q \) is positive, the first best will be implemented and the manager is left with an expected utility equal to his reservation utility. However, if this is strictly negative, the investor must promise the manager rent to ensure that he reports truthfully.
A6 Proof of theorem 1

We attempt to show the condition at which it is possible to construct a contract that will be accepted by the good agent, but disregarded by the poor. Stremme (2001 theorem 5.2 p26) shows that in a dynamic setting and under the assumption of no short selling or leverage, this condition boils down to a simple constraint on the reservation utilities $\Gamma_g < (1 + p^-) \Gamma_b$. Simple calculations show that this also holds under the static situation. Our situation changes these conditions as we will show, giving the investor a unique state of which he can satisfy the participation of the high guy.

First we make some important observations. By reducing the payment promised in the 0 state, the investor gains flexibility when binding the participation constraint of the high type. To see this, consider the case when $W(0) \neq 0$. For the low type, this implies that $W(+) < (\Gamma_b - P^0 W(0))/P^+$. We know that for any $W(\pm)$ we must have that $W(+) > W(\pm)$ to ensure optimal portfolio selection. Thus, the investor wants to reduce $W(0)$ as much as possible.

**Case 1: Mimicking the leverage strategy**

First, for the good managers to accept and the bad to decline contract the following two conditions must hold:

\[
P^+ W(+) + (1 - P^+) W(0) - P^- W(0) + P^- W(\mp) \geq \Gamma_g \tag{1.1}
\]

\[
P^+ W(+) + (1 - P^+) W(0) - P^- W(0) < \Gamma_b \tag{1.2}
\]

We then have

\[
P^+ W(+) + (1 - P^+) W(0) - P^- W(0) \geq \Gamma_g - P^- W(\mp) < \Gamma_b \tag{1.3}
\]

Next, we must check the conditions on $W(\mp)$ that implies participation only for the high type. Suppose optimal portfolio selection is satisfied. In proposition 3 we show that only the largest possible return matters when defining a floor of payments. That is, if optimal leverage constitutes the strategy with the potential
highest return, then transfers in any other state cannot be larger than this. Hence, we have the wage at \( Y(\pm) \) must satisfy two conditions:

\[
W(\pm) < \begin{cases} 
W(+) & \text{if } P^+ \geq P^- \\
\frac{P^+ W(+) + P^- W(-)}{P^-} & \text{if } P^+ < P^- 
\end{cases} 
\quad (1.4)
\]

These are the conditions for optimal portfolio selection and leverage strategy for the low type. Let’s now consider the case where \( P^+ \geq P^- \). Then we can rewrite (1.11) to:

\[
\Gamma_g < \Gamma_b + P^- W(\mp) < \Gamma_b \left( 1 + \frac{P^-}{P^+} \right)
\]

Hence, the condition for screening is:

\[
\Gamma_g < \Gamma_b \left( 1 + \frac{P^-}{P^+} \right) 
\quad (1.5)
\]

Now, consider the case where \( P^+ < P^- \). Now 1.11 instead writes to:

\[
\Gamma_g < \Gamma_b + P^- W(\mp) < 2\Gamma_b
\]

And we end up with the condition:

\[
\Gamma_g < 2\Gamma_b 
\quad (1.6)
\]

**Case 2: Mimicking the short sale strategy**

We now attempt to verify that these conditions also hold when the bad manager commits to a short sale strategy. Using the same calculations as in the first case we rewrite (1.11) to:

\[
P^- W(\pm) + (1 - P^+) W(0) - P^- W(0) \\
\geq \Gamma_g - P^+ W(+) < \Gamma_b
\]

Similarly, (??) becomes:
We then end up with the conditions:

\[ W(+) < \begin{cases} W(\pm) & \text{if } P^- \geq P^+ \\ \frac{P^- W(+)}{P^+} & \text{if } P^- < P^+ \end{cases} \quad (1.8) \]

We then end up with the conditions:

\[ \Gamma_g < 2\Gamma_b \quad \text{for } P^- < P^+ \]
\[ \Gamma_g < \Gamma_b \left( 1 + \frac{P^+}{P^-} \right) \quad \text{for } P^- \geq P^+ \quad (1.9) \]

**A7 Proof of Proposition 6**

Conversely to the case where returns were observable, all returns in the support except the lower bound must be strictly positive. This means we can write the condition for non-active management for the bad agent as:

\[ P^+ W(+) + (1 - P^+)W(0) + P^- W(-) - W(0) < \Gamma_b \quad (1.1) \]

And the participation condition for the high type is:

\[ P^+ W(+) + (1 - P^+)W(0) + P^- W(\mp) - W(0) \geq \Gamma_g \quad (1.2) \]

This in turn implies that:

\[ \Gamma_b - P^- W(-) > \Gamma_g - P^- W(\mp) \]

Rewriting:

\[ \Delta \Gamma < P^- (W(\mp) - W(-)) \quad \text{where } \Delta \Gamma = \Gamma_g - \Gamma_b \quad (1.3) \]

Now we can solve for the monetary value the signal has for the high guy using the minimum payments required to ensure truthful reporting remembering that the investor could potentially push \( W(\mp) \) as high as marginally below \( W(+) \), but this will again be pinned down by the participation constraint of the low type. To characterize the feasible space we can first use definition 3 and bind truth telling to the states \( W(-) \) and \( W(0) \). This and the participation constraint for the low type imply:
Moreover we have

\[ \text{Sup} W(+) - W(-) = \frac{\Gamma_b}{p^+} + \frac{a^*(1 - \Sigma)}{p^+} \left( (R^0 - R^-)(p^+ + p^-) - (R^+ - R^0) \right) \]  

(1.5)

Since \( \max W(+) < \text{Sup} W(+) - W(-) \) we can combine (1.3) and (1.5) and have our result.

\[ W(+) < \frac{\Gamma_b - (1 - p^+)(1 - \Sigma)(R^+ - R^0)a^*}{p^+} + \frac{p^-(1 - \Sigma)(R^0 - R^-)a^*}{p^+} \]  

(1.4)

A8 Proof of Proposition 7

Denote \( E[Y^{FB}] \) the expected gross portfolio return under first best management and \( E[Y^{SB}] \) gross portfolio return under management by a bad manager. Then for any strategy, the bad managers return with delegation is \( E[Y^{FB}] - \Gamma_g \). Hence, the investor is left with: \( E[Y^{FB}] - (1 - \Sigma)(E[Y^{FB}] - \Gamma_g - Y) - \Gamma_g = E[\Sigma(Y^{FB} - \Gamma_g)] + (1 - \Sigma)Y \). For active management on the other hand, the investor is left with: \( E[Y^{SB}] - (1 - \Sigma)(E[Y^{SB}] - Y) \). Taking the difference between these two expressions we get: \( \Sigma \left( Y^{FB} - (Y^{SB} + \Gamma_g) \right) \). Rewriting and we have our result.

\[ W^+ = E\left[ (Y^{FB} - Y)(1 - \Sigma) \right]. \]

Also let \( W^j \) be the payment to the jth manager. We can write the expected return generated by the first manager as:

\[ W^1 = \frac{(1 - \Sigma)(R^+ - R^0)a^*}{p^+} \]

A9 Proof of Theorem 2

Proof: First we notice that \( W^+ = E\left[ (Y^{FB} - Y)(1 - \Sigma) \right]\). Also let \( W^j \) be the payment to the jth manager. We can write the expected return generated by the first manager as:
Next, we attempt to find expressions for all $W$. Consider a bad manager who has delegated his portfolio to another bad manager who then reaches a good manager. First the good manager needs to be compensated leaving $E(Y^{FB}) - W^+$, then the second bad manager needs to be compensated: $(E(Y^{FB}) - W^+ - Y)(1 - \Sigma)$. But this is the same as $W^+ - (W^+)(1 - \Sigma) = \Sigma W^+$

If there was another manager also needed compensation this would simply be:

$$E(Y^{FB} - W^+ - \Sigma W^+ - Y)(1 - \Sigma) = \Sigma^2 W^+$$

And another

$$E(Y^{FB} - W^+ - \Sigma W^+ - \Sigma^2 W^+ - Y)(1 - \Sigma) = \Sigma^3 W^+$$

We can now rewrite the series to:

$$\Pi^B(E(Y^{FB}) - W^+) + (1 - \Pi^B) \left( \Pi^B(E(Y^{FB}) - W^+ - \Sigma W^+) \right)$$

$$+ (1 - \Pi^B) \left( \Pi^B(E(Y^{FB}) - W^+ - \Sigma W^+ - \Sigma^2 W^+) + \cdots + (1 - \Pi^B)(\Pi^B(E(Y^{FB}) - W^+ - \sum_{j=1}^{n} \Sigma^j W^+) \right)$$

We notice that $W^+$ must be paid with certainty, hence we can take $E(Y^{FB}) - W^+$ out of the series. Moreover, the other part can be written as:
Using rules for geometric series, we see that each of these terms can be written as geometrical series. Thus we can write this as:

\[-\left( \sum_{j=1}^{\infty} (1 - \Pi^B)^j \Sigma^j W^+ + \sum_{j=2}^{\infty} (1 - \Pi^B)^j \Sigma^j W^+ + \cdots + \sum_{j=\infty}^{\infty} (1 - \Pi^B)^j \Sigma^j W^+ \right)\]

We see this can be written:

\[-\frac{(1 - \Pi^B) \Sigma W^+}{1 - (1 - \Pi^B) \Sigma} \left( 1 + (1 - \Pi^B) \Sigma + (1 - \Pi^B) \Sigma^2 + \cdots \right)\]

Using the rules for a geometric series once more we arrive at our \( K \), that is

\[ K = E(Y^{FB}) - W^+ - \frac{(1 - \Pi^B) \Sigma W^+}{(1 - (1 - \Pi^B) \Sigma)^2} \]

For the rest of the theorem we subtract the expected utility when delegating from expected utility from self-managing and find our result

\[ K > E[Y^{SB}] \]

\[ \square \]

A 10 Preliminary Thesis Report

1.0 Introduction

In economics we are often concerned with how information asymmetries will affect the outcome of some fiduciary relationship between two or more parties. If the objects of all parties were the same, there would be no inefficiencies and no formalization of the agreement will be needed. However, this is often not the case. A worker will have objectives that do not correspond to the objectives of the company, a manager of a mutual fund will want to maximize his own payment or an insured agent may take action he would not otherwise take if not protected. It is not difficult to see that examples like this could be found in all aspects of the economy. If the objectives between the players differ, the principal could propose a contract which would protect his interest such that the
agent is given the incentives to take actions the principal would have done in the absence of the relationship (Laffont and Martimort 2001). Contract theory is a field of economics which is concerned with the design of such incentive compatible contract under various information structures (Bolton and Dewatripont 2005). A contract is an agreement between parties (individuals, businesses, organizations or government agencies) which may or may not be legally binding and involves a promise to do something in return for a valuable benefit (exchange of promises). The legal nature of the ‘Contract’ (‘Contract law’) is derived from the principle expressed in the Latin phrase pacta sunt servanda, which literally means "pacts must be kept". Moreover, a contract specifies the terms and conditions under each parties commitment.

In contract theory two important cases of information asymmetry are described. The first case is referred as ‘Adverse Selection’, where because some parties have more information than others regarding the state of the world, results in market inefficiency and possibly a full market breakdown. In an influential paper by Akerlof (1970), he shows that, using the market for used cars as an example, car owners with good quality cars will never put their car up for sale since the buyers are only willing to pay an amount equal to an estimate based on the distribution of quality (value). Thus reducing the amount the buyers are willing to pay for a car given that they only know the distribution. The idea is that this looped mechanism may then, in the worst case, lead to a market breakdown, or possibly, “a market for lemons”.

The second case is known as ‘Moral Hazard’ which occurs when one party have private information the actions taken that would influence the fiduciary relationship. Examples of this may be a worker, which is costly to monitor for the employer, could work hard or shirk, or a manager of mutual fund could do thorough analytical work prior to his investments or he could more or less arbitrarily pick a portfolio matching his own risk profile. These cases are often explained and analyzed in the literature using a Principal-Agent model, in which a principal that hires an agent to pursue his interests’ faces difficulties arising from asymmetric information and this problem is called ‘agency problem’ or ‘principal

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32 Reflecting the power of enforcement
agent problem’. There exists a large literature that discusses analytical properties of principal agent framework and provides us with an enlightening understanding of how optimal contracts are designed when faced with agency problem.

2.0 Literature Review

In our thesis, we will focus on the problem of moral hazard in the hedge fund industry, where the manager of the fund takes action which is hidden to the investor. Before we go more into the institutional setting, we present a general review of the theory of moral hazard (and to some extent adverse selection) in different contexts.

2.1 The Static Case

Laffont and Martimort (2001) present a thorough description of the binomial case where effort and production can take on two values, high or low. In the basic model presented, the optimal rent extraction-efficiency trade-off faced by the principal when designing his contractual offer to the agent is characterized. They point out that in the case of adverse selection where an agent may get access to information which is not available to the principal when a task is delegated, in order to achieve efficiently, an incentive compatible contract must be prepared such that it entices the agent to share the private information. In the literature, this is referred as the “revelation principle”. The term first appeared in Baron and Myerson (1982), where the authors considered the problem of regulating a monopolist with cost unknown to the regulator. In their proposition they define the revelation principle as:

*Without any loss of generality, the regulator may be restricted to regulatory policies which require the firm to report its cost parameter θ and which give the firm no incentive to lie.*

In economics, the term moral hazard was first introduced by Pauly (1968), when considering the problem in the insurance industry, where he found that even if the agent is risk averse, it is not optimal to provide him with full insurance. The model was first formalized by (Zeckhauser 1969). In the same institutional setting
as Pauly, he presented the optimization problem of the individual. If we consider a more general setting, the problem of moral hazard can be formalized in both a continuous and a discrete way with respect to effort and output. In the standard model, output is made conditional on effort and the state of nature. Laffont and Martimort (2001) presents a general model under the assumption that effort and production can take on two values, i.e., high or low. Under these assumptions they analyze various cases. First when the agent is protected by limited liability, they show that moral hazard is not a problem when the agent is risk neutral despite of the non-observability of effort. This is because the principal can choose incentive compatible transfers (high and low), which relaxes the agent's participation constraint and leave no rent to the agent. The Agent is rewarded when production is high and punished when the production is low. This is also in line with Biais (2009a). The principal who is also risk neutral makes an expected payment which is equal to the disutility of effort for the agent. This simple case implies that delegation will be costless to the principal. In the literature this is referred as First Best Cost.

Another important case is when the risk neutral agent has limited liability. This deduces the optimal contract to the point that the delegation will no longer be costless to the principal, because agent is now only liable to some extent and agent has to be given transfers greater than some exogenous level which implies that agent has limited liability. If this exogenous given level is smaller than the optimal transfers given some realization of nature, the agent must be provided positive limited liability rent. This means that cost of the effort will be the second best cost to the principal (higher than first best cost). The same holds in case of when the agent is risk averse. Here, because the agent is risk averse and the optimal transfers would entail that the agent has to be paid a risk premium to induce participation. Therefore the delegation would again be costlier than first best cost, which will increase with the degree of risk aversion.

This analysis are made in a binary world, but could easily be extended to allowing for continuous effort or production. If however effort is defined continuously, then solving the program of the principal becomes very involved as it must be

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33 They also expand, allowing for more effort or production levels
solved with respect to an infinite number of incentive constraints. Laffont and Martimort (2001) consider such a model where the production is defined over a closed interval with a distribution and density conditioned on effort equal to $F(q|e)$ and $f(q|e)$ respectfully. The infinitely many incentive constraints are the replaced with the first order condition. This approach was criticized by Mirrles (2009), by the difficulty of proving global maxima of expected utility when employing the first order approach. However, Jewitt (1988) presented a procedure for validating this methodology. The first step is to ensure that the multiplier on the incentive constraint is positive, secondly, ensure that the utility of the agent falls within a known class, and finally, make sure that the utility function of the agent is concave.

2.2 The dynamic case
In the dynamic case of moral hazard, the payoff to the agent could be made conditional on past performance. Biais (2009a) and Laffont and Martimort (2001), presents similar, discrete time, two period models, In these models, incentive compatibility implies an increase in the expected utility of the agent after success and decrease after failure. Here the principal can take advantage of the dynamic nature of the relationship in order to relax the incentive constraints. For instance - the transfer promised at the second period after two successes is useful to entice effort at both first and second period. Moreover, Biais allows for scaling of the project between the periods, and finds that closing the firm at the second period if production is low in the first period can help avoiding full market breakdown. The intuition is that the agent rationally expects the project to be liquidated (or the investor exercising his walkaway option in the case of money management) when the project performs poorly. While such liquidation is good to provide the agent with incentives, it hurts efficiency. Therefore, by introducing this extension to the model it still reflects the rent/efficiency tradeoff. Moreover, if the old manager can be replaced, Biases postulates that the principal needs to weigh this alternative in relation to their current state and cost of finding a new manager.

Under the discrete time infinite time horizon case the continuation expected utility of the agent and the value function of the principal are evaluated given the
information available at the beginning of period n. All the other assumptions are same as in one and two period case. Here, the continuation utility (CU) of the agent evolves as a function of the cash flow realizations. The idea is that following a negative (positive) realization of output, the continuation utility of the agent must be reduced (increased), such that the incentive compatibility holds. Moreover, Biais (2009b) discusses that due to the sensitive nature of the continuation utility of the agent to the performance of the firm, incentive compatibility requires that this sensitivity be equal to the magnitude of the moral hazard problem (reflected by the agents outside option and his cost of effort).

### 2.3 Allowing for more dimensions

In all generality, effort can be thought of as a vector consisting of n choice variables, each with its own unique technology linking it to the returns or production. Some papers focus explicitly on this property. Livdan and Tchistyj (2010), investigates a contract where owner of a financial institution/investor tries to align manager’s objectives with his own. They consider a two dimensional moral hazard problem in which risk neutral manager privately chooses the risk of the investments and his binominal effort level. Both these actions are unobservable to the principle. In the model, a high risk project increases the probability of a high cash flow realization, but it also results in significant losses in a bad state of nature, which is referred to as the “disaster” state.

In this model, the incentive compatible contract must reward the agent for reporting the high cash flow, otherwise the agent would just report low returns and steal the difference. However, conditioning the agent's reward on the reported cash flows also creates an incentive for the agent to take the high-risk project. The reason for that is that the agent benefits from the high cash flow and is protected by the limited liability in the “disaster” state. The contract that induces the agent to choose the low-risk project without stealing cash flows requires significant payoffs for the agent for the “non-disaster” outcomes. Finally the authors find that if it is impossible to write a contract conditional on the state of nature, it may be optimal to increase the investments. An increase in the
project size should be used as a reward for the manager for high cash flow realizations.

2.4 Applications

The principal agent model can be used to study a wide selection of markets in the economy. We will now present some of the most famous applications of the methodology.

2.4.1 Credit rationing

This phenomenon refers to a situation where lenders limit the supply of additional credit to borrowers, even if the borrowers are willing to pay higher interest rates. Tirole (2006) uses a principal agent model to explain credit rationing, with a focus on moral hazard. A common idea is that an increase in the interest rate has no effect on the borrower in the event of bankruptcy since he is protected by the limited liability. A higher interest rate would mean a lower stake for the borrower, since he now has to pay more than the normal rate to get access to certain amount of money. This reduced stake motivates the borrower to pursue projects with high private benefits, which in turn may reduce the probability of the reimbursement. Another way to look at the problem, and more in line with an adverse selection argument, is that higher interest rates tend to attract low quality borrowers and since these low-quality borrowers are more likely to default on their loan, they are less affected by a rise in the interest rate than high quality borrowers. Therefore to keep a good sample of borrowers, lenders may want to keep the interest rates low.

Tirole studies the phenomena in a simple agency model and uses it to illustrate the role of net worth. In this model, an “entrepreneur” does not have enough money to finance a fixed-size project and must therefore resort to outside funding. The entrepreneur controls his effort level and may also chose between projects with different probability of success. Both players are assumed risk neutral, but due to the competitive nature of the market, the lenders make zero profits. This assumption will act as a constraint in the maximization problem of the lender. Finally, the borrower who is protected by limited liability is ensured non negative transfers from the relationship. Tirole shows under which
conditions the entrepreneur receives and, if so, how the profit is shared between the lenders and the borrower.

Due the zero profit condition of the investors, when project is funded borrower receives the entire surplus if the project is funded. Tirole show that a necessary condition to receive financing is that the break even constraint holds. This constraint say that if the entrepreneurs own stake in the project is less than some limit dependent on the severity of the moral hazard problem, the probability of high return and the required lending, the lender cannot afford to incentives the agent to exert high effort (which is assumed to be the only case where it is possible to get financing in the first place). Thus, he will not receive financing

2.4.2 Investor-manager relationships

The principal agent framework has also been prominent in the hedge fund literature when analyzing the effect of managerial effort on returns. Kouwenberg and Ziemba (2007) presents a theoretical study of how incentives affect hedge fund risk and returns and compare their findings to empirical data. In their paper, they model the behavior of a representative hedge fund manager and derived an optimal investment strategy under the assumption of an option like compensation scheme. Doing this they find that performance fees, which originally provides the manager with the incentives to boost returns, also increases his risk appetite. The same notion is also shared by investment experts such as Warren Buffet, who believes that hedge funds shares only profits but not losses.

In their empirical analysis Kouwenberg & Ziemba show that hedge funds with performance fees have lower mean returns and worse risk-adjusted performance. Their results do not indicate increased volatility in hedge funds employing performance fees, but they do find significant results when focusing on the downside risk and not volatility, indicating increased risk appetite. Finally, they find out that fund of funds that charge higher performance fee are more risky and have higher average returns, indicating that the manager’s responds to the performance fee by allowing more risk. This need not imply that these managers are more skillful, since this does not explain why managers with high
(if that’s true), have more risky returns on average. If skill is a indeed factor in earning better returns, then high performing managers should also be able to do so with less risky investments as well.

3.0 The Institutional Setting

Hedge funds are open ended investment vehicles that pursue strategies that deliver absolute return regardless of market turbulence. Moreover, hedge funds differ from mutual funds and other investment vehicles by employing more sophisticated investment strategies and by being subject to a more liberal regulatory environment allowing them to hold both short and long positions, and to leverage their investments (Strömqvist 2009). Furthermore, hedge funds often have a high minimum investment and target institutional investors as well as wealthy individuals.

Traditionally hedge funds have remained robust through periods of financial instability compared to other investment vehicles (Strömqvist 2009), but the recent financial crisis hit the hedge fund industry hard and many funds still struggle to reach their pre-crisis watermark, although most (57%) surpassed the watermark during 2010 (Suisse 2011). Nevertheless, the rapid growth in many countries superannuation may ensure the industry a steady flow of funds in the years to come. In fact, the number of pension funds investing in hedge funds has increased by 50% since 2007 (Prequin 2011).

Many questions have been raised in the aftermath of the global financial crisis. Critics have claimed that the industry’s use of leverage and short positions enhanced the volatility in the markets. These are however, by no means new to the industry, which faced similar accusations in the 90s crisis. Others marginalise the industry’s potential to influence the financial markets in a crisis situation and argue that hedge funds where greatly negatively affected by the crisis, and the fact that short selling was temporary banned during the crisis (Strömqvist 2009). Moreover, Hasan et al. (2009) finds support for their hypothesis that short selling is more profound for financial institutions with greater exposure to the subprime market, indicating that banning short selling may reduce the disciplining effect on investors holding risky subprime assets.
More interesting is the industry’s new focus on the remuneration system, and particularly on the HWM structure, where the traditional 2%/20% structure has received much attention both by academics and practitioners. Zhan (2010) proposes that it may be in the best interest of the investor to reset the watermark in order to provide the manager with the right incentives following a period with large losses, while Richard Beales are more concerned about the industry’s ability to attract the best managers by relying solely on the management fee until the industry is out of the waters. He claims that the financial crisis has highlighted the need for a reform of the fee structures commonly applied by the industry. One solution according to Beales is to make incentives depended on several periods of performance and to stop payments of illiquid gains. This is also in line with Biais et al. (2010), where they find that in a dynamic moral hazard situation, compensation to the agent should be based on long term track record.

Another possible solution presented by Beales is to introduce a lone pine model. In this compensation structure, the incentive fee is not reduced to zero following a loss, but the manager receives only half the rate when recovering out of the waters. He receives this reduced fee until he has recovered 150% of the loss. Thus, fees paid by the investor are reduced as well as the volatility in fees received by the manager. A contract based on this can at the end of any given period be described as:

\[
W_t = \begin{cases} 
\beta(Y_{t-1}(1 + 2r_t) - L_{t-1})/2, & \text{if } r > 0 \text{ and } Y_{t-1}(1 + r_t) \geq L_{t-1} \\
\beta(Y_{t-1}(1 + r_t) - Y_{t-1})/2, & \text{if } r > 0 \text{ and } Y_{t-1}(1 + r_t) < L_{t-1} \\
0, & \text{if } r \leq 0
\end{cases}
\]

First note that \( \alpha = \alpha \) in this example (it could easily be included, but we leave it out for now to maintain the focus on the more interesting performance fee). Moreover, \( Y_{t-1} \) is the AuM at the beginning of the period, \( r_t \) is the realised return of period \( t \), \( L_{t-1} \) is the mark that needs to be reached in order to collect full fees. When the value of the fund surpasses this level, the manager starts collecting his usual fee \( \beta \) again.

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34 This structure implies a usually annual fee on the assets under management of 2% and an asymmetric incentive fee of 20% on the returns
3.0 Motivation

3.1 Moral hazard and the hedge fund industry

In general, investors delegating the management of their assets to an agent cause frictions due to the different object functions of the investor and the manager. The manager, will privately make decisions which may not be in the best interest of the investor. There are several factors which may help mitigate these frictions and (Ackermann, Mcenally, and Ravenscraft 1999) identify four such factors in the investor manager relationship.

The first factor is the contract. That is, the contract can be designed such that the manager is given the incentives to make effort and risk decisions that are optimal for the investor. The problem for the investor is that if he cannot observe the effort or risk choices of the manager. If the agent is risk averse or protected by limited liability and he cannot directly dictate the effort or risk level in the contract, the first best cannot be reached. The investor will instead have to settle for the second best, implying a positive expected rent to the manager36.

Ownership structure may also help mitigate the problem of moral hazard. Often, especially in hedge funds, managers have substantial investments in their own fund. This joint ownership structure exposes the manager to the same risks as the investors and thus helps align the objects of the investors and managers. However, as pointed out by Ackermann, Mcenally, and Ravenscraft (1999), given a risk averse manager, he may end up choosing a risk level below that is preferred by the investors. Moreover, Kouwenberg and Ziemba (2007) show that risk taking by the manager is greatly reduced if the manager has a substantial investment of his own wealth into his own fund (at least 30%).

Thirdly, government regulation may put restrictions on the extent at which the managers may adopt certain risky strategies, lockup periods and investments criteria (such as allowing for smaller minimum investment). This is less relevant for the hedge funds than the more regulated mutual funds industry. One argument often used to rationalize the minimal regulation facing hedge funds, is

36 If the manager could make effort and risk part of the contract he could extract all rent from the agent.
that investors of such investment vehicles are assumed to be more rational and informed than the average investor (considering the high minimum investment and the large degree of institutional clients), and thus does not need the same protection as for instance investors in mutual funds. Nevertheless, there has been increasing concern regarding the level of regulation in the industry, magnified by the financial crisis, where short selling in many financial institutions was temporarily banned in order to reduce the negative impact short selling can have in an already vulnerable sector.

Finally, market forces may also help reduce the principal agent problem. The idea is that well informed, rational investors, will exit funds where effort or risk does not match their optimal preferences and invest in higher performing funds with better matching risk profiles. Obviously, these selection mechanisms scale with the degree of fund transparency. Thus, for hedge funds, especially with a joint ownership where replacing the manager may prove difficult, this effect is limited (Ackermann, Mcenally, and Ravenscraft 1999).

3.2 The fee structure
Hedge funds often employ a different compensation structure compared to other more heavily regulated investment vehicles such as mutual funds. A common practice in the industry has been to charge an annual management fee on the assets under management (usually 2%) and a performance fee on returns (usually 20%). Many hedge funds also use hurdle rates as part of their remuneration scheme. That is, incentive fees are only paid on returns exceeding a benchmark such as the LIBOR plus some spread. Critics have claimed that an option like structure like described above gives the manager incentives to increase the volatility of the portfolio above what is preferred by the investor, and Starks (1987), finds by using an agency theoretic approach that a symmetric compensation structure dominates the option like structure in terms of aligning the desired risk level for the players, where the latter structure incentivizes the agent to choose risk level above what is optimal for the principal.
3.3 High water mark

Another common characteristic of many hedge funds remuneration schemes is the use of a high water mark\(^\text{37}\). This implies that the manager needs to recover previous losses in order to keep collecting incentive fees\(^\text{38}\). If we consider a fictional fund X with initial investment €10,000. The chart below represents the simulated annual performance of fund X. The performance is modeled as a random walk. That is:

\[
NAV_t = NAV_{t-1} + \varepsilon_t, \text{where } \varepsilon_t \text{ is i.i.d Gaussian white noise}
\]

In this case, investments are made at the beginning of period 1 and the watermark is set at the initial value €10,000. At time 2, the value of the fund starts dropping and the HWM is set at the highest historical NAV, that is point a on the chart. Assuming an annual asymmetric compensation structure given by:

\[
W_t = aV_t + \beta(V_t - h_t)^+
\]

Where \(a\) is the management fee, \(\beta\) is the performance fee and \(V\) and \(h\) are the value of the fund and the watermark respectfully. Thus the manager is protected by limited liability ensuring positive transfers\(^\text{39}\). However, in order to collect performance fee once the value starts rising again, the value of the fund must exceed the watermark, \(a\). At this point he will collect fee until the next turning point \(c\), which again will be the new watermark.

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\(^{37}\) Figures from CISDM 2008 shows that out of 872 respondents, 96% employed a watermark structure in their remuneration scheme (Ray and Chakraborty 2009).

\(^{38}\) He will still collect his annual management fee on the total assets of the fund.

\(^{39}\) Alternatively we could assume a symmetric remuneration scheme where the agent would be directly punished from poor performance with negative transfers.
The watermark structure have been heavily debated in the literature, and Zhan (2010) shows by simulating a principal agent model over five and ten periods, that funds employing this compensation scheme significantly underperforms funds using a traditional asymmetric compensation structure. The reason according to Zhan is that following large losses, the cost of effort to the manager in order to reach the last watermark becomes large and the manager reduces his effort the next period, thus further increasing the distance to the watermark. Chakraborty and Ray (2008) found similar results when analyzing empirical data in order to predict the impact on the distance from the watermark on effort, walkaway option carried out by the investor and risk appetite of the manager. Using realized return as a proxy of the effort level of the manager they find evidence that increasing the distance to the watermark leads to lower effort, investors will more commonly exercise their walkaway option and managers are more likely to accept greater risks.

4.0 The research questions and Methodological approach

4.1 The research question(s)

In order to analyze the investor-manager relationship in the hedge fund industry, we identify the following three research questions:
1. How does the high watermark compensation structure influence the manager’s effort and risk choices?

2. How does a high watermark contract influence a hedge fund’s ability to attract the best managers?

3. What effect will alternative contract structures have on the manager’s effort and risk choices?

4.2 The model

We plan to analyze different hedge fund compensation structures using a principal agent model. To develop the model we begin in a simple static environment assuming that the investor have all the bargaining power and propose a take it or leave it offer to the manager. Given that the manager accepts the contract, he privately chooses whether and to what degree to exert effort and the volatility of the investments. Both these choice variables are private information and hidden to the investor who only can observe the realized reported return. Thus, there is moral hazard. Moreover, both the agent and principal are risk neutral, but the manager is protected by limited liability. That is, there exist some exogenous level limiting the extent at which the principal can contract negative payments from the agent (Laffont and Martimort 2001) (given by $T(x^*) \geq -\zeta$ where $\zeta = 0$ implies non negative transfers to the agent). Finally, we will assume that both the principal and the agent can commit to the contract.

The basic problem of the investor can be written in line with the standard method in the literature. That is, he will maximize his expected utility, subject to a participation constraint, which ensures the agent expected utility above his exogenous given reservation utility, and a set of incentive constraints, ensuring that the agent selects the appropriate level of effort and risk. If we allow effort to be continuous on a closed interval (i.e. $e \in (0, \infty)$, then we will necessary have to solve the program with infinitely many effort incentive constraints. Thus, a common practice in the literature is to replace the general constraint with its first order condition. Finally, the investor must optimize his object function
subject to a set of limited liability constraint, ensuring the manager transfers larger or equal to the exogenous level explained above.

In order to capture the dynamic nature of the HWM contract, we plan to expand to a two period model. In this situation the choices of the manager in period one will affect his utility in period two. Moreover, it will allow us to analyze the effect of distance to the watermark on risk appetite and choice of effort and test for the robustness of the results obtained in the static situation.

We also want to extend the model to capture different dynamics. If we allow for replacement of the manager after period 1, the expected compensation facing the new (if desirable) manager will be influenced by the funds past performance. That is, he may have to recover previous losses if the fund is below its watermark in order to collect performance fee. Furthermore, we can also allow for the manager to observe a binominal signal in addition to the reported return in line with Laffont and Martimort (2001). Here, the signal observed is dependent on the effort level (or risk choice) of the agent and provides the principal another source of information of which to condition his transfers. Another expansion is to allow the investor to scale down or up the investment (i.e. relax the assumption of a lock out period). In this case, the manager may scale down investments in order to provide the manager with additional incentives because he can threaten with downsizing if the fund performs poorly (Biais et al. 2010). Moreover, introducing a situation where the investor may increase his investment in period two could help incentivize the manager. The idea is that this new capital will have a new watermark reference, so the manager will in effect be able to collect performance fee on the return of this investment, even if the initial investment is below the watermark. Finally we may assume a joint ownership structure where the manager invests a substantial amount of his own wealth into the fund (as many funds require). Thus, his income will no longer depend solely on the fees he collect, but also on the appreciation (depreciation) of the fund’s assets. That is, his effort and risk decision will now also affect his equity stake.
5.0 Shortcomings of the methodology

The principal agent framework rests on a set of assumptions which greatly reduces the technical difficulty of solving the model, but also limits the generality of the methodology. Laffont and Martimort (2001), identifies seven such assumptions in their comprehensive work “Theory of incentives”. Below follows a discussion of some of these assumptions, which we will include in our methodology:

5.1 The ability to costless enforce contracts

In the principal agent framework, a contract signed by the agent and the principal can be costless enforced by the principal, leaving no room for the agent to fail to carry out his commitment. If the optimal contract requires the agent to be punished for some realization of output, it could be in his best interest to renege on the agreement. This is however, not possible with the assumption of costless enforcing of the contractual agreement, since the principal could make use of the judicial system in order to force the agent to comply with his commitment. However, in reality we know that there are considerable costs related to run a case through the judicial system. Laffont and Martimort (2001) also highlight the need for the court system to be able to verify that the agent has violated the contractual agreement and the ability for enforce punishment.

5.2 The assumption of full commitment by the agent and the principal

This assumption states that we do not allow for renegotiation of the contract at the time when the agent has made his choices regarding effort and risk, and before the state of nature is revealed, even if such a renegotiation may be pareto improving (Stole 2001). Moreover, when considering an intertemporal model, the players can commit to the contract such that there will be no renegotiation of the contract between two periods, following the realization of some output. As argued by Laffont and Martimort (2001), Actions may be taken ex-post which will be beneficial for both players, but by allowing for such renegotiation, the agent will rationally anticipate this and take actions which will reduce the ex-ante optimality.
5.3 All players are rational

In the standard principal agent literature it is assumed that all players are rational utility maximizers. In reality, we observe that people may not act as predicted by the expected utility theorem when faced with choices under uncertainty. An alternative to this theorem was presented by Kahneman and Tversky (1979). In their paper entitled *Prospect Theory: An Analysis of Decision under Risk*, They present a theory, where the standard utility function of the agent is replaced with a value function, representing the value of an outcome for the agent. The value function is assumed to be concave for monetary gains and convex for monetary losses. Furthermore, the probabilities are replaced by a decision weighting function which need not be the same as the probabilities (i.e. \( f(\pi) = \pi \), the agent will evaluate the probability in line with the expected utility, but if this is not the case, expected utility will not hold).