Reward redemption effects in a loyalty program when customers choose how much and when to redeem

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REWARD REDEMPTION EFFECTS IN A LOYALTY PROGRAM
WHEN CUSTOMERS CHOOSE HOW MUCH AND WHEN TO
REDEEM ¹

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ABSTRACT

The redemption of loyalty program (LP) rewards has an important impact on LP members’ behavior, particularly on purchase behavior before and after redeeming a reward. However, little is known about the interplay between members’ purchase and redemption behavior when members are not pressured with points expiration and they choose for themselves when and how much to redeem. In this context, the effects of redemption are not straightforward, as little additional effort is required from an LP member to obtain the reward. Analyzing the behavior of 3,094 members in such an LP, we find that the mere decision to redeem a reward significantly enhances purchase behavior before and after the redemption event, even when members redeem just a fraction of their accumulated points. Conceptually, we refer to this enhancement as the redemption momentum, which is an alternative and novel explanation of the existence of pre-reward effects that does not depend on points-pressure. In addition to the overall impact of redemption on purchases, prior purchase behavior also enhances redemption decisions. Finally, we find a number of moderating effects on purchase and redemption behavior that derive from the length of LP membership, age, income and direct mailings. Our study’s most important managerial implication is that firms can avoid imposing binding thresholds and/or points expiry and still enhance members’ purchase behavior.

Keywords:
Customer Relationship Management, Loyalty program, Customer loyalty, Reward redemption, Purchases, Points pressure, Rewarded behavior, Direct mailings
Introduction

In recent years, loyalty programs (LPs) have become the dominant tool for loyalty marketing worldwide. In the United States alone, the number of LP memberships exceeded 2.65 billion in 2012, increasing by 26.7% since 2010 (Berry, 2013). LPs aim to engage program members by rewarding their repeated purchases of a firm’s product through (the redemption of) loyalty points that members collect on their purchases. Therefore, the benefits of an LP for a member become the most salient when redeeming a reward (Nunes & Drèze, 2006; Smith & Sparks, 2009a). Yet, as much as one-third of $48 billion worth of LP currency issued in 2010 remained unredeemed (Gordon & Hlavinka, 2011); likewise, *The Economist* estimated that “the total stock of unredeemed miles was worth more than all the dollar bills in circulation” (The Economist, 2005). To reduce liability, LPs introduced points expiration; however, this may undermine loyalty building efforts and engender customer frustration (Land, 2013; Stauss, Schmidt, & Schoeler, 2005). For example, point expiration is common in the airline industry where, due to restrictions on the availability of “award seats,” LP points often expire before members have an opportunity to cash in points (average award seat availability is only about 60% at major airlines (McCartney, 2012)). On the other hand, LPs are increasingly opting for a no-expiration (or long-term expiration) policy to avoid negative customer experiences. For instance, 96% of credit-card programs promote “no expiration” as their key sales feature (Land, 2013). On the other hand, without the expiration pressure to redeem points, firms fear that members’ active engagement may decline and that their loyalty will fade in turn. Whether firms should encourage reward redemption and consider long-term expiration policies ranks among the least understood aspects of LPs (CRMtrends, 2012; Shugan, 2005).

Reward redemption may have an important impact on members’ behavior, particularly on purchase behavior just before and after redeeming a reward. Having to reach a pre-
specified threshold on time to obtain a reward motivates members to increase their expenditures—an effect known as *points pressure* (Kivetz, Urminsky, & Zheng, 2006). However, if a customer already has enough points or (s)he has too few points to be able to reach the threshold, the points pressure becomes negligible (Hartmann & Viard, 2008; Lewis, 2004). The question, then, is whether firms can expect redemption effects in LPs without significant binding deadlines that “require customers to jump through hoops to receive a reward” (Blattberg, Kim, & Neslin, 2008, p. 566). Unfortunately, the prevailing theoretical mechanisms to explain such effects are equivocal.

If firm-imposed motivators leading to point pressure are removed, then the presence of redemption effects depends on whether the redemption decision by itself impacts behavior. In LPs with continuous and linear rewarding schemes, members obtain a certain amount of LP currency for each dollar/euro spent and choose when to redeem (redemption timing) and what to redeem (redemption amount), based on their personal reward preferences and the collected balance of points (cf. Stourm, Bradlow, & Fader, 2013). Moreover, in continuous LPs, the program itself and/or its points typically do not expire for a longer period of time (e.g., retail LPs). This context allows us to investigate whether redemption effects on behavior in pre- and post-reward period can be evoked by the act of redeeming itself in the absence of firm-imposed thresholds. The decision to redeem points may precede the moment at which the reward is redeemed or it may occur at a point-of-sales without much prior planning, which has direct consequences on behavior.

Analyzing the purchase and redemption behavior of 3,094 members in a Dutch continuous LP, we find that in as much as 70% of redemptions, the decision to redeem is made a short time ahead of the redemption. Having made the decision motivates customers within the LP, resulting in an increase in purchase behavior prior to the redemption event, even when customers subsequently redeem just a small fraction of their overall points balance.
We label this effect *redemption momentum* and note that this effect complements the *points pressure* effect, which may occur for members who have an insufficient amount of points in the weeks before a redemption.

In the post-reward period, the redemption enhances feelings of gratitude, importance, satisfaction or obliged reciprocity, which may in turn spur purchase behavior (Palmatier, Jarvis, Bechkoff, & Kardes, 2009). However, empirical findings on the post-reward effects on members’ behavior are scarce and the results are mixed in the literature. In some cases, points pressure shifts purchases in time and creates post-redemption dips due to stockpiling. This is not expected to occur when members can choose timing and redemption amounts. Our study provides support for positive post-reward effects when customers do not face binding deadlines and can choose the redemption timing and amount.

Finally, redemption effects on purchase behavior may vary across LP members (Kopalle, Sun, Neslin, Sun, & Swaminathan, 2012; Stourm et al., 2013; Zhang & Breugelmans, 2012). In particular, the effects may be moderated by members’ prior experience with the LP (length of LP membership) and various socio-demographic aspects (age, income, etc.), as well as the amount of direct mailing promotions that members obtain (Lewis, 2004). Yet, those interaction effects have not been extensively investigated. In response, we provide an integrated analysis of the main and interaction effects.

In summary, the contribution of this paper is threefold. First, we explore whether LPs can foster redemption effects without imposing restrictive deadlines. To this end, we examine alternative mechanisms that drive (pre-)redemption effects and propose the novel *redemption momentum* mechanism, which goes beyond the traditional points pressure explanations. Second, this study tackles the interrelatedness of purchase and redemption decision-making by simultaneously modeling purchase incidence, purchase amount, redemption decision and redemption amount. Moreover, our model studies the interplay between redemption and
purchases, accounting both for endogeneity of redemption and endogeneity of personalized mailings to LP members. Third, this study provides an integrated analysis of potential moderating effects, such as relationship length, socio-demographics and direct mailings, on the relationship between redemption and purchases. In this way, our paper answers the call to simultaneously model diverse LP mechanisms to better understand the underlying processes and sources of incremental sales in LPs (Blattberg et al., 2008; Kopalle et al., 2012).

The paper proceeds by discussing the theoretical background and existing studies on the effects of reward redemption. It then continues with the model formulation, a description of the data, the empirical analyses and the results. We conclude with a discussion of key findings and managerial implications.

2. Prior Literature
Marketing literature has extensively studied the effects of LPs on customer behavior (Leenheer, van Heerde, Bijmolt, & Smidts, 2007; Liu, 2007). A synthesis of available evidence indicates that, overall, LPs enhance LP members’ behavior (Dorotic, Bijmolt, & Verhoef, 2012) through increases in purchase volume/frequency (Drèze & Hoch, 1998; Lewis, 2004; Liu, 2007; Taylor & Neslin, 2005) and share of wallet at the LP provider (Leenheer et al., 2007; Verhoef, 2003). However, the role that reward redemption itself plays in this increase is not clear. Existing research on LP rewards has mainly focused on the attractiveness of different reward types and their impact on profitability (Kim, Shi, & Srinivasan, 2001; Kivetz & Simonson, 2002; Zhang, Krishna, & Dhar, 2000), while reward redemption effects themselves have received relatively less attention (Dorotic et al., 2012; Smith & Sparks, 2009a).

Below we separately review the literature on three key aspects: pre-reward effects, post-reward effects, and the impact of mailings and other main moderators. Table 1 provides
an overview of (selected) prior research, summarizes their main findings, and positions our study.

2.1 Pre-reward Effects

Literature to date almost exclusively links pre-reward effects to the goal-pursuit theory and the points-pressure mechanism (Kivetz et al., 2006; Kopalle et al., 2012; Taylor & Neslin, 2005). Points pressure suggests that pre-reward effects are driven by members’ anticipation of obtaining future rewards and/or by switching costs, which together constitute the pressure to collect a sufficient amount of points for a reward (Hartmann & Viard, 2008; Kopalle et al., 2012; Lewis, 2004). Researchers provide evidence of pre-reward effects in short-term LPs, in which members must reach a spending threshold during a time-limited period to obtain a pre-specified reward (e.g., “Spend X on groceries within 3 months, get a free turkey” or “Buy 10, get 1 free”) (Kivetz et al., 2006; Lal & Bell, 2003; Taylor & Neslin, 2005). In such sales promotion-like LPs, the points pressure is high due to the high potential sunk costs and saliency of explicit goals.

In continuous LPs, empirical support for pre-reward effects is found for those LPs with distinctive customer tiers (Kopalle et al., 2012; Drèze & Nunes, 2011) and for retailers with specific, firm-defined redemption thresholds (Lewis, 2004; Zhang & Breugelmans, 2012). These studies reaffirm that pre-reward effects occur through explicit threshold reward structures set by a firm (e.g., LP tiers or “for each 500 collected points customers obtain a voucher/discount”). Such a known external threshold may induce pressure to build up purchases to reach the threshold, thereby spurring the points pressure.

Nonetheless, Smith and Sparks (2009a) found that in a typical continuous retail LP, where customers endogenously choose how much and when to redeem, only the smallest
group of analyzed redeemers (approximately 10%) demonstrated a planning behavior of saving points in order to reach a higher-value reward. The majority of redemptions seemed to be driven by the notion of rewarding and treating oneself from the accumulated balance, sometimes on impulse (Smith & Sparks, 2009a, 2009b). Moreover, recent psychological insights indicate that goal-pursuit may not be the only mechanism driving LP behavior (Henderson, Beck, & Palmatier, 2011; Wiebenga & Fennis, 2014). The findings of Stoum et al. (2013) indicate that in the absence of firm-driven restrictions on the amount and timing of redemption, members may form latent thresholds of redemption based on their subjective perceptions of their points’ value relative to cash. Therefore, the points-pressure mechanism alone may not be sufficient in explaining the impact of redemption on pre-reward purchase behavior. We posit that in the absence of external thresholds (points pressure), members form internal, latent states that affect their behavior before and after redemption, as explained in subsequent sections.

2.2 Post-reward Effects

Post-reward effects are mostly attributed to the rewarded-behavior mechanism (Blattberg et al., 2008; Taylor & Neslin, 2005). Reward redemption enhances subsequent purchase frequency and volume through behavioral learning that ties repurchases to rewards (Rothschild & Gaidis, 1981). Furthermore, a reward obtained through an LP can evoke the belief of a windfall gain or good deal (Arkes et al., 1994; Smith & Sparks, 2009b), a sense of appreciation from the firm that evokes reciprocal feelings (e.g., gratitude, indebtedness) in customers (Gwinner, Gremler, & Bitner, 1998; Palmatier et al., 2009), a sense of belongingness (Dowling & Uncles, 1997), or an elevated sense of status (Drèze & Nunes, 2009). Therefore, reward redemption may induce positive post-reward effects by reinforcing attitudinal attachment, which then affects purchase behavior (Haisley & Loewenstein, 2011;
Taylor & Neslin, 2005). This post-reward effect is instrumental for building long-term relationships with LP members (Kumar & Shah, 2004; Palmatier et al., 2009).

However, the empirical support for post-reward effects is mixed. Some studies reveal positive post-reward effects on purchase behavior in short-term LPs, albeit mainly among light users or for particular types of rewards (Lal & Bell, 2003; Roehm, Pullins, & Roehm, 2003; Taylor & Neslin, 2005). Kivetz and colleagues (2006) found no support for such effects in an experimental study; they instead found evidence for post-reward resetting (i.e., a dip in the purchase behavior after redeeming a reward when purchases return to their pre-reward baseline levels). In a continuous LP setting, Drèze and Nunes (2011) also found post-reward resetting in an airline LP, but not to the initial level, which implies some positive post-reward effect. However, they studied a customer tier program, where reaching a higher tier entitles members to preferential treatment and higher status. It is therefore hard to judge whether increased baseline behavior after redemption is due to the new benefits or the redemption itself. In a similar setting, Kopalle and colleagues (2012) did not find the rewarded behavior effect for a customer tier-oriented segment of members in a hotel LP. Conversely, the study found a positive post-reward effect for the price-sensitive segment attracted to free hotel stays.

Table 1 provides an overview of these mixed research findings and highlights the need for additional empirical evidence in continuous reward settings where members do not have to increase their effort in pre-reward periods and, consequently, they may not feel a particular sense of accomplishment after redeeming.

2.3 Moderating Effects of Mailings, Length of Membership and Socio-demographics

It is beneficial for LP providers to leverage the information they have and target members with personalized mailings (Blattberg et al., 2008; Lewis, 2004). However, the current literature lacks a systematic examination of the impact of personalized marketing
efforts on reward redemption behavior (Blattberg et al., 2008). Yet, it is important to control for the impact of mailings on members’ purchase and redemption behavior in order to accurately delineate the influence of various other drivers (like goal attainment and points pressure). A complicating factor is that the possible target selection by the LP makes the mailings an endogenous decision. Such endogeneity needs to be taken into account when mailings are included as a driver of purchase behavior.

Beside mailings, various individual characteristics may influence the interplay between redemption and purchase. Members respond differently to LPs depending on their usage or spending levels (Kim et al., 2001; Liu, 2007), their experience with the LP (e.g., length of LP membership) (Bolton, Kannan, & Bramlett, 2000), or socio-demographic characteristics (Leenheer et al., 2007; Lemon & von Wangenheim, 2009; Magi, 2003).

The impact of socio-demographic differences in LPs is still ambiguous (Dorotic et al., 2012). In particular, little knowledge exists on the discrete moderating impacts of socio-demographics and the length of LP membership on pre- and post-reward effects. Differences in individual characteristics may influence the size of the reward redemption effects: higher income members have greater purchasing power and may therefore be more flexible with their purchasing levels and respond more strongly to reward incentives. Additionally, long-term members have more experience with the LP, which may lead to higher responsiveness to the LP (Bolton et al., 2000).

3. Conceptualization of the Interplay between Redemption and Purchase

To understand the interplay between redemption and purchase behavior in a continuous and linear rewarding context, it is important to enrich the existing explanations in order to account for diverse motivations and bi-directional relationships. Rewarding may affect purchase behavior, while purchases (i.e., points collection) may in turn affect redemption. In this
context, the sequence of decision-making concerning redemptions may help to explain the reward redemption effects, as illustrated in Figure 1.

<INSERT FIGURE 1 HERE>
This figure outlines the sequence of decision-making that guides our research. The solid arrows indicate the decisions that members make: from the decision to redeem to the purchases after redemption. The dashed arrows, pointing at the box surrounding the process, indicate the influence of the related concept on all aspects of the process (e.g., the overall influence of an accumulated points balance).

If members have a choice to redeem all or just a fraction of their accumulated balance of LP points without being pressured or incurring sunk cost, then a potential increase in purchase behavior in the pre-redemption period is driven by an internal state rather than the points pressure. We posit that the decision to redeem a reward may by itself act as a driver of pre-redemption effects. We coined the term redemption momentum to refer to the redemption decision’s impact on purchase behavior when members do not feel the points pressure. The redemption momentum is active from the point in time that a redemption is planned until it occurs. The decision to redeem a reward may precede the actual redemption and induce excitement for and salience of the benefits of LP membership. This in turn may increase motivation and enhance purchase behavior before the actual redemption takes place. Applied to the LP setting, the situational benefit salience (cf. Petty & Cacioppo, 1979; Ratneshwar, Warlop, Mick, & Seeger, 1997) may refer to a temporary increase in the salience of redeeming points for a reward, which may originate from the anticipation of a specific usage situation related to the redemption (e.g., a decision to redeem points for a visit to an amusement park that reinforces the subsequent motivation for utilizing the program). Dhar, Huber, & Khan (2007) found support for a similar shopping momentum effect where the propensity of subsequent purchases is enhanced merely by an initial decision to purchase.
Once the redemption decision has been made by a member, the actual redemption will typically follow within a short period of time. At the redemption event, the redemption momentum may still exist because customers can make a decision to redeem a reward in the same week as when s/he makes a purchase, or even during the purchase trip itself. After the redemption event, the post-reward effects may enhance behavior, like elaborated earlier.

Previous discussion outlines the impact of redemption decision on purchase behavior. However, purchase behavior may also affect the likelihood of redemption. Since points are directly related to purchases in an LP, obtaining points and bolstering one’s balance increases awareness of the LP, i.e., increases the accessibility of the LP in memory (cf. Higgins, 1989). This in turn increases the likelihood of redeeming one’s collected points. At each points-saving event, the LP becomes more mentally represented (accessible) since the customer is reminded of the LP. If the LP is accessible in members’ minds, a positive redemption decision becomes more likely. In case a member does not make a purchase in a particular week, and therefore does not obtain LP points, the mental accessibility of the LP decreases. In summary, purchase behavior (evident in the balance of points) increases the probability that a redemption decision will be made, which may in turn lead to the redemption momentum effects on subsequent purchases, as illustrated in Figure 1.

4. Data Description

4.1 Loyalty Program Description

The data for our study are derived from a nationwide coalition LP in the Netherlands. Program members can collect points by purchasing at more than ten LP partners, including both online and offline retailers, as well as service providers. Participating vendors function in the grocery retail, gas retail, insurance, and travel agency industries, among other sectors. The number of points awarded reflects spending amounts, and one LP point equates on average to
a euro spent. Given that we are primarily interested in insights at the LP level (i.e., interplay between redemption and purchase behavior within the LP rather than for individual vendors), we aggregate points saved and redeemed across LP vendors.

The LP provider runs periodic promotions in order to allow members to collect additional LP points or to encourage them to redeem the promoted awards. The promotions are personalized and mailed to members, highlighting accumulated points and promotional offers.

Members can redeem points for a variety of awards, ranging from kitchen utensils to travel and holidays. Therefore, the available redemption options are very heterogeneous and range from very small amounts to large awards like holiday packages. At any time, LP members can decide to redeem any amount from their accumulated balance of points to obtain rewards. Collected points do not expire.

4.2 Data and Descriptive Statistics

We analyze longitudinal weekly data on members’ collection of loyalty points and redemptions over the course of three and a half years (184 weeks). The weekly purchase behavior reflects the number of points collected, aggregated across LP vendors per member. The LP membership card provides information on socio-demographic characteristics (age and household income) and the date that each member joined the LP. The final sample contains information on the behavior of 3,094 LP members over 184 weeks. Selected members have to show at least 30 purchases and at least one redemption within the observation period. The first 10 weeks are used to initialize some dynamic variables. To initialize the post-reward variable, we also make use of redemption data prior to the start of our estimation sample. In fact, we

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2 Points are not vendor-specific and so redemption does not depend on members saving points from a particular vendor. Also, the coalition LP does not include competitors among vendors from the same industry (it rather has complementary vendors), so points saving at one vendor does not attenuate purchases at other vendors.
have information on redemptions up to 560 weeks before the start of our sample. Such prior data is not available for purchases.

On average, LP members made 0.72 purchases with the LP card per week and redeemed rewards once every 10 months (42 weeks). On average, members received 0.59 mailings per week (ranging from 0 to 2 per week across members). An average member has participated in the LP for more than 11 years, is 49 years of age, and earns a disposable annual income close to the average for the Netherlands (€17,000; Statistical Yearbook of the Netherlands, 2009).

Remarkably, there is a large variation in the number of points redeemed at a particular redemption. Although the majority of rewards obtained are worth less than 5,000 points, the right-hand tail of the distribution reaches up to 60,000 points. Figure 2 depicts the frequency distributions of the redemption amounts conditional on the amount being less than 1,000; and conditional on the amount being between 1,000 and 5,000. The figure shows large variability in the selected (internal) redemption thresholds among LP members and yet it also indicates that certain amounts are much more common than others.

<INSERT FIGURE 2 and 3 HERE>

The interplay between the redemption amounts and the available points for redemption (balance) at the redemption occasion is critical to understanding the (theoretical) drivers of pre- and post-rewarding effects. In Figure 3 we compare the empirical distribution of the redemption amount to the distribution of the number of points available at each moment in time across all members. Note that the horizontal axis has a log-scale. The distribution of the number of available points is clearly to the right of the distribution of the redemption amounts. Further investigation shows that, on average, a member spends 26% of his/her balance of points upon redemption. In only 3% of the cases is more than 90% of the accumulated balance spent. Therefore, in almost all redemption occasions, redeemers utilize
much fewer points than they have at their disposal. This indicates that possible purchase acceleration in the pre-reward periods cannot occur purely due to the lack of points needed for the redemption. Theoretically, if the points pressure effect is driven by the urge to accumulate a “sufficient” amount of points to redeem the reward, our data suggest that in 97 percent of observed redemption cases, the theoretical arguments of “points pressure” and “sunk costs” are not applicable or at least insufficient explanations.

Before specifying the model, we provide some model-free evidence of the presence of reward effects. Figure 4 shows the average points-saving behavior in the periods close to a redemption. The graph clearly shows an increase in average purchase behavior as redemption approaches and that behavior after the redemption stays at higher levels than average for one to two weeks. In this way this figure clearly shows the existence of pre- and post-reward effects in our LP.

5. Model

5.1 Model Specification

In this section, we model the members’ redemption and purchase decisions. We denote the number of saved LP points\(^3\) in purchases by individual \(i\) in week \(t\) as \(S_{it}\); the number of redeemed points in week \(t\) by the same individual \(i\) is denoted by \(R_{it}\). Both actions are related to the number of loyalty points that a member \(i\) has in the beginning of week \(t\) (the balance of points), which is denoted as \(B_{it}\). Given the redemptions, purchases and the number of points at the beginning of the week, we can calculate the number of points at the end of the week. If a member returns a purchase to the store, the balance will be corrected accordingly. We denote

\(^3\) Given that LP points-saving is directly related to purchase behavior, we refer to points savings as purchases.
this correction by $C_{it}$. We do not model these returns, but we incorporate them in the calculation of the number of points. The updating equation for the number of points becomes:

$$B_{i,t+1} = B_{i,t} + S_{i,t} - R_{i,t} - C_{i,t}. \tag{1}$$

To address the possible bi-directional dependence (endogeneity) between purchases and redemption, we explicitly model the sequence of decision-making (as outlined in Figure 1). The moment in time when a positive redemption decision is made may not coincide with the actual moment in time when the redemption incidence occurs. However, as researchers, we only observe the actual occurrence of the redemption; the timing of the decision is unobserved. The redemption decision may be made at any moment in week $t$. Once a member has planned a redemption, we assume that (s)he will not consider planning another redemption until the redemption actually happens. Next, the member decides whether to make a purchase at a participating store and use the loyalty card. Conditional on the purchase incidence, the member decides on the purchase amount. If the member decided to redeem in week $t$, (s)he finally decides on the redemption amount at the redemption incidence. As an illustration, a redemption (incidence and amount) that occurs in week $t$ may be the result of a redemption decision at time $t-2$ (two weeks before the redemption incidence). The purchases that occurred between those two events (in weeks $t$, $t-1$ and $t-2$) will be affected by the redemption decision from week $t-2$. This impact on purchases before the redemption incidence contributes to the pre-reward effect. In fact, it is the shape of the pre-reward effect that identifies the redemption timing decision (see also Figure 4). Note that by making the timing of the redemption decision endogenous, our assumptions of the order of decisions become less restrictive than they may seem at first. Although the redemption incidence and amount are placed last in the sequence of decisions, the redemption decision may have actually happened before the purchase decisions. However, we do not impose this. A member
could also decide to redeem at the point-of-sale. In this case, we allow for a potential impact of redemptions in week $t$ on the purchases in the same week.

5.2 Operationalization and Modeling of Main Dependent Variables

We introduce four main dependent variables: purchase behavior is analyzed through purchase incidence and purchase amount, while redemption behavior is analyzed through redemption decision and the redemption fraction (amount redeemed from the total balance).

We model purchase behavior with a hurdle or two-part model (Cragg, 1971; see Cameron & Trivedi, 2005 for a textbook treatment). In this model, the decision to purchase is modeled separately from the purchase amount. In other words, we model the log points savings amount conditional on the points-savings incidence. The log transformation on the purchase amounts ascertains that purchase amounts remain positive. The redemption decision is modeled using a standard probit model. Then, conditional on redemption incidence, we model the logit transformation for the redeemed fraction of the available number of points. This transformation ensures that the redemption amount is bounded by zero and the number of available points. Note that the number of points that can be spent in week $t$ equals the initial number of points plus the saved points in that week. The redemption fraction is given by

$$f_{it} = \frac{R_{it}}{B_{it} + S_{it} + 1},$$

where we add 1 to the number of points available to ensure that the logit transformation of $f_{it}$ exists even if all available points are redeemed, that is, $R_{it} = B_{it} + S_{it}$.

In the section below, we first discuss our modelling approach for determining the impact of redemption on purchases; afterward, we specify the purchase equations. As explained earlier, members may make a redemption decision ahead of the actual redemption incidence. The model for the timing of redemption decisions by member $i$ at time $t$ consists of
two parts. First, we use a probit model to describe whether a new redemption is planned at a particular point in time. This probit model is described in terms of a latent variable $RD_{it}^*$, which symbolizes redemption decision. Next, in case a redemption is planned (i.e., a member has made a decision to redeem in a future), we model the time until the redemption incidence; this time is denoted by $k_{it}$. The two variables $RD_{it}^*$ and $k_{it}$ together completely describe the redemption incidence. To summarize the member’s position in the redemption process, we introduce the redemption timing variable ($RT_{it}$). $RT_{it}$ can take on the following finite set of values $RT_{it} \in \{-1, 0, 1, \ldots, m\}$. If $RT_{it}=-1$, no redemption is planned for the near future. If $RT_{it} \neq -1$, the variable gives the number of purchase opportunities until the next redemption event (counting from the beginning of the week). Hence, if $RT_{it}=1$ or $RT_{it}=0$, the redemption occurs in week $t$ itself. In the former case, the decision to redeem was made before the purchases were made in this week; in the latter case, the decision was made after the purchase. Note that in the case where $RT_{it}=0$, the timing of the redemption decision does not induce a pre-reward effect, because the decision to redeem occurs after the purchase. Finally, when $RT_{it}>1$, a redemption event will occur in the near future, e.g., if $RT_{it}=2$, the redemption happens in the next week. In this way $RT_{it}$ summarizes the decisions that member $i$ has made, the likes of which may impact current or future behavior. Of course, the variable $RT$ can only be partly observed. For example, if no redemption occurs at time $t$ for individual $i$, we know that $RT_{it}$ does not equal 0 or 1 and that $RT_{i,t-1}$ does not equal 2, etc. However, the exact timing of each redemption decision remains unobserved. Therefore, $RT_{it}$ should be seen as a latent variable.

The dynamic process for $RT_{it}$ can be formally represented by

$$RT_{it} = \begin{cases} RT_{i,t-1} - 1 & \text{if there is a previously planned redemption } (RT_{i,t-1} > 1) \\ k_{it} & \text{if a future redemption is planned now } (RT_{i,t-1} \leq 1 \text{ and } RD_{it}^* > 0) \\ -1 & \text{if no future redemption is planned } (RT_{i,t-1} \leq 1 \text{ and } RD_{it}^* \leq 0) \end{cases}$$

The first line in (3) corresponds to the case where a redemption was already planned at (or before) $t-1$, so the time until the redemption incidence needs to be updated by reducing it by 1.
The second and third lines correspond to the case where a new redemption could be planned (i.e., a redemption incidence occurred in the previous week or no redemption was planned before; both cases correspond to the condition $RT_{i,t-1} \leq 1$). This decision is governed by the latent variable $RD_{it}^*$ (redemption decision). A new redemption will be planned if $RD_{it}^* > 0$, whereas no new redemption will be planned if $RD_{it}^* \leq 0$. In case of a positive redemption decision, the variable $k_0$ gives the number of purchase occasions until the redemption and it is modeled as a draw from the set of numbers $0,1,\ldots,m$, with probabilities $\pi_0, \pi_1, \ldots, \pi_m$. The number $m$ will be relatively small; based on the model-free evidence of the pre-reward effect, we expect $m$ to equal 2 or 3 at most.

The redemption decision is modeled by a probit model. The latent redemption decision $(RD_{it}^*)$ variable therefore follows

$$RD_{it}^* = \mu_{i1} + \gamma_{i1} t + Z_{it}^R \beta_{i1} + W_t \delta_1 + \xi_{it}, \text{ with } \xi_{it} \sim N(0,1),$$

where

$$Z_{it}^R = \left( \begin{array}{c} \log B_{it} \\ PntPre_{it} \\ PostRed_{it} \\ Access_{it} \\ Mailings_{it} \end{array} \right).$$

In this vector of explanatory variables, $B_{it}$ gives the balance at the start of week $t$, while $PntPre_{it}, PostRed_{it}, Access_{it}$ and $Mailings_{it}$ respectively give the points pressure (for eligible members), post-reward effect following a redemption incidence, accessibility of the LP due to purchases, and mailing decay variables. The exact operationalization of these variables will be discussed later. Finally, the variable $t$ denotes a time trend and $W_t$ captures seasonal dummies. For each member the time trend is defined relative to the moment at which the member subscribed to the LP. This variable therefore captures the length of the membership in the LP.
The logit transformed redemption fraction is modeled as

$$\log(\frac{f_{it}}{1-f_{it}}) = \mu_{i2} + \gamma_{i2}t + Z_{it}^R\beta_{i2} + W_{it}'\delta_2 + \nu_{it},$$

for all $t$ where $RT_{it} = 0$ or $1$,

with $\nu_{it} \sim N(0, \sigma_{R,i}^2)$.

To model purchases, we denote model purchase (points-saving) incidence by the binary variable $SI_{it}$. This variable is modeled using a probit model, that is,

$$SI_{it} = \begin{cases} 0 & \text{if } SI_{it}' \leq 0 \\ 1 & \text{if } SI_{it}' > 0, \end{cases}$$

with

$$SI_{it}' = \mu_{i3} + \gamma_{i3}t + Z_{it}^S\beta_{i3} + W_{it}'\delta_3 + \varepsilon_{it},$$

with $\varepsilon_{it} \sim N(0,1)$, (7)

where

$$Z_{it}^S = \begin{pmatrix} I(RT_{it} \geq 1) \\ \log B_{it} \\ PntPre_{it} \\ PostRed_{it} \\ Access_{it} \\ Mailings_{it} \end{pmatrix}.$$ (7a)

The first row of this vector gives the pre-reward effect due to redemption momentum (as an indicator related to the previously specified $RT_{it}$); the other rows correspond to the variables used in (4a). The indicator in the first row equals 1 if a redemption was planned before the focal purchase decision, which would allow for redemption momentum to occur. The corresponding parameter measures the impact of having made the decision to redeem on the purchase incidence.

Based on the purchase incidence ($SI_{it}=1$), we model the purchase (points-saving) amount as

$$\log S_{it} = \mu_{i4} + \gamma_{i4}t + Z_{it}^S\beta_{i4} + W_{it}'\delta_4 + \eta_{it},$$

for all $t$ where $SI_{it} = 1$,

with $\eta_{it} \sim N(0, \sigma_{S,i}^2)$. 

20
The complete set of heterogeneous parameters is related to member-specific explanatory variables ($V_i$) such as the individual’s age, income and membership duration at the start of the dataset. Denote $\theta_i = (\mu_{i1}, \mu_{i2}, \mu_{i3}, \mu_{i4}, \gamma_{i1}, \gamma_{i2}, \gamma_{i3}, \gamma_{i4})'$ and $\beta_i = (\beta_{i1}', \beta_{i2}', \beta_{i3}', \beta_{i4}')'$. The vector $\theta_i$ contains all member-specific intercepts and member-specific trends; for this vector we specify a model including random effects, that is,

$$\theta_i = \Gamma_1 V_i + \omega_i,$$

where $\omega_i \sim N(0, \Omega)$. For parsimony, we do not include random effects for the parameters in $\beta_i$ and we set $\beta_i = \Gamma_2 V_i$. In other words, we include interaction effects between the variables in $Z_i$ and those in $V_i$. Therefore, the heterogeneity in $\beta_i$ is only related to observed characteristics. For the ease of interpretation, we have standardized all moderating variables in $V_i$ to have mean 0 and variance 1.

In the purchase and redemption equations above, we have introduced four error terms. The two error terms in the purchase (or redemption) equations are assumed to be independent. In principle, a correlation between the two errors can be specified; such a correlation is often included in sample selection models. In these cases, there is usually a separate process that determines whether an observation is sampled—for example, if someone participated in a job training program, then including the correlation would allow one to draw conclusions regarding the potential impact of the training program on those who decided to forgo the training. Unlike that setting, however, behavior within the LP program is not susceptible to sample selection and represents a corner-solution model (Wooldridge, 2011). Corner-solution (two-part) models separately describe the incidence and the amount conditional on incidence. The error terms in both equations are usually assumed to be independent (see the discussion in Wooldridge, 2011, p. 691). In theory, an identified correlation would quantify the impact that unobserved factors may jointly have on the incidence and quantity decision, but in practice such a correlation is usually very difficult to estimate without imposing exclusion restrictions.
However, models in practice often yield similar insights with or without a correlation—see for example, Madden (2008) and Konus, Neslin, & Verhoef (2014).

Another possible correlation is the one between the redemption decisions and the purchase decisions. This correlation would capture the impact that unobserved events may have on redemption and purchase decisions simultaneously. However, there are already three processes in the model that link redemption decisions to purchase decisions: (i) All decisions are tied together through the balance variable: one cannot redeem points that were not saved; (ii) The decision to redeem may precede the actual redemption moment and this has an impact on purchase behavior (redemption momentum); (iii) We allow for correlated, individual-specific parameters. The latter link captures certain patterns—for example, that members who purchase a lot may also redeem often. Furthermore, for each individual there tends to be only a few redemptions. This circumstance, together with the rich dependence between redemption and purchases that is already in the model, leaves little scope for estimating additional correlations.

In tandem with members’ intertwined purchase and redemption decisions, LPs usually send mailings to a selected group of members to encourage redemption and purchase. In other words, these mailings may be endogenous. Without correction, this may lead to biased estimates (Franses, 2005). For example, if the mailings are sent to those who are likely to purchase, we would overestimate the impact of the mailings on purchase. From discussions with the LP manager, we know that only the frequency of the mailings is endogenous; its timing is not set based on individual behavior. This observation allows us to easily correct for the endogeneity, namely by including the average number of mailings received in the vector $V_i$ (see Mundlak, 1978 and Risselada, Verhoef, & Bijmolt, 2014 for an application in marketing). By doing so we identify the true impact of mailings on the redemption and purchase decisions. Note that the parameters in $\Gamma_1$ and $\Gamma_2$ related to the number of mailings
should not be seen as measuring the causal impact of mailings. In most cases, these parameters will mainly provide information on how the mailing strategy is set.

5.3 Operationalization of Main Explanatory Factors

In this subsection, we discuss how we operationalize some of our main explanatory factors. We acknowledge that pre-reward effects may occur through the points-pressure effect for those members who have an insufficient balance for their preferred redemption amount. The points-pressure effect is the result of members’ internal redemption thresholds, which are based on the members’ preferences for the available awards. If the points pressure is active, then the member is closer to a threshold, and thus (s)he is more inclined to wait and save points until the threshold is reached. However, these preferences, and by extension their thresholds, are not observed. In our LP, there is a reward available for almost every number of points; nonetheless, some common redemption thresholds can be observed across all redemptions. We therefore operationalize the internal thresholds using the most common amounts of points spent across the entire population (see Figure 2). In our specification, we used all redemption amounts that occur more than 200 times in our sample.

We next specify the points-pressure effect as a function of the relative distance between the current balance and the next redemption threshold, that is,

\[ PntPre_{it} = \begin{cases} 
0 & \text{if } B_{it} > \tau_K \\
\left( \frac{B_{it} - \tau_{k-1}}{\tau_k - \tau_{k-1}} \right)^\alpha & \text{if } \tau_{k-1} \leq B_{it} < \tau_k, \text{ for } k = 1, \ldots, K, 
\end{cases} \]

where \( \tau_k, k=1,\ldots,K \) denote the internal thresholds. Given that we aim to explore the shape and duration of the points-pressure effect, we specify the shape of the effect using parameter \( \alpha > 0 \). If \( \alpha > 1 \), the points-pressure effect starts relatively close to the redemption threshold. If \( \alpha < 1 \), the points-pressure effect starts relatively early.
The post-reward effect following from the redemption incidence may influence members’ purchases. The post-reward effect potentially lasts for a number of weeks after the redemption incidence. In our model, we capture this effect using an exponentially weighted average of lagged redemption, that is,

\[ \text{PostRed}_{it} = R_{i,t-1} + \lambda_1 \text{PostRed}_{i,t-1}, \]  

where \( R_{it} \) denotes a redemption incidence indicator, and \( 0 \leq \lambda_1 \leq 1 \) gives the decay rate of the post-reward effect. We use data before the start of our estimation sample to initialize this post-reward variable.

In line with our conceptual model, the mental accessibility of the LP due to prior purchases is operationalized as an exponential decay of purchase incidence (i.e., stock of purchases), that is,

\[ \text{Access}_{it} = S_{i,t-1} + \lambda_2 \text{Access}_{i,t-1}, \]  

where, as before, the parameter \( 0 \leq \lambda_2 \leq 1 \) controls the decay rate. The notion that periods with increased purchases may enhance the accessibility of the LP and thereby produce a spillover effect on behavior is in line with the literature on RFM models, direct mailings, and decay effects in both advertising recall and purchase history in household scanner data (Leone, 1995; Gönül, Kim, & Shi, 2000).

Finally, we include the dynamic impact of mailings sent to the members by an exponentially weighted average of current and past mailings, that is,

\[ \text{Mailings}_{it} = M_{it} + \lambda_3 \text{Mailings}_{i,t-1}, \]  

where \( M_{it} = 1 \) if member \( i \) received a mailing in week \( t \). Like before, we use pre-sample information to initialize this variable.

5.4 Parameter Estimation
We opt for Bayesian techniques for parameter estimation, as our model is highly nonlinear and contains many member-specific latent variables. More specifically, we use Markov chain Monte Carlo [MCMC] sampling, where we combine Gibbs sampling and Metropolis Hastings [MH] sampling. We sample the latent variables $RT_{it}$, $RD^*_{it}$, $k_{it}$, and $SI^*_{it}$ alongside the other model parameters. The estimated parameters include the decay rates $\lambda_1$, $\lambda_2$, $\lambda_3$ as well as the probabilities $\pi_0, \pi_1, \ldots, \pi_m$ that determine the time between a redemption decision and the actual redemption incidence. In the technical appendix we present the details of our sampler\textsuperscript{4}.

We generated 60,000 draws from the Markov Chain and removed the first 20,000 draws as a burn-in period. Of the remaining draws, we retained every 5th draw to reduce autocorrelation. As discussed before, we set the thresholds $\tau_k$ equal to all unique redemption amounts that occur more than 200 times in our sample. Using this rule we set the thresholds to 100, 200, 300, ..., 1,000, 1,200, 1,500, 2,000, and 3,000.\textsuperscript{5} Finally, we set $m=2$. This limits the pre-redemption effects to a maximum of two weeks before the redemption incidence. This choice is mainly motivated by Figure 4. However, we have also considered a model with $m=3$ and found no substantive difference with the presented results\textsuperscript{6}.

6. Results

We first consider a model with only the main effects (including correction for endogeneity of mailings) and then consider the full model that accounts for interactions. The estimation results for both models are presented in Table 2. The main effects are very robust, and the overall effects stay the same even after controlling for moderating variables.

\textsuperscript{4} We checked the performance (and implementation) using a simulation experiment. We generated data using known parameters and tested whether the estimation procedure is able to retrieve these parameters. The MCMC sampler proves to perform well. Details and results of this experiment are provided in the Web appendix.

\textsuperscript{5} As a robustness check, we have considered models with higher and lower numbers of redemption thresholds. The obtained results are very robust. Therefore, we choose to present the option that includes the majority of commonly selected thresholds and avoids three thresholds chosen relatively frequently, but less than 200 times (1,400, 1,700, and 2,400 points).

\textsuperscript{6} More specifically, in the model with $m=3$, the estimated value for $\pi_3$ was close to 0. This effectively reduces the model to a model with $m=2$. 

25
In the discussion below, we differentiate between the effects on purchase (LP points-saving) incidence, purchase amount, redemption decision/timing and redemption amount/fraction. This provides a fruitful environment for discussing the diverse mechanisms underlying the relationship between redemption and purchase behavior. Whenever we discuss a particular parameter estimate from Table 2, we present the posterior mean and refer to it as $\gamma$; if necessary we add a subscript that refers to a particular model component. Note that there are separate coefficients for all four decisions. These parameters are all part of the matrices $\Gamma_1$ or $\Gamma_2$; see equation (9).

6.1 Timing of the Redemption Decision

The starting point in analyzing the interplay between the decision to redeem points and purchase behavior is understanding whether a redemption decision precedes purchase or vice versa (i.e., the redemption occurs as a consequence of increased purchases in some period). We find that in an overwhelming majority of redemptions (around 70%), the redemption decision is made before the purchase decision. In other words, approximately 31% of redemption decisions are made at the point of redemption: members decide to redeem ad hoc and do so immediately. In the model, this percentage is represented by $\pi_0$, which indicates the proportion of members for whom the purchase decision (in the same week when redemption occurs) is not affected by the redemption decision. Sixty-four percent of redemptions are planned ahead in the same week: customers go to a store, make a purchase and then redeem their points ($\pi_1$). At this point, redemption momentum exists because the decision to redeem may still affect the purchase. Around 6% of redemptions are planned a full week ahead and subsequently affect purchase behavior until the redemption event ($\pi_2$). We emphasize that this LP is used on a weekly basis (groceries, etc.), which adds face validity to these estimates.
6.2 Pre-reward Effects

We find support for the existence of pre-reward effects even when members are not “pressured” with points expiration. Positive pre-reward effects are driven both by the points-pressure effect for members with insufficient balances and the redemption momentum that goes beyond the points pressure. In terms of effect size, the redemption momentum is the most important pre-reward effect (based on the evidence presented in Table 2 and in Section 7).

6.2.1 Points Pressure Effects on Purchase and Redemption Behavior

For approximately 3% of members who may have experienced points pressure before the redemption, there is an average increase in the likelihood of purchase ($\gamma_{PntPre} = 0.053$). The points-pressure effect starts early after passing a previous threshold (the posterior mean for log $\alpha = -2.492$, which corresponds to $\alpha = 0.083$). However, points pressure primarily affects purchase incidence, not the purchase amount ($\gamma_{PntPre}$ for purchase amount is not significant).

As members approach the next available internal threshold, they become less likely to redeem ($\gamma_{PntPre} = -0.086$). This negative effect is expected since members likely postpone redemption until they pass the threshold. This also reinforces the notion that members are driven by an internally set threshold behavior and redeem rewards after reaching this internal threshold. Accordingly, when approaching a redemption threshold, the redeemed amount tends to be a smaller fraction of the total balance ($\gamma_{PntPre} = -0.131$). In other words, if members do decide to redeem before the threshold, they redeem a smaller part of their balance.

6.2.2 Redemption Momentum Effects

As mentioned before, the effects of redemption go beyond the points-pressure effect; the mere decision to redeem a reward affects members’ subsequent purchase behavior (creating the
redemption momentum). When the decision to redeem a reward occurs before the actual redemption (for 69.5% of members), members increase their frequency of purchase ($\gamma_{\text{RedMom}} = 1.763$) as well as their purchase amounts ($\gamma_{\text{RedMom}} = .325$) in periods between the redemption decision and the redemption event. As expected, the redemption follows shortly after members make the decision to redeem. Hence, the pre-reward effect due to redemption momentum stretches to a maximum of one week before the redemption (evident from $\pi_0$, $\pi_1$, and $\pi_2$ estimates discussed in 6.1).

6.3 Post-reward Effects

In the post-reward periods, members tend to purchase more often ($\gamma_{\text{PostRed}} = .033$) and they increase their purchase amounts per purchase ($\gamma_{\text{PostRed}} = .031$). We thus provide empirical support for positive post-reward effects in the continuous reward setting. The estimated redemption decay parameter in post-reward periods is $\lambda_1 = .734$. The impact of redemption therefore lasts relatively long after the redemption. The post-reward effect is maximal in the week after the redemption; the effect reduced to 73.4% two weeks later, to 53.9% three weeks later ($0.734^2$), and so on.

On average, post-reward effects have a positive impact on the subsequent likelihood of redeeming, since the impact of post-reward effects on the subsequent decision to redeem is positive and significant ($\gamma_{\text{PostRed}} = .227$). However, a non-significant, negative post-reward effect is found on redemption fraction ($\gamma_{\text{PostRed}} = -.023$).

6.4 Purchase Behavior Reinforces Redemption

Our conceptual model proposes that increased purchases in a certain period may encourage members to make the decision to redeem, since purchasing increases the mental accessibility of the LP due to prior purchases. As the members’ average “stock-of-purchases” increases, it
reinforces purchase frequency and spending amounts ($\gamma_{Access} = .282$ and .064, respectively). This increase also boosts the likelihood of the redemption decision ($\gamma_{Access} = .039$), but if members decide to redeem, it reinforces the redemption of smaller fractions (rather than a redemption of all/majority of collected points) ($\gamma_{Access} = -.057$). This finding suggests that LP members become more cognizant of the ability to redeem their collected points as a result of purchases enhancing the LP’s mental saliency; however, relatively larger redemptions would be planned ahead. In addition, the estimated accessibility decay parameter between purchases is .847, which indicates that the decay in accessibility between two purchase incidents is slow. In other words, members slowly forget about the LP if they do not use it.

6.5 Trends and Moderating Effects

The simultaneous estimation of the four dependent variables allows us to assess the associations between the individual-specific effects arising from the four purchase and redemption responses. The correlations between the eight individual-specific effects (four intercepts plus four trends) are presented in Table 3. Combining those results with the results of the moderating effects presented in Table 2 reveals interesting trends for LP managers. We discuss these insights below.

6.5.1 Decreasing Responsiveness to the LP

Findings in Table 3 reveal important concerns for LP managers due to the strong negative correlation between the baseline effects and the trend for purchase and redemption behavior. LP members with a high purchase propensity (frequent buyers) tend to decrease their purchase incidence over time ($\rho(SI_{base}, SI_{trend}) = -.684$). The same holds for purchase amount ($\rho(logS_{base}, logS_{trend}) = -.582$). Similar negative correlations are observed in the redemption incidence and redemption amount parameters. The propensity to redeem is likely to decrease
over time for those who, at first, decide to redeem relatively often ($\rho(RD_{base}, RD_{trend}) = -0.708$).
But even more, the slope of the trend in the redemption fraction is negatively correlated with
the base redemption fraction ($\rho(\log R_{base}, \log R_{trend}) = -0.537$). This implies that, with time, high-
baseline members become less likely to redeem, and even if they decide to redeem, their
redemption amount also decreases over time. In other words, there is a mean-reversion
process. Members who are initially very active become less active over time (and vice versa).

<INSERT TABLE 3 HERE>

An analysis of the moderating effects in Table 2 (full model) further supports the
finding of negative trends in purchase frequency and amount over time for all members ($\gamma$’s
are -0.071 and -0.252, respectively). The decline in purchase responsiveness to the LP over time
is particularly pronounced among older members and long-term loyal members. Spending
patterns worsen for those groups even more than for an average member: older members
show stronger declining trends both in purchase frequency and amount (coefficients -0.032 and
-0.016, respectively); meanwhile, long-term members particularly decrease their likelihood of
purchase more so than their purchase amounts (coefficients -0.021 and 0.034, respectively).

6.5.2 Moderating Impact on Pre- and Post-reward Effects

Overall, we find strong heterogeneity in the baseline purchase and redemption behavior of LP
members (given the relatively large variances in the baseline estimates for all dependent
variables reported in Table 2). Importantly, long-term members seem to be less responsive to
LP mechanisms. Points pressure, accumulated balance and prior purchases have less impact
on the purchase incidence and redemption decision of long-term members, since the positive
main effects of these variables are negatively moderated by the number of years as an LP
member ($\gamma_{PntPre\cdotMemberYrs} = -0.022$ (purchase incidence) and -0.037 (redemption decision);
$\gamma_{Access\cdotMemberYrs} = -0.020$ (purchase incidence) but positive 0.004 (purchase amount;
In the post-reward periods, rewarded behavior has less positive effects on purchase amounts for long-term members relative to others ($\gamma_{\text{PostRed} \times \text{MemberYrs}} = -0.016$).

In addition, long-term members show a more rational redemption behavior once they decide to redeem. Long-term members are even less likely than others to redeem just before reaching the preferred threshold ($\gamma_{\text{PntPre} \times \text{MemberYrs}} = -0.037$) and even if they do, their redemption amounts tend to be a smaller fraction of their total accumulated balance ($\gamma_{\text{PntPre} \times \text{MemberYrs}} = -0.107$). Also, the amount accumulated in the balance does not increase the likelihood of redemption and its amount ($\gamma_{\text{Balance} \times \text{MemberYrs}} = -0.017$ and -0.062). These results may be explained by long-term members’ experience in the LP.

We also observe a positive moderating effect of age. The redemption momentum is stronger for older members, as the redemption momentum increases both their purchase frequency ($\gamma_{\text{RedMom} \times \text{Age}}$) and spending amounts ($\gamma_{\text{RedMom} \times \text{Age}} = 0.026$). In contrast, the redemption momentum is weaker for higher-income members ($\gamma_{\text{RedMom} \times \text{Income}} = -0.021$). Both age and income reinforce the impact of post-reward effect on the likelihood of a new redemption ($\gamma_{\text{PostRed} \times \text{Age}} = 0.038$ and $\gamma_{\text{PostRed} \times \text{Income}} = 0.035$).

6.5.3 Mailing Effects

The impact of mailings appears in two distinct manners in the model. First, there is the direct impact of mailings through the mailing decay variable. Second, there is the moderating impact of the average number of mailings that a member received. The impact of the latter variable on the baselines is likely attributable to the LP’s target selection. Our results show that those members who purchase frequently and in higher amounts tend to receive more mailings (estimated coefficients are 0.061 and 0.104 in the full model, respectively). But conversely, more frequent redeemers receive fewer mailings on average (coefficient equals -0.078).
The estimated mailing decay parameter of equation (12) equals $\lambda_2 = 0.743$. The impact of a mailing is strongest in the week when the mailing is received, while in the second week the carry-over effects reduce to 74.3%, and three weeks later they reduce to 55.2%, and so on. This weekly decay parameter is in line with previously reported decay parameters on advertising effects (Clarke, 1976; Leone, 1995) and direct mailing effects (van Diepen, Donkers, & Franses, 2009a).

Overall, mailings have a direct positive impact on purchase incidence ($\gamma_{Mail} = .022$) and amount ($\gamma_{Mail} = .003$). The effect is marginally larger for long-term members ($\gamma_{Mail*MemberYrs} = .002$). Furthermore, mailings seem to encourage redemption for older members ($\gamma_{Mail*Age} = .006$). The impact of mailings on the purchase likelihood is marginally enhanced for high-income members ($\gamma_{Mail*Income} = .003$), but it is negatively moderated by the total number of mailings received ($\beta_{MailsReceived} = -.012$). Therefore, the effectiveness of mailings in encouraging purchase and redemption declines as the number of mailings increase ($\gamma_{Mail*MailsReceived} = -.012$ and -.020, respectively).

7. Effect Size Simulations

To further analyze the impact of redemption on purchase behavior, we conducted a series of simulations in which the behavior of an average member is repeatedly generated. The effects of different model components are analyzed by switching off one component at a time (i.e., setting its parameter to zero). Only for the impact of accessibility due to purchases (stock-of-purchases) do we set the accessibility to its average value over time. Given the high frequency of purchases, the accessibility variable is always much larger than zero.

For each scenario, we analyzed the average purchase behavior around the moment of redemption. In addition, we calculated the average purchases around the moment of redemption for those redemptions where the balance is below our highest points-pressure
threshold. In this way we fully explore the differences between the points-pressure and redemption momentum mechanisms in the pre-reward effects. The findings from these simulations are illustrated in Figure 5.

Looking at the difference between the base scenario (our full model) and the effects without points pressure, we can see that the points pressure presents a rather small and limited contribution to the overall rewarding effects. The same conclusion holds when we analyze the average behavior of members whose balance is below the highest points-pressure thresholds. Overall, although there does seem to be a significant points-pressure effect, we find its effect size to be relatively small. One explanation is that the large majority of members have a large balance of points at the time of redemption (more than sufficient for their redemption). However, even when that is not the case, the magnitude of the effect is still relatively small.

The redemption momentum clearly has the largest impact. The largest part of the peak in purchases at and before the redemption moment can be attributed to this pre-redemption effect beyond points pressure. We therefore posit that the mere decision to redeem triggers a substantial increase in purchase behavior among LP members.

The post-reward effect is significant in the model and the simulations show its substantial impact in the periods after the redemption. Our simulations indicate that post-reward effects limit the potential dip after obtaining a reward, particularly in situations where customers have a lower balance than the highest threshold.

Finally, we recommend caution when interpreting the effects in Figure 5 that relate to the results labelled ‘without the accessibility of prior purchases.’ Note that the effects derived from switching off a model component are complex; all current decisions are connected to all future decisions through the accumulated balance. This also explains why the overall purchase levels are substantially lower when accessibility of prior purchases is switched off (i.e.,
accessibility is set to an average level). Moreover, the effects go beyond just purchase behavior: the model components also affect the redemption decision itself. For example, by switching off points pressure, we also observe a slight increase in redemption frequency (and slightly larger redemptions) at low balances. This in turn may reduce the average balance that members have, leading to less frequent purchases. So in order to understand (and explain) all effects, one needs to take the entire model into account.

8. Discussion

This study aims to better understand the LP members’ reward-redemption behavior and its impact on purchase behavior—in particular, the behavior directly preceding and following a redemption. The study examined a typical LP with a continuous and linear rewarding structure (i.e., one point per euro spent), which is common among retailers of frequently purchased items. Importantly, in LPs without point expiry deadlines, members endogenously choose when and how much to redeem from a broad spectrum of potential reward options. Little is known regarding whether redemption effects occur when no expiration or binding policies exist. Because obtaining a reward in such LPs requires only little additional effort from members, some authors have postulated that pre-reward effects would not occur (Blattberg et al., 2008). Using an extensive data set of 3,094 members involved in such an LP, we simultaneously modeled purchase incidence, purchase amount, redemption decision and redemption amount (as a fraction of available balance). This allowed us to empirically investigate such pre-reward effects. We summarize and discuss our key findings below.

For the majority of members (approximately 70%), the decision to redeem occurs before the actual redemption and affects their subsequent purchase decisions. Therefore, the interplay between redemption and purchases occurs in this order: (i) customer makes a decision to redeem which (ii) increases the salience of the LP and its benefits and (iii)
encourages (pre-reward) purchase behavior. Once the decision to redeem is made, (iv) the redemption occurs within a short period of time (one week). After the redemption, (v) the (post-reward) purchase behavior is enhanced by the rewarded behavior effects.

Reward redemption leads to important pre-reward increases in LP members’ purchase even when they do not face points expiry or binding thresholds. The drivers of such increases go beyond points pressure. Even when the majority of LP members (97%) redeem just a (small) fraction of their overall accumulated balance at a redemption incidence, we still find strong evidence for pre-reward effects. Hence, our findings counter the notion that members’ purchase behavior prior to reward redemption is motivated solely by the points-pressure mechanism (in other words, that members only increase purchases in order to accumulate a sufficient number of points for their preferred reward). Our findings strongly emphasize the power of redeeming a reward in LPs: The decision to redeem motivates members and reinforces their subsequent behavior. Importantly, we theoretically introduce the novel concept of redemption momentum as an additional explanation for the existence of the pre-reward effects beyond points pressure. Hence, pre-reward effects are a general phenomenon driven by multiple underlying processes (e.g., goal attainment, increased LP engagement, salience), which occur for LPs with diverse designs. That said, alternative reward mechanisms (like redemption momentum) have substantially larger influence than external thresholds (points-pressure mechanism) in a context of continuous, linear LP rewarding.

The effects of redemption also enhance behavior in post-reward periods. Hence, we provide empirical support for the reinforcing effects of the rewarded behavior mechanism. Members who just redeemed a reward demonstrate a higher purchase incidence and higher purchase amounts. This finding supports the notion that redeeming rewards may create positive attitudes and feelings that drive members to purchase more frequently and obtain higher amounts of LP points even in the absence of external pressures from the firm.
(Blattberg et al., 2008; Palmatier et al., 2009). It also empirically supports and extends the findings of Kopalle et al. (2012) on the existence of the post-reward effect in a continuous LP, but for a broader range of LP members (we find that the effects on purchase behavior mostly hold across various customer groups). On the other hand, our findings counter the notion of the post-reward resetting mechanism reported by Drèze and Nunes (2011), at least for the retail setting with no LP tier structure. In this respect, our findings help to clarify equivocal empirical evidence on post-reward effects in continuous LPs.

We also found support for the reinforcing impact of previous accumulated purchases on the redemption likelihood. Prior purchases not only positively affect subsequent purchases, but they also increase the redemption probability, which speaks in favor of the increased salience. Although accumulated purchases in a certain period enhance the likelihood of redemption, they do not affect the redemption fraction.

Both the pre- and post-reward effects on purchase incidence and purchase amount substantially differ between members. An important moderator is membership length. In general, the effects on purchase and redemption behavior are less pronounced among long-term LP members. This might be due to the learning effects in tandem with the more strategic redemption behavior among such members (Lal & Bell, 2003; Liu, 2007). These members have extensive experience with the LP and they may be less prone to change their purchase behavior when they redeem an award. We also observe some interesting moderating effects of age and income, which have not been shown before.

Mailings have an overall positive impact on the purchase and redemption behaviors (both on incidence and amount of purchase and redeeming). However, the impact of mailings on purchase incidence, redemption likelihood and redeemed fraction declines with the total number of mailings received. This may indicate a worrying trend of LP members becoming increasingly unresponsive to the LP’s personalized mailings, or it may be the result of the
LP’s targeting policy (e.g., van Diepen, Donkers, & Franses, 2009b). Those members who purchase often as well as those who purchase a lot tend to receive more mailings. By contrast, those members who are more likely to redeem tend to receive relatively fewer mailings than others. Given this finding, there is possibly some untapped potential in terms of tailoring promotional strategies to increase redemption incidence.

9. Managerial Implications

Firms increasingly try to remove hurdles in their LPs to improve members’ experiences. As a result, LPs that preserve balances for a long time, or simply forgo the expiration of points and miles entirely, are increasingly common among retailers (e.g., Tesco’s Clubcard, Nectar, Airmiles), car rental agencies (e.g., Hertz Gold Rentals), hotels (e.g., Intercontinental’s Priority Club Rewards), airlines (e.g., Delta Airlines’ SkyMiles, JetBlue Airlines’ TrueBlue) and financial institutions (e.g., American Express Membership Rewards, Wells Fargo Rewards). Answering the question of whether firms should encourage redemption without imposing points expiry and binding thresholds is relevant for three main reasons. First, the lack of redemption limits the LP’s power to build and sustain loyalty due to missed opportunities to strengthen relationships and engage members (Levey, 2011). Second, the lack of redemption may lead to a potential decrease in LP involvement and diminish perceived program value over time, exacerbated by the trend of decreasing active participation in enrolled LPs (Gordon & Hlavinka, 2011). Third, the difference between issued and redeemed points has a profound impact on profit, since unredeemed points create an accounting liability for the firm (i.e., debt to members) (Levey, 2011; The Economist, 2005).

Our findings suggest that companies should actively try to encourage redemption in order to sensitize consumer responsiveness and increase the salience of the LP. This is particularly relevant given our findings on the overall trends of declining purchase and
redemption activity. Members’ accumulation of points is far above the highest common reward values, and upon redemption, members on average redeem just 26% of their available balance. Yet even in this setting, reward redemption plays an important role in increasing the purchase behavior in periods before and after the redemption.

There are two main reasons why LP managers should actively influence redemption incidence and redemption amount and thereby increase members’ engagement in the LP. Firstly, lagged purchase incidence positively affects redemption incidence, as does the accumulated balance of points. This means that encouraging purchasing also increases the probability of members redeeming a reward. Secondly, redemption incidence and fraction can be stimulated with mailings. Notably, customers who recently redeemed an award are more likely to subsequently redeem another reward, suggesting that stimulating reward redemption can be a rather powerful way to increase purchase incidence directly in the short term, as well as in the long run through increased redemption incidence. But we caution here that increasing the number of mailings also has a negative implication, as it may reduce the effectiveness of each subsequent mailing due to factors such as irritation (van Diepen et al., 2009b).

To encourage the redemption frequency of LP members, companies should consider offering a wide range of potential rewards from which members can choose (see Figure 2). On one hand, encouraging the redemption of larger amounts decreases liability for the LP provider, but on the other hand, managers may feel they are putting the company’s financial solvency at risk. However, as long as members do not decide to redeem the rewards all at the same time, firms should not experience strong problems in this respect.

Furthermore, our findings provide valuable insights on policies for managing the relationships with long-term, loyal LP members relative to more recent customers. Interestingly, long-term members are relatively more frequent purchasers (they have higher
points-saving incidence); however, they have comparably lower purchase amounts (see Table 2). Over time, though, these loyal members tend to decrease their purchase incidence even more than other members. Retailers should use this insight to design promotional strategies targeted at long-term versus more novel LP members. Long-term members are relatively (albeit marginally) more apt to increase the amount spent in response to promotional strategies. This group is therefore an important target segment for policies intending to encourage redemption. Because both pre- and post-reward effects seem to be harder to evoke among long-term members, managers are advised to carefully tailor their personalized marketing strategy to encourage redemption effects.

10. Limitations and Further Research
This study has mainly focused on the continuous types of reward structures. Our focal LP is analogous to many LPs with a continuous rewarding structure and no points expiration. Nevertheless, this study has analyzed the effects of rewarding in only one LP in one country, which limits its generalizability. Though the analyzed LP’s structure is typical of coalition LPs in other counties, some conclusions may not automatically transfer. Moreover, being interested in effects within the LP as a whole, we aggregated LP data collected across various vendors that participate in the LP. Further research could analyze the differences in reward behavior effects across individual vendors in the context of partnership LPs, as well as for sole proprietary LPs.

Our empirical analysis of reward redemption effects is limited to LP members who had redeemed at least once in the observation period. This choice may have created a selection effect (relative to non-redeemers). However, this selection was necessary to analyze reward redemption effects. In addition, we examined point collection behavior rather than the
exact amounts spent. These measures might not correspond perfectly if a member does not use his or her LP card for every purchase.

Since we could not obtain the information on the cost structure in the observed LP, we could not fully analyze the profit potential of rewarding effects. It would be beneficial to evaluate the profit implications of the rewarding effects analyzed in this study.

Our study has provided evidence that the drivers of pre-reward effects are complex and may go beyond the rational expectations of the points-pressure effect. To this end, internal reward thresholds, and especially redemption momentum, may play an important role. More in-depth theoretical evidence is required on the mechanisms that drive the effects of rewarding in continuous rewarding structures without expiry deadlines. Such an investigation would require setting up a series of experimental studies. In particular, diverse psychological drivers may exist under redemption momentum; the size of this effect and its importance in engaging LP members warrant in-depth analysis of the underlying psychological mechanisms. One aspect involves the notion that members may want to maintain their accumulated balance after redemption, which may induce them to speed up purchases in pre-reward periods.

In general, we know relatively little regarding the emotional drivers of LP behavior. Future research needs to explore the notion that deciding to redeem a reward may induce excitement about and salience of the benefits of LP membership. Arousal (excitement) and valence of feelings may both be signals for action in the LPs (cf. feelings as information theory). Unfortunately, we do not have attitudinal data that would allow us to further explore these issues. Furthermore, even without explicit expiration dates, there can be a pressure to accelerate purchasing (and point accumulation) if customers believe (1) that the company will go bankrupt or (2) there will be a devaluation in points, both of which have occurred in the airline industry (see The Economist, 2005). While this is not the case in the analyzed LP, the
issue of how customers perceive the value of their reward currency is an important research question.

In our analysis we had to make the assumption that points-pressure thresholds are common across all members. Most likely there are differences across members. However, these differences are very difficult, if not impossible, to identify. Redemptions are relatively rare events and it is also rare that the balance of a randomly selected member is close to a particular threshold. This is due to the fact that most redemptions correspond to only a fraction of the balance. Nevertheless, we think it important for future research to explore the topic of heterogeneous point-pressure thresholds in LPs. Moreover, since we find that redemption momentum dominates points pressure in continuous and linear LPs, it is also important to consider the existence of such alternative mechanisms in other LP designs.

Furthermore, in our analysis we assessed the number and timing of mailings that LP members received, without looking deeply at the contents of said mailings (e.g., Feld et al., 2013). Further research may better account for the contents of mailings. Finally, in this study we included multiple relevant moderators of the reward effects on our studied dependent variables. Future research might include other moderators, specifically some soft moderators such as attitudes towards the program and the participating retailers.
**Technical Appendix**

In this appendix we discuss all the steps of our MCMC sampler in detail. We first introduce some common notation. The four main equations (redemption decision, redemption fraction, purchase (points saving) incidence, and purchases (points saving) amount) are summarized in vector notation; that is, we group all observations of a single member. We write these equations such that we can simplify the derivations below. For redemption decision we write

\[ y_i^1 = M_i^1 \left( \frac{\mu_i}{Y_{1i}} \right) + Z_i^1 \beta_{i1} + W_i^1 \delta_1 + \zeta_i^1, \]  
(A1)

where \( y_i^1 \) denotes a \( T_i x 1 \) vector with elements \( RD_{it}^* \), \( M_i^1 \) equals a \( T_i x 2 \) matrix consisting of a column of ones and a column with a trend. \( Z_i^1 \) collects all relevant row vectors \( Z_{it}^R' \), and \( W_i^1 \) is a matrix obtained by stacking all relevant row vectors \( W_{it}^* \). In (A1) we collect all weeks at which a redemption decision is made (positive or negative), that is, all weeks \( t \) for which \( RT_{it,t-1} \leq 1 \). Finally \( \zeta_i^1 \) is a vector of normal distributed error terms with variance \( \sigma_i^2 = 1 \).

For the redemption fraction we write

\[ y_i^2 = M_i^2 \left( \frac{\mu_i}{Y_{2i}} \right) + Z_i^2 \beta_{i2} + W_i^2 \delta_2 + \zeta_i^2, \]  
(A2)

where \( y_i^2 \) is a \( T_i x 1 \) vector of \( \log(f_{it}/(1-f_{it})) \), only for those observations where \( RL_i=1 \). The matrices \( M_i^2, Z_i^2, \) and \( W_i^2 \) are defined analogous to \( M_i^1, Z_i^1, \) and \( W_i^1 \). \( \zeta_i^2 \) is a vector of random errors each with variance \( \sigma_i^2 = \sigma_R^2 \).

For the points savings incidence we define

\[ y_i^3 = M_i^3 \left( \frac{\mu_i}{Y_{3i}} \right) + Z_i^3 \beta_{i3} + W_i^3 \delta_3 + \zeta_i^3, \]  
(A3)

where \( y_i^3 \) is a \( T_i x 1 \) vector containing the elements \( SI_{it}^* \). The elements of the error term have variance \( \sigma_i^2 = 1 \). Note that \( T_i = T \), for all \( i \).

Finally for the point savings amount we have

\[ y_i^4 = M_i^4 \left( \frac{\mu_i}{Y_{4i}} \right) + Z_i^4 \beta_{i4} + W_i^4 \delta_4 + \zeta_i^4, \]  
(A4)

with \( y_i^4 \) a \( T_i x 1 \) vector of \( \log(S_{it}) \), only for those observations where \( SL_i=1 \). The obvious definitions apply to \( M_i^4, Z_i^4, \) and \( W_i^4 \). The elements of \( \zeta_i^4 \) have variance \( \sigma_i^2 = \sigma_S^2 \).

**Sample \( \lambda_1, \lambda_2 \) and \( \lambda_3 \)**

To sample \( \lambda_1, \lambda_2 \) and \( \lambda_3 \) we employ a random walk Metropolis Hastings [RW-MH] sampler. The candidate values are obtained as \( \lambda_k^{\text{rand}} \sim N(\lambda_k^{\text{current}}, s_k^2) \), for \( k=1,2,3 \), where the \( s_k^2 \) are set such that we obtain an acceptance rate
between 15% and 40%. As the candidate density is symmetric, the acceptance probability depends only on the likelihood of the data. In principle we could take this likelihood conditional on all other parameters, including the effect-size parameters of the mailings, accessibility, and the post-reward effect. However, these effect-size parameters are expected to be quite dependent on \( \lambda_k \). Therefore, in this step, we integrate out the effect-size parameters of mailings and the post-reward effect to obtain better mixing.

We first split \( \Gamma_2 \) in four parts \( (\Gamma^1_2, \ldots, \Gamma^2_2) \), one for each equation. Next, each \( \Gamma^k_2 \) is split in two parts: the part including the mailing, accessibility, and post-reward effects \( (\Gamma^k_2) \), and the remainder \( (\Gamma^{k^*}_2) \). Our approach can be seen as sampling from the distribution of \( (\lambda_1, \lambda_2, \lambda_3, \tilde{\Gamma}_2^k (k = 1, \ldots, 4)) \) given all other parameters (including \( \Gamma^{k^*}_2, k = 1, \ldots, 4 \)), by first sampling from \( (\lambda_1, \lambda_2, \lambda_3) \) given the other parameters and next sampling from \( \tilde{\Gamma}_2^k \) given \( \lambda_1, \lambda_2, \lambda_3 \) and the other parameters. This first step is discussed below; the other step is discussed later in this appendix.

The acceptance probability in the RW-MH sampler depends on

\[
l_k(\lambda_1, \lambda_2, \lambda_3) = \prod_{k=1}^4 \int_{\tilde{\Gamma}_k^k} \left( \prod_{i=1}^N \pi(y^k_i | \lambda_1, \lambda_2, \lambda_3, \tilde{y}_k, \text{other parameters}) \right) \pi(\tilde{y}_k) d\tilde{y}_k, \tag{A5}\]

where \( \tilde{y}_k = \text{vec}(\tilde{\Gamma}_k^k) \), \( \pi(\tilde{y}_k) \propto 1 \), and

\[
\pi(y^k_i | \lambda_1, \lambda_2, \lambda_3, \tilde{y}_k, \text{other parameters}) = (2\pi)^{\frac{T_{ik}}{2}}(\sigma_{ik})^{-T_{ik}} \exp\left( -\frac{1}{2\sigma_{ik}} (v^k_i - H_{ik}\tilde{y}_k)'(v^k_i - H_{ik}\tilde{y}_k) \right), \tag{A6}\]

where \( v^k_i = y^k_i - M^k_i(\tilde{y}_k) - Z^k_i\beta^*_ik - W^k_i\delta^*_i \), and \( H_{ik} = (V_i' \otimes \tilde{Z}_i^k) \). \( Z_i^k \) and \( Z_i^k \) are defined such that they separate the post-reward, accessibility, and mailing variables from the other variables, respectively. Note that \( Z_i^k \) is a function of \( \lambda_k \). The product over all \( i \) of the density in (A5) is proportional to \( \exp(\frac{-1}{2}(\hat{v}^k - \hat{H}_k^k\tilde{y}_k)'(\hat{v}^k - \hat{H}_k^k\tilde{y}_k)) \), where \( \hat{v}^k \) is obtained by stacking the vectors \( v^k_i/\sigma_{ik} \) and \( \hat{H}_k \) is obtained by stacking the matrices \( \frac{1}{\sigma_{ik}} H_{ik} \). Next we observe that

\[
\exp\left( -\frac{1}{2} (\hat{v}^k - \hat{H}_k^k\tilde{y}_k)'(\hat{v}^k - \hat{H}_k^k\tilde{y}_k) \right) = \exp\left( -\frac{1}{2} \left[ (\tilde{y}_k - \tilde{y}_k)'(\hat{H}_k^k\tilde{y}_k)(\tilde{y}_k - \tilde{y}_k) + \hat{v}^k'\hat{v}^k - \hat{v}^k'\hat{H}_k^k(\hat{H}_k^k)'^{-1}\hat{H}_k^k\hat{v}^k \right] \right), \tag{A7}\]

with \( \tilde{y}_k = (\hat{H}_k^k)'^{-1}\hat{H}_k^k\hat{v}^k \). The integral in (A5) is therefore proportional to

\[
\exp\left( \frac{1}{2} \hat{v}^k'\hat{H}_k(\hat{H}_k^k)'^{-1}\hat{H}_k^k\hat{v}^k \right) \int_{\tilde{y}_k} \frac{1}{\tilde{y}_k} |\hat{H}_k^k\tilde{y}_k|^2 \exp\left( -\frac{1}{2} (\tilde{y}_k - \tilde{y}_k)'(\hat{H}_k^k\tilde{y}_k)(\tilde{y}_k - \tilde{y}_k) \right) d\tilde{y}_k. \tag{A8}\]
The integral above is the kernel of a multivariate normal and therefore the integral is proportional to 1. Therefore we get

$$l_j(\lambda_1, \lambda_2, \lambda_3) \propto \prod_{k=1}^3 \exp\left(\frac{1}{2}v_k^T H_k (H_k H_k)^{-1} H_k v_k\right) |H_k H_k|^{-\frac{1}{2}}.$$  \hspace{1cm} (A9)

Finally, the acceptance rate becomes

$$\min\left\{1, \frac{l_j(\lambda_{\text{current}})}{l_j(\lambda_{\text{current}})}\right\}.$$  \hspace{1cm} (A10)

**Sample α**

To sample α we also use a RW-MH sampler. The procedure is similar to that presented above. However, now we split $\Gamma_k^2$ into the pre-reward effect size ($\Gamma_k^2$) and the remainder ($\Gamma_k^2$). The derivation of the acceptance probability is equivalent to the derivation above.

**Sample θ**

We sample the elements of this vector in four steps, one for each equation. We sample $\mu_{ik}$ and $\gamma_{ik}$ by combining

$$y_i^k - Z_i^k \beta_{ik} - W_i^k \delta_k = M_i^k (\mu_{ik}) + \zeta_i^k,$$  \hspace{1cm} (A11)

with the hierarchical distribution for $\mu_{ik}$ and $\gamma_{ik}$ conditional on the other parameters, which follows from $\theta_i \sim N(\Gamma_i V_i, \Omega)$. Denote the conditional mean for $(\mu_{ik}, \gamma_{ik})$ by $m_i^k$ and the conditional variance by $V_i^k$. We now draw $\mu_{ik}$ and $\gamma_{ik}$ from a multivariate normal with mean

$$\left(\frac{1}{\sigma_{ik}^2} M_i^k + (V_i^k)^{-1}\right)^{-1} \left(\frac{1}{\sigma_{ik}^2} M_i^k (y_i^k - Z_i^k \beta_{ik} - W_i^k \delta_k) + (V_i^k)^{-1} m_i^k\right).$$  \hspace{1cm} (A12)

and variance

$$\left(\frac{1}{\sigma_{ik}^2} M_i^k M_i^k + (V_i^k)^{-1}\right)^{-1}.$$  \hspace{1cm} (A13)

**Sample $\sigma_{\theta, i}^2$ and $\sigma_{\gamma, i}^2$**

Conditional on the other parameters, $\sigma_{\theta, i}^2$ has an inverted $\chi^2$-distribution with degrees of freedom equal to $\nu + T_{i,2}$, where $\nu$ gives the prior degrees of freedom (set to 5). The scale parameter equals $\zeta_i^2 \zeta_i^2 + vs$, where $s$ controls the scale under the prior (set to 1). The sampling of $\sigma_{\gamma, i}^2$ follows equivalent steps, with the same prior settings.

**Sample $R_{T_i}$ and $k_i$**
For every redemption occasion, we sample the moment at which the redemption decision was made. This moment defines \( k_t \) and \( RT_{it} \). This moment is sampled without conditioning on \( RD_{it}^* \). In other words, we sample from the joint distribution of \( RD_{it}^* \), \( RT_{it} \) and \( k_t \) by first sampling from the marginal distribution of the latter two variables and next from the conditional for the first variable (see the step below).

To sample the moment of the redemption decision, we calculate the conditional probabilities for all possible number of purchase occasions between the moment of redemption and the redemption decision. This number is denoted by \( k^* = 0, 1, \ldots, m \). Each value of \( k^* \) corresponds to a particular sequence of \( RD_{it}^* \) and \( k_t \).

In the rare case where there are two redemptions in \( m \) weeks, the upper bound of \( k^* \) equals the number of weeks between the redemptions, that is, 1 if the redemptions are in two consecutive weeks. To reduce notation, below we assume the upper bound equals \( m \).

Consider a redemption happening at time \( t^* \), the conditional probability for a particular value of \( k^* \) is proportional to

\[
\begin{cases}
\pi_0 \Pr[RD_{it}^* > 0] \prod_{k=1}^{m-1} \Pr[RD_{it}^* - k \leq 0] \prod_{k=0}^{m-1} \left[ \phi \left( \frac{\zeta_{it}^3 - k}{\sigma_{t,i}} \right) \right] & \text{if } k^* = 0 \\
\pi_1 \Pr[RD_{it}^* > 0] \prod_{k=1}^{m-1} \Pr[RD_{it}^* - k \leq 0] \prod_{k=0}^{m-1} \left[ \phi \left( \frac{\zeta_{it}^3 - k}{\sigma_{t,i}} \right) \right] & \text{if } k^* = 1 \\
\pi_k \Pr[RD_{it}^* - (k^* - 1) > 0] \prod_{k=k-1}^{m-1} \Pr[RD_{it}^* - k \leq 0] \prod_{k=0}^{m-1} \left[ \phi \left( \frac{\zeta_{it}^4 - k}{\sigma_{t,i}} \right) \right] & \text{if } k^* = 2, \ldots, m,
\end{cases}
\]

where the final product gives the likelihood contribution of the points savings decisions at and before the moment of redemption. The terms \( \zeta_{it}^3 \) and \( \zeta_{it}^4 \) are defined in (A3) and (A4) and implicitly depend on \( k^* \) through the dependence on \( RT_{it} \).

### Sample SI*_{it} and RD*_{it}

Given the other parameters and \( RT_{it} \), the latent variables \( SI^*_{it} \) and \( RD^*_{it} \) have a truncated normal distribution. The latent variable \( SI^*_{it} \) (\( RD^*_{it} \)) is negative if individual \( i \) does not make a purchase (positive redemption decision) at time \( t \). Otherwise, it is positive. Note that a redemption decision can only be made at time \( t \) if \( RT_{it-1} \leq 1 \). In case \( RT_{it-1} > 1 \), \( RD^*_{it} \) is not sampled. \( RD^*_{it} \) is sampled from the appropriate truncated normal with mean \( \mu_{it} + \gamma_{it} t + Z_{it}^{\beta_{11}} + W_{it}^{\delta_1} \) and variance 1. The mean for \( SI^*_{it} \) equals \( \mu_{it} + \gamma_{it} t + Z_{it}^{\beta_{13}} + W_{it}^{\delta_3} \).

### Sample \( \pi_0, \ldots, \pi_m \)

To sample \( \pi_0, \ldots, \pi_m \), we first count the number of times the “time gap” between redemption decision and redemption occasion equals \( j \); we denote this count by \( c_j \). The prior distribution for the vector \( \pi \) is set to a
Dirichlet \((1,1,\ldots,1)\) distribution. This distribution is quite uninformative. The conditional distribution of the vector \(\pi\) now becomes a Dirichlet distribution with parameters \(1+c_{i0}, 1+c_{i1}, \ldots, 1+c_{in}\).

**Sample \(\Gamma_i\) and \(\Omega\)**

Given all \(\theta_i\) vectors, the sampling of \(\Gamma_i\) and \(\Omega\) follows the standard results for the multivariate regression model (see Rossi, Allenby, & McCulloch, 2005). In order to improve performance we have an inverted Wishart prior on the variance. We set the degrees of freedom to 10 and the location parameter such that the expected value of the distribution equals 0.5 times a unit matrix.

**Sample \(\delta\)**

Given all latent variables and the other parameters, \(\delta_i\) has a multivariate normal distribution with mean

\[
\left(\sum_{i=1}^{N} \frac{1}{\sigma_{ik}^2} W_i^k W_i^k\right)^{-1} \sum_{i=1}^{N} \frac{1}{\sigma_{ik}^2} W_i^k' \left(y_i^k - M_i^k \left(\mu_{ik}\right) - Z_i^k \beta_{ik}\right),
\]

and variance

\[
\left(\sum_{i=1}^{N} \frac{1}{\sigma_{ik}^2} W_i^k W_i^k\right)^{-1}.
\]

**Sample \(\Gamma_2\)**

We split the matrix \(\Gamma_2\) in four parts, the part related to equation \(k\) is denoted by \(\Gamma_2^k\). We now use the fact that

\[
Z_i^k \beta_{ik} = Z_i^k \Gamma_2^k V_i = (V_i' \otimes Z_i^k) \text{vec}(\Gamma_2^k). \]

This allows us to write

\[
y_i^k - M_i^k \left(\mu_{ik}\right) - W_i^k \delta_k = (V_i' \otimes Z_i^k) \text{vec}(\Gamma_2^k) + \zeta_i^k.
\]

Collecting the equations across all members we obtain a multivariate normal distribution for \(\text{vec}(\Gamma_2^k)\) with mean

\[
\left(\sum_{i=1}^{N} \frac{1}{\sigma_{ik}^2} (V_i V_i' \otimes Z_i^k Z_i^k)\right)^{-1} \left(\sum_{i=1}^{N} \frac{1}{\sigma_{ik}^2} (V_i V_i' \otimes Z_i^k) \left(y_i^k - M_i^k \left(\mu_{ik}\right) - W_i^k \delta_k\right)\right),
\]

and variance

\[
\left(\sum_{i=1}^{N} \frac{1}{\sigma_{ik}^2} (V_i V_i' \otimes Z_i^k Z_i^k)\right)^{-1}.
\]
REFERENCES


### Table 1.

Effects of reward redemption: Overview of findings and study positioning

<table>
<thead>
<tr>
<th>Study</th>
<th>Design</th>
<th>Purchase incidence</th>
<th>Spending Pre-reward effects</th>
<th>Spending Post-reward effects</th>
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<tr>
<td>Kopalle et al. (2012)</td>
<td>Empirical (Hotel chain)</td>
<td>✓</td>
<td>✓</td>
<td>+</td>
<td>0/+ (only for price</td>
<td>2 redemption options (free stay vs. free upgrade)</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>sensitive segment)</td>
<td>×</td>
</tr>
<tr>
<td>Drèze &amp; Nunes (2011)</td>
<td>Empirical (Airlines)</td>
<td>✓</td>
<td>✓</td>
<td>+</td>
<td>0 (resetting but not</td>
<td>×</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>to the initial level)</td>
<td>×</td>
</tr>
<tr>
<td>This study</td>
<td>Empirical (Retailing)</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>Income</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>✓</td>
<td>Age</td>
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<td></td>
<td></td>
<td></td>
<td>Relationship duration</td>
</tr>
</tbody>
</table>

**Notes:** ✓ analyzed effect; × effect not analyzed; + positive effect; 0 post-reward dip (resetting).

The study by Zhang and Breugelmans (2012) is not included in this table because the design of the analyzed LP is not directly comparable.
**Figure 1.**

Conceptual model of the interplay between redemption and purchases when customers choose what and when to redeem in a continuous, linear LP.
Figure 2.

Distribution of redemption amounts
Figure 3.
Empirical cumulative distribution function of redemption amount and available points
(log scale)
Figure 4.

Average points-saving behavior in periods close to redemption
Figure 5.

Results of effect-size simulations
<table>
<thead>
<tr>
<th>Table 2. Model estimation results</th>
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<td>Baseline</td>
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<tr>
<td>Constant</td>
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<tr>
<td>Average income</td>
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<tr>
<td>Age</td>
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<tr>
<td>Membership yrs</td>
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<tr>
<td>Avg. no. mailings</td>
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<tr>
<td>Variance</td>
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<tr>
<td>Trend</td>
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<tr>
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<tr>
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<td>Redemption momentum</td>
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<td>Average income</td>
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<tr>
<td>Avg. no. mailings</td>
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</tbody>
</table>

\(^a\) Defined as log savings amount; \(^b\) Defined as logit transformed redemption fraction of the total amount of accumulated points (balance); \(***\), \(**\), \(*\): 99%, 95%, and 90% highest posterior density regions that do not contain zero, respectively.

Purchase=LP points saving; \(\pi\) (red. decision just after purchase), \(\pi\prime\) (red. decision before the purchase), \(\pi\prime\prime\) (red. decision a week before purchase), Points pressure logit(s), Post redemption decay, Accessibility decay.
Table 3.
Correlation matrix of individual-level saving and redemption parameters (full model)

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<th>SI₀</th>
<th>SIₜ</th>
<th>logS₀</th>
<th>logSₜ</th>
<th>RD₀</th>
<th>RDₜ</th>
<th>logRF₀</th>
<th>logRFₜ</th>
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<td>-0.023</td>
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