Optimal Equity Based Incentives
A Norwegian Perspective

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Abstract

We analyze optimal executive compensation in a principal agent framework using two sample firms from the Norwegian market, design and solve a bi-level principal agent optimization problem. Our analysis reports three important findings. First, our unambiguous results show that stock options should be a part of the optimal contract in addition to a certain base salary. The options, rather than restricted shares produce the right incentives. Second, indexed options should be granted instead of traditional options in cases where firms have a volatility higher than the market index and where it is a good correlation between those two. Third, our solutions show that exercise price of options should be at or near the stock price at the granting time. We confirm the robustness of our model by optimizing for alternative risk aversion degrees of the CEOs and the disutility factor associated with their effort.
Sammendrag

Preface

This paper is the final master thesis at Norwegian University of Science and Technology, for the 5-year MSc study in Industrial Economics and Technology Management, specialization in Applied Economics and Optimization, with Computer and Communication Technology.

As my master thesis, I mainly wanted to write something more technical and use my knowledge in optimization and programming. Also, choosing a highly interesting topic of CEO pay, incentives and principal agent framework motivated my work even more. In this paper, like a majority of the research on incentive contracts, I focus solely on chief executive officer, even though Corporate Management consists of more than just the CEO.

I would like to thank my supervisor Einar Belsom for great professional support, my friends and family for unconditional love and for their support in various difficult situations. Finally, I am sure this work will be of interest and as a guideline for further research.
# Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td>iii</td>
</tr>
<tr>
<td>1 Introduction</td>
<td>1</td>
</tr>
<tr>
<td>2 Optimal compensation scheme</td>
<td>5</td>
</tr>
<tr>
<td>2.1 Intrinsic and extrinsic motivation</td>
<td>5</td>
</tr>
<tr>
<td>2.2 Principal agent framework</td>
<td>7</td>
</tr>
<tr>
<td>2.3 Prior research</td>
<td>10</td>
</tr>
<tr>
<td>3 Our principal agent model</td>
<td>15</td>
</tr>
<tr>
<td>3.1 Assumptions</td>
<td>15</td>
</tr>
<tr>
<td>3.2 The complete model</td>
<td>22</td>
</tr>
</tbody>
</table>
CEO compensation contracting is a heated and controversial topic. The board members should in theory design these contracts on behalf of the shareholders for maximizing the firm’s value. However, this has not always worked as intended and optimal contracting is still an unresolved issue.

There has been lot of focus on equity-based compensation for the top executives the last decades. The compensation scheme for the CEOs have in countries like the US almost always stock options as an essential part. In contrast, this type of compensation is not particularly common in Norway. Here, mainly bonuses and restricted shares are granted. While the CEO pay and wealth have increased tremendously in the US, the Norwegian CEOs are in average having a salary equal to a small fraction of the American CEOs’ salary.

Even with this highly increasing popularity of equity-based compensation, there has been criticisms against this kind of rewarding system.
While most professionals within the field point out that the stock options and shares provide a great incentive to the CEO matching shareholders’ goals, others claim the opposite (Meulbroek 2001, Hall and Murphy 2002). Although the size of the compensation is only slightly related to the executive’s effort, some defend these cases with extremely high CEO pay with that most of the CEOs are paid for their talents and skills (Gabaix and Landier 2008).

On the road to the optimal contract, there has been a lot of talk about filtering exogenous risk, i.e. external and uncontrollable factors like market fluctuations and inflation. However, this is rarely taken care of in the actual contracts. This absence has been of concern to the researchers and they have often suggested indexed stock options and shares without any breakthrough (Bebchuk and Fried 2004). Perhaps this is due to the shortcomings in corporate governance (e.g. Bertrand and Mullainathan 2001) or because of the trade-off between risk and incentives in the CEO’s compensation (Maug et al. 2012).

In the agency theory for executive compensation contracts, the literature is mainly based on that the shareholders (principal) are risk-neutral while the CEO (agent) is risk-averse. Among others, He (2008) shows that the choice of the CEO as risk-averse can be qualitatively explained, through the negative relation between incentives and the firm value.

We review relevant literature on incentive contracts and principal-agent framework for compensating executives, and design a standard principal agent model based on Armstrong et al. (2007), Dittmann et al. (2010) and Maug et al. (2012) as the main sources to determine optimal equity-based compensation scheme for Norwegian executives. Furthermore, we improve these previous works in several ways.

Our compensation contract can consist of a certain amount base salary, restricted shares and stock options. We take into account that the CEO has pre-existing wealth and invested some of it in the firm. The con-
tracting period is a year, in line with most of prior researches, while the options granted have a maturity of 5 years. Our special case, valuing the options according to their market value, gives the CEO possibility to sell those call options after the contracting period.

Our contribution to the field of optimal CEO pay consists of a robust model, making use of the most important and latest theories proposed by previous researches. The most important contribution is our bi-level model taking both the principal’s and the agent’s preferences into account. This is something that have been absent in most of the previous works, including all of Dittmann & co.’s researches. These works implement a first order approach, where they assume that the CEO already performs at the optimal effort level and also fix various other variables. Another important contribution is that we include restricted shares, indexed options and traditional options, which gives a great picture of the overall equity-based compensation. Last, but not least, our model implements the CEO’s ability to affect the firm’s performance, where most of the previous works do not.

The stochastic problem in our model is transformed and simplified into a deterministic equivalent, giving us a model solvable in a matter of minutes. This model is also greatly expandable, thus being of highly relevance in further research on optimal CEO compensation scheme.

The rest of this paper is structured as follows. Chapter 2 presents some theory and highlights previous and related works. Chapter 3 introduces our model and methodology. The results, evaluation and further analysis is presented in chapter 4. Finally, chapter 5 concludes with discussion, weaknesses of our approach, overall assessment and possible directions for future work. Some technical materials, including graphs, tables and the programming code, are gathered in the appendix.
Chapter 2

Optimal compensation scheme

What is the reason behind an action? What does motivate and induce incentives to a specific person? These are two fundamental questions in a principal-agent framework where the principal wants the agent to do some work, by motivating or giving incentives to choose the best action. In this chapter, we will look at the importance of motivation and incentives, before introduction the principal-agent framework. We close this chapter by highlighting previous related works on this field.

2.1 Intrinsic and extrinsic motivation

There are numerous interpretations and no common accepted definition of "motivation". According to Arnold et al. (1991), the word motivation is derived from "motive", which refers to a purpose or intention, a mo-
tive, as the cause behind an action. Likewise, Kaufmann and Kaufmann (1998) defines motivation as the driving force behind human behavior; it is something that causes us to act, to maintain this activity and give it meaning and purpose. This driving force arises from our desires and our need to perform a specific action.

Traditional and behavioral economics have two very different views on this topic. The former assumes that each individual will maximize his own profit or utility, i.e. take rational decisions that best reflect his financial interests. Behavioral economics, on the other hand, takes into account the complexity and limitations of the human being. There is a cause, goal and purpose, together with the degree of intensity behind our actions. Thus, behavioral economics try to explain why two people with seemingly the same background, knowledge and duties, perform differently.

Behavioral economics distinguishes between intrinsic and extrinsic motivation. If a person is motivated to give an effort because of something else than the financial compensation, it is seen as intrinsic motivation. A such motivated employee will genuinely care about his work, find his tasks meaningful and identify himself with the organization. Thus, he will likely perform and act accordingly to the organization’s goals and interests (Murdock, 2002, Akerlof and Kranton, 2005).

Contrary, the level of effort for an extrinsic motivated person will depend on the reward he gets for his performance. This is what we usually associate with the word incentive; a tangible or verbal reward for a specific behavior, designed to encourage this behavior. Extrinsic incentives motivates an employee to perform a certain action or activity, rather than something else. An example is a real estate agent, who has incentives to sell for a highest possible amount, as a result of his provision-based compensation. In industry, financial compensation is the most commonly used incentive scheme, but in theory it can be any factor, financial or non-financial.
2.2 Principal agent framework

An organization (the principal) appoints a person (the agent) to perform one or more tasks. Principal agent theory, an important building block in the performance management and incentive schemes, assume that an employee will shirk his duty if the process or the results are not related to his financial self-interest. Furthermore, it is assumed that only through monitoring, combined with well-defined contracts, that this form of evasion can be averted (Simon, 1991).

As we see, this is a great simplification of the reality, without taking into account the agent’s identity and intrinsic motivation, among other factors. Nevertheless, this framework is very extensive, with numerous lines of research that attempt to solve various dimensions of the principal agent problem. We focus in this section on three main problems this frameworks tries to solve, namely information asymmetry, conflicting objectives and risk sharing. We also look at different types of contracts and prepare for our main work.

"By definition the agent has been selected for his specialized knowledge and the principal can never hope to completely check the agent’s performance" (Laffont and Martimort, 2001, page 12).

Moral hazard, an important aspect in the principal agent theory, is about that a person with full information can take advantage of the other party because the latter possess less information. In most cases, only the agent knows how much effort he is putting into the work. Such information asymmetry will make it difficult and costly for the principal to observe the agent’s actions, and thus be difficult to reward him properly as he deserves (Holmstrom, 1979).

Goal incongruence arise when the agent and the principal have different goals. The agent wants to maximize his utility by giving an effort so
that it maximizes his reward. The principal, on the other hand, has objectives to maximize the firm’s profit, which in turn is dependent on the agent’s effort.

Third, there are usually some uncertain external factors related to the agent’s performance. It may be fluctuations in the market, popularity, weather conditions or other unforeseen circumstances. This means that the agent is exposed to some kind risk he can not control, thus giving rise to the risk-sharing problem. We know that people have different risk preferences, and as a basis in the principal agent theory, it is assumed that the agent is risk averse, while the principal is risk neutral. In addition, different agents have different utility functions, i.e. utilities depending on the degree of risk aversion.

Agency theory looks therefore into all these factors and aim to optimize the incentive scheme. It tries to solve the problem with goal incongruence by designing the best contract that links the agent’s compensation to the performance. It is also common for the least risk averse, the principal, to take most of the risk. Furthermore, to deal with information asymmetry and moral hazard problem, including risk sharing, the principal can offer a set of optimal contracts for various utility functions for the agent. Thus, the agent choose the contract best suited his preferences and disutilities of effort, which also helps the principal understand the agent’s preferences.

The principal can offer many different types of contracts. We limit ourselves to look at (1) fixed salary, (2) pure performance-based, and (3) fixed salary including performance-based compensation. The aim of the principal is to design and offer an optimal contract the agent will accept. In order to provide performance-based compensation, the agent’s actual performance needs to be measured, which can be very difficult. A number of research articles, Holmstrom and Milgrom (1991), Prendergast (1999), Baker (2002), among others, points out that any performance measure will include the effects of random, uncontrollable
and external factors that shape the final result. A good performance measure will therefore have a low impact of such factors and most of the actual performance. This can result in more motivated agents who will give higher effort when they know that the principal can control the performance and also expose them to less risk.

We have a firm with its board of directors who acts accordingly to the interests of the shareholders. This is the principal. Then we have executives who perform tasks for the firm. In reality, every executive are agents, but we specialize on the case where the chief executive is the agent. The principal wants to maximize the firm’s value while the agent maximizes his utility. The firm’s value depends more or less on the agent’s effort. There is of course some external factors which the CEO can not control, market fluctuations and trends are two examples. The agent’s utility function tells us that his performance depends on the compensation.

It can be shown that the agent, in most cases, will not give an optimal effort if he gets compensated with just a fixed base salary. Since it is costly for him to put in more effort than necessary to fulfill the contract, he will deviate from the firm’s goal, which is to maximize its profit (Heinrich and Marschke 2010). Notably, these are the cases where the intrinsic motivation does not influence.

By just compensating with a fixed salary, the principal will have difficulties to control the agent’s efforts. This is preferred only in cases where the agent’s effort has no impact on the result. If however, the CEO receives a compensation solely depending on the firm’s value, for instance the firm’s stock price, the principal can to some extent get more out of the agent. The latter will give higher effort to increase his compensation. Such a performance-based rewarding scheme is therefore more preferable. However, with just a performance-based compensation, the CEO risks to get zero, or even negative, salary. Even when he gives a decent effort, there are factors in the stock price he can not control, thus
giving no guarantee for an increased firm value. Here, we will encounter problems with risk sharing. The principal is not aware of the agent’s risk preferences, and with uncertain external factors present, it will be impossible to create an optimal contract. The principal is likely able to offer a performance-based compensation that the CEO accepts, but it will be far from optimal as the agent will demand higher salary to take the risk.

Performance-based compensation, without any fixed base salary, is not preferable in work situations where the agent is exposed to risk. Thus, combining fixed base salary and a reward depending on the performance, is one of the simplest and most popular compensation schemes (Heinrich and Marschke, 2010). The variable performance-based part aims to induce incentives to give a higher effort, but is balanced with a base salary for not pushing too much risk on the agent (Baker, 1992).

There are a number of performance measures, each befitting various cases. In our road to optimal executive contract, we will focus on equity-based measures. These measures, which are based on the stock price, will reduce the goal incongruence between the agent and the principal. By granting shares or stock options as the performance-based part, the CEO will likely get incentives to perform at a higher level, which increases his compensation depending on the stock price. Next, we will highlight previous work done by researchers and other experts on this topic.

### 2.3 Prior research

We look at some of the main prior research on optimal executive contracts, based on fixed salary and equity-based compensation for inducing incentives. Particularly, as it have been very popular in the US to grant stock options, this issue is being taken up and it is asked whether the
observed contracts can be justified. Further, they discuss the question if the CEO pay is inefficient and provide some interesting results for different types of models and risk preferences of the executives.

Of the earliest work we look at, Hall and Murphy (2002) uses the certainty equivalent approach in line with R. Lambert and Verrecchia (1991), and find optimal solutions where restricted shares dominates stock options. Here, the CEO is assumed to have preferences with constant relative risk aversion (CRRA), and the stock price is lognormally distributed. The major weakness in this model is however that their analysis is based on partial equilibrium, i.e. that it only takes into account certain parts of the contract. They try to grant the CEO equity-based compensation by dropping fixed base salary. Thus, this model does not include the actual incentives provided by the options.

Next out is Aseff and Santos (2005), where the authors consider if stock options induce incentives to increase performance. Here, the CEOs can give either high or low effort, and it is assumed that the agent’s problem can be addressed with the first order approach (FOA). Furthermore, the compensation contract can consist of only fixed salary and stock options, so no restricted shares. They show that stock options should be a part of the optimal contract to make the CEO give a high effort. The cost of moral hazard (when the CEO chooses the low effort) is high for the principal, but can be prevented by granting options and thus inducing incentives for high effort.

Similarly, Kadan and Swinkels (2006), concludes that stock options should be part of the optimal contract where the non-viability risk (a probability that the stock price becomes equal to zero) is low, which is true in most cases. In addition to that the stock options are preferred to restricted shares, they also show that low bankruptcy risk is correlated with less use of restricted shares. A major weakness in their model is that the compensation scheme can consist of either options or shares, but not both. Another point to note is a constraint with minimum
Dittmann and Maug (2007) is one of the reference papers in recent times, which in spirit of Holmstrom (1979), develops a single-period standard principal-agent model where the agent is risk averse and have preferences with CRRA. Their optimal solutions differ considerably from the observed compensation schemes. They report that optimal contracts should have no or minimal stock options in the scheme. Further, the base salaries should be lower and incentives are to be provided through restricted shares. Even though they use a number of realistic futures, they assume that the CEOs in observed compensation scheme already give an optimal effort and try to minimize the firms’ cost for the given effort-level. They also use the FOA and with that consider only a partial optimization problem; whether if the observed compensation contracts are optimal.

As their results contradict with the empirical estimations, (the CEOs in the observed cases are being granted a lot of stock options), this have been origin to several different models. Dittmann et al. (2010) looks at compensation contracts when executives are loss-averse. Their optimal contracts explain the observed contracts, the compensation schemes granting stock options and including a high base salary. They find out that the more loss-averse the agent is, the more options are granted. This is because an increase in loss aversion also increases the risk-tolerance. As we see, this is the total opposite of when the CEO is just risk-averse.

Further, Dittmann and Yu (2011) considers a model where a risk-averse CEO is provided with risk-taking incentives in addition to effort incentives. Calibrating the model to the observed contracts, they find that this matches better with the observed contracts than a model without risk-taking incentives. They also find that the optimal contract protects the CEO from losses for bad outcomes and that the options should be granted in-the-money in these cases.
In their latest work, Maug, Dittmann, and Spalt (2012), include indexed options in the compensation scheme. Their findings do not support indexing equity-based compensation. For most of their CEOs, indexed options take away their incentives because they reduce the chances of bad outcomes.

As we mentioned above, the common factor for most of this research, especially the works by Dittmann & co., is that all of them use actual observed contracts as reference, i.e. they assume the CEO performs an optimal action in the observed cases, and try to reduce the firm’s cost for the same action. Using a first order approach like this can be problematic. The principal can likely increase the CEO’s effort level and also maintain the same cost. Thus this differs from a typical agency model where we should consider both the agent’s and principal’s optimization, not just the principal’s.

Unlike these, Armstrong et al. (2007) solves a complete bi-level optimization problem between the agent and the principal. They find that stock options are almost always an important part of the executive compensation scheme. A tiny weakness in their model is that they consider only options granted at-the-money.

We see from all these works that there is ongoing research on whether to use stock options in an optimal executive compensation scheme. To get a better insight, we apply some of the approaches from this literature, especially Armstrong et al. (2007) and Maug et al. (2012), and construct our own agency model.
Chapter 3

Our principal agent model

Our principal agent model is a contracting problem between the CEO (agent) and the shareholders (principal). The agent’s action is unobservable and costly, but there is a positive correlation between the agent’s effort and the firm’s value at end of the period. The general optimization model below introduces our problem:

\[
\begin{align*}
\max_{\phi, n_{st}, n_{or}} & \quad z = n_{tot} \cdot S_T(a) - \pi_T \\
\text{s.t.} & \quad EV(W_T(a)) - D(a) \geq U \\
& \quad a \in \arg\max_{a_i} \{EV(W_T(a_i)) - D(a_i)\}
\end{align*}
\]

(3.1) (3.2) (3.3)

3.1 Assumptions

The principal maximizes the firm’s expected value \(z\); total shares outstanding \(n_{tot}\), times the stock price \(S_T\) at end-of-period T, less the
agent’s total compensation, $\pi_T$. Latter consists of fixed base salary $\phi$, restricted shares $n_{sr}$ and stock options $n_{or}$. The first requirement for the principal to satisfy is to offer a compensation which outnumber the agent’s outside options, namely his reservation utility $U$. This is taken care of in the restriction 3.2, where $EV$ is the expected wealth-utility. Both the agent’s expected end-of-period wealth $W_T$ and his disutility $D$ is function of his effort $a$. The restriction 3.3 shows that the agent will choose his optimal effort $a$ according to his expected utility for the corresponding end-of-period wealth $W_T$.

**Agent’s wealth**
The agent’s end-of-period wealth $W_T$ consists of his pre-existing wealth and his total compensation for this period, $\pi_T$.

$$W_T = W_0(1 - \omega) * \exp(r_f T) + n_{su} S_T + \pi_T \tag{3.4}$$

The agent has a fraction $\omega$ of his pre-existing wealth invested in the firm, in terms of unrestricted shares $n_{su}$. The rest, $1 - \omega$, is invested at risk-free rate. The CEO’s compensation, $\pi_T$, is given by:

$$\pi_T = \phi * \exp(r_f T) + n_{sr} S_T + n_{or}(\psi * JS + (1 - \psi) * BS) \tag{3.5}$$

Here, the fixed salary $\phi$ is invested at the risk-free rate. The CEO can be granted an amount of restricted shares $n_{sr}$ and stock options $n_{or}$ as a part of the compensation. The value of the latter is given by Johnson and Tian (2000) for indexed options and Black and Scholes (1973) for traditional options. The indexation-level $\psi \in [0, 1]$ determines the fraction of options indexed.

**Stock Price**
The firm’s expected stock price at end of the period is defined as a
function of the CEO’s effort $a_t$, which in turn is represented as the total expected return $\mu_p$. The end-of-period stock price $S_T$ is given by:

$$S_T = S_0 \times \exp\{ (\mu_p - \frac{\sigma_p^2}{2})T + u_p \sigma_p \sqrt{T} \}, \quad (3.6)$$

$S_0$ is the firm’s stock price at $t=0$ and $\sigma_p$ is its volatility. We also include a part with uncontrollable noise, $\exp\{u_p \sigma_p \sqrt{T}\}$. Here, $u_p$ is a number from the normal distribution of $\mu_p$ and $\sigma_p$. As it can be seen, we formulate $S_T$ so that the CEO affects both the mean and standard deviation of the stock price distribution. Next, we define a market price $M_T$, in the same manner as above.

$$M_T = M_0 \times \exp\{ (\mu_m - \frac{\sigma_m^2}{2})T + u_m \sigma_m \sqrt{T} \}, \quad (3.7)$$

We need this market index price later, for valuing indexed options. $M_0$ is the market price at $t=0$, $\mu_m$ and $\sigma_m$ are market’s expected return and volatility, respectively. $u_m$ is analogous to $u_p$. Both are correlated with a coefficient of correlation $\rho$, and the CAPM gives us:

$$\beta = \rho \frac{\sigma_p}{\sigma_m}, \quad \mu_p = r_f + \beta (\mu_m - r_f) \quad (3.8)$$

**Agent’s effort**

One of the crucial choices in this model is the definition of the agent’s effort. This effort is basically not observable, but the principal can to some extent track it, as it will affect the end-of-period stock price $S_T$. The principal has then to design a contract keeping in mind that the agent will perform an action which maximizes his utility. A simple but power-consuming approach is to define the effort as a continuous value
greater than zero, $0 < a < \infty$. To reduce the complexity of our model, we define 100 different actions the agent can take. This goes in line with Armstrong et al. (2007), where these discrete values for the agent’s effort, $a_i \in \{1, 2, ..., 99, 100\}$, provide a perfect correlation with $\mu_p$.

We define the total return $\mu_p$ as a piecewise linear function of the agent’s effort. At the lowest possible effort, $a_i = 1$, we set the firm’s return equal to the risk-free rate. This will give a negative $\mu_p$ since the risk-free rate is much lower than the firm’s cost of capital. Next, we assume that at the maximum effort level, the highest return is 3 times the firm’s cost of capital. In between the highest and lowest effort level, we need a breakpoint for zero return. We get that if we set the return equal to the cost of capital, giving us $\mu_p = 0$. As shown in figure 3.1, this is located at $a_i = 30$.

![Figure 3.1: Total return as a function of effort](image)

This seemingly arbitrarily scaling of total expected return $\mu_p$ will appear a bit controversial at the first glance and is something that should be discussed. At the maximum effort level 100, the total return of our first firm is 14 %. For our second firm, this equals to 18 %. We defend this
choice by referring to Armstrong et al. (2007) and ValueLine (2012). The former report from the high long-term Value Line forecast of annualized return, their mean being equal to 20.05 % with a standard deviation of 8.76 %. As I was unable to find any similar forecast for our Norwegian firms, I estimate a lower value for our first firm and 4 % higher value for our second firm. As we will see later from the data of our firms, this fits well when it comes to the sizes of our firms. Additional we needed to scale a bit down when we compared our Norwegian firms to the average American firms’ size. Noteworthy, there is still doubtful whether there is this strong connection between the CEO’s effort and the firm’s return. Nevertheless, we will go on with this assumption in line with previous research.

CEO’s utility
The agent’s expected utility is additively separable in wealth and effort. Expected utility of wealth, $EV(W_T(a_i))$ and the disutility of effort, $D(a_i)$ gives us the CEO’s total utility:

$$EU(W_T(a_i)) = EV(W_T(a_i)) - D(a_i)$$  (3.9)

In a principal-agent framework, it is usually assumed that the principal is risk-neutral and the CEO is risk-averse (Eisenhardt, 1989, Prendergast, 1999). One of the most used utility function for the agent’s wealth is the constant relative risk aversion, CRRA:

$$V(W_T(a_i)) = \frac{1}{1 - \gamma} W_T(a_i)^{1-\gamma}$$  (3.10)

Here, $\gamma$ is the coefficient of relative risk aversion. The agent is risk-neutral if it equals to zero. The higher the value, the more risk-averse the agent is. Initially, we set $\gamma = 2$. Maug et al. (2012) and Armstrong et al. (2007) are two of several prior works which support this choice.
We define the disutility of effort function as given in the equation above. The choice of this is a controversial and well-debated topic in the compensation literature. While \(a_i^2\) is commonly used, the same can not be said about the value of the disutility scaling parameter \(\lambda\). This parameter has to ensure that \(D(a_i)\) is of the same order of magnitude as \(V(W_T(a_i))\), and also preserve the significance of the disutility of effort. We will later on discuss the choice and calibration of this parameter.

**Stock options**

The CEO is granted a composition of traditional stock options and indexed stock options. The traditional stock options’ payoff is valued accordingly to Black & Scholes formula for call options. The latter is based on the performance and links the strike price to an equity index (Johnson and Tian [2000], Wu [2002]).

\[
BS = S_T * N(d_1) - Ke^{-r_f \hat{T}} N(d_2),
\]

\[
d_1 = \ln\left(\frac{S_T}{Ke^{-r_f \hat{T}}}\right) + \frac{\sigma_p \sqrt{\hat{T}}}{2}, \quad d_2 = d_1 - \sigma_p \sqrt{\hat{T}},
\]

Equation 3.12 gives us the Black & Scholes valuation for stock options. We choose dividend-protected options, as dividend pay will just reduce the firm’s value and also the valuation of both traditional and indexed options. \(\hat{T}\) is options’ maturity, \(N\) is the cumulative standard normal distribution function and \(K\) is the strike price. In our initial solution, we set \(K = S_0\), which gives at-the-money options at the time they are issued.
Noteworthy, we set the maturity to go further beyond the contracting period, to a maturity of 5 years. Additionally, we value the Black & Scholes options using the end-of-period stock price $S_T$, thus making these traditional options related to the stock price at the end, and not the initial stock price as sometimes used in old valuations. Both these modeling choices show that we get the market value of the options, depending on the $S_T$. This also implies that the CEO can sell the options after the end of the contracting period of an year. Even though these choices are not widely used, they are greatly of interest to evaluate. We expect this dependency with $S_T$ will provide the CEO incentives for giving a better performance and thus gaining a higher stock price $S_T$.

Indexed options are valuated using Johnson & Tian’s formula [2000], in the same procedure as Schnusenberg and McDaniel [2000], Duan and Wei [2003] and Meulbroek [2001] among others.

\[
JT = S_T * N(d_1^{indx}) - H_T * N(d_2^{indx}), \quad (3.13)
\]

\[
d_1^{indx} = \frac{\ln(S_T/H_T) + 0.5\sigma_I^2 \hat{T}}{\sigma_I \sqrt{T}}, \quad d_2^{indx} = d_1^{indx} - \sigma_I \sqrt{T},
\]

Here, we can see many similarities to the Black & Scholes formulation. For traditional options, we have a simple strike price, depending only on whether the options are granted in-the-money (ITM), at-the-money (ATM) or out-of-the-money (OTM). Here, we have a more complicated strike price, $H_T$, the value of the index exercise price.

\[
H_T = S_0 \left( \frac{M_T}{M_0} \right)^\beta \exp\{\eta \hat{T}\}, \quad (3.14)
\]

where $\eta = (1 - \beta)(r_f + 0.5\rho_s \sigma_p)$. $\beta$ and $\rho$ are given from equation 3.8 on page 17. To vary the exercise price, we can replace $H_T$ with $\xi H_T$, where
ξ ≤ 1 if options are granted ITM, ξ = 1 if ATM and ξ ≥ 1 if OTM. As we see, the indexed options also have a maturity different from the contracting period, which gives us similar valuation choice as we used for traditional options.

### 3.2 The complete model

In this section, we present our complete optimization model. For the sake of clarity, we do not include every bi-formulations presented in the previous section. Both $S_T$, $W_T$ and $\pi_T$, which are dependent on the agent’s chosen effort, are expected values. The indexation degree, $\psi$, which is omitted here, is also a variable as mentioned earlier.

**Sets**

$N$ set of actions

**Parameters**

- $a_i$ action i
- $n_{tot}$ total shares outstanding at $t=0$
- $S_T(a_i)$ end-of-period stock price
- $\pi_T$ total CEO pay
- $W_T(a_i)$ total CEO wealth

**Variables**

- $\phi$ fixed salary
- $n_{sr}$ number of restricted shares
- $n_{or}$ number of stock options
- $\delta_i$ binary for action i
Objective

\[
\min \sum_{i \in N} \delta_i [n_{tot} S_T(a_i) - \pi_T(\phi, n_{sr}, n_{or})] \tag{3.15}
\]

Constraints

\[
\delta_i E(U(W_T, a_i) \geq U \quad \forall i \in N \tag{3.16}
\]

\[
\sum_{i \in N} \delta_i EU(W_T, a_i) - EU(W_T, a_j) \geq 0 \quad \forall j \in N \tag{3.17}
\]

\[
\sum_{i \in N} \delta_i = 1 \tag{3.18}
\]

\[
n_{sr} + n_{or} \leq n_{tot} \tag{3.19}
\]

\[
\phi, n_{sr}, n_{or} \geq 0 \tag{3.20}
\]

\[
\delta_i \in \{0, 1\} \quad \forall i \in N \tag{3.21}
\]

This mixed-integer non-linear programming problem, MINLP problem, which is formulated as a bilevel optimization program, takes the agent’s optimal response to the principal’s decision into account when producing the optimal solution.

We define binary variables \(\delta_i\) to ensure that one and only one of the agent’s actions is selected. If for instance action \(a_i = 30\) is selected, only \(\delta_{30}\) is active. Equations 3.16 to 3.18 ensure this connection. Furthermore, equation 3.17 tells us that the CEO chooses the action which gives him the highest utility.

### 3.3 Implementation

Our optimization model is implemented using GAMS IDE [Rosenthal, 2011]. It is one of the most versatile tools for this kind of problems.
has a high-level and easy-to-learn syntax system, and includes numerous solvers, befitting anything from easy LP problems to the most complex optimization problems.

Our model given in the previous section is implemented as a nonlinear mixed integer programming problem, using COUENNE (Belotti, 2009, Vigerske, 2012) and BONMIN (Bonami and Lee, 2007) as the solvers. The latter is an open-source MINLP solver which uses exact algorithms for convex problems and variants of branch-and-bound algorithm for the problem with non-convex constraints. Similarly, COUENNE makes use of branching techniques, bound tightening and heuristics among others to find a global optimum to the non-convex problems.

We have used a powerful stationary computer for running our program. Intel(R) Core(TM) i7-3770K CPU @ 3.50GHz as the processor, 16GB memory, 64 bits OS and a SSD as the tertiary cache drive. The runtime varies between 120 and 1500 seconds, with a mean running time of about 300 seconds. The model code is included in appendix B.

Financial choices

As our first sample firm, we use Hafslund ASA, a Norwegian company listed on Oslo stock exchange. It is one of the largest listed utility companies in Scandinavia, as well as Norway’s largest power grid owner and leading player within electricity sales. All parameters and figures are fetched from public available data and information, mainly Hafslund (2011) and Oslo Børs’ website (2012).

Norwegian CEOs had in average a monthly salary of 70 900 NOK in 2011 (SSB, 2012). We use this information to get our basic solution. As CEO’s pre-existing wealth, we use his taxable income fiscal year 2010, fetched from Skatteetaten (2012). The initial stock price $S_0$ is set to the traded value first date in 2011.
We calibrate the wealth-utility scaling parameter $\kappa$ and effort-utility scaling parameter $\lambda$. To get reasonable results, we need to determine these critical parameters to the best degree. Without $\kappa$, the wealth-utility would have be very close to zero, which will give us numerical difficulties. For instance, with $W_T = 5 \times 10^6$ and $\gamma = 2$, we have a wealth-utility equal to $-0.0000002$. The choice of $\kappa$ will ensure that the CEO’s end-of-period wealth is deflated, giving us manageable numbers.

The choice of $\lambda$ depends on two factors. For the first, it has to give a disutility at the same scale as wealth-utility. Consequently it has to be a positive but small number. Second, it has to be high enough so the disutility of great effort is prominent. Basically, this means that the CEO should not be able to do the highest possible efforts without getting adequately rewarded for that performance.

The reservation utility $\bar{U}$ is the CEO’s total utility for other possible offers. We use the basic Norwegian CEOs’ mean salary, with our specific values of $\kappa$ and $\lambda$, as the basis for determining this reservation utility. Table 3.1 shows us the parameters we set for $\gamma = 2$, our initial problem.

<table>
<thead>
<tr>
<th>Firm &amp; market</th>
<th>The CEO</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{T}$</td>
<td>5</td>
</tr>
<tr>
<td>$r_f$</td>
<td>0.0312</td>
</tr>
<tr>
<td>wacc</td>
<td>0.06</td>
</tr>
<tr>
<td>$\sigma_p$</td>
<td>0.236</td>
</tr>
<tr>
<td>$\sigma_m$</td>
<td>0.263</td>
</tr>
<tr>
<td>$S_0$</td>
<td>70</td>
</tr>
<tr>
<td>$n_{tot}$</td>
<td>195 000 000</td>
</tr>
<tr>
<td><strong>Strikeprice</strong></td>
<td>at-the-money</td>
</tr>
</tbody>
</table>

Armstrong et al. (2007) uses a disutility multiplier equaling to 0.000075
and a reservation utility at $-2.247$ in their model. As we see, our choices are very close to theirs, which gives a better validity to both our and their choices.

As we are handling uncertainty in the form of unknown effort for the proposed contract, we work with expected values. Both our end-of-period firm stock price $S_T$ and market stock price $M_T$ are lognormally distributed. We have handled the stochastic by transforming the continuous distributions to discrete ones. We use the standard normal distribution as our starting point and derive other discrete distributions from it. Ergo, the stochastic program is transformed and simplified into a deterministic equivalent.

To get a decent approximation, we divide the interval in 100 equal parts, giving us 100 possibilities for each of the 100 effort levels. Further, we use the cumulative distribution function for the standard normal distribution to estimate all the expected values, including that of $S_T$, $M_T$ and $\pi_T$. All this give us a model which takes into account certainty equivalence and risk premium.

We compute our first results with constraints on salary types. First of all, we construct a basic contract with a fixed salary equal to the Norwegian CEOs’ mean fixed salary, 850 800 NOK. Then we find solutions for unfixed base salary, shares and indexed options. We omit fixing for traditional options, as they are a part of our optimal contract. The entire results for this initial problem are shown in the appendix, while table 4.1 in the next chapter shows the results for the basic and the optimal solution.
Chapter 4

Evaluation and analysis

In this chapter we will evaluate and analyze our results: can the principal safely include equity-based compensation and be sure it will provide the CEO incentives for performing as desired? We have made our model as close as possible to a real-world problem, but nevertheless, it is still a model. Certain assumptions had to be made and we could not have succeeded within the set time frame without our few simplifications.

4.1 Initial solution

Our basic solution, as shown in table 4.1 gives an effort level at 24 with the mean fixed salary. Here, with low base salary, no shares and stock options, the results show us that there is no incentive for the CEO to give a better effort. This effort level gives an end-of-period stock price 3.6 % lower than the initial stock price, thus reducing the firm’s value by about 500 million NOK.
We see that the CEO has some incentives to give a little effort in our basic solution. One main reason for this is likely due to the unrestricted shares he owns in the firm before the start of the contracting period.

<table>
<thead>
<tr>
<th>Table 4.1: Results for $\gamma = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>BASIC</strong></td>
</tr>
<tr>
<td>Effort</td>
</tr>
<tr>
<td>Fixed salary</td>
</tr>
<tr>
<td>Shares</td>
</tr>
<tr>
<td>Options</td>
</tr>
<tr>
<td>$S_T$</td>
</tr>
<tr>
<td>Cost of option</td>
</tr>
<tr>
<td>CEO’s total wealth</td>
</tr>
<tr>
<td>Firm’s total cost</td>
</tr>
<tr>
<td>Firm value</td>
</tr>
</tbody>
</table>

The CEO’s expected utility is the same, -2.85, for all the solutions except the basic one (see table A.1 in the appendix). As we see, and as a matter of course, this expected utility equals to the reservation utility. It is here the true restriction lies. Since the principal wants to maximize the firm’s value, and thus hold the CEO’s compensation to the lowest possible level, the principal will offer a contract which gives the CEO same utility as his reservation utility.

The firm’s cost of an option correspond to either Black-Scholes or Johnson-Tian’s expected valuation of a stock option, depending on whether the granted options are traditional or indexed, respectively. This valuation is almost the same for the CEO; the latter values options a bit lower.

Furthermore, the CEO’s total wealth is the sum of his compensation and pre-wealth. Obviously, his compensation approximately equals the firm’s total cost. The latter is the principal’s cost for offering the
given compensation composition, without taking the profit into account. Thus, the total cost is $[\phi e^r + n_{sr}S_T + n_{or}(\text{firm's cost of options})]$.

As we can see from the graph in the appendix (figure A.2), it is only at an effort level around 41 we retain the initial value of the stock price. Also, the value of the firm scales accordingly. Since the fixed salary basically does not affect the firm value in any way, we will see a moral hazard problem. The principal can not verify the agent’s effort, so the latter will likely perform according to his utility. So, the principal will want to give the right incentives, which will be of benefit for both of the parties.

This bi-level model gives an unrestricted optimal solution at an effort level at 88, consisting of stock options and a fixed base salary as a compensation for the risky options.

Comparing to the basic solution, the firm’s expected value improves roughly by 16 %, about 2 billion NOK. The CEO’s expected compensation is 2.3 million NOK, thus an increase of about 1.4 million NOK from the basic contract. For the principal, the total cost of the compensation contract increases by almost 1.5 million NOK. However, this should be completely acceptable since the firm is expecting this extreme growth.

We question how realistic this is. Can a person, with all his talents and skills, be behind this unbelievable growth? Therefore, we should be cautious when analyzing all the numbers. The main weakness here is likely the expected return being too high for the related disutilities and utilities. Nevertheless, our main purpose does not lie in that detail.

An important point to notice in our optimal solution is that the stock options are not indexed at all. Traditional options with a special variant of Black & Scholes valuation is granted. The CEO is granted a certain number of options, and at end of the period he is allowed to sell the options in the market, thus giving him this Black & Scholes value. The
most common case is that the CEO is not allowed to sell, but can either exercise the options or get an equivalent cash refund. Unlike this, we get an incentive-creating compensation in our variant where the CEO’s valuation of options consists of end-of-period stock price instead of initial stock price.

The indexed options, which will try to reduce the uncontrollable market noise, is absent in our results. We suspect the main reason for choosing traditional and not including indexed options are the low volatility of the firm. This volatility is even lower than the volatility of index. It is then likely it will not lower the risk premium. We will later check if this can be proved. Also, both the CEO and the principal value indexed options somewhat lower than the traditional options. Expected value of the Johnson & Tian option is 16.8, versus 25.6 for the Black & Scholes options.

In our model, both the restricted shares and stock options give the CEO incentives for increasing the firm’s value, namely give an effort to raise the stock price. The results show that the principal chooses to grant options rather than shares, as the firm’s cost of option is considerably lower than the cost of granting shares. Also, since there is some uncertainty related to shares and the CEO’s effort decides more or less the outcome of the stock price, fixed salary is used as the certain salary, which is needed for the CEO to accept those incentive-creating uncertain options. We will further in the next sections consider different aspects of our model, the effects of our model assumptions.

4.2 Exercise price

In the last section, we had an optimal solution with the options at-the-money. Granting options out-of-money, i.e. with an exercise price
higher than the initial stock price $S_0$, will obviously give a lower cost for the firm. Will that also induce the CEO incentives to give a higher effort?

Setting a strike price at 77, 10% higher than $S_0$, we get the results shown in table 4.2. Notably, these results are when we only include options, disregard fixed salary and the accompanying risk of the options. An optimal solution with OTM exercise price is presented in the appendix, table A.2.

**Table 4.2: At-the-money or out-of-the-money**

<table>
<thead>
<tr>
<th></th>
<th>Traditional options</th>
<th>Indexed options</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ATM</td>
<td>OTM</td>
</tr>
<tr>
<td>Effort</td>
<td>88</td>
<td>87</td>
</tr>
<tr>
<td>Options</td>
<td>90 989</td>
<td>94 105</td>
</tr>
<tr>
<td>Cost of option</td>
<td>25.585</td>
<td>22.070</td>
</tr>
<tr>
<td>Total cost</td>
<td>2 327 963</td>
<td>2 076 900</td>
</tr>
<tr>
<td>$S_T$</td>
<td>78.61</td>
<td>78.42</td>
</tr>
<tr>
<td>Firm value</td>
<td>15 326 730 569</td>
<td>15 289 137 040</td>
</tr>
</tbody>
</table>

It is to be expected that the CEO’s and the principal’s valuation of both traditional and indexed options decreases. This means the CEO would want more options to retain the same effort level. As we see, that is not happening even though the CEO is granted more options.

The problem is the risk aversion of the agent. A strike price of 77 is very close to the stock price for the effort level at 88. With such a high risk, the agent will want a much greater proportion of certain salary, risk reward, to go along with this.

Additionally, there have been a few discussions about the optimal exercise price. Bebchuk and Fried (2004) argues for out-of-money options as they only pays off upon strong performance, while Dittmann et al. (2010)
justifies in their optimal contract that for loss-averse CEOs, the principal should grant as much in-the-money options as out-of-the-money options, and Dittmann and Yu (2011) suggests in-the-money options for CRRA preferences and if the CEO can affect the firm risk. However, these findings report marginal savings from at-the-money options and Hall and Murphy (2000), among others, show that for risk-averse executives, the incentives are maximized with exercise price at, or near, the grant date stock price.

4.3 Correlation with market

So far, indexed options have not been worth considering. We check whether it is due to our sample firm’s low volatility. Our second firm is a firm also listed on Oslo stock exchange, but with a higher volatility than Hafslund. Inserting the appropriate parameters (table 4.3), including the right expected returns for each effort level, \( \mu_p(a_i) \), we rerun our optimization problem.

| \( n_{tot} \) | 3 182 575 000 | \( \sigma_p \) | 0.287 |
| \( S_0 \) | 140 | \( \rho \) | 0.84483 |
| wacc | 0.09 | \( \beta \) | 0.919989 |

While table A.3 in the appendix show all the results for our second firm, table 4.4 shows the two important solutions. The first one has only fixed base salary, while the second one is our optimal solution. The basic solution has a high amount of fixed salary, while the effort level does not get any higher than 63. This effort level can be a bit misleading as we are in a moral hazard problem. Except for the unrestricted shares he owns from before, the CEO has no incentives to give a real effort. However, the stock price does only increase marginally compared to the
initial stock price. We can therefore understand, in addition to the incentive-creating shares he owns, that this effort level can be gained.

On the other hand, the optimal solution is a mix of fixed base salary and stock options. The latter provide the desired incentives for this high level of effort.

<table>
<thead>
<tr>
<th>Table 4.4: Results for our second firm</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>FIXED SALARY</strong></td>
</tr>
<tr>
<td>Effort</td>
</tr>
<tr>
<td>Fixed</td>
</tr>
<tr>
<td>Shares</td>
</tr>
<tr>
<td>Options</td>
</tr>
<tr>
<td>$S_T$</td>
</tr>
<tr>
<td>Cost of option</td>
</tr>
<tr>
<td>CEO’s total wealth</td>
</tr>
<tr>
<td>Firm’s total cost</td>
</tr>
<tr>
<td>Firm value</td>
</tr>
</tbody>
</table>

The equity-based incentives increase the firm value by 10.4 %, and at the same time having a marginal increase in the cost, 0.5 million NOK. This is indeed a very interesting result, even if disregard the unbelievable growth. In particular, what stock options, and even shares, can do to induce the right incentives. Another interesting point is that all the options granted are indexed.

What are the reasons for choosing indexed and not traditional options? A possible explanation can be that the firm’s cost of traditional options for this effort level is 63.46, which is considerably higher than the cost of indexed options.

Another, but the significant explanation is the volatility of the firm and the correlation with the market index. With an almost perfect
correlation \((\rho = 0.85 \text{ and } \beta = 0.92)\), we see that the indexed options will be worth considering as they will behave as a catalyst. From the formula of \(H_T\) (equation 3.14 on p.21), we get that for \(\beta = 1\), then \(H_T = \left(\frac{S_0M_T}{M_0}\right)\). With a \(\beta = 0\), then \(H_T = S_0e^{\gamma \hat{T}}\). Ergo, the correlation between the firm’s volatility and the market’s volatility should be as strong and positive as possible to get the best out of indexed options, i.e. filter out most of the uncontrollable market noise. In addition, the firm has a higher volatility than the index. Therefore, we can say this result supports our hypothesis. Indexed options should be chosen to lower the risk premium and cope with the risk aversion of the agent.

4.4 Risk aversion degree

So far, it has been in favor of an equity-based compensation scheme. As the risk aversion degree is another well-debated topic, we should also look into that. What will happen if the agent is less risk averse? What about a higher risk aversion?

Our further analysis will be based on the second firm, as presented on the previous section. We consider another 2 risk aversion degrees, namely \(\gamma = 0.5\) and \(\gamma = 4\), and report the results (table 4.5).

There are a few things to note here. Firstly, the difference in reservation utilities and disutility scaling parameters \(\lambda\). For \(\gamma = 0.5\), we have a high reservation utility, and correspondingly lower \(\lambda\). For the other two, we have the reservation utility and the \(\lambda\) at almost the same level.

Secondly, all the stock options granted are fully indexed, for all 3 levels of risk aversion degree. This indicates that the indexed options are the best choice for our second firm. Some other similarities we see in these optimal solutions are that they all include a base fixed salary and
Table 4.5: Risk aversion degrees

<table>
<thead>
<tr>
<th></th>
<th>$\gamma = 0.5$</th>
<th>$\gamma = 2$</th>
<th>$\gamma = 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effort</td>
<td>98</td>
<td>93</td>
<td>90</td>
</tr>
<tr>
<td>Fixed</td>
<td>162 826</td>
<td>800 397</td>
<td>1 087 705</td>
</tr>
<tr>
<td>Shares</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Options</td>
<td>54 071</td>
<td>40 158</td>
<td>21 031</td>
</tr>
<tr>
<td>Reservation utility</td>
<td>2.0</td>
<td>-2.85</td>
<td>-2.33</td>
</tr>
<tr>
<td>Disutility parameter</td>
<td>0.000022</td>
<td>0.000235</td>
<td>0.00026</td>
</tr>
<tr>
<td>ST</td>
<td>168.77</td>
<td>166.00</td>
<td>164.36</td>
</tr>
<tr>
<td>Cost of options</td>
<td>43.00</td>
<td>41.21</td>
<td>40.16</td>
</tr>
<tr>
<td>CEO’s total wealth</td>
<td>8 397 407</td>
<td>8 335 063</td>
<td>7 793 150</td>
</tr>
<tr>
<td>Firm’s total cost</td>
<td>2 493 037</td>
<td>2 480 468</td>
<td>1 966 696</td>
</tr>
<tr>
<td>Firm value</td>
<td>537 131 414 654</td>
<td>528 314 805 810</td>
<td>523 094 964 005</td>
</tr>
</tbody>
</table>

stock options, have high optimal effort levels and accordingly high firm value. We still miss the absence of shares in the compensation scheme. Apparently, stock options are a better choice as the incentive-providing, but risky compensation.

As we see, the CEO does better performance with lesser certain salary for the low risk aversion degree $\gamma = 0.5$. For $\gamma = 4$, the risk aversion is much higher. Here, the principal has to offer a greater proportion of certain salary, and at the same time include equity-based compensation to induce incentives to some extent.

4.5 Scaling parameters

The reservation utility, the disutility scaling parameter $\lambda$ and the wealth-utility scaling parameter $\kappa$ are all connected strongly together. We can not change the value of one of them without affecting the other two. Therefore, there will be no reason to vary more than one of them.
All our solutions above have the same κ, our choice of the reservation utility and λ for each of the risk aversion degrees, are discussed earlier.

For each of the 3 risk aversion degrees, we change the λ several times, and report the results in the appendix (table A.4). We could also have varied the reservation utility, but then we needed to also re-calibrate the λ, which basically would have been pointless.

We can by looking at different λs confirm our results from earlier. Stock options are used to induce incentives to do a better performance and attain a high stock price. All these granted options are indexed, thereby telling us the importance of these in our case with the second firm.

Another point to note is that increasing the λ also makes it harder for the CEO to give high efforts. For instance, with a risk aversion degree at 2, and \( \lambda = 0.00029 \), only 0.000055 higher than our optimal solution, the CEO will not be able to do a better effort than 85. This is even though he is being compensated with almost 1 million more.

We look at the different disutility parameters and their correlation to the effort-levels and thereby implicitly the total return of the firm \( \mu_p \).
For our initial \( \lambda = 0.000235 \), we have \( \mu_p = 16.46\% \). This is clearly an overestimation of the power of incentives. On the other hand, with \( \lambda = 0.00029 \) we get \( \mu_p = 14.14\% \), which can be acceptable. For our first firm, however, we get more realistic returns. The optimal solution for the first firm has an effort-level at 88, \( \lambda = 0.000235 \), and gives \( \mu_p = 11.6\% \).

From this, we can see that the choice of λ is extremely sensitive to changes. With only an increase or decrease of just one thousandths, we get substantial changes in effort level. As we calibrated our initial model using the Norwegian CEO’s mean salary and effort level at equilibrium, we got those high optimal results. Setting a more strict foundation, higher disutility scaling parameter and reservation utility, would have given lower effort levels at optimum.
Chapter 5

Discussion and assessment

Finally in this work, we will discuss our results and compare to the related literature, highlight some of the weaknesses before we conclude with an overall assessment and directions for future work.

5.1 Discussion

Our analysis show us that the performance is not at the desired level without any equity based incentives. Granting stock options seems to be the best choice. In most of the cases, except for the cases where it is too expensive or impossible for the principal to induce the agent to give a high effort, stock options are the dominant source of equity based incentives. A somewhat surprising result is that shares are almost never granted.

Shares are basically stock options with strike price equal to zero, and
are therefore more costly. Since stock options are riskier than shares, the marginal value of stock options is less than of shares, they are more expensive for the firm. But since stock options induce more incentives than shares, 1 NOK worth stock options provide more incentives than 1 NOK worth shares, the stock options are in overall cheaper.

Another explanation for granting stock options rather than restricted shares, is that the latter get extremely costly the higher the effort-level. Also, when the expected end-of-period stock price is high, this provides the CEO a payoff in most of the outcomes of the stock price, i.e. for a range of the right-sided effort levels. Therefore, the principal will be better off with stock options. The firm’s cost in granting stock options are considerably lower than for shares, and in addition resulting in a payoff for the CEO only when the $S_T$ is higher than the exercise price.

In the CEO’s perspective, he will not mind getting stock options as a part of the compensation, as long as it is not too costly for him to give a high effort and as long as he is also getting paid a certain base salary. A particular case in our model is where the maturity of options is beyond the contracting period. Basically, this gives the CEO possibilities to sell the call options after the end of the contracting period. We see that this induces additional incentives and also provides a higher valuation of stock options.

For firms where it is possible to filter away some of the exogenous market noise, the CEO would rather prefer indexed equity-based compensation, rather than traditional compensation. To mitigate the risk, market index options which are tied to some kind of index, OSEBX in our case, is a good choice to induce incentives. Our unambiguous results show that the indexed options are less costly than the traditional options, and that they are always preferred to traditional options in cases where it is good correlation with the market index and a higher than average firm volatility.
There has been a lot of concern about that some executives are being rewarded for general upswings in the market, which is totally out of their control (Betrand and Mullainathan, 2001). This pay for luck is something that should be taken care of, and using indexed stock options or even shares to filter out such noise can be of great importance.

Our results are consistent with some of the recent research. Even though the earlier work by Dittmann & co. (Dittmann and Maug, 2007) and Hall and Murphy (2002) does not support using stock options in the optimal contract, later research like Dittmann et al. (2010) and Dittmann and Yu (2011) conclude with granting stock options as the greatest incentive-creating compensation. Likewise, Armstrong et al. (2007), which our work relates most to, closes with a similar conclusion. Stock options are almost always part of the optimal contract. Furthermore, Maug et al. (2012) look whether options should be indexed to induce more incentives. Here our results differ; they report that indexing options are not a good choice for most of the cases and will destroy the CEOs' incentives. Our findings show that whether to index the options or not depends on the volatility of the firm and its correlation with the market. Therefore, there can not be a common solution to all the firms; indexing will just be beneficial for a range of firms and not for every single firm.

Comparing our results to the practiced compensation schemes in the Norwegian market, we see great inconsistence. Mostly, stock options are not a part of CEO pay, which is also the case for our first firm. The question is then why there is this contrasting differences from the US firms. Several explanations can be found here, but one of the main reasons are the regulations and the way of thinking of the Norwegian government, let alone the Norwegian people. In the US, they have nearly a free market while in Norway the government has ownership in a lot of the largest firms. They can therefore constrain executive pay and set benchmark for the rest of the country. Lately, all the government-owned firms have been pushed to drop granting options to their executives, thus
focusing more on base salary and other bonuses. Nonetheless, the CEO pay in Norway increased 18 % the last year (Aftenposten 2012), and there have been a constant debate in the Norwegian society regarding the executives earning tenfold more than average workers.

In this case, we suggest indexing the options. Granting traditional call options can result in large salaries the executives do not deserve or have earned through pure skills and talents. Therefore, indexing options will remove a great deal of the uncontrollable factor with the market fluctuations and thereby paying the CEO for what he deserves.

5.2 Shortcomings of the model

The greatest weakness in our model is probably the correlation between the CEO’s effort and the firm’s expected return. Even though this method is implemented by Armstrong et al. (2007) for US firms, we get a controversial model for our Norwegian firms without any reference point. Nevertheless, this has been an acceptable starting point for an equity based principal-agent model set in Norway and should be developed further to correspond with real observed and forecasted data.

We select two firms as our sample. The first being a mediocre but less risky firm, while the second being a huge but more risky. Even though they are diversified that way, these may not be enough to determine the optimal compensation for the Norwegian firms and represent the total Norwegian market. All the previous research, set in the US and with easy access to a large database of firms and executives, have used a sample of hundreds of firms. However, as we perform some kind of sensitivity analysis in addition to our sample firms, we get a good picture of different cases in Norway and the results should hold.
We estimate the volatilities of our firms using daily historical data for a year. This most likely gives poor estimates of the real volatilities, thus affecting the correlation with the market and the beta form CAPM. Using monthly historical volatilities, over a longer period, would have given a better estimation.

Our valuation of options are very primitive. Usually, options are awarded several times during the contracting period, they have different exercise prices and also different time to maturity. Including all these real-life mechanisms would have given higher incentives, as shown by Armstrong et al. (2007) among others. An increased frequency of granting options will give strike prices in accordance with the stock price at the granting time, and thereby reduce the risk of options. Our results show already that there is a great deal of incentives in options and by sacrificing a bit accuracy to get a less complicated model, can be a good compromise.

Finally, our technical modeling choices can have some weaknesses and possibilities for improvement. As we implement both the principal’s and agent’s decision, to the other party’s choices, in the same optimization problem, there can be unobserved issues. A possibility would have been to model the problem as a real two-stage stochastic programming problem; the principal offers a contract in the first stage and the CEO answer with his effort level for the proposed contract. The second stage is then the outcome, i.e. firm’s value for given effort level. This will have complicated our model considerably, and as we assume that the reservation utility is know to both agent and principal, our model as a single bi-level problem is more than acceptable.
5.3 Overall assessment

We suggest the optimal CEO compensation contract should include stock options in addition to fixed base salary. Our unambiguous results show that there is a lot to fetch from such contract. The equity based compensation gives the CEO incentives to perform at a higher level and also reduces the total agency cost for the firm. Furthermore, the CEO should invest some of his wealth in the firm’s assets, giving an increased ownership which in turn will be a source for intrinsic motivation.

Stock options should be indexed if the firm has a higher volatility and good correlation with the market index. This will reduce noises which the CEO can not affect, thus giving a fair compensation. A weakness in our model is that the CEO’s effort and the expected return of the firm is tightly connected, which results in that the CEO can affect the outcome too much. Also, our basic solutions give somewhat unrealistic high expected returns, which is likely the case of using weakly calibrated parameters.

We see that the Norwegian firms do not offer the executives optimal contracts. As the Norwegian government are against granting options and is opposed to high CEO pay, indexing options are worth considering. Just offering a high base salary will be more costly for the firm than offering cheaper equity based compensation in addition to a lower base salary.

The goal of our work was to base on the research on American firms, by Dittmann et al. (2010) and Armstrong et al. (2007) among others, and model a contract for Norwegian CEOs. Similar to the previous works, we use the standard principal-agent framework where the principal wants an agent to perform a task and the agent’s effort is unobservable. We differ from the previous work in several areas. First, in contrast to Dittmann & co., we implement a complete bi-level problem where both the agent and
principal maximizes their utility and firm value, respectively. This gives a more realistic model where we do not need to assume that the observed contracts have optimal effort level or be affected by the shortcomings of the first order approach. Secondly, we do not allow negative base salary and facilitates granting both restricted shares and stock options in addition to a base salary. Thirdly, we do several sensitivity analysis, including granting options out-of-the-money and at-the-money. Finally, we index options using a market index and check whether it is better to index than grant normal call options.

Even though we specialized in equity based compensation and extrinsic incentives, we should mention that highly intrinsically motivated CEOs will require less compensation for high performance. It is therefore crucial to facilitate for genuinely motivating the executives. Another benefit from this, is that by paying lower wages for great effort, the controversy of enormous executive compensation can be avoided.

5.4 The road ahead

This work can be seen as a first step in designing the optimal CEO compensation contract for a Norwegian CEO. With the shortcomings and the simplicity of our model, there should be several interesting directions for future work.

Some of recent works report that the CEOs indeed are loss-averse and our model can easily be extended to include loss-aversion. In the same line, there is also possible to check out different types of risk aversion, for instance decreasing relative risk aversion.

Another direction is to implement something similar to Dittmann & co. for Norwegian firms, i.e. assume that the observed contracts have
optimal effort level and formulate an optimization problem which minimizes the firm’s costs. Here, it should also be possible to calibrate the parameters better with real observed and quantitative data.

Further, designing a more complex model including different options, with different exercise prices and maturity, and including more compensation alternatives, for instance debt, pension plans and severance pay, will give a more realistic model.

Finally, there should be possible to design a model with a real two-stages stochastic programming problem. Without knowing the CEO’s preferences, which is likely the true case, the principal can offer several contracts, the so-called non-linear contracts, for different possible preferences. By taking this into account, we will be on the road to a perfect contract.
Appendix A

Figures and tables

Figure A.1: CDF used in our programming model
Figure A.2: The expected stock price as a function of effort. The initial stock price 70 is preserved at an effort level around 41.
Table A.1: All results for $\gamma = 2$, for our first firm. The first one is basic solution with base salary fixed to the Norwegian CEOs’ mean salary. The second is where we include only base salary in the compensation scheme, while the third only includes shares. Thereafter, we have a compensation scheme with only indexed options where we disregard the risk aversion and the agent’s displeasure in uncertain salary. Finally we have the optimal solution, which includes a base salary and non-indexed options. The disutility scaling parameter $\lambda$ is 0.000235 and the reservation utility is -2.85 in all these models.

<table>
<thead>
<tr>
<th></th>
<th>BASIC</th>
<th>FIXED SALARY</th>
<th>SHARES</th>
<th>INDEXED</th>
<th>OPTIMAL</th>
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<tbody>
<tr>
<td>Effort</td>
<td>24</td>
<td>50</td>
<td>86</td>
<td>87</td>
<td>88</td>
</tr>
<tr>
<td>Fixed</td>
<td>850 800</td>
<td>2 256 446</td>
<td>-</td>
<td>-</td>
<td>1 310 683</td>
</tr>
<tr>
<td>Shares</td>
<td>-</td>
<td>-</td>
<td>23 618</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Options</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>124 615</td>
<td>38 137</td>
</tr>
<tr>
<td>Indexation degree</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>ST</td>
<td>67.55</td>
<td>71.56</td>
<td>78.22</td>
<td>78.416</td>
<td>78.61</td>
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<tr>
<td>Cost of options</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>16.666</td>
<td>25.585</td>
</tr>
<tr>
<td>Compensation</td>
<td>875 394</td>
<td>2 321 672</td>
<td>1 842 641</td>
<td>2 071 772</td>
<td>2 321 883</td>
</tr>
<tr>
<td>CEO’s total wealth</td>
<td>5 160 669</td>
<td>6 487 390</td>
<td>6 127 916</td>
<td>6 360 523</td>
<td>6 614 118</td>
</tr>
<tr>
<td>Firm’s total cost</td>
<td>877 763</td>
<td>2 327 957</td>
<td>1 847 464</td>
<td>2 076 828</td>
<td>2 327 949</td>
</tr>
<tr>
<td>obj. (firm value)</td>
<td>13 170 899 250</td>
<td>13 954 740 618</td>
<td>15 251 615 258</td>
<td>15 289 137 073</td>
<td>15 326 730 583</td>
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Table A.2: Out-of-the-money optimal solution for our first firm

<table>
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<td>Shares</td>
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<tr>
<td>Options</td>
</tr>
<tr>
<td>Indexation degree</td>
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<tr>
<td>ST</td>
</tr>
<tr>
<td>Cost of options</td>
</tr>
<tr>
<td>CEO's total wealth</td>
</tr>
<tr>
<td>Firm's total cost</td>
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<td>obj. (firm value)</td>
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Table A.3: Results for our second firm ($\gamma = 2$)

<table>
<thead>
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<th>TRADITIONAL</th>
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<tr>
<td>Effort</td>
<td>63</td>
<td>92</td>
<td>92</td>
</tr>
<tr>
<td>Fixed</td>
<td>2 005 067</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Shares</td>
<td>-</td>
<td>12 502</td>
<td>-</td>
</tr>
<tr>
<td>Options</td>
<td>-</td>
<td>-</td>
<td>37 551</td>
</tr>
<tr>
<td>Indexation degree</td>
<td>-</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>ST</td>
<td>150.31</td>
<td>165.46</td>
<td>165.46</td>
</tr>
</tbody>
</table>

| Cost of option  | - | - | 63.018 | 41.205 |
| CEO’s total wealth | 7 642 406 | 7 914 266 | 8 227 930 | 8 335 063 |
| Firm’s total cost  | 2 068 611 | 2 068 516 | 2 366 382 | 2 480 468 |
| Firm value  | 478.375 E+9 | 526.569 E+9 | 526.569 E+9 | 528.315 E+9 |
Table A.4: Results for different disutility scaling parameters $\lambda$. For each of the three risk aversion degrees $\gamma$ we report results with 3-4 variety of $\lambda$s. Here, $a$ is the effort-level, base is fixed salary, op-c is the firm’s cost of granting options, tot-c is the firm’s total cost and finally f-value is the firm’s total value. We also use the same reservation utilities $U$ associated with each of the risk aversion degrees.

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$U$</th>
<th>$\lambda$</th>
<th>$a$</th>
<th>BASE</th>
<th>SHARES</th>
<th>OPTIONS</th>
<th>OP-C</th>
<th>ST</th>
<th>TOT-C</th>
<th>F-VALUE</th>
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<tr>
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<td>2.0</td>
<td>0.000012</td>
<td>100</td>
<td>594 420</td>
<td>3 319</td>
<td>14 063</td>
<td>43.734</td>
<td>169.89</td>
<td>1 792 110</td>
<td>540.700 E+9</td>
</tr>
<tr>
<td>0.5</td>
<td>2.0</td>
<td>0.000022</td>
<td>98</td>
<td>162 826</td>
<td>-</td>
<td>54 071</td>
<td>43.00</td>
<td>168.77</td>
<td>2 493 037</td>
<td>537.131 E+9</td>
</tr>
<tr>
<td>0.5</td>
<td>2.0</td>
<td>0.000024</td>
<td>93</td>
<td>-</td>
<td>511</td>
<td>58 968</td>
<td>41.21</td>
<td>166.00</td>
<td>2 514 654</td>
<td>528.315 E+9</td>
</tr>
<tr>
<td>2</td>
<td>-2.85</td>
<td>0.00021</td>
<td>100</td>
<td>-</td>
<td>3 499</td>
<td>58 718</td>
<td>43.73</td>
<td>169.89</td>
<td>3 162 401</td>
<td>540.698 E+9</td>
</tr>
<tr>
<td>2</td>
<td>-2.85</td>
<td>0.00023</td>
<td>94</td>
<td>1 806 975</td>
<td>-</td>
<td>14 537</td>
<td>41.92</td>
<td>166.55</td>
<td>2 473 570</td>
<td>530.066 E+9</td>
</tr>
<tr>
<td>2</td>
<td>-2.85</td>
<td>0.000235</td>
<td>93</td>
<td>800 397</td>
<td>-</td>
<td>40 158</td>
<td>41.205</td>
<td>166.00</td>
<td>2 480 468</td>
<td>528.315 E+9</td>
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<tr>
<td>2</td>
<td>-2.85</td>
<td>0.000239</td>
<td>85</td>
<td>7 252</td>
<td>-</td>
<td>84 406</td>
<td>38.45</td>
<td>161.67</td>
<td>3 253 148</td>
<td>514.507 E+9</td>
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<tr>
<td>4</td>
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<td>0.00022</td>
<td>98</td>
<td>1 574 500</td>
<td>-</td>
<td>8 014</td>
<td>43.00</td>
<td>168.77</td>
<td>1 969 004</td>
<td>537.132 E+9</td>
</tr>
<tr>
<td>4</td>
<td>-2.33</td>
<td>0.00026</td>
<td>90</td>
<td>1 087 705</td>
<td>-</td>
<td>21 031</td>
<td>40.16</td>
<td>164.36</td>
<td>1 966 696</td>
<td>523.095 E+9</td>
</tr>
<tr>
<td>4</td>
<td>-2.33</td>
<td>0.0003</td>
<td>84</td>
<td>898 049</td>
<td>-</td>
<td>32 218</td>
<td>38.12</td>
<td>161.13</td>
<td>2 154 642</td>
<td>512.808 E+9</td>
</tr>
</tbody>
</table>
Appendix B

The GAMS code

Here, we have included our full GAMS code for the first firm. There are some explanations in the code itself. Apart from that, [Rosenthal (2011)](#) explains in great detail the syntax, semantics and structure of the GAMS programming language.
Chief Executive Officer Compensation
- Optimal Equity-based Incentives

Model by Visnu Manoharan, 2012-

function cdfNormal /stolib.cdfnormal /;

Option solprint = off, limrow=0, limcol=0;
Option reslim = 5000, optcr=0.00001;
$offsymxref
$offlisting

* Set i for actions/efforts while sets t and l are for the sampling/cdf process

Set i action i /1*100/  
t number of discrblocks /1*100/
Alias (i,j);

Scalar r, v, w, totF; r=-3; v=1; w=1;

Parameters

* CEO-specific parameters

cT    "length of contracting period" /1/  
W0    "initial CEO wealth" /4000000/  
rutil "reservation utility" /-2.85/
omega    "fraction of initial wealth invested" /0.0003/
gamma   "risk aversion degree" /2/
kappa    "utility scaling parameter" /6850800/
lambda   "disutility scaling parameter" /0.000235/
nsu      "unrestricted shares CEO own" /18000/

*****************************************************************
* Firm- and Market-specific parameters
*****************************************************************

oT       "options' maturity time" /5/
rf       "risk-free rate" /0.0312/
wacc     "cost of capital" /0.06/
muin     "expected return market index" /0.07/
sigma    "volatility, sigma" /0.235934181/
sigmain  "volatility, market index" /0.263084684/
rho      "coefficient of correlation" /0.2738247/
beta     "CAPM beta" /0.245572789/
S0       "initial stock price" /70/
ntot     "number of shares outstanding" /195000000/
M0       "initial value of stock market index" /400/
MT       "end-of-period value of stock market index"
MR(t)    "samples for MT"

*****************************************************************
* Below are some more parameters which are computed later on, or
* loaded from external data-files.
*****************************************************************
F(t)      "cumulative for snd"
SR(i,t)   "samples for ST(i)"
ST(i,t)   "discrete stock prices"
exST(i)   "expected ST(i)"
pwealth(i,t) "CEO's wealth at time zero"
a(i)      "effort for action i"

#include effort.inc

mu(i)     "expected total return, dep. on effort"
#include return.inc

assigns for some already-declared parameters
loop(t,
    F(t) = cdfNormal(r+0.06,0,1) - cdfNormal(r,0,1);
    r=r+0.06;
); 
loop(t,
    SR(i,t) = S0*exp((mu(i)-(sigma**2)/2)*cT + ((-3+0.06*w)*sigma+mu(i))*sigma*sqrt(cT));
    MR(t) = M0*exp((muin-(sigmain**2)/2)*cT + ((-3+0.06*w)*sigmain+muin)*sigmain*sqrt(cT));
    w=w+1
  );
exST(i) = sum(t,F(t)*SR(i,t));
MT = sum(t,F(t)*MR(t));
loop(t, ST(i,t)= sigma*(-3+0.06*v) + exST(i);
    v=v+1;
  );
pwealth(i,t) = (W0*(1-omega))*exp(rf*cT)+ nsu*ST(i,t);
dd1(i) = (log(exST(i)/K)+(rf+0.5*sigma**2)*oT)/(sigma*sqrt(oT));

dd2(i) = dd1(i) - sigma*sqrt(oT);

BS(i,t) = ST(i,t)*errorf(d1(i,t))- K*exp(-rf*oT)*errorf(d2(i,t));

BnS(i) = exST(i)*errorf(dd1(i)) - K*exp(-rf*oT)*errorf(dd2(i));

exBS(i) = sum(t,F(t)*BS(i,t));

*****************************************************************
* Johnson-Tian valuation for indexed stock options
*****************************************************************

Parameters

H  T  "strike price of indexed options"
eta  "parameter for return valuation"
dlin(i,t)  "d1_indx"
ddlin(i)  "d1_indx principal"
d2in(i,t)  "d2_indx"

dd2in(i)  "d2_indx principal"
JT(i,t)  "Johnson-Tian value"
JnT(i)  "expected JT principal"
exJT(i)  "expected value JT"


eta = (1-beta)*(rf+0.5*rho*sigmain*sigma);

HT = S0*((MT/M0)**beta)*exp(eta*(oT));

dlinc(i,t) = (log(ST(i,t)/HT)+0.5*(sigmain**2)*(oT))

/ddlin(i) = (log(exST(i)/HT)+0.5*(sigmain**2)*(oT))

/dd2in(i) = ddlin(i) - sigmain*sqrt(oT);

JT(i,t) = ST(i,t)*errorf(dlin(i,t))-HT*errorf(d2in(i,t));

JnT(i) = exST(i)*errorf(ddlin(i)) - HT*errorf(dd2in(i));

exJT(i) = sum(t,F(t)*JT(i,t));

Variables

pi(i,t)  "CEO compensation"

expi(i)  "expected compensation"

EUtil(i)  "wealth-utility with crra"

DUtil(i)  "disutility"

phi  "fixed salary"

exw0(i)  "expected pre-wealth"
Positive Variables phi, nsr, nor, pi(i,t), psi;

Binary Variables d(i);

Equations

obj "objective function"
exutil(i) "CEO's positive utility function"
disutil(i) "disutility function"
compens(i,t) "CEO's compensation this period"
excompen(i) "expected compensation for action i"
expwealth(i) "expected pre-wealth for action i"
twealth(i,t) "WT for each i and t"
firmcost(i) "total cost for firm"
particicon(i) "CEO participation constraint"
incentcon(j) "incentivite compatibility constraint"
onebincon "only one active binary constraint"
totalsharecon "restrict total shares given CEO"
indexlvlcon "restrict fraction of options indexed"

compens(i,t).. pi(i,t) =e= phi*exp(rf*cT)+nsr*ST(i,t) + nor*(psi*JT(i,t) + (1-psi)*BS(i,t));
excompen(i) .. expi(i) =e= sum(t,F(t)*pi(i,t));
exw0(i) =e= sum(t,F(t)*pwealth(i,t));
tw(i,t) =e= pi(i,t) + pwealth(i,t);
firmcost(i) ..  
  fcost(i) =e= phi*exp(rf*cT)+ nsr*exST(i) 
  + nor*(psi*JnT(i) + (1-psi)*BnS(i));

exutil(i) ..  
  EUtil(i) =e= \sum(t, 
  F(t)*((1/(1-gamma))*((phi*exp(rf*cT)+nsr*ST(i,t)
  + nor*(psi*JT(i,t) + (1-psi)*BS(i,t))
  +(W0*(1-omega))*exp(rf*cT)+ nsu*ST(i,t))
  /kappa)**(1-gamma) 
  )); 

disutil(i) ..  
  DUtil(i) =e= lambda*a(i)**2;

*****************************************************************
* The objective function and the restrictions.
*****************************************************************

obj ..  
  z =e= \sum(i, d(i)*
  ntot*exST(i)- (phi*exp(rf*cT)+ nsr*exST(i)
  + nor*(psi*JnT(i) + (1-psi)*BnS(i))
  ) 
  );

particicon(i) ..  
  d(i)*(EUtil(i)-DUtil(i)) =g= rutil;

incentcon(j) ..  
  \sum(i, d(i)*(EUtil(i)-DUtil(i)))
  - (EUtil(j)-DUtil(j)) =g= 0;

onebincon ..  
  \sum(i,d(i)) =e= 1;

totalsharecon ..  
  nsr + nor =l= ntot;

indexlvlcon ..  
  psi =l= 1;

*****************************************************************
* Define model, allocate memory, solve and display solution
*****************************************************************

model contract /all/;

contract.workspace = 1450;

Option MINLP = BONMIN;
solve contract using minlp maximizing z;

display expi.l, exw0.l, EUtil.l, exJT, exBS, JnT, BnS, exST, HT;
display d.l, phi.l, nsr.l, nor.l, z.l, psi;
Bibliography


# MASTERKONTRAKT
- uttak av masteroppgave

## 1. Studentens personalia

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## 2. Studieopplysninger

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<td>Institutt for industriell økonomi og teknologiledelse</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Studieprogram</th>
<th>Hovedprofil</th>
</tr>
</thead>
<tbody>
<tr>
<td>Industriell økonomi og teknologiledelse</td>
<td>Anvendt økonomi og optimering</td>
</tr>
</tbody>
</table>

## 3. Masteroppgave

<table>
<thead>
<tr>
<th>Oppstartsdato</th>
<th>Innleveringsfrist</th>
</tr>
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<tbody>
<tr>
<td>16. jan 2012</td>
<td>11. jun 2012</td>
</tr>
</tbody>
</table>

**Oppgavens (foreløpige) tittel**

**Optimal Equity Based Incentives**

**Oppgavetekst/Problembeskrivelse**
Evaluate various equity based incentives through development, estimation and analysis of principal agent model.

**Main contents:**

1. Review and discussion of theoretical and empirical literature related to equity based incentive plans.

2. Development of principal agent model enabling comparative analysis of various classes of equity based incentives, estimation of model parameters based on empirical data and presentation of model simulation results.

3. Overall assessment of the insights gained through model analysis.

<table>
<thead>
<tr>
<th>Hovedveileder ved institutt</th>
<th>Medveileder(e) ved institutt</th>
</tr>
</thead>
<tbody>
<tr>
<td>Førsteamanuensis Einar Belsom</td>
<td></td>
</tr>
</tbody>
</table>

**Merknader**

1 uke ekstra p.g.a påske.
4. Underskrift

Student: Jeg erklærer herved at jeg har satt meg inn i gjeldende bestemmelser for mastergradsstudiet og at jeg oppfyller kravene for adgang til å påbegynne oppgaven, herunder eventuelle praksiskrav.

Partene er gjort kjent med avtalens vilkår, samt kapitlene i studiehåndboken om generelle regler og aktuell studieplan for masterstudiet.

Trondheim, 16.01.12

Sted og dato

[Signature]
Student

[Signature]
Hovedvevleder

Originalen lagres i NTNU's elektroniske arkiv. Kopi av avtalen sendes til instituttet og studenten.