Abstract

Norwegian legislation prevents banks from sharing specific kinds of information that might have been used to better predict the creditworthiness of their customers. We construct a simple market participation model to describe how good methods of estimating default risk is likely to result in increased customer surplus and more fair, effective credit allocation. We use a game theory-framework to describe why these gains can only be realized if these estimations can be shared with other banks, as the customer will otherwise be able to reset his or her risk assessment by switching banks. We propose and evaluate three possible implementations, and remark that our analysis suggests that the full gains of improved risk assessment that is introduced with CRD IV and Basel III cannot necessarily be realized without changes in the banks ability to share information about their customers.

Our results are sensitive to assumptions about price competition between banks, rationality of customers and distribution of default probabilities.
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1 Acknowledgments

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2 Introduction and Research Question

2.1 Purpose of Thesis

The purpose of this thesis is to analyze the effects of the legislation prohibiting Norwegian banks from sharing internal risk-assessments of private customers between each other, using an analytical framework. Moreover, we aim to illustrate potential effects of enabling more open sharing of this information, and describe the conditions under which these effects may be realized.

2.2 Motivation for Thesis

When we started writing this thesis, we intended to concentrate on the pricing of risk in the loan portfolios of Norwegian commercial banks, hoping to illustrate how Norwegian banks might systematically underprice their risk because customers have increased incentive to re-negotiate their loan terms when their default-risk decreases, and less incentive when it increases. What we discovered early on was that customers did not seem to be rewarded for becoming less risky or punished for becoming more risky, at least not in terms of changed interest rates.

We have later been explained that this apparent mispricing stems from the fact that banks do not possess the same information about each customer - In fact, when a customer switches banks, much of the information that enters into the risk-assessment of a customer is lost because of Norwegian privacy legislation prevents banks from sharing some kinds of
information, such as statistics about account movements (spending, saving) and pending defaults on other loans (Sparebanken Vest, 2015; Nordea, 2015b). This effectively allows customers to reset their risk-assessment by switching to a new bank with less information. Because of this option, Norwegian commercial banks are not fully able to change interest rates in accordance with internal risk assessments without losing the business of their customers to other banks (Nordea, 2015a).

Another implication of this observation is that banks cannot always correctly assess the risk of new customers, as some information will be missing until the bank has been able to observe the customer over time (Nordea, 2015a).

2.3 Thesis Summary and Main Findings

In this thesis, we describe this situation, which we argue has been created largely because banks are prohibited from exchanging their internal risk assessments, and examine how the situation would change if information could be shared more freely.

Section 3 is a description of the Norwegian credit market with special emphasis on private customers, as well as the risk assessment procedures of Norwegian banks. Much of this section will be known to most readers, but still provides important context for our later analysis. Also note that the section also includes the definition of the bond that we will use to represent loans to private customers later in the thesis.

In section 4, we construct and use the Simple Market Participation Model to describe how the amount of information available to an individual bank affects the properties of the credit market, and describe the gains that might result from improving the information of the bank.

In section 5, we use a game theoretical framework to describe the conditions that have to be met for these gains to be realized, with special emphasis on the conditions under which a private customer will switch banks, and how this affects the bank when determining interest rates.

After we have constructed and analyzed our models, we spend section 6 summarizing our results. In section 6.1 we spend some time repeating key attributes of the Norwegian credit market, and use these to discuss the potential impact of increasing the ability Norwegian banks have to share information.

We conclude by suggesting three real-world methods to allow for better information sharing in section 7, and suggest a set of research topics that would be useful to explore in relation to this thesis in section 8.
2.3.1 Simple Market Participation Model

We have dubbed our first model as the Simple Market Participation Model. This model illustrates key properties of the credit market, with special emphasis on how many customers will choose to participate in the credit market, and which interest rates they receive. We show that a bank with much information about its customers will be able to offer lower average interest rates than a bank with little information, and find that this will increase the overall market size and consumer surplus. We also illustrate how a bank with little information will have to set interest rates in a way that is favourable to high-risk customers, and unfavourable to low-risk customers. This leads to an inefficient allocation of credit and interest costs, where the safe customers subsidize the risky customers. Simply put, we find that the cost of asymmetric information is to a large degree carried by the low-risk customers. An argument can also be made that some of the cost is carried by the bank, who under certain conditions cannot issue as much credit as they wish.

The model is largely our own work, but builds on insights from Akerlof (1970) and Stiglitz and Weiss (1981).

2.3.2 The Customer-Bank Game

The second model describes the interaction between banks and customers in terms of game theory, and illustrates how one bank will be unable to make use of good information unless the other banks on the market has access to the same information - In simple terms, because the customer could otherwise just switch to a bank with less information if his interest rate is adjusted.

Simply put, the first model describes the benefits of better information by comparing a case with complete information with a case that has asymmetric information, while this model describes why all banks need to have access to the same information for these benefits to be realized.

2.3.3 Main Findings

Given that the banks were already subject to effective price-competition, we claim that the costs of asymmetric information is actually carried by the customers who participate in the market, but whose internal risk assessment points to a lower risk than what their current interest rate suggests (henceforth referred to as good customers). This can potentially lead to a smaller credit market and a higher overall interest rate charged. Customer surplus
is in either case reduced. We show that improving the information available to the banks alleviates these problems, and suggest a set of real-world implementations.

We argue that the implementations can be expected to lead to less credit being allocated to bad customers, and more credit being allocated to good customers, subject to our definitions of “good” and “bad”. We argue that this makes the average private economy more stable, as borrowing more than your risk profile should allow for becomes more expensive. We also note that the increased information sharing will make switching banks easier for good customers, meaning that these will get an increase in bargaining power, and therefore lower interest rates. At the same time, bad customers are unable to reset their risk-assessment by switching to banks with less information.

While much of the cost of incomplete information is carried by the good customers, banks also have something to lose if asymmetric information leads to a reduction in market size due to excess supply. We show that the conditions of having anything less than a full market are less likely to be met as information flows between banks more freely, and are thus enabled to invest more capital into the private loan market.

Our implementations will, for the purposes of risk-assessment, make some of the internal information into public information, meaning that interest rates offered to a customer should change less when switching banks. The implication is that interest rates given to each customer will correlate more closely with the default risk of that customer. We also argue that this gives commercial banks better incentives to report their true risk when calculating regulatory capital requirements.

We argue that the more effective allocation of credit has a positive effect on economic growth, and that more correct pricing of risk leads to a more stable economic environment. This is because the more accurate setting of interest rates means that credit will be allocated in closer accordance with the default probability of each private customer, assuming that our models hold. This agrees with similar findings by Houston, Lin, Lin and Mae (2010), which is discussed in section 2.4.

2.3.4 Relation to new Capital Regulations

While not central to this thesis, the analysis we do is highly relevant to the new regulations introduced with CRD IV, which is a set of new capital regulations for the European banking sector based on the new Basel III framework. CRD IV is meant to assure that bank liquidity and risk is kept satisfactory, and banks are incentivized to improve their internal risk assessment and management departments. Banks that can show to sophisticated and
reliable methods are granted softer restrictions on leverage. (Sparebanken Vest, 2014).

In this thesis, we conclude that better internal risk assessment procedures can lead to several benefits, but that some of these can only be realized on the condition that this improved risk assessment is shared by all the banks that give loans to private customers\(^1\). It follows that the banks that invest large sums in risk assessment and management will get the benefit of increased leverage, but, as our model in section 5 suggests, the benefits of improved allocation of credit and setting of interest rates will not be fully realized unless this improved analysis of their customers is also shared with their less sophisticated competitors.

### 2.4 Relation to Existing Literature

There is already a sizeable body of literature concerning asymmetric information and information sharing within the banking sector. With this thesis we hope to make an addition to existing literature, in that while we draw significant inspiration and support from several excellent papers on similar subjects, they mostly either (1) treat the corporate credit market, or (2) examine the predictive power of bank information with respect to default probabilities. Broadly speaking, our thesis differs in that we concentrate on the private credit market, and in its more applied focus. Our perhaps most interesting finding is the thought that gains associated with better information cannot necessarily be realized unless this information also known to the banks competitors, which we hope is an interesting contribution to existing literature.

As already mentioned, we have applied elements and intuition from Akerlof (1970) and Stiglitz and Weiss (1981) to build a model that illustrates the potential value of information sharing with respect to customer risk assessment, and theory from Gibbons (1992) to explain which conditions must be in place to realize this potential value. The intuition from these sources is central to the rest of our thesis, so we discuss these more closely as they are applied.

Papers such as Sharpe (1990) has regarded the same subject. Sharpe describes customer relationships in the banking market, and gives a possible explanation for why and how lenders capture some of the rent generated by their old or existing customers. He is mainly concerned with the corporate credit market, but also draws parallels to the private credit market. He explains the lock-in effect (old borrowers is more inclined to borrow for the same lender rather than someone else) by information asymmetry, and not, say, that the current lender treats them particularly well. He referred to it as being ”informationally captured” in

\(^1\)CRD IV treats the sharing of financial information, but not individual customer information.
the current bank. This is largely similar to the lock-in effect\textsuperscript{2} we observe in our model given a lack of information sharing. As a solution to this problem, he suggests, lenders should try to develop reputation and thus lead to so-called "implicit contracts"\textsuperscript{3}. We, on the other hand, argue that it should rather be their information that is shared.

Another paper, with a very similar title to our own, *Information Sharing in Credit Markets* by Pagano and Jappelli (1993) finds reason to believe that lenders have incentives to share information with other lenders. They build a model consisting of a finite number of towns with a continuum of households within each town. Through this model they show how banks are incentivized to share information by factors such as mobility and heterogeneity of their customers, the size of the credit market, and advances in information technology. The same effect is, however, limited by fear of increased competition by new entrants into the market. This paper, at least its title, seems to be very similar to our thesis, but, evidently, their focus is entirely different. However, it is worthwhile to note that the idea of a positive financial value to be derived from information sharing is not a new one.

An elaboration of this point is found by Houston et al. (2010). The paper finds, among other things, that the benefits of information sharing among creditors appear to be uniformly positive. However, it seems like this is along the dimensions regarding bank profitability, bank risk, likelihood of financial crisis, and economic growth; evidently not entirely nuanced. We enrich this discussion by viewing how the private customers may be affected by information sharing.

Jankowitsch, Pichler and Schwaiger (2007) examine the economic value of credit rating systems, in that, whether or not banks benefits from improving their (internal) credit rating systems. They find that they do, which offers some credence to our claim that banks will be able to better predict default probabilities if they gain excess to other banks’ internal information regarding a given customer. However, they focus on the statistical power of a bank’s internal rating system, but it should also apply to predict the effects of better information. Nevertheless, a key factor driving this gain, they say, is the regulatory capital\textsuperscript{4} which is affected by the accuracy of the predicted default probabilities, (and presumably also loss given default and exposure at default). The regulatory capital the bank must hold is determined by the Basel committee’s regulatory framework, presented through Basel II and, more recently, Basel III. Similar results were also found by Kallberg and Udell (2003),

\textsuperscript{2}Good customers is unable to convince other banks of their lower-than-expected default probability, and thus be offered a lower interest rate.

\textsuperscript{3}For further discussion, please see the cited source.

\textsuperscript{4}The capital the bank must hold to ensure liquidity.

Another paper addressing the role of information within the banking market is Dell’Ariccia and Marquez (2004). They analyze how the capture effect\textsuperscript{5}, caused by asymmetric information, affects the loan portfolio of informed lenders. They find that banks (1) charge a higher interest rate the greater the information asymmetry is, and (2) that banks reallocate credit towards existing borrowers when faced with greater competition. They call the second effect ”flight to captivity”. Put succinctly, we find that this paper lends some credence to one of the results that follow from our models, which is that a bank will usually prefer a somewhat low interest rate over a high one in the presence of information asymmetry to reduce the costs of incomplete information. The discussion regarding competition between banks and “flight to captivity” falls outside the scope of our thesis, and we will operate with the simplifying assumption of perfect price competition for the remainder of this thesis.

3 The Norwegian Credit Market

In this section, we briefly describe the basics of the banking system and the risk-assessment and pricing of loans. The section continues with a discussion regarding the kinds of information a bank has access to and how this affects the interest rate. We also define the terms good and bad customers, and discuss the situation in the credit market as it is today, with respect to information sharing. We conclude this section by defining an illustrative bond that will be used to represent a loan given to private customers in our thesis.

A more detailed description of the Norwegian banking system and risk assessment procedures can be found in appendix C.1 and C.2, as well as a description of common properties of Norwegian credit. Most importantly, Norwegian mortgages tend to have low default probabilities and high recovery rates. Mortgages make up a large portion of total credit issued to Norwegian private customers.

3.1 The Basics of Banking

A common definition of a bank, as set forth by Freixas, Xavier, Rochet and Jean-Charles (1997), is “an institution whose current operations consists in granting loans and receiving deposits from the public”. This is also the definition regulators use when they decide whether a financial intermediary has to submit to the prevailing prudential regulations for banks (Basel, 2005).

\textsuperscript{5}The same as being ”informational captured” as put forth by Sharpe (1990).
The societal purpose of a bank is to provide credit to firms and private customers. More specifically, the bank converts highly liquid cash deposits into illiquid bonds - That is, it allows customers to deposit and withdraw cash on a very short time-horizon, while at the same time lending out that same money on a very long time-horizon. In this way, the bank allows for money that needs to be kept liquid to still be activizied by long-term projects, such as property purchases and corporations. In order to do this, the bank must have a large enough amount of depositors to be able to accurately predict how much money it needs to have on hand to allow for withdrawals and defaults, and how much it can lend out to the wider market. The commercial banks also distribute credit from the National Bank to the private markets. The money required to be kept on hand is dubbed regulatory capital, and is a set fraction of the risk-weighted total of all loans given out by the bank. (Sparebanken Vest, 2014).

3.2 Customer Scoring and Risk Grades

Customer scoring refers to the process of assigning a risk grade to each customer according to their default (credit) risk. The scoring process is a statistical one, and includes both public information and internally observed variables. The estimated risk grade is used both for risk management, early warning of defaults and as a part of the credit approval process. (Nordea, 2015b).

The risk grade of each customer is frequently re-evaluated. This often results in that the score distribution changes. Such changes are mainly due to three factors. These are (1) changes in the risk grade of existing customers, (2) customer turnover (i.e. new customer have a different risk grade relative to existing/leaving customers) and (3) increased or decreased exposure per risk grade to existing customers. The risk grade may also migrate, that is, the overall risk grade may change. Such migration is affected by, among other things, macroeconomic development and thus changes in the overall financial situation to the customers in general (customers repayment capacity). (Nordea, 2015b).

For our purposes, the key observation here is that the banks apply both public and internal information in the scoring process, and that risk grade frequently changes. The details of the customer scoring process are not directly relevant to our analysis, and are explained in greater detail in appendix C.2.
3.3 Interest Rates and Information

The bank has access to two kinds of information about their customers, the first kind being public information, such as income and assets, which is known at the outset of any loan. The banks also observe the behaviour of their customers to get what we dub as internal information. This internal information is significant when predicting the default-risk of a customer, but cannot be shared between banks due to privacy legislation. Typically, a bank considers its internal variables to be statistically significant with respect to the default probability of a customer after having observed the customer for approximately three months. (Nordea, 2015a).

Say now that a customer turns out to be a *good customer*. A good customer is defined as a customer who will get a more favourable risk assessment if the bank has access to internal information, than if it only has access to public information. When the bank has estimated the variables that make up internal information, a good customer will then ideally get a lower interest rate, having been established to have lower risk than originally expected. The bank might agree to lower the interest somewhat, but does not necessarily have to lower it as much as might be "fair", or even lower it at all.

The reason for this is that the customer cannot credibly threaten to switch banks - After all, other banks do not have access to the internal information of the first bank, meaning they will assign an interest rate based only on public information. By definition, a good customer will therefore get a less favourable risk assessment when switching banks. We may say that being a good customer actually creates a lock-in effect, as the internally measured variables are of positive value to a good customer, value that would be relinquished upon switching to another bank.

Conversely, the bank cannot charge any extra interest from customers who, when the internal information becomes known, are shown to have higher risk than what the public information suggested. The reason for this is that such customers, we dub them *bad customers*, can easily switch to a bank with less information, so any attempt at charging the bad customer extra will lead to losing that customer. While an argument can be made that these observations must mean that switching banks frequently must be a strong signal of being a bad customer, current practice is to not charge a higher interest rate from customer based on frequent switching of banks. (Nordea, 2015a).

The implication of the current legislation is that the bank cannot meaningfully price-discriminate between good and bad customers. Instead, the bank must charge an interest rate, which can be understood as the price of the loan, that is based on the publicly known
variables plus an expected extra risk. It follows that good customers must on average be paying too high interest, while bad customers pay too low interest - The good customers subsidize the bad. We note that this means that the banks will essentially only use public information to set the interest rates, because using only information that other banks have access too also means that the customers cannot get different terms in other banks. This is a central observation to this thesis, and is described in great detail in section 4 and 5 and appendix B.

From a customer-perspective, we feel safe in saying that this is not a very fair situation. It also means that customers that have low enough risk to justify a bigger loan might be unable to get one, while customers whose risk is actually too great to borrow more might still get to do so. This means that, in addition to unfair allocation of costs and credit, the customer might indirectly get hurt by being incentivized to take more debt than is adviseable\(^6\), or by not getting access to credit that the customer can afford.

### 3.4 Definition of Illustrative Bond

In this thesis, we will construct two models to illustrate the effect changes in information sharing between banks can have on the Norwegian credit market. To make our findings easier to compare and interpret, as well as to limit the amount of notation used, we define a standard, simple bond that we will use to represent the loans given to private customers.

We hold that the additional complexities that are usual on the Norwegian credit market will only serve to make our analysis less straightforward, without illustrating any effects that are interesting to our thesis.

In this thesis, we focus on private customers, as the problems we examine in this thesis do not apply in the same way for the corporate loan market. Private customers mainly take loans for investing in property (Finanstilsynet, 2013), also referred to as mortgages. That said, Norwegian mortgages are secured against all assets of the borrower, so for the purposes of this thesis we claim it is reasonable to assume that both mortgages and other credit acts like an ordinary bond. A closer discussion of this claim can be found in Lucchino and Morelli (2012).

We use the definition given by Mishkin (2013) which states that a bond is a debt security\(^7\) that promises to make payments periodically for a specific period of time, and, moreover, typically consists of the money lent (the principal) and payment (interest) for the given

\(^6\)Which is also the view expressed by Bennardo, Pagano and Piccolo (2014).

\(^7\)Also referred to as a financial instrument (Mishkin, 2013).
service, which generally is determined by a rate\(^8\) set at the outset of the loan. In our thesis we build upon this definition, and further define the bond used as follows.

**Definition:** A loan is, in this thesis, defined as a customer \(i\) selling a one-period bond to the bank. The bond has a face value of 1, default probability of \(Q_i\) and interest of \(r\). The customer derives utility \(U\) from making this loan. The bond is always sold at par value, which is 1. The value of the bond to the bank is defined as \(V_{B,i}\), the value to customer \(i\) is defined as \(V_{C,i}\), and is the value of the cashflows the transaction represents to each party.

\[
\begin{align*}
V_{B,i} &= -1 + (1 + r)(1 - Q_i) \\
V_{C,i} &= 1 + U - (1 + r)(1 - Q_i)
\end{align*}
\]

On the Norwegian credit market, we usually do not observe that the customer has any financial gains from taking bank-loans on the Norwegian credit market, which implies that there is some non-monetary gain for the customer. This observation is the motivation for including a term \(U\), which represents the utility the customer derives from taking a loan. A detailed analysis of this assumption falls outside the scope of this thesis, but an argument can be made that this is either due to a customer surplus being associated with whatever investment the loan is intended to cover or that the loans enable the customers to smoothe their consumption over time.

For simplicity, we assume that all customers derive the same utility \(U\) from taking the loan\(^9\). However, this is not a necessary assumption, as we will later find that the criterion for market participation is merely that utility is non-negative, which we claim is a reasonable assumption to make. The implication is that setting utility to be constant for all customers lets us avoid handling an additional probability distribution in our calculations without changing our main findings in any meaningful way.

### 4 The Simple Market Participation Model

#### 4.1 Model Introduction

In this section we construct the simple market participation model, which is a model we have created to give a stylized description of the credit market with respect to loans given to

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\(^8\)Fixed or adjustable (Sundaresan, 2009).

\(^9\)This can be justified by regarding \(U\) as the representative customer’s utility.
private customers. Our purpose with this model is to illustrate how interest rates and market size is affected not only by the default probabilities of the customers, but also the information about default probabilities that is available to the bank. We do this by evaluating one case with full information, and one with asymmetric information. We then compare the properties of the markets that emerge from each assumption to illustrate the effect information, or the lack thereof, with respect to the value of both the customers and banks.

This model is inspired by Akerlof (1970), which discusses how a market with asymmetric information might function, as well as Stiglitz and Weiss (1981), which describes how a model similar to Akerlofs might react if market size also affects the value of the product or service that is exchanged. Our contribution is to assume that the product exchanged is a bond such as the illustrative bond described previously, allowing us to more accurately model how asymmetric information affects the credit market. We also use a continuous distribution of default probabilities, as opposed to the two possible default probabilities assumed by Stiglitz and Weiss (1981).

One of the problems we encounter is that, under asymmetric information, increases in market size also decrease the average value of loans if the banks cannot meaningfully differentiate between high- and low-risk customers. This leads to a situation where we do not necessarily find a classic equilibrium where supply equals demand, but rather get a case of permanent excess supply or excess demand. Drawing on the findings of Stiglitz and Weiss (1981), we describe how such an equilibrium may still be stable and natural for the credit market.

While not a problem treated by this model, it is important to note that good information can only be used by the bank if it is also available to all other banks as well. If not, customers that are identified as high-risk will be able to switch to banks with less information to reduce their interest rates, thus preventing the bank from making use of all the information it has about its customers (Nordea, 2015a). The full consequences of this is discussed in detail in the game theory section (section 5) of this thesis.

In short, the market participation model explores the potential gains of improved information about the default risk of customers, and the conditions that has to be in place to realize these gains are explored in in the game theory section of this thesis.

4.2 Model Outlines

The simple market participation model is based on the intuition from Akerlof (1970). Akerlof described a situation where people trade used cars of either high or low quality. We
build on this by modeling the credit market as a group of consumers selling (or not selling) bonds to the bank. We have chosen to solve this assuming continuously distributed default probabilities, because we feel this gives a more dynamic and useful model. That said, we have made a simple and discrete example in appendix B, which can be used to gain a clearer understanding of the effects we observe when studying this model closer in section 6.3 and 6.4.

The bond has a monetary value, $V_{B,i}$ for the bank. We set the value to a customer $i$ to be, $V_{C,i}$, which includes both the monetary value to the customer, and the possibility of an additional value from the utility of getting a loan.

**Definition:** The credit market consists of a large number, $N$, private customers, of which $n \in (0, N)$ choose to participate in the credit market if their value of doing so, $V_{C,i}$, is non-negative. For simplicity, we have chosen to let the default probabilities be uniformly distributed between 0 and 1. Note that this is a simplifying assumption, and that the real distribution is skewed towards lower default probabilities (Sparebanken Vest, 2015). This makes no difference to the kinds of effects we observe, merely their magnitude.

$$Q_i \sim U(0, 1)$$

The bank is subject to perfect price-competition, and will therefore always set interest $r$ so that $E(V_{B,i}) = 0$.

When unable to differentiate between customers, such as in the case of asymmetric information, we assume that the bank will still be aware of the probability distribution of default probabilities, and will thus be able to set an interest rate based on expected default probability that the bank will charge all customers that choose to participate in the market. Given the uniform distribution, we can also say that the default probability of the least risky customer that wants to participate in the market, $Q_M$, will be equal to the fraction of the customers that do not wish participate in the market at a given interest $r$. Therefore, we may also say that the fraction of customers that will participate in the market at a given interest is equal to $1 - Q_M$, and that the number of participating customers $n$ can also be given as $N(1 - Q_M)$.

We remark that customers must necessarily be indexed such that a lower index $i$ corresponds to a higher default probability $Q_i$. This means customer $i = N$ is the safest customer on the market, while customer $i = 1$ is the riskiest customer on the market, and customer $i = n$ is the safest participating customer.
Also note that the distinction between good and bad customers from section 3.3 is not a necessary distinction to make in this model, although it is certainly possible to think in these terms when interpreting the situations that arise when we look at the outcomes of the model later. However, this model is mainly a stylized representation intended to illustrate the effects that arise from asymmetric information between bank and customer. Therefore, we will assume that the bank can either distinct between all customers, or none of them - The real scenario is somewhere in between, and is that the bank can meaningfully divide their customers into a set of risk grades.

The results of the simple market participation model will later be applied in section 5, where the distinction between good and bad customers is re-introduced and becomes central to the analysis.

We summarize the model as we have described it before continuing:

\[
V_{B,\text{Total}} = \sum_{i=1}^{n} V_{B,i} = N(1 - Q_M) \left( -1 + (1 + r)(1 - \hat{Q}) \right) \\
V_{C,i} = 1 + U - (1 + r)(1 - Q_i)
\]

\[Q_i = \text{Default Probability of Customer } i \in (0, N)\]

\[Q_M = \text{Default Probability of least risky customers take the loan at a given interest}\]

\[\hat{Q} = \frac{1}{n} \sum_{i=1}^{n} Q_i = \text{Expected Default Probability given Market Participation} = \frac{1 + Q_M}{2}\]

\[r = \text{Interest Rate}\]

\[U = \text{Utility of Taking Loan}\]

\[N = \text{Total Number of Customers, Max Market Size}\]

\[n = \text{Number of Participating Customers} = N(1 - Q_M)\]

As mentioned, a numerical example can be found in appendix B.

Having established our framework, we can proceed to see how our model behaves under conditions of full and asymmetric information.

### 4.3 The Case of Full Information

If the bank has full information, it is able to differentiate between customers, as the bank knows the default risk of each individual customer. This means that the bank can correctly
set an interest rate that suits the individual default probability of each customer, so that we can say that:

\[ V_{Total}^B = N(1 - Q_M)\left(-1 + (1 + r)(1 - Q_i)\right) \]

We know that the bank will attempt to set the interest so that it derives a value of zero from buying bonds, so we must have that \( V_{Total}^B = 0 \). With this condition, we find that the interest charged to each customer must be:

\[ N(1 - Q_M)\left(-1 + (1 + r)(1 - Q_i)\right) = 0 \]
\[ -1 + (1 + r)(1 - Q_i) = 0 \]
\[ (1 + r)(1 - Q_i) = 1 \]
\[ r = \frac{1}{1 - Q_i} - 1 \]

This gives us that the value \( V_{C,i} \) to each customer is:

\[ V_{C,i} = 1 + U - \left(1 + \frac{1}{1 - Q_i} - 1\right)(1 - Q_i) = U \]

We note that customer value, \( V_{C,i} \), actually does not depend on the default risk of the customer at all, because the bank is able to price each customer perfectly. Thus, we have that all customers will participate given that \( U \geq 0 \), meaning that we have \( Q_M = 0 \). In the opposite case, \( U < 0 \), we have \( Q_M = 1 \). This gives us a set of possible stable states for the credit market.

**4.3.1 \( U \geq 0 \)**

Under this condition, we have that \( Q_M = 0 \) so that:

\[ V_{B,i} = -1 + (1 + r)(1 - Q_i) = 0 \]
\[ V_{C,i} = U \geq 0 \]

The total customer surplus will here be \( NU \), while total bank surplus will be 0. Average interest rate paid will be given by:

\[ E(r) = E\left(\frac{1}{1 - Q_i} - 1\right) = \frac{1}{1 - E(Q_i)} - 1 = \frac{1}{1 - 0.5} - 1 = 1 \]
4.3.2 Other Stable States

For the sake of completeness, we note that stable market states also occur when \( N = 0 \) or \( U < 0 \), as we then have a stable state where no loans are taken. While these cases might be interesting in their own right, they both result in a market with zero participants, which we have not observed on the market we are working with, and which we believe would have reasons that could not easily be analyzed within the framework we are building. Therefore, these stable states are not interesting to this thesis.

4.3.3 Concluding Remarks on the Case of Full Information

Because we observe a market for credit in Norway, we must obviously have that \( U \geq 0 \). Therefore, we argue that we can assume that this holds for the remainder of this thesis.

4.4 The Case of Asymmetric Information

Under asymmetric information the bank must offer all customers the same interest rate based on the expected default probability. Because of this, it makes sense to do the analysis in two parts, examining in turn the supply and demand of credit. We are back to giving our values as:

\[
V_B^{Total} = N(1 - Q_M)\left(-1 + (1 + r)(1 - \hat{Q})\right)
\]

\[
V_{C,i} = 1 + U - (1 + r)(1 - Q_i)
\]

4.4.1 The Supply of Credit

We introduce the notation \( S = 1 - Q_M \) and observe that

\[
\hat{Q} = \frac{1 + Q_M}{2} = 1 - \frac{S}{2}
\]

These are rewritings we do for the sake of mathematical convenience, although we note that \( S \) has a much more intuitive interpretation than \( Q_M \): \( S \) can be understood as the fraction of the market that the bank is willing to offer credit to. We can then rewrite \( V_B^{Total} \) as:

\[
V_B^{Total} = N(1 - Q_m)\left(-1 + (1 + r)(1 - \hat{Q})\right)
\]

\[
= NS\left(-1 + (1 + r)(1 - (1 - \frac{S}{2})))\right)
\]

\[
= NS\left(-1 + (1 + r)\frac{S}{2}\right)
\]
Because $V_{B,i} = 0$, we have that:

$$NS \left( -1 + (1 + r) \frac{S}{2} \right) = 0$$

$$(1 + r) \frac{S}{2} = 1$$

$$S = \frac{2}{1 + r}$$

We interpret $S$ as the fraction of the market that the bank will offer credit to given an interest rate $r$ is equal to $S = \frac{2}{1 + r}$. It follows that the total supply of credit will be given by $SN = \frac{2N}{1 + r}$.

### 4.4.2 The Demand for Credit

To identify the fraction of the customers that demand credit at a given interest rate, we recall that the least risky, participating customer will have that $V_{C,i} = 0$. For this equation we say that $D = 1 - Q_M$ and that total demand is then given by $DN$.

$$1 + U - (1 + r)(1 - Q_M) = 0$$

$$1 + U - (1 + r)D = 0$$

$$D = \frac{1 + U}{1 + r}$$

It follows that total demand for credit will be given by $DN = N \frac{1 + U}{1 + r}$.

### 4.4.3 Equilibrium Conditions

A classic equilibrium is found when supply equals demand, that is to say that:

$$SN = DN$$

$$S = D$$

$$\frac{2}{1 + r} = \frac{1 + U}{1 + r}$$

$$U = 1$$

We note that the equilibrium is not given by the interest rate $r$ at all, but rather the utility the customers derive from taking a loan. If we have $U > 1$ we would in fact always have excess demand no matter the interest rate, while $U < 1$ would give a permanent excess supply. This is due to a problem first described by Stiglitz and Weiss (1981), namely that the imperfect information of the bank leads to a scenario where increasing the interest rate
also changes the average value of loans given, as the average default probability goes up. Therefore, we observe that both supply and demand of credit decreases as interest increases, meaning a classic equilibrium is by no means a certainty.

\[ S, D \]

\[ U > 1 \]

\[ U < 1 \]

\[ r \]

\[ N \]

\[ S, D \]

**Figure 1**: Total Supply and Demand of credit as a function of interest rate \( r \). Note that the Demand-curve shifts when utility \( U \) is changed, while Supply remains unchanged.

We note that the figure 1 shows that we will either have constant excess supply, constant excess demand, or equilibria at all interest rates, depending on the value of \( U \). If \( U \neq 1 \), we converge towards an equilibrium as interest \( r \) is decreased, but we never get to a stable state by changing the interest, especially not when considering that the maximum market size is, in this case, constrained between \( 0 \) and \( N \). We get several possible market states.

### 4.4.4 U>1, Excess Demand

What determines interest-rate and market size when utility is such that demand exceeds supply? Stiglitz and Weiss (1981) assumes that the return of the bank is concave with respect to interest rate because they assume that an increased interest rate will push the least risky firms out of the market, meaning that there is some profit-maximizing interest rate, and that the bank will just distribute credit at random until this point is reached, being unable to differentiate between customers. This is clearly not the case given the way we have specified our model, as the assumption of price competition means that the bank will always set the interest so that expected value to the bank is zero. In fact, given that the bank is
always able to estimate the average default probability on the market, there is no case that follows from our model where the bank has anything to gain by limiting the supply of credit.

To gain an indication of how the bank will act in this situation, we can gain some understanding by taking a step back and consider the competition between banks. We have previously stated that the bank in the model is subject to price-competition. However, there is one implication of this which we have not discussed, which is the fact that whatever bank offers the lowest interests will necessarily grab all of the market, as a high-risk customer can always avoid the cost of his risk by taking loans with a bank that has lower average default probability. We will discuss this more closely later in the thesis, but for now, we state that a bank that sets its interest rate high will be pushed out of the market, leaving us with only banks that minimize their interest rate. This changes if we allow for the banks to have differences in branding, service and so forth, which would mean that $U$ varies with the bank. This is however outside the scope and interest of this thesis.

Because we have excess demand, we know that the bank the utility of the customers is such that the minimum interest the bank can set, while keeping $V_B^{Total}$ non-negative, still makes the entire potential market, $N$, take up loans. This can be written as $SN = N$, which is the same as saying that $S = 1$.

$$S = \frac{2}{1 + r} = 1 \iff r = 1$$

We note that this means we have a market size of 1 given $U \geq 1$ and the same interest-rate as was offered under full information.

4.4.5 U<1, Excess Supply

The case of excess supply means that the bank is offering more credit than the customers demand at any interest-rate. Such a market will always converge to zero in our model, because the bank will, at any interest-rate, find that the market has a higher default-probability than what is priced in at the current interest-rate.

A proof is given as follows:

$$V_B^{Total} = N(1 - Q_M)(-1 + (1 + r)(1 - \hat{Q}))$$

$$V_{C,i} = 1 + U - (1 + r)(1 - Q_i)$$

We hold that the bank will set $V_B^{Total} = 0$, and that a customer only participates if $V_{C,i} \geq 0$. 

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We find that the interest \( r \) can be given as:

\[
V_B^{Total} = N(1 - Q_M)\left(-1 + (1 + r)(1 - \hat{Q})\right) = 0
\]

\[
r = \frac{1}{1 - \hat{Q}} - 1
\]

We substitute in this expression for \( r \) in \( V_{C,i} \) and find a market participation condition for each customer:

\[
V_{C,i} = 1 + U - (1 + r)(1 - Q_i) \geq 0
\]

\[
1 + U - \left(1 + \frac{1}{1 - \hat{Q}}\right)(1 - Q_i) \geq 0
\]

\[
1 + U - \frac{1 - Q_i}{1 - \hat{Q}} \geq 0
\]

\[
Q_i \geq 1 - (1 + U)(1 - \hat{Q})
\]

Recalling that \( \hat{Q} \) is the expected default probability of all participating customers, we can say that:

\[
\hat{Q} = E(\{Q_i \mid Q_i \geq 1 - (1 + U)(1 - \hat{Q})\})
\]

\[
= \frac{1 + 1 - (1 + U)(1 - \hat{Q})}{2}
\]

\[
= 1 - \frac{1}{2}(1 + U)(1 - \hat{Q})
\]

Solving this expression for \( \hat{Q} \) gives us the stable expectation of market default probability:

\[
\hat{Q} = 1 - \frac{1}{2}(1 + U)(1 - \hat{Q})
\]

\[
\hat{Q} - \frac{1}{2}(1 + U)\hat{Q} = 1 - \frac{1}{2}(1 + U)
\]

\[
\hat{Q} = 1
\]

Knowing that this also implies that \( Q_M = 1 \), this is the same as saying we have market size zero. This is the same as what we see in Akerlof (1970): Because increased interest also reduces the expected credit quality (increased default probability), adverse selection leads to a state where no transactions happen.

4.4.6 \( U=1 \), Equilibrium

In this case, the utility is such that it barely justifies the interest payment for the least risky customer on the market. In this case, the price competition-mechanism forces the bank to
minimize interest, meaning that we get the same interest as in the case of excess demand: An interest rate of \( r = 1 \), which gives complete market participation of \( N \).

### 4.4.7 Other Stable States

For the sake of completeness, we state that we also get a stable state if there is no market, \( N = 0 \).

### 4.5 Outcome Comparison

Having established how the market looks both with complete and asymmetric information, we take a step back to summarize what we are trying to illustrate: If there is a difference between the two cases, then we will be able to reasonably assume that as the banks gain better information, we will move away from the state resulting from asymmetric information, and closer to the state resulting from complete information. Finally, we treat how we can realize these effects in the game theory section of the thesis.

The relevant variables at first glance is the average interest paid and the market size.

#### 4.5.1 Market Size

We note that the bank will set interest so that it can either supply credit to the entire market, meaning that supply is set to \( SN \), or none of it. If we have \( U \geq 1 \), this gives a market size of 1, which is equal to the market size we get under complete information. However, because all customers get the same interest rate under asymmetric information, some customers will drop out of the market if their utility from taking a loan is low - In our case, this threshold is at 1. The mechanics of adverse selection will then lead to a state where no transactions occur because of steadily deteriorating credit quality, the same as one of the extreme cases put forth by Akerlof (1970).

Summarized, if utility from taking a loan is high, we get full market participation regardless of whether we have complete or asymmetric information. If utility is low, we get no market due to adverse selection, although we might have a period of with some market participation as the banks work their way towards an interest-rate that is too high for the potential customers. This state is not necessarily stable, as the required interest rate for the bank increases as marked size decreases due to increasing average default probability in the remaining market. A stable state must therefore either mean that we have \( U = 1 \) or excess demand for credit.
4.5.2 Average Interest Rate

Given high utility, we have market size equal to 1, which means that average interest rate will be 1 regardless of if the bank gives all customers the same interest rates or not. Given our distribution of default probabilities, this gives $E(r) = 1$, which makes sense - The bank should require an interest at 100 % given the 50 % average default probability given in our simplified case.

4.5.3 Customer Surplus

The customer surplus is the difference between what a customer is willing to pay, and what the customer actually pays. In our case, this is the difference between the interest the customer was willing to pay, and what the customer actually paid.

Under full information, each customer gets an unique interest rate that sets the value of that specific loan to zero for the bank. The remaining value then befalls the customer. We have already shown this value to be equal to $U$, meaning that the customer surplus under full information is $NU$ if there is a market, and zero if there is not.

Under asymmetric information, all customers are offered the same interest rate. If there is a market, this interest rate has been shown to be 1. In this case the least risky customer has a customer surplus of zero, while the most risky customer has a surplus of $1 + U$. Knowing that we have an uniform distribution of probabilities, we find that the sum of customer surplus under asymmetric information, given that $U \geq 1$, is equal to $\frac{1}{2}N(1 + U)$.

We note that when $U > 1$, then $NU > \frac{1}{2}N(1 + U)$. We can therefore say that customer surplus is lower under asymmetric information, except in the case of $U = 1$. It follows that it is in fact the customer that has to carry the cost of asymmetric information.

Having examined the most obvious characteristics of the credit market, we examine two more subtle factors: The fairness of interest expense division and the overall market risk.

4.5.4 Fairness of Interest Expense Division

Under complete information, one interest rate is given to each customer so that the bank has an expected value zero from each loan. Under asymmetric information, all customers are offered the same interest rate based on the average default probability of the entire population of participating customers. This implies that, under asymmetric information, low-risk customers have a higher interest expense than they would under complete information, while
high-risk customers pay less. In effect, the good credit is subsidizing the bad.

If this is fair or not is subject to an ethical discussion outside the scope of this thesis. However, we can briefly state that it seems reasonable that high-risk customers will usually have a lower income or less assets than low-risk customers. A valid perspective might be that these should also be allowed credit to finance a place to live and so forth, but it is equally valid to discuss whether there are better and more controllable ways to allow for this. An equally valid question is if this burden should be carried by the low-risk customers, or if it is more effective if this burden is carried by the state. Another possible point is that a customer whose default risk is very high should perhaps not be incentivized to take a large loan in the first place.

4.5.5 Overall Market Risk

While not always the outcome of the model, we see a tendency that asymmetric information incentivizes high-risk customers to borrow money, as these are subsidized by the low-risk customers, who will therefore be less willing to borrow money. A real concern is then that credit is in fact allocated to high-risk customers rather than low-risk customers. Given the model as it stands, this is only a concern in a case of stable over-supply, which should only occur under very skewed probability distributions (which may well be the case on the Norwegian credit market).

4.6 Conclusion of the Simple Market Participation Model

We have now shown how, under asymmetric information, the low-risk customers have to subsidize the high-risk customers, leading to a state where we either have no market at all, or a market that superficially looks like the one under full information, but with lower customer surplus. This is because the customer, having no way of signaling low risk, has to carry the cost of asymmetric information by accepting an interest rate that is so high that it eliminates most of or all of the customer surplus that would have occurred under full information. Bank surplus remains the same, as the bank in any case has to minimize interest due to price competition.

Based on this analysis, we claim that moving towards a state closer to full information will result in “better” allocation of credit, overall reduction of credit risk and increased customer surplus. We will now have a closer look on how better information cannot necessarily be utilized by banks unless this information is also available to other banks, highlighting why
good information in itself is not necessarily enough to realize the gains described in the simple market participation model.

5 The Customer-Bank Game

We now move on to the analysis of those who do participate, that is, we examine what happens when the banks’ belief about the customer’s risk class is updated with private information, using game theory. We have, however, not succeeded in finding a similar example for such a game or model in the literature. Instead, we draw on the insights from Gibbons (1992) and other models we have encountered in our field of study, and thus develop and propose a stylized and formalized model describing the customer-bank interaction.

The purpose of this section is multiple. First, we need to describe and analyze the situation as it is today, that is, we need a model which can describe the dynamics leading to a so-called pooled equilibrium in which all customers within each public risk class is charged the same interest rate. Second, we need to be able to give some predictions about the outcome given sharing of internal information. And, third, we further look at two different ways of sharing information, forced and voluntary. This is done because we wish to provide a somewhat nuanced discussion regarding what kind of legislation-change that might be optimal.

In this process, we consider an economy that consists of a finite number of public information classes, while we use two different approaches to the classification of the private information: First, two possible risk classes within each segment is considered, which is dubbed the discrete case. Second, we expand the model to include a continuum of possible private risk classes, which is dubbed the continuous case.

Our work build upon one basic tenet of traditional economic theory, namely, that market participants are rational in their decision making. However, we realize that this is not necessarily the case in the private credit market, and provide extensive discussion regarding this.

This section thus evolves as follows. We begin with a short description of game theory and discuss central tools and topics necessary for the model. Next, we describe the game and the players, in which we focus on defining the scope and limitations of the game. This is followed by a thorough analysis of both cases, in which we consider three different approaches regarding the sharing of information. First, we say that the private information cannot be shared. Second, we consider the situation in which the banks are forced to share the private
information. And, third, for the sake of discussion, we consider the situation in which the customers can voluntary choose to share information. As will become clear, the most ambiguous case is the last one, because we must make some further assumptions about the players (i.e., the banks and the customers).

We recognize that the approach used in this section may be somewhat excessive, but we wish to provide a thorough and comprehensive analysis of this interaction, and thus find it appropriate.

5.1 The Basics of Game Theory

As Gibbons (1992) so precisely puts it: Game theory is the study of multiperson decision problems. Which is exactly what we consider in this section, and is the reason for why we have chosen game theory as the underlying theory.

We start out with a distinction between static and dynamic games. In static games, the players move simultaneously, then the players receive payoffs that depend on the combination of actions just chosen. In dynamic games, however, players move in a specific order, in that, some of the players observe the actions taken by other players before they choose action. (Gibbons, 1992)

Dynamic games have either perfect or imperfect information, and either complete or incomplete information. Perfect information is when the player with the move knows the complete history of the game thus far. If at least one player is uncertain about which node the game has come to, we say that the game has imperfect information. Complete information is when the players’ payoffs are common knowledge. Thus, if at least one player is uncertain about the payoffs, the game is said to be of incomplete information. (Gibbons, 1992).

Games can be expressed in three different forms, verbal, normal-form, or extensive-form. Normal-form representation uses matrices and is convenient for static games. Extensive-form representation uses decision- or game-trees and is suited for games that have several stages, in effect dynamic games. The normal-form representation of a game specifies the players and their strategies, and the payoffs (payoff is simply the value associated with a possible outcome (Pindyck and Rubinfeld, 2013)) received by the players for each feasible combination of actions. The extensive-form representation specifies the players, when each player has the move, what each player can do and knows at each stage, and the payoff received by each player. A combination of verbal and normal and/or extensive-form is generally provided. (Gibbons, 1992).

Contingent on the type of game, it exists four equilibrium concepts; Nash equilibrium
(NE), subgame-perfect Nash equilibrium (SPNE), Bayesian Nash equilibrium (BNE), and perfect Bayesian equilibrium (PBE); where NE and BNE is solution concepts for static games, whereas SPNE and PBE is solution concepts for dynamic games (Gibbons, 1992). Bayesian simply refer to games of imperfect information.

The following text is concerned with formalities, in that, we provide definitions of the central terms used in this section.

First, the notation as used by Gibbons (1992): $n$ is the number of players in the game. $G$ denotes the game (the set of strategy spaces and utility functions the players’ face), $S_i$ the player $i$’s feasible set of actions or strategy space, and $u_i$ is player $i$’s utility function. $s_i$ is player $i$’s chosen action or strategy, with $s_i \in S_i$. * simply denotes optimal strategy, that is, maximizes the player’s utility.

The formal definition of a Nash equilibrium as stated by Gibbons (1992) goes as follows.

**Definition:** In the $n$-player normal-form game $G = \{S_1, \ldots, S_n; u_1, \ldots, u_n\}$, the strategies $(s^*_1, \ldots, s^*_n)$ are each a **Nash equilibrium** if, for each player $i$, $s^*_i$ is (at least tied for) player $i$’s best response to the strategies specified for the $n-1$ other players, $(s^*_1, \ldots, s^*_{i-1}, s^*_{i+1}, \ldots, s^*_n)$:

$$u_i(s^*_1, \ldots, s^*_{i-1}, s^*_i, s^*_{i+1}, \ldots, s^*_n) \geq u_i(s^*_1, \ldots, s^*_{i-1}, s_i, s^*_{i+1}, \ldots, s^*_n)$$

for every feasible strategy $s_i \in S_i$; that is, $s^*_i$ solves

$$\max_{s_i \in S_i} u_i(s^*_1, \ldots, s^*_{i-1}, s_i, s^*_{i+1}, \ldots, s^*_n)$$

The intuition is that, in a NE, all the $n$ players choose whatever action which maximize their utility given that everybody else choose the action which maximize their utility given that everybody else maximize their utility, and so on. As Pindyck and Rubinfeld (2013) explains it, a NE is when “I am doing the best I can given what you are doing, and you are doing the best you can given what I am doing”. In other words, if each player’s strategy is a best response to the other players’ actions, we have a NE.

Next, we present the definition of strategy as given by Gibbons (1992):

**Definition:** A **strategy** for a player is a complete plan of action - It specifies a feasible action for the player in every contingency in which the player might be called upon to act.

Followed by the definition of SPNE, also as given by Gibbons (1992):
Definition: A **subgame-perfect Nash equilibrium** is when the player’s strategies constitutes a NE in every subgame.

The formal definition of BNE is excluded since we do not use it in our thesis, but we do, however, use PBE. A perfect Bayesian equilibrium consists of strategies that fulfill 4 requirements, these requirements are (Gibbons, 1992):

1. The player with the move must, on each information set, have a belief about which node the game has come to. That is, the players have assigned a probability distribution over the nodes in the information set.

2. And, next, given the players beliefs, the players strategies must be what is called *sequential rational*, in that, for each information set the action taken by the player with the move must be optimal given the player’s belief at that information set, and the other players subsequent strategies. In which subsequent strategies is defined nearly identical to strategy, with the only difference being that the subsequent strategy covers the strategy from that information set until the end of the game.

3. At information sets on the equilibrium path (the path the game goes trough the game tree given that it is the equilibrium strategies that is played), beliefs are determined by Bayes’ rule and the players equilibrium strategies.

4. At information set off the equilibrium path, beliefs are determined by Bayes’ rule and the other players’ equilibrium strategies where possible.

Thus, also as presented by Gibbons (1992):

**Definition:** A **perfect Bayesian equilibrium** consists of strategies and beliefs satisfying requirement 1 through 4.

And, lastly, the definition (Gibbons, 1992) of information set is provided.

**Definition:** A **information set** for a player is a collection of decision nodes satisfying

(a) the player has the move at every node in the information set, and

(b) when the play of the game reaches a node in the information set, the player with the move does not know which node in the information set has (or has not) been reached.
Credibility is central in dynamic games, that is, the players must be able to credible threat of punishing the other player for making certain moves (Gibbons, 1992). This is also crucial in signaling games, which are discussed next.

5.1.1 Information Asymmetry and Signaling

Information asymmetry occurs, as Copeland and Shastri (2004) describes it, when one group of participants have superior information relative to other groups. And, in many cases, such information is difficult, if not impossible, to fully reveal by others (Eeckhoudt, Gollier and Schlesinger, 2005). As the example set forth by Eeckhoudt et al. (2005) goes: Consider a store which offers a free piano to those who cannot play the piano. While it is easy to check if a person can play the piano, it is difficult, if not impossible, to check if someone cannot play the piano (since everyone can fake it).

In such situations, it is possible for the group with the superior information to do actions that affect the probabilities the other groups assign to each possibility (that is, the players belief at each information set). For example, in the already cited paper by Akerof, Eeckhoudt et al. (2005) points out that the seller can offer a limited warranty, so that the seller will have lower expected income when offering a low quality product. Knowing this, the customers will perhaps assume a greater probability of the product being of high quality than before, i.e., the buyer updates his or hers beliefs. Here, the warranty is a signal to the buyer. Such signals have a central role in situations with asymmetric information, because it can be used to increase the level of information in the game. As Copeland, Weston and Shastri (2004) define it: A signal is an action taken by the more informed that provides credible information to the less informed. That is, the more informed may signal his or her type to the less informed using certain actions.

In our case, both the customer and the bank can be the more informed part. Thus, we consider how signaling (knowingly or not) may affect the outcome.

5.1.2 Backward Induction

Backward induction is a solution-method which can be applied to dynamic games. It is the process of reasoning from the end of the game to determine a sequence of optimal actions. It proceeds by first considering the last step a decision might be made and determine what to do in any situation at that time. Using this information, one can then determine what to do at the second-to-last time of decision. This process continues backwards until one has determined a set of optimal actions for every possible situation at every point in the
game (i.e. for every possible information set). This solution concept was first put forth by Von Neumann and Morgenstern (2007), and have ever since been used in the literature (see e.g. Tirole (1988) and Gibbons (1992)). A simple theoretic example follows (Gibbons, 1992):

**Example:**
There are two players, player 1 and player 2. Both have a number of feasible actions to choose from, and both maximize utility. The game they play is a two-stage (dynamic) game of complete and perfect information, which evolves as described next:

1. Player 1 chooses an action \( a_1 \) from a feasible set \( A_1 \)
2. Player 2 observes \( a_1 \) and then chooses an action \( a_2 \) from the feasible set \( A_2 \)
3. Payoffs are \( u_1(a_1, a_2) \) and \( u_2(a_1, a_2) \)

When player 2 gets the move at the second stage of the game, the player will face the following problem.

\[
\max_{a_2 \in A_2} u_2(a_1, a_2)
\]

We go on by assuming that for each \( a_1 \), player 2’s optimization problem has a unique solution, denoted by \( R_2(a_1) \). This is referred to as player 2’s reaction function to player 1’s action, i.e., player 2’s best response to player 1’s action. Since this is a game of complete and perfect information, player 1 can solve this reaction function as well as player 2 can. Thus, 1’s maximization problem at the first stage is

\[
\max_{a_1 \in A_1} u_1(a_1, R_2(a_1))
\]

Assume that this optimization problem for player 1 also has a unique solution, denoted by \( a_1^* \). (The * indicates the optimal action.) We call \( (a_1^*, R_2(a_1^*)) \) the backwards-induction outcome of the game (Gibbons, 1992). SPNE is the equilibrium concept associated with such an outcome.

Note the crucial assumptions that players maximize utility and make rational decisions. This may not be the case in the private credit market, but we argue that our description of the game sufficiently incorporates this complication. We discuss this thoroughly in the following.
5.2 Description of the Game

5.2.1 The Players

We consider a person in need of credit and a counterpart extending the demanded credit, henceforth referred to as the customer and the bank, respectively. We keep the assumption made about the banking market in section 4, namely, both players face a competitive banking market\textsuperscript{10}, which implies that the bank is price taker. We are interested in the dynamics associated with the updating of private information\textsuperscript{11} and, thus, it is not necessary to complicate the game by introducing a third, fourth or more banks. We will, however, analyze a case in which the customers can choose to share information, and, as will become clear, we must explicitly discuss how the other banks (the market) will react to different actions taken by the customers. We will return to this later on.

We note that our previous distinction was that public information is information available to all the banks, while internal information is observed by each bank over time, and to this date cannot be shared. Note, also, that using a distinction between the two different information types is a common approach in the literature, and we refer to the likes of Rothschild and Stiglitz (1992).

At first the bank segments the customers with respect to public information (income, age, profession, leverage and so forth) in $M$ groups. We assume that all banks will put the same customer in the same group, given that they use the same information\textsuperscript{12}. And we say that the banking market consists of $L$ banks, and $l \in \mathcal{L} = \{1, 2, \ldots, L\}$ indicates a given bank.

That is, we say that a customer within the public information risk class $i$ is of $j_i$ private information risk class, with probability of default denoted as $\text{PD}_{i,j_i}$, where $i = 1, 2, \ldots, M$, and each segment consists of $m_i$ customers. Each customer has a private risk class $j_i$, and we distinct between so-called good ($G$) and bad ($B$) customers, where, as a reminder, good customers are those with a risk class implying a lower risk than what their public information class implies, and bad customers are those with a higher risk than what their public risk class implies. That is,

\[ j_i \in [G_i, B_i] \]

\textsuperscript{10}Whether or not it exits some form of tacit collusion, or inefficient competition in the banking market is not the focus of this section. See, e.g., the Competitive Authorities’ report \textit{Konkurranse i Boliglånsmarkedet} (Competition in the Mortgage Market) (2015), for such discussion.

\textsuperscript{11}Or, for that matter, the existence of private (asymmetric) information

\textsuperscript{12}This assumption mostly holds in practice on the Norwegian retail/mortgage credit market (Sparebanken Vest, 2015)
In the case where the bank cannot identify the private risk class at the outset of a loan (i.e. the situation as it is today), but knows the distribution of $j_i$, and if we assume that the interest rate charged is a function of PD, that is,

$$r = r(PD_{i,j}),$$

then such adverse selection results in that the interest rate offered to public information risk class $i$, $r_i$, is priced based on expected default probability. That is, the interest charged to all customers in segment $i$ at the outset of the loan is $r_i^E$, where superscript $E$ denotes expectation. So,

$$r_i^E = r(E(PD_{i,j_i})) \quad (5.1)$$

Hence our discussion in section 4. Consequently, there is not just one interest rate charged, but several, which seems to be the case in the real world as well. So, the market can be said to be efficient of semistrong-form\(^{13}\). In our analysis, we look at customers within the same public information based risk class.\(^{13}\)

The observant reader has noted that we, in the outlines above, have simplified the world and assumed that the only relevant parameter determining the interest rate is PD. Since credit risk is among the most important parameters for any creditor, it is not unreasonable to focus exclusively on this.

While the purpose of the market participation model was to point out that some customers may fall out of the market due to asymmetrical information and how asymmetric information affects the general interest in the economy, this section aims to explain why the interest is set predominantly by the public information. The implication of this is of course that if more of the internal information is made into public information, then the interest can be set more accurately.

### 5.2.2 Order of Moves and Feasible Actions

The game begins when the bank has updated its belief about the customer’s risk class: After $t$ periods\(^{14}\), the bank may wish to discriminate further with respect to the customers’ true risk, which the bank estimates using models with private information collected through customer-bank relationship as input (Nordea, 2015b). The bank moves first, and the customer observes this before he or she chooses action. For instance, if the bank chooses to increase the interest

\(^{13}\)See appendix D on page 79 for a concise explanation of "semi-strong form" and the market efficiency hypothesis.

\(^{14}\)Nordea reports this to be about three months, but for generality we simply say $t$. 

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rate, the customer is able to get an offer from another bank before deciding to stay at the current bank or switch. The bank to which the customer switches cannot deviate from its offered interest rate (at least not for \( t \) periods).

We also assume that the bank chosen by the customer at the outset is random, that is, the banks are perceived as being equal by the customer because they are assumed to offer the same interest rate given the same information. We realize that this is, most likely, not the case.

As already mentioned, we will discuss a case in which customers can choose to share or not. In that case, the order of moves is somewhat altered, but we defer the description of this to later in the text.

### 5.2.3 Payoff Structure

For simplicity, we assume that each customer borrows 1 unit at date 0, pays back the principal and interest at date \( T = 1 \), and we ignore discounting. This follows our definition of the illustrative bond in section 3.4 on page 14. However, for the purpose of this model, we assume that the bank is able change the interest rate during the lifetime of the loan\(^{15}\). This simplifies the exposition, and allows us to view relevant payoffs as the interest rate.

We assume that the customer experiences some costs related to bank switch. We follow (Konkurransetilsynet, 2015) and consider search (se) and switching (sw) cost, denoted as \( C_{se} \) and \( C_{sw} \), respectively. Such costs, they say, can be monetary, time and/or psychological costs, and we assume they are non-negative. Furthermore, as Klemperer (1995) points out, so-called ”brand loyalty” can also affect this (perceived) cost. It can be argued that search costs is both small and sunk, and, thus, not relevant for the final choice. But, it may be the case that customers still regard such costs as relevant for the final payoff. It is, however, often the case that banks assists in the transfer, effectively reducing \( C_{sw} \) and \( C_{se} \). Note also that these costs evidently are highly subjective, and should therefore be next to impossible to estimate on an individual level. Which is why we say that \( C_{sw} \) captures all relevant search and switching costs for the representative bank customer\(^{16}\).

We include these costs as they are stated above in order to include every possible dis-

\(^{15}\)Alternatively, the customer borrows for one period at the time, and need to renew its loan for many periods.

\(^{16}\)As an extension of this approach, it could be argued that different customer types have different perception of this cost. For example, people with a higher education within economics (or, someone that have written their master thesis with this as topic) may find it less stressful to switch bank than someone without such background.
utility the customer experiences during a bank switch. As we shall see, this is very useful in our analysis.

One further simplifying assumption, we assume that the players utility functions are linear in its arguments, because doing otherwise (probably) complicates the game more than what needs be.

Following the likes of Rothschild and Stiglitz (1992), we assume the banks\textsuperscript{17} have financial resources such that they are willing and able to take on any contract that they think will make an expected profit.

### 5.2.4 Possible Weak Points

The incentives of both parties can be questioned. Because, in our conversation with banks (Sparebanken Vest (2015) and Nordea (2015a)), we have been told that they do not actively price risk into the offered interest rate, but, rather, take the market rate at face value. At the same time, banking customers in Norway are known to be notoriously loyal to their bank (Konkurransetilsynet, 2015). It may be that many customers are not interested in financial matters, and they perceive the gain of bargaining the loan terms as too small relative to perceived costs. We have, however, calculated that a 25 bps (0.25 percentage points) decrease in an initial annual interest of 4.25% paid on a level-payment fixed-rate mortgage of 2 millions NOK over 20 years, have a present value of about 50 000 NOK\textsuperscript{18}. Clearly, renegotiating the interest have considerably value\textsuperscript{19}, which means that they have either greatly undervalued the gains, their perceived costs are at such levels, or, more likely, a combination.

Nevertheless, if it is true that banks do not actively price the risk in, it simply supports our equilibrium in the limited information game analyzed in section 5.3.1. It is possible that the historical low default probability in Norway in recent years (Nordea, 2015b), influences banks’ incentives. However, in the game with full information, the above mentioned may attenuate the outcome of the game. Further discussion about both in section 5.3 and 5.4.

Another element worth mentioning is that the customers themselves may not know their own type, that is, their probability of default (PD). In that, they may overestimate their own ability to repay the loan. For instance, it might be that credit demand increases in PD, ceteris paribus.

\textsuperscript{17}Insurance companies in the case of Rothschild and Stiglitz (1992).

\textsuperscript{18}See appendix A on page 73 for exposition.

\textsuperscript{19}We recognize that, with the purpose of convincing an average customer to renegotiate his or hers interest rate, it might be better to use yearly undiscounted savings.
5.3 The Discrete Case

We start with the case where the customer in each segment $i$ can be either good or bad, or, equivalently, low or high private information risk class relative to their public information risk class, respectively. To be clear, bad customers have a a higher probability of default than their public information risk class implies, while good customers have a lower PD than their public risk class implies. That is, $j_i = G_i, B_i$. So,

$$\text{PD}_{i,B_i} > \text{PD}_i > \text{PD}_{i,G_i}$$

We assume the bank knows the share of high and low private information risk class in each public information risk class\textsuperscript{20}, with initial probabilities denoted as $p_i$ for good customers and $(1 - p_i)$ for bad customers, but cannot identify each customer’s private information risk class.

After the bank updates its belief about the customer’s risk, it continues to charge $r_i^E$ or changes the interest rate. That is, the bank faces the choice of either increase (I) or do nothing (DN) in the case of bad customer, and either decrease (D) or do nothing in the case of good customer. Thus, the bank’s feasible actions, $A_B$, are D, DN and I. The customer, being either good or bad, can choose to stay (S) at the current bank or switch (Sw). The customer’s set of feasible actions, $A_C$, thus consists of S and Sw.

Clearly, we could include the option of increasing the interest rate in the bank’s feasible set of actions in the case of low risk, which we discuss further in both the discrete and the continuous case below. And, for the high risk case, we could include the option to decrease the interest rate, but it makes little sense. Thus, we analyze changes in the interest following an update of the customer’s risk class, not changes due to, e.g., changes in the overall interest level in the economy.

As already mentioned, interest rate $r_i^E$ offered to segment $i$ is priced after the expected default probability. After $t$ periods, the bank is able to update its belief about the customer. If the customer is high risk, the bank can either increase the interest rate or do nothing. On the other hand, if the customer is low risk, the bank can either decrease the interest rate or do nothing. Thus, potential interest rates are $r_{i,G_i}$, $r_{i,B_i}$ or $r_i^E$, and, given our assumptions, it is reasonable to assume that

$$r_{i,B_i} > r_i^E > r_{i,G_i}.$$ \hspace{1cm} (5.2)

The bank, from which the customer gets an offer, is assumed to face the choice between I, D or DN, and uses public information in the pricing process. Also, the bank has some

\textsuperscript{20}This can be on basis if historic data.
beliefs about the customer’s (private information) type, denoted as $q_i$ for good customers and $(1 - q_i)$ for bad customers. To begin with, we distinct between $p_i$ and $q_i$ because the action of getting an offer may be a signal to the bank, and, consequently, this action may be used to update the bank’s beliefs. However, according to our contact person in Nordea Nordea (2015a), this action is too ambiguous to be interpreted as a high risk customer switching banks, or in any other way. So, since the action $Sw$ is too ambiguous to be interpreted as a signal in any way, the bank cannot update its belief based on this signal alone, so, presumably, $q_i = p_i$. Therefore, we say that the bank offers the same rate to segment $i$ as the current bank.

Customers is assumed to minimize interest and switching expenses, that is, they get an offer from one or more other bank(s) (OB) and one offer from the current bank (CB), and uses

$$\min(r_{i,j_i}^{CB}, r_{i,j_i}^{OB} + C_{sw})$$

as decision rule.

For simplicity, we describe two different games. One starting contingent on the customer being low risk, and the other starting contingent on the customer being high risk. We use this approach to simplify the game-trees, because it would become more complex if we were to draw both types in the same figure.

Backward induction is used as solution method, which means that we start out with analyzing what the customer will do contingent on the bank’s action.

We follow the convention of listing payoffs in the extensive-form representation at the terminal nodes of the game, with the first mover’s payoff on top, and the second player’s payoff under, and so on. Equilibrium and the following payoff combination is denoted as $(player 1’s action/payoff, \ldots, player n’s action/payoff)$. As we shall see shortly, the process it is to find the solution(s) of the game mainly consists of discussion regarding what the bank thinks the customer will do given its different possible actions.

We identify the game as a two-stage dynamic game with complete and perfect information, and two players, with the following dynamics.

1. The bank "observes" the player’s private information risk class $j_i$, and then chooses an action $a_B \in A_B = I, D, DN$

2. The customer observes $a_B$ and then chooses an action $a_C \in A_C = S, Sw$

3. Payoffs are $u_B(a_B, a_C)$ and $u_C(a_B, a_C)$
As it is stated above, this game is not hard to solve. Since it is complete and perfect information, the bank can always infer what the customer will *rationally* do. This game is complicated because customers are not rational in a traditional sense. We isolate this complication by letting utility losses be a part of $C_{sw}$.

5.3.1 Information Sharing Limitations

We start by analyzing the situation as it is today. Banks cannot share internal information with each other, and customers are not able to share such information regardless if they want to or not. Figure 2 show the extensive-form representation of the game starting after

![Figure 2: Extensive-form representation of the game starting contingent on the customer being $j_i = G_i$](image)

the customer’s type is revealed to be low risk. As already explained, if the customer switch to another bank, he or she is offered $r_i^E$ because the other bank(s) still price after expected default probability.

Reading of this figure, if bank plays D, the customer must choose to either stay (S) or switch (Sw). Since we assume that $C_{sw} > 0$ and $r_{i,G_i} < r_i^E$ (from (5.2)), it will always be the case that

$$-r_{i,G_i} > -(r_i^E + C_{sw}),$$

and given (5.3), customer plays S. Stated in another way:

$$S \succ Sw \iff -r_{i,G_i} > -(r_i^E + C_{sw}) \iff C_{sw} > 0 \iff r_{i,G_i} < r_i^E$$

(5.4) (5.5)
In words: Action S is strictly preferred over Sw given that the payoff from playing S is higher, and condition (5.4) and (5.5) is satisfied, which they always are. The same reasoning is applied to the following analysis as well, but we do not mathematically state the customer’s preferences in the remainder of this analysis. Next, if bank plays DN, the customer plays S, since

\[-r_i^E > -(r_i^E + C_{sw})
\]

\[\Downarrow\]

\[C_{sw} > 0\]

Using backward induction, the customer’s reaction function thus becomes

\[R_C(a_B) = S.\] (5.6)

That is, regardless of the bank’s action, the customer chooses S. Thus the customer’s strategy that describes what the customer will do in every contingency the customer may be called upon to act is (S,S)\(^{21}\): If the bank plays D, the customer plays S (this what the the first notation inside the parenthesis describes), and if the bank DN the customer plays S (the second notation in the parenthesis). This notation will be used for the equilibria in the remainder of this section.

Next we look at the optimization problem for the bank.

\[
\max_{a_B \in A_B} u_B(a_B, R_C(a_B))
\] (5.7)

Since the bank assesses the possible actions taken by the customer as we have done above, and \(r_i^E > r_i,G_i\), the subgame-perfect Nash equilibrium of this game is as follows. The bank plays DN and the customer play S, expressed as

\[
\{a^*_B, R_C(a^*_B)\} = \{DN, (S,S)\},
\]

with payoffs \((r_i^E, -r_i^E)\).

The resulting outcome shows us that the low risk customers are charged a higher interest rate then what their risk class implies, since the asymmetric information reduces their bargaining power. This is also the view expressed by (Fama, 1985), but instead of dubbing it private information, he refers to it as inside information. Although he mainly discusses the corporate loan market, he also recognize that private customers, i.e., individuals, gets a

\(^{21}\)Recall that this fulfills the definition of strategy given in section 5.1.
lower contracting cost for inside debt, relative to outside debt (which we capture by $C_{sw}$). But in our model, it is the bank, not the customer, who reap this gain.

Next we look at the case of $j_i = B_i$. Figure 3 show the extensive form representation of the game starting after the customer’s type is revealed to be high risk. Reading of this figure, we can see that if the bank plays I, the customer can play either Sw or S. In this case, the customer’s action depends on $C_{sw}$. If

$$-r_{i,B_i} > -(r_i^E + C_{sw})$$

the customer plays S. If $C_{sw} < r_{i,B_i} - r_i^E$, the customer instead plays Sw. If the bank plays DN, the customer chooses S (if $C_{sw} > 0$, which we have assumed it to be). The customer’s reaction function becomes:

$$R_{i,j_i}(a_B)|_{j_i = B_i} = \begin{cases} 
S, & \text{if } a_B = I \text{ and } C_{sw} > r_{i,B_i} - r_i^E \\
Sw, & \text{if } a_B = I \text{ and } C_{sw} < r_{i,B_i} - r_i^E \\
S, & \text{if } a_B = DN
\end{cases}$$

Next we look at the optimization problem for the bank.

$$\max_{a_B \in A_B} u_B(a_B, R_{C_{Bi}}(a_B)) \quad (5.8)$$
As mentioned, the costs related to a bank switch is subjective, and, thus, it is difficult for the bank to assess these correctly. Two things follows, the bank may choose an action with the least uncertainty attached to it, or the bank assess the market at large and estimates a representative $C_{sw}$. In either approach, it is reasonable to start by checking whether the bank has a dominant strategy (Gibbons, 1992). Since it is unclear what the customer does if the bank plays I (it depends on $C_{sw}$), such strategy does not exist. So, the bank’s action depends on its belief about the $C_{sw}$. So,

$$a_B^* = \begin{cases} I, & \text{if } C_{sw} > r_{i,B_i} - r_i^E \\ DN, & \text{if } C_{sw} < r_{i,B_i} - r_i^E \end{cases} \quad (5.9)$$

It is of our belief that $C_{sw}$ can be endogenous, meaning that the customer may be offended if the bank chooses to increases the rate, and changes bank in some form of protest. Nevertheless, the situation today as described by Nordea is that the bank chooses not to increase the rate since they believe that customers switch in that case. This implies that at least Nordea assumes $C_{sw} < r_{i,B_i} - r_i^E$ and plays DN, since $r_i^E > 0$.

The SPNE of this game is

$$\{a_B^*, R_{CH}(a_B^*)\} = \{DN, (Sw, S)\}, \quad \text{if } C_{sw} < r_{i,B_i} - r_i^E,$$

with payoffs $(r_i^E, -r_i^E)$.

Evidently, due to the information asymmetry caused by information sharing limitations, high risk customers are charged a lower interest rate than what their risk class implies.

Thus far we have analyzed the outlined games with information sharing limitations. The outcomes of these games shows us that both high and low risk customers are charged the same interest rate. This outcome can be described as a pooling equilibrium (Rothschild and Stiglitz, 1992), since both types are charged the same interest rate. We saw this occur in the simple market participation model, where uncertainty about default probabilities (risk) caused a situation where all customers paid the same interest. Moreover, we saw how such a market ended up having a disproportionate share of high risk customers, as both Eeckhoudt et al. (2005) and Akerlof (1970) points out. Such a phenomenon is, as mentioned, called adverse selection.

In addition, this is also the case as described by Nordea and Sparebanken Vest, and thus our model has succeeded in describing the situation as it is today. We note, however, that this is no guarantee for our model being ”good”, but it lends some credence to it.
5.3.2 Forced Information Sharing

In this game, the banks are required to share their internal assessments with other banks, meaning that all information is now public information. Note that the payoff reached after the customer chooses Sw in Figure 4 and 5 is changed. This reflects the fact that customers under these conditions are not able to change their interest rate by switching bank, as all banks have already been assumed to give the same interest given the same information.

![Diagram](image.png)

**Figure 4:** Extensive-form representation of the game starting contingent on the customer being low risk

We begin by analyzing the game starting after the customer’s type is revealed to be low risk. See Figure 4 for extensive form representation. If the bank plays DN, the customer plays Sw if:

\[-(r_{i,G_i} + C_{sw}) > -r^E_i\]

\[\iff\]

\[C_{sw} < r^E_i - r_{i,G_i} > 0\]

If the bank plays D the customer plays S, since \(-r_{i,G_i} > -(r_{i,G_i} + C_{sw})\), and \(C_{sw} > 0\). This results in the following reaction function:

\[R_{CG}(a_B) = \begin{cases}  
Sw, & \text{if } a_B = \text{DN and } C_{sw} < r^E_i - r_{i,G_i} \\
S, & \text{if } a_B = \text{DN and } C_{sw} > r^E_i - r_{i,G_i} \\
S, & \text{if } a_B = \text{D} \end{cases}\]

The bank still maximizes its objective function as stated in (5.7), and, again, the optimal action depends on \(C_{sw}\). If the bank assumes \(C_{sw} < r^E_i - r_{i,G_i}\), they will infer that the
customer plays $S$ if they play $D$, and $Sw$ if $DN$. Thus, if the bank chooses $DN$ the received payoff is zero, but if they play $D$ the payoff is $r_i,G_i$. On the other hand, if they believe that $C_{sw} > r_i^E - r_i,G_i$, and they play $D$ the customer still plays $S$ with $r_i,G_i$ as resulting payoff, but if they play $DN$, the customer plays $S$ resulting in $r_i^E$ as payoff. This results in following subgame-perfect Nash equilibria:

$$\{a_B^*, R_{CL}(a_B^*)\} = \begin{cases} 
\{D, (S,Sw)\}, & \text{with payoffs } (r_i,G_i, -r_i,G_i), \quad \text{if } C_{sw} < r_i,G_i - r_i^E \\
\{DN, (S,S)\}, & \text{with payoffs } (r_i^E, -r_i^E), \quad \text{if } C_{sw} > r_i,G_i - r_i^E
\end{cases}$$

Figure 5: Extensive-form representation of the game starting contingent on the customer being high risk

Evidently, it is not possible to say with certainty what the final outcome would be. If, however, we have that $C_{sw} < r_i,G_i - r_i^E$, it is clear that the bank can charge the customer such an interest rate that the customer is marginally better of by staying. That is, instead of $r_i,G_i$ the bank can demand:

$$r_{i,G_i}^* = r_i,G_i + C_{sw} - \varepsilon,$$

where $\varepsilon$ is something very small. Ergo, the bank is still able to capture the switching cost. This will be further discussed in the continuous case.

Next we analyze the game starting after the customer’s type is revealed to be high risk. The extensive-form representation is presented in Figure 5. If the bank plays $DN$, the customer plays $S$ since $-r_i^E > -(r_i,B_i + C_{sw})$, and if the bank plays $I$ the customer plays $S$ since $-r_i,B_i > -(r_i,B_i + C_{sw})$. The resulting reaction function:

$$R_{CH}(a_B) = S$$
That is, the customer stays at the current bank regardless of whether the bank plays I or DN. Thus, the optimal action for the bank according to (5.8) is I, since \( r_{i,B_i} > r^E_i \). The equilibrium is:

\[
\{a^*_B, R_{C_{ii}}(a^*_B)\} = \{(I, (S,S))\},
\]

with payoffs \((r_{i,B_i}, -r_{i,B_i})\). And, here as well, the bank may increase the interest up to the point where the customer is marginally better off by staying:

\[
r^*_i = r_{i,B_i} + C_{sw} - \varepsilon
\]

Again, this is further discussed in the continuous case.

As we can see, removing the information sharing limitations results, in this game, in a more efficient (correct) pricing of risk. The low risk customer is still charged a higher interest rate than what his or her risk class implies, due to \( C_{sw} \) being strictly positive and not easily measured. The major difference, however, is that the high risk customer now is charged a the correct interest rate, or even a higher interest rate than what his or her risk class implies. Assuming decreasing credit demand in interest rate, the result may be a higher aggregate credit demand by low risk customers, while the high risk customers’ credit demand falls. In the simple market participation model, we showed how the average interest rate falls when the public information is improved, which leads to a larger market for credit. This outcome can be described as a separating equilibrium (Rothschild and Stiglitz, 1992), in which the different types are charged different interest rates.

5.3.3 Voluntary Information Sharing

In this section, we analyze the case with voluntary information sharing. “Voluntary” means that the customers themselves choose to either share internal information or not with other banks (or, equivalently, the market). As we shall see, this increases the necessity of both parties (the customer’s, current bank and the other banks) being sophisticated, for the efficiently priced outcome to be obtained, that is, when all customers are priced efficiently with respect to both public and internal information. The banks must have some belief about both the customers that switch and the other banks. The customers, on the other hand, must somehow know that they may be able to improve their loan terms by sharing their information. More discussion about both later on. We remark that sharing with the market is equivalent with the customer sharing information with a third party that again shares with the banks, as we discuss in section 7.
Compared to previously in this section, the order of moves is slightly changed: The current bank, bank 1, still estimates the customer’s true private risk class, and may wish to change the interest rate. The customer is still able to collect an offer before deciding to stay or switch, but we include this node explicitly because it may affect the other bank’s actions. Evidently, the customer is called upon to act twice in this game, first to share or not share, and second, as before, to either stay or switch. It is important to analyze the possible actions taken by the ”other bank” in depth. This is because, when the other bank receives the request from the customer, this action may be interpreted as a signal (opposite to the previous cases). Hence, the bank may update its belief about the customer, and thus deviate from offering $r_j^E$. Finally, the customer observes the action taken by the other bank and then makes his or her final choice to stay or switch, after which the game ends.

It follows that the customer now has two sets of feasible actions, one for the first node at which he or she is called upon to act, $A_{1C}$, and one for the last node, $A_{2C}$. The customer’s feasible set of actions consists of two possible choices at the first node: They can choose to either be open (Op), i.e., share information with the market; or closed (Cl), i.e., not share information with the market. Other than this, all the players feasible set of action is the same as before.

The banking market is still assumed to be competitive and without tacit collusion.

We identify the game as a 4-stage dynamic game of complete but imperfect\textsuperscript{22} information. The game evolves as follows.

1. The current bank ($B_1$) ”observes” the player’s private information risk class $j_i$, and then chooses an action $a_{B1} \in A_{B1} = I,D,DN$

2. The customer chooses to be either open or closed. That is, chooses $a_{1C} \in A_{1C} = Op,Cl$

3. Bank 2 or the market observes the action by the customer, and chooses $a_{B2} \in A_{B2} = I,D,DN$

4. The customer observes $a_{B1}$ and $a_{B2}$, and then chooses an action $a_{2C} \in A_{2C} = S,Sw$

5. Payoffs are $u_{B1}(a_{B1},a_{1C},a_{B2},a_{2C})$, $u_{C}(a_{B1},a_{1C},a_{B2},a_{2C})$ and $u_{B2}(a_{B1},a_{1C},a_{B2},a_{2C})$

The banks are thus uncertain about the other bank’s chosen action (since they do not know the customer’s type).

We now move on to the attempt of solving this case, which is more of an analysis than solution in a traditional sense. As approach, we have chosen to do a stepwise analysis of

\textsuperscript{22}Since bank 2 or the market is uncertain about which action bank 1 has chosen.
the case, using two different assumptions about the customers information set, and two assumptions about the banks. We view the customers as having either full or no knowledge about their own \( j_i \) and the resulting \( r_{i,j_i} \). We regard both as plausible, but a combination of the two may be the most realistic approach. The banks, on the other hand, are assumed to be either naive or sophisticated. Naive is when the banks do not update their belief about customers that do not share information, and sophisticated is when they do.

We say that the customers choose to be either open or closed regardless of collecting an offer or not, alternatively, all customers are assumed to collect an offer, whereas the only decision they must make at this point are to either share information or not.

The Naive, Full-knowledge Case

We begin with assuming that customers have full knowledge about their own type and the market is naive. Thus, bank 2 receives either of two possible messages. The first being "I want an offer, here is my (internal) information", or, the other, "I want an offer, but you get only the public information". Bank 2 must next give its offer, and the interest rate still depends solely on PD.

\[
r = r(E\{PD_{i,j_i}\}) = r(q_i(PD_{i,C_i} + (1 - q_i)PD_{i,B_i}))
\]  

(5.10)

Given that the bank is naive, in that the bank will set the interest rate to \( r^E \) unless the customer agrees to share his or her internal information, its reaction function looks like this:

\[
R_{b2}(a^1_C) = \begin{cases} 
  r_{i,j_i}, & \text{if } a^1_C = \text{Op} \\
  r^E, & \text{if } a^1_C = \text{Cl} 
\end{cases}
\]  

(5.11)

Bank 2 chooses to offer \( r_{i,j_i} \) if the customer share information, because doing otherwise results in the customer going to another bank. In other words, the market offers the customer an interest, and, following the same arguments as in section 5.4.1, the market always offers \( r_{i,j_i} \) to the customer with characteristics \((i, j_i)\).

The customer still minimizes interest and switching costs. The optimal action is thus easy to deduce: The customer chooses the option that yields the lowest total cost, and we assume that the customer stays if the total cost is the same for either option.

In this naive, full-knowledge case, the following is the outcome. All customers with \( r_{i,j_i} < r^E \) will share their information to obtain a lower interest rate. From (5.2) and (5.3) we see that all good customers will share and all bad customers will retain. That is,

\[
a^1_C = \begin{cases} 
  \text{Op}, & \text{if } j_i = G_i \\
  \text{Cl}, & \text{if } j_i = B_i 
\end{cases}
\]
Thus, (5.11) becomes

$$R_{B_2}(a^1_C) = \begin{cases} r_{i,G_i}, & \text{if } a^1_C = \text{Op} \\ r^E_i, & \text{if } a^1_C = \text{Cl} \end{cases}$$

This leads us to figure 6 and 7 as the resulting game-trees for the good and the bad customer, respectively. Note that figure 7 is identical to figure 3, which means that we have already found the equilibrium of this game. While the good-customer game needs to be solved explicitly. This is, however, not that complicated: If we assume that the bank believes that $r_{i,G_i} + C_{sw} \leq r^E_i$, the bank will play D, the customer thus plays S ($C_{sw} > 0$). On
the other hand, if the bank assumes \( r_{i,G_i} + C_{sw} > r_i^E \), the bank plays DN and the customer plays S. And, in either case, \( B_2 \) plays D. So, the SPN equilibria of this game are

\[
\{a_{B1}^*, R_{B2}(a_1^C), R_{CG}(a_{B1}^*, a_{B2}^*)\} =
\begin{cases}
\{D, D, (S, Sw)\}, \text{ with payoffs (} r_{i,G_i}, 0, -r_{i,G_i} \text{)} & \text{if } r_{i,G_i} + C_{sw} \leq r_i^E \\
\{DN, D, (S, S)\}, \text{ with payoffs (} r_i^E, 0, -r_i^E \text{)} & \text{if } r_{i,G_i} + C_{sw} > r_i^E
\end{cases}
\tag{5.12}
\]

The final result is, as previously, that the outcome depends on \( C_{sw} \). It also implicitly depends on the distribution of private risk classes. However, for some values of \( C_{sw} \), good customers is charged their true interest rate, while the bad customer is charged \( r_i^E \). This should also mean that \( r_i^E \) is somewhat higher.

We discuss this more closely in section 5.4. For now, we remark that this is not central to this section, as we have anyways assumed that the bank either offers the true interest rate or the same expected rate. In conclusion, we can say that this outcome is slightly more fair than the the outcome under information sharing limitations, provided that good customers are charged their true interest rate. This concludes our analysis regarding the naive, full-knowledge case.

**The Sophisticated, Full-knowledge Case**

We proceed to consider the case of sophisticated banks.

If all customers who gain from sharing information decide to do so, this tells us that the customers who do not share their information must be bad customer. If bank 2 gets information, it is able to update its belief about that customer (\( q_i \) in equation (5.10) above), and therefore, the same is true if the customer does not share information. As already mentioned, the customers who may lose from sharing, retain their information. So, given that the banks understand this, they know that

\[
q_i = \begin{cases} 1, & \text{if } a_C = \text{Op (and } j_i = G_i) \\ 0, & \text{if } a_C = \text{Cl} \end{cases}
\tag{5.13}
\]

Hence,

\[
r(a_1^C = \text{Cl}) = r(PD_{i,B_i}) = r_{i,B_i}
\]

Consequently, (5.11) becomes

\[
R_{B2}(a_1^C) = \begin{cases} r_{i,G_i}, & \text{if } a_C = \text{Op} \\ r_{i,B_i}, & \text{if } a_C = \text{Cl} \end{cases}
\tag{5.14}
\]
See the game-tree in figure 8 for illustration. Note that the current bank is uncertain about what the other bank has offered to the customer\(^{23}\). That is, the current bank does not know if the customer faces the payoffs illustrated in figure 7 or 8\(^{24}\).

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{game_tree.png}
\caption{The sophisticated, full-knowledge case, for the \(j_i = B_i\) customer.}
\end{figure}

Recall that this corresponds to the current bank’s information set, as described in section 5.1: It satisfies (a) that the bank has the move on both nodes at this informations set, and (b) the bank does not know which node the game as come to. The two nodes consist of one for the bad and for the good customer, at which the bank must choose to increase, decrease or do nothing. Which node the game comes to depend on what the other bank does.

If the current bank believes that figure 8 is the case, it will play I since \(r_{i,B_i} > r_{i}^{E}\), and DN otherwise. In other words, if the current bank believes that the other banks reaction function is (5.14), it will play I given bad customer, and thus leading to the outcome in which bad customers are charged their appropriate interest rate. If all actors in the banking market believe that other banks use (5.14) as their reaction function, the following is the equilibrium.

\begin{align*}
\{a_{B1}^*, R_{B2}(a_{C1}^*), R_{CG}(a_{B1}^*, a_{B2}^*)\} &= \{I, I, (S, Sw)\} \\
\text{with payoffs } (r_{i,B_i}, 0, -r_{i,B_i}). \text{ Otherwise,} \\
\{a_{B1}^*, R_{B2}(a_{C1}^*), R_{CG}(a_{B1}^*, a_{B2}^*)\} &= \{DN, DN, (Sw, S)\}
\end{align*}

\(^{23}\)We could (or perhaps should) illustrate this, but when we sketched the game-tree, it became unnecessarily complicated.

\(^{24}\)\(B1\) does, however, know if the customer is open or closed (since they must share the information).
with payoffs \((r_i^E, 0, -r_i^E)\). Note the necessary condition for both (5.15) and (5.16), namely \(C_{sw} < r_{i,B_i} - r_i^E\).

We now introduce a new notation, \(\mu_l\), which is bank \(l\)’s belief about the sophistication level of other banks. For simplicity we assume \(\mu_l = 1, 0\), that is, either the bank believes that other banks are sophisticated, or not, respectively. By doing so, we can in theory correctly state the resulting Bayesian equilibria. (5.12) and (5.15) together becomes

\[
\{(D,I),(D,I),(S,S), \mu_l = 1\} \quad \forall l \in \mathcal{L} \tag{5.17}
\]

In (5.17) the first parenthesis is the strategy set of the “current bank” or simply “Bank”, and describes its optimal strategy given a good customer (the first notation inside the parenthesis) and the optimal strategy given bad customer (the second notation in the parenthesis). Both are given that \(\mu_l = 1\). The second parenthesis describes the ”other bank’s” optimal strategies in the same manner as the former. The last parenthesis describes the customer’s optimal strategy contingent on their own type in same manner as the other two. \(\mu_l = 1\) states that this outcome is valid given that bank \(l\) believes that all other banks are sophisticated, and, lastly, we say that all the banks do so.

Thus, the other possible equilibrium, by combining (5.12) and (5.16) we get

\[
\{(D,D_N),(D,D_N),(S,S), \mu_l = 0\} \quad \forall l \in \mathcal{L} \tag{5.18}
\]

Which is read in the same way as (5.17)

This leads us to the discussion regarding the pricing strategy of the customer’s current bank. If they choose to capture the switching cost through an increased interest rate, the end-game interest rate (denoted with a \(*\)) is as in the previous case

\[
r_{i,j_i}^* = r_{i,j_i} + C_{sw}
\]

When the final outcome is (5.17) bad customer must pay a higher interest rate, while the good customer pays a lower interest rate, both relative to \(r_i^E\). Note that this is given that (a) the customers have full knowledge of what they gain if they are open, and (b) the bank understands all customers that gain from being open will choose to do so. This concludes the sophisticated, full-knowledge case.

The No-knowledge Case

In this case, we regard the game given that the customers do not know their own type. In other words, they do not know their own \(j_i\). We do not provide a formal analysis for this case, but we do, however, offer some discussion.
How will this affect their behavior? Most likely it will depend on each individual. Some will fight changes for the worse in the loan terms, and some will perhaps acquiesce with the change. Evidently, it is difficult to say anything without making some further assumptions. For instance, we can regard the customers as being reluctant to changes and interested in knowing their own type, and assume that everyone will share their information to gain such knowledge. This will in turn lead to the case where the situation is as described in figure 6 for the good customer and either figure 7 or 8 for the bad customer, contingent on the banks’ belief about other banks. This concludes our discussion regarding the no-knowledge case.

Concluding Remarks

Evidently, the final result highly depends on both the customer and the bank. If the bank assumes that at least one other bank does not increase given no information, they may charge \( r_E \). And, interestingly, how do the good customer in fact know that he or she is a good customer without the bank saying so? In sum, it may be that voluntary information gives the good customers increased bargaining power, but they must take the initiative themselves and also be aware of their type. In the case of the bad customers, they being charged the appropriate interest rate highly depends on all the banks being sophisticated, or, alternatively, it depends on the banks belief about the other banks, if each bank believes that every other bank is sophisticated, then all banks becomes just that. In that case, it may be that the profits from deviating from I given Cl, is so high that the market never will have (5.17) as the equilibrium.

5.3.4 Summary of the Discrete Case

We started out be viewing the current legislation, the case in section 5.3.1, and explained how the observed situation may occur, in which good and bad customers are charged the same interest. Such an outcome is referred to as a pooling equilibrium. That is, high risk customers pay too low interest, and low risk customers pay too high interest, relative to their true interest rate, given asymmetric information. We have shown further, through section 5.3.2 and 5.3.3, how this is because the banks can only utilize public information to set interest, and that this information is not sufficient to create a separating equilibrium. In that, the good and bad customer pays different interest rates. The analysis performed in

\[^{25}\text{Of course, given that it exists internal or private information.}\]
section 5.3.3 shows us that the outcome given voluntary information sharing is difficult to predict.

This leads us to one of the central take-aways of this thesis, which is that banks can only meaningfully use public information to set interest rates, implying that making as much information shareable between banks as possible is crucial for efficient setting of interest rates.

5.4 The Continuous Case

Here we relax the assumption of \( j_i \) being either \( B_i \) or \( G_i \), and more realistically assume that \( j \) in each group \( i, j_i \), is somehow distributed between some high and low levels of private information risk class, that is

\[ j_i \in [B_i, G_i], \]

The expected probability of default is

\[ E(PD_{i,j_i}) = \sum_{j_i=B_i}^{G_i} PD_{i,j_i}f(PD_{i,j_i}) \]

in the discrete case, and

\[ E(PD_{i,j_i}) = \int_{G_i}^{B_i} PD_{i,j_i}f(PD_{i,j_i})dj_i \]

in the continuous case, where \( f(\cdot) \) is the probability density function, which we have not yet defined. The interest rate is still assumed to be dependent solely on PD, so

\[ r_i^E = r(E(PD_{i,j_i})), \]

and the banks assess each customer in the same way.

5.4.1 Perfect Capital Markets

We start out by analyzing the case of full information and no friction \( C_{sw} = 0 \), which is dubbed the case of perfect capital markets. But, for consistency, the private information risk class must still be estimated using internal data. Consequently, as soon as one participant finds \( (i, j_i) \) all participants in the market know each customer’s risk class. The bank charges \( r_i^E \) at the outset of the loan, and the bank updates its belief after \( t \) periods. Since we are assuming perfect competition in the banking market, the other banks cannot set its interest rate lower than \( r_{i,j_i} \) without resulting in a net present value (NPV) \(< 0\), hence \( r_{i,j_i}^{OB} = r_{i,j_i} \).
The customer chooses the lowest interest rate of the current bank’s (CB) offered rate and the other bank’s (OB) offered rate, i.e.,

$$\min(r_{i,j}^{CB}, r_{i,j}^{OB})$$

Since the current bank wants to keep the customer (otherwise the payoff is 0), and uses

$$\max(0, r_{i,j})$$

as decision rule, they match the other bank’s offer, and we rule out tacit collusion. So, each customer is offered (O) his or hers “true” interest rate,

$$\frac{r_{i,j}^{O|PCM}}{\forall (i,j)}$$

This outcome is dubbed the perfect capital market (PCM) outcome\textsuperscript{26}, in that, it is full information and no friction (no switching costs). We derive this result because it will be used as a benchmark for the remainder of this section. See the orange line in Figure 9 for illustration, using uniformly distributed non-specified values.

In the PCM case, and given that $j_i$ is uniformly distributed, the bank still charges the same average interest rate, but each customer is charged the correct interest rate for his or hers risk class. If we further assume that each customer has linear and identical credit demand, then the bank still extends the same total amount of credit, with the only difference being a more correctly priced risk.

Moreover, the probability of default is often assumed to increase in the interest rate. Obviously, if the interest payment increases, the probability of default (not being able to fulfill the payment required) must be the same or increase. In the text above, we have implicitly assumed that all customers gets a loan, i.e., their risk class is not over the critical risk class in terms of Stiglitz and Weiss (1981) even after being charged a higher interest rate. This will also be an underlying assumption in the remainder of this section.

5.4.2 Information Sharing Limitations

We continue the analysis by describing the current situation, in which it is prohibited by law to share internal information, and is dubbed the case of information sharing limitations (ISL). Each customer’s risk class is not common knowledge, while $C_{sw} > 0$\textsuperscript{27}. The bank charges

\textsuperscript{26}This is also known as the Bertrand paradox (Tirole, 1988) (perfect competition with only two actors).

\textsuperscript{27}We can justify this by observing that banks price after expected default probability, and they are able to estimate the representative customer’s switching cost.
at the outset of the loan. For the sake of discussion and clarity, we analyze two different cases, (a) when the bank increases the interest offered to customers with true interest rate higher than \( r^E_i \), and (b) when the bank maximizes the rate charged to all customers.

Starting with the former: When the bank updates its belief, it wants to increase the interest rate offered to only those whose risk class implies an interest rate above \( r^E_i \). Customers are assumed to minimize interest and switching expenses, that is, they get an offer from one or more other bank(s) (OB) and one offer from the current bank (CB), and (still) use (5.3) as decision rule. And the current bank is (for now) assumed to use

\[
\max(r^E_i, r_{i,j_i})
\]

as their decision rule. Since everyone can ”reset” their interest rate by switching banks, all customers with

\[ r_{i,j_i} > r^E_i + C_{sw} \]

will credibly threat to switch bank. Thus, the bank offers

\[ r^O_{i,j_i} = r^E_i + C_{sw} \]

to all of these customers, while

\[ \max(r^E_i, r_{i,j_i}) \]

to everyone else. That is,

\[
r^O_{i,j_i}^{ISL} = \begin{cases} 
  r^E_i + C_{sw} & \text{if } r_{i,j_i} > r^E_i + C_{sw} \\
  r_{i,j_i} & \text{if } r^E_i < r_{i,j_i} < r^E_i + C_{sw} \\
  r^E_i & \text{if } r_{i,j_i} < r^E_i 
\end{cases}
\]

See the blue line in figure 9 for illustration.

Moving on to (b), where the bank maximizes the interest rate offered to all customers. Assuming exogenous \( C_{sw} \), and given that other banks price based on expected default probability, that is, offer \( r^E_i \) to all customers, then the bank uses

\[ \max(r^E_i + C_{sw}, r_{i,j_i}) \]

as decision rule. The condition \( r_{i,j_i} < r^E_i + C_{sw} \) captures that some customers can credible threat to switch, and, consequently, the bank does not increase their interest rate further (over \( r^E_i + C_{sw} \)). Evidently, the optimal interest rate (denoted with *) is

\[ r^*_{i,j_i}^{ISL} = r^E_i + C_{sw} \quad \forall (i, j_i), \]

\[ \text{As discussed earlier, customers may be offended by an increased interest rate.} \]
Figure 9: Illustrating possible outcomes in the case of information sharing limitations. \( r_{i,j} \) is the corresponding interest rate to characteristics \((i, j)\), \( r^{*\text{ISL}}_{i,j}\) is the set of highest possible interest rates the bank can charge on average, \( r^{O\text{ISL}}_{i,j}\) is the other possible set of interest rates, both as deduced in section 5.4.2, and \( r^E_i \) is the interest rate priced after the expected default probability in segment \(i\).

i.e., everyone is charged \( r^{*\text{ISL}}_{i,j} \), illustrated by the black line in Figure 9.

### 5.4.3 Forced Information Sharing

All banks are now obligated to share their internal assessment/information, this scenario is dubbed forced information sharing (FIS). \( C_{sw} > 0 \), its value is still assumed to be estimated as the average switching cost, and is known by all market participants. The customer still uses (5.3) as decision rule. The bank still charges \( r^E_i \) at the outset of the loan. Here, also, two scenarios are analyzed: (a) when the bank increases the interest charged to customers with a risk class implying an interest rate higher than \( r^E_i \), and (b) when the banks charges the highest possible interest rate to all customers.

Starting with (a), the bank uses (5.19) as decision rule. That is, they wish to change the interest rate charged to those with risk class implying \( r_{i,j} > r^E_i \). Customers with a "real" interest rate lower than \( r^E_i - C_{sw} \) is now able credibly threat to switch (note how this has
Figure 10: Illustration of possible outcomes in the case of forced information sharing, with switching costs ($C_{sw}$). $r_{i,j_i}$ is the corresponding interest rate to characteristics $(i, j_i)$, $r_{i,j_i}^{s|FIS}$ is the set of highest possible interest rates the bank can charge on average, $r_{i,j_i}^{O|FIS}$ is the other possible set of interest rates, both as deduced in section 5.4.3, and $r_i^E$ is the interest rate priced after the expected default probability in segment $i$.

shifted from the bad customer to the good customer), while everyone else is charged either the same $r_i^E$ or their true interest rate. That is,

$$r_{i,j_i}^{O|FIS} = \begin{cases} 
  r_{i,j_i} & \text{if } r_{i,j_i} + C_{sw} < r_i^E \\
  r_i^E & \text{if } r_{i,j_i} + C_{sw} > r_i^E \text{ and } r_{i,j_i} < r_i^E \\
  r_{i,j_i} & \text{if } r_{i,j_i} > r_i^E 
\end{cases}$$

See the blue line in figure 10 for illustration.

In scenario (b) where the bank maximizes the interest rate offered to every customer, the interest rate charged to characteristics $(i, j_i)$ is

$$r_{i,j_i}^{s|FIS} = r_{i,j_i}^* = r_{i,j_i} + C_{sw} \ \forall (i, j_i)$$

Which is the PCM outcome plus the representative customer’s switching costs. See the black line in figure 10 for illustration.
5.4.4 Voluntary Information Sharing

Lastly, we analyze the voluntary information sharing (VIS) case, which is assumed to evolve much in the same way as the discrete case described in section 5.3.3 on page 46. The only difference is that, rather than I, DN, and D, the banks have a basket of infinitely many interest rates to choose from. The customers still choose to be either open or closed, that is, to either share their internal information or not. $C_{sw} > 0$, its value is still assumed to be estimated for the average customer, and is known by all market participants. The customers still use (5.3) as decision rule. The banks still charge $r^E_i$ at the outset of the loan, provided that they do not have access to internal information.

Due to the complexity of this case, we solve it using a stepwise analysis. We do this in the same way as in section 5.3.3 starting with the naive, full-knowledge case, followed by assuming sophisticated banks. Because we face a continuum of private risk classes (and thus a continuum of interest rates), we also look at how the customers will react to the banks’ updating of beliefs. Finally, we discuss how the customers’ current banks behave.

The Naive, Full-knowledge Case

We start by saying that the customers have full knowledge of their own type, and know their corresponding market interest rate. The market always uses all available information to price the customers. The implication of this is that all customers with $r_{i,j_i} < r^E_i$ choose to be open because the market interest rates for these types are below $r^E_i$ (the market will offer $r_{i,j_i}$ to all open customers, by the same arguments as in section 5.4.3), while everyone else choose to be closed:

$$a^1_C = \begin{cases} \text{Op} & \text{if } r_{i,j_i} < r^E_i \\ \text{Cl} & \text{if } r_{i,j_i} > r^E_i \end{cases}$$ (5.20)

The open customer will switch bank if the offered interest rate from the current bank is greater than the total cost of switching. The open customer credibly threatens to switch when

$$r_{i,j_i} + C_{sw} < r^E_i$$

Which is the same as the blue line for values below $r^E_i$ in figure 10.

Next we look at the offered interest rate to those with $r_{i,j_i} > r^E_i$. The outcome depends on the market, as discussed in section 5.3.3, if the market is what we call naive, the interest
rate offered is as follows.

\[
\begin{align*}
    r_{i,j_i}^{O|VIS}(N) = \begin{cases} 
    r_{i,j_i}, & \text{if } a_C^1 = \text{Op} \text{ and } r_{i,j_i} + C_{sw} < r_i^E \\
    r_i^E, & \text{if } a_C^1 = \text{Op} \text{ and } r_{i,j_i} + C_{sw} > r_i^E \\
    r_i^E, & \text{if } a_C^1 = \text{Cl}
    \end{cases}
\end{align*}
\] (5.21)

See the blue line in figure 11. That is, some of the good customers are charged a lower interest relative to the case with information sharing limitations, while bad customers are still charged \( r_i^E \). The good customers are now able to share their internal information which, given our assumptions, leads to them being charged what we call their true interest rate, except those who may fall in the region in which the switching cost is too great to gain from switching. The resulting interest rates for the bad customers, on the other hand, comes from our assumption of naive banks, in that they do not automatically assume that customers who retain their information are bad customers.

We proceed by discussing how assuming sophisticated banks may alter the final outcome.

The Sophisticated, Full-Knowledge Case

We now assume that the banks are sophisticated, in that they assume that customers who retain information are of higher risk than the average customer, i.e., they are bad customers. Because, if all customers that gain from being open are just that, then it must mean that all other customers must be of a higher risk class then the average customer in segment \( i \). This gives us the possible case in which the interest rate offered to those with \( r_{i,j_i} > r_i^E \) is priced using the updated expected default probability:

\[
r_i^{E(Cl_i)} = r(E\{PD_{i,j_i}|a_C^1 = Cl\})
\]

Where \( Cl_i \) denotes a closed customer in segment \( i \). Remember that we have assumed that the bank knows the distribution of \( j_i \), but cannot identify each customer’s \( j_i \). If the banks take this effect into consideration, in which case we call the bank sophisticated (So), it results in the following possible values of the offered interest rate.

\[
r_{i,j_i}^{O|VIS}(So) = \begin{cases} 
    r_{i,j_i}, & \text{if } a_C^1 = \text{Op} \text{ and } r_{i,j_i} + C_{sw} < r_i^E \\
    r_i^E, & \text{if } a_C^1 = \text{Op} \text{ and } r_{i,j_i} + C_{sw} > r_i^E \\
    r_i^{E(Cl_i)} & \text{if } a_C^1 = \text{Cl}
    \end{cases}
\] (5.22)

Illustrated by the red line in figure 11. The outcome for the good customers is the same as with naive banks, but the outcome for the bad customers is changed: They are now charged a higher interest rate, but still the same interest rate for every customer the falls under the bad customer-definition.
Figure 11: Illustration of possible outcomes in the case of voluntary information sharing, with switching costs ($C_{sw}$). $r_{i,j_i}$ is the corresponding interest rate to characteristics ($i, j_i$), $r_{i,j_i}^{O|VIS}(N)$ is the set of interest rates the customers are offered given full knowledge and naive banks, $r_{i,j_i}^{O|VIS}(So)$ is the possible set of interest rates offered to customers given full knowledge and sophisticated banks, both as deduced in section 5.4.4, and $r_i^E$ is the interest rate priced after the expected default probability in segment $i$. 
The Customers’ Behavioral

If we believe that the above is the case, all bad customers are (for the moment) charged a higher interest rate. In other words, the interest rate charged to those whom do not share their information is priced after the expected default probability among these customers, and, since we have seen that the customers that can gain from sharing do so, it is reasonable to think that the average risk in the retain-group is higher.

This may lead to a process in which everyone with a market interest rate below the interest rate given by the expected default probability choose to be open, starting over. Observe that some customers now may find themselves in this situation:

\[ r_t^E < r_{i,j_i} < r_t^{E\{Cl_i\}} \]

In this case they may now gain from being open, and thus be offered \( r_{i,j_i} \) in the market, rather than \( r_t^{E\{Cl_i\}} \). The reference interest rate, so to say, is no longer \( r_t^E \), but the new and updated \( r_t^{E\{Cl_i\}} \).

Remember that the customers first choose the action which minimize their type’s market interest rate, and second choose to stay or switch, which means that the switching costs becomes relevant after the choice of being open or closed is made.

If this is true, we may observe an iterative process which leads to every customer optimally being open. The intuition here is the same as above, everyone with a market interest rate under the current interest rate of \( r_t^E \) will choose to be open. That is, (5.20) now look like this:

\[
a_C^1 = \begin{cases} 
Op & \text{if } r_{i,j_i} < r_t^{E\{Cl_i\}} \\
Cl & \text{if } r_{i,j_i} > r_t^{E\{Cl_i\}} 
\end{cases} 
\]  

(5.23)

Since \( r_t^{E\{Cl_i\}} \) will increase or stay the same as some change from closed to open (at least in this framework), it is reasonable to believe that such a iterative process may occur. In that, as some change to being open, more customers will find it beneficial to change their type as well. Note, also, that this is a logic result of our model, and seems intuitively very plausible. In figure 12 we have plotted a such situation, in which we plot the market interest rate for the customers as more and more customers changes type from closed to open, still using uniformly distributed non-specified values. We find it sufficient to plot three iterations to illustrate the idea, but the process continues (forever, in theory) until all customers have changed their type to open. The last customer in each segment is of course indifferent between being open or closed, since the market, in this model, in effect know their type. The end result is thus that all customers choose to be open, and, consequently, is offered their
true interest rate in the market.

The Current Bank’s Behavioral

So far we have discussed how the market interest rate is set. Next, we discuss the interest rate offered by the current bank.

This rate still depends on which approach the bank is using. If the bank aims to capture the customers’ switching cost through increased interest rate, the final interest rate is $r^*_{i,j_i}$ as before, and just $r_{i,j_i}$ if not. The relationship between the two is still

$$r^*_{i,j_i} = r_{i,j_i} + C_{sw}$$

Concluding Remarks

Evidently, the outcome of the voluntary information sharing case is unclear, and depends on the actions taken by both the customers and the market. If the banks are so-called sophisticated and the customers actively share information to reveal their own type and choose bank on basis of interest rate alone, then we get the last case described above in which all customers are charged their true interest rate.

5.4.5 Summary of the Continuous Case

We started out with a short deduction of the outcome given perfect capital markets, see section 5.4.1. We did this to have benchmark for the set of true interest rates.

We followed with an analysis given today’s situation, i.e., the case with information sharing limitations, see section 5.4.2. We showed that we not necessarily end up with one interest rate, but, due to switching costs and bad customer’s switching-option, rather a set of different interest rates. The good customer is overpriced, some of the bad customers is correctly priced while the customers representing the highest default probabilities are still underpriced. On the other hand, if the bank aimed to maximize the interest rate charged to all customers, we ended up with one interest rate charge to all customers within each segment $i$. The good customers became even more overpriced, and the previously correctly priced segment of bad customers also became overpriced.

We proceeded to look at possible outcomes stemming from forced information sharing in section 5.4.3. Here we saw that this leads to an outcome in which bad customers are either correctly priced or overpriced, contingent on the bank’s pricing practice. Here as well, the switching cost did cause some disturbance.
Figure 12: The iterative process, in which more and more customers benefits from being open, that may occur in the case of voluntary information sharing, illustrated with three iterations.
Lastly, in section 5.4.4 we examined possible effects of voluntary information sharing. This was the most complicated case, and we argued that the final outcome depends on the market participant’s behavior, and, therefore, is difficult to predict. Given some assumptions, this case is similar to the case with forced information sharing, differing only in the fact that the customers may experience the system as less enforced due to its voluntary nature. Furthermore, the good customer’s bargaining power is greatly increased independent of the banks being either naive or sophisticated.

The main take-away from this analysis is that we see that private information moves us from a pooling to a separated equilibrium, which gives a more accurate setting of interest rates, provided that all banks have access to the same information.

6 Conclusion and the Benefits of Information Sharing

In section 4, we constructed a model that illustrated how asymmetric information between bank and customer leads to a situation where good customers have to either subsidize the loans of bad customers, or drop out of the credit market entirely, which finds support in Stiglitz and Weiss (1981) and Pagano and Jappelli (1993).

It appears intuitive that this situation can be alleviated by improving the information of the bank. However, as we have now shown in section 5, the benefits of improved information is only realizable provided that this improved information is also known to the other banks on the market.

Having reviewed papers discussing similar subjects, we find that some have theorized that additional benefits can be derived from improved information sharing, although they are not treated in this thesis.

- **Countering moral hazard**: When banks share their information between them, borrowers will experience a higher default cost, possibly leading an increased incentives to repay the loan. So, credit-sharing institutions can increase borrowers’ cost of defaulting, thus increasing debt repayment (Padilla and Pagano (2000)).

- **Countering information monopoly**: Conversely, sharing of credit-related information has the additional benefit of reducing the information monopoly a lender has on its borrowers. For instance, banks with long-standing relationships with their customers know the credit history of those borrowers, while other lending institutions do not have access to this information. This allows the bank to charge higher interest rates and
extract other rents from those high quality borrowers, see, e.g., Padilla and Pagano (1997) and Sharpe (1990).

- **Countering over-indebtedness**: Information sharing between lenders reveals borrowers’ debt exposure to all participating lenders, eventually reducing aggregated indebtedness as highly indebted individuals receive less credit (Bennardo et al., 2014).

### 6.1 Impact Analysis

When considering the impact of improved information sharing, we consider a few key observations.

- **Similarity of Risk Assessments**: We have already established that, given the same information, Norwegian commercial banks will generally arrive at the same risk assessment of a customer. Additionally, we have that the external and internal risk assessments are highly correlated. (Sparebanken Vest, 2015).

- **Early Warnings**: Risk assessment based on both public information and internally observed variables reveal increases and decreases in risk earlier than assessments based on public information only, allowing for more rapid updates in bank risk profile. (Sparebanken Vest, 2015).

- **Low Default Probabilities**: An important property of the Norwegian private credit market is the very low default probabilities of the loantakers. As we have already discussed, this can probably be attributed to the way Norwegian loantakers have to put up the entirety of their earnings and assets as security, meaning that it is almost exclusively preferable to service your debt rather than defaulting. Put succinctly, loans to Norwegian private customers tend to have low default probabilities and high recovery if a default occurs (Sparebanken Vest, 2015).

The fact that internal and external risk-assessments have highly correlated results implies that the value of increased information sharing would not primarily be in improved risk assessment, although there certainly are cases where the internal rating might deviate from the external one. We also note that as long as the average default probability on the Norwegian credit market remains very low, the significance of better risk assessments is questionable.

The most important effect of increased information sharing would rather be shorter reaction time to increased credit risk on the Norwegian private market in general. Typically, failure to pay other obligations is not revealed to the bank before it has entered court, which
typically takes about nine months. Internal assessment, however, can be shown to reveal such trouble much more rapidly (Nordea states that this takes approximately three months).

Very broadly then, we propose that under current conditions, the average interest rate paid in the Norwegian private credit market would not be very much changed by increased information sharing. However, the effect on bank solidity might be positively affected, as the banks will be able to register and react to adverse changes in credit risk at a much earlier stage than they are currently able to.

In conclusion, we also repeat our statement from section 3.2, which is that our findings imply that the potential gains related to the new capital regulations, introduced through Basel III generally and CRD IV in Norway, are unlikely to be fully realized under today's legislation. While CRD IV appears to successfully incentivize banks to improve their risk assessment and management processes by offering them softer leverage restrictions in return, the effects this might have on credit allocation and interest rates will still, to some degree, be restricted by the least sophisticated banks on the credit market. To alleviate this, we refer to our implementation suggestions in section 7, which each allows for the realization of some or all of these gains.

7 Implementation Suggestions for Information Sharing

So far we have described how lacking information about customer risk can potentially have an adverse effect on credit market size, credit allocation and overall market risk. A system where banks are allowed, or even required, to share their internal risk-assessments of private customers has been shown to alleviate this problem somewhat, as it puts each bank closer to the optimal case of full information.

We have also shown that lacking information-sharing allows bad customers to avoid paying for the full sum of their risk, meaning that they are in effect subsidized by the good customers participating in the market. This is because they always have the option to switch to a bank with less information, effectively resetting their risk-assessment. A system where the banks are allowed to share internal information, and thus have the same risk-assessment for each customer, would remove this option and enable the banks to set interest rates in closer accordance with customer default risk. As we have described in section 2.4, Pagaon and Jappelli (1993) describes a set of additional gains a bank may derive from sharing information more freely, providing additional incentives.

It is therefore our recommendation that some measure be taken to ease the sharing of
information between banks.

7.1 Change in Legislation

The simplest, but perhaps also most problematic, way to implement information sharing is to change the legislation that is currently prohibiting it. This involves changing Norwegian privacy laws so that banks are allowed to share recorded information, such as bank account movements and pending defaults. The legal feasibility of this is outside the scope of our thesis, but provides an interesting subject for future research.

It is also a point to somehow standardize which variables are measured, and how they are shared. This is both to ensure consistency in the risk-assessment the same customer receives in each bank, as well as to make sure that the bank have less opportunity to cherry-pick the information that they choose to share, either to gain a competitive advantage or to make switching banks less attractive to their customers.

A problem with this implementation is that some of the banks might not be capable of delivering information of the same quality of information as others. This can conceivably give worse terms to customers switching from banks with relatively simple risk-assessments to more technically advanced banks, as the risk-assessment of these customers gives less information than that of other customers. We think it is reasonable to assume that this could be a disadvantage to these “simple” banks, and thus might harm the competition between banks. It is also more resource-demanding for each bank if the system is not standardized, so that individual considerations has to be made for each incoming customer.

We conclude that a change in legislation, if feasible, will have to be followed by legislation requiring the bank to share a set of standardized variables that all banks are equally able to record.

The greatest weakness of this solution is that important information might be kept internal because some banks lack the ability to produce or make use of it, and that these banks cannot be expected to have the capital to improve their risk-assessment procedures on the short term.

We maintain that it is still likely to alleviate the problem of information asymmetry and subsidization of customers with higher-than-expected risk, even though the gains of very sophisticated risk-assessments are unlikely to be realized.
7.2 Change in Legislation, Third-Party Rating Bureau

We have already established that a change in legislation alleviates the problem of asymmetric information, and at it at the very least reduces the bargaining power of bad customers. We described that a problem occurs because banks have varying degrees of technical sophistication in their risk-assessment methods, meaning that the information that can be shared in any organized fashion must be the lowest common denominator to avoid putting the more simple banks at a competitive disadvantage.

We suggest that in addition to the changes in legislation already suggested, the shared risk-assessment process can be outsourced to a third-party organization. This leaves the banks responsible for the collection of data on their customer, but lets the risk-assessment be done by an independent organization and made available to the banks. This is not necessarily a cost to the banks, as the average cost of risk-assessment might reasonably be expected to go down when the process is centralized in this manner. It also allows for a common system of higher quality than what we have seen previously, as the third-party assessment might hold higher quality than what banks with relatively simple risk-assessment procedures are able to do themselves.

A third-party assessment also dodges the problem of moral hazard typically seen when banks are free to do their own assessments, as the third-party organization itself has no incentive to under- or over-report the risk that they observe.

An important discussion is how this third-party bureau fits into the current regulations with regard to risk-assessment. We argue that, because there is no guarantee that the bureau holds the quality previously held by the assessments made by the more advanced banks in the first place, the assessments made by the bureau should only take the place of the standardized system that is currently provided to the “simple” banks by the authorities (Nordea, 2015b), while still allowing for more advanced banks to supplement with their own assessments, provided that their methods are approved in the same manner as is currently required. However, these internal assessments will still be augmented by data provided by the third-party bureau, thus achieving equal or better results than in the first suggested implementation.

We argue that this implementation has the same kind of effect as our first suggested implementation, but with equal or greater impact. This comes at a cost, a third-party bureau would presumably be somewhat costly to run. In addition, we might have the additional consequence of more even competition through less economies of scale in risk-management, and potentially landing slightly closer to the full-information case.
7.3 No Change in Legislation, Volunteer-Based Third-Party Solution

The perhaps least controversial implementation we suggest is the introduction of an optional third-party solution that requires small or no changes in current legislation. We have two variations of this implementation:

7.3.1 Third-Party Data Repository

This implementation involves offering all private customers to let banks share their internal information on them with a third-party data repository available to all the commercial banks. The trade here would be that they give up some measure of privacy in exchange for a fairer risk-assessment, and thus fairer loan terms.

We argue that a customer that shares data with the third-party data repository must be a good customer, as there would otherwise be no reason to share. If this practice becomes common, then not sharing data will become a strong signal of being a bad customer. This, in addition to the improved access to customer data, allows the banks to more accurately gauge the quality of each customer.

To ensure that this becomes common practice, we propose that some additional and more obvious incentive, such as a small improvement to loan terms, is offered to customers that agree to share their information freely.

In section 5.3.3. we discuss a case like this, and find that a bank will only want to adjust interest rates in accordance with the information from the repository if it believes that all other banks would use this information to update their interest rate offered in the same manner.

It follows that this implementation, while presumably cheap and simple to implement, is not guaranteed to realize much of the potential gains of information sharing.

7.3.2 Third-Party Rating Bureau

We return to the third-party rating bureau from our second implementation, although with the important difference that letting the bureau handle customer data is a decision left up to each individual customer.

This alleviates the problems of the third-party data repository, as the standardized risk-assessment done by the bureau ensures that the banks know that all other banks will use an updated risk-assessment for each participating customer that is of at least the same quality.
as the one provided by the third-party bureau.

While this will presumably be a more expensive solution than the third-party data repository, we claim that this also means we will be much less likely to get the situation from section 5.3.3., where improved sharing of information does not necessarily affect the setting of interest rates for bad customers.

Both volunteer-based implementations rely on becoming commonly used, so that a customer that does not share information is giving a strong signal of being a bad customer. This depends on the customers actually being able to determine their own risk more accurately than the bank is able to when only possessing public information.

The volunteer-based implementations have the advantage of demanding less or no changes in legislation, and are as such more feasible implementations than the first two.

8 Further Research

Subjects that fell outside the scope of this thesis, yet would have been of great interest to us, include:

8.1 Legal Feasibility of Implementation Suggestions

Information sharing has implications on Norwegian privacy legislation. An evaluation of what legal changes has to be made, and how this might happen, is important when considering the feasibility of each suggestion.

8.2 Political Feasibility of Implementation Suggestions

Privacy legislation can be said to be a sensitive subject in Norway, and previous attempts to make private information more easily available to law enforcement agencies have been met with public outcry. This might have implications of the feasibility of making the necessary legal changes to allow for freer information sharing between banks. Such a discussion can also give concrete ideas for how such changes should be presented, and which modifications that can be made should the public find them unpalatable.
8.3 Empirical Impact Analysis

We have not been able to gain access to good data, despite cooperation from two major Norwegian banks. This is due to the fact that such data is private information, both to the customers and banks, and thus cannot easily be shared outside the bank (as is the subject of our thesis to begin with). A study with access to data, such as a study done internally in one or more banks, might shed some light on the impact freer information sharing might actually have on the Norwegian credit market.
Appendices

A  Gains from Renegotiating the interest rate

The relationship between monthly mortgage payments (MMP), mortgage rate \( (r_d) \), maturity \( (M = 12 \times T) \) and face value \( (B(0)) \), in a level payment fixed-rate contract, is expressed in the following equation.

\[
\text{MMP} \sum_{n=1}^{12T} \frac{1}{(1 + \frac{r_d}{12})^n} = B(0)
\]

The interest component on the payment on date \( n + 1 \) is simply \( B(n) \frac{r_d}{12} \). We introduce two interest rates and interest components, \( r_d^i \) and \( B_i(n) \), \( i = 1, 2 \), where \( r_d^1 > r_d^2 \). Thus, the present value of retained interest (RI) payments over the remaining lifetime of the loan is as follows.

\[
PV(\text{RI}) = \sum_{n=1}^{12T} \frac{B_1(n) \frac{r_d^1}{12} - B_2(n) \frac{r_d^2}{12}}{\left(1 + \frac{r_d^2}{12}\right)^n}
\]

With values \( r_d^1 = 4.25\% \), \( r_d^2 = 4.00\% \), \( T = 20 \), \( B(0) = 2 000 000 \text{ NOK} \); \( PV(\text{RI}) = 47 831.01 \text{ NOK} \)

B  Simple Market Participation Model, discrete example

We have a market with a bank and two customers, customer 1 and customer 2. Customer 1 and 2 each have a bond they wish to sell, which are equal in all things except their default probabilities. Each customer also derives an utility \( U \) from taking the loan. This value represents non-financial gains, such as the opportunity to smoothe consumption over time, and the access to capital for purposes such as buying property, which would otherwise not be possible.

Default probability of customer 1, \( Q_1 = 0.1 \)

Default probability of customer 2, \( Q_2 = 0.2 \)
Depending on the information of the bank and the utility gained by the customers, we have a set of three possible stable states for this credit market. We go through an example of each stable state.

**B.1 Bank has full information, \( U > 0 \)**

Because the bank knows the default probability of each customers, they get different interest rates. The rates are found by the same method as the one used in section 6.3, and is such that the value to the bank is zero.

\[
\begin{align*}
    r_1 &= \frac{1}{1 - Q_1} - 1 = \frac{1}{0.9} - 1 = 0.111... \\
    r_2 &= \frac{1}{1 - Q_2} - 1 = \frac{1}{0.8} - 1 = 0.25
\end{align*}
\]

We then find the values to the customers, \( V_{C,1} \) and \( V_{C,2} \) to be:

\[
\begin{align*}
    V_{C,1} &= 1 + U - (1 + 0.111...)(1 - 0.1) = U - 0.05885 \\
    V_{C,2} &= 1 + U - (1 + 0.25)(1 - 0.2) = U + 0.05885
\end{align*}
\]

We note that because the bank can separate between high- and low-risk customers, all customers gain a positive value if they gain any utility from sources such as consumption smoothing or better access to funds, but gain no financial value from the transaction. Total consumer surplus is \( 2U \).

**B.2 Asymmetric information**

We have the same market, but the bank is now unable to tell which customer has which default probability. It will therefore set the same interest rate \( r \) for both customers, applying the method used in section 6.4:

\[
\begin{align*}
    \hat{Q} &= \frac{0.1 + 0.2}{2} = 0.15 \\
    r &= \frac{1}{1 - 0.15} - 1 \sim 0.1765
\end{align*}
\]

In this case, the customers do not get the same value from taking the loan:

\[
\begin{align*}
    V_{C,1} &= 1 + U - (1 + 0.1765)(1 - 0.1) = U - 0.05885 \\
    V_{C,2} &= 1 + U - (1 + 0.1765)(1 - 0.2) = U + 0.05885
\end{align*}
\]
We note that in this case, both customers will still participate provided utility $U$ is large enough to make sure they both have non-negative value from taking the loan. However, we see very clearly that value has been transferred from the low-risk customer to the high-risk customer. Total customer surplus, however, remains the same as in the case of full information.

It is also useful to note that if the utility from taking a loan is such that $V_{C,1}$ is negative and $V_{C,2}$ is positive, the interest rate will increase to 0.25, as the average default probability of the market, $\hat{Q}$, has increased. This effect is also observed in the more general case we describe in section 6, where the market size steadily moves towards 0 when utility is lower than 1. This does of course depend on the distribution of default probabilities - In a discrete case, we arrive at a smaller, but still non-zero market, such as in this example.

In conclusion, we should also briefly discuss that given high utilities, it looks like if the bank could have taken a higher interest-rate. We assume that the bank is subject to price-competition, thus the constraint $E(V_{B,i}) = 0$ that we introduce in section 6, and it follows from this that a higher interest rate would result in a loss of customers. If this was a monopoly or the bank in question had some sort of advantage over other banks though, it would be conceivable that the bank takes a positive value.

C The Norwegian Credit Market

C.1 The Norwegian Credit Market

Given the Norwegian population of about 5 million people, the Norwegian credit market is large relative to those of comparable countries. There are roughly 2.1 millions households, most of which are owned by the inhabitants (see Figure 13). The high share of homeowners, in addition to the cash-out-refinancing\textsuperscript{29}, is a main factor explaining the high household debt to disposable income ratio, which has been at over 200\% since 2011 (Almaas, Bystrøm, Carlsen and Su, 2015). In contrast, the same ratio in the U.S. has been mostly under 140\%, even at its peak in 2007, the year before the subprime crisis.

One key attribute of the Norwegian market that might help explain why the banks seemingly approves the current situation is the full-recourse loan policy (Krogh, 2010), which means that the mortgage follows, not the property itself, but the borrower. This implies that

\textsuperscript{29}Such re-financing may explain why increases in house prices are correlated with increased consumption (kilde: Langsiktig makro)
if a borrower defaults, the creditor has a claim on all of that person’s assets, not just the house itself.

Intuitively, this gives the borrower strong incentives to honor his or her obligations to the borrower. Norway also has high social welfare and income-equality (Lucchino and Morelli, 2012). On average, this contributes to a lower credit-risk relative to abroad, as well as very high recovery rates in the event of default. In addition, governmental guidelines in Norway state that a customer should not be able to borrow more than 85% of the market value of the house, although exceptions are made. Such a limit makes it less likely that banks experience significant losses on defaulted mortgages, as the property that makes up much of the security still can cover the principal in the event of a default, except in the case of very large price-changes. During the recent years there has been historically low default probability on the Norwegian market. The factors discussed above might provide a partial explanation to this observation.

C.2 Nordea’s Risk Assessment Approach

The following is a very short summary of Nordea’s report regarding risk and capital management from 2015 (Nordea, 2015b). Most of Nordea’s risks, about 85% of the total risk exposure amount (REA) originates within Wholesale and Retail Banking. The dominant risk category is credit risk, representing around 83% of REA; this is capitalized by a net interest income 10 times higher than net loan losses. Nordea limits the risk appetite by setting boundaries for concentration risk, probability of default, loan losses, and expected loss. Retail mortgages currently represents 27% of Nordea’s total exposure. The size of the Retail market is 175 146 EURm, with an average risk weight of 13%, and REA of 21 940

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30Risk appetite within Nordea is defined as the level and nature of risk that the bank is willing to take in pursuit of the articulated strategy on behalf of shareholders. Risk appetite is defined by constraints reflecting the views of shareholders, debt holders, regulators and other stakeholders.
EURm. The Norwegian Retail Credit market was 27 092 EURm or roughly 235 000 NOKm (EUR/NOK = 8.7 (4.1.2015)) as of 31 December 2014. It follows that Nordea’s retail credit exposure in general represents a significant share of Nordea’s total risk exposure. All of which offer support to the viability and necessity of our thesis. At the end of 2014 Nordea’s capital ratios was as follows. CET1 (Common Equity Tier 1) 15.7%, tier 1 17.6% and total capital 20.7 %31.

As a side comment, Nordea reports that the housing market are currently stable, and loan losses are decreasing in all of Nordea’s markets32. But they do recognize that the Norwegian market is sensitive to changes in market conditions due to elevated debt to income ratios amongst borrowers (see section 3), and they believe that the market may be negatively affected by the extensive regulatory agenda with regards to to mortgage lending in, among other countries, Norway.

The goal of rating and scoring of customers, and thus IRB models, is to predict default and rank customers according to their default (credit) risk. Ratings are used for corporate and institution exposure, while scoring is used for retail exposure. Both of which is typically used in the credit risk management and decision making process. Such processes can include the credit approval process; calculation of REA, economic capital and expected loss (EL); monitoring and reporting of risk; and performance measurement using the economic profit (EP) framework.

In the household market Nordea uses AIRB and, so-called risk grade is used. Nordea uses internal models in the risk grade estimation process, they seek a high standard of risk management, that is, applying available techniques and methodologies. These models is based on statistical techniques to predict the probability of customer default. Noreda, too, regards payments past due more than 90 days as in default. The models are based in internal data and take into account customer characteristics and behavioral information. This is the type of information we refer to as private information. The estimated risk grade is used in both the risk management process (for instance it can provide a early warning for high risk customers), and

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31 Minimum capital requirement as of 31 December 2014: 4.5% CET1, 6.0% tier 1, and 8% own funds
32 These markets include the Nordic countries, ++
the credit approval process (including both automatically approval and decision support). In addition to these models, credit bureau information is used in the credit process. Which represents such information we regard as public. The risk grade assigned to each customer consist of 18 grades, A+ - F- for non-defaulted customers and 0+ - 0- for defaulted customers. See Figure 14 for the risk grade distribution of Nordea’s overall IRB retail portfolio. The average default probability was 0.85%.

The current regulation demands a annual validation of the internal models (Vest, 2015). In accordance with CRR requirements, Nordea has developed a validation process with the aim to ensure and improve the performance of the risk assessment models, procedures and systems and to ensure the accuracy of the PD estimates (Nordea, 2015b). They validate the scoring models once a year, both quantitative and qualitative, by evaluating the models’ discriminatory power, that is, the models’ ability to predict default levels and distinguish default risk on a relative basis.

Furthermore, the risk class is frequently re-evaluated, this results in that risk grade distribution changes. Such changes are mainly due to three factors, changes in the risk grade of existing customers; customer turnover (i.e. new customer have a different risk grade relative to existing/leaving customers); increased or decreased exposure per risk grade to existing customers. The risk grade may also migrate, that is, the overall risk grade may change. Such migration is affected by, among other things, macroeconomic development and thus changes in the overall financial situation to the customers in general (customers repayment capacity). In Figure 15 and 16 we shown the number of notches up or down exposures are re-rated in Nordea’s retail portfolio. Evidently, at least during 2014, the risk grade was rather volatile, which means that re-scoring frequently occurs, providing further strength to our assertions.

Figure 15: Retail re-scored exposure at default
Figure 16: Retail re-scored number of customers (%), as displayed by Nordea in (Nordea, 2015b).
We have now presented the key attributes of Nordea’s IRB model necessary in our work. That is, how Nordea assess the risk of its customers, what types of information is used in their models, and whether or not they frequently re-classifies customers. For further discussion and details regarding Nordea’s risk management please see (Nordea, 2015b).

D Efficient Market Hypothesis

A market is efficient if the prices in that market fully reflects all available information. It is normal to distinguish between 3 levels of market efficiency: (i) weak-form, (ii) semistrong-form, and (iii) strong-form. The market is said to be efficient of weak-form if prices reflect all information that can be derived from technical analysis (analyzing historic market data, such as trading volume and prices). As an example, if the market is efficient of weak-form, then it should not be possible to gain abnormal returns by analyzing and trading on such things as price trends, because all investors have already learned to exploit the signals. If the market is efficient of semistrong-form, then the prices reflect all available public information, such as prospects of a firm. Note that this form also include the weak-form. Thus, if all investors have access to the same information, then this should be reflected in the market price (i.e., it is not possible to trade on public available information). Lastly, if the market is said to be efficient of strong-form, then all available information (public and inside/internal) relevant for the asset/commodity is reflected in the price. And, here as well, strong-form includes both weak and semistrong-form. Bodie, Kane and Marcus (2011).
References


