The Impact of CO₂ Tax: A Simulation Analysis for the Norwegian Petroleum Sector

CHEN Zhi & Fei MA

Supervisor: Professor Stein Ivar Steinshamn

Master thesis, Economics and Business Administration, Energy, Natural Resources and the Environment

NORWEGIAN SCHOOL OF ECONOMICS

This thesis was written as a part of the Master of Science in Economics and Business Administration at NHH. Please note that neither the institution nor the examiners are responsible – through the approval of this thesis – for the theories and methods used, or results and conclusions drawn in this work.
Abstract

In the late March, Norway submitted its climate target to the UN Framework Convention on Climate Change, where it stated its commitment to a target of at least 40 percent of emission reduction by 2030 compared to the 1990 level. As a main policy to reduce the emissions, the CO₂ tax has already been implemented in Norway since 1991. In this paper we analyze the impact of CO₂ tax on emissions, specially focusing on the emissions from the Norwegian Petroleum sector. We start our analysis with a simple Hotelling-type model to simulate the Norwegian petreolem sector. Following the model, we do simulations using different values of carbon tax rates (represented by the percentage of unit cost). The analysis indicates that there is a linear relationship between the carbon tax rates and the amount of petroleum extracted: each percentage of increase in carbon tax may reduce the emissions by nearly 0.1083 percent in the first year. Besides, the impacts also include a loss of approximately 0.1974 percent of total resource wealth and an 0.0825 percent of extraction period extension. In addition, we conclude that the change of interest rate and total resource reserves have a significant impact on the carbon emissions as well. However, it should be noted that all the results of our analysis are subject to a series of assumptions in the model and the simulations.

Key Words: CO₂ tax, CO₂ emissions, the Norwegian petroleum sector, simulation analysis
Acknowledgements

First, we would like to thank our supervisor Professor Stein Ivar Steinshamn for his inspiring guidance as well as the insightful advice throughout the thesis writing. Moreover, we would like to express our gratitude to all our dear friends for their support and company during the entire period.
## Contents

Abstract .................................................................................................................................. - 1 -

Contents................................................................................................................................... - 2 -

1. Introduction ....................................................................................................................... - 4 -

2. Literature Review and Theoretical Background ............................................................... - 7 -
   2.1 Literature Review ........................................................................................................ - 7 -
   2.2 Theoretical Background ............................................................................................ - 11 -
   2.2.1 The Greatest Market Failure ............................................................................... - 11 -
   2.2.2 Carbon Taxes Principles ..................................................................................... - 13 -
  2.3 Concerns on Free-riding and Carbon Leakage.......................................................... - 16 -

3. Carbon Tax and the Norwegian Petroleum Sector .......................................................... - 17 -
   3.1 Norway’s Climate Policy........................................................................................... - 17 -
   3.2 Carbon Taxes in the Norwegian Petroleum Sector ................................................... - 18 -
   3.3.1 Norway’s Petroleum Production ......................................................................... - 19 -
   3.3.2 Emissions from the Petroleum Sector .................................................................. - 19 -

4. Model Structure and Parameters ..................................................................................... - 21 -
   4.1 The parameters .......................................................................................................... - 21 -
   4.2 A specific utility function .......................................................................................... - 27 -

5. Simulation Analysis ........................................................................................................ - 29 -

6. Conclusions ..................................................................................................................... - 35 -

Reference................................................................................................................................ - 36 -
1. Introduction

The increasing awareness of global warming has drawn people’s attention to human-induced greenhouse gas emissions. As the International Panel of Climate Change (IPCC) stated in its latest report (Climate Change 2014 Synthesis Report) ‘Anthropogenic greenhouse gas emissions have increased since the pre-industrial era. This has led to atmospheric concentration of carbon dioxide (CO2) that are unprecedented in at least the last 800,000 years. Their effect have been detected throughout the climate system and are extremely likely to have been the dominant cause of the observed warming since the mid-20th century.’ As a major greenhouse gases, the anthropogenic CO2 stems mainly from combustion of fossil fuels and other industrial processes. On the other hand, the vast amount of literatures discuss about the peak oil theory\(^1\), which concerns the energy supply security in the future resulting from the resource scarcity. Moreover, much of them is alarmism (Helm, D. (2011a)).

The politicians have taken actions to try to reconcile these two debates with a wide variety of climate polices. The basic principle is to cut the demand for fossil fuels in order to reduce greenhouse gas emissions, hence to enhance climate change mitigation. At the same time, the substitutes of fossil fuels are to be developed and energy efficiency improvement to be encouraged as well, to resolve the issue of energy supply security. Among those, a carbon tax has been considered as one of the main economic instruments. As a type of Pigovian tax, it is levied on the carbon content of fuels, given that the greenhouse gas emissions are closely related to the carbon content of the fuels.

Norway is a country quite depending on its rich natural resources, including oil and gas. As the largest industry, the petroleum industry has been of high significance for the Norwegian economy since its production started in the early 1970s on the continental shelf. In 2013, the petroleum sector’s share of national GDP reached 21.5 percent; share of state revenue amounted to 29.1 percent, share of total investment and total exports were 30.7 percent and 48.9 percent. Moreover, the importance of the petroleum industry is also represented by the

---

\(^1\) Peak oil refers to a time when the petroleum extraction rate starts to decline. It represents a decline of available petroleum supply, although there have been arguments against the theory based on discoveries of new oil reserves as well as confidence in substituting oil.
Government Pension fund (corresponding to about one million kroner per Norwegian citizen), as well as employment.

On the other hand, the petroleum industry is a major contributor to the national greenhouse gas emissions. According to Greenhouse Gas Emissions 1990-2012 National Inventory Report, the national total CO$_2$ emissions increased by 27 percent from 1990 to 2012. Among the total, emissions from the oil and gas industry increased by 77 percent in the period. In 2012, CO$_2$ accounted for 84 percent of the national total greenhouse gases, 30 percent of which originated from the oil and gas industry. The report suggested the increase in emissions came from the growing production of oil and gas as well as the export of natural gas pipelines.

In order to reduce the emissions, Norway was among the first countries that introduced carbon taxes over two decades ago, in the beginning of 1990s. It is consequently of great interest to discuss the effectiveness of carbon taxes in Norway, especially in the petroleum sector.

Do the carbon taxes work in reducing carbon production hence emissions in Norway, especially in the petroleum sector? In 1994, ECON Energy and SINTEF (Stiftelsen for industriell og teknisk forskning$^2$) jointly conducted an ex post evaluation studying the socio-economic effects of the carbon tax on oil and gas extraction. It pointed out that emissions would probably increase in the future resulting from a rise in production, greater use of gas and phasing out of more fields. In 1997, a follow-up report concluded that the total effect was 8 percent less in CO$_2$ emissions from continental shelf than they would have been, and 3 percent of the reduction could be ascribed to CO$_2$ tax. Other studies (e.g. Sinclair (1992), Sinn (2008)) however argued that these climate policies are not useful to reduce emissions.

In this paper, we first give an overview of selected literature that studies the effectiveness and designing of carbon taxes in Section 2, as well as the relevant theoretical background. Norway’s climate policy, specifically carbon taxes and the Norwegian petroleum sector are studied in Section 3. In Section 4, we use a simple Hotelling-type model to simulate the Norwegian petroleum sector. Based on the model, we do simulations analysis to illustrate how the envisaged carbon taxes affect the amount of petroleum extracted hence emissions in

---

$^2$ The largest independent research organization in Scandinavia, for more information http://www.sintef.no/
Norway in Section 5. The effects of the change in interest rate and total resource reserves are also analyzed in our simulations. At last, Section 6 concludes the paper.
2. Literature Review and Theoretical Background

2.1 Literature Review

Climate polices are designed by the institutions to reduce the emissions since it is usually considered as an effective tool to reduce the carbon demand. But do the carbon taxes work on reducing emissions? And how do we design an optimal carbon tax? Many studies have showed great interest in analyzing the effectiveness and design of carbon taxes. Sinclair (1992) for instance noted that such climate policies as carbon taxes could be ‘pointless or even damaging’. The conclusion is based on the point that both the demand and supply side affect the resource extraction, unlike the one-sided analysis with only the demand side. It believed that we should also consider the fuel producers’ time profile of production, considering the depletion rate as endogenous.

No matter what is the initial level of carbon tax, the study suggested a steadily decreasing rate of carbon tax over time, since the falling tax rate could encourage the suppliers to delay resource extraction in order to increase their net present values (NPVs). In contrast, Hoel and Kverndokk (1996) concluded that the optimal carbon tax should initially be increasing but ultimately fall and approach zero when time goes to infinity. It should start reducing before the carbon stock in the atmosphere (negative externalities) reaches its maximum. It combined the theories of emission externalities, modelled by a positive relation with the carbon stock in the atmosphere and the exhaustibility of fossil fuels, represented by the increasing extraction costs in the model.

Similar to Sinclair (1992), Sinn (2008) concluded that the measures aiming at reducing carbon consumption are useless, as they are unlikely to flatten the carbon supply path. What is more, if the discounted value of the carbon price is reduced more at present than in the future as a result of these measures, the resource owners will probably choose to accelerate their extraction. This could thus exacerbate the global warming. This phenomenon is called Green paradox. The successful policies have to be able to flatten the carbon supply path in the energy market; otherwise, the Green paradox would occur. In addition, the study also explained the strong link between the extraction of carbon and the production of CO2, as a consequence of both the law of chemistry and the lack of technologies (such as carbon capture
and storage CCS). In conclusion, the demand reducing policies should also be able to reduce the carbon supply.

Sandal and Steinshamn (2008) points out that such studies as Sinclair (1992) are not correct to demonstrate that the tax rate should be decreasing over time. It suggests that the time path of the optimal tax is quite sensitive to the form of the pollution decay function and discounting. The paper develops an optimal feedback control law, which allows deriving the optimal corrective tax as a function of cumulative emission levels.

On contrast, Hoel (2010) noted that the carbon taxes could lead to declining emissions no matter what time profile the carbon taxes have. According to the analysis, the carbon taxes that rise at a rate above the discount rate may induce the near-term emissions to increase. If the growth rate is significantly high, the carbon tax is still probably desirable although it may lead to an increase in present and near-term emissions.

Focusing on Norway, the study *Towards a More Cost-effective Environmental Policy in the 1990s* conducted an ex ante analysis using Statistics Norway’s macro-economic model MODAG in 1992. The study included three scenarios, compared to a reference alternative with no new measures introduced. The result showed that the emissions would increase by 30 percent by 2000 in the reference alternative. On contrary, the other scenarios showed that the unilateral taxes were high enough to meet the target by 2000³, however would reduce the industrial growth by 0.5 to 1 percent per annum. Hence, the unilateral Norwegian taxes would make little sense. The report concluded that it would be much cheaper to reduce emissions in other countries.

As we mentioned earlier, an ex post analysis conducted by ECON Energy and SINTEF in 1994 was based on interviews with companies from Norwegian continental shelf (NCS) regarding their operational and investment decisions in the period 1991-1993. The study concluded that emissions per unit of oil/gas reduced by about 8 percent in the period as a consequence of the measures implemented in this period. It also stated that there is no great potential for further emission reduction per unit. A removal or reduction of the tax would not have any great effect on emissions per unit. A follow-up report in 1997 concluded that the

---

³ The Norwegian national target for emissions was to stabilize the growth of CO₂ emissions gradually during 1990s at the latest by 2000.
total effect was that the CO₂ emissions from continental shelf were 8 percent less than they would have been. And three percent of the reduction could be ascribed to CO₂ tax.

Different from ECON report, Lindeberg (1996) (cited in Andersen et al. (2000)) concluded that there was potential for further reduction in emissions from the petroleum sector, only if the tax was retained and the policy certainty was ensured. It also pointed out that the CO₂ tax had great impacts on the operational decisions in oil production on the continental shelf, leading to only 18 percent rise in the sector’s emissions (in comparison with 55 percent increase in oil production).

Both the Ministry of Finance (1998) and Statistics Norway (1998) conducted ex ante analysis and noted that wide-ranging taxes would be needed. To achieve large reductions in emissions specially from oil companies on the continental shelf, the Ministry of Finance (1998) using the macro-economic model MODAG pointed out that it would cost around NOK 1,000/ton of CO₂. While Statistics Norway (1998) suggested a CO₂ tax of NOK 700/ton CO₂ on all types of fossil fuels from all sectors, given that the result of the dynamic general equilibrium model MSG-6 showed a fall in CO₂ emissions of 13.3 percent, with 0.4 percent fall of welfare.

From sectoral level, compared to other sectors in Norway, Dragsund, etc. (1999) pointed out that it would be very costly to conduct future measures in the oil industry compared to opportunities in other sectors. This study combined both ex post and ex ante analysis comparing the cost from the aspects of effects of measures implemented as well as potential future measures. It described the individual emission reducing measures introduced by companies and stated how much CO₂ reduction could be reached. The result showed that the costs of future measures in the oil industry would be very high in comparison with measures in other sectors.

ECON (1999) recommended that the CO₂ tax should be repealed or reduced for the oil industry because the sector is heading for a crisis, especially as a result of low oil prices at that time. The tax has an unfortunate effect on marginal projects. A recent study, Ekaterina et al. (2015) found that oil and CO₂ price might have affected emission intensities on the Norwegian continental shelf. It compared different policy alternatives for the reduction target: CO2 tax, more restrictive permission to improved oil recovery (IOR) projects, fields’
electrification requirement, pointing out that CO$_2$ price is more cost-effective. On this perspective, the study suggested to increase the CO$_2$ price to reduce emissions.
2.2 Theoretical Background

The general consensus on climate change is that it is an example of market failure involving negative externality and public goods. Nevertheless, it has its own features different from others. In this section, we start with the descriptive discussion about the climate change, which has been considered as the greatest market failure by the Stern Review. To correct the market failure, a wide range of policies have been designed. Carbon taxes and command and control regulation are the main environmental economic instruments for emissions reduction hence climate change mitigation. In this section, we also make a comparison between these two instruments, following the economic theory behind the implementation of carbon taxes.

2.2.1 The Greatest Market Failure

The natural greenhouse effect refers to the greenhouse gases acting as a partial blanket for the longwave radiation from the Earth’s surface. It leads to an appropriate global mean surface temperature of 14 °C, instead of -19 °C that is much colder for mankind to live (Le Treut et al. 2007). This blanketing effect has however been intensified by human activities due to the release of greenhouse gases, resulting from the combustion of fossil fuels and forest removal. The Stern Review estimated that the stock of greenhouse gases in the atmosphere could reach 550 ppm CO₂e, which is double pre-industrial levels by 2035, and continue increasing thereafter. It will reach about 900 ppm until 2100. Consequently, the global average temperature is to increase over 2°C and even over 5°C in the longer term. This radical rise is equivalent to the change from the last ice age to today.

The report stated that climate change is the greatest market failure the world has ever seen, due to its causes and impacts globally, as well as considerable costs-related uncertainties. The climate change could have serious impacts on all the countries from all aspects: development, human life and the environment. The poorest countries and populations would be the most vulnerable, although they have never been the major contributor to climate change. As for the total cost of climate change under BAU emissions, the report estimated it would be equivalent to an average reduction in global consumption of at least 5 percent.

In environment economics, anthropogenic climate change as a result of greenhouse gases emissions is considered a negative externality. This is due to that most of the effects of
emissions will fall on the whole society, next generation who are not usually directly involved. The producers conducting activities however do not take responsibilities for these external negative effects. Instead, they make decisions by considering only their private marginal cost. From another perspective, to reduce emissions is often considered as a public good, for which the producers do not have much incentive. This leads to market failure by over production therefore emissions.

In order to show the market failure arising from negative externalities of emissions, we use a simple diagram below (see Figure 1). Due to the external cost from emissions on the society, the marginal social cost (MSC) is larger than the marginal private cost (MPC). As we can see in Figure 1, the MSC curve is higher than the MPC curve. In an unregulated market, the producers decide their production level based on marginal private cost instead of marginal social cost. Consequently, the price and production level in a free market are represented by $P_1$ and $Q_1$. However, $Q_2$ (lower than $Q_1$) instead represents the optimal output level. For the production level higher than $Q_2$, the marginal social cost is larger than the marginal private cost.

**Figure 1. Negative externalities of emissions**
2.2.2 Carbon Taxes Principles

Given that the human-induced climate change is an example of market failure involving negative externalities and public good, the government intervention is needed to correct it. Policies cover a wide range of alternatives: emission pricing, performance standards and technology mandates. Emission pricing have been considered by the economists to be among the most cost efficient alternative to reduce emissions, as a consequence of its potential to achieve the emission reduction target with the lowest cost. Emission pricing includes carbon taxes and cap and trade.

Carbon Taxes Principles

The analysis of environmental economists is based on the competitive market model, using supply and demand curves. As we discussed in 2.2.1, the market failure exist due to the marginal private cost fails to represent the marginal social cost, which includes the external cost, resulting in higher level of emissions. In order to achieve the socially optimal level of emissions, let us consider the competitive model commonly used in environmental economics. As more emissions produced, their marginal external costs (MEC) increase. We analyze MEC by relating the levels of emissions with their associated marginal damages (MD). The marginal damage function represents the relation between the change in damages imposed on society from one unit change of emission, denoted by an up-sloping curve. The area under the marginal damage curve denotes the total damages.

On the other hand, the polluters have abatement cost if they tend to reduce the emissions, which covers the costs of every possible way of reducing emissions. The marginal abatement cost (MAC) curve is downward sloping. The polluter is expected to choose the easiest (cheapest) way first to reduce the emission, with the current emission level (where MAC =0) as the starting point. In this framework, the socially optimal level of emissions is achieved when the marginal damages of emissions equal the marginal abatement costs (MD = MAC).

As a type of Pigovian tax, carbon taxes are levied on the carbon content of fuels, given that the CO₂ emissions are closely related to combustion of the carbon content. The basic principle of CO₂ tax is to reduce emissions by raising the price of inputs and services that contributes to such emissions (AC Christiansen, 2001). It charges the CO₂ emissions at a price equal to the marginal damage caused by the negative externality, so that decision makers take account of
the external cost, thus leading to an optimal production level. In theory, the taxes that are cost effective ought to cover all the sources from all the sectors. Moreover, the carbon price should be set equal to the external social cost of emissions. Nevertheless, there has been no consensus on how large the external social cost is. There have been extensive discussion about the optimal carbon tax rate, with quite different results. Stern (2006) for instance suggested the external cost exceeding $300 per ton, while Nordhaus (2007) estimated a carbon tax at $30 per ton of carbon, less than one tenth of the Stern number. The big difference is partly due to how one should discount the future, discount rate.

Carbon Tax vs. Cap and Trade System

It is consensus that carbon pricing, including carbon tax and cap and trade system, is most favored by economists to reduce CO₂ emissions. Both of the systems encourage the producers to reduce emissions during their production processes, by exerting an extra cost for emitting. Consequently, a higher price of affected goods relative to other goods will promote the consumption patterns, leading to less consumption of the carbon intensive goods.

Under a carbon tax, the price of the carbon is established directly by the regulatory authorities in form of carbon tax rate. Hence, the emission reduction is uncertain under the tax system. Relatively, under a cup and trade system, the overall emission level is given by the authority in form of allowances. The price of carbon is thus yielded in the market for allowances, thus is characterized by uncertainty. Theoretically, despite of the difference in establishment of carbon price, those two systems enable all the affected producers to face the same price of emissions. All will reduce their emissions up to the point where the abatement cost equals the emission price.

Although the two market-based instruments have obvious advantages over direct mandates, there are disagreements as to which form is better, carbon taxes or cap and trade. Goulder and Schein (2013) stated that the two instruments have equal potential in terms of even burden distribution between producers and consumers, preserving international competitiveness and avoiding emission offsets, if they are properly designed. The design of the instruments is as important as the choice of which one to employ. The paper also pointed out policies that consider emission prices exogenously have advantages relative to others. Those polices include carbon tax and a hybrid system (a cap and trade system with a ceiling or floor price).
The reason is that exogenously specified prices could avoid volatile emission price and minimize the expected policy errors from aspects of uncertainties about costs and benefits.

In aspect to administration fee, the cap and trade system might generally occur higher costs relative to carbon tax, since the regulatory authorities must monitor the emission levels, establish a registry for allowances and keep track of allowance trades and the corresponding changes in ownership of allowances. It is also necessary to note that the administrative costs along with monitoring emissions could be considerably lower under an upstream system (Goulder and Schein (2013)).

Another difference is the volatility of emission price. Under the carbon tax, the emission price is the tax rate. It is therefore not volatile. Under the cap and trade system, the volatility of emission price is definitely an issue. The regulatory authorities set the total emission allowances in the market, absolutely inelastic supply. The demand from the carbon intensive producers are changing. As a result, the allowance (emission) price is volatile. This leads to further uncertainties as to costs and benefits of emission reduction. Some researchers show preference to carbon tax based on this perspective.

Speaking of the coordination with other policies, under the cap and trade system, the implementation of an additional emission-reducing policy would bring no further emission reduction. While under the carbon tax, further reduction in emission could be achieved with another policy. This significant advantage of carbon tax system was firstly analyzed in Fischer and Preonas (2010), and then Goulder and Stavins (2012). The reason is that under cap and trade system, the overall emissions is certain and determined by the cap. The reduction in emissions due to the additional policy would cause the price of allowance to fall, but the overall emissions will not change since all the allowances will be demanded in the end.

Revenue raising is another potential advantage of carbon tax over cap and trade system. A carbon tax could raise revenue for the governments. It then can be used to for instance finance emission reduction programs. It can also be to lower distortionary taxes, which could thus reduce the aggregate net costs of the policy.
2.3 Concerns on Free-riding and Carbon Leakage

It has been proved difficult to build programs to encourage countries join international climate agreements. One important reason is that the provision of emission reduction, as a public good is subject to free-riding problem. The benefits of emission reduction by other countries can be enjoyed by one country without taking any domestic abatement measures, and then free-riding occurs. The climate change mitigation is considered as a typical transnational free-riding problem. The failure of the Kyoto Protocol, the only important international climate treaty, can be seen as the result of free-riding problem (Nordhaus 2015).

Given the existence of reciprocal externalities in emission reduction between countries, the outcome of emission reduction in one country or area does not only depend on its own actions, also on what other countries and areas do. Consequently, the country make its climate polices after observing how other countries acted or building its expectation about how others will act.

Suppose other countries or areas will not have such contingent carbon tax. A higher carbon tax in Norway would probably induce an increase in emissions outside the country, and then carbon leakage occurs. It will cause an increase in the total abatement costs, resulting in a less effective climate policy. The introduction of a higher carbon tax in Norway will cause price difference on greenhouse gas emissions, which also indicates the existence of carbon leakage (Kallbekken et al., 2007).

In the short term, carbon leakage may make the industry lose international market share to the competitors that are not covered by the higher carbon tax. Moreover, the firms have incentives to relocate its capital to other countries with less contingent policies, which applies to a longer period. With the additional cost associated with a higher carbon tax, the ability to pass through this additional cost on product prices plays important role.
3. Carbon Tax and the Norwegian Petroleum Sector

3.1 Norway’s Climate Policy

Norway’s climate policy involves mainly the Framework Convention on Climate change and the Kyoto Protocol. Since the end of 1980s, climate change and greenhouse gas emissions have been the concern of Norway. The first national target for CO₂ emissions in the SIMEN report in 1989 was scheduled by the Norwegian Parliament (Hovden and Lindseth, 2002). It was to stabilize the growth of CO₂ emissions gradually during 1990s at the latest by year 2000. This target was then tightened to a stabilization of CO₂ emissions at 1989 level by 2000 as a consequence of arguments from the opposition parties for more ambitious target. It must be mentioned that there were two fundamental assumptions that SIMEN report made. First, Norway’s economic development will be dependent on other sectors instead of oil and gas industry. Second, CO₂ taxes would be universally introduced in Norway. Those two assumptions obviously did not hold.

Sometime later in 1992, a statement from the Ministry of the Environment (MoE) showed a less committed attitude towards the emission stabilization target: ‘the stabilization target should not become an incantation requiring disproportionately greater efforts from Norway than from other nations’ (Nilsen 2001: 164). In 1995, White Paper 41 (MoE 1995) confirmed to give up the national CO₂ emission stabilization target by 2000, given the development of oil industry and lack of an international climate regime.

According to the latest White paper (Report No. 21 (2011–2012) to the Storting (white paper) Summary) on Norwegian policy, Norway is committed to 30 percent of national emission reduction up to 2020 compared to 1990. This national target is consistent with its commitment under the second period of Kyoto Protocol, which corresponds to average annual emission reduction at 84 percent of the 1990 level by 2020. Moreover, the Norwegian government suggests that Norway reduce greenhouse emissions by at least 40 percent compared to 1990 level, which increases the level of ambition in Norwegian climate policies. Another objective of Norwegian climate policy is that Norway will be carbon neutral in 2050.

In order to meet its target, Norway adopted carbon taxes in 1991 and confirmed its position as a pioneer in climate change mitigation and greenhouse gas emission reduction. It now covers
about 60 percent of the total greenhouse gas emissions of the country (Norway’s Sixth National Communication). More about the Norwegian carbon taxes will be presented in the following part. Beside of carbon taxes, Norway introduced a national emission tradable permits system (ETS) in 2005. It mainly covered the same sources as the EU quota system based on free quotas, accounting for 11 percent of the national total greenhouse gas emissions. The exemption are emissions that are already regulated by carbon taxes. The system was later expanded to include some of the industries that are exempted from the carbon tax, for instance parts of the process industry (NOU 2007, Norwegian Climate and Pollution Agency 2012), increasing the proportion of coverage to nearly 40 percent. The carbon tax rate of offshore industry is reduced to compensate the rising cost as a consequence of ETS. From 2013, the EU ETS covers about a half of the total emissions.

3.2 Carbon Taxes in the Norwegian Petroleum Sector

The Norwegian carbon tax was regulated for the first time in Act 21 December 1990 no 72 relating to tax on discharge of CO2 in the petroleum activities on the continental shelf in 1990. It stated that CO2 tax is charged on petroleum that is burnt and natural gas that is discharged to air and CO2 separated from petroleum and discharged to air, on installation used in connection with production or transportation of petroleum. Carbon taxes has been the most significant alternative for reducing emissions from the petroleum sector, bringing in improvement of technologies and development of other emission reducing measures.

An optimal tax system requires a uniform tax rate for all sources from all sectors (Hoel 1996). However, extensive exemptions and wide differentiation of tax rates characterize the Norwegian carbon taxes, just like most cases. The carbon taxes started at a rate of US $ 40.1 per ton of CO2 on gasoline, diesel, mineral oil, oil and gas connecting to extraction activities in North Sea (OECD 1997). Some industries with relatively high emissions are partly or totally exempted, for instance metal producing process industries. Other industries exempted are fishing, air and ocean transport, cement manufacturing and land based use of gas (Bruvoll and Larsen 2004).

As of 1 January 2013, the CO2 tax on emissions from offshore petroleum activities rose by NOK 200 per tonne, almost twice the previous level. The goal was to increase the use of electricity generated onshore in the offshore industry (Norwegian Climate Policy).
3.3 The Norwegian Petroleum Sector

As Norway’s largest industry, the Norwegian petroleum sector has been of great significance to the country from aspects of value creation, state revenue and export value (Facts_2014). In 2012, the petroleum sector (excluding services) contributed to about 29 percent of Norway’s total revenues. The Government Pension Fund, where the revenues from the petroleum activities are deposited into, was valued to NOK 5038 billion as of the end of 2013. It corresponds to about 1 million NOK per Norwegian citizen. Moreover, the petroleum industry also shows its importance in terms of employment and investment. The number of people employed directly and indirectly in the petroleum industry is about 250000. In 2012, the investments achieved nearly 29 percent of the national total fixed capital investments.

3.3.1 Norway’s Petroleum Production

In 2012, Norway was ranked as the fifteenth largest oil producer and sixth largest gas producer in the world. The total petroleum production in 2014 was about 218.6 million standard cubic meters oil equivalent. Currently there are 78 fields in production on the Norwegian continental shelf, producing about 1 925 000 barrels of oil, NGL and condensate per day (indicated by the production figures in February 2015).

About 44 percent of the total estimated recoverable resources on the Norwegian continental shelf have been produced. The petroleum production is expected to slightly increase in the coming years, followed by a decrease in a longer-term perspective (Norway’s Sixth National Communication). The production level will rely largely on the number of new discoveries and their sizes in the long term.

3.3.2 Emissions from the Petroleum Sector

Emissions from the petroleum sector generally come from combustion of natural gas in turbines, flaring and combustion of diesel (see Figure 2 (Facts_2014)). We can see that nearly 80 percent of the emissions stem from combustion of natural gas in turbines. In Norway, the petroleum activities accounted for around 26 percent of emissions in 2012. The report also stated that emissions from the petroleum sector are expected to increase up to around 2017 and then drop gradually.
Figure 2. CO₂ emissions from the petroleum activities in Norway in 2012
4. Model Structure and Parameters

Instead of studying on how the different sectors in the economy will respond, in our thesis we choose to focus on the fossil fuel market and how the implementation of the carbon taxes will affect the whole market, especially the amount of petroleum extracted.

We are going to use a Hotelling-type model to simulate the Norwegian fossil fuel market. This model is developed by Rosendahl (1994). In this model, we will make some assumptions to simplify the situation. Firstly, we ignore the market imperfections like the existence of OPEC in the oil market and also the technology changes. So under our time frame, the relationship between the amount of fossil fuel used and the associated carbon emissions is fixed. In addition, we assume that the unit extraction cost of the fossil fuel is constant. And since CO₂ has made up the majority of the greenhouse gases, we ignore other GHGs. Furthermore, we also ignore the fact that the carbon tax could have influences on other fossil fuel markets, which will cause unclear substitution effects between fossil fuels. In this thesis, we use petroleum to represent all the fossil fuels in the real market, and study how the carbon taxes affect the petroleum market.

4.1 The parameters

We are modeling a competitive petroleum market. We use $c$ and $v$ to represent the constant unit cost of petroleum extraction and the unit carbon tax respectively. In our analysis, there is only one big producer in the market and $c$ is its unit cost. And we assume there is no information asymmetry between the government and the producer, so the carbon tax is imposed directly on the cost. Also, a limited amount $A$ of the fossil fuel resource is assumed. Because we have already ignored the technical progress, the relationship between the consumption amount of petroleum and the carbon emissions is also fixed in our analysis. We use $x_t$ to stand for the extraction or consumption rate of the petroleum at time $t$, and use $u(x)$ to represent the total social utility by consuming $x$. Finally, we use $r$ as the discount rate.

The optimal extraction path for the society as a whole is found by maximizing

$$\int_0^\infty e^{-rt} (u(x_t) - cx_t - vx_t) dt, \quad x_t \geq 0$$

(1)
with respect to $x_t$, over a possibly infinite time horizon $[0, \infty]$. Here we assume that the total social utility $u(x_t)$ includes both producers and consumers’ surplus, and also the externalities caused by the consumption of the petroleum. Therefore, the social value we maximize here is the total social utility subtracts the total costs of the production and the tax revenue of the government. Again, the maximization is constrained by the condition:

$$\int_0^\infty x_t \, dt = A$$  \hspace{1cm} (2)

The amount of the remaining reserves $a(t)$ will decrease due to the petroleum production, the relationship between $a(t)$ and $t$ is as follow

$$\frac{da(t)}{dt} = -x_t$$  \hspace{1cm} (3)

We use the optimal control theory to find out the solution to (1) and (2). The Hamiltonian of these two equations is:

$$H(t, A, x_t, \lambda_t) = [u(x_t) - (c + v)x_t]e^{-rt} - \lambda_t x_t$$  \hspace{1cm} (4)

According to the necessary conditions, the optimal extraction path must fulfill the following relation:

$$[u'(x_t) - (c + v)]e^{-rt} \leq \lambda_t \quad (= \lambda_t \ for \ x_t > 0)$$  \hspace{1cm} (5)

The shadow price of the resource stock at time $t$, $\lambda_t$, is constant, because the extraction costs do not depend on the accumulated production. We have $\frac{\partial H}{\partial A} = 0$. For positive extraction, (5) can then be written as
\[
\frac{d}{dt}(c + v) = \lambda e^{rt}
\]  
(6)

If at the equilibrium point, the consumer price is higher than the marginal utility, the consumers will choose not to consume that much petroleum. If at the equilibrium point, the consumer price is lower than the marginal utility, the consumers will choose to consume more petroleum to maximize their welfare. Thus, at the equilibrium point, the consumer price must equal to the marginal utility. Therefore, the optimal path is realized by letting the consumer price \( p_c \) equal to the marginal utility \( u'(x_t) \).

\[ p_c = u'(x_t) \]  
(7)

In order to make this model to be more representative to the reality, we make some assumptions about the utility function. The utility function \( u(x) \) is assumed to be a monotone increasing function, which means \( u'(x) > 0 \). And we assume that the marginal utility function is a monotone decreasing function, \( u''(x) < 0 \). Moreover, we require \( u'(0) > (c+v) \), because otherwise the producers will choose not to produce in the very beginning. Finally, we assume that \( u'(0) < \infty \) and \( u'(\infty) = 0 \). The shape of the utility function should be like the one in the Figure 3.

**Figure 3. The shape of the utility function**
Before we go any further about this model, we need to discuss about the economic rent first. In economics, rent is a surplus value after all costs and normal returns have been accounted for, i.e. the difference between the price at which an output from a resource can be sold and its respective extraction and production costs, including normal return. So here in our analysis we define the unit resource rent \( \pi_t \) to be the difference between the consumer price \( p_c \) and the aggregate of unit cost and unit tax \( (c + v) \). Furthermore, we use producer price \( p_p \) to represent the price producers receive net of tax. And the equations for these two prices are shown as follows

\[
p_c = u'(x_t) = c + v + \pi_t \tag{8}
\]

\[
p_p = p_c - v = c + \pi_t \tag{9}
\]

From the two equations above, we can see that the only difference between the consumer price and the producer price is the carbon tax.

Our main focus here is how the solution path can be characterized, and how a shift of the carbon tax rate will affect the solution path. If we combine equation (6) and (8) together, we can derive that:

\[
\pi_t = \lambda e^{rt} \tag{10}
\]

And if we set \( t=0 \), then

\[
\pi_0 = \lambda \tag{11}
\]

where \( \lambda \) is the shadow price of the resource stock at time 0.

Hotelling rule is obtained from the optimization problem, i.e. the unit resource rent \( \pi_t \) is growing exponentially with rate \( r \) until the resource is depleted:
\[ \pi_t = \pi_0 e^{rt} \quad (\text{For } x_t > 0) \tag{12} \]

If we combine the equations (8), (9) and (12) together we can derive that both the consumer price and the producer price are increasing over time. Then, what we can infer from (8) is that \( x_t \) will decrease over time, because that the marginal utility will be higher when there is less resource in the market. And because we have already assumed that \( u'(0) < \infty \), at a finite point of time \( T \), \( x_t \) will reach zero. Moreover, we combine the equations (8) and (12), and derive the following equation:

\[ u'(0) = c + v + \pi_0 e^{rT} \tag{13} \]

What we can derive from the function \( p_c = u'(x_t) \) is that the extracted amount \( x_t \) is implicitly a function of the consumer price \( p_c \). Here we use \( F() \) to represent the inverse function of \( u'(x_t) \), i.e. \( F(p_c)=[u']^{-1}(p_c) \). Then we get:

\[ x_t = F(c + v + \pi_0 e^{rt}) \tag{14} \]

Because of the assumption we made earlier (\( u'(0) >(c+v) \)), the petroleum producer will choose to extract the whole resource. Therefore we can transform the constraint in the optimization problem into an equation, and replace \( \infty \) with \( T \). After that, substituting (14) into this constraint, and using (13), we can characterize the solution in the following way, and also determine the two unknown variables \( \pi_0 \) and \( T \) uniquely:

\[ \int_0^T F(c + v + \pi_0 e^{rt}) \, dt = A \tag{15a} \]
In this model, we are also going to check how the total resource wealth responds due to the implementation of the carbon taxes. The value of the resource wealth II (at time t=0) can be expressed in the following way:

\[
II = \int_{0}^{T} e^{-rt} \pi t x_t dt
\]  

(16)

And if we substitute equation (12) into (16), we can obtain:

\[
II = \int_{0}^{T} \pi_0 x_t dt
\]  

(17)

Since the definition of A is the total amount of the resource, and we will deplete the whole resource at time T.

\[
A = \int_{0}^{T} x_t dt
\]  

(18)

From (17) and (18), we can get the equation below:

\[
II = \pi_0 A
\]  

(19)

By the help of Hotelling rule we reach the conclusion that the resource wealth is simply equal to the resource rent at time t=0 times the amount of the resource. Since A is fixed, we can just use \( \pi_0 \) to examine the change of the resource wealth.
4.2 A specific utility function

In order to satisfy the assumptions we made before, we introduce the following specific utility function. It is an exponential function, where $u'(0) = \beta < \infty$:

\[ u(x) = 1 - e^{-\beta x} \tag{20} \]
\[ u'(x) = \beta e^{-\beta x} \]

In order to be consistent with earlier assumptions, we require that $(c + v) < \beta$. The relative risk aversion (RRA) of this utility function is defined as follows:

\[ RRA = -\frac{d(u'(x_t))}{dx_t} \frac{x_t}{u'(x_t)} = -\frac{u''(x_t)}{u'(x_t)} \tag{21} \]

The intertemporal elasticity of substitution $\sigma_t$ at time $t$ is the reciprocal of the relative risk aversion:

\[ \sigma_t = \frac{1}{RRA} = \frac{1}{\beta x_t} \tag{22} \]

Thus, since $x_t$ is a monotonic decreasing function, the intertemporal elasticity $\sigma_t$ will not be constant along the extraction curve. It will be monotonically increasing, and reaches infinity when the whole resource is extracted. In the later session, $\sigma_t$ will help us find the suitable values for the other parameters, so that we can do the simulation in the right direction.

And the unique solution of the optimization problem, given by relation (15a) and (15b), can now be characterized in the following way:

\[ \int_0^T \frac{1}{\beta} \ln\left(\frac{\beta}{c+v+\pi_0 e^{rt}}\right) dt = A \tag{23a} \]
\[ T = \frac{1}{r} \ln\left(\frac{\beta-c-v}{\pi_0}\right) \tag{23b} \]

Since obviously the total amount of the fossil fuel in the reserves will not change due to the introduction of the carbon taxes, the total amount of the GHG emissions generated during the
whole period will remain the same level. However, the carbon taxes do influence the extraction path of the petroleum industry. Especially in the first few years, the extraction amount will decrease due to the implementation of the carbon taxes because the carbon taxes have increased the production costs. Therefore, in this thesis we mainly focus on $x_0$, which is the extraction or consumption amount of the fossil fuel in the first year after the implementation of carbon taxes. And we assume the emission amount of GHG is proportional to the extraction amount of petroleum. The expression of $x_0$ is as follows:

$$x_0 = \frac{1}{\beta} \ln \left( \frac{\beta}{c+v+\pi_0 e^0} \right)$$  \hspace{1cm} (24)$$

The model developed in Rosendahl (1994) is mainly focused on the relationship between the carbon taxes and the total petroleum wealth. Combining his model and the Hotelling model, we build a new small model mainly focused on the relationship between the carbon taxes and the carbon emissions in the first year.
5. Simulation Analysis

The equation set (23a/23b) we obtained in the last section is the objective function of the simulation. In order to simulate the market we make the sum of the unit cost and unit tax \((c+v)\) equal to 5 (Rosendahl (1994)). The main discount rate we use here is 0.05, as this is the normal rate of return used in Norwegian government projects\(^4\). And we still need to endow the values for the two exogenous variables \(A\) and \(\beta\). \(A\) represents the amount of the resources but the size of it is difficult to interpret. If we combine (7) and (20) together, we can derive that

\[
p_c = \beta e^{-\beta x}
\]

Since \(x\) is always non-negative, the maximum amount of \(e^{-\beta x}\) is 1. Therefore, the utility parameter \(\beta\) indicates the maximum price. And \(\beta\) also determines the curvature of the utility function in Figure 3.

Here we use other indicators to help us find the appropriate values of \(A\) and \(\beta\). These indicators are the unit resource rent \(\pi_0\), the extraction time \(T\) and the elasticity at time zero, \(\sigma\). We try different values of \(A\) and \(\beta\) to do several sets of simulations, the results are shown in the Table 1.

<table>
<thead>
<tr>
<th>(\beta)</th>
<th>(\pi_0)</th>
<th>(T)</th>
<th>(\sigma)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>4.966</td>
<td>13.9995</td>
<td>2.46</td>
</tr>
<tr>
<td>25</td>
<td>8.8555</td>
<td>16.3166</td>
<td>1.68</td>
</tr>
<tr>
<td>50</td>
<td>15.3062</td>
<td>21.5682</td>
<td>1.09</td>
</tr>
<tr>
<td>100</td>
<td>21.6912</td>
<td>29.5394</td>
<td>0.74</td>
</tr>
</tbody>
</table>

Table 1. Simulate values of endogenous variables\(^5\)

---

\(^4\) We use the recommended discount rate in Norwegian transport projects by Harald Minken

\(^5\) The units for \(\beta\), \(c\), \(\pi_0\) and \(v\) are dollar per barrel, the unit for \(T\) is year.
When the values of $A$ and $\beta$ are 0.2 and 50 respectively, the values of those indicators are most close to those in the Norwegian petroleum market. After the first simulation, we will also adjust the values of those parameters for sensitivity analyses.

So now we have all the parameters we need. Firstly, we are going to use 5% as the discount rate. And we increase the carbon tax rate gradually from 0% to 40%.

<table>
<thead>
<tr>
<th>$r$</th>
<th>0.05</th>
<th>0.05</th>
<th>0.05</th>
<th>0.05</th>
<th>0.05</th>
<th>0.05</th>
<th>0.05</th>
<th>0.05</th>
<th>0.05</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>$A$</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>$c$</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0</td>
<td>0.25</td>
<td>0.5</td>
<td>0.75</td>
<td>1</td>
<td>1.25</td>
<td>1.5</td>
<td>1.75</td>
<td>2</td>
</tr>
<tr>
<td>$x_0$</td>
<td>0.018022</td>
<td>0.017924</td>
<td>0.017826305</td>
<td>0.01772862</td>
<td>0.017631024</td>
<td>0.017534</td>
<td>0.017436</td>
<td>0.017339</td>
<td>0.017241</td>
</tr>
<tr>
<td>$\Delta% (x_0)$</td>
<td>-0.54%</td>
<td>-1.09%</td>
<td>-1.63%</td>
<td>-2.17%</td>
<td>-2.71%</td>
<td>-3.25%</td>
<td>-3.79%</td>
<td>-4.33%</td>
<td></td>
</tr>
<tr>
<td>$\Delta% (\pi_0)$</td>
<td>-0.98%</td>
<td>-1.96%</td>
<td>-2.94%</td>
<td>-3.91%</td>
<td>-4.89%</td>
<td>-5.86%</td>
<td>-6.82%</td>
<td>-7.79%</td>
<td></td>
</tr>
<tr>
<td>$\Delta% (T)$</td>
<td>0.40%</td>
<td>0.80%</td>
<td>1.21%</td>
<td>1.62%</td>
<td>2.03%</td>
<td>2.45%</td>
<td>2.87%</td>
<td>3.30%</td>
<td></td>
</tr>
</tbody>
</table>

From Table 2, we can see how the values of $T$, $\pi_0$ and $x_0$ change because of the implementation of the carbon tax (from 0% to 40%). First, we are going to find out the relationship between the tax rate $\gamma$ and the percent change of the oil extraction amount in the first year $x_0$. We made a regression out of these two sets of data. The equation is as follows

$$y_1 = -0.1083x - 0.00003$$

$$R^2 = 1$$

where $y_1$ is the percent change of extraction amount of petroleum and $x$ is the change of carbon tax rate.

From the equation above, we conclude that it is a linear relationship between these two variables. As the carbon taxes rate starts to increase, the oil producers will tend to decrease the extraction amount of petroleum, which is consistent with our previous analysis. Every time the carbon taxes rate goes up by 1%, the carbon emissions will go down by about

---

5 The values of indicators here are taken from Rosendahl (1994).
0.1083%. So if we implement a 20% carbon tax on the oil, the total carbon emissions that the whole sector generated in the first year will go down by about 2.17%.

The carbon tax does not only have an effect on the carbon emissions, but also on the total resource wealth. Another important relationship we can derive from the data above is one between the total resource wealth and the carbon tax rate. Because of the decrease of the oil production and the rise of the oil price, the economy will be affected. And here is the regression equation

\[
y_2 = -0.1947x - 0.0001
\]

\((-858.40)\)

\[R^2=1\]

where \(y_2\) is the percent change of the resource wealth and \(x\) is the carbon taxes rate.

The relationship between the percent change of the resource wealth and the carbon tax rate is also linear. As the carbon tax rate increases, the total resource wealth will decrease. When the carbon taxes rate goes up by 1%, the resource wealth will decline by about 0.1947%. Suppose we implement a 20% carbon tax on the oil, the total resource wealth that the economy could get will go down by about 3.91%. Combining this with the results above, we can derive that if we want to reduce the carbon emission by 2.17%, the total resource wealth will also decline by about 3.91%, at the same time.

Another parameter that will also be affected by the change of the carbon tax is the total extraction period of petroleum \(T\). The implement of the carbon tax will extend the total extraction period \(T\), since with a higher extraction cost, the producer will tend to delay the production of the petroleum. And the regression equation between these two variables is as follows

\[
y_3 = 0.0825x - 0.0002
\]

\((243.11)\)

\[R^2=0.99\]

where \(y_3\) is the percent change of the total extraction period and \(x\) is the carbon taxes rate.
We can see from the equation that every 1% increase of carbon taxes rate will cause the total extraction period to extend by 0.0825%. The implement of 40% carbon tax will prolong the period by 3.3%. This is also quite straightforward to understand. As the government starts to increase the carbon tax, the petroleum producer will intend to lower the extraction amount in the first few years to balance the costs. But as we can see from the equation, the effect is not so obvious. As a matter of fact, after some point of time the production amount will be higher than the tax-free scenario, we can see the extraction path in the Figure 4.

![Figure 4. Extraction path before and after tax increase](image)

From the outcome above, we know how the change of carbon tax rate will affect the carbon emissions, the resource wealth and the petroleum extraction period. In order to have a deeper understanding of these relationships, we need to do some sensitivity analysis about these parameters.

### Table 3. Results of the sensitivity analysis (r=3%, 7%)

<table>
<thead>
<tr>
<th>t</th>
<th>0</th>
<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
<th>1</th>
<th>1.25</th>
<th>1.5</th>
<th>1.75</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δ%(x_0) 0.03</td>
<td>-21.37%</td>
<td>-21.31%</td>
<td>-21.25%</td>
<td>-21.19%</td>
<td>-21.13%</td>
<td>-21.07%</td>
<td>-21.01%</td>
<td>-20.95%</td>
<td>-20.89%</td>
</tr>
<tr>
<td>Δ%(π_0) 0.03</td>
<td>28.18%</td>
<td>28.34%</td>
<td>28.50%</td>
<td>28.67%</td>
<td>28.83%</td>
<td>29.00%</td>
<td>29.17%</td>
<td>29.35%</td>
<td>29.52%</td>
</tr>
<tr>
<td>Δ%(T) 0.03</td>
<td>28.30%</td>
<td>28.20%</td>
<td>28.22%</td>
<td>28.18%</td>
<td>28.14%</td>
<td>28.09%</td>
<td>28.05%</td>
<td>28.01%</td>
<td>27.97%</td>
</tr>
<tr>
<td>Δ%(x_0) 0.07</td>
<td>-18.51%</td>
<td>-18.61%</td>
<td>-18.71%</td>
<td>-18.81%</td>
<td>-18.91%</td>
<td>-19.01%</td>
<td>-19.11%</td>
<td>-19.21%</td>
<td>-19.31%</td>
</tr>
<tr>
<td>Δ%(π_0) 0.07</td>
<td>16.60%</td>
<td>16.60%</td>
<td>16.52%</td>
<td>16.44%</td>
<td>16.36%</td>
<td>16.27%</td>
<td>16.20%</td>
<td>16.11%</td>
<td>16.03%</td>
</tr>
<tr>
<td>Δ%(T) 0.07</td>
<td>-15.01%</td>
<td>-14.90%</td>
<td>-14.90%</td>
<td>-14.91%</td>
<td>-14.91%</td>
<td>-14.89%</td>
<td>-14.89%</td>
<td>-14.89%</td>
<td>-14.89%</td>
</tr>
</tbody>
</table>
We first do the sensitivity analysis with the interest rate $r$. We keep other variables fixed and use the interest rate 3% and 7% instead of 5%. The results are shown in Table 3. The average change rate of $x_0$ under different carbon taxes rates is -21.13% when the interest rate changes from 5% to 3%, and 16.36% while the interest rate goes up to 7%. As the interest rate decreases from 5% to 3%, the average change rates of $\pi_0$ and $T$ are 28.84% and 28.14% , meaning that both $\pi_0$ and $T$ increase by 28.84% and 28.14% respectively. As the interest rate goes from 5% up to 7%, the average change rates of $\pi_0$ and $T$ are -18.91% and -14.91%, both of them decreasing by -18.91% and -14.91% respectively.

When the interest rate goes up, the petroleum producers intend to shorten the production period and increase the extraction amount in the first year. And when the interest rate goes down, the petroleum producers intend to prolong the production period and decrease the extraction amount in the first year. Moreover, it should be noted that the decrease of the interest rate $r$ would have greater impact than its increase on the extraction amount, total resource wealth and the extraction period.

<table>
<thead>
<tr>
<th>Table 4. Results of the sensitivity analysis (A=0.1, 0.3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
</tr>
<tr>
<td>$\Delta % (x_0)$</td>
</tr>
<tr>
<td>$\Delta % (\pi_0)$</td>
</tr>
<tr>
<td>$\Delta % (T)$</td>
</tr>
<tr>
<td>$A$</td>
</tr>
<tr>
<td>$\Delta % (x_0)$</td>
</tr>
<tr>
<td>$\Delta % (\pi_0)$</td>
</tr>
<tr>
<td>$\Delta % (T)$</td>
</tr>
</tbody>
</table>

After the interest rate, we use the resource amount $A$ as the target of our sensitivity analysis. Instead of 0.2, we use 0.1 and 0.3 as the total resource amount A and assume other variables constant. The sensitivity analysis results are shown in Table 4. This time $x_0$ and $T$ change in the same direction as the total resource amount changes. As the total resource amount declines from 0.2 to 0.1, the average change rates of $x_0$ and $T$ under different taxes rates are -27.65% and -29.96% respectively. The resource wealth will however increase by 38.88%. While as the resource amount $A$ goes from 0.2 to 0.3, the average change rates of $x_0$ and $T$ under different taxes rates are 19.95% and 23.50% respectively. The resource wealth will decrease by 22.71%.

When the total petroleum reserves goes up, the producers will choose to increase the extraction amount in the first year and also prolong the extraction period, but not as much as
the change of the total reserves. Because the demand for the petroleum will not change too much just because we have discovered more petroleum. The consumption of the resource is still determined by the demand curve and the supply curve. In addition, now since there are more resources, the supplies of the resources will surplus the demands, the price of the petroleum will decline. As a result, the total resource wealth will decrease due to the discovery of new reserves.

From the sensitivity analysis, we conclude both the changes of interest rate and the total resource amount will have a significant effect on the amount of carbon emissions, total resource wealth and the extraction period. The rise of the interest rate will decrease the amount of the carbon emissions, and in the same time will raise the resource wealth and extend the resource extraction period. Besides, the increase of the total petroleum reserves will increase the amount of the carbon emissions and extend the resource extraction period, but in the same time will reduce the total resource wealth. And the change of interest rate plays more important role than the change of the total reserves amount in affecting the carbon emission amount.
6. Conclusions

The analysis presented in this paper uses a simple Hotelling-type model to simulate the Norwegian petroleum sector. Considering the strong link between the extraction of carbon and the production of CO\textsubscript{2} emissions, our analysis focuses on how the amount of the fossil fuel extracted is affected with the implementation of carbon taxes. Assume that there is only one producer in the market and the government has access to information on its production cost. In this case, we are able to employ the percentage of unit cost to represent the level of carbon tax rates, for instance from 0\% to 40\%.

With the simulated values we observe, we conclude that there is a linear relationship between the carbon tax rate and the carbon emissions generated in the first year. In specific, the oil producer may decrease the amount of petroleum extracted (by 0.1083\%) as the carbon tax increases (by 1\%). In addition, the increase of carbon tax (by 1\%) also has an effect on the total resource wealth (declining by 0.1974\%) and total extraction period (prolonging by 0.0825\%). It should be noted that the figures in the results are all subject to our assumptions in the model.

The simulations in Section 5 indicate that the change of interest rate and total resource reserves have a significant impact on the emissions, total resource wealth and the extraction period. The decline of interest rate (from 5\% to 3\%), for instance may reduce the emissions by 21.13\%, increase the resource wealth by 28.84\% and prolong the extraction period by 28.14\%.
Reference


Le Treut, Hervé, Ulrich Cubasch, and Myles Allen. "Historical Overview of Climate Change Science."


