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OPTIMAL FARE AND QUALITY IN PASSENGER TRANSPORT UNDER DIFFERENT REGULATORY REGIMES

FINN JØRGENSEN* · BERNER LARSEN**
TERJE ANDREAS MATHISEN***

ABSTRACT: The article studies how travel distance and the weight a transport operator places on profit versus consumer surplus on the one hand, influences the level of fare, quality of transport supply and travellers’ generalised costs on the other hand. The analysis starts with a regulatory regime in which both fare and transport quality are controllable for the transport firm. Secondly, the fare is endogenous for the firm whilst the quality is set by the authorities. Thirdly, the transport firm can control quality when the fare is fixed. The analysis applies power relationships in a general model describing the market for public passenger transport. The study provides relevant knowledge for transport authorities regarding how transport firms with different goal functions respond to changes in regulatory regimes.

JEL Classification: D21, D42, L11, L91.

1. Introduction

How will fare and quality of transport supply on the one hand vary with passengers’ travel distance and transport operators’ objectives on the other hand? Although travelling distance varies considerably amongst passengers using the same mode of transport and transport operators’ goals vary due to different ownership structure (see for example Nash (1978) and Jørgensen and Preston (2007)), the above issues have been somewhat neglected in transport research. Analysing the above issues also shows how generalised travel costs, given by the sum of fares and time costs (Button, 2010), vary with travel distance when fares and quality are designed in accordance

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with transport operators’ goals. Since generalised travel costs are regarded as an indicator of “distance barriers” such analyses enable a discussion of how these barriers vary with ownership structure in the transport industry. Another two issues which also will be addressed in this paper are how the well-being of the travellers is influenced when the authorities can either regulate the quality of transport supply or the fares.

During the last 10 years both theoretical and empirical studies have, admittedly, dealt with many of these issues. Tsai et alii (2008) optimised distance-based fare schemes and service headway under certain quality restrictions such as service capacity and fleet size. Jørgensen and Preston (2003) and Jørgensen et alii (2004) analysed empirically how marginal costs are linked to trip length, the first work using data from the Norwegian bus industry and the latter from the Norwegian ferry industry. These cost estimations are important in order to find optimal relationships between fares and travel distance no matter whether the transport operators maximise profits or pursue other objectives for which cost effectiveness is important. Neither of these works focused, however, empirically on the interrelationships between marginal costs, trip length and the quality of transport supply, implying that they give insufficient empirical information to design optimal fare and quality plans simultaneously.

The simultaneous influences of travel distance and the transport operator’s objectives with regard to fares, transport quality and generalised travel costs are discussed theoretically in Jørgensen and Pedersen (2004). The transport firm’s pay-off function is a weighted average of consumer surplus and profit. With reasonable restrictions being placed on the actual functions, their model derives, however, few unambiguous results. Later on, Jørgensen and Preston (2007) used the above-mentioned work as a starting point but simplified the analysis by imposing special functional forms and by assuming that quality requirements are regulated by the transport authorities and thereby treated as exogenous for the transport firm. Given a linear cost function, Jørgensen and Preston (2007) demonstrated that whether one assumes linear, power or exponential relationships between transport users’ demand and their generalised travel costs is critical in determining how fare and generalised travel cost relate to transport operator objectives and travel distance. Finally, Clark et alii (2011) focus on how fares are related to travel distance between locations under different competitive situations between two transport firms. More precisely they analyse how the equilibrium fares arising from Collusion, Cournot, Stackelberg, Bertrand and sequential price competition are linked to trip length assuming the firms produce symmetrically differentiable services, have identical costs and treat quality as exogenous. The
main conclusions here are that more intense competition between the firms and higher demand from the authorities regarding the quality of transport supply result in fares becoming more dependent on travel distance. Moreover, the firms’ competitive situation has less influence on fares, the longer the routes the operators compete on.

The two first-mentioned works above are seen in the light of empirical studies analysing the relationships between fares and trip length for a selection of transport modes in Norway (Mathisen, 2008). For all modes Mathisen’s study (2008) shows close positive relationships between fare and travel distance; the squares of the multiple correlations coefficients exceeded 0.90 in all cases, meaning that more than 90 per cent of the variation in the fare can be explained by distance. Other studies also show positive significant relationships between the fare and the distance travelled; for example McCarthy (2001) for international flights and Ning (2008) for local bus transport in the British Isles. Neither of these studies analyse, however, empirically how the quality of supply and generalised travel costs relate to distance travelled for passengers using the respective modes of transport. To our knowledge such studies are lacking at the present time. One obvious reason is that transport quality is a difficult but not an impossible variable to operationalise. Characteristics of transport modes such as the age of the vehicles, their average speed, service interval (for example frequency) and the waiting environment are, for example, observable and in some cases good indicators of the quality of transport supply. See for example the discussion of service quality by Hensher et alii (2003) and the effects of quality of service on demand by Paulley et alii (2006) and Rojo et alii (2012).

Also empirical studies aiming to quantify how fares and the quality of transport depend on transport firms’ objectives are lacking, probably due to the difficulties of providing precise enough indicators of the firms’ goals, see Jørgensen and Preston (2007) for a more thorough discussion of this matter. On the other hand the theoretical models mentioned above produce unambiguous results as far as the transport firm’s objectives influence on fares and generalised travel costs is concerned; fare and generalised costs increase the more weight the firm puts on profit both when quality and fares are endogenous and when quality is regarded as exogenous for the transport operator. The weight put on profit has, however, an ambiguous effect on the quality of transport supply alone, given that the operator can control quality.

Summing up, theoretical works have, so far, given inconclusive answers regarding how fare and the quality of transport supply are related to travel distance and the transport operator’s objectives; at least in cases when both fare and quality are endogenous for the transport firm. Empirical studies
show, as expected, positive relationships between fare and travel distance for most modes, but travel distance influence on the quality of transport supply and transport firms’ objectives influence on both fare and transport quality remain unanswered issues empirically.

The aim of this article is to elaborate on previous analyses made by Jørgensen and Pedersen (2004) and Jørgensen and Preston (2007) by imposing more restrictions on the functions representing the relationships in the model. This enables us to conclude the outcomes for different chosen classes of functions and minimum necessary restrictions on them in order to produce unambiguous results. In addition to analysing the two regulatory regimes dealt with in the previous works, namely when: 1) fare and quality both are endogenous for the transport operators and 2) fare is endogenous and quality exogenous for them, we will also focus on the case when 3) quality is endogenous and fare is exogenous for them. The study provides relevant knowledge for transport authorities about how a transport firm will respond to changes in regulatory regimes. Even though deregulation of the transport sector has given the transport operators more freedom, cases 2 and 3 are still relevant; at least as far as the Norwegian transport sector is concerned, see Section 4.

The structure of this paper is as follows: in Section 2, we describe briefly the model presented in Jørgensen and Pedersen (2004) and its most relevant results. The choice of functional forms and reasonable restrictions placed on parameter values when operationalising the above model further is presented and discussed in Section 3. Then, Section 4 derives how optimal fare, transport quality and generalised travel costs are influenced by trip length and the transport operators’ objectives under different assumptions regarding regulatory regimes. Finally, conclusions and implications are offered in Section 5.

2. The basic model and its main results. A brief review

2.1. The model

In this section we will briefly review the general model for the passenger transport market with one supplier developed by Jørgensen and Pedersen (2004). For a thorough discussion of the model, its assumptions and results, we would like to refer to the above work.

The total cost, $C$, for a transport operator is given by the following function:

$$C = C(X, Q, D) \text{ where } C_X, C_Q, C_D > 0, C_{XX}, C_{QQ}, C_{DD} \geq 0, C_{XQ}, C_{XD}, C_{QD} > 0 \quad [1]$$
in which \( X \) is the number of transported passengers, \( Q \) is a variable denoting the quality of transport supply and \( D \) is the mean distance travelled by the passengers. In [1] the notations \( C_x \) and \( C_{xx} \) represent, respectively, the first- and second order partial derivative of \( C \) with respect to \( X \). It is evident from [1] that costs are assumed to increase non-concavely with the number of passengers, \( X \), quality of supplied services, \( Q \), and travel distance, \( D \). Moreover, it follows from the signs of the cross-derivatives that the marginal costs related to serving passengers increase as the travel distance and quality increase and vice versa.

The generalised travel cost, \( G \), is the sum of monetary costs \( (P) \) and time costs \( (T) \) as defined in [2]. In this specification quality only affects time costs implying that the average willingness-to-pay for quality is equal to the marginal willingness-to-pay for quality.\(^1\)

\[
G = P + T(Q,D) = G(P,Q,D) \tag{2}
\]

where \( G_P = 1, G_Q, T_Q < 0, G_D, T_D > 0 \)
and \( G_{QQ}, T_{QQ} > 0, G_{DD}, T_{DD} \geq 0, G_{QD}, T_{QD} < 0 \)

The time cost is reduced convexly with the quality of service and increases non-concavely with the distance travelled. The signs of the second order cross-derivatives in [2] imply that the reduction in time costs for improved quality increases with distance.

The demand for passenger transport is assumed to be reduced convexly with respect to generalised travel cost as specified in equation [3].

\[
X = X(G) \text{ where } X_0 < 0, X_{GG} > 0 \tag{3}
\]

Based on [1], [2] and [3], the transport operator’s profit, \( \pi \), from serving passengers can be written as:

\[
\pi = P \times X\left(G(P,Q,D)\right) - C(X(G(P,Q,D)),Q,D) = \pi(P,Q,D) \tag{4}
\]

Moreover, the consumer surplus arising from the transport activity, \( CS \), can be derived as:

\[
CS = \int_{G(P,Q,D)}^{\infty} X(g)dg = CS(P,Q,D) \tag{5}
\]

\(^1\) From [2] follows \( P = G^{-1}(X) - T(Q,D) \Rightarrow \frac{\partial^2 P}{\partial X \partial Q} = 0 \) which is the condition for the average willingness to pay for quality is equal to the marginal willingness to pay for quality.
In many European countries local businesses, local authorities and national states have substantial equity interests in transport firms (Blauwens et alii, 2008; Button, 2010). A reasonable assumption is that these groups of owners are not just concerned about the firms’ profits but also about the travellers’ wellbeing. In order to take this into account we assume that the transport operator’s utility function, \( U \), is a weighted average of consumer surplus (\( CS \)) and profit (\( \pi \)); that is

\[
U = (1 - \alpha) \times CS(P, Q, D) + \alpha \times \pi(P, Q, D) = U(P, Q, D)
\]

where \( 0.5 \leq \alpha \leq 1 \)

The restrictions placed on \( \alpha \) imply that we disregard cases in which the transport firm places greater weight on consumer surplus than on profits (\( \alpha < 0.5 \)). Values of \( \alpha < 0.5 \) imply, according to the first order conditions in [7] below, that operators will set the fare (\( P \)) lower than the marginal cost (\( C_X \)). This is clearly unreasonable. A value of \( \alpha = 1 \) indicates profit maximization, while \( \alpha = 0.5 \) indicates equal weight put on consumer surplus and profit. An example of an intermediate case with \( \alpha = 0.7 \) implies that the firm places 2.33 times greater weight on profit than consumer surplus. Since public owners are more concerned about the standard of the transport services the firm offers than private owners, \( \alpha \) is likely to decrease when the proportion of shares held by public bodies increases. Maximising social surplus implies that \( \alpha = 0.5 \) (\( \alpha > 0.5 \)) when the costs of raising public fund is zero (positive), see for example Lewis and Sappington (1988) and Jørgensen and Preston (2007).

Inserting [4] and [5] in [6] and maximising \( U \) with regard to the fare level (\( P \)) and the quality of transport service (\( Q \)), give the following first order conditions:

\[
\frac{p - C_X}{p} = \frac{1 - 2\alpha}{\alpha} \times \frac{1}{\varepsilon} \quad \text{where} \quad \varepsilon = \frac{X_G}{X} \quad \frac{p}{X}
\]

\[
-XG_Q = C_Q
\]

in which \( \varepsilon \) is the elasticity of travel demand with respect to fare (note that \( X_G = X_p \) according to equation [2]).

The first equation in [7] gives a regular monopoly solution when \( \alpha = 1 \) and the well known condition for maximising social surplus (\( P = C_X \)) when \( \alpha = 0.5 \). The last equation tells us that the increase in revenue stemming from the final unit of quality supplied (\( -XG_Q \)) is equal to the extra cost of supplying this unit (\( C_Q \)). It is worth noting that the latter condition holds for all values of \( \alpha \). Hence, when the marginal costs of providing quality (\( C_Q \)) are constant a
monopolist will provide the same level of quality as the social optimum level. The reason for this is that our model, as emphasised previously, assumes that the marginal willingness to pay for quality is equal to the average willingness to pay for quality, see Spence (1975) and Beil et alii (1995).

2.2. Main model results

The model’s main results are characterised by what happens to the optimal fare ($P^*$) and the optimal quality supply ($Q^*$) and the resulting generalised travel cost ($G^*$) when travel distance ($D$) and the weight that the transport firm puts on profit ($\alpha$) change. By differentiating [7] and [2] with respect to $D$ and $\alpha$, the model derives the following results:

- The signs of $\partial P^*/\partial D$, $\partial Q^*/\partial D$ and $\partial G^*/\partial D$ are all ambiguous
- The signs of $\partial P^*/\partial \alpha$ and $\partial G^*/\partial \alpha$ are positive, but the sign of $\partial Q^*/\partial \alpha$ is ambiguous.

Summing up, with the restrictions imposed on the actual functions, characterised by the signs of their first- and second order partial derivatives, we cannot conclude unambiguously with regard to how fare, quality of supply and generalised travel costs develop when the transport distance increases. Or in other words: the model does not unconditionally support the view commonly held that the level of fare, quality of supplied transport services and travellers’ generalised costs are greater on longer journeys than on shorter journeys. Changes in the transport firm’s utility function (change in $\alpha$) give, however, more unambiguous results; both fare and generalised travel costs will increase the more weight the firm puts on profit as opposed to consumer surplus. The firm’s objectives influence on the quality of transport supply is, however, still uncertain.

Also when quality ($Q$) is predetermined for the transport operator, the model with the original restrictions on the functions gives few unambiguous conclusions regarding the response on $P^*$ and $G^*$ as a result of marginal changes in $D$, $\alpha$ and $Q$.

The above results together with the new results stemming from more restrictions placed on the actual functions are summarised in Table 1 in Section 4.4.

3. Specifying the functional forms

In order to move further in the analysis and deduce more unambiguous results regarding travel distance and transport firms’ objectives influence on fare, quality of transport and general travel costs, we impose more restrictions on the general functions. The criteria for choosing types of functions
are that they are in accordance with the restrictions placed on the first- and second derivatives in Section 2, that they are mathematically tractable and that they are reasonable in relation to the problem in question. These considerations generated the following power functions:

\[ C(X, Q, D) = \tau_0 X^{\tau_1} Q^{\tau_2} D^{\tau_3} \quad \text{where} \quad \tau_0 > 0, \tau_1, \tau_2, \tau_3 \geq 1 \quad [8] \]

\[ T(X, Q, D) = \gamma_0 Q^{-\gamma_1} D^{\gamma_2} \quad \text{where} \quad \gamma_0, \gamma_1 > 0, \gamma_2 \geq 1 \quad [9] \]

\[ X(G) = \beta_0 G^{-\beta_1} \quad \text{where} \quad \beta_0 > 0, \beta_1 > 1 \quad [10] \]

The signs and magnitudes of the \( \tau \) (tau), \( \gamma \) (gamma) and \( \beta \) (beta) parameters stated in [8], [9] and [10], respectively, secure that the functions’ first- and second order partial derivatives correspond to the assumptions made in the general model in Section 2. The cost function in [8] implies that cost increases non-concavely with the number of passengers, quality and average travel distance. Moreover, marginal costs (\( C_X \)) increase as the quality of transport supply and travel distance increase; that is \( C_{XQ}, C_{XD} > 0 \). The time cost function in [9] implies that time costs decrease convexly in quality and increase non-concavely in travel distance and the reductions in time cost when quality increases will be higher as travel distance increases; that is \( T_{QD} < 0 \). The demand function in (10) decreases convexly in generalised travel costs. The condition that \( \beta_1 > 1 \) secures that the optimal price (\( P^* \)) is positive for all values of \( \alpha \), see first equation in [7].

Power functions are commonly used in the transport field mainly because they often provide great explanatory power and their parameters are easy to estimate and interpret. The \( \tau_1, \tau_2 \) and \( \tau_3 \) parameters in [8] are elasticities and denote percentage change in the firm’s costs when the number of passengers, the quality of transport supply and the travel distance increase by one per cent, respectively. Similar, \( \gamma_1 \) and \( \gamma_2 \) show a percentage decrease (increase) in time costs when the quality of transport supply (distance) increases by one per cent. Finally, \( -\beta_1 \) in [10] denotes the percentage decrease in the number of passengers transported when generalised travel costs increase by one per cent.

Our chosen specification of cost and time functions is, of course, a debatable topic. The cost function in [8] implies that costs approach zero when either \( X, Q \) and \( D \) approach zero. Hence, this disregards fixed costs. For further discussion of the properties of different cost functions see for example Baumol et alii (1988), Braeutigam (1999) and Pels and Rietveld (2008). The critique that can be raised of the time function in [9] resembles this close-
ly. It disregards time costs that are independent of the distance travelled by the mode. These would include walking time, waiting time, and the time spent boarding and alighting the mode of transport. Despite their showing some shortcomings the chosen functions are, however, reasonable and consequently do provide interesting results.

4. Model results when using power functions

4.1. Fare and quality of supply are controllable for the transport firm

In many local areas and between certain destinations one supplier that to a large extent can control both fare and transport quality is commonplace; for example as far as bus transport and rail transport in Norway are concerned. Using the first order conditions in [7] that maximise the transport operator’s utility function in combination with the functions [2], [8], [9], and [10] give, after some mathematical computations, optimal values of fare ($P^*$), quality of transport supply ($Q^*$) and generalised travel costs ($G^*$). These expressions are presented in Appendix A. The restrictions imposed on the parameters in the power relationships in combination with the fact that $0.5 \leq \alpha \leq 1$ imply that $(\alpha \beta_1 + 1 - 2\alpha) > 0$. The above results ensure that $P^*$, $Q^*$ and $G^*$ are positive.

In order to infer how the optimal values of $P^*$, $Q^*$ and $G^*$ are influenced by travel distance ($G$) and the weight the transport operator puts on profits ($\alpha$), we derive the following elasticity expressions from the formulas in Appendix A:

$$ EL_D P^* = \frac{\gamma_2 \beta_3}{\gamma_1 \beta_1 (\tau_1 - 1) + \gamma_1 + \tau_2} $$
$$ EL_D Q^* = \frac{\gamma_3}{\gamma_1 \beta_1 (\tau_1 - 1) + \gamma_1 + \tau_2} $$
$$ EL_D G^* = \frac{\gamma_2 \beta_3}{\gamma_1 \beta_1 (\tau_1 - 1) + \gamma_1 + \tau_2} $$

and

$$ EL_\alpha P^* = \frac{\beta_1 \alpha \beta_1^2 + \alpha \beta_1^2 \tau_1 + \alpha \gamma_1 \beta_1 \tau_2 (\tau_1 - 1) + (3\alpha - 1) \gamma_1 \tau_2 + (1 - \alpha) \gamma_1 \tau_2}{(\alpha \beta_1 + 1 - 2\alpha) [\alpha \beta_1 \gamma_1 \tau_1 + (2\alpha - 1) \gamma_1 \tau_2] [\gamma_1 \beta_1 (\tau_1 - 1) + \gamma_1 + \tau_2]} $$
$$ EL_\alpha Q^* = \frac{\beta_1 \alpha \beta_1^2 + \alpha \beta_1^2 \tau_1 + \alpha \gamma_1 \beta_1 \tau_2 (\tau_1 - 1) + (3\alpha - 1) \gamma_1 \tau_2 + (1 - \alpha) \gamma_1 \tau_2}{(\alpha \beta_1 + 1 - 2\alpha) [\alpha \beta_1 \gamma_1 \tau_1 + (2\alpha - 1) \gamma_1 \tau_2] [\gamma_1 \beta_1 (\tau_1 - 1) + \gamma_1 + \tau_2]} $$

$$ EL_\alpha G^* = \frac{\beta_1 \alpha \beta_1^2 + \alpha \beta_1^2 \tau_1 + \alpha \gamma_1 \beta_1 \tau_2 (\tau_1 - 1) + (3\alpha - 1) \gamma_1 \tau_2 + (1 - \alpha) \gamma_1 \tau_2}{(\alpha \beta_1 + 1 - 2\alpha) [\alpha \beta_1 \gamma_1 \tau_1 + (2\alpha - 1) \gamma_1 \tau_2] [\gamma_1 \beta_1 (\tau_1 - 1) + \gamma_1 + \tau_2]} $$

The elasticity expressions in [11] and [12] prove to be simpler and, thus, easier to interpret than the derivatives. Moreover, the signs related to a specific variable are equal for the elasticities and the derivatives. Given the restrictions imposed on the parameters, it follows from the elasticity expressions above that:

$$ EL_D P^* = EL_D G^* > 0 \text{ and } EL_D Q^* \geq 0 \text{ when } (\gamma_2 \beta_3 (\tau_1 - 1) + \gamma_2 - \tau_3) \geq 0 $$
$$ and $$ EL_\alpha P^*, EL_\alpha G^* > 0 \text{ and } EL_\alpha Q^* = 0 \text{ when } \tau_1 = 1 \text{ and } EL_\alpha Q^* > 0 \text{ when } \tau_1 > 1 $$

We can, thus, conclude unambiguously that an increase in travel distance, $D$, and the weight the transport operator places on profit, $\alpha$ (alfa), will increase fare
and generalised travel costs. The fact that $EL_D P^* = EL_D G^*$ means that a certain percentage increase in travel distance increases the fare and generalised travel costs at the same rate. Moreover, the quality of transport supply is non-decreasing in $\alpha$; i.e. it increases with $\alpha$ (is constant) when the firm’s costs increase convexly (linearly) with the number of passengers transported ($\tau_1 > 1$, $\tau_1 = 1$).

When the relationship between the costs and the number of passengers transported is linear, ($C_{xx} = 0$) our modelling, thus, implies that the quality provided is the same no matter how the transport firm weights profit versus consumer surplus (value of $\alpha$). The increase in generalised travel costs when $\alpha$ increases means that reduced time costs due to possible better transport quality does not outweigh the disadvantages for the travellers of higher fares. In other words: travellers prefer transport operators putting some weight on their wellbeing (consumer surplus) instead of just being pure profit-maximisers.

One ambiguous result above is the sign $EL_D Q^*$; i.e. how the quality of transport is influenced by the travel distance. When $\tau_1 > 1$ it follows from [11] that a sufficient but not necessary condition for $EL_D Q^* > 0$ is that $\gamma_2 \geq \tau_3$. This means that a given percentage increase in travel distance has relatively greater influence on travellers’ time costs than on operator’s costs. When the operator’s cost function is linear in the number of passenger transported ($X$) such that $\tau_1 = 1$, $\gamma_2 > \tau_3$, this is a necessary condition for $EL_D Q^*$ being positive. Without having more accurate information about the magnitudes of $\gamma_2$, $\beta_1$, $\tau_1$, and $\tau_3$, it is thus difficult to come up with firm statements regarding the sign of $EL_D Q^*$. What we can say, however, is that: (1) the more transport quality influences travellers’ time costs ($\gamma_2$ increases), (2) the more demand is influenced by generalised travel costs ($\beta_1$ increases), (3) the more firm costs are influenced by the number of passengers ($\tau_1$ increases) and (4) the less firm costs are influenced by the travel distance ($\tau_3$ decreases), the more likely it is that passengers travelling on longer routes will enjoy higher quality of transport supply than those travelling on shorter routes.

Two other important things worth noting are, firstly, that the weight the transport operator places on profit as opposed to consumer surplus (values of $\alpha$ between 0.5 and 1) does not influence the signs of either of the elasticities above. Moreover, the magnitudes of $EL_D P^*$, $EL_D Q^*$ and $EL_D G^*$ are independent of $\alpha$.

As mentioned earlier Spence (1975) concluded that a necessary condition for a monopolist providing an optimal social level of quality is that the marginal costs of providing quality, $C_Q$, is constant. Our model specification implies, however, that a monopolist can provide the same level of quality as the social optimal level even though the marginal costs of providing quality varies. The cost function in [8] implies namely that $C_Q$ varies for all positive values of $\tau_2$. 

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1 As mentioned earlier Spence (1975) concluded that a necessary condition for a monopolist providing an optimal social level of quality is that the marginal costs of providing quality, $C_Q$, is constant. Our model specification implies, however, that a monopolist can provide the same level of quality as the social optimal level even though the marginal costs of providing quality varies. The cost function in [8] implies namely that $C_Q$ varies for all positive values of $\tau_2$. 

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4. 2. Quality is predetermined and fare controllable for the transport firm

An example of this regulatory regime is regional air transport operated by public service obligation (PSO) contracts (e.g. Williams and Pagliari, 2004). The authorities set quality standards and the operator can design his own fare system within certain limits. When quality \((Q)\) is exogenous for the transport operator, optimal fare \((P^*)\) and generalised travel costs \((G^*)\) can be inferred by using the first equation in [7] in combination with the functions [2], [8], [9], and [10]. Unfortunately, the system is not solvable in the sense that \(P^*\) and \(G^*\) cannot be fully expressed by \(\alpha, \tau, \gamma\) and \(\beta\) parameters alone. We can, however, write the optimal \(P^*\) and \(G^*\) values in the following ways:

\[
P^* = \frac{(2\alpha-1)T+\alpha\beta_1 C_X}{\alpha\beta_1 + 1 - 2\alpha} \tag{13}
\]

\[
G^* = \frac{\alpha\beta_1}{\alpha\beta_1 + 1 - 2\alpha} \times (T + C_X) \tag{14}
\]

in which \(C_X\) is the transport firm’s marginal costs and \(T\) the traveller’s time costs on board. The previous restrictions placed on the \(\alpha, \tau, \gamma\) and \(\beta\) values ensure that \(P^*\) and \(G^*\) are both positive.

In order to discuss how the values of \(P^*\) and \(G^*\) are related to travel distance, \((D)\), the weight that the transport operator places on profit, \((\alpha)\) and quality demands, \((Q)\), we differentiate implicitly [13] and [14] with regard to \(D, \alpha\) and \(Q\). We then obtain the following elasticity expressions:

\[
EL_D P^* = \left[\frac{2(2\alpha-1)\beta_1 (\tau_1 - 1) (\alpha\beta_1 + 1 - 2\alpha) C_X}{(2\alpha-1)T + \alpha\beta_1 C_X + \alpha\beta_1 (\tau_1 - 1) C_X} \right] \times \frac{\gamma_1 T + \alpha\beta_1 \tau_1 C_X}{T + C_X} \times \frac{T + C_X}{T + C_X + \alpha\beta_1 (\tau_1 - 1) C_X} \tag{15}
\]

\[
EL_D G^* = \frac{\alpha\beta_1}{T + C_X + \alpha\beta_1 (\tau_1 - 1) C_X} \times \frac{\alpha\beta_1}{T + C_X + \beta_1 (\tau_1 - 1) C_X} \times \frac{1}{\alpha\beta_1 + 1 - 2\alpha} \tag{16}
\]

\[
EL_{\alpha}P^* = \frac{\alpha\beta_1 (T + \beta_1 C_X)}{(2\alpha-1)T + \alpha\beta_1 C_X + \alpha\beta_1 (\tau_1 - 1) C_X} \times \frac{T + C_X}{T + C_X + \alpha\beta_1 (\tau_1 - 1) C_X} \times \frac{1}{\alpha\beta_1 + 1 - 2\alpha} \tag{16}
\]

\[
EL_{\alpha}G^* = \frac{\alpha\beta_1}{T + C_X + \beta_1 (\tau_1 - 1) C_X} \times \frac{1}{\alpha\beta_1 + 1 - 2\alpha} \tag{17}
\]

\[
EL_Q P^* = \left[\frac{2(2\alpha-1)\beta_1 (\tau_1 - 1) (\alpha\beta_1 + 1 - 2\alpha) C_X}{(2\alpha-1)T + \alpha\beta_1 C_X + \alpha\beta_1 (\tau_1 - 1) C_X} \right] \times \frac{\gamma_1 T + \alpha\beta_1 \tau_2 C_X}{T + C_X} \times \frac{T + C_X}{T + C_X + \alpha\beta_1 (\tau_1 - 1) C_X} \tag{17}
\]

\[
EL_Q G^* = \frac{-\gamma_1 T + \tau_2 C_X}{T + C_X + \beta_1 (\tau_1 - 1) C_X} \tag{17}
\]
Given the conditions previously placed on the parameters, it is straightforward to verify from the equations under [16] that $EL_\alpha P^* > EL_\alpha G^* > 0$. Also when the quality of transport supply is exogenous for the transport operator, both fares and thereby generalised travel costs increase the more weight the operator puts on profit, and the fare will increase most, in relative terms. From [15] it follows that $EL_D G^* > 0$ implying that generalised travel costs increase with travel distance. In contrast to the case in which both the fare and the quality of transport are controllable for the operator, we cannot conclude unambiguously that the fare will increase with trip length. Only in cases where the cost function is linear in $X (\tau_1=1)$ will $EL_D P^*$ always have a positive value. A further inspection of the expressions under [17] shows that the signs of both $EL_Q P^*$ and $EL_Q G^*$ are ambiguous when the operator places more weight on profit than consumer surplus; i.e. when $\alpha > 0.5$. Hence, it is uncertain whether higher quality requirements imposed by the regulator will lead to a higher or a lower fare ($P^*$) and generalised travel cost ($G^*$).

Only in the special case when the transport firm aims to maximise social surplus ($\alpha = 0.5$), will higher quality demands unambiguously lead to higher fares. The sign of $EL_Q G^*$ is, however, still uncertain implying that we cannot conclude unambiguously whether the travellers will benefit ($G^*$ reduces) or not ($G^*$ increases) when the regulators impose higher requirements to the quality of transport supply. Generalised travel costs will decrease (increase) if the reduction in travellers’ time costs due to better transport quality is higher (lower) than the increase in fares.

The restrictions placed upon $\alpha$ imply that the signs of the above-mentioned elasticities do not depend on the weight put by the transport operator on profit versus consumer surplus, except for the sign of $EL_Q P^*$; it is more likely to be negative the more weight the firm places on profit ($\alpha$ increases).

### 4.3. Fare is predetermined and quality controllable for the transport firm

In this case quality is treated endogenously by the firm whereas the fare is set by an external authority. This case is, for example, relevant as far as subsidised ferry transport is concerned (e.g. Baird and Wilmsmeier, 2011). In Norway ferry fares are decided on by the Norwegian Public Roads Administration and are the same all over the country (Mathisen, 2008). The ferry operators can, however, to a large extent influence the quality of supply on different services.

The optimal values for $Q$ and $G$ cannot be solved explicitly, but by using equations [2], [8], [9] and [10] in combination with implicit differentiation of the second equation in [7] with regard to $\alpha$, $D$ and $P$ we can derive that:
where the definitions of $U_{QQ}$, $U_{QD}$, $U_{Qa}$ and $U_{QP}$ are given in Appendix B.

With the restrictions placed on the parameters in [8], [9] and [10] it follows from Appendix B that $U_{QQ}, U_{Qa} < 0$. Then it follows from [19] that $(\partial Q^*/\partial \alpha) < 0$ and $(\partial G^*/\partial \alpha) > 0$ since $T_Q < 0$. Hence, when the transport firm puts more emphasis on profit, the quality of transport supply is reduced whilst generalised travel costs increase.\(^1\) The sign of $U_{QD}$ can be both positive and negative resulting in $(\partial Q^*/\partial D) \leq (>) 0$. Inserting the $(\partial Q^*/\partial D)$ expression derived from Appendix B in the right-hand equation in (18) gives $(\partial G^*/\partial D)$. Increasing the trip length will, thus, increase generalised travel costs but its influence on the quality of transport supply remains ambiguous. This means that the effect on travellers’ time cost due to increasing trip length will always dominate the positive effect arising from a possible better transport quality.

The sign of $U_{QP}$ is also uncertain implying that the sign of $(\partial Q^*/\partial P)$ is ambiguous. Also the sign of $(\partial G^*/\partial P)$ is not so transparent, but it can be seen from [20] that $(\partial Q^*/\partial P) < 0$ implies that $(\partial G^*/\partial P) > 0$. With the restrictions so far imposed on the functions concerned, we are, thus, unable to conclude how increasing predetermined fare will influence transport quality and generalised travel costs. Further inspection of the $U_{QP}$ expression in the appendix in combination with (20) shows, however, that a sufficient but not necessary condition for $(\partial Q^*/\partial P)$ being positive is that $\beta_1 > 1/(\tau_1 - 1)$. Since $\beta_1 > 1$ this inequality always holds when $\tau_1 \geq 2$ or when the cost elasticity with respect to the number of passengers transported is equal to or greater than 2. Such a high value of $\tau_1$ is unlikely in practice except during peak periods. However, this condition on $\tau_1$ does not produce an unambiguous sign of $(\partial G^*/\partial P)$.

4.4. Summary of the comparative analyses

The specification of power functions in Section 3 to be used in the general model by Jørgensen and Pedersen (2004) presented in Section 2 provides

\[^1\] Using more general functions Spence (1975) proved that a firm maximizing profit will always supply less quality than the social optimum level when the price is fixed. Hence, this result is in accordance with our finding as far as the impact on optimal quality of increasing weight on profit is concerned.
a number of additional unambiguous results with respect to the relationships between the variables involved in the model. The achievements made by operationalising the model using power functions are demonstrated in Table 1 by comparing the ability to determine unambiguous signs for the relevant partial derivatives in the original model (left column, Section 2) by the operationalised model (right column, Section 3). The results are divided into three rows in Table 1 according to the three regulatory regimes; i.e. when the transport operator (1) controls both fare and service quality, (2) when the fare is controllable and the service quality is exogenous and (3) when the service quality is controllable and the fare is exogenous.

As shown in Table 1, many ambiguous relationships exist in the original model when transport companies can set both the fare and the quality freely. Only positive relationships between weight put on profit, α, and the variables fare, $P^*$, and generalised costs, $G^*$, could be unambiguously identified. Hence, the influence of the weight put on profit on quality ($Q^*$) and the influence of distance on all endogenous variables ($P^*$, $Q^*$ and $G^*$) can both be positive and negative. In the operationalised model, however, three more important unambiguous results are identified; 1) both fare and generalised costs increase with trip length 2) the quality of transport supply increases (is constant) the more weight the transport firm puts on profit versus consumer surplus when its costs increase convexly (linearly) with the number of passengers transported 3) the quality of transport supply always increases with distance when distance influences travellers’ time costs more than the firm’s costs, in relative terms.

In the case when fare can be controlled by the firm while quality is exogenously given by the authorities, it is clear in both the original model and, consequently, in the operationalised model that both fare and generalised costs are positively related to α. The only additional clear sign provided by the operationalised model is that generalised costs increase with respect to distance. Fare will, however, increase with distance when the transport firm’s costs increase linearly with regard to the number of passengers transported. Fare also increases when transport firms aiming to maximise social surplus meet higher demands regarding transport quality. Another result worth bearing in mind is that the influence of changes in quality demands on generalised travel costs is uncertain, both in the operationalised and the basic model. It is thus uncertain whether higher quality demands resulting in lower time costs for passengers are outweighed by higher fares.
<table>
<thead>
<tr>
<th>Regulatory regime</th>
<th>The basic model (Section 2)</th>
<th>Operationalised model (Section 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Both fare and quality are endogenous (for the transport company)</td>
<td>$\frac{\partial P^<em>}{\partial \alpha} &gt; 0, \frac{\partial Q^</em>}{\partial \alpha} \geq (&lt;)0, \frac{\partial G^<em>}{\partial \alpha} &gt; 0$ $\frac{\partial P^</em>}{\partial D} \geq (&lt;)0, \frac{\partial Q^<em>}{\partial D} \geq (&lt;)0, \frac{\partial G^</em>}{\partial D} \geq (&lt;)0$</td>
<td>$\frac{\partial P^<em>}{\partial \alpha} &gt; 0, \frac{\partial Q^</em>}{\partial \alpha} \geq 0^a, \frac{\partial G^<em>}{\partial \alpha} &gt; 0$ $\frac{\partial P^</em>}{\partial D} &gt; 0, \frac{\partial Q^<em>}{\partial D} \geq (&lt;)0^b, \frac{\partial G^</em>}{\partial D} &gt; 0$</td>
</tr>
<tr>
<td>Endogenous fare and exogenous quality</td>
<td>$\frac{\partial P^<em>}{\partial \alpha} &gt; 0, \frac{\partial Q^</em>}{\partial \alpha} &gt; 0$ $\frac{\partial P^<em>}{\partial D} \geq (&lt;)0, \frac{\partial Q^</em>}{\partial D} \geq (&lt;)0$ $\frac{\partial Q^<em>}{\partial \alpha} &gt; 0$ $\frac{\partial Q^</em>}{\partial D} \geq (&lt;)0^d, \frac{\partial G^*}{\partial D} \geq (&lt;)0$</td>
<td>$\frac{\partial P^<em>}{\partial \alpha} &gt; 0, \frac{\partial Q^</em>}{\partial \alpha} &gt; 0$ $\frac{\partial P^<em>}{\partial D} \geq (&lt;)0^e, \frac{\partial G^</em>}{\partial D} \geq (&lt;)0$</td>
</tr>
<tr>
<td>Endogenous quality and exogenous fare</td>
<td>$\frac{\partial Q^<em>}{\partial \alpha} &gt; 0, \frac{\partial G^</em>}{\partial \alpha} \geq (&lt;)0$ $\frac{\partial Q^<em>}{\partial D} \geq (&lt;)0, \frac{\partial G^</em>}{\partial D} \geq (&lt;)0$ $\frac{\partial Q^<em>}{\partial \alpha} &lt; 0, \frac{\partial G^</em>}{\partial \alpha} &gt; 0$ $\frac{\partial Q^<em>}{\partial D} \geq (&lt;)0, \frac{\partial G^</em>}{\partial D} \geq (&lt;)0$</td>
<td>$\frac{\partial Q^<em>}{\partial \alpha} &lt; 0, \frac{\partial G^</em>}{\partial \alpha} &gt; 0$ $\frac{\partial Q^<em>}{\partial D} \geq (&lt;)0, \frac{\partial G^</em>}{\partial D} \geq (&lt;)0$</td>
</tr>
</tbody>
</table>

$^a\frac{\partial Q^*}{\partial \alpha} = 0$ when $\tau_1 = 1$ and $\frac{\partial Q^*}{\partial \alpha} > 0$ when $\tau_1 > 1$.

$^b$ A sufficient but not necessary condition for $(\frac{\partial Q^*}{\partial D}) > 0$ is that $\gamma_2 > \gamma_3$.

$^c$ A sufficient but not necessary condition for $(\frac{\partial P^*}{\partial D}) > 0$ is that $\tau_1 = 1$.

$^d$ A sufficient but not necessary condition for $(\frac{\partial P^*}{\partial Q}) > 0$ is that $\gamma = 0.5$.

$^e$ A sufficient but not necessary condition for $(\frac{\partial Q^*}{\partial P}) > 0$ is that $\tau_1 \geq 2$. 
For the third regulatory regime when quality is controllable and fare is exogenous for the transport operator, Table 1 shows that the signs of all derivatives are ambiguous in the basic model, meaning that we cannot conclude in which directions the weight the transport operators puts on profit ($\alpha$), travel distance ($D$) and predetermined fare ($P$) influence the optimal quality of transport supply ($Q^*$) and generalised travel costs ($G^*$). In the operationalised model we move a little further; the greater the weight the operator puts on profit, the lower the quality of transport supply and the higher the generalised costs. Moreover, the model shows that generalised costs increase unambiguously with trip length. Finally, only in the rather unrealistic case when the elasticity of the transport firm’s costs with respect to the number of passenger transported is greater than 2, can we conclude unambiguously that the quality of transport supply will increase when the regulators impose higher fares. The signs of the other derivatives are, nevertheless, still ambiguous; i.e. in which ways distance influences transport quality and predetermined fare influences generalised travel costs.

5. Conclusions and implications

In this study we have developed a model aimed at discussing how travel distance and the weight a transport operator puts on profit versus consumer surplus influence:

1. The fare level, the quality of supplied transport services and generalised travel costs when both fare and transport quality can be controlled by the transport operator.

2. The level of fare and generalised travel costs when fare can be controlled by the transport firm and quality is set by the authorities. For this regulatory regime we also analyse how changes in the authorities’ demands regarding transport quality influence fare and generalised travel costs.

3. The level of transport quality and generalised travel costs when quality can be controlled by the transport firm and fares is set by the authorities. We also infer how different levels of fares imposed by external bodies influence the quality setting by the transport operators and subsequently travellers’ generalised costs.

This article expands on a general model for a public transport market developed by Jørgensen and Pedersen (2004) in two ways: first, instead of specifying the cost function ($C(X, Q, D)$), the passengers’ time costs function ($T(Q, D)$) and the demand function ($X(G)$) using only the signs of the first and second order derivatives, we specify all these three functions by power func-
tions giving signs of the derivatives in accordance with the basic model. In this way we illustrate to what extent reasonable specifications of the actual functions provide more unambiguous results. Second, the model is expanded to analyse the interesting regulatory regime in which fare is uncontrollable for the transport firm while quality is controllable. As mentioned earlier, this regime applies, for example, in the case of Norwegian ferry transport.

The results of the analyses suggest making several comments as far as regulation of the transport sector is concerned. If we keep to the operationalised model we can conclude that when both fare ($P$) and quality ($Q$) can be controlled by the transport firm, travellers will experience higher fares ($P^*$), higher or constant quality ($Q^*$) and higher generalised travel costs ($G^*$) when the firm places more weight on profits versus consumer surplus ($\alpha$ increases). The positive effect for travellers related to possible increased quality is, thus, outweighed by the firm charging higher fares. Increasing travel distance ($D$) will increase fares and generalised travel costs. The level of quality ($Q^*$) the travellers enjoy onboard will increase with the trip length if the elasticity of travellers’ time costs ($T$) with respect to travel distance is greater than the elasticity of the transport firm’s costs ($C$) with respect to travel distance; i.e. $\gamma_2 > \gamma_3$. When $\gamma_2 < \gamma_3$, the opposite may occur. The common view held that transport operators allocate the best modes to the longer routes, is, thus, not so clear cut. It is also worth noting that all the results above apply irrespective of the weight the transport firm puts on profit compared to consumer surplus. Increases in $D$ and $\alpha$ will, thus, influence $P^*$, $Q^*$ and $G^*$ in the same directions, regardless of whether the transport firm maximises profit ($\alpha=1$) or social surplus ($\alpha=0.5$).

Moreover, when quality is exogenous for the transport firm, fares, and generalised costs increase when the transport firm puts more weight on profit as opposed to consumer surplus. The influence of trip length on fare is ambiguous under this regulatory regime; only when the transport firm’s costs increase linearly with the number of passenger transported ($\tau_1=1$) will fares unambiguously increase with travel distance. Generalised travel costs will, however, always increase with trip length meaning that increased time costs due to longer travel distance will always dominate the effects of an eventual fare reduction. What happens with fare and generalised travel costs when the authorities impose tougher quality standards also remains uncertain. Only in the case when the firm maximises social surplus ($\alpha=0.5$) will fare definitely increase when quality requirements increase. The influence on the travellers’ wellbeing, as measured by their generalised costs is still, however, uncertain. Irrespective of the weight the transport firm places on profit versus consumer surplus, it is, thus, uncertain whether travellers will benefit when
the authorities impose higher quality standards. The reduced time costs enjoyed by travellers due to higher standards may indeed be outweighed by the operator charging higher fares.

When the transport firm faces exogenous fares, but controls quality, the quality level of supplied transport services will be reduced if more weight (\( \alpha \) increases) is put on profit as compared to consumer surplus. Consequently, generalised travel costs increase. The first result contradicts the result found under the regulatory regime in which both quality and fare can be controlled by the firm; then quality is non-decreasing in \( \alpha \). In other words, profit-maximising firms will probably offer a higher quality of transport supply than firms also focusing on consumer surplus when they can control both fare and quality but lower quality when they purely control quality. The influence of travel distance on transport quality is ambiguous but generalised travel costs do increase unambiguously with trip length, meaning that higher time costs due to longer distance travelled always outweigh lower time costs from possible higher mode standards. It is also uncertain how the increasing fare set by external bodies will influence transport quality and generalised travel costs. Only in the rather unrealistic case when the elasticity of the transport firm’s costs with respect to the number of transported passengers is greater than 2 (\( \tau > 2 \)) will the passengers certainly enjoy higher quality. The influence on travellers’ generalised costs still stays uncertain.

Summing up then, irrespective of the three regulatory regimes stated above, the travellers’ wellbeing (measured by \( G \)) will be reduced when the transport firm puts more weight on profit as compared to consumer surplus. As emphasised in Section 2, a common view is that publicly owned transport firms are more inclined to put less weight on profit than privately owned firms do (see for example Jørgensen and Preston, 2007). The above results explain, thus, why transport users often wish for public bodies to hold a significant proportion of the shares in such firms, in particular when they are monopolists. Two other important measures following from the paper are that higher quality demands from the authorities when fare can be controlled by the transport operator and lower fare set by the authorities when quality can be controlled by the transport firm do not necessarily result in lower generalised travel costs. Imposing higher quality standards and lower fares aiming to increase travellers’ wellbeing can, thus, turn out to be counterproductive. The latter conclusions apply irrespective of the weight the transport firm puts on profit versus consumer surplus.

Finally, we would like to emphasise that even though all our adopted functions are reasonable, they do limit the generality of the above results. Despite these limitations, the paper nevertheless has established the feasibility of ana-
lysing the fare and quality setting under different regimes in transport firms with different goals offering services over different distances. Moreover, it illustrates how the imposition of specific functions leads to more unambiguous results.

References


Appendices

Appendix A: Expressions of optimal price ($P^*$), quality ($Q^*$), and generalised costs ($G^*$), when both fare and quality are endogenous variables for the transport company.

\[
P^* = y_0 \left[ \alpha \beta_1 y_1 \tau_1 + (2 \alpha - 1) \tau_2 \right] \times \left( \frac{y_0 y_1}{\tau_0 \tau_2 \beta_0^{\tau_1-1}} \right)^{-\frac{\gamma_1}{\gamma_2}} \times \left( \frac{y_0 \alpha \beta_1 (y_1 \tau_1 + \tau_2)}{(\alpha \beta_1 + 1 - 2 \alpha) \tau_2} \right)^{-\frac{\gamma_2}{\gamma_1 (\tau_1-1) + \gamma_1 + \tau_2}} \times \frac{\gamma_2}{\gamma_1 \beta_1 (\tau_1-1) + \gamma_1 + \tau_2} \times \frac{1}{\gamma_2 \tau_1 + \gamma_1 + \tau_2} 
\]

\[
Q^* = \frac{y_0 y_1}{\tau_0 \tau_2 \beta_0^{\tau_1-1}} \times \left( \frac{y_0 \alpha \beta_1 (y_1 \tau_1 + \tau_2)}{(\alpha \beta_1 + 1 - 2 \alpha) \tau_2} \right)^{\frac{\beta_1 (\tau_1-1)}{\gamma_1 (\tau_1-1) + \gamma_1 + \tau_2}} \times \frac{\gamma_2}{\gamma_2 \beta_1 (\tau_1-1) + \gamma_2 + \tau_2} \times \frac{\gamma_2}{\gamma_1 \beta_1 (\tau_1-1) + \gamma_1 + \tau_2} 
\]

\[
G^* = \frac{y_0 y_1}{\tau_0 \tau_2 \beta_0^{\tau_1-1}} \times \left( \frac{y_0 \alpha \beta_1 (y_1 \tau_1 + \tau_2)}{(\alpha \beta_1 + 1 - 2 \alpha) \tau_2} \right)^{\frac{y_1 + \tau_2}{\gamma_1 \beta_1 (\tau_1-1) + \gamma_1 + \tau_2}} \times \frac{\gamma_3}{\gamma_3 \beta_1 (\tau_1-1) + \gamma_3 + \tau_2} \times \frac{\gamma_3}{\gamma_2 \beta_1 (\tau_1-1) + \gamma_2 + \tau_2} 
\]

Appendix B: Cross-derivatives of the utility function in the case of exogenous fare and endogenous quality.

\[
U_{QQ} = \frac{1}{\sigma^2 Q^2} \left[ (\alpha - 1) y_1^2 X G^2 + (-2 \tau_1 \beta_1 + \beta_1 + 1) \alpha \tau_2 y_1 C G T - \alpha \tau_1 (\tau_1 - 1) \beta_1^2 y_1^2 C T^2 \right] < 0 
\]

\[
U_{QD} = \frac{1}{\sigma^2 D Q} \left[ (1 - \alpha) y_1 y_2 X G^2 + \alpha \beta_1^2 y_1 y_2 \tau_1 (\tau_1 - 1) C T^2 + \alpha \tau_2 (y_2 - \tau_3) C G^2 \right] \geq (\alpha) < 0 
\]

\[
U_{QA} = -\frac{\gamma_1}{\alpha Q} X \times T < 0 
\]

\[
U_{QP} = y_1 (\alpha (\beta_1 - 1) + 1) \frac{X T}{\sigma Q} + \alpha \tau_2 [\beta_1 (\tau_1 - 1) - 1] C G + \alpha \beta_1^2 y_1 \tau_1 (\tau_1 - 1) \frac{C T}{\sigma^2 Q} \geq (\alpha) < 0 
\]

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