

# Merger simulations with observed diversion ratios<sup>\*</sup>

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## Abstract

A common approach to merger simulations used in antitrust cases is to calibrate demand from market shares and a few additional parameters. When the products involved in the merger case are differentiated along several dimensions, the resulting diversion ratios may be very different from those based upon market shares. This again may affect the predicted post-merger price effects. This article shows how merger simulation can be improved by using observed diversion ratios. To illustrate the effects of this approach we use diversion ratios from a local grocery market in Norway. In this case diversions from the acquired to the acquiring stores were considerably smaller than suggested by market shares, and the predicted average price increase from the acquisition was 40 % lower using this model rather than a model based upon market shares. This analysis also suggests that even a subset of observed diversion ratios may significantly change the prediction from a merger simulation based upon market shares.

**JEL codes:** K21, L11, L41

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# 1. Introduction

Merger simulation attempts to estimate the price effects of horizontal mergers in oligopolistic markets. Recently there has been a considerable growth in the use of merger simulation models in cases investigated by antitrust authorities.<sup>1</sup> The common approach when doing merger simulations, is to calibrate demand from market shares. Market shares can be poor predictors of substitution between products. The diversion ratio – how large fraction of customers leaving product *A* that would buy product *B* – are useful in defining the closest substitutes and therefore in measuring how closely products or firms compete. Thus, since diversion ratios comprise more information about substitutability than market shares, we propose to calibrate demand from observed diversion ratios.

Although this approach has not been used in merger simulations, diversion ratios have been accepted as an important input to define relevant markets.<sup>2</sup> They have been widely used in merger cases, for example by competition authorities in the UK on mergers between retail stores.<sup>3</sup> The authorities were skeptical to the traditional method of counting the number of stores within isochrones, as differentiation between various types of stores is not captured. Instead of relying on market shares – or simply the number of rivals – a survey to reveal diversion ratios was conducted. A natural next step would be to use this information also to estimate the price increase following the merger. This is in line with the approach proposed in the new horizontal merger guidelines in the US.<sup>4</sup> Of course, in a

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<sup>1</sup> Walker (2005) and Budzinski and Rohmer (2009) describe the use of merger simulation models.

<sup>2</sup> O'Brien and Wickelgren (2003) reformulated the critical loss analysis for defining the relevant market to employ diversion ratios rather than price elasticities. See also Katz and Shapiro (2003).

<sup>3</sup> The first merger where diversion ratios were used by the Competition Commission was Somerfield's acquisition of Morrison's 115 grocery stores in 2005. See Walters (2007) and Reynolds and Walters (2008) for a description of more recent merger cases in the UK where this method was used.

<sup>4</sup> The guideline states: 'The Agency rely much more on the value of diverted sales than on the level of the HHI for diagnosing unilateral price effects in markets with differentiated goods. Where sufficient data are available, the Agencies may construct economic models designed to quantify the unilateral price effects resulting from the merger' (US 2010, page. 21.) Note that HHI is an index that is calculated from market shares.

merger case a simulation should be supplemented by a broader analysis of among other things the prospects for entry and repositioning of products.

The diversion ratio is an intuitive measure of how the pricing of one product is constrained by another product and may be helpful in situations where own- and cross-price elasticities are unavailable. Note, however, that the diversion ratio informs us about the customers' second choice, not about the reduction in demand following a price increase. Diversion ratios as such are thus suitable for revealing which products are close substitutes, but less suitable for predicting the price increase following a merger. Shapiro (1996) combined the diversion ratio and the price elasticity of market demand into a formula for predicting the price increase following a merger. His formula is derived from a symmetric duopoly model with single-product firms, and it has been used by competition authorities in the UK to estimate what they have characterized as an 'illustrative price increase'.

Most markets, however, are characterized by asymmetries both in demand and costs, and single- as well as multi-product firms. In such cases simple formulas are unavailable and the evaluation of an exact, but complex formula may come close to performing a full merger simulation. Constructing a merger simulation model involves three steps: Establishing a demand system, specifying the mode of producer behavior and calibrating marginal costs. We are concerned with establishing a demand system.

There are two approaches to derive demand. The first approach is an econometric estimation where one exploits a large data set. This approach has been applied in, among others, Bordley (1993), Hausman *et al.* (1994), Nevo (2000), Pinkse and Slade (2004), Ivaldi and Verboven (2005) and Ivaldi and Lörincz (2009). Although this method is the preferred one, in some cases there are time and resource constraints that make this approach inapplicable. This makes it natural to apply a second approach, which is to calibrate demand from a minimal data set – often only market shares, reference prices and a few elasticity

parameters.<sup>5</sup> In such cases the popular approach has been to assume that diversion ratios are proportional to observed market shares. While this route provides a quick first prediction of a price increase, it may introduce serious biases. As pointed out by Willig (1991), inferences of the nature of competition from market shares are problematic if the products are differentiated by characteristics salient to consumers and when relevant products vary in their similarities to one another in regards to these characteristics. In markets with differentiated products he suggested collecting information beyond market shares.

Our approach to constructing a simulation model follows this idea. We calibrate demand from observed diversion ratios and assessed market price elasticities. This is a methodology that makes it possible to exploit information from a simple survey, an approach that is attractive when an econometric estimation is not feasible. To the best of our knowledge this routine is new. It involves formulations also employed by econometricians – after all the common focus is on establishing a demand system – but in spirit our routine differs. We calibrate partial price elasticities from observations of diversion ratios, whereby all elasticities get the right signs; econometricians let their larger data set speak, but often constrain parameters in order to obtain correct signs.

To illustrate the applicability of this approach, we analyze an acquisition in a local grocery market in Norway, where we have pairs of diversion ratios between eight stores.<sup>6</sup> These diversion ratios differ significantly from those that would follow from the assumption of diversion ratios being proportional to market shares. We argue that a combination of

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<sup>5</sup>Werden and Froeb (1994) and Epstein and Rubinfeld (2002) employ calibration. See Werden and Froeb (2008) for a survey of the literature on merger simulations and Budzinsky and Ruhmer (2009) for a discussion of the role of such models in competition policy cases.

<sup>6</sup> In August 2007 Norway's largest retail chain Norgesgruppen (NG) acquired the smaller chain Drageset. The acquisition raised anticompetitive concerns in several local markets and was notified to the Norwegian Competition Authority. Based on market shares the local market at Voss, a small village in the western part of Norway, was the most problematic. NG controlled four of the eight largest stores, while the acquired company had one. With the acquisition NG would increase its market share from less than 50 % to more than 60 %. The acquisition was cleared in an early phase. We do not take a stand on whether the clearance was correct or not; we just use this case to illustrate our approach. Note, however, that our merger simulation indicates that the anticompetitive effect of the merger is more limited than what follows from a traditional approach based on market shares.

location and the differences in product variety of the various stores can explain the observed diversion ratios better than what market shares are able to do. To check whether these observed differences matter for the price prediction from a merger simulation, we apply the model (denoted OBS) to the mentioned acquisition and compare its predictions with a model (denoted MS) where demand is based upon diversion ratios that are proportional to market shares. The predicted average price increase of the OBS model is 40 % lower than the prediction of the MS model. We think this difference is substantial and that it illustrates the need to go beyond market shares when assessing the anticompetitive effect of a merger in a differentiated products market. In this case, the lower prediction follows because the sum of observed diversion ratios from the acquired to the acquiring stores is much lower than predicted from market shares; 39% versus 54% respectively.

Due to time and resource constraints or for other reasons, one may have to rely on less detailed information than we got. We have investigated how price predictions would change if we only used a subset of this full information. It turns out that observed diversion ratios from two out of eight stores explain 2/3 of the gap in predictions, which suggests that even a subset of the (actual) diversion ratios may contribute significantly to improve the prediction from a merger simulation.

The article is organized as follows. Section 2 describes the local grocery market at Voss, Norway. In Section 3 a simulation model is calibrated from observed diversion ratios and market demand elasticities. In Section 4 this model is used to simulate the price effect following an acquisition, and these predictions are compared to the predictions from a merger simulation model based only on market shares. Section 5 concludes.

## 2. A local grocery market: An example of large asymmetries

A grocery market is differentiated along several dimensions. Stores are located throughout the community leading to geographic differentiation. Some stores have a limited space and a low number of products while others have large space and a wider range of brands. This leads to differentiation in their offer, which we will call their product. Differentiation in space, product, and possibly other dimensions as well, may cause large asymmetries in diversion ratios that cannot be inferred from market shares.

To illustrate such asymmetries, we consider the local grocery market in the village Voss in the western part of Norway. Twelve stores were located reasonably close to each other, while there were long distances to stores elsewhere. In February 2008, after Norgesgruppen's (NG) acquisition of Drageset was cleared by the Norwegian Competition Authority, Halleraker and Wiig (2008) made a survey among 800 shoppers.<sup>7, 8</sup> They focused on the eight largest stores with a joint market share of more than 90 % of annual turnover.<sup>9</sup> The market share of the largest store was approximately three times that of the smallest (of the eight). For our analysis, the most important question in the survey was which other store the shopper would have chosen if this store was unavailable, thus revealing his second choice. This information was aggregated to find the (revenue weighted) diversion ratios  $d_{ji}$ , *i.e.*, how large fraction of diverted revenue from store  $i$  that would go to store  $j$ ,  $j \neq i$ , if store  $i$  was not available. As a background for interpreting these numbers we present some data about location of stores in Voss and the answers from the sample of respondents.

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<sup>7</sup> Judged from the age profile the sample of respondents is representative for the population at Voss.

<sup>8</sup> Note that at the time of the enquiry NG had not yet changed the profile of the acquired store. Thus, the reported diversion ratios should provide good indications of the diversion ratios prior to the acquisition.

<sup>9</sup> At Voss Norgesgruppen controlled four stores (Meny, Spar, Kiwi Vangen and Kiwi Palmafossen), while Coop had two stores (Coop Mega and Coop Prix), and the ICA chain had the store Rimi. See also Table 1.

[Figure 1: Map with location of grocery stores at Voss]

The eight stores fall into three geographical segments (Figure 1). Five stores are clustered in the centre of Voss. Drageset - the acquired store - and Coop Prix are located about four km to the north of the centre, while Kiwi Palmafossen is located about three km to the east.

[Table 1: Descriptive statistics from the survey]

Table 1 shows that the average bill per respondent is largest for Drageset, Meny, and Coop Mega, with Kiwi Vangen and Rimi at the lower end (Column 1). These latter two stores have the largest shares of customers saying price is important for their choice (Column 2), while Drageset, Meny, and Coop Mega have customers for whom price is reported to be of less importance; their customers say that product range is more important for their choice (Column 3). A picture emerges of stores falling into two groups: “Low-price” (Kiwi Vangen, Kiwi Palmafossen, Rimi, and Coop Prix) and “Product range” (Coop Mega, Meny, and Drageset), with Spar in between and probably closer to the second group.

Drageset and Meny have the lowest shares of Voss residents, 77.8 and 76.5 % respectively (Column 4). Few local customers mean more out-of-town customers. This interpretation is partially corroborated by respondents stating that their second choice is not among the other seven surveyed stores (Column 5). Meny and Coop Mega have somewhat larger diversions than the average to the outside of the surveyed market at Voss. These differences in what we call out-of-market diversion ratios may be caused by Voss being a (weekend) vacation resort for people from the nearby city of Bergen. Their second choice (to Meny or Coop Mega) may not be a low price store at Voss, but a store in Bergen with a larger assortment of brands.

Diversion ratios vary of course with differences in market shares. A firm with a larger market share will (on average) receive a larger share of firm  $i$ 's diverted revenue. Let  $D_{ji} \equiv s_j/(1-s_i)$  denote a diversion ratio from  $i$  to  $j$  that is proportional to firm  $j$ 's share  $s_j$ . Our point is that observed diversion ratios  $d_{ji}$  are very different from  $D_{ji}$ .  $I_{ji} \equiv d_{ji}/D_{ji}$ ,  $i \neq j = 1, \dots, n$  indicate the extent to which market shares explain observed diversion ratios. If market shares are good predictors, the  $I_{ji}$ 's should come close to one. Figure 2 displays these ratios in increasing order and shows large deviations from unity; the ratio varies from 0.02 to 3.54, with 21 observations smaller than 0.5, 22 between 0.5 and 1.5, and 13 above 1.5.

[Figure 2: Observed diversion ratio divided by inferred diversion ratio]

The observed diversion ratios differ quite substantially from what is obtained from market shares.<sup>10</sup> Beyond any doubt there are considerable differences between customers' first choice as represented by market shares and their (contingent) second choice as indicated by observed diversion ratios. Since the responses described in Table 1 seem to corroborate the observed diversion ratios, we reject the explanation that respondents' answers are random or systematically biased.<sup>11</sup>

In order to identify explanations for these differences see Table 2. Consider Drageset and its geographically closest rival – Coop Prix. The  $I_{ji}$ 's between these stores show that both receive much larger shares of diverted customers from each other than

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<sup>10</sup> A chi-square test on the differences between observed ratios and ratios inferred from market shares makes us reject the null hypothesis and conclude that observed and inferred diversion ratios are not equal.

<sup>11</sup> Using McFadden's choice model, see McFadden (1974), it can be shown that respondent characteristics explain second choice at least as well as their first choice. This is no surprise, since we expect that consumer characteristics explain the ordering of first and second choices.



implied by their market shares. This suggests that location can be important for determining diversion ratios.<sup>12</sup>

[Table 2: Observed diversion ratio divided by inferred diversion ratio]

Consider now stores located at the centre of Voss, in particular Coop Mega, and look at the corresponding row. Three out of four of the reported  $I_{ji}$ 's in Table 2 are above 2, showing that diversions *into* Coop Mega (from stores in the centre, see Figure 1) are much higher than implied by its market share. Except for Spar, these high numbers do not correspond to equally high numbers *from* Coop Mega to the other stores. This pattern is consistent with the notion of a store's product. As already noted, Coop Mega, Meny, and Drageset are judged by their customers to have a broad product range. The remaining stores focus on low price, with Spar in between. Thus, there are two reasons why for example diversion ratios between Coop Mega and Spar are high ( $I_{ji}$ 's about 2.5). Customers consider their product ranges as reasonably equal, and the two stores are next neighbors. This illustrates that store characteristics can partly mitigate the differentiation caused by location, since similarity in two store's product range can make those stores close substitutes even if they are not located close to each other.

A third dimension to differentiation is probably also present in the data. Coop Mega has a much larger diversion to the low-price store Coop Prix *outside* the centre (indicator 1.62) than to low price stores Rimi and Kiwi Vangen *at* the centre (indicators of about 0.5).

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<sup>12</sup> We have run a linear regression which confirms that distance matters for the diversion ratio even controlling for market shares. The observed revenue weighted diversion ratio is the dependent variable. We recognize that the customers are different and therefore include dummies for the stores from which the customers potentially would leave, market share of the store that the customers would potentially divert to, and distance between the first and second choice stores. The coefficient of distance is approximately 0.03 (and statistically significant). It indicates that an increased distance of 1 km would induce a 3 %-age point reduction in the diversion ratio.

Coop's membership policy whereby customers get a discount on yearly spending in all Coop stores, *i.e.*, a lock-in effect, is a plausible explanation.

The  $I$ -indicators show that the main rivals to the acquired store are the Coop-stores, and that observed diversion to the other four NG-stores is lower than predicted by market shares. Hence we would expect a merger simulation based upon observed diversion ratios to predict a lower price increase than a simulation based upon market shares.

### **3. Model calibration**

We construct a simulation model where demand is calibrated from observed diversion ratios in the Voss grocery market. (For details, see Appendix B).

Step 1: Observed diversion ratios are not proportional to market shares. This is taken as given, and the novelty of our approach is that we let own and cross price elasticities be determined by the observed diversion ratios. One can think of the  $n(n-1)$  diversion ratios as determining the corresponding cross-price elasticities. By stipulating  $n$  market price elasticities  $\varepsilon_i$ , we are able to calibrate the  $n$  own-price elasticities ( $\varepsilon_{ii}$ ).

It is useful to compare this approach to the established one in Werden and Froeb (1994) and Epstein and Rubinfeld (2002), where diversion ratios are assumed to be proportional to market shares. In addition to observed prices and volumes (market shares) these functions are calibrated with the stipulation of the market price elasticity and one measure of the partial price elasticities.

The value of the market price elasticity may be obtained from econometric estimation or other sources. Epstein and Rubinfeld (2002) suggest stipulating the market price elasticity at -1 when nothing else is known. This is what we have done, *i.e.*, stipulated  $\varepsilon_i = -1$ .

Step 2: It is well known that the price increase from a merger simulation depends on the chosen demand function, and that linear demand will predict a lower price increase than other functions that are typically used.<sup>13</sup> In order to have conservative estimates of the price increase and difference in predictions from the two models we use linear demand

$$(1) \quad x_i = a_i + \sum_j b_{ij} p_j, \quad i = 1, \dots, n,$$

where  $x_i$  is demand for product  $i$ ,  $a_i$  and  $b_{ij}$  are parameters<sup>14</sup> to be calibrated from the partial price elasticities, reference prices  $P_i = 1$ ,<sup>15</sup> and volumes  $X_i$ .<sup>16</sup>

Step 3: The marginal costs are inferred from the first-order conditions for optimal pricing in a Bertrand price-setting oligopoly. We assume constant variable cost ( $c_i$ ) and fixed cost ( $F_i$ ) and distinguish between one-store and multi-store companies. For a one-store company the marginal cost is computed as:

$$(2) \quad c_i = P_i(1 + 1/\varepsilon_{ii}).$$

The marginal cost is a function of the stipulated reference price and the calibrated own-price elasticity of the product, and it is consistent with the assumptions of Bertrand behavior and that data constitute equilibrium. The marginal costs for a firm controlling  $g$  stores are

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<sup>13</sup> Crooke *et al.* (1999) demonstrated the very diverse predictions for post merger price increases following the choice of linear, logit, AIDS, or isoelastic demand functions.

<sup>14</sup> From  $\varepsilon_{ij} \equiv (\partial x_i / \partial p_j)(p_j/x_i) = b_{ij}(p_j/x_i)$ , we find  $b_{ij} = \varepsilon_{ij}(X_j/P_i)$  and next  $a_i = X_i - \sum_j b_{ij} P_j = X_i(1 - \varepsilon_i)$ .

<sup>15</sup> We argued above that some stores focus on low price implying that others have higher prices. This might be reflected in different reference prices. First, these differences are small, probably less than 10%. Next, the calibration of marginal costs from first order conditions reflects price levels whereby the marginal cost transmits reference price level into the equilibrium price level. Stipulating  $P_i = 1$  also makes the revenue based diversion ratios from the enquiry immediately available as volume indicators.

<sup>16</sup> Lower-case symbols ( $x$  and  $p$ ) denote variables in the model, while upper-case  $X$  and  $P$  denote levels that are observed or taken as references.

found by solving  $g$  linear equations in  $g$  unknown marginal costs  $c_i$ .<sup>17</sup> Table 3 displays the calibrated marginal costs corresponding to each demand system - one based upon observed diversion ratios (OBS), the other based upon diversion ratios assumed to be proportional to market shares (MS).

[Table 3: Calibrated marginal cost and own price elasticities of stores]

The calibrated marginal costs ( $c_i$ ) of the MS model show little variation. The two one-store companies are imputed the highest costs; the stores of the two-store company slightly lower costs, while the stores of the four-store group get the lowest cost. Differences on company level stem from the coordination of pricing within companies. The cost variations between stores within a company or between the two one-store companies stem from their sizes (sales revenue); the larger the sales, the lower the imputed marginal cost  $c$ .

The calibrated marginal costs ( $c_i$ ) of the OBS model differ considerably more across stores than the corresponding numbers for the MS model. This follows from the larger differences in calibrated partial price elasticities (compare Tables A1 (a) and (b)), which again reflects differences in diversion ratios between these models. In the MS model, diversion ratios in a row are of similar values and calibrated own-price elasticities become roughly equal. Observed diversion ratios, however, convey large asymmetries, whereby calibrated own-price elasticities in the OBS model show larger variation.

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<sup>17</sup> We have included diversion ratios between the four stores owned by NG prior to the merger, because these stores can be substitutes for the consumers. Obviously, NG will internalize diversion between its stores, for example by identical price increases. This price-increasing mechanism is taken care of in the calibration of marginal costs, since it is assumed that NG set prices jointly in all its stores. The higher the diversion ratios between the stores in the NG group, the larger are the price-cost margins that NG can set and the larger are the calibrated price-cost margin for the NG stores prior to the merger.

#### 4. The price effects of a merger

As usual we assume that cost structures are unchanged from before the acquisition. Also, we only consider the computation of unilateral effects by merging parties and the immediate response from rivals. In an actual merger case, some of the parties may subsequently invest in promotion, product revision, etc. Modifications along these lines can be incorporated in our approach as in other models. Such extensions would require additional and case-specific information, they may be based on speculative, or at least, less quantifiable and directly measurable effects and they are disregarded here. Alternatively, the merger simulations should be focusing on the price effect alone and be supplemented by a more qualitative analysis such as the prospects for entry and any possible reposition of products after the merger.

The magnitude of price increases following a merger depends upon the behavior of the most price sensitive customers. In our survey all customers – not only the marginal ones – are asked about their second choice. Since diversion ratios may differ between customers, this may create a bias. However, we do not think there will be any systematic bias. All customers have a second choice, and we see no obvious reason why a price sensitive customer's second choice is more likely to be, say, store number one than what is represented by this store's diversion ratio in the sample as a whole. Therefore, we have no a priori reason to believe that the observed diversion ratios of the whole sample understate or overstate the diversion ratios of the marginal consumers.<sup>18</sup>

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<sup>18</sup> This conclusion is supported by results reported in Bordley (1985) and further discussed in Bordley (1993). He finds that under certain conditions – that are met for a large class of demand functions – the diversion ratio is independent of the amount of the price change. This differs from what we expect to find for price elasticities. The marginal customer is by definition more price sensitive, and an average measure would imply that we would underestimate the price sensitivity following a marginal price increase. For further discussion, see Appendix D in Competition Commission's investigation of Somerfield plc/Wm Morrisson Supermarket plc in 2005.

## 4.1 Merger simulation

The post merger model consists of the same equations as the pre-merger model, with the same calibrated demand and cost parameters, with one exception. The acquired firm, Drageset, is not an individual store, but part of the acquiring company.

[Table 4: Predicted %-age price increases following the acquisition]

There are similarities in predictions from these two models. First, in both models prices increase more for the stores involved in the acquisition (Meny, Spar, Kiwi Vangen and Kiwi Palmafossen together with the acquired store Drageset) than for outside stores. This is line with theory, see Deneckere and Davidson (1985). A merger implies that the merging parties no longer compete against each other. Rather they coordinate their behavior by setting higher prices on all their products. The outsiders, on the other hand, have at the outset no reason to change their price setting. They only respond to the price increases by the merging parties and increase both their prices and volumes. Second, the price increase of the acquired store Drageset is predicted to be the highest in both models. To understand this, note that even though it has the second largest market share (less than 20 %), it is small compared to the sum of the other four stores involved in the acquisition (with a combined market share of more than 40 %). About 39 % of Drageset's customers have one of the four stores as their second choice, while 23 % of the customers at these stores have Drageset as their second choice. The large diversion from Drageset to the other four parties implies that it is optimal to increase its price substantially after the acquisition. The other stores have internalized diversions between themselves before the acquisition and the additional internalization of diversion is low warranting only a smaller increase.

Despite some similarities, there are also important differences in predictions from these two models. First, the predicted average price increase of model OBS is 40 % lower than that of model MS. By comparing market shares and diversion ratios, we can explain (some of) the difference. The observed diversion from Drageset to the four stores involved in the acquisition is 39 %, while the corresponding diversion inferred from market shares is 54 %. Thus, the MS model overestimates the extent to which the acquisition internalizes diversion. The MS model considers Drageset to be a closer rival to the other merging stores than what follows from the observed diversion ratios. Drageset's main rival, Coop Prix, is not involved in the acquisition. There are large diversion ratios between Drageset and Coop Prix, the two stores that are located close to each other, and they compete fiercely also after the acquisition. Competition is thus considered as more intense in the OBS model and implies a stronger competitive constraint on the price setting after the acquisition. Second, the between-store variation in predicted price increases is much larger in model OBS than in model MS. This feature is similar to what was observed above with respect to calibrated marginal costs. Model OBS simply contains more diverse data (asymmetries) than does model MS.<sup>19</sup>

The very diverse marginal costs that were inferred for the OBS-model may be interpreted as invalidating our approach by signaling that there is some inconsistency in data and assumptions behind eq. (2).<sup>20</sup> Given our confidence in the observed diversion ratios and the calibrated demand, an alternative approach to establishing the model is to stipulate marginal costs. Assume we choose  $c_i = 0.75$  for all  $i$ , which is about the average of

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<sup>19</sup> In order to facilitate the comparison of price predictions from the two demand systems and focus on the consequences of the diversion ratios between the stores, we have used the observed diversions out of the market in both models. We have rerun the models with the alternative assumption that the diversion out of the market is equal (to the average 18.7 %) for all stores. It turns out that the predicted price increases are hardly affected by this change of assumption.

<sup>20</sup> The drawback of calibrating the marginal costs residually is that all deficiencies in data or assumptions are transmitted into the calibrated marginal costs. See Werden and Froeb (2008) for a discussion of the method. Given our goal to compare two models that differ with respect to how demand is derived these drawbacks should not discriminate or favor any of the two approaches.

marginal costs for both models.<sup>21</sup> The price increases predicted from this version of the models are 2.6% for OBS and 5.1% for the MS-model. These numbers are not very different from those obtained above, 3.2% and 5.3% (OBS and MS, respectively). Thus, the difference between predictions remains.<sup>22</sup>

As a last robustness check of our calibration we varied the prices for the various stores according to observations of the prices in various chains' stores. The stores were put into three categories with prices 5% respectively 10% above the lowest. What we found was that when the price level of a store is increased, its imputed marginal cost is increased by the same amount, which is also seen from eq. (2) (for a one-store company). Thus, the resulting price increase from a merger was lowered compared to the base case.

## 4.2 The effects of partial information on diversion ratios

In other cases one might have some, but not a complete set of the diversion ratios. Moreover, because it is costly to obtain this information, one might wonder whether it will pay in terms of improved predictions to acquire (more) diversion ratios. In order to illustrate the potential value of more information, consider a sequence of simulations where we initially use model (MS) that is based only on market shares and iteratively revise it by employing increasingly more information in terms of observed diversion ratios.

Theory says that diversion ratios between the merging parties are the most important factors for explaining price increases. This is explained in Farrell and Shapiro (2010), and

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<sup>21</sup> We now dispense with the idea that observed prices ( $P_i = 1$ ) constitute equilibrium. Rather we calibrate pre-acquisition equilibrium prices that in general will differ from 1.0. Next, we compute post-merger prices and then derive price changes.

<sup>22</sup> We have also looked into the effect of using other values than -1 for the market price elasticity. When the external elasticity is halved, the predictions of price-increases in both models are doubled. This should not come as a surprise looking at the relationship between the external elasticity and the partial price elasticities in (A5) in the appendix. Doubling  $\varepsilon_i$  means doubling all partial elasticities as well, and thus doubling the own price elasticity  $\varepsilon_{ii}$ . Conversely, a doubling of the external elasticity leads to predicted price-increases that are halved compared to the base case.



also pointed out in the US horizontal merger guidelines.<sup>23</sup> Consider therefore first a modified MS model where diversions from the acquired store are replaced by the corresponding observed numbers. The predicted price increase of this model is lower than that of the modified MS model and explains 46 % of the difference in average price predictions between two models.<sup>24</sup> Introducing next observed diversion ratios for one of the four stores already controlled by the acquiring chain reduces the gap between model predictions by another 20 %. Including observed diversion ratios of only these two stores contributes 2/3 of the difference in prediction from a full set of observed diversion ratios versus no such data (Figure 3), suggesting that just a subset of the (actual) diversion ratios may contribute significantly to improve the prediction from a merger simulation.

[Figure 3: Average price increases from subsets of observed diversion ratios]

Shapiro (1996, 2007) advocates a totally different approach basing price prediction on simple formulas assuming the industry is described by a single-product duopoly. He suggests using such a formula to screen mergers.<sup>25</sup> Competition Commission applied his formula to predict what they called ‘illustrative price increases’ following Somerfield’s acquisition of Morrison’s 115 grocery stores in 2005.<sup>26</sup> In the Voss case a multi-store company acquires another store.<sup>27</sup> Although a simple formula may not suit this more

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<sup>23</sup> In US (2010) it is stated that ‘diversion ratios between [merging firms’ products] can be very informative for assessing unilateral price effects. ... Diversion ratios between products sold by merging .. and non-merging firms have at most secondary predictive value.’

<sup>24</sup> This is explained by the observed diversion ratios. Table 2 shows that all diversion ratios from Drageset to NG stores are lower than those that are estimated from market shares.

<sup>25</sup> Farrell and Shapiro (2010) have proposed an alternative way to screen mergers. They do not estimate any price increase following a merger, but rather ask whether a merger will lead to an upward pricing pressure (UPP). This will be the case if the UPP from loss of direct competition is not offset by a reduction in marginal costs. Their approach has been criticized in Schmalensee (2010), who proposes to use simple merger simulations instead of an UPP approach.

<sup>26</sup> For a critique of their price predictions, see RBB (2006).

<sup>27</sup> Moresi *et al.* (2008) discuss how the existence of multiproduct firms will affect the definition of the relevant market. However, they do not provide any simple formula for the price increase following a merger in such a market.

complex situation it is of interest to illustrate the approach. When only the price of one of the two products is increased, the increase is  $mD/2$ , where  $m$  is the price-cost margin and  $D$  is the diversion from this product to the other. Filling in our numbers, the acquired firm would increase its price by 5.7 % according to this simplified Shapiro approach.<sup>28</sup> In comparison, model OBS predicts a price increase of 7.5 % (Table 4) and model MS predicts an even larger increase of 11.4 %. One obvious reason for the difference in predictions between a simple formula and a simulation model is that the models (OBS and MS) allow prices on all products to be increased (adjusted). Anticipating such adjustments, it becomes profitable to increase the price of the one product even more.

## 5. Some concluding remarks

We have presented an example from a local market where customers' stated diversion ratios (their second choice) differ considerably from diversion ratios that are proportional to market shares (customers' first choice). This case shows that market shares can be poor predictors for the actual competition between products, and it is not surprising that the price predictions from the simulation based upon observed diversion ratios differ significantly from predictions of the simulation based only on market shares. It illustrates that the traditional approach to merger investigation – by considering market shares – can be misleading.

As far as we know, no other merger simulation model is based upon observed diversion ratios. Our approach can be valuable where diversion ratios are easily obtained. Even information on some, but not all ratios can significantly improve the price predictions. Information on diversion ratios between some or all of the merging parties would provide a quick and easy extension of the existing, calibrated merger simulation models.

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<sup>28</sup> The diversion ratio from the acquired store to the other stores involved in the merger is 38.7 %, and we apply the price-cost margin (1 - 0.705) which was used in model OBS (see Table 3).

We have found that in this particular case a model based on market shares would predict a higher price increase than a model based on diversion ratios. One reason could be the way the local market has been selected for investigation by competition authorities. The actual acquisition involved several local markets, and the one we have investigated is the one which the competition authority selected because it had the highest market shares of the firms involved in the acquisition. Such a screening can lead to a systematic bias, since the mergers with large market shares by the involved parties are not necessarily the ones with high diversion ratios among those parties. In other instances, the market share approach may underestimate the anticompetitive problem. A merger would have the largest anticompetitive effect if two firms with high diversion ratios between their products merge. Firms behaving strategically might find it profitable to merge two units with relatively low market shares, but high diversion ratios between their products. The low market shares may result in an early clearance by competition authorities, while the merger internalizes intense competition due to high diversion ratios between their products.

**Table 1: Descriptive statistics from the survey**

	Average bill (NOK)	Price	Product range	Residence at Voss	Leakage out of Voss
Rimi (ICA)	183.05	22.8	8.9	92.1	11.9
Drageset (the acquired store)	428.56	1.0	29.3	77.8	10.1
Coop Mega	299.96	4.1	34.0	87.6	17.5
Coop Prix	254.06	20.8	0.0	91.7	17.7
Meny (Norgesgruppen)	443.55	0.0	37.8	76.5	20.4
Spar (Norgesgruppen)	221.44	2.0	13.1	84.9	14.1
Kiwi Vangen (Norgesgruppen)	154.84	28.6	2.0	89.8	17.4
Kiwi Palmafossen (Norgesgruppen)	280.83	17.0	2.2	94.3	14.8

Notes: The names in parentheses indicate the retail chain of the various stores.  
*Average bill* denotes average bill per respondent (1 NOK  $\approx$  1/9 Euro)  
*Price* denotes the share of the respondents saying price is important for their choice  
*Product range* denotes the share of the respondents saying product range is important for their choice  
*Residence at Voss* is the share of the respondents being a Voss resident.  
*Leakage out of Voss* denotes the share of the respondents saying that their 2<sup>nd</sup> choice is not among one of the eight surveyed stores

**Table 2. The relative diversion ratios:  $I_{ij} = d_{ij}/D_{ij}$   $j \neq i$ .**

To	From							
	Rimi	Drageset	Coop Mega	Coop Prix	Meny	Spar	Kiwi Vangen	Kiwi Palmaf.
Rimi		0.36	0.53	0.34	0.46	0.25	1.65	1.59
Drageset	0.14		0.15	2.83	0.89	0.24	0.02	0.22
Coop Mega	2.18	1.21		0.96	2.19	2.48	1.40	0.82
Coop Prix	0.94	3.54	1.62		0.32	0.29	0.12	0.38
Meny	0.92	0.97	1.14	0.49		1.01	0.26	1.78
Spar	0.48	0.53	2.66	0.31	0.82		2.00	0.70
Kiwi Vangen	1.13	0.33	0.49	0.52	0.09	1.84		1.17
Kiwi Palmafossen	1.66	0.61	0.22	0.89	1.33	0.62	2.80	

Notes:  $d_{ji}$  denotes observed diversion ratios, while  $D_{ji}$  denotes a diversion ratio that is proportional to firm  $j$ 's market share.

**Table 3: Calibrated marginal cost and own price elasticities of stores**

	Own price elasticity		Marginal cost	
	OBS	MS	OBS	MS
Rimi	-3.65	-5.63	0.726	0.823
Drageset	-3.39	-5.19	0.705	0.807
Coop Mega	-7.73	-5.44	0.852	0.804
Coop Prix	-5.89	-5.80	0.812	0.802
Meny	-4.81	-5.07	0.722	0.725
Spar	-5.98	-5.43	0.696	0.708
Kiwi Vangen	-4.26	-5.75	0.594	0.707
Kiwi Palmafossen	-5.14	-5.70	0.642	0.711

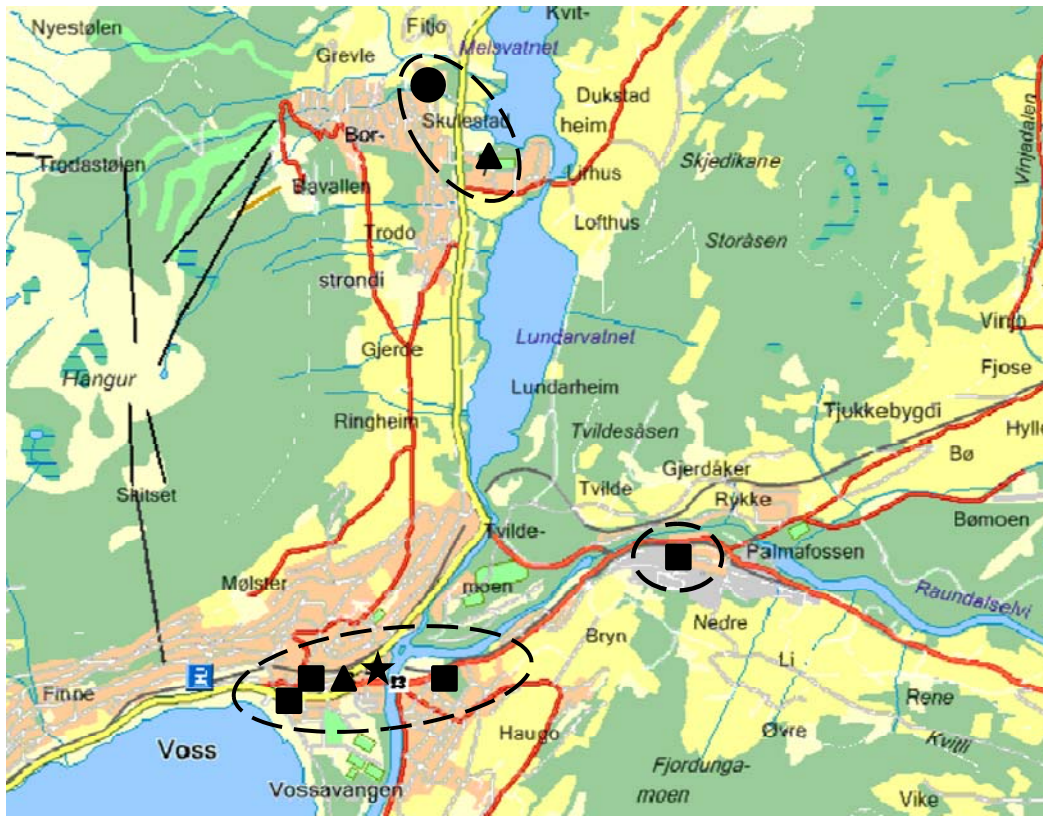
Notes: OBS denotes that numbers come from the model based on observed diversion ratios, while MS denotes that numbers come from the model where diversion ratios are proportional to firms' market shares. Pre merger prices are set equal to 1, such that price-cost margins are equal to  $(1 - \text{marginal cost})$ .

**Table 4. Predicted percentage price increases following the acquisition**

	OBS	MS
Rimi	1.1	2.7
Drageset	7.5	11.3
Coop Mega	1.6	2.7
Coop Prix	1.9	2.6
Meny	4.1	5.7
Spar	2.5	6.2
Kiwi Vangen	1.8	6.3
Kiwi Palmafossen	2.9	6.1
Average	3.2	5.3

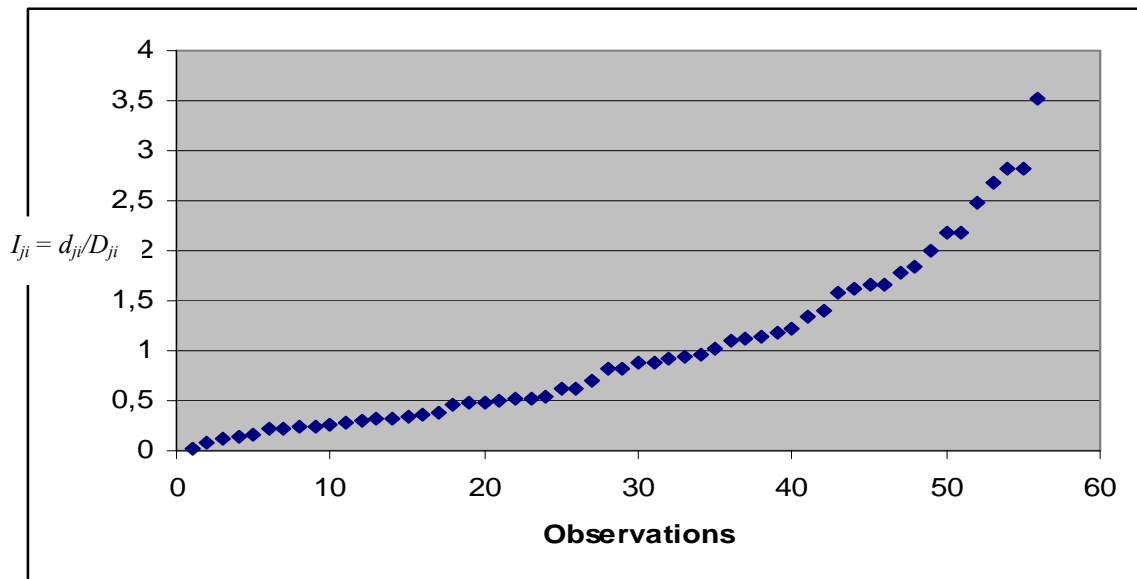
Note: See note to Table 3.

**Figure 1: Grocery stores at Voss**

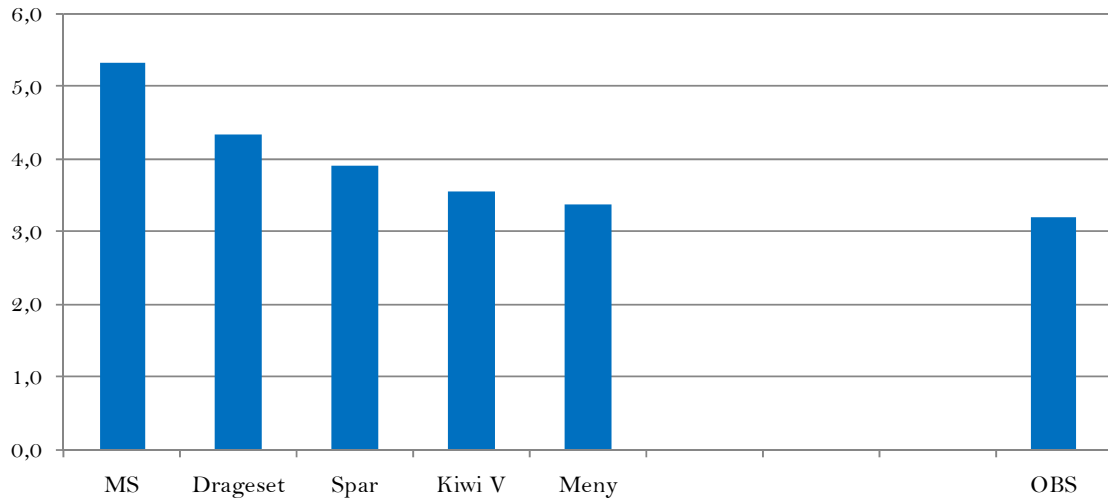


Note: The four stores owned by the retail chain Norgesgruppen are marked with a black square; the two Coop stores are marked with a black triangle; Drageset, acquired by Norgesgruppen, is marked with a circle, and Rimi, a store in the retail chain ICA is marked with a star. The five stores clustered in the centre of Voss are – counted from west to east - Kiwi Vangen, Spar, Coop Mega, Rimi, and Meny. Drageset and Coop Prix are located about four km to the north of the centre, while Kiwi Palmafossen is located about two km to the east.

**Figure 2: Observed diversion ratio divided by diversion ratio from market shares.**



**Figure 3. Predicted average price increase given subsets of observed diversion ratios.**



**Notes:** MS denotes the prediction of the MS model. Drageset denotes the prediction of a modified MS model where we use the observed diversion ratios from Drageset, that is, this column of diversion ratios is changed. Similarly, Spar shows the prediction of a modified MS-model where also the observed diversion ratios from Spar are used. OBS denotes the prediction if we use the observed diversion ratios between all eight stores.

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## Appendix A.

**Table A1 (a). Partial price elasticities at benchmark values – the OBS model.**

To	From							
	Rimi	Drageset	Coop Mega	Coop Prix	Meny	Spar	Kiwi Vangen	Kiwi Palma.
Rimi	-3.65	0.22	0.53	0.12	0.47	0.19	0.48	0.63
Drageset	0.05	-3.39	0.15	1.00	0.91	0.18	0.01	0.09
Coop Mega	0.72	0.75	-7.73	0.34	2.23	1.94	0.41	0.33
Coop Prix	0.31	2.20	1.63	-5.89	0.33	0.22	0.04	0.15
Meny	0.31	0.61	1.15	0.17	-4.81	0.79	0.08	0.71
Spar	0.16	0.33	2.68	0.11	0.84	-5.98	0.59	0.28
Kiwi Vangen	0.37	0.20	0.49	0.18	0.09	1.44	-4.26	0.47
Kiwi Palmafossen	0.55	0.38	0.22	0.32	1.36	0.49	0.82	-5.14

**Table A1 (b). Partial price elasticities at benchmark values – the MS model.**

To	From							
	Rimi	Drageset	Coop Mega	Coop Prix	Meny	Spar	Kiwi Vangen	Kiwi Palma.
Rimi	-5.63	0.95	0.71	0.35	1.07	0.71	0.40	0.44
Drageset	0.51	-5.19	0.71	0.35	1.07	0.71	0.40	0.44
Coop Mega	0.51	0.95	-5.44	0.35	1.07	0.71	0.40	0.44
Coop Prix	0.51	0.95	0.71	-5.80	1.07	0.71	0.40	0.44
Meny	0.51	0.95	0.71	0.35	-5.07	0.71	0.40	0.44
Spar	0.51	0.95	0.71	0.35	1.07	-5.43	0.40	0.44
Kiwi Vangen	0.51	0.95	0.71	0.35	1.07	0.71	-5.75	0.44
Kiwi Palmafossen	0.51	0.95	0.71	0.35	1.07	0.71	0.40	-5.70

## Appendix B. Model calibration

There are three steps in our calibration of demand. First, the full set of partial price elasticities are derived from diversion ratios and assumed market price elasticities.<sup>29</sup> This step is general and holds for any demand system. Next, a choice of functional form is made, and finally, parameters of this functional form are derived from the partial price elasticities.

### 1. A routine for calibrating partial price elasticities from diversion ratios

Assume there are  $n$  differentiated products (stores in our application), and let  $x_i$  and  $p_i$  denote volume respectively price of product  $i$ . Demand for product  $i$  is

$$(A1) \quad x_i = f_i(p_1, \dots, p_n), \quad i = 1, \dots, n,$$

The *diversion ratio* says which fraction of diverted customers from product  $i$  that switch to product  $j$  because of a price increase of product  $i$

$$(A2) \quad d_{ji} \equiv (\partial x_j / \partial p_i) / (\partial x_i / \partial p_i) = \varepsilon_{ji} x_j / \varepsilon_{ii} x_i, \quad i \neq j = 1, \dots, n.^{30}$$

The diversion ratio says nothing about how many customers or what fraction of product  $i$ 's customer base will leave. The own-price elasticity contains such information. Eq. (A2) suggests that given the values of  $\varepsilon_{ii}$ ,  $i=1, \dots, n$ , all cross-price elasticities  $\varepsilon_{ji}$  can be derived.

$$(A3) \quad \varepsilon_{ji} = d_{ji}(x_i/x_j)\varepsilon_{ii}, \quad j \neq i = 1, \dots, n \quad \text{or equivalently} \quad \varepsilon_{ij} = d_{ij}(x_j/x_i)\varepsilon_{jj}, \quad i \neq j = 1, \dots, n.$$

A demand system for  $n$  imperfect substitutes in  $n$  prices has  $n^2$  partial price elasticities. Often one has insufficient data to pin down that many parameters. Our observed diversion ratios indicate non-symmetric differentiation, which we take as given. One can think of the  $n(n-1)$  diversion ratios as determining the corresponding cross-price elasticities as shown in

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<sup>29</sup> Observing that all partial price elasticities may be derived from own-price elasticities and diversion ratios (fractions), Bordley (1993) suggests estimating price elasticities of demand by first estimating diversion fractions. We take observed diversion ratios as given and derive demand that is consistent with these observations. While all our price elasticities have the right signs, they will typically not satisfy conditions of individual consumer demand theory, as imposed by Bordley.

<sup>30</sup> The partial price elasticity  $\varepsilon_{ij} \equiv (\partial x_i / \partial p_j)(p_j/x_i)$  is called own-price elasticity when  $j=i$  and cross-price elasticity when  $j \neq i$ .

eq. (A3). Next we stipulate the  $n$  market price elasticities ( $\varepsilon_i$ ), whereafter the  $n$  own-price elasticities ( $\varepsilon_{ii}$ ) are determined from the following relationship between price elasticities<sup>31</sup>

$$(A4) \quad \varepsilon_i = \varepsilon_{ii} + \sum_{j \neq i} \varepsilon_{ij}, \quad i = 1, \dots, n.$$

To demonstrate this routine, set  $d_{ii} = 1$  and use eq. (A3) to substitute for cross-price elasticities in eq. (A4)

$$\varepsilon_i = \varepsilon_{ii} + \sum_{j \neq i} \varepsilon_{ij} = d_{ii}(x_i/x_i)\varepsilon_{ii} + \sum_{j \neq i} d_{ij}(x_j/x_i)\varepsilon_{jj} = \sum_j d_{ij}(x_j/x_i)\varepsilon_{jj} \quad i = 1, \dots, n.$$

Assume that in addition to diversion ratios  $d_{ji}$  and stipulated market price elasticities  $\varepsilon_i$ , we have observed quantities ( $X_i$ ). Substitute  $X_i$  for  $x_i$  and multiply by  $X_i$  to obtain

$$(A5) \quad X_i \varepsilon_i = \sum_j (d_{ij} X_j) \varepsilon_{jj}, \quad i = 1, \dots, n.$$

(A5) is a set of  $n$  linear equations that is solved in order to obtain  $n$  own-price elasticities  $\varepsilon_{jj}$ . Define  $v_i = X_i \varepsilon_i$  and  $r_{ij} = d_{ij} X_j$ . On vector notation eq. (A5) is  $\mathbf{v} = \mathbf{R}\boldsymbol{\chi}$ , where the vector  $\mathbf{v}$  and the matrix  $\mathbf{R}$  represent data (observations and stipulations/best guesses) and  $\boldsymbol{\chi}$  is a vector of unknown variables representing own-price elasticities (the scales of diversion). In order to solve for  $\boldsymbol{\chi}$  the matrix  $\mathbf{R}$  has to be non-singular, and furthermore, for the solution values to have the right sign, i.e.,  $\varepsilon_{jj} = \chi_j < 0$ ,  $\mathbf{R}^{-1}$  should to be positive. Both these requirements follow from the assumption that  $\sum_{i \neq j} |d_{ij}| < 1, j=1, \dots, n$ . Then  $\mathbf{R}^{-1} > 0$  and  $\boldsymbol{\chi} = \mathbf{R}^{-1}\mathbf{v} < 0$ .

The critical assumption is that there is diversion out of the market, that is, some customers at store  $j$  would rather buy outside the market than from any of the  $n-1$  other stores in this market. With  $d_{ii} = 1$  each column of  $\mathbf{R}$  has a positive sum of elements, which implies a positive dominant diagonal. With negative off-diagonal elements,  $\mathbf{R}^{-1}$  is then non-negative, and because  $v_j < 0$ ,  $\varepsilon_{jj} = \chi_j < 0$ , i.e., own-price elasticities are negative. From eq. (A3) we now obtain all cross-price elasticities, which are positive.

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<sup>31</sup> Let  $p_j = P_j t$ ,  $j=1, \dots, n$ , with  $P_j$  reference price and  $t$  a price change. The market price elasticity is defined as  $\varepsilon_i \equiv (dx_i/dt)(t/x_i)$ . Totally differentiating demand,  $f_i(p_1(t), \dots, p_n(t))$ , we obtain  $dx_i/dt = \sum_j (\partial x_i / \partial p_j)(\partial p_j / \partial t)$ . Multiplying through the equation by  $(t/x_i)$  gives us the market elasticity on the left, whereas the right hand side can be written as the sum of products of two elasticities,  $\varepsilon_i = \sum_j \varepsilon_{ij} \varepsilon_{jt}$ , and since  $\varepsilon_{jt} \equiv (\partial p_j / \partial t)(t/p_j) = P_j(t/P_j t) = 1$  for all  $j$ , it follows that  $\varepsilon_i = \sum_j \varepsilon_{ij}$ .

It is useful to compare our approach to the established one as exemplified by the ALM model of Werden and Froeb (1994) and the PCAIDS model of Epstein and Rubinfeld (2002). Each of them employs a functional form that implies that diversion ratios are proportional to market shares.<sup>32, 33</sup> Together with observed prices and volumes (or market shares) these functions are calibrated with the stipulation of only two parameters related to price elasticity of demand. In both the ALM and the PCAIDS model the average market price elasticity ( $\varepsilon$ ) is stipulated.<sup>34</sup> In addition, Werden and Froeb (*op. cit.*) stipulate a parameter  $\beta$  representing the scale of partial elasticities, while Epstein and Rubinfeld (*op. cit.*) stipulate one of the  $n$  own-price elasticities. All (remaining) partial price elasticities then follow from the functional form, the observed prices and volumes, and these two stipulated parameters.

Before closing this section observe the following implication of eq. (A5).<sup>35</sup>

$$(A6) \quad \sum_i s_i \varepsilon_i = \sum_j s_j \varepsilon_{jj} d_{0j}$$

The average market price elasticity ( $\varepsilon$ ) on the left equals an average product of own-price elasticity and diversion out of the market. In their ALM model Werden and Froeb (1994) define the market elasticity  $\varepsilon \equiv \beta P_O p^*$  where  $\beta$  is a parameter representing the level of partial price elasticities,  $P_O$  is the fraction of customer that buys outside the market, and  $p^*$  is the average price level. Thus, when both the values for  $\varepsilon$  and  $\beta$  are stipulated, and  $p^*$  follows from data,  $P_O$  is determined. In our case the  $d_{0j}$ 's (diversion out of the market) are observed, whereby we need to stipulate only one of the two prices sensitivities  $\varepsilon$  or  $\beta$ . A relevant question is which of these parameters we do have most confidence in. Our OBS method is based on an enquiry into diversion ratios, including responses on buying outside the market. Of the two elasticities we think that the market price elasticity is an easier number to intuit than own-price elasticities.

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<sup>32</sup> Both systems allow for hierarchical decomposition (or nesting) such that clusters of products can be handled; products within a nest are equally close substitutes and each of them is an equally close substitute to any product in another nest. Nesting improves the fit to underlying non-symmetry, but suffers from treating products as either in a particular nest or not, while often there is gradual change in location.

<sup>33</sup> Proportionality is a manifestation of the property of independence of irrelevant alternatives (IIA) and is a useful definition of what it means that products are equally close substitutes to each other. As an illustration, consider firms A, B, and C with market shares 0.5, 0.3, respectively 0.2. Based on these shares the diversion ratio from A to B is  $0.3/(1-0.5) = 0.6$  and from A to C is  $0.2/(1-0.5) = 0.4$ . Hausman *et al.* (1994) point out that IIA is often rejected in empirical analyses.

<sup>34</sup> In PCAIDS all  $\varepsilon_i = \varepsilon$ , while in ALM  $\varepsilon = \sum_i s_i \varepsilon_i$ , where  $s_i$  is market share of product  $i$ .

<sup>35</sup> Let  $s_i \equiv X_i / \sum_j X_j$ . Summing over  $i$  in eq. (A5), the left hand side is immediate. On the right hand side switch summation order and observe that  $\sum_i d_{ij} = 1 - \sum_{i \neq j} |d_{ij}| \equiv d_{0j}$ , which is diversion out of the market.

In the Voss case we stipulate market elasticities  $\varepsilon_i = -1, i=1, \dots, 8$ .<sup>36</sup> The derived own price elasticities (from eq. (A5)) and the cross-price elasticities (from eq. (A3)) are reported in Table A1 (a). The row-sums are -1 as assumed (cf. eq. A4.) Table A1 (b) shows the partial price elasticities that follow from proportionality with market shares. Also in this matrix are the row-sums -1. Furthermore, cross price elasticities of row  $i$  are proportional to market shares, and cross price elasticities in a column are identical. These features are shared with the PCAIDS and ALM models and follow from the assumption of proportionality.

## 2. Calibration of demand for the Voss case

In order to have a conservative estimate of the price increase we use linear demand

$$x_i = a_i + \sum_j b_{ij} p_j, \quad i = 1, \dots, 8,$$

where  $a_i$  and  $b_{ij}$  are parameters to be calibrated from the full set of partial price elasticities.<sup>37</sup> We define reference prices  $P_i = 1$  and volumes to equal observed market shares  $X_i = s_i$ .

## 3. Calibration of marginal cost

We follow the established approach to calibration of marginal costs by inferring these from the first-order conditions for optimal pricing. We assume constant variable cost ( $c_i$ ) and fixed cost ( $F_i$ ). As there are both one-store companies and multi-store companies we have to distinguish between these types. The *one-store company* profit is given by  $\Pi_i = (p_i - c_i)x_i - F_i$ , and the first order condition is:

$$(A7) \quad \partial \Pi_i / \partial p_i = x_i + (p_i - c_i) (\partial x_i / \partial p_i) = x_i + (p_i - c_i) b_{ii} = 0.$$

Using eq. (A7), observed values of  $x_i$  and  $p_i$  ( $X_i$  respectively  $P_i$ ) and the calibrated value of  $b_{ii}$ , the (assumed constant) marginal cost is computed as

$$(A8) \quad c_i = (X_i + b_{ii} P_i) / b_{ii} = X_i / b_{ii} + P_i.$$

<sup>36</sup> Epstein and Rubinfeld (2002) suggest that absent independent information,  $\varepsilon_i = -1$  is a good starting point for a preliminary merger simulation.

<sup>37</sup> From  $\varepsilon_{ij} \equiv (\partial x_i / \partial p_j)(p_j / x_i) = b_{ij}(p_j / x_i)$ , we find  $b_{ij} = \varepsilon_{ij}(X_j / P_i)$  and next  $a_i = X_i - \sum_j b_{ij} P_j = X_i(1 - \varepsilon_i)$ .

A *multi-store company* controls a group ( $G$ ) of  $g$  stores. Profit is  $\Pi_G = \sum_{j \in G} (p_j - c_j)x_j - F_G$  and the first order condition

$$(A9) \quad \frac{\partial \Pi_G}{\partial p_i} = x_i + \sum_{j \in G} (p_j - c_j) \frac{\partial x_j}{\partial p_i} = 0, \quad i \in G.$$

Using observed values we get

$$(A10) \quad X_i + \sum_{j \in G} (P_j - c_j) b_{ji} = 0, \quad i \in G.$$

These are  $g$  linear equations in  $g$  unknown marginal costs  $c_i$ , whose values are obtained by matrix inversion.



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