

NHH



# Pricing in Offshore Shipping Markets

*A Two-Regime Mean Reverting Jump Diffusion Model with Seasonality*

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Master thesis in Economic Analysis

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## Abstract

The academical research on offshore shipping markets is very limited. This thesis is an attempt to improve our understanding of the spot rates for offshore shipping markets and consequently our ability to do more accurate pricing. I perform an empirical analysis of the most significant characteristics of the spot rates for Platform Supply Vessels (PSV) and Anchor Handling Tug Supply (AHTS) vessels and propose a model able to capture these dynamics. The proposed spot rate model is an extension to the simple geometric mean reversion model, incorporating two-regime mean reversion, jumps and a deterministic seasonal function. Parameters are estimated based on the historical spot rates and the model is calibrated for the North Sea market. Using modern derivatives techniques I derive the risk adjusted spot rate process and adopt Tvedt's [30] approach to pricing vessels as a spot rate contingent claim on cash flows, where the pay-off structure can be described as a continuous American call option. The proposed spot rate model is then applied to the problem and the partial differential equation satisfying the value function of a vessel is derived.

## Preface

This thesis is written as a concluding part of my Master of Science in Economics and Business administration at NHH. The master thesis is written within the field of my major in Economic Analysis.

Writing this thesis has been a rewarding experience. I have a strong interest in mathematical finance, time-series analysis and derivatives pricing. This interest have motivated a thesis on a subject which from an academical stand point lacks these quantitative insights. The work has greatly improved my insights on how the framework for contingent claim analysis can be applied to pricing of financial assets. I have also developed a more deeper understanding of time series analysis and implementations in R.

I would like to thank my supervisor, Roar Adland, for providing valuable insights and advice. I would also like to thank Liv Anette Ramberg for proofreading the thesis and providing useful comments and support.

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# 1 Introduction

Platform Supply Vessels (PSV) and Anchor Handling Tug Supply (AHTS)-vessels represents one of the largest cost elements in the upstream oil and gas industry [1]. A PSV-vessel is a ship specially designed to supply offshore oil and gas installations. Installations are dependent on regular supplies from PSV-vessels to ensure continuous production. AHTS-vessels are mainly built to handle anchors for oil rigs, but can also be used as a substitutes for supply ships. They differ from PSVs in being fitted with winches for towing and anchor handling, having an open stern to operate such equipments. Even though PSV and AHTS-vessels may be concerned with fairly different operations, they both operate in the same offshore market and the dynamics of the underlying spot rate process share many of the same characteristics. It would therefore be natural to include both in the same analysis. Note that unless otherwise stated, the term "spot rate" refers to the timecharter-equivalent (TCE) spot rate<sup>1</sup>.

The contribution of this paper is threefold. Firstly, I examine four stylized facts about the spot rate for PSV and AHTS-vessels. Secondly, I present a model able to capture the most significant characteristics of the spot rate and estimate the parameters for the North Sea market. Thirdly, I propose a framework for valuation of PSV and AHTS spot rate contingent claims.

## 1.1 Motivation

The validity of the general arbitrage pricing framework depends on the capability of investors to follow a dynamic portfolio strategy that replicates the payoff to the spot rate contingent claim. As first pointed out by Merton[20], the critical assumption required for the replicating portfolio strategy to be feasible is that the spot rate process can be described by a stochastic process. A good model for the underlying spot rate process is therefore crucial. As far as I know, there has not been conducted any research on the spot rate for the offshore segment. The most relevant paper on this subject is written by Aas et al. [1]. They explore the role of PSV-vessels in offshore logistics, revealing some of the dynamic properties behind the spot rate generating process.

In the next subsections I present the stylized facts of the spot rate and discuss methods for modelling and pricing of spot rate contingent claims,

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<sup>1</sup>The TCE spot rate equals the net spot rate for a trip or operation less all costs related to fuel and port costs, divided by the trip duration in number of days

with review of relevant research on these subjects in other fields with applications to the offshore shipping market.

## 1.2 Modelling the Spot Rate

Even though the research on offshore shipping markets are limited, the spot rate of PSV and AHTS-vessels have many similar characteristics with electricity prices and the larger ocean-going shipping segments. The spot rate in shipping markets in general is, from an academic stand point, fairly well covered. The most common approach to modelling spot rates and electricity prices are to use simple parametric stochastic differential equations: Geometric Brownian motion (Dixit and Pindyck [9]), Ornstein-Uhlenbeck process (Vasicek [32], and Bjerk Sund and Ekern [7]) and the Geometric Mean reversion (Schwartz [25] and Tvedt [30]).

A weakness of such one-factor Markovian models are that they often ignore higher order autocorrelations in both the autoregressive and the moving average part. As will be shown later, most one-factor Markovian mean reversion models, such as the Ornstein-Uhlenbeck, are continuous time analogues of the discrete time AR(1)-process. Application of such models are often imposing too simplistic dynamics on the underlying resulting in significant autocorrelation in residuals. As an example Adland [4] identifies lag effects in the conditional variance and short term momentum in the spot rate, indicating significant higher order autoregressive and moving average terms in the underlying spot rate-process. However, the specification of any model will always be subject to a trade-off between analytical tractability and forecast accuracy. This class of models is usually preferred because these models are analytically solvable and offers closed form solutions for certain freight rate contingent claims.

As pointed out by Adland [2] and Bandi [5], there have more recently been a tendency to turn to non-parametric approaches to estimation of scalar diffusion models. The advantage of this approach is that by not imposing a specific parametric structure, the extent of misspecification can be reduced. Another approach is to model the spot rate in a stochastic equilibrium framework (see, for instance, Adland and Strandenes [3] and Tvedt [29]).

### 1.2.1 Stylized Facts About the Spot Rate

#### a) Jumps

Spot rates for PSV and AHTS vessels are defined by very large price movements. When all vessels in the market are fully employed, short term supply

can only be increased through higher utilization of the current fleet. As discussed in [4] this can be achieved by higher vessel speed, reduced port time, shorter ballast legs, and delaying regular maintenance. However, this strategy is limited by technical constraints. When the fleet is sailing at maximum capacity, the supply function becomes almost perfectly inelastic. The demand curves for offshore shipping services are, as discussed in [1], inelastic due to the enormous shortage costs that can arise if an installation is not kept operational. These factors, combined with short spot rate contracts and a small regional market, induce the very high volatility observed in the North Sea spot rate.

In markets with such extreme volatility there is an inherent limitation to models with dispersion terms relying only on the Gaussian distribution. This problem can be overcome by modeling the diffusion as a stochastic process containing jumps. Jump diffusion models of electricity prices are quite common. See, for instance, Lucia and Schwartz [17], Escibano et al. [10] and the Vilaplana extension to Schwartz and Smith [33] two-factor model. Jump diffusion models in shipping are however less common, but as shown by Nomikos et al. [21] jump-extended models yield important improvements over the basic log normal setting.

### **b) Seasonality**

The North-Sea offshore industry are subject to particular harsh weather conditions. The ability of the vessel to perform in bad weather are mainly determined by the machinery, propulsion arrangement and hull design [1]. Traditionally in shipping, loading and unloading capabilities is associated with ports. This is not the case for PSV-vessels where the main challenges in this regard are to be found offshore. The AHTS-vessels are similarly doing most of the operations offshore. Combined with gradually more demanding operations, weather becomes a relevant factor for supply and demand. Weather conditions in the North Sea follow seasonal fluctuations, implying a deterministic seasonality function for the spot rate. This will be discussed further in Section 2.2.

### **c) Stationarity**

Another important characteristic of shipping markets in general, is the stationary dynamics of the spot rates. The implied asymptotically explosive behaviour of a non-stationary processes is not consistent with the notion of mean reversion and the observed properties of the spot rate. The main

argument for suggesting that the spot rate follows a mean reverting process is capacity adjustments[4]. Persistent high spot rates are not sustainable in the long run as exit from lay up and new ordering will increase the supply and bring spot rates down. Similarly, when the spot rates are low supply is expected to decrease due to lay-up and scrapping. Following the discussion of non-linearity in [2] Kokkebakker et al. [15] argues that the spot rate is only mean reverting in the extremes of the distribution, exhibiting non-stationary behaviour over most parts of the empirical range. Stationarity is tested in Section 2.3

While the prevailing notion in maritime economic literature have been that the spot freight rate processes of ocean-going shipping are mean reverting, most empirical research have concluded that the spot freight rate is non-stationary. As discussed in [2] this is often a consequence of incorrectly assuming that the spot rate exhibits linear dynamics. Non-linear properties are for instance level effects in the conditional variance or volatility clustering not due to the spot rate level. This will be discussed in the subsections e) and f) and investigated in Section 2.4 by fitting a linear time series model and checking for autocorrelation in squared residuals.

#### **d) Two-Regime Mean Reversion**

Modelling mean reversion with jumps introduces a few issues related to the rate of mean reversion. As often observed in electricity prices (see, for instance, Weron et al. [34]), random fluctuations in the spot rate tends to revert slowly back to the equilibrium, while price jumps revert quickly. In classic maritime literature, Koopmans [16] characterized the short-term supply curve in shipping by two distinct regimes, distinguished by whether the fleet is fully employed or not. He suggested a short-term supply function that is very elastic when tonnage is unemployed (i.e. low spot rates), and very inelastic when the fleet is fully employed (i.e. high spot rates). Furthermore, demand is also assumed to be very inelastic with respect to the spot rate. Thus, combined with short spot rate contracts, a shock to the system would cause the spot rate to jump and then revert quickly back to equilibrium. Contrary to classical jumps, events with such characteristics are often classified as spikes<sup>2</sup>. Random fluctuations during periods of unemployed tonnage, on the other hand, revert rather slowly. Modelling mean reversion with standard error correction model would therefore impose too fast reversion for the process under the normal regime and too slow reversion

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<sup>2</sup>I will denote these "extreme" events as jumps in the remainder of this thesis.

under the jump regime. Using insights from Geman [13] and Mayer et al. [19] I derive a solution to the problem using a two-regime mean reversion model.

#### **e) Level Effect in the Conditional Variance**

Adland [4] argues that the price elasticity of the supply and demand functions with respect to the spot rate will influence the volatility of the spot rate. As the demand for offshore shipping services reaches the lower break-even limit, short term supply curves becomes almost perfectly inelastic as discussed in d). The conditional probability distribution in this theoretical interval will be upward biased and zero at the break-even level. Equivalently the upper limit of the spot rate will exhibit a downward bias such that the theoretical conditional probability density function is "hump"-shaped. The phenomenon can be captured by modelling the spot rate as a log normal process, as done by Tvedt [30].

#### **f) Short-Term Momentum**

There is a prevailing notion of shipping cycles in the maritime economic literature (see, for instance, Stopford[27] and Zannetos [35]). The concept of shipping cycles, mean reversion and seasonality suggests that spot rate trends tend to persist in the short run. This violates the assumptions of Markov processes which are, by definition, only dependent on the current spot rate level. As argued by Adland [4] the existence of such trends are not irrational, because the spot rate for PSV and AHTS segments cannot be stored or traded, implying that it is difficult to exploit these trends by constructing arbitrage portfolio's. This will be discussed more in the Section 1.3. In particular, only the charterers can potentially profit from a negative trend, while only shipowners can profit from a positive trend, in both cases by delaying the fixtures of vessels. Moreover, Adland argues that the alternative costs related to delaying fixtures may outweigh the potential profit for both charterers and shipowners if spot rates were to increase or decrease in the short term. Thus, relaxing the constraints of the efficient markets hypothesis [11] and extending the insights from Zannetos [35].

#### **g) Lag Effects in Conditional Variance**

As identified by Adland [4] the lag effects in conditional variance is a consequence of the level effect due to the characteristic shape of short-terms supply and demand functions. The two-regimes identified by Koopmans

induce different types of volatility on the spot rate. In the low-volatility regime spot rates are low and the supply function is very elastic, and in the high-volatility regime spot rates are high and the supply function is very inelastic. Tvedt incorporates the level effect by using the geometric mean reversion model, where spot rates are log-normally distributed.

Kavussanos [14] tested the performance of Generalized Autoregressive Conditional Heteroskedasticity (GARCH)- models in capturing higher order lag effects in the condition variance, revealing that there are lag effects in the conditional variance not due to the level effect. He argues that there is significant volatility clustering during and after external shocks. An example is, for instance, political events. Jumps are often a consequence of such information reaching the market. During such periods the increased uncertainty related to information will induce higher volatility. There is however no direct link between the spot rate level and its volatility in such models.

### 1.3 Pricing of Spot Rate Contingent Claims

When a model for the spot rate is obtained, PSV and AHTS-vessels can be valued as a claim on the cash flow from the uncertain spot rate. As identified by Tvedt [30] and Martinussen [18], at any point in time, there are three alternatives for an offshore vessel; operation, lay-up and scrapping. The pay-off function from a vessel is consequently similar to that of a continuous American option. The pay-off function for the option captures both the value of the flexibility related to lay-up and scrapping, and the value of the uncertain claim on cash flow from operation. The option is continuous in the sense that exercising it, i.e. laying-up the vessel, does not keep the investor from re-entering into the market and thus reactivating the same option. The maturity of the option is at the maximum age of the vessel, where the vessel is scrapped and the shipowner receives the value of the vessel as sold to a demolition yard.

In the standard option pricing framework, derivatives securities are priced by solving stochastic differential equations with dynamics equal to the underlying asset. In maritime economic literature, Bjerksund and Ekern [7] is a pioneer contribution on this subject. They apply contingent claims analysis to the pricing of shipping derivatives, using arbitrage arguments on the spot rate to derive closed form solutions. However, this approach relies on the assumption that investors are able to follow dynamic replicating portfolio strategies. An implicit assumption is the notion of complete markets, and consequently that investors are able to replicate pay-offs from the security in

all possible states of the market. This is known as the fundamental theorem of asset pricing. As pointed out by Adland [4], contrary to commodities and financial assets, the spot rate is the price of a service that cannot be stored or traded. This implies that the usual arbitrage arguments used for pricing purposes does not apply to the spot rate. Because we cannot construct risk neutral probabilities, pricing of spot rate contingent claims from PSV and AHTS-vessels are dependent on investors utility functions. However, Nomikos et al. [21] identifies a more recent fundamental transformation of the market for ocean-going shipping freight, from being a service market, to a market where freight rate can be bought and sold for investment purposes. This is due to the a rising derivatives market for ocean-going freight rate. Currently there are no such derivatives market for the offshore market. Under such conditions, a common approach is to assume that we are already working under risk neutral probabilities, and thus proceed with pricing directly. This approach relies on calibration of the model through implied parameters. A different approach by Lucia and Schwartz[17], consists of incorporating the market price of risk in the drift term.

The remainder of this thesis is organized as follows. In section 2 I present the data and perform an empirical analysis to investigate the above-mentioned characteristics in the spot rate. In section 3 I present the theoretical framework and derive a model for the spot rate, replicating the underlying dynamics. In section 4 I estimate the parameters for the North Sea market and in section 5 I apply Tvedt's approach to pricing spot rate contingent claims. Section 6 concludes.

## 2 Data Analysis

In this section i will perform an empirical analysis of the dynamics of the North Sea spot rate for PSV and AHTS-vessels. The spot rate for PSV-vessels are divided into the segments  $6 - 800m^2$  and  $> 800m^2$ . The AHTS spot rate segments are  $13 - 18\ 000bhp$  and  $> 18\ 000bhp$ . The spot rates are provided by Clarksons SIN-database and are quoted weekly, deducted for fuel and port costs. The spot rate analysed can therefore be thought of as the time charter equivalent(TCE)-spot rate and therefore includes two sources of uncertainty, i.e. the  $\pounds$ /ton spot rate and the North Sea fuel price. A plot of the spot rates are shown in Figure from August 1996 to October 2014 are shown in Figure 1.

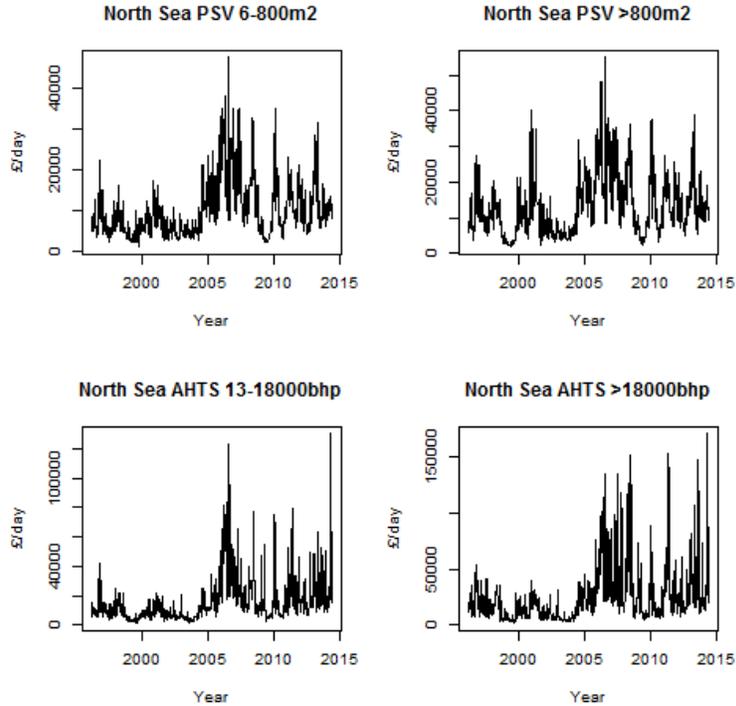


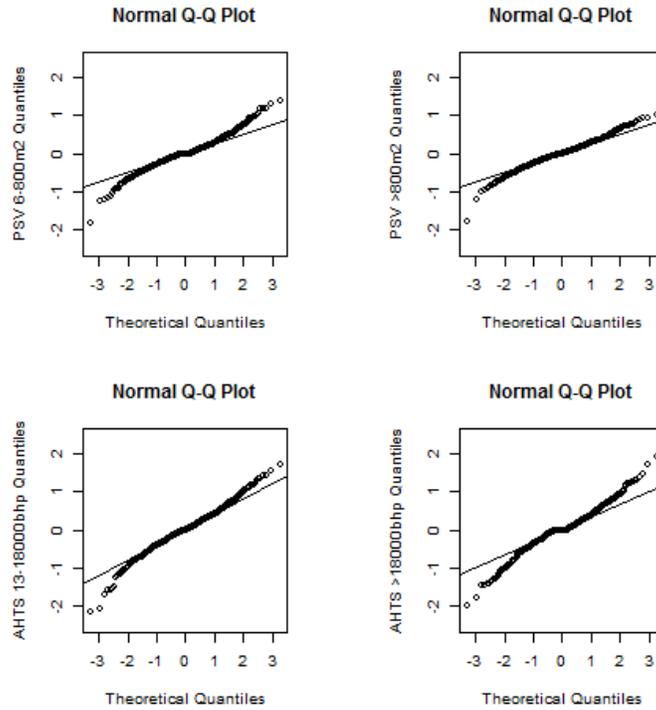
Figure 1: Spot rates

## 2.1 Normality Test

In financial modelling prices are often assumed to be log-normally distributed, in order to account for the limited liability feature of publicly listed companies and level effect of the stock price. This is equivalent to saying that returns have a Normal or Gaussian distribution. The large spot rate movements<sup>3</sup> observed for North Sea PSV and AHTS-vessels indicates that the probability of "extreme" events is higher than what we would expect under the Normal distribution. Jumps in the spot rate are provoked by shocks to the system and reflects the supply side's short-term inability to react to demand fluctuations. As discussed in the introduction, according to [16] the supply function is very inelastic when the fleet is fully employed (i.e. when the spot rate is high). When the fleet is not fully employed (i.e. when the spot rate is low) the supply function is very elastic. Marginal returns from

<sup>3</sup>Spot rate movements, i.e. spot rate returns,  $R_t$ , are defined as  $\ln(\frac{S_t}{S_{t-1}})$

an oil-platform are much higher than to the marginal cost of chartering a PSV/AHTS vessel, making the demand inelastic with respect to the spot rate. These factors combined with highly weather dependent operations, a small regional market and short spot rate contracts allow the spot rate to exhibit extreme volatility.



**Figure 2:** Normality probability test for spot rate returns

In Figure 2 I have plotted the spot rate returns against the theoretical quantiles of the Normal distribution. It can clearly be observed that both the PSV and AHTS-segments show a significant departure from the Normal distribution, which is represented by the straight line. This implies that the Normal distribution is not suitable for modelling the diffusion process of the spot rate.

In order to extract jumps I deseasonalize the data by subtracting the mean of every day across the series according to

$$R_t = r_t - \bar{r}_d \tag{1}$$

where  $R_t$  is the deseasonalised spot rate return,  $s_t$  the spot rate return at time  $t$  and  $\bar{s}_d$  is the corresponding mean. The seasonal values are removed and the remainder is smoothed to find the trend.

I extract the jumps by writing a numerical iterative algorithm that filters the de-seasonalised, de-trended spot rate movements with absolute value above 1,5 times the interquartile range (IQR) from the 25th percentile. IQR is defined as the difference between the 75th and 25th percentiles, and is a standard robust measure of scale. The filter is defined as

$$R_t \notin [1.5 * IQR - Q1] \cap [1.5 * IQR + Q3] \quad (2)$$

which is the analogue to a filter of three standard deviations from the mean. Figure 4 shows a plot of the jumps in the spot rate defined by this filter.

In Figure 3 I have plotted the filtered returns against the theoretical quantiles of the Normal distribution. We can clearly observe that the normality test improves, indicating the jump diffusion-model is able to capture the dynamics of the spot rate diffusion.

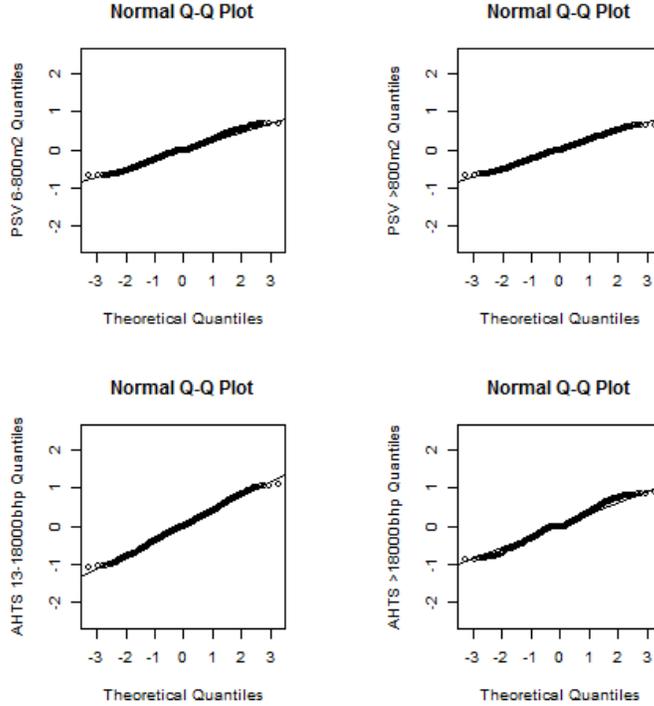
## 2.2 Seasonality

The North-Sea offshore industry is subject to particular harsh weather conditions. As previously stated, PSV and AHTS-vessels are doing most of the operations offshore. Combined with gradually more demanding operations, weather becomes a relevant factor the supply and demand. Weather conditions in the North Sea follow seasonal fluctuations, implying that a deterministic seasonality function would be able to capture spot rate fluctuations caused by weather conditions.

Seasonality can be evaluated with an autocorrelation test. Figure 5 shows a plot of the autocorrelation functions of the respective spot rates on level form. A visual inspection reveals no clear seasonal properties in the spot rate. But a slight oscillation in the AHTS segments is observed.

The Fourier series is suitable to model the wave-like function of seasonal fluctuations in weather conditions. It decomposes the periodic function in the spot rate into a sum of oscillating, sine and cosine-functions. In order to test the significance of seasonality I will fit an ARMA-model with a Fourier series of order  $K$  to capture the seasonality in the spot rate

$$y_t = a + \sum_{k=1}^K [\alpha \sin(2\pi kt/m) + \beta \cos(2\pi kt/m)] + N_t \quad (3)$$



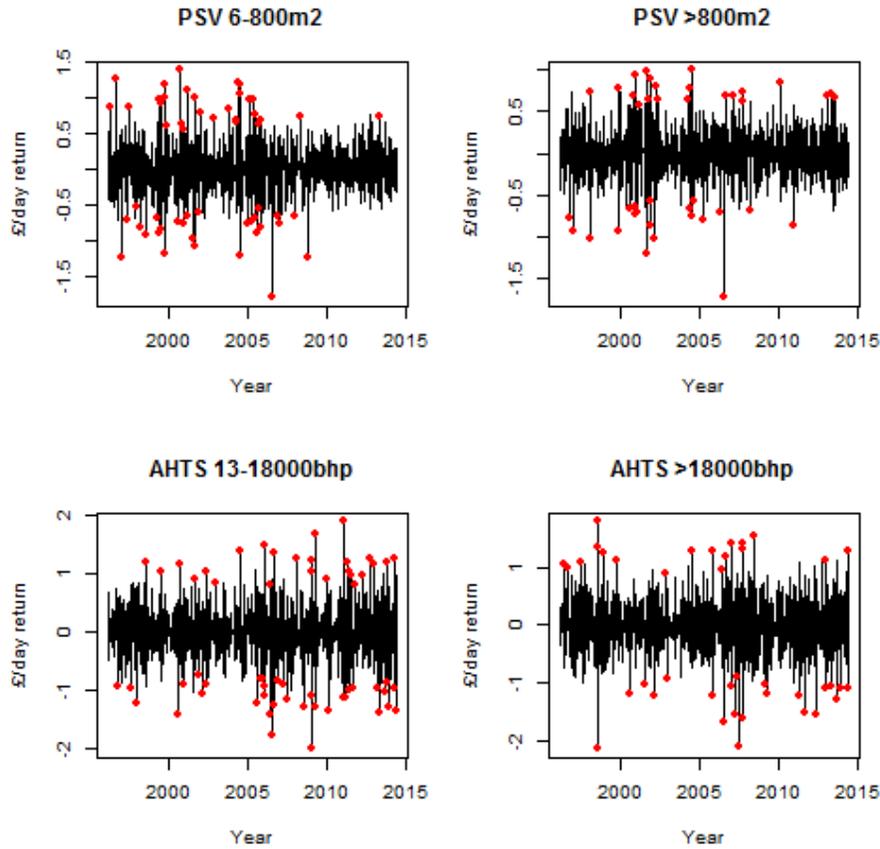
**Figure 3:** Normality probability test for filtered spot rate returns

where  $m$  is the seasonal frequency,  $N_t$  is an  $ARMA(p, q)$ -model containing  $p$  autoregressive parts and  $q$  moving average parts

$$N_t = \alpha_0 + \sum_{i=1}^p \alpha_i y_{t-i} + \sum_{i=0}^q \beta_i \epsilon_{t-i} + \epsilon_t. \quad (4)$$

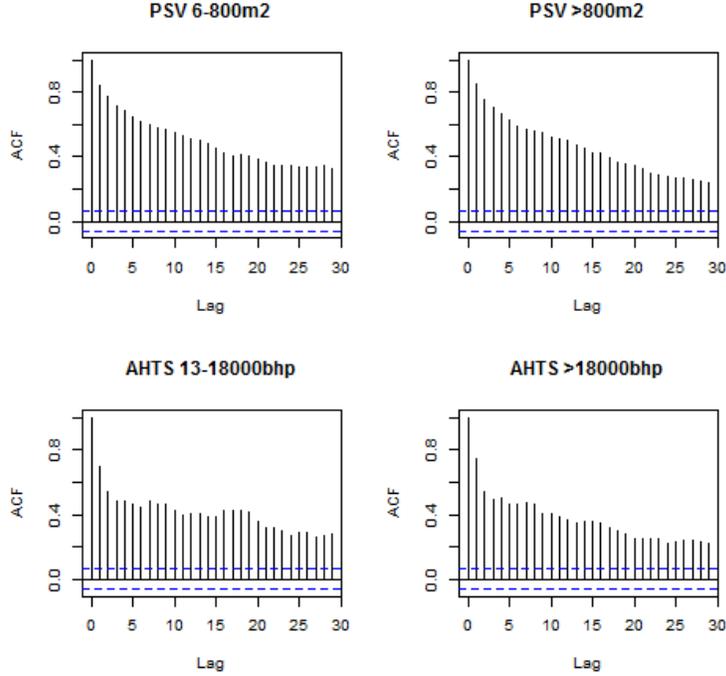
The ARMA-model captures all the linear dynamic relationships in the spot rate  $y_t$  and the random factor  $\epsilon_t$ , while the Fourier series captures the seasonality. The model with seasonality will be compared to an equivalent model without seasonality by using the Diebold-Mariano test to determine the significance of the seasonal component. The test determines whether the seasonal component improves the prediction accuracy of the model, i.e. whether it has a significant lower mean squared prediction error (MSPE) than the model without the seasonal component.

The ARMA-model is fitted by implementing the algorithm proposed by Gardner et al [12] in R to compute the exact maximum likelihood via



**Figure 4:** Jumps

the Kalman Filter, subject to minimizing the Akaike Information Criterion (AIC). AIC deals with the trade-off between goodness of fit and the complexity of the model. The order of  $K$  in the Fourier series for the seasonal component is equivalently chosen by minimizing the AIC. In figure 6 I have plotted the AIC under Fourier series of different order  $K$ 's. Observe that  $K = 2$  gives the lowest AIC for all segments except AHTS 13-18000bhp. Selecting  $K$  accordingly, fitting the model and applying the Diebold-Mariano test to get output shown in table 1. The test reveals that the seasonal component is significant at 5% significance level for PSV  $>800m^2$  and AHTS 13-18000bhp. The test is not significant for AHTS  $>18000bhp$ , while the PSV  $6-800m^2$  segment pass at 10 %confident level.

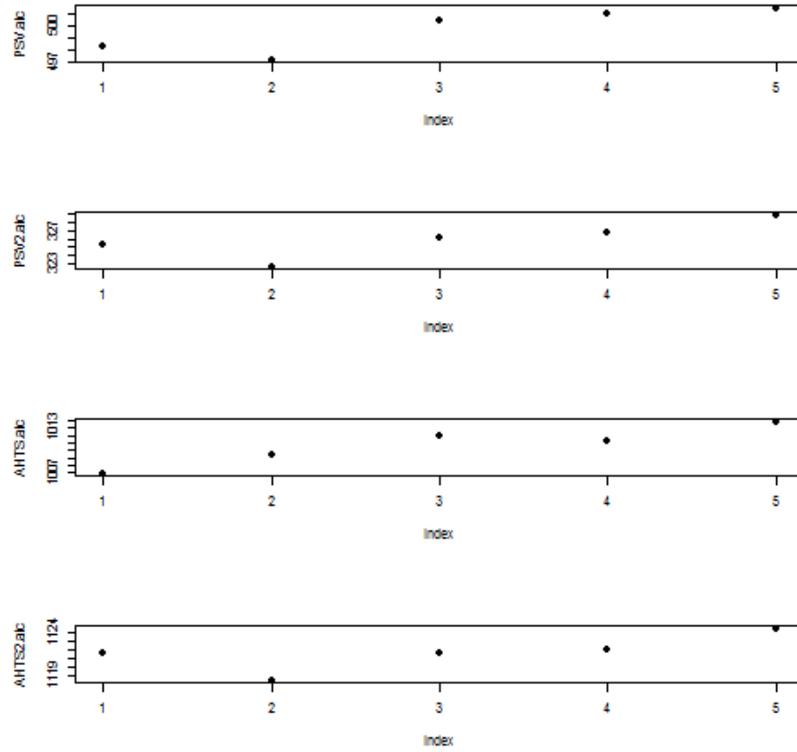


**Figure 5:** Autocorrelation function for PSV and AHTS spot rates

The test is not conclusive, but points in the direction of a seasonal component. However, the models with the seasonal component have a lower AIC, compared to the models without a seasonal component. In table 2 we can see that this is true for all segments. Shibata [26] showed that minimizing the AIC is asymptotically equivalent to minimizing the one-step-ahead out-of-sample mean squared error. This indicates that the seasonal component improves the forecast accuracy of the model for all segments.

The significance of the seasonal function can further be tested by discussing the causal relationship between different weather conditions on the spot rate, and see if that corresponds with the estimated seasonal functions.

Weather conditions tends to be more harsh during the autumn/winter months, and more easy during the summer months. Bad weather will typically bottleneck the supply and reduce the demand. The question is which of these effects is the dominating the other. If the supply effect dominates, bad weather would imply higher spot rates during the winter months, ceteris



**Figure 6:** AIC of different order K in Fourier series

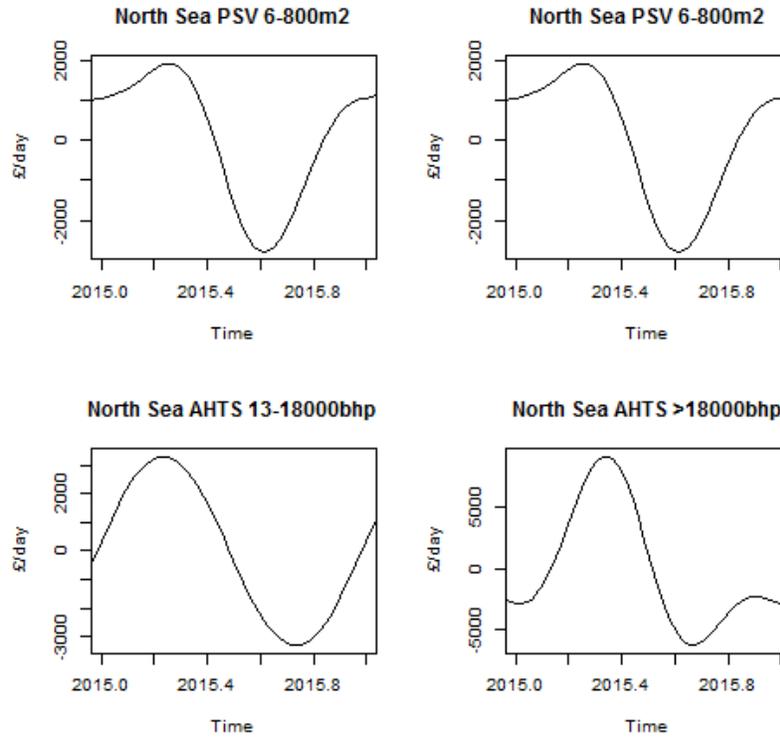
paribus. On the other hand, if the demand effect is dominating it would lead to lower spot rates in months associated with bad weather. The seasonal functions estimated by the Fourier series are plotted in Figure. 7.

Segment	MSPE		DM-Statistic	P-Value
	With Seasonality	Without		
PSV 6-800m <sup>2</sup>	0.0970	0.0986	1.4497	0.0737
PSV >800m <sup>2</sup>	0.0809	0.0825	2.0878	0.0185
AHTS 13-18000bhp	0.1663	0.1885	1.8736	0.0306
AHTS >18000bhp	0.1865	0.1885	1.0651	0.1436

**Table 1:** Diebold-Mariano test

Segment	AIC	
	With Seasonality	Without
PSV 6-800m <sup>2</sup>	497.167	505.724
PSV >800m <sup>2</sup>	322.5872	334.1109
AHTS 13-18000bhp	1006.849	1126.093
AHTS >18000bhp	1118.347	1126.093

**Table 2:** Aikake Information Criterion



**Figure 7:** Deterministic seasonality functions

The seasonal functions are more or less monotonically increasing from the minimum point in July until the global maximum point in February, with a saddle point for the PSV segments and the biggest AHTS segment around January. The spot rates are then a decreasing functions until the turning point in July. The low spot rates during the summer months reflects that the easy weather conditions in this period increase the efficiency of the fleet, and consequently the supply, more than demand. The saddle point in January, reflects the effect of bad weather on the spot rate. A rig-move, for instance, can generally be delayed or scheduled according to weather conditions. This would increase the backlog, demand, and consequently the spot rate in the subsequent periods. This effect can be observed with seasonal maximum points in February. The largest AHTS segment lags the smaller segment by a few months, with maximum as late as March/April, and with a local saddle point in January. This illustrates that the sensitivity of demand with respect to bad weather for more heavy duty offshore operations.

While weather conditions likewise represent a bottleneck for PSV supply, it does not necessarily change the demand. Offshore installations are dependent on consistent delivery of supplies to stay operational[1]. As a consequence, the spot rates for PSV-vessels show a more persistent high level throughout the winter months. During the summer months rates tends to go down as weather conditions are more easy and vessels are becoming more efficient. The latter holds for both PSV and AHTS-vessels.

### 2.3 Stationarity

In finance stationarity is often referred to as mean reversion. As discussed in the Section 2.1 the spot rate for PSV/AHTS-vessels are subject to frequent shocks driven by the oil industry's inelastic demand for offshore supply and AHTS-services. The notion of mean reversion in shipping markets imposes an implicit condition of stationarity on the spot rate. A stationary time series has a finite mean and variance, which makes us able to predict future values of the underlying stochastic process. A shock to a stationary system would only have temporary effect as the underlying dynamics would exhibit mean reversion towards a long term equilibrium. If the spot rate had an infinite variance a shock would amplify in magnitude with respect to a change in the time regime, exhibiting asymptotically explosive behaviour making the variance dominate the deterministic seasonality, trend, drift or mean.

As discussed in the introduction, the main argument for suggesting that the spot rate follows a mean reverting process is capacity adjustments[4]. Persistent high spot rates are not sustainable in the long run as exit from

lay up and new ordering will increase the supply and bring spot rates down. Similarly, when the spot rates are low supply is expected to decrease due to lay-up and scrapping, and thus increasing the spot rates. Koekebakker et al. [15] argues that there must, from a theoretical point of view, exist a lower and upper bound to spot freight rates. The lower bound reflects the break-even level of the spot freight rates and the upper bound represents the maximum price for which charterer is willing to ship his goods. The existence of such upper and lower bounds is not consistent with the implied properties from a non-stationary spot rate process. Although the prevailing notion in shipping literature has been that the spot rate is stationary, most empirical research have concluded otherwise. A common demoninator of these researches have been that they assume linear properties on the spot rate. Adland and Cullinane[2], on the other hand, conclude that the spot rate follows a non-linear process. By imposing non-linear properties on the spot rate, Koekebakker et al. [15] show that the spot rate process is mean-reverting only at the extremes of the distribution, exhibiting non-stationary behaviour over most parts of the empirical range.

### 2.3.1 Augmented Dickey-Fuller Test

I can check if the spot rate is mean reverting by testing whether the time series process is stationary. Stationarity can be assessed by checking for unit root in the spot rate using the Augmented Dickey-Fuller(ADF)- test. The ADF-test imposes a linear stochastic difference equation on the underlying process to check if the coefficients sum to one.

Consider an  $p$ 'th-order autoregressive process for the spot rate

$$y_t = a_0 + \sum_{i=1}^p a_i y_{t-i} + \epsilon_t \quad (5)$$

taking the first order difference,  $y_t - y_{t-1}$ , to obtain

$$\Delta y_t = a_0 + \left( \sum_{i=1}^p a_i - 1 \right) y_{t-1} + \sum_{i=2}^p a_i \Delta y_{t-i} + \epsilon_t \quad (6)$$

The Augmented Dickey-Fuller test for our time series is thus specified as

$$H_0 : \sum_{i=1}^p a_i = 1, \text{ the series contains a unit root,}$$

$H_1 : \sum_{i=1}^p a_i < 1$ , the series is stationary.

The Dickey-Fuller test assumes that residuals are independent and have a constant variance. This creates problems as the underlying spot rate process may contain both autoregressive and moving average components. Fortunately, Said and Dickey [24] have shown that we can solve this problem by using a finite order autoregression to approximate the true data-generating process. Thus, an unknown ARIMA(p,n,q) process can be approximated by an ARIMA(n,1,0) process of order  $n < T^{\frac{1}{3}}$ , where T is the total number of observations. I select number of lags by minimizing the AIC.

The ADF-test statistic is shown in 3. The test statistic reveals that the spot rates are stationary on level form for all segments at < 1% significant level. I.e. the test confirms statistically significant mean reversion in the spot rate for PSV and AHTS-vessels.

Segment	Test statistic
PSV 6-800	-3.8179
PSV 800	-3.8849
AHTS 13-18000	-6.757
AHTS 18000	-7.1122

**Table 3:** Augmented Dickey-Fueller test

## 2.4 Nonlinearity

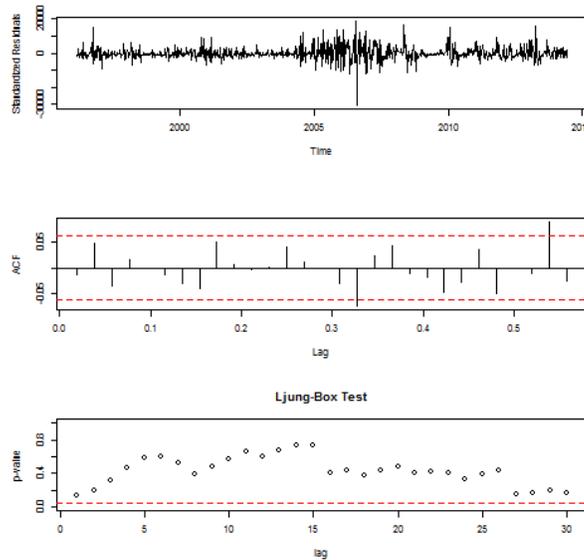
The non-linear dynamics of the spot rates can be investigated by performing a Ljung-Box test for autocorrelation on the squared residuals from a linear time series model fitted to the spot rate level. I will use the Box-Jenkins framework to select the specification of a linear stochastic difference equation, containing  $p$  autoregressive parts and  $q$  moving average parts. This is known as the ARMA(p,q)-model:

$$y_t = \alpha_0 + \sum_{i=1}^p \alpha_i y_{t-i} + \sum_{i=0}^q \beta_i \epsilon_{t-i} + \epsilon_t \quad (7)$$

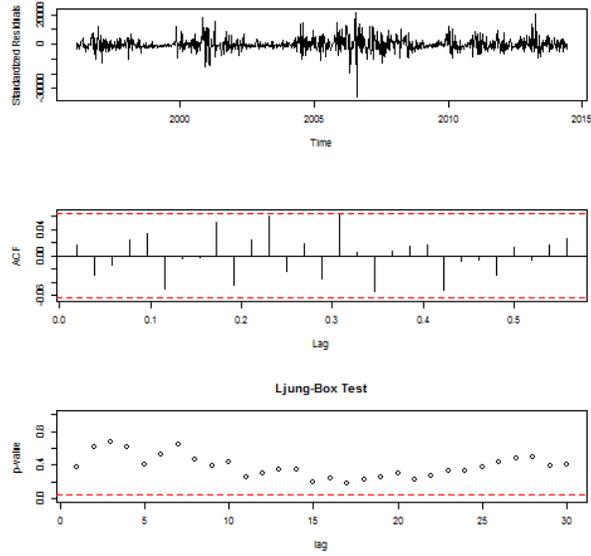
The ARMA-model is fitted as in Section 2.2 with the algorithm proposed by Gardner et al [12]. The estimated ARIMA-model captures the linear dynamic relationships in both the spot rate  $y_t$  and the random factor  $\epsilon_t$ . A

visual indication on the suitable numbers of autoregressive terms can be obtained by looking at a plot of the partial autocorrelation function (PACF). There is however a trade-off between goodness of fit and the number of parameters we introduce. The AIC deals with the trade-off between goodness of fit and the principal of parsimony, and is a measure of the relative quality of time series model. Using the Box-jenkins framework subject to minimum AIC to arrive at the linear ARIMA models with diagnostics shown in figure 8,9,10 and 11. The red-line indicates 5%-significance level. The Ljung-Box p-values implies that we do not reject the null hypothesis that the residuals are independently distributed. This means that there are no autocorrelation in the residuals from the model and that residuals are driven by a white-noise process. In other words, the fitted models are able to capture all the linear dynamics in the spot rate. However, there are still some significant autocorrelations left in higher order lags for the smaller AHTS-segment. The plot of the residuals shows indications of volatility clustering, meaning that there are autocorrelation in squared residuals.

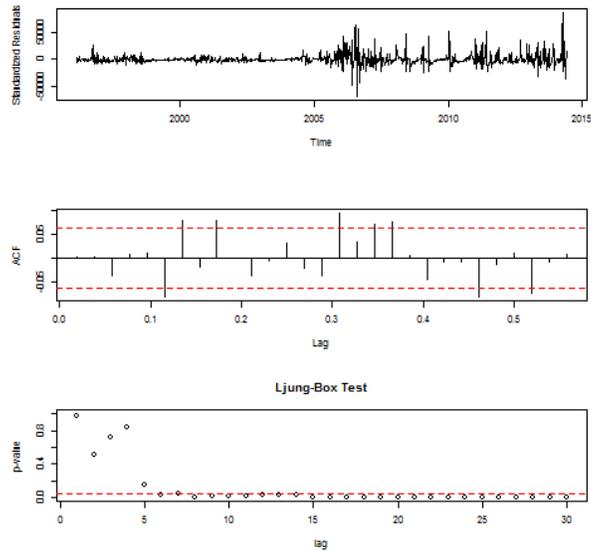
**Figure 8:** ARIMA(2,0,1) fitted to PSV 6-800m<sup>2</sup>



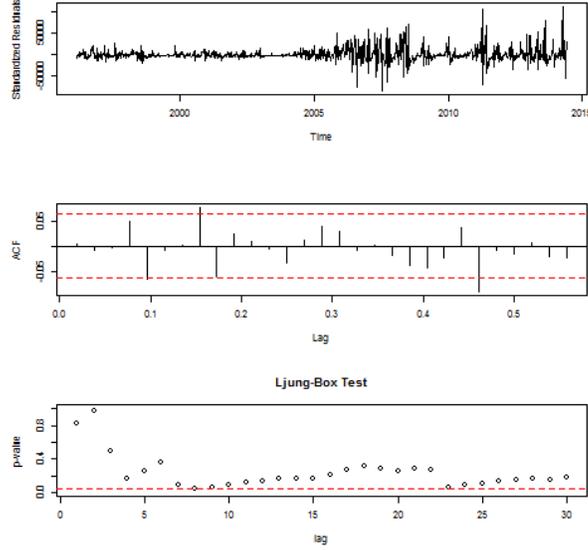
**Figure 9:** Diagnostics of ARMA(2,0,1) fitted to PSV  $>800m^2$



**Figure 10:** Diagnostics of ARMA(2,0,2) fitted to AHTS 13-18 000bhp



**Figure 11:** Diagnostics of ARMA(3,0,1) fitted to AHTS >18 000bhp



I can test for non-linearity by applying the Ljung-Box test on the squared residuals from these models. A plot of the Ljung-Box p-values for the squared residuals are provided in figure 12. The p-values reveal that there are significant autocorrelation for lags of the squared residuals, indicating significant lag effects in the conditional variance. The question is whether there are lag effects in the conditional variance that are not due to the level effect. The effect of volatility clustering can be investigated by using the McLeod-Li test to identify the order of lags in a autoregressiv conditional heteroscedastic (ARCH)-process:

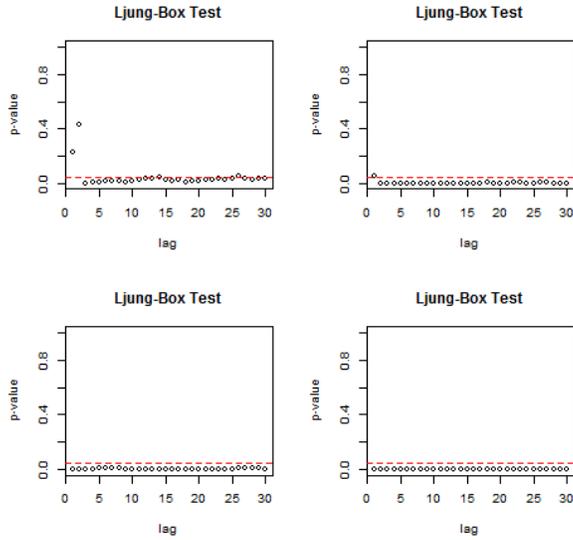
$$\hat{\epsilon}_t^2 = \hat{\alpha}_0 + \sum_{i=1}^q \hat{\alpha}_i \hat{\epsilon}_{t-i}^2 \quad (8)$$

where

$$H_0 : \sum_{i=1}^q a_i = 0, \text{ there is no autocorrelations in } \epsilon_t^2,$$

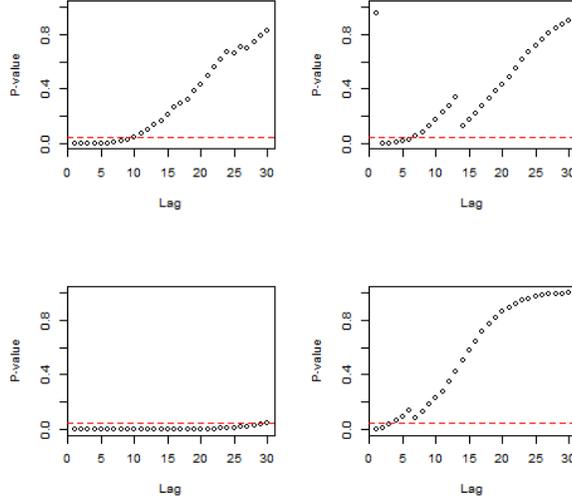
$H_1$ : At least one  $\alpha \neq 0$ , there exist autocorrelations in  $\epsilon_t^2$ .

The test statistic in Figure 13 reveals significant lag effects in conditional variance for all the segments. However, in (G)ARCH-models there are no direct link between the level of the spot rate and its volatility. In other



**Figure 12:** Ljung-Box test of squared residuals

words, the test will reveal autocorrelations due to the level effect, but an ARCH-process specified on the spot rate will not be able to capture it. To capture the level effect on autocorrelations in the conditional variance I can model the spot rate as a log-normally distributed process. To arrive at a log normally distributed mean reversion process for the spot rate  $S_t$  I can, for instance, apply the Ornstein-Uhlenbeck-process to the spot rate return  $X_t$ , such that  $\ln(S_t) = X_t$ . In figure 14 I have applied the McLeod-Li test to the log transformed spot rate. The test reveals that the lag effects in conditional variance is only statistically significant for the largest AHTS-segment. This implies that the geometric mean reversion-process is able to capture most of the autocorrelations in the conditional variance, and that these autocorrelations mostly comes from the level effect. The left over lag effect in the largest AHTS-segment is consequently volatility clustering. I.e. large changes tends to be followed by large changes, independent of current spot rate levels.



**Figure 13:** McLeod-Li Test. From top left to down right: PSV 6 – 800m<sup>2</sup>, PSV > 800m<sup>2</sup>, AHTS 13 – 18 000bhp and AHTS > 18 000bhp

## 2.5 Model selection

Mean reversion properties for a stationary process can be modelled by using error correction models such as the Ornstein–Uhlenbeck (OU)-process

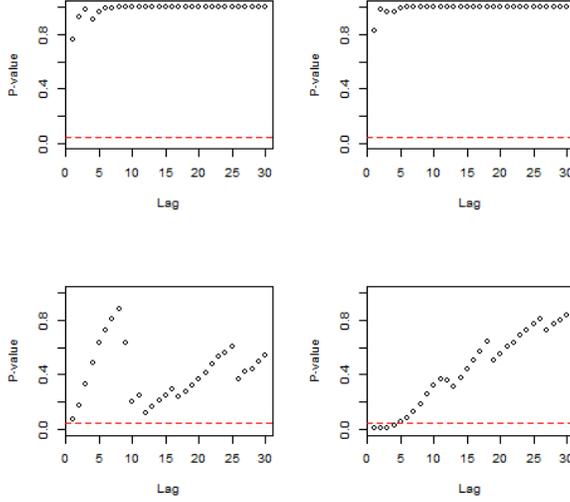
$$dX_t = \theta(\mu - X_t)dt + \sigma(t)dW_t \quad (9)$$

In order to capture the level effect of spot rates, I apply the OU-process to the log transformed spot rate  $X_t$ , such that  $\ln(S_t) = X_t$ . The OU-process can be considered a continuous time analogue to the AR(1)-process. To show this, assume we want to solve the OU-process numerically. This can be done by applying the Euler-Maruyama scheme to  $X_t$  in 9:

$$\Delta X_t = X_{t+\Delta t} - X_t = \theta(\mu - X_t)\Delta t + \sigma\sqrt{\Delta t}\epsilon_{t-\Delta t} \quad (10)$$

which can be re-written as an AR(1)-process

$$X_{t+\Delta t} = \theta\mu\Delta t + (1 - \theta\Delta t)X_t + \sigma\sqrt{\Delta t}\epsilon_{t-\Delta t} \quad (11)$$



**Figure 14:** McLeod-Li Test. From top left to down right: PSV 6 – 800m<sup>2</sup>, PSV > 800m<sup>2</sup>, AHTS 13 – 18 000bhp and AHTS > 18 000bhp

where  $\theta\mu\Delta t$  is the intercept-term and  $(1 - \theta\Delta t) \equiv \rho$  is the coefficient of the AR-term. The OU-process is mean reverting when  $|\rho| < 1$ , which can only happen if

$$1 - \kappa\Delta t < 1 \Leftrightarrow \left[0 < \Delta t < \frac{2}{\alpha}\right]. \quad (12)$$

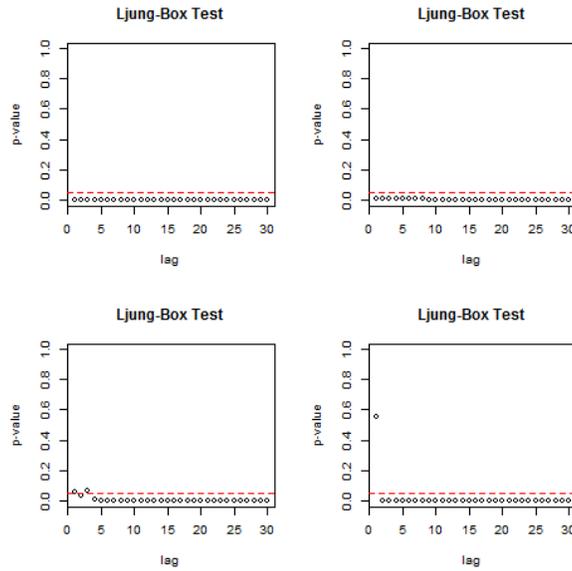
Selecting the time step  $\Delta t$  accordingly is therefore crucial in the simulation.

Moreover, if I choose to model the spot rate return as a geometric mean reversion process I am implicitly making an assumption that the spot rate can be explained by an AR(1)-process. The validity of this assumption can be evaluated by fitting an AR(1)-process to the log transformed spot rate level. The coefficients of this model are shown in table 4, where  $\alpha_0$  is the intercept and  $\alpha_1$  is the mean reversion coefficient of the AR(1) process.

The Ljung-Box test of the residuals and the squared residuals from the AR(1)-models fitted to the spot rates using the Kalman-filter algorithm in [12] is plotted in figure 15 and 16. The Ljung-Box test reveals that there are significant linear and non-linear autocorrelation in the residuals from all the AR(1)-models fitted to the log transformed spot rate level. This reflects the momentum dynamics and the lag effects in the conditional variance. As

Segment	$\alpha_0$	$\alpha_1$
PSV 6-800	8.9929	0.8720
PSV 800	9.2872	0.8941
AHTS 13-18000	9.3718	0.8253
AHTS 18000	9.7933	0.8491

**Table 4:** Parameters of fitted AR(1)-process

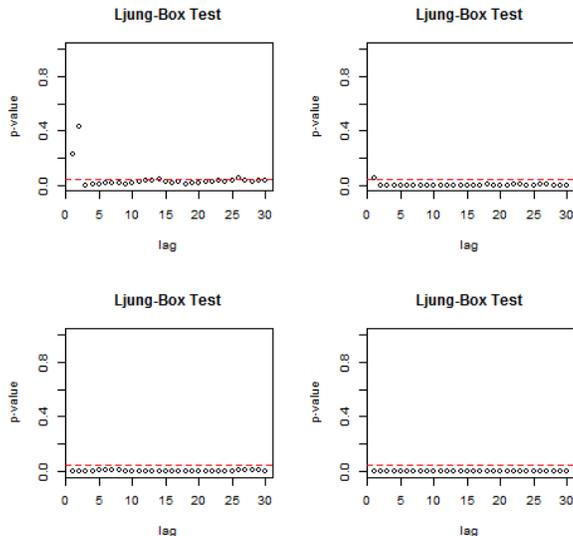


**Figure 15:** Ljung-Box test on residuals from AR(1). From top left to down right: PSV 6 – 800m<sup>2</sup>, PSV > 800m<sup>2</sup>, AHTS 13 – 18 000bhp and AHTS > 18 000bhp

discussed, the level effect is a contributing factor to the lag effects in the conditional variance. The level effect can be captured by modelling the spot rate as a log normal process.

It is evident that an AR(1)-model is not able to capture all the dynamics in the spot rate. But the geometric mean reversion-model is among a small class of analytical solvable stochastic differential equations. The specification of the model will always be subject to a trade-off between analytical tractability and prediction accuracy.

The spot rate for PSV and AHTS-vessels share some of the same dynamic properties as electricity prices. Random fluctuations of the spot rate will revert slowly back to the equilibrium in times without extreme events while



**Figure 16:** Ljung-Box test on squared residuals from AR(1). From top left to down right: PSV  $6 - 800m^2$ , PSV  $> 800m^2$ , AHTS  $13 - 18\ 000bhp$  and AHTS  $> 18\ 000bhp$

price spikes revert very quickly. The two-regime characteristics for mean reversion becomes a problem when the model is based on the standard mean reversion processes. I would consequently impose an unrealistically high reversion rate for the diffusion process and too slow reversion for the jump process. A solution to this problem has been described by Benth et al[6] and Geman and Roncoroni[13]. The solution involves separating the mean reversion factors for the process under the spike regime and normal regime. Such two-regime models serves as an extension to the standard autoregressive models, allowing for higher order of flexibility through a change of the mean reversion parameter. This problem will be dealt with in Section 3.2.

### 3 Theoretical Framework

The data analysis revealed four characteristics that should be accounted for in the model. First, the tendency of spot rate returns to exhibit frequent jumps with greater magnitude than what the Normal distribution would predict. This can be captured by a diffusion model containing jumps.

The second observation is that spot rate jumps tend to revert quickly

while random standard normal fluctuations revert rather slowly. This can be modelled by a two-regime mean reversion model.

The third characteristic is the non-negative spot rates and the level effect in the conditional variance. As discussed the level effect of spot rates is also inducing a lag effect in the conditional variance. These properties can be captured by modelling the spot rate with a log-normally distributed process.

The fourth observation is a significant seasonal component in the spot rate. This can be modelled by using a Fourier series of order  $K$  to estimate a deterministic seasonal function for the spot rates.

### 3.1 Mean-Reverting Jump Diffusion With Seasonality

As in [17] I can prevent negative spot rates and capture the level effect in the conditional variance, by assuming that the log freight rate process,  $S_t$ , can be written as

$$\ln S_t = X_t + g(t), \quad (13)$$

such that the spot freight rate can be expressed as

$$S_t = e^{X_t} G(t) \quad (14)$$

where  $G(t) \equiv e^{g(t)}$  is the deterministic seasonality function and  $X_t$  is a mean reverting jump diffusion process for the underlying spot freight rate whose dynamics are given by the Ornstein-Uhlenbeck process

$$dX_t = (\mu - \alpha X_t)dt + \sigma(t)dW_t + \ln J dN_t. \quad (15)$$

In Equation 15,  $\mu$  is the drift parameter (intercept), representing the long term equilibrium of  $S_t$ ,  $\alpha$  is the mean reversion rate,  $\sigma$  is the time dependent volatility,  $dW_t$  is the increment of the Wiener process,  $J$  is the random jump size, and  $dN_t$  is a poison process subject to

$$dN_t = \begin{cases} 1 & \text{with probability } \lambda dt, \\ 0 & \text{with probability } 1 - \lambda dt \end{cases} \quad (16)$$

where  $\lambda$  is the arrival frequency of jumps <sup>4</sup>. Additionally,  $dW_t$ ,  $dN_t$  and  $J$  are assumed to be independent. The random jump size,  $J$ , has the following

---

<sup>4</sup>The arrival frequency of jumps are defined as number of jumps per unit time.

properties

$$J = e^\phi, \phi \sim N(\mu_J, \sigma_J^2) \quad (17)$$

$$\mathbb{E}[\ln(J)] = \frac{1}{2}\sigma^2 \quad (18)$$

$$\text{Var}[\ln(J)] = \sigma^2 \quad (19)$$

The jump risk is non-systematic and can consequently be hedged by diversification. Furthermore, by assuming that  $\mathbb{E}(J) \equiv 1$ , I can ensure that there are no excess returns to this strategy.

Applying Itô's Lemma to the continuous part in Equation 14 and an analogous lemma for the jump part to get the dynamics for the spot freight rate

$$dS_t = \left( \frac{dg(t)}{dt} + \mu - \alpha \ln S_t + \alpha g(t) + \frac{1}{2}\sigma \right) S_t dt + \sigma S_t dW_t + (J - 1) S_t dN_t \quad (20)$$

which can be expressed as a Geometric Mean Reversion (GMR)-process with jumps

$$dS_t = \alpha(\mu(t) - \ln S_t) S_t dt + \sigma S_t dW_t + (J - 1) S_t dN_t \quad (21)$$

where the equilibrium level of the spot rate process is given by

$$\mu = \frac{1}{\alpha} \left( \frac{dg(t)}{dt} + \frac{1}{2}\sigma^2(t) \right) + g(t) \quad (22)$$

The model in Equation 21, is an extension of the log-normal mean reversion model first applied to spot rates by Tvedt [30]. The extended model incorporates both seasonality and jumps. In the next section I will also account for two-regime mean reversion dynamics in the spot rate.

### 3.2 Two-Regime Model

Smooth Transition Autoregressive (STAR)-models are extensions to standard autoregressive models, allowing for higher order of flexibility through a change of the mean reversion parameter.

The general STAR-model for a univariate time series  $y_t$  with  $p$ 'th order autoregressive lags and transition function  $T(s_t; \gamma, c)$  is given by [28] as:

$$\begin{aligned}
y_t = & (\phi_{1,0} + \sum_{i=1}^p \phi_{1,i} y_{t-p}) (1 - T(s_t; \gamma, c)) \\
& + (\phi_{2,0} + \sum_{i=1}^p \phi_{2,i} y_{t-p}) T(s_t; \gamma, c) + \epsilon_t, \quad t = 1, \dots, T
\end{aligned} \tag{23}$$

Implementing the two-regime framework can easily be done by splitting the spot rate process  $dS_t$  in Equation 15 into a normal diffusion process  $dY_t$  and an jump diffusion process  $dZ_t$ :

$$dX_t = dY_t + dZ_t \tag{24}$$

$$dY_t = (\mu - \alpha_Y Y(t))dt + \sigma(t)dW(t) \tag{25}$$

$$dZ_t = -\alpha_Z Z(t)dt + dN(t) \tag{26}$$

where  $X_t$  is the log transformed spot rate,  $\alpha_Y$  is the mean reversion rate for the normal diffusion process and  $\alpha_Z$  be the mean reversion under the jump process.

### 3.3 Risk-Adjusted Dynamics of the Spot Freight Rate

Suppose that the brownian motion can be defined on the underlying filtered probability space  $(\Omega, \mathcal{F}, \mathbb{P})$ . Let  $\mathbb{P}$  be the function that return the probabilities of different events defined by the  $\sigma$ -algebra  $\mathcal{F}$  on the outcome space  $\Omega$ .

In a complete market, it is possible to replicate pay-offs from all financial instruments in all potential states of the market. This is known as the fundamental theorem of asset pricing. In a complete market we can consequently hedge all freight rate contingent claims. Thus, we are able to construct risk-neutral probabilities  $\mathbb{Q}$  for different events on our outcome space  $\Omega$ . The risk neutral probability measure  $\mathbb{Q}$  is often referred to as the Equivalent Martingale Measure.

Offshore shipping markets have several properties that violates the assumption of a complete market. As stated earlier, the spot rate are defined by jumps that cannot be explained by the normal distribution. In the presence of jumps, returns from spot freight rate contingent claim are discontinuous and the market is, by definition, incomplete. This will be further discussed in Section 5.

The presence of complete markets in shipping does not only depend on the assumption of continuous returns, but also on the assumption that investors are able to construct replicating portfolios. As pointed out by Adland [4], contrary to commodities and financial assets, the freight rate is the price of a service that cannot be stored or traded. This implies that the typical cash-and-carry arbitrage arguments used for pricing purposes does not apply to the spot rate. Because we cannot construct risk neutral probabilities, pricing of spot rate contingent claims from PSV and AHTS-vessels are dependent on investors utility functions.

In incomplete markets the equivalent martingale measure is not unique, and we need to select an appropriate measure. Under such market conditions a common approach is to assume that we are already working under an equivalent martingale measure and calibrate the model with implied parameters.

For now assume that we are able to construct risk neutral probabilities. Then the risk-adjust the spot rate process  $S_t$  can be obtained by deflating with the risk-free asset, represented by government bonds, and applying the Girsanov's theorem.

As in [17] let  $r_t$  be the risk free rate and  $B_t$  the risk free asset, with the following dynamics

$$B_t = e^{\int_t^T r_u du}$$

Let  $S_t$  represents the mean reverting jump diffusion process under the probability measure  $\mathbb{P}$  for the underlying spot freight rate, with dynamics equal to what I derived in Equation 21

$$dS_t = \alpha(\mu(t) - \ln S_t)S_t dt + \sigma S_t dW_t + (J - 1)S_t dN_t$$

where  $\mu(t) = \frac{1}{\alpha} \left( \frac{dg(t)}{dt} + \frac{1}{2}\sigma^2(t) \right) + g(t)$ .

As in Merton's Jump Diffusion model[20] the drift component of the spot rate  $\mu$  can be decomposed into the instantaneous expected return  $\tilde{\mu}$  and the market price of jump risk,  $\lambda\kappa$ , where  $\lambda$  is defined as the arrival frequency of jumps per unit time and  $\kappa \equiv \mathbb{E}(J - 1)$  is the expected percentage change in the spot rate.

If I deflate the freight rate process  $S_t$  with the risk free asset  $B_t$

$$\frac{S_t}{B_t} \equiv \frac{S_t}{e^{\int_t^T r_u du}} \equiv S_t^* \tag{27}$$

I can modify the mean and the diffusion by applying Itô's Lemma and Girsanov's Theorem to arrive at the risk-adjusted dynamics for the spot

freight rate process  $S_t$  under the equivalent martingale measure  $\mathbb{Q}$ . Applying Itô's Lemma to  $S_t^*$  to arrive at

$$dS_t^* = (\alpha\tilde{\mu} - \alpha \ln S_t - r - \lambda\kappa)S_t^* dt + \sigma S_t^* dW_t + (J - 1)S_t^* dN_t \quad (28)$$

Now having adjusted the mean of  $S_t$  I need to apply Girsanov's Theorem to ensure that our diffusion is still a Wiener process. Girsanov's Theorem says that for any market price of risk,  $\theta$ , there exist risk neutral probabilities  $\mathbb{Q}$ 's such that

$$d\tilde{W}_t = dW_t + \theta dt \quad (29)$$

is a Wiener process under the new probability measure  $\mathbb{Q}$ . Equivalently will the compound poisson process  $\sum_{n=1}^N J_n$  have a new intensity rate  $\tilde{\lambda} = \lambda(1 + \kappa')$ .

$$dS_t^* = (\alpha\tilde{\mu} - \alpha \ln S_t - r - \lambda\kappa - \sigma\theta + \tilde{\lambda}\tilde{\kappa})S_t^* dt + (J - 1)S_t^* dN_t + \sigma d\tilde{W} \quad (30)$$

which is a martingale if and only if the drift is equal to zero, which implies that the risk premium equation is

$$\frac{\alpha(\mu - r - \ln S_t) - \lambda\kappa + \tilde{\lambda}\tilde{\kappa}}{\sigma} = \theta \quad (31)$$

where the jump risk premium is  $\lambda\kappa - \tilde{\lambda}\tilde{\kappa}$  and the Wiener risk is  $\sigma\theta$ . Applying Girsanov's theorem to Equation 30 to obtain the risk-adjusted spot rate process under the equivalent martingale measure  $\mathbb{Q}$ :

$$dS_t^{\mathbb{Q}} = (r - \tilde{\lambda}\tilde{\kappa})S_t dt + (J - 1)S_t dN_t + \sigma S_t d\tilde{W}_t^{\mathbb{Q}} \quad (32)$$

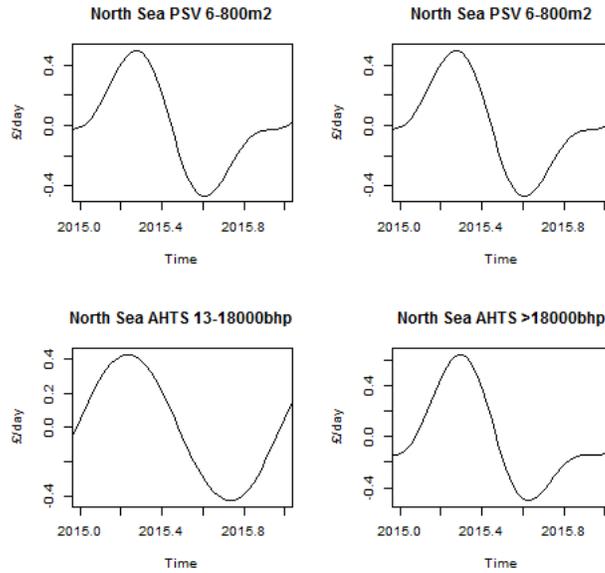
Observe that the drift of the risk-adjusted process follows the risk free rate  $r$ , deducted for market price of jump risk under the risk neutral measure. This is consistent with the drift we would expect given replicating portfolios and a risk-neutral position in the underlying.

## 4 Calibration

### 4.1 Seasonality

The deterministic seasonality function in Equation 13,  $g(t)$ , is estimated by fitting a Fourier series of order  $K$  to the log-transformed spot rate  $X_t$

by minimizing the AIC.  $K$  is of the same order as estimated in Figure 6. The deterministic approach to the seasonality function offers more reliability than, for instance a seasonality function that depends on parameters from the historical data. Such a non-deterministic function would add unreliability to the calibration of the model which is based on a particular noisy data set with fairly few observations ( $N=952$ ). The estimated seasonal functions are plotted in Figure 7.



**Figure 17:** Jump vector

## 4.2 Mean reversion

After removing seasonality from the spot rate I can estimate the parameters of mean reversion. The mean reversion effect will be modelled by the Geometric mean reversion model derived in last section. The discrete-time representation of 24 becomes

$$\Delta X_t = \Delta Y_t + \Delta Z_t \tag{33}$$

Using equations 25 and 26, I can rewrite Equation 33 using the same procedure as in Mayer et al. [19]

$$\begin{aligned}
\Delta X_t &= (\mu_Y - \alpha_Y Y(t))\Delta t + \sigma(t)\Delta W(t) - \alpha_Z Z(t)\Delta t + \Delta N(t) \\
&= (\mu_Y - \alpha_Y Y(t) - \alpha_Z Z(t))\Delta t + \sigma(t)\Delta W(t) + \Delta N(t) \\
&= \left( \frac{\mu_Y}{Z(t) + Y(t)} - \alpha_Y \frac{Z(t)}{Z(t) + Y(t)} + \alpha_Z Z(t) \frac{Y(t)}{(Z(t) + Y(t))} \right) (Z(t) + Y(t))\Delta t + \sigma(t)\Delta W(t) + \Delta N(t) \\
&= \alpha_X X(t)\Delta t + \sigma(t)\Delta W(t) + \Delta N(t)
\end{aligned} \tag{34}$$

The mean reversion rate  $\alpha_Z$  can now be estimated as a linear regression of log-prices against the log returns

$$\Delta X(t) = \phi Z(t)\Delta t + \epsilon_t \tag{35}$$

where  $\phi$  is the autoregressive parameter and  $\epsilon$  is the changes in the spot rate not caused by mean reversion. Assuming that  $\alpha_Z = -\phi$ , I estimate the mean reversion rates as listed in table 5.

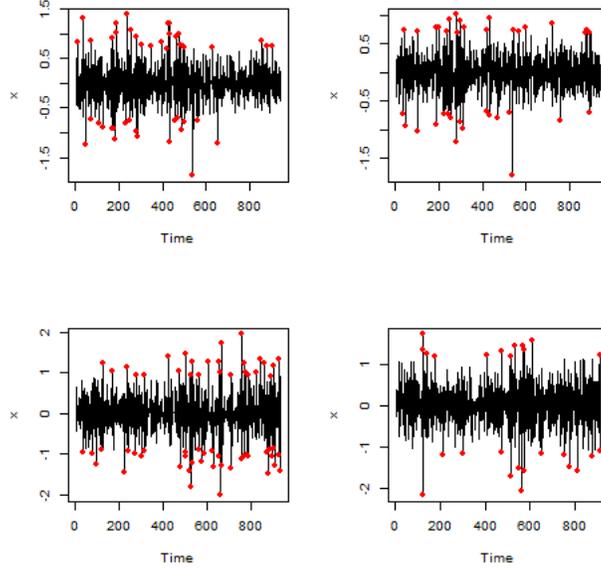
Segment	$\alpha_X$
PSV 6-800	0.1332
PSV 800	0.1054
AHTS 13-18000	0.1804
AHTS 18000	0.1542

**Table 5:** Combined mean reversion rate

After removing the mean reversion effect from the spot rate I obtain the diffusion process  $\epsilon$ , shown in figure 18. I can determine the jump vector by applying a numerical algorithm that filters out jumps defined as absolute value spot rate movements above 1,5 times the interquartile range (IQR) from the 25th percentile. Filtered jumps are marked by red.

The vector of jumps are then used to estimate the mean reversion,  $\alpha_Z$ , under the jump diffusion process in Equation 26, and the filtered spot rates are used to estimate the mean reversion,  $\alpha_Y$ , under the normal diffusion process in Equation 25.

The mean reversion of the normal diffusion process,  $Y_t$ , follows a Ornstein-Uhlenbeck type mean reversion. The parameters of this process are calculated using maximum likelihood estimation. The log-likelihood function of the spot rate can be derived from the conditional density function as done in [31]. I write an algorithm that estimates the mean reversion by solves



**Figure 18:** Jump vector

Segment	$\mu$	$\alpha_Z$	$\epsilon$
PSV 6-800	0.6887	0.0769	0.2527
PSV 800	0.6523	0.0702	0.2450
AHTS 13-18000	0.8638	0.0911	0.3392
AHTS 18000	1.0574	0.1079	0.3849

**Table 6:** Parameters of the normal spot rate process

the maximum likelihood equations in appendix A. The maximum likelihood estimated parameters of mean reversion for  $Y_t$  are shown in table 6.

The mean reversion for the jump diffusion process,  $Z_t$ , can be estimated by the same procedure. The drift of the jump process is set to zero, allowing the effects of a jump to revert back to zero. The estimated mean reversion parameters for the jump diffusion process are listed in table 7

### 4.3 Jumps

The arrival frequency of jumps,  $\lambda$ , are defined as the number of jumps per unit time. The arrival frequency for the different segments are calculated by dividing the number of jumps, defined by the filter in Equation 2, on the

Segment	$\alpha_Z$	$\epsilon$
PSV 6-800	0.3208	0.2614
PSV 800	0.2996	0.2572
AHTS 13-18000	0.6218	0.3524
AHTS 18000	0.5687	0.4053

**Table 7:** Parameters of the jump spot rate process

number of observations. The probability of a jump in the spot rate is thus equal to  $\lambda$ , listed in Table 8.

Segment	$\lambda$
PSV 6-800	0.042
PSV 800	0.025
AHTS 13-18000	0.050
AHTS 18000	0.021

**Table 8:** Parameters of the jump spot rate process

Observe that the probability of jumps are higher in the smaller segments. As noted in [1], the probability that a smaller vessel is in harbour when something urgently needs to be taken care of is higher, than with a larger vessel. In a small regional market smaller vessels are therefore more flexible and efficient in dealing with demand uncertainties, allowing more frequent positive jumps in the spot rate of the smaller segments.

#### 4.4 Simulation

As discussed earlier, a problem with applying the Euler-Maruyama scheme to the spot rate level, is that it can generate negative rates. A solution to this problem is to simulate returns  $X_t = \ln(S_t)$ . The continuous time process of the Ornstein Uhlenbeck can be approximated by the Euler scheme in discrete time. As discussed earlier an AR(1)-process is only stationary if  $|\alpha| < 1$ , which can only happen if

$$1 - \kappa\Delta t < 1 \Leftrightarrow \left[ 0 < \Delta t < \frac{2}{\alpha} \right] \quad (36)$$

Selecting the time step  $\Delta t$  accordingly, and writing an algorithm that simulates price return paths

$$\ln S_t = X_t + g(t),$$

such that the spot freight rate can be expressed as

$$S_t = e^{X_t} G(t) \quad (37)$$

where  $G(t) \equiv e^{g(t)}$  is the deterministic seasonality function and  $X_t$  has dynamics equal to those in Equation 15

$$dX_t = (\mu - \alpha X_t)dt + \sigma(t)dW_t + \ln J dN_t. \quad (38)$$

In order to capture the two-regime mean reversion dynamics I split the normal diffusion process and the jump process as done in Equation 24:

$$\Delta S_t = \Delta S_t^Y + \Delta S_t^Z$$

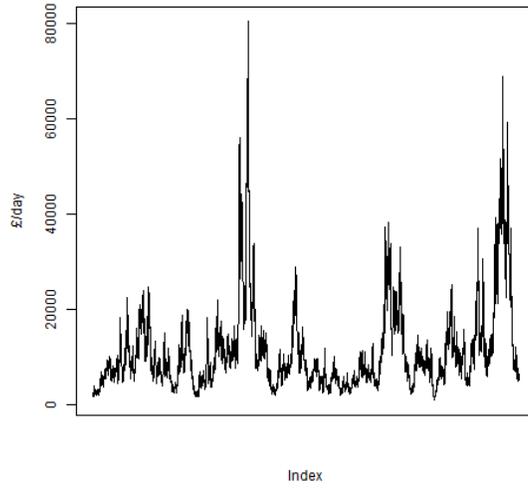
The discrete time representation of the normal diffusion process,  $S_t^Y$ , is

$$\begin{aligned} \Delta S_t^Y &= S_{t+1}^Y - S_t^Y = e^{(\mu - \alpha X_t)\delta t + \sigma(t)\sqrt{\Delta t}\epsilon_t} \\ S_{t+1}^Y &= S_t^Y + e^{(\mu - \alpha X_t)\delta t + \sigma(t)\sqrt{\Delta t}\epsilon_t} + g(t) \end{aligned} \quad (39)$$

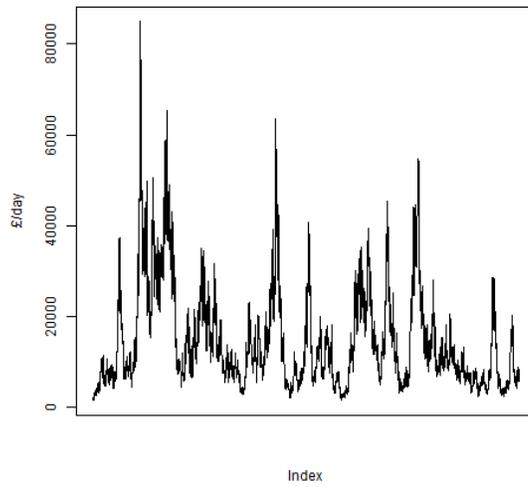
and the discrete time jump diffusion process,  $S_t^Z$ , with drift is

$$S_{t+1}^Z = S_t^Z + \alpha_Z S_t^Z + (J - 1)S_t dN_t. \quad (40)$$

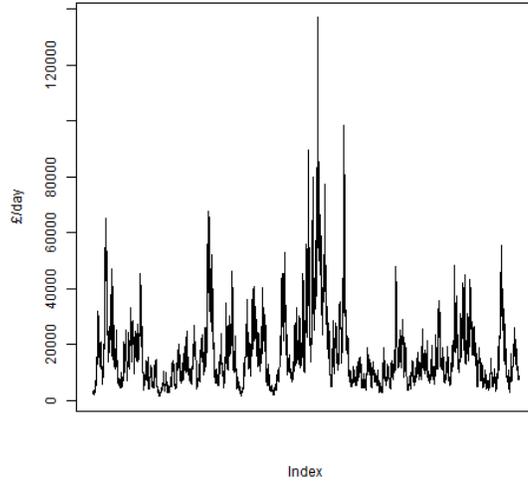
In Figure 19, 20, 21 and 22 I have plotted a simulated a path of each of the spot rates using the parameters estimated in Section 4. Each plot consists of 8 000 points and has a time step,  $\Delta t$ , of 0.1 such that time horizon corresponds to approximately 15 years.



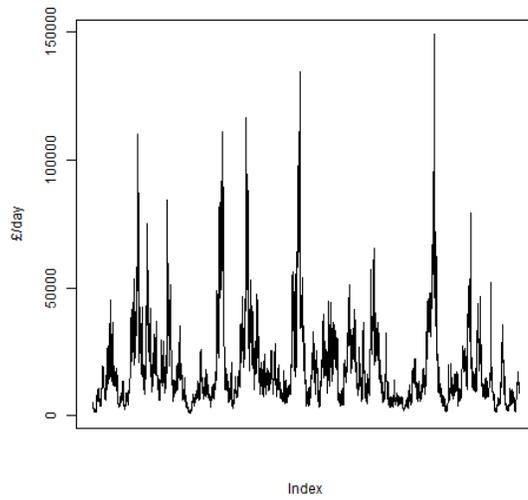
**Figure 19:** Simulated spot rate PSV 6-800m<sup>2</sup>



**Figure 20:** Simulated spot rate PSV >800m<sup>2</sup>



**Figure 21:** Simulated spot rate AHTS 13-18000bhp



**Figure 22:** Simulated spot rate AHTS >18000bhp

## 5 Application

Martinussen [18] focuses on the importance of lay-up and scrapping in valuation of tanker vessel. Just like in tanker markets, at any given time, there are three alternatives for an offshore vessel; operation, lay-up and scrapping. As suggested by Tvedt [30], the cash flow from PSV and AHTS-vessels have the same structure as an American option. The vessel is a continuous option on the cash flow, until the maturity to scrap the vessel and receive the value of the vessel as sold to demolition. The American option captures the value of the spot rate contingent claims on cash flow, but also the value of flexibility related to lay-up and scrapping of the vessel.

Adopting Tvedt's approach to pricing spot rate contingent claims, I can derive the solution to the extended problem where the underlying spot rate follows a geometric mean reverting jump diffusion process. As in [30] the model assumes that there are zero costs related to laying up a vessel and re-entering the market. It also ignores costs associated with re-classing and docking a vessel. The cash flow from operation and lay-up until a vessel is scrapped can be described by the continuous American call option

$$\begin{aligned} C_t &= \max(S_t - w, -m) \\ &= (S_t - w)\mathbb{1}_{[S_t - w > -m]} - m(1 - \mathbb{1}_{[S_t - w > -m]}) \end{aligned} \quad (41)$$

where  $S_t$  is the time charter equivalent spot rate,  $w$  is the operating costs excluding voyage related costs and  $m$  is the cost of keeping the vessel idle. The option described above is continuous in the sense that exercising the option and laying-up the vessel does not keep the investor from re-entering into the market with an option of the same payoff structure.  $\mathbb{1}_{[S_t - w > -m]}$  is an indicator function of the event that the cash flow from keeping a vessel operational is larger than cash flow of keeping the vessel idle. The optimal policy when this event does occur is to keep the vessel operational, and receive cash flow  $S_t - w$ . Otherwise it is optimal to exercise the option of laying-up the vessel with cash flow  $-m$ .

When the age of the vessel,  $t$ , reaches the maximum lifespan,  $T$ , the value of the vessel converges with the scrapping value  $\Psi_t$ . The vessel may be sold at any point in time before  $T$ . Assuming a risk neutral investor, the optimal policy is to scrap the vessel if the present value of  $C_t \leq \Psi_t$ . The time at which the vessel is sold or scrapped is equal to the stopping time,  $\tau$ , of the option  $C_t$ . The optimal policy of termination is, as in [30], given by

$$\tau = \inf(0 < t < T; \Phi_{t,s} \leq \Psi_t) \quad (42)$$

where  $\Psi$  is the market value of the vessel defined below. I.e. the optimal time of termination is the first point in time where the present value of the vessel as a going concern is less than or equal to the scrapping value of the vessel.

Assuming complete markets, or equivalently, that we can replicate pay-offs in all states of the economy, it is possible to hedge the cash flow  $C_t$ , and introduce the equivalent martingale measure  $\mathbb{Q}$ . As in [30] the market value of the vessel is then equal to the risk neutral present value of the cash flows generated from time  $t$  to  $\tau$ , given by

$$\Phi_{t,s} = \mathbb{E}^{\mathbb{Q}} \left[ \int_t^T e^{-r(u-t)} C_u du + e^{-r(\tau-t)} \Psi_\tau \right] \quad (43)$$

Assuming complete markets we can hedge the cash flow from  $C_t$  and estimate the value of a vessel by the use of arbitrage pricing arguments. The cash flow from  $C_t$  can be hedged by constructing a self-financing replicating portfolio, by trading in the underlying spot rate,  $S_t$ , and the risk free asset,  $B_t = e^{\int_t^T -r du}$ , with dynamics

$$dB_t = r_t B_t dt. \quad (44)$$

Let  $\varphi$  denote the amount invested in the underlying asset and,  $\theta$ , the amount in the risk-free asset. The replicating portfolio becomes

$$C_t = \varphi_t S_t + \theta_t B_t. \quad (45)$$

where the spot rate  $S_t$  has the same dynamics as in Equation 21. Thus, the dynamics of the replicating portfolio becomes

$$d(\varphi_t S_t + \theta_t B_t) = (\varphi \mu_{S_t} + \theta r_t B_t) dt + \varphi (\sigma_{S_t} dW_t + (J-1) S_t dN_t) \quad (46)$$

Assuming no jumps ( $\lambda = 0$ ), by Itô's Lemma,  $C_t$  has dynamics

$$dC_t = \left( \frac{\partial C}{\partial t} + \mu_S \frac{\partial C}{\partial S} + \frac{1}{2} \sigma_S^2 \frac{\partial^2 C}{\partial S^2} \right) dt + \frac{\partial C}{\partial S} \sigma dW_t. \quad (47)$$

I can make sure that the portfolio is replicating by ensuring the drift and dispersion terms are everywhere equal

$$dC_t = d(\varphi_t S_t + \theta_t B_t). \quad (48)$$

By manipulations of the above equation, I can find the portfolio weights that satisfies the self-financing replicating argument to arrive at the partial

differential equation in the Black-Scholes framework. The dispersion term is replicating if and only if

$$\begin{aligned}\frac{\partial C}{\partial S}\sigma dW_t &= \varphi\sigma_{S_t}dW_t \\ \varphi &= \frac{\partial C}{\partial S},\end{aligned}\tag{49}$$

and drift term is replicating if and only if

$$\begin{aligned}\varphi\mu_{S_t} + \theta r_t B_t &= \frac{\partial C}{\partial t} + \mu \frac{\partial C}{\partial s} + \frac{1}{2}\sigma_S^2 \frac{\partial^2 C}{\partial s^2} \\ \theta &= \frac{1}{r_t B_t} \left( \frac{\partial C}{\partial t} + \frac{1}{2}\sigma_S^2 \frac{\partial^2 C}{\partial s^2} \right).\end{aligned}\tag{50}$$

Inserting the portfolio weights,  $\varphi$  and  $\theta$ , back into Equation 45 to arrive at the Black-Scholes PDE

$$rC_t = rS \frac{\partial C}{\partial S} + \frac{\partial C}{\partial t} + \frac{1}{2}\sigma_S^2 \frac{\partial^2 C}{\partial S^2}\tag{51}$$

However, in the presence of jumps, it is not possible to hedge away the idiosyncratic risk due to jumps. This is because the replicating portfolio is a linear hedge, given an infinitesimal change in the spot rate, but the option price is a non-linear function of the spot rate. The Black-Scholes "hedge" is done in continuous time and requires an uninterrupted dynamic rebalancing of the portfolio given changes in the underlying spot rate, and consequently the option delta. Moreover, when jumps are present, the spot rate follows a discontinuous process, and the Black-Scholes hedge will not eliminate risk even in the continuous limit. As a consequence Merton [20] chose to leave the jump-risk unpriced, resulting in no change in the jump terms when changing the probability measure. A different method is to incorporate market price of jump-risk by using two options expiring at different maturities to hedge the underlying jump diffusion [23]. For application, see for instance Cheang [8].

I can adjust Tvedt's model [30] to allow for jumps in the spot rate. Let  $Y_t$  denote the state of the system

$$Y_t = \begin{bmatrix} u + t \\ S_t \end{bmatrix}\tag{52}$$

Then it follows that the risk-adjusted increments of  $Y_t$  is given by

$$dY_t = \begin{bmatrix} 1 \\ \mu_{S_t} \end{bmatrix} dt + \begin{bmatrix} 0 \\ \sigma_{S_t} \end{bmatrix} dW_t + \begin{bmatrix} 0 \\ (J-1)S_t \end{bmatrix} dN_t\tag{53}$$

The Itô diffusion  $Y_t$  has an infinitesimal generator  $\mathbb{A}$ , given by

$$\mathbb{A}f(t, s) = \frac{\partial f(t, s)}{\partial t} + \mu_{S_t} \frac{\partial f(t, s)}{\partial s} + \frac{1}{2} \sigma^2 \frac{\partial^2 f(t, s)}{\partial s^2} + \lambda(f(t, x + (t, x))f(t, x)). \quad (54)$$

From Equation 43, it follows that the present value of a vessel,  $\Phi_{0,s}$ , is a solution to a combined Dirichlet-Poisson problem (see, for instance, [22]). The value function  $\Phi_{t,s}$  satisfies the two conditions

$$\mathbb{A}\Phi_{t,s} = -e^{rt}C_t, \quad \text{when } \Phi_{t,s} > \Psi_t \quad (55)$$

$$\lim_{t \rightarrow \tau} \Phi_{t,s} = e^{rt}\Psi_t, \quad \text{when } \Phi_{t,s} \leq \Psi_t \quad (56)$$

Assume that the function for the market value of a vessel can be expressed as  $\Phi_{t,s} = V_t e^{-rt}$ , when  $\Phi_{t,s} > \Psi_t$ . Then it follows from Equation 55 that the value of the vessel in this interval satisfies the following partial differential equation

$$-rV + \frac{\partial V}{\partial t} + \mu_S \frac{\partial V}{\partial S} + \frac{1}{2} \sigma_S^2 \frac{\partial^2 V}{\partial S^2} + \lambda \mathbb{E}[V(J) - V] dN_t = rC \quad (57)$$

where  $\mu_S$  is the drift and  $\sigma_S$  is the standard deviation of the underlying spot rate. Thus, if we assume that we are able to construct risk neutral portfolios in the underlying vessel the spot rate dynamics becomes equal to those in Equation 32 and the partial differential equation becomes

$$-rV + \frac{\partial V}{\partial t} + (r - \tilde{\lambda}\tilde{\kappa})S_t \frac{\partial V}{\partial S} + \frac{1}{2} \sigma^2 S_t^2 \frac{\partial^2 V}{\partial S^2} + \tilde{\lambda} \mathbb{E}_J^{\mathbb{Q}}[V(J) - V] dN_t = rC \quad (58)$$

The optimization problem given by the equations 55 and 56, can be solved numerically. A closed form solution, is to my knowledge, not available. Moreover, when departing from the simple geometric Brownian motion, and incorporating mean reversion and jumps in a fairly complex pay-off function, closed form solutions becomes very hard, if not impossible, to obtain. The performance of the numerical method applied to such problems depends ultimately on the ability of the model to capture the dynamics of the spot rate process.

A numerical solution to the problem can be obtained by the Monte Carlo method. This method involves simulating random paths of the spot rate according to the procedure in Section 4.4, with the estimated parameters

from Section 4, to arrive at different present values of a vessel given by the problem in equations 55 and 56. Given the central limit theorem and the law of large numbers, the mean of the implied probability density function of vessel values will converge with the true value of a vessel as the number of simulated paths goes to infinity.

## 6 Conclusions

In this thesis I have analysed some of the most significant characteristics of the spot rates for PSV and AHTS-vessels. I have proposed a model able to capture these dynamics, estimated the parameters for the North Sea market, and derived a framework for pricing of PSV and AHTS-vessels.

The empirical analysis revealed a tendency of the spot rate to exhibit frequent jumps with greater magnitude than what the Normal distribution would predict. By incorporating a jump process I was able to capture these price movements, as shown in Section 2.1. Furthermore, the analysis in Section 2.2 confirmed existence of statistically significant mean reversion for all the segments. As discussed in Section 3.2, spot rate jumps tend to revert quickly while random standard normal fluctuations revert rather slowly. In order to account for this phenomenon I estimated different reversion rates for the normal diffusion process and the jump process in Section 4.2. I also identified a significant seasonal component in the spot rate in Section 2.2, which was estimated by using a Fourier series of order  $K$  to arrive at a deterministic seasonal function for the spot rates in Section 4.1. In Section 2.4, assuming that spot rates are log-normally distributed I was able to capture the level effect in the conditional variance and consequently some of the autocorrelations in the conditional variance due to this level effect.

The characteristics revealed in the empirical analysis was then used to derive an extension to the geometric Mean reversion model used by Tvedt [30], incorporating both jumps, seasonality and two-regime mean reversion, in Section 3. In Section 5, the model was then implemented in Tvedt's framework for pricing of VLCCs, where the cash flow is modelled as a continuous American call option. The option consequently captures the value of the spot rate contingent claim on cash flow, and the value of flexibility related to lay-up and scrapping of the vessel. Furthermore, I derive the partial differential equation satisfying the value function of the vessel, where the solution to this problem can be solved numerically. Closed form solution for this optimization problem does not exist. Moreover, when departing from the simple geometric Brownian motion, and incorporating mean rever-

sion and jumps in a fairly complex pay-off function, closed form solutions becomes very hard, if not impossible, to obtain. The performance of the numerical method applied to such problems depends ultimately on the ability of the model to capture the dynamics of the spot rate process. Even though, the proposed model is able to capture the most significant characteristics of the spot rate, there are still significant linear and non-linear autocorrelations left in the residuals. This confirms the existence of short-term momentum in the conditional mean of the spot rate, represented by trends caused by shipping cycles, mean reversion and seasonality. Such trends violates the theory of efficient markets, but are able to persist in the short-run because spot rates are non-tradeable assets and therefore arbitrage portfolios are not easily obtained. Following the discussion in [4], there are also transaction costs for both charterer and shipowner related to delaying fixtures when the spot rate is trending.

According to the McLeod-Li test there are also significant autocorrelations in the conditional variance. The same test applied to the log-transformed series revealed that only the largest AHTS-segment showed significant lag effects in the conditional variance not due to the level effect. This implies that the geometric mean reversion model is able to capture all the non-linear dynamics in the conditional variance of the spot rate for PSV-vessels and the smaller AHTS-segment. The largest AHTS-segment, on the other hand, needs an autoregressive conditional heteroscedastic (ARCH)-model to be fully specified in the moving average part of the process.

However, there is a trade-off between analytical tractability and prediction accuracy. The former is of great importance, but because the model is already solved in a numerical framework the analytical tractability is arguably less relevant and the value of a more complex model is worth quantifying. A model incorporating higher order continuous time ARMA-terms for the linear dynamics and ARCH-terms for the non-linear part may be appropriate.

# Appendix

## A Maximum Likelihood Equations for the Ornstein Uhlenbeck-Process

$$\lambda = \frac{S_1 S_{22} - S_1 S_{12}}{n(S_{22} - S_{12}) - (S_2^2 - S_2 S_1)}$$

$$\phi = -\frac{1}{\delta} \ln\left(\frac{S_{21} - \mu S_2 - \mu S_1 + n\mu^2}{S_{22} - 2\mu S_2 - S_2 S_1}\right)$$

$$\tilde{\epsilon}^2 = \frac{1}{2}[S_{11} - 2\alpha S_{21} - \alpha^2 S_{22} - 2\mu(1 - \alpha)(S_1 - \alpha)(S_1 - \alpha S_2) + n\mu^2(1 - \alpha)^2]$$

$$\epsilon^2 = \tilde{\epsilon}^2 \frac{2\lambda}{1 - \alpha^2}$$

where  $\mu = \lambda\phi$  and

$$\alpha = e^{-\lambda\delta}$$

$$S_2 = \sum_{i=1}^n S_{i-1}$$

$$S_1 = \sum_{i=1}^n S_i$$

$$S_{22} = \sum_{i=1}^n S_{i-1}^2$$

$$S_{21} = \sum_{i=1}^n S_{i-1} S_i$$

$$S_{11} = \sum_{i=1}^n S_i^2$$

$$\mu = \lambda\phi$$

$$\alpha = e^{-\lambda\delta}$$

$$S_2 = \sum_{i=1}^n S_{i-1}$$

$$S_1 = \sum_{i=1}^n S_i$$

$$S_{22} = \sum_{i=1}^n S_{i-1}^2$$

$$S_{21} = \sum_{i=1}^n S_{i-1} S_i$$

$$S_{11} = \sum_{i=1}^n S_i^2$$

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