Citizen coproduction and efficient public good provision: theory and evidence from local public libraries

Kristof De Witte
Maastricht University
University of Leuven

Benny Geys
BI Norwegian Business School
Vrije Universiteit Brussel

This is the authors’ final, accepted and refereed manuscript to the article published in


DOI: http://dx.doi.org/10.1016/j.ejor.2012.09.002

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Citizen Coproduction and Efficient Public Good Provision:  
Theory and Evidence from Local Public Libraries*  

Kristof De Witte a and Benny Geys b  

 a Top institute for Evidence Based Education Research, Maastricht University, Kapoenstraat 2, 6200 MD Maastricht,  
the Netherlands, k.dewitte@maastrichtuniversity.nl; and University of Leuven (KU Leuven), Naamsestraat 69, 3000  
Leuven, Belgium, kristof.dewitte@econ.kuleuven.be.  

 b Corresponding author: Norwegian Business School BI, Nydalsveien 37, N-0442 Oslo, Norway, email:  
Benny.Geys@bi.no; and Vrije Universiteit Brussel (VUB), Pleinlaan 2, B-1050 Brussel, Belgium; email:  
Benny.Geys@vub.ac.be.  

Abstract  
In both public administration and economics, efficiency is brought forward as an important  
criterion for evaluating administrative actions. Clearly, its value as an assessment principle  
depends on our ability to adequately measure efficiency. This article argues that citizen's  
coproduction in public services requires a careful reassessment of how we approach the  
measurement of productive efficiency in public service delivery. Theoretically, we illustrate that  
using observable outcomes (e.g., library circulation, school results, health outcomes, fires  
extinguished, crimes solved) as output indicators is inappropriate and leads to biased estimates of  
public service providers' productive efficiency. This bias arises because citizens co-determine  
final outputs, leaving them at least partly beyond the service providers' control. Empirically, we  
find supportive evidence of both the existence and importance of such 'demand-induced' bias.  

Keywords: Citizen coproduction, Public service provision, Technical efficiency, Local  
government, Libraries.  

JEL-classification: C14, C61, I21  

* The authors gratefully acknowledge insightful comments from Stefano Della Vigna, Leif Helland and Alexander  
Kalb. This work was supported by the Fonds Wetenschappelijk Onderzoek Vlaanderen (grant number G.0022.12 to  
the second author).
Introduction

Efficiency has been a central criterion for evaluating administrative actions since many years (Ostrom and Ostrom, 1971). A vast academic literature has subsequently developed aiming to understand the level and/or determinants of productive or technical efficiency - understood in terms of providing a maximum amount of output for a given level of inputs (Koopmans, 1951; Fried et al., 2008) - in (local) public good provision.¹ This attention is likely to increase further as (local) public service providers in many Western countries are facing both increasing demands (e.g., due to demographic change; Geys et al., 2008) and tightening budgets (e.g., due to governments' financial constraints). From such policy perspective, it is clear that understanding governments' performance allows for the detection of best practices and is a prerequisite for evaluating ways of performance improvement.

Yet, the value of efficiency as an assessment principle depends critically on our ability to adequately measure it. Two aspects are paramount in this respect. The first relates to the technical (or econometric) instruments required to estimate public service providers' efficiency. While this literature goes back, at least, to the pioneering contribution of Farrell (1957), recent years have witnessed a fast development with respect to the toolbox of efficiency measurement. Parametric as well as semi-parametric and non-parametric techniques have developed rapidly, allowing researchers and practitioners to deal more appropriately with output complexity, exogenous contextual variables, and so on (recent contributions include Daouia and Simar, 2007; Balaguer et al., 2010; Thanassoulis et al., 2012; De Witte and Kortelainen, 2013). The second aspect is of a more operational nature, and concerns the need for "clear conceptual measures" of inputs and outputs (Ostrom and Ostrom, 1971, 204; Balaguer et al., 2010). In the extensive empirical literature on public sector productive efficiency, one common characteristic is the reliance on final outcomes - e.g., school results, health outcomes, library circulation, waste collected, taxes collected, water or energy delivered, crimes solved - as the main output measure. This article argues that this choice ignores a key characteristic of the production process for public services: namely, the central role of citizens as 'coproducers' of public services (e.g., Whitaker, 1980; Parks et al., 1981). Such citizen coproduction has - with the increasing financial constraints on (local) governments - received renewed academic and political interest in recent years (Pestoff, 2006; Meijer, 2011). Yet, while the academic literature on coproduction discusses both the determinants and consequences of such coproductive activities (see below), it does not highlight its critical importance for the measurement of public service providers' productive efficiency.

In combining the literatures dealing with the measurement of public sector efficiency and with citizens' coproduction in service provision, this article provides three main contributions. First, theoretically, we argue that citizens as coproducers of public services make that final outcomes are inappropriate output variables in efficiency studies. The reason is that final outputs - due to such coproduction - are at least partly beyond the control of the service provider. That is, schools can "supply little education without inputs from students", police forces cannot maintain community safety without citizens reporting crimes or testifying in court (Parks et al., 1981, 1003) and tax collection is eased with citizens "submitting tax returns" (Alford, 2002, 39). As a consequence, estimates of public service providers' productive efficiency using such output measures may become biased. Briefly stated, low levels of citizen activity (e.g., low study investment by pupils or few requests for library books) imply relatively low levels of final outputs, which leads a high-input service provider to be designated as inefficient. Yet, it might be

¹In the remainder of this paper, we will use the terms productive and technical efficiency interchangeably.
that this provider is very effective at translating basic inputs into service potential. If so, the use of final outputs as performance measures unduly punishes this service provider for the restricted nature of citizens' coproductive activity in this area.\(^2\) This is not an argument to support the location of high-cost service providers in areas with low citizen participation, which constitutes a clear waste of public resources (i.e., allocative inefficiency). Rather, the argument is that, from a purely productive efficiency perspective, this service provider should not be described as an underperformer for an element beyond its control (i.e., citizens' coproduction or consumption decisions).

Second, using a rich dataset of municipal public libraries, our empirical contribution lies in evaluating the existence and importance of the bias deriving from ignoring citizens' role in the provision of public services. We find that high (low) levels of citizen participation in a given area generate high (low) estimates of productive efficiency when the selection of output variables ignores the extent of citizen coproduction. This relation vanishes when accounting for citizen's involvement in public good provision. These findings provide substantial support for our theoretical proposition that ignoring citizen coproduction generates biased inferences on public service providers' technical efficiency, and confirm that careful consideration of the production process and concomitant selection of output variables is crucial to make accurate inferences.

Third, from a methodological perspective, we contribute to the literature by employing a specially tailored non-parametric efficiency model (relying on recent work by Daraio and Simar, 2007, and its extention to discrete models by De Witte and Kortelainen, 2013). The model is rooted in the Free Disposal Hull (FDH) approach (Farrell, 1957; Deprins et al., 1984), which does not assume any a priori distribution on the production frontier. This is crucial as information on the production function is often lacking (Yatchew, 1998). The model also avoids two earlier drawbacks of FDH. First, it allows for outlying observations through the order-m technique of Cazals et al. (2002). Second, it captures heterogeneity among public service providers by immediately incorporating the exogenous environment into the estimate of the efficiency scores (thus avoiding a 'separability' condition which assumes that service providers' operating environment does not influence the level of the basic inputs and service potential). The results illustrate how this novel non-parametric methodology can help mitigate demand-induced bias.

Our results have important practical implications since properly characterizing and measuring performance is the first step towards discovering ways of improving performance. That is, evaluating whether public rather than private sector provision is more efficient (e.g., Andrews et al., 2011), or how, say, fiscal decentralization (e.g., Barankay and Lockwood, 2007), corruption (e.g., Dal Bó and Rossi, 2007), government ideology and/or fragmentation (e.g., De Witte and Geys, 2011) or citizen engagement (e.g., Borge et al., 2008; Geys et al., 2010) affect the (in)efficiency of public service providers can only succeed if the measurement of efficiency itself proceeds appropriately. Given the increasing efficiency-requirements on public policy-makers (see above) and the ensuing need to uncover pathways to improved performance, the benefit of increased knowledge on such measurement issues - as provided in the current article - is beyond doubt.

In the next section, we provide a more detailed theoretical discussion of our main argument. Then, in section 3, we present an empirical evaluation using data on Flemish local public

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\(^2\)Throughout the article, we will often refer to citizens' co-productive activities as 'demand' or 'citizen participation'. The term 'demand' is thereby used in a broader sense than the mere passive desire for a good or service, and implies active involvement (as reflected in, for example, study hours, book borrowing requests, fire prevention activities or tax form submission).
Theoretical background
The Concept of Citizen Coproduction
Reflecting the sharp distinction in economic theory between consumers and producers, the traditional view of public service provision is one where "the citizen reverts to a consumer and evaluator role, while government performs" (Sharp, 1980, 108). Although such clear role-division is arguably appropriate for the manufacturing sector of the economy (e.g., televisions, sofas, iPads), a similarly sharp distinction between consumers and producers is harder to maintain for (public) services. Indeed, for most services, it holds that "without the productive activities of consumers nothing of value will result" (Parks et al., 1981, 1002). Such effects are obvious in, for instance, the education sector. Bandiera et al. (2010, 1379) indeed estimate that "underlying ability or motivation to succeed are the single most important determinant of academic achievement" and "account for around 56% of the overall variation in test scores". The same also holds, however, in numerous other settings. For instance, patients should respect doctors' and nurses' orders if their health is to improve, one cannot get a decent haircut without sitting still in the barber's chair, and active engagement of the (long-term) unemployed underlies successful unemployment assistance programs.

In the late 1970s and early 1980s, public administration scholars developed the concept of citizen coproduction to capture this direct involvement of citizens in the design and delivery of public services (Ostrom and Ostrom, 1971; Sharp, 1980). It constituted an explicit attempt to move away from a "relieving logic" (a top-down view where citizens merely consume services) towards an "enabling logic" (a bottom-up view in which providers enable beneficiaries to coproduce the service) (Neumann, 1984). Most basically, the concept thereby highlights the "conjoint responsibility of lay citizens and professional government agents for the delivery of public services" (Sharp, 1980, 105). In other words, citizens - whether as individuals or as (in)formal groups - are seen as part of the public service production function, an issue we return to in more detail below.

In the ensuing coproduction literature, much attention has gone to the reaction of government officials to citizens' active involvement. This shows that government officials often fear the disruption of routines, their professional expertise and autonomy, and "balk at sharing their turf with citizen volunteers" (Sharp, 1980, 117). More recent work has also analyzed the conditions (un)favorable to coproduction. For instance, Van Ryzin (2011) points to the critical role of values such as citizens' trust in government institutions and perceptions of fairness, while Bifulco and Ladd (2005) provide evidence that appropriate, supportive organizational and institutional structures can stimulate coproduction.

Coproduction and Productive Efficiency
From the perspective of measuring public service providers' productive efficiency, consumers' active presence in the production function for public service outputs becomes of critical importance. Indeed, this "special nature of public sector production" (Ruggiero, 1996, 499) implies that final outcomes (e.g., school results, crimes resolved, library circulation, and so on) are not fully under the control of the public service provider. In other words, they are not 'produced' in a strict sense by the latter. Consequently, even though commonly employed (see above), they are inappropriate when evaluating service providers' technical (in)efficiency (see
also Ruggiero, 1996; Cordero-Ferrera et al., 2008; De Witte and Geys, 2011).
In fact, taking citizen coproduction seriously, accurate estimation of public service provider's
technical efficiency requires separating out the active involvement by the recipients of public
services (i.e., citizens' coproduction) from the production independently executed by the service
provider. More specifically, it suggests the existence of a two-stage production process that is
reminiscent of the distinction proposed by Becker (1965) and Lancaster (1966) between 'goods'
and 'commodities'; where 'goods' are intermediate outcomes provided by firms (e.g., food items
in a supermarket) and subsequently transformed by consumers into 'commodities' that fulfill their
desires (e.g., an individuals' meals or nutrition) (see also Kiser, 1984).

In the first stage, basic inputs -- such as labour and capital -- are translated into 'service potential'
-- such as available materials and machinery. At this stage, the public service provider is fully
accountable for the amount of service potential generated through a given amount of public
expenditures and, as such, represents a natural environment for productive efficiency analyses.
Referring once more to Becker (1965) and Lancaster (1966), this amounts to arguing that the
productive efficiency of private-sector firms is best evaluated using 'goods' - and not 'commodities' - as output measures. This is exactly what scholars of private-sector productive
efficiency have done for decades.

Then, in the second stage, service potential is transformed into observable outputs - such as
school outcomes, library circulation or crimes solved. This second-stage process is (at least
partly) beyond the control of the public service provider as it crucially depends on citizens
decisions. For example, available fire extinguishing potential is only exercised when there are
fires to be put out, while libraries only engage in book-lending when there are requests for library
books by potential readers. As such, the second-stage process could be interpreted as reflecting
how the supply of service potential by the service provider is met by the consumption activity (or 'demand') of potential service users to generate final outcomes.3

Importantly, this two-stage delineation not only defines indicators of 'service potential' rather
than 'final outputs' as appropriate output measures in public sector efficiency studies (as recently
argued by De Witte and Geys, 2011), it also helps clarifying that using final outputs produces
biased inferences - with the extent of mis-estimation depending on the importance of citizens'
role in bringing such final outputs about. To see this, imagine that public service providers can
employ one available input (e.g., Money) to generate service potential (SP): i.e., \( SP = f(Money) \). For simplicity, we assume that inputs always translate one-to-one into service potential (i.e.,

\[ SP = f(Money) \]

). This implies that in the input-output space, all jurisdictions will be on the
production frontier (independent of their size), which generates a benchmark situation where
everyone is perfectly productive efficient in this stage of the production process.4 However, the
provided service potential may or may not be taken up by the population, depending on the
equilibrium effort of citizens in coproduction. The final output (denoted Outcome) thus depends
on both service potential and citizens' activity (or Demand): i.e., \( Outcome = f(SP, Demand) \). Assuming a simple multiplicative functional form (i.e., \( Outcome = SP \times Demand \)), the relation
between the service provider's input and final outputs can be characterized as

\[ Outcome = SP \times Demand \]

. The second term of the right-hand side is 1 (by assumption, see

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3Remember that our use of the term 'demand' does not refer merely to the passive desire for the existence of a public
service, but, rather, to citizens' consumption decisions and activity.
4Constraining the relation between inputs and service potential to be one-to-one is immaterial to our results, but
simplifies the argument.
above), while the first term reflects the role of citizens' consumption decisions. The result is that the placement of a jurisdiction in the input-output space now becomes a direct function of the level of citizens' demand for the public good in this jurisdiction. Consequently, the estimate of public sector efficiency using the final outcome as an output indication will likewise be a direct function of the level of citizens' involvement, generating what we will call 'demand-induced bias'.

One might argue that we implicitly assume public service providers to provide services irrespective of local preferences. This clearly need not hold in reality and we agree that a library providing unwanted services (e.g., books no one has an interest in reading) should not be designated as fulfilling its task, even when it provides the maximum possible amount of services (i.e., service potential) for a given budget. However, this argument introduces the appropriateness of services or the responsiveness of the local service provider into the analysis (Dunn, 2004), and thus goes beyond productive efficiency in a strict sense. Although we are not claiming that these additional elements are less important than productive efficiency, nor that the quality of the outputs produced can be ignored, we do maintain that analyses of pure productive efficiency should regard the appropriate framework and not implicitly encompass such effects.

Empirical Analysis

Our empirical evaluation of the existence and importance of demand-induced bias in public sector efficiency measurement proceeds in three stages. First, in section 3.2, we estimate library technical efficiency in two ways: once in the 'traditional' way (i.e., relating basic inputs to final outputs) and once following the theoretical discussion above (i.e., relating basic inputs to service potential). This provides the main dependent variables for the remainder of the analysis. Then, in section 3.3, we evaluate what drives citizens' consumption decisions (or demand) for the local public service analyzed here (i.e., public libraries), as this information is central to assess - in section 3.4 - whether variation in citizen participation indeed explains the differences between the efficiency estimates obtained in section 3.2. Still, before we turn to these analyses, section 3.1 describes the institutional environment we study (i.e., local public libraries in Flanders) as well as the definition and sources of the main data.

Institutional setting and data

We employ a rich dataset on local public libraries in Flanders previously exploited by De Witte and Geys (2011). While our central argument applies more broadly than public libraries (see below), this case has some particularly attractive characteristics. First, there is a legal requirement for municipalities surpassing a certain population threshold to have a local public library, which provides us with a large and diverse dataset (N=291). Moreover, these libraries all face the same institutional environment; that is, they are similarly financed (i.e., mostly through tax-financed funds from the local government and subsidies from higher-level governments) and have similar (legal) obligations. Yet, importantly, the library management retains significant discretionary influence on, for example, the selection of books and media. While the former element makes the

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5We implicitly assume that popular demand is independent of the level of service potential (e.g., people are not stimulated to read by the fact that more books are made available to them). Relaxing this assumption does not affect our argument, as long as demand is not perfectly positively correlated with service provision.
libraries directly comparable for the purpose of our study, the latter issue provides a crucial source of variation across libraries. Also, which is important given that we use expenditures as a measure of input, libraries across Flanders face very similar input prices because book prices are the same for all buyers and the wages of public sector employees are strictly regulated in Belgium (via uniform collective labour agreements that induce strong wage homogeneity). Finally, local public library services are generally irrelevant to individuals' choice of residence (for recent evidence, see Bhatt, 2010), unlike, for example, a jurisdiction's education, tax policy or public safety provisions. Vice versa, selection of consumers by public libraries is unlikely to occur, which clearly does not hold in, for example, education or health care (Parry, 1996). These issues are important since they strongly mitigate potential concerns about endogeneity, self-selection bias and identification problems (we return to this below).

Our dataset - which derives from the Department 'Social Development and Local Cultural Policy' (Afdeling Volksontwikkeling en Lokaal Cultuurbeleid) of the Flemish Regional government - contains information on library revenues (from subsidies, fines and fees), expenditures (on personnel, infrastructure, library collection maintenance), collection size (e.g., books, CDs, DVDs, and so on) and operations (i.e., circulation, requests) for all 291 municipal public libraries in Flanders in 2007. This provides us with information about inputs, service potential and final outputs as well as information on some institutional characteristics of the libraries. To allow for the analysis of the role of citizens' decisions on library outputs, we merge this dataset with information on a number of background characteristics of the municipalities in which these libraries operate.

Given that efficiency measurement approaches based on the Free Disposal Hull methodology - such as the ones employed below (see section 3.2 for details) - face a curse of dimensionality (inclusion of more inputs and outputs increases the number of efficient observations; see Kneip et al., 1998), we opt for three input indicators, three measures of service potential and three final output variables. This restriction of the dimensionality of the analysis is consistent with the rules of thumb provided in the literature (e.g., Banker et al., 1989). As inputs, we use expenditures on (1) personnel, (2) operating expenditures (Opex; mainly maintenance of the collection) and (3) infrastructure. It is important to note at this point that charitable donations to and employment of volunteers in public libraries are uncommon in Flanders (in contrast to, for example, the US) and are, as such, not included in the analysis. These inputs, which fully exhaust the library expenditure budget, are used to provide (1) youth books, (2) fiction and non-fiction books and (3) other media (CD, DVD, CD-ROM, and so on). Hence, we use three collection-related variables (expressed in number of books or media) as indicators of library service potential.

6Our analysis relies on a cross-sectional setting and it would be interesting to extend the analysis to a panel setting (e.g., to capture non-observed heterogeneity through library fixed effects). This is, unfortunately, unfeasible at present due to both a lack of sufficiently time-wise comparable data and important methodological constraints (i.e., fixed effects estimators have not been developed in production theory - in which our efficiency model is rooted, see below).

7This infrastructure spending does not refer to big investment projects (such as major renovations or additions to the library buildings), which tend to be lumpy and time-specific. Instead, it measures the annual expenditure on infrastructure that occurs because books must be housed in an enclosed space and larger book collections require a larger space with higher maintenance costs.

8Clearly, the service potential of a library goes beyond these three variables and might also include the amount and quality of assistance programs, courses, lectures and/or exhibitions offered, the availability of computer terminals, internet access, and so on. Unfortunately, however, data for such outputs are unavailable (this also holds for more indirect data such as the number of visits to each library, which could proxy availability of such additional services). Hence, to the extent that basic inputs are employed for the provision of such services, our analysis is likely to over-
these variables correspond to stock measures, which may benefit older libraries, the variable returns to scale approach employed in our efficiency model smoothly accounts for this (Fried et al., 2008). Finally, in line with earlier work on library efficiency (Hammond, 2002; Hemmeter, 2006), our three final outputs are (1) total youth borrowers (<16 years), (2) total fiction and non-fiction book circulation and (3) total media circulation.

Descriptive statistics of all variables employed are presented in Table 1. These statistics first of all indicate that personnel costs are the main input for the libraries. On average, personnel costs are almost five times higher than operational expenditures. Secondly, in the average library, the total number of youth books (i.e., fiction, non-fiction and comics) is higher than the number of (non-youth) fiction and non-fiction books. Thirdly, media circulation is relatively high compared to its share in the libraries' portfolio: i.e., each media item is, on average, borrowed 2.6 times while books have a circulation rate of 3.2. Finally, more libraries (82%) have a media borrowing fee than a membership fee (72%).

<table>
<thead>
<tr>
<th>Table 1: Descriptive statistics (year=2007; n=291)</th>
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<tr>
<td><strong>Inputs (in €)</strong></td>
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<td>Personnel</td>
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<td>Opex</td>
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<td>Infrastructure</td>
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<td><strong>Service potential (in absolute amounts)</strong></td>
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<td>Youth books</td>
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<tr>
<td>Fiction and non-fiction books</td>
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<td>Media (CD, DVD, CD-ROM)</td>
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<td><strong>Final outputs (in absolute amounts)</strong></td>
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<tr>
<td>Young borrowers (&lt;16 years)</td>
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<tr>
<td>Book circulation</td>
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<tr>
<td>Media circulation</td>
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<tr>
<td><strong>Demand variables</strong></td>
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<tr>
<td>Education (% higher education)</td>
</tr>
<tr>
<td>Population density (per km²)</td>
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<tr>
<td>Membership fee (dummy)</td>
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<tr>
<td>Media borrowing fee (dummy)</td>
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Estimating (in)efficiency

To estimate performance of public entities, we apply three efficiency models based on the Free Disposal Hull (FDH) methodology (Deprins et al., 1984; Cazals et al., 2002; Daraio and Simar, 2005). Before intuitively describing these approaches, it is important to point out that they are well-suited to our setting. First, as a fully nonparametric approach, FDH does not require any information on the production technology. This is crucial as such information (i.e., how inputs are transformed into outputs) is often unavailable to researchers (Yatchew, 1998) and wrongly specified models lead to biased estimation results (Hjalmarsson et al., 1996). Second, our model estimate true technical inefficiency in stage one of the public service production process (since we observe less than the true output level for a given level of inputs).
does not require information on prices as the FDH approach relies on shadow prices to aggregate inputs and outputs (Deprins et al., 1984). This resolves the fact that in public sector settings, prices are often unobserved or unreliable because of subsidies. Third, the FDH methodology has recently been extended to allow corrections for the impact of outlying observations that in the current context might arise from atypical observations (referred to as 'robust FDH' or 'order-m'; Cazals et al., 2002) and exogenous variables (thus addressing that public organizations work in different environments and with different restrictions; referred to as 'conditional FDH'; Daraio and Simar, 2005). The latter is also important from an econometric perspective since the resulting estimates have recently been shown to have additional attractive properties: i.e., they are consistent (i.e., estimate the 'true' inefficiency) and have a fast rate of convergence (Jeong et al., 2010). Given the purpose of our paper, we present only an intuitive description of the model(s). A formal discussion can be found in, for instance, Cazals et al. (2002), Daraio and Simar (2007) or Fried et al. (2008), while a graphical representation is provided in, for instance, Fried et al. (2008) and Muller (2008). Krüger (2012) points out that the robust FDH approach can be worse than the more traditional FDH approach when measurement errors are not excessively large and outliers are absent. Unfortunately, as presented in the descriptive statistics in Table 1, our data includes various outlying observations. Moreover, to capture the heterogeneity among observations, De Witte and Marques (2010) show by simulated data that the robust conditional efficiency approach is a superior model. Therefore, we apply the robust and conditional efficiency approach.

The Free Disposal Hull model

The seminal paper of Farrell (1957) inspired researchers to consider performance as a relative estimate, whereby any given decision-making unit (DMU) is evaluated against a frontier of best practice observations. Consider DMUs, which are using heterogeneous and non-negative inputs to produce heterogeneous and non-negative outputs. The FDH model relies on a free disposability assumption of inputs and outputs: if and then [where denotes the production technology set:
while for an inefficient observation One can interpret \((1-\gamma)\) as the potential percentage increase in output for the evaluated observation were it to produce as efficient as its best practice counterpart.

The robust FDH model

As can be observed from equation (1), the FDH frontier in the standard FDH setting of Deprins et al. (1984) is deterministic in the sense that all observations from the sample potentially constitute the frontier. However, a-typical observations might create outliers in the data. Such outlying observations influence the production frontier and, consequently, the efficiency estimates. Cazals et al. (2002) suggest to mitigate the impact of outlying observations by drawing with replacement subsamples of size among those DMUs with fewer inputs than the evaluated DMU (i.e., among those so that ). Relative to this smaller sample of observations, performance is assessed. After repeating the sampling and efficiency evaluation times, where is sufficiently large (larger than 2,000), the robust efficiency (also denoted by 'order-') scores are obtained by taking the arithmetic average of the inefficiencies.\(^9\)

Due to the smaller reference set and the fact that the evaluated DMU ( ) does not form part of its own reference set in every of the drawings, the robust FDH model can result in 'super-efficient' efficiency scores (i.e., ). In our case, these super-efficient observations obtain efficiencies below 1, which indicates that the evaluated DMU is performing better than the average DMUs in its reference sample (see Cazals et al., 2002 and Daraio and Simar, 2007 for an extensive discussion). The larger , the less super-efficient observations in the data. In the extreme case where , there is no super-efficiency estimated and the traditional deterministic FDH model is estimated. Following Daraio and Simar (2005), the size of is determined as the value for which the percentage of super-efficient observations (i.e., ) declines only marginally with an increase in . To determine this value, we count for various values of the percentage of super-efficient observations. In the setting at hand, from onwards, the percentage of super-efficient observations declined only marginally with

As a result of the repeated draws, outlying observations will not be part of the reference sample in every draw. This has two important consequences. First, the partial frontier of size will shift inwards relatively to the full frontier of size (i.e., ), such that (or, in

\(^9\)Cazals et al. (2002) also suggest a perfectly equal (if is sufficiently large) integral formulation. As the integral formulation is faster to compute, the R code underlying our analysis (which is available upon request) uses this alternative version of the robust FDH model. Note also that we use the terminology 'robust FDH' and 'order-m' interchangeably throughout the remainder of the paper.
words, inefficiency will be lower in the order- model). Second, the influence of outlying observations - in both the input and output dimension - on the performance estimates is mitigated.

The conditional robust FDH model
An attractive feature of the order- model arises from the ease with which exogenous variables can be included. We refer to exogenous variables as background characteristics that are beyond the influence of the evaluated DMU, but which influence its performance (Ruggiero, 1996).¹⁰ Though suggested in Cazals et al. (2002), Daraio and Simar (2005) were the first to implement so-called conditional robust FDH by adapting the robust FDH model in a way that the subsamples of size are now drawn with a particular probability. The latter probability is determined by a Kernel function around the exogenous variables . Observations ( ) that are similar to the evaluated observation ( ) are drawn with a higher probability than dissimilar observations. Formally, the conditional robust FDH model draws times the reference sample of size with replacement and with a probability among those such that ; where denotes a Kernel function and the appropriate bandwidth (estimated by cross-validation). Finally, the efficiency evaluations are averaged to obtain the robust conditional efficiency estimates .¹¹ The interpretation of the efficiency scores is similar to the order- model. De Witte and Kortelainen (2013) extend the conditional efficiency approach of Daraio and Simar (2005) to include discrete variables. They estimate a mixed kernel which treats continuous (denoted by ), discrete ordered (denoted by ) and discrete unordered (denoted by ) variables differently in the estimations. The generalized product kernel function reads as:
The results of the efficiency estimations are summarized in Table 2. We denote the efficiency scores by, respectively, and for the 'service potential model' (which follows our theoretical argument above) and the 'service delivery model' (which follows the traditional approach using final outputs). In the first three columns, we present the results of the Free Disposal Hull method, while the next three columns use the robust version of the same model (i.e., robust FDH) and the last three columns also account for exogenous variables (i.e., conditional robust FDH). In each case, we show summary statistics for and as well as the difference between both estimates. Note also that in the conditional robust FDH estimation, we follow De Witte and Geys (2011) in controlling for the ideological stance of the local government (using a Left-Right scale from 0 to 10 based on a self-placement survey; Bue lens et al., 2008), the share of women in the local council, the wealth of the municipality (measured as income per capita, in 1000€), population size and the source of public library funding (defined as the share of regional subsidies in the total library budget). The first two variables capture the municipality's political environment, which may affect library policies as right-wing parties are often argued to have different preferences and priorities compared to left-wing ones (Hibbs, 1977). The same has more recently been argued to hold for male compared to female politicians (Edlund and Pande, 2002; Geys and Revelli, 2011). The wealth of the municipality captures the idea that high-income residents are generally more in favour of wide-ranging cultural provisions (Werck et al., 2008), and may therefore demand maximization of libraries' service potential given budgetary constraints. A large population size increases the group of potential users of the library's services. Its effect is a priori uncertain as a large population may increase the group interested in efficient libraries, but it could also engender congestion concerns (lowering the perceived benefit of the library and reducing efficiency demands) (De Witte and Geys, 2011). Finally, the source of public library funding is likely to influence the local population's monitoring activity as people care more about inefficiency in the provision of public resources when they pay for such services more directly (through fees and municipal subsidies as compared to regional-level grants) (Hemmeter, 2006; De Witte and Geys, 2011).

For the FDH model, we find an average (in)efficiency score of 1.162 for the 'service potential model' and 1.142 for the 'service delivery model'. This suggests that, on average, Flemish local public libraries could, with a given input vector, increase their outputs by respectively 16% and 14% if they were to work as efficient as the best practices. The average difference between both estimates lies around two percentage points, but, importantly, it displays significant variation. The bottom two rows of Table 2 indeed illustrate that for nearly 60% of the 291 libraries in our sample (72+93=165), both measures do not generate the same result. These differences can at times be quite substantial, as indicated by their minimum and maximum values. While the data and conditional robust efficiency framework employed at this point are similar as in De Witte and Geys (2011), we differ from their analysis by including all five controls simultaneously. Moreover, their analysis aims to explain public sector performance, rather than estimate the demand bias.

Although one could clearly consider additional exogenous controls - e.g., the age of the library, the age distribution of the population it serves, the geographical features of the municipality (which may affect ease of access to the library) - data for such indicators were unfortunately unavailable.

Efficiency scores are considered as equal if the estimates differ by less than 1 percentage point. If we do not impose
confirmed by the Spearman rank correlation between both efficiency estimates, which amounts to only 0.42.

<table>
<thead>
<tr>
<th></th>
<th>$\lambda_{\text{poten.}}$</th>
<th>FDH $\lambda_{\text{def.}}$</th>
<th>Difference</th>
<th>$\lambda_{\text{poten.}}$</th>
<th>Robust FDH $\lambda_{\text{def.}}$</th>
<th>Difference</th>
<th>Conditional robust FDH $\lambda_{\text{poten.}}$</th>
<th>$\lambda_{\text{def.}}$</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>1.162</td>
<td>1.142</td>
<td>0.020</td>
<td>1.104</td>
<td>1.078</td>
<td>0.026</td>
<td>1.045</td>
<td>1.056</td>
<td>-0.010</td>
</tr>
<tr>
<td>St. Dev.</td>
<td>0.288</td>
<td>0.291</td>
<td>0.029</td>
<td>0.293</td>
<td>0.304</td>
<td>0.011</td>
<td>0.221</td>
<td>0.233</td>
<td>-0.012</td>
</tr>
<tr>
<td>Minimum</td>
<td>1</td>
<td>1</td>
<td>-1.651</td>
<td>0.475</td>
<td>0.436</td>
<td>-0.169</td>
<td>0.541</td>
<td>0.450</td>
<td>-0.083</td>
</tr>
<tr>
<td>Maximum</td>
<td>3.135</td>
<td>2.932</td>
<td>1.551</td>
<td>3.078</td>
<td>2.926</td>
<td>1.428</td>
<td>2.320</td>
<td>2.517</td>
<td>0.735</td>
</tr>
<tr>
<td>$\lambda_{\text{poten.}} &lt; \lambda_{\text{delivery}}$</td>
<td>72</td>
<td></td>
<td></td>
<td>126</td>
<td></td>
<td></td>
<td>118</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda_{\text{poten.}} &gt; \lambda_{\text{delivery}}$</td>
<td>93</td>
<td></td>
<td></td>
<td>92</td>
<td></td>
<td></td>
<td>103</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As expected, the robust FDH model leads to lower estimated inefficiency levels (1.104 and 1.078, respectively) and some efficiency scores now also lie below 1. Hence, some observations are super-efficient: they perform better than the average observations in their reference sample. Yet, the difference between both estimates increases slightly (to 2.6% on average) and now about 75% of the libraries in our sample (126+92=218) fail to generate the same result on both measures. This follows from the fact that fewer libraries achieve an efficiency score equal to 1 (which is a relatively frequent occurrence in the standard FDH model). The Spearman rank correlation likewise remains rather low at 0.48.

Finally, turning to the conditional robust efficiency model's findings, we observe that the average inefficiency level as well as its standard deviation decline. After controlling for exogenous factors, the average inefficiency level lies around 5%. Moreover, while the variation in the difference between both efficiency models now declines substantially (reflected in both a lower standard deviation and less extreme minimum/maximum values) compared to the unconditional models, the number of libraries providing different estimates across both models edges up a little further (118+103=221; or 76% of our sample). This is also confirmed by the lower Spearman rank correlation of 0.32. As the estimation of these three different models not only serves as a robustness exercise, but also gives information about how their underlying assumptions affect our findings, we will return in more detail to these differences below.

Determinants of library lending

The results in the previous section clearly illustrate that although the difference between using service potential or final outputs is not excessive on average, it can be substantial for a significant number of observations. This in itself does not imply that one or the other set of estimates is 'better' or 'more accurate'. Yet, from a theoretical point of view, the estimates relying on service potential can be designated as most appropriate if we can show that the difference between both sets of efficiency measures is related to citizens' active role in procuring the service providers' final outputs (as discussed in section 2). To this end, we first evaluate how library lending is affected by citizens' proclivity to borrow books. The motivation for this analysis lies in the threshold, the number of libraries where the results differ across both estimates naturally increases. Still, all results reported below are robust to this alternative operationalisation, indicating that our results are not driven by the exact definition of the efficiency-difference (full details available upon request).
empirical verification of a number of variables - derived from earlier work by Locher (2005) and Løyland and Ringstad (2008) - that may serve as proxies for the demand for library books. Specifically, our estimation equation here takes the following form:

\[
\text{(4)}
\]

where \( y \) denotes the dependent variable and is either a) total circulation of books (youth, fiction and non-fiction) or b) total circulation of other media (CDs, DVDs and CD-ROMs) (Locher, 2005; Løyland and Ringstad, 2008). As explanatory variables, we include the education level of the municipal population (measured as the share of inhabitants over 18 with a university or college degree), population density (measured as inhabitants per km²) as a proxy for the average travel time to the library and two indicator variables for the presence of an overall library membership fee and a fee-system for borrowing other media. In line with previous work, we expect education and population concentration to have a positive effect on library demand (and thereby book and media circulation), while the existence of fees is expected to have a negative effect. The library stock (measured through both the total stock of books and other media available at the library) is included as a scaling measure (Løyland and Ringstad, 2008).

Table 3 presents results both from the estimation in linear form in Columns (1) and (2) (Locher, 2005) and loglinear form in Columns (3) and (4) (Løyland and Ringstad, 2008). Several observations follow from Table 3. First, there is high congruence between the linear and loglinear specifications, such that the results are not driven by either of these functional forms. Second, higher presence of other media does not have a crowding-out effect on book circulation, while a larger book stock actually seem to stimulate media borrowing. This suggests that books and other media are complements rather than substitutes (for a similar observation in Norwegian public libraries, see Løyland and Ringstad, 2008). Finally, and most important for our analysis, we find that library book circulation (though not media circulation) in Flemish municipalities is significantly positively related to the population's education level and population concentration (proxying lower average travel time to the library). Membership and/or borrowing fees are - in line with earlier work by Locher (2005) - linked with lower circulation of books and other media. Overall, these results indicate that these variables can serve as valid proxies for citizens' library-demand.

15 Equivalent results to those presented in Table 3 are obtained when estimating both equations jointly using 3-Stage Least Squares (3SLS) or seemingly unrelated regression (SUR). Note also that, unfortunately, we lack separate circulation data for youth, fiction and non-fiction books. Hence, while the availability of separate stock data allowed us to make this distinction in the efficiency analysis in section 3.2, we are constrained to estimate one joint demand function for all types of books.

16 We also tried replacing education with average income per capita (in 1000€). This, however, generally provided somewhat weaker results (available upon request). Note also that we unfortunately lack data on loan length, which may affect a library's circulation rate.

17 To the extent that fees might be used to ration access to a limited resource, their introduction (and level) may be influenced by demand conditions. Although this creates a reverse-causality issue, it is of limited concern here since it suggests a positive association between demand and fees. Finding a negative effect despite this countervailing effect would thus only strengthen our interpretation of fees as depressing demand.
Demand-induced bias?

We now bring the results of Sections 3.2 and 3.3 together to assess whether the difference in efficiency estimates obtained by relying on final outputs rather than service potential is affected by citizens' active role in the public good production process. To this end, we compare the outcomes of the 'service potential model' and the 'service delivery model', and link their difference to variables reflecting citizens' demand. That is, we estimate:

\[
\text{(5)}
\]

where \( \phi \) denotes a monotonic link function, \( \mathbf{x}_i \) is a vector of variables proxying the level of citizens' demand in jurisdiction \( i \) (i.e., education level, population concentration and the presence of fees), and \( u_i \) a continuous iid random variable, independent of \( \mathbf{x}_i \). The functional specification of \( \phi \) is thereby subjected to robustness analysis. A first specification consists of an OLS regression on the raw difference between \( \mathbf{x}_i \) and \( u_i \), using heteroscedasticity-consistent standard errors. Using OLS regression is feasible as (1) by construction, this difference is not bounded; and (2) thanks to the use of the (conditional) robust FDH model, the efficiency estimates are consistent and not serially correlated (Jeong et al., 2010). Still, in a second specification, we map the outcomes of \( \mathbf{x}_i \) to a variable

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18We include all four demand proxies in the analysis here since each of them affects at least some part of overall library demand (by affecting either book or media circulation, or both; see section 3.3.). Hence, each of them will affect the (in)efficiency estimates of the 'service delivery model' that we calculated in section 3.2, based on both book and media circulation as final outputs (see section 3.1).
This re-operationalization not only allows testing whether outliers in the true values of the efficiency difference affect our results, but it also goes some way towards controlling for the uncertainty involved in measuring the exact level of inefficiency. Moreover, by collapsing the dependent variable into such three-point scale, we reduce variation in the dataset, making it harder to obtain statistically significant estimates. As such, the latter approach provides a harsher test of any structural effects in our data. Clearly, the relationship between and requires an ordered logit or probit model. Given the definition of as service-potential-efficiency minus service-delivery-efficiency, we hypothesize that high citizen participation (reflected in high education, high population concentration, no fees) lowers the value of (see section 2).

It is important to note that in our test of this hypothesis we make use of a two-step approach in line with Banker and Natarajan (2008), McDonald (2009) and Banker (2011). This is different from the alternative bootstrap approach suggested by Simar and Wilson (2007), who claimed that the (double-)bootstrap deals more adequately with a number of statistical issues associated with explanatory analysis. Our choice of the two-step approach is not intended to indicate a preference for either method (for a critical comparison of both methods, see Banker and Natarajan, 2008; Banker, 2011; Simar and Wilson, 2011). It is rather driven by the fact that we evaluate the role of demand variables on the difference in efficiency scores from the service potential and service delivery models. We consider the use of the (double-)bootstrap approach as a robustness check in further research.

Before turning to the results, two further comments regarding our identifying assumptions are likewise required. First, the true efficiency difference as defined above will be under-estimated when decisions on service potential anticipate the nature and extent of citizens' consumption decisions (e.g., by buying more copies of books a library expects to be popular). Fortunately, this is unlikely to be a major concern in our setting as libraries are urged to buy a broad selection of books and to spread their collection sufficiently. This not only reduces the budget available for buying multiple books of expected 'best-readers', but also leads libraries to, by definition, stock a large number of single-copy books. Second, while our cross-sectional dataset admittedly does not warrant strong causal inferences, it is important to reiterate that both our sample selection and estimation procedure strongly mitigate concerns of reverse causality. Reverse causality would indeed imply that the difference in both efficiency measurements affects population concentration, education level or the presence of fees. This does not appear very likely. More specifically, while efficient public services have been argued to attract in-migration and become capitalized into housing prices (e.g., Brueckner, 1979), Bhatt (2010) shows that public library services are irrelevant to individuals' choice of residence, making the efficiency of such provision an unlikely driver of location choice (and thus population concentration). Similarly, while the

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19Remember that in all regressions presented, efficiency scores are considered as equal when their difference is below 1 percentage point. Nonetheless, as mentioned above, relaxing this assumption leaves our results qualitatively unaffected. Note also that, as a further robustness check, we ran all regressions using the ratio between both efficiency measures - i.e., rather than their difference as the dependent variable. This leaves our findings unaffected.

20Maarten Vandekerckhove, Flemish administrator responsible for public libraries, personal communication, 27 September 2010.
number of books available in a library may arguably affect the (potential) educational attainment of the local population, it is much harder to see why the technical efficiency with which such books are made available has a similar effect. Finally, inefficient libraries may be driven to introduce fees to address budgetary problems. Even here, however, the exact measurement of such efficiency - and thus the difference between and - is unlikely to affect that decision.

The results are presented in Table 4. Regressions (1) through (4) display the results when using standard FDH, while regressions (5) through (8) and (9) through (12) use the efficiency estimates obtained from the robust and conditional robust FDH models, respectively. All three sets of results follow the same pattern. The first column, i.e., regressions (1), (5) and (9), provide the results using the entire dataset. The following column provides a robustness check by excluding all libraries where there is no difference between the 'service potential model' ( ) and the 'service delivery model' ( ). The final two columns present results using an ordered logit and ordered probit, respectively, on the efficiency difference redefined as a three-point scale (i.e., the variable ).

The results in regressions (1) through (4) provide substantial support for our argument that ignoring citizens' coproductive decisions leads to biased estimates of service providers' productive efficiency. Indeed, education (when using the transformed difference between both efficiency measures), population concentration (when using the untransformed efficiency difference) and the membership fee variable (in all regressions) are all linked to a higher estimate of local public library productive efficiency when using final outputs (rather than service potential) as the outcome measure. Moreover, tests evaluating the joint significance of all explanatory variables in the model firmly reject the null hypothesis of no effects (p<0.05 in each case). This result substantiates the main theoretical argument in section 2. Indeed, it implies that high (or low) service-delivery-efficiency relative to service-potential-efficiency is driven to a significant extent by high (or low) demand for the services provided in the jurisdiction under study.

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21Education and population concentration are significantly positively correlated (r=0.19; p<0.01), which may explain why only one of them reaches significance in either model. This appears substantiated by the fact that dropping either tends to raise the explanatory power of the other. Still, as we are interested predominantly in the joint effect of our demand proxies - and are less concerned with their individual effects - we retain both variables in all models.
Interestingly, while largely similar results are obtained when using robust FDH to estimate (regressions (5) through (8)) - suggesting that our results are not driven by the distributional assumptions underlying FDH - accounting for a number of important exogenous variables does have an important effect on our findings (regressions (9) through (12)).
Indeed, controlling for exogenous factors directly in the efficiency-estimation (i.e., applying conditional robust FDH) makes that the effect of demand-side variables on the efficiency-difference weakens substantially. More specifically, we find that the statistical significance of all such variables is strongly mitigated in this specification and that they are no longer jointly statistically significant at conventional significance levels (p>0.10 in all cases). Although citizen-induced effects are not completely eliminated, the latter observation does suggest an important practical implication. It indicates that one possible way to (partially) control for demand-side effects - and mitigate the consequences of demand-induced bias - is to account for the exogenous environment directly in the estimation of jurisdictions' technical (in)efficiency. While accounting for such exogenous factors has been advised for diverse reasons before (e.g., Ruggiero, 1996; Yu, 1998; De Witte and Kortelainen, 2013) and several different techniques to do so have been brought forward (for critical overviews, see Balaguer-Coll et al., 2010; De Witte and Kortelainen, 2013), our results provide one more reason to heed such advise - i.e., to mitigate the influence of, and bias induced by, demand-side factors on the efficiency estimates.

Conclusion
With the increasing financial constraints on (local) governments in recent years, the coproductive activities of citizens in public service delivery obtained renewed academic and political interest (Pestoff, 2006; Meijer, 2011). In this article, we analyse the role such citizen coproduction plays for the measurement of public service providers' productive efficiency. Particularly, we argued that taking account of the 'coproductive' activities of citizens is crucial for appropriately defining inputs and outputs in the analysis of productive efficiency (and for the accuracy of the results from such analyses). Indeed, as final outcomes are influenced by citizens' active participation in the public service production process, they are inappropriate to evaluate the pure productive efficiency of public service provision, and their use will lead to biased inferences. Empirical evidence using data on local public libraries in Flanders provides significant support for the presence and importance of the resulting 'demand-induced' bias.

Clearly, our argument goes beyond the particular setting empirically evaluated in this paper. If public entities are evaluated on output indicators which are influenced by citizen coproduction, they loose discretion over their performance. Even stronger, in many service sectors, it is a responsibility of the public service provider to reduce certain outcomes: e.g., fire departments prevent fires, water utilities urge consumers to reduce water consumption, police officers try to minimize the number of interventions required, competition authorities aim to reduce cartel formation. If public entities are assessed only on second stage outcomes, blurred performance scores will be obtained. In Table 5, we illustrate this broader applicability of our argument by providing, for a selected number of public service sectors, what is the key observable output generally used in existing efficiency estimates, and what we believe to be more appropriate measure(s) of service potential unaffected (or, at least, much less affected) by citizen coproduction.
Finally, it is important to reiterate that this paper addresses pure technical or productive efficiency and not allocative efficiency, equity, responsiveness, adequateness or appropriateness of public services provided. Hence, it may seem troubling to some readers that, say, a library which chooses to purchase undesirable, cheap books becomes the most "efficient" in our analysis. To some extent, we agree. That is, while this library is economically (or productively) efficient, it is clearly not the best one could do. However, to evaluate the latter - i.e., optimal public service provision in a broad sense - an array of indices should be applied. Productive efficiency is not any more or less important than other dimensions, and governments are likely to be (or ideally should be) interested in some optimal mix of all dimensions, but a valid and appropriately characterized evaluation of public service provision stands or falls with a correct measurement of all aspects involved (Ostrom and Ostrom, 1971). This requires indices that are not implicitly contaminated by elements beyond what is being measured. The approach presented in this paper provides a step towards producing valid estimates of public service providers' technical efficiency.

<table>
<thead>
<tr>
<th>Sector</th>
<th>Observable output</th>
<th>Service potential</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water utilities</td>
<td>Water volume</td>
<td>Peak volume water</td>
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<tr>
<td>Electricity services</td>
<td>Observed consumption</td>
<td>Peak load</td>
</tr>
<tr>
<td>Sewerage utilities</td>
<td>Sewerage collected</td>
<td>Peak volume sewerage</td>
</tr>
<tr>
<td>Waste services</td>
<td>Waste collected</td>
<td>Waste collection capacity</td>
</tr>
<tr>
<td>Police</td>
<td>Crimes solved</td>
<td>Hours of patrolling in street</td>
</tr>
<tr>
<td>Tax authority</td>
<td>Tax revenues, Tax returns investigated</td>
<td>Tax effort</td>
</tr>
<tr>
<td>Education</td>
<td>Test scores</td>
<td>IT services for students, Teacher quality</td>
</tr>
<tr>
<td>Health</td>
<td>Health outcomes</td>
<td>Number of available beds</td>
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<tr>
<td>Public transport</td>
<td>Number of passengers, Revenues</td>
<td>Network length, Average service frequency</td>
</tr>
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References


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