Nonlinear Observer and Control Design for Electropneumatic Clutch Actuator

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Summary

This thesis treats position control of an electropneumatic clutch actuator operated by simple on/off solenoid valves. This clutch actuator is intended for automatic manual transmission systems or clutch-by-wire systems in heavy-duty trucks. Pressurized air is already present as a part of the brake system in such vehicles, and pneumatics is therefore the natural choice for actuating the manual transmission in these clutch actuator systems. Production cost is a crucial factor in all parts of the automobile industry. For the clutch actuator system this influences the choice of number of sensors present in the production system, limiting it to only a position sensor. It also lead to that a set of on/off solenoid valve is chosen for actuation over more commonly used proportional valves, even though it complicates the control task. Production quality sensors are considered in this thesis, and these are sensors which are more influenced by noise than sensors found in test rigs made for research and development. Motor vibration and possible other mechanical influence also have to be taken into consideration when testing in a full-scale truck as done in this thesis.

Motivated by the choice of on/off solenoid valves as the control valves in the clutch actuator system, switched control design is considered. Switched controllers exploit the on/off solenoid valves discrete behavior, switching the valves between fully open and fully closed in a manner such that desired supply or exhaust of air from the actuator chamber is obtained, ensuring the demanded piston position. Based on this line of action, two switched controllers are developed, and possible other mechanical influence also have to be taken into consideration when testing in a full-scale truck as done in this thesis.

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Summary

This thesis treats position control of an electropneumatic clutch actuator operated by simple on/off solenoid valves. Switched controllers are well suited for actuator piston position control of the clutch actuator system. Full-state feedback availability is assumed in the design of the switched controllers. Adaptive nonlinear observers are derived to provide estimates of unmeasured states and parameters for the clutch actuator system. These adaptive observers are deterministic with linear output-injection, and have adaptation laws for estimation of clutch load characteristics and friction. A full-order adaptive observer, including filtering of position for suppression of noise, shows best performance. The adaptation of unknown parameters accounts for wear of the clutch and variations in temperature, ensuring a robust design of the clutch actuator model.

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and improving state estimates. Theoretical analyses of convergence are derived under persistence of excitation condition, and performance of the adaptive observers are validated by experimental data.

The switched controller and the adaptive nonlinear are combined into an observer-based switched controller. Simulation of this is provided, comparing the results with actual truck measurements. These analyses indicate that the pressure sensor can be exchanged with an adaptive nonlinear observer without significant loss of performance for nonlinear state feedback based control designs for the system.
Acknowledgments

This thesis present the research results of my doctoral studies at the Department of Engineering Cybernetics (ITK) at the Norwegian University of Science and Technology (NTNU) from August 2005 to June 2010, interrupted by parental leaves. The funding have been provided by the Research Council of Norwegian (SNF) and Kongsberg Automotive ASA (KA).

First of all I would like to thank my supervisor, Professor Tor Arne Johnsen at the Department of Engineering Cybernetics, NTNU. He has been a great advisor, always being available for discussions and directions. I am still amazed by how fast he responds all my questions, patiently providing me with new ideas or solutions to what I thought to be big problems.

I am also very thankful to my co-supervisor Dr. Glenn-Ole Kaasa. First of all for taking the initiative to the collaboration between the ITK and Kongsberg Automotive ASA, making this thesis possible. His knowledge on every aspects of the clutch systems, which he share willingly, still impress me greatly. Steen Roar Sørve, Christian Bratli and Håkon H. Solberg at KA also deserves thanks for providing me technical information on the clutch system, and helping me set up the truck for experimental testing. KA even allow me to tests after their part of the research project was finished, without these results the thesis would not been complete.

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than me, and having time to listen to my complaints.

I also need to thank my family. My mother and father who always have supported me and my two sisters, and always arrange things such that we could pursue whatever education we liked. Thanks to both my sisters, Hilde, for being my mathematical alibi, and Siv for proofreading. You both undergoing the same type of education as me, have made you supportive conversation partners. My dear friend Kristin Johansson also deserves thanks for helping me with the proofreading.

My husband Trond-Evan deserves the biggest gratitude. He has spent hours listening to my worries about my progress, and somehow still believes in me and thinking that I can accomplish anything. At last, I need to mention our two beautiful children, Maren and Sakris. My thesis would have been finished a couple of years before without you, but you have provided me with a new perspective in life. Your unconditional love have encouraged me through some tough days the last year.

Kongsberg, February 2011

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Chapter 1
Introduction

This thesis considers position control of an electropneumatic clutch system for heavy duty trucks. The main task is to obtain clutch actuator position control, which is accurate enough to provide smooth driving of the vehicle, at the same time as the controller does not introduce unnecessary wear of the clutch or the actuator, and adapts to tear-and-wear and environmental changes. This is achieved by designing controllers and observers suitable for application to the electropneumatic clutch actuator. The intended applications for the clutch actuator are automated manual transmission (AMT) and clutch by wire (CBW) solutions for heavy duty trucks.

1.1 Motivation
Demand on reduction in the fuel consumption and the associated CO₂ emission for vehicles is highly relevant in the discussion on global warming. Large amounts of goods are transported by trucks every day, and due to the high volume even small decreases in the fuel consumption will both improve exhaust emissions and reduce cost considerably. One option for enhancing fuel efficiency is development of better transmission solutions.

Manual Transmission (MT) systems perform better than conventional Automatic Transmission (AT) systems in terms of fuel efficiency, typically a 10% or better fuel mileage can be expected, Kuroiwa et al. (2004). However, the AT systems provide better driving comfort and simplified vehicle operation, and the drivers prefer these conveniences. This is a part of a clear trend in the motor industry, the market share for MT systems is decreasing while the market share for AT systems is increasing. Several semi-automatic systems are developed as a response to this progress. Automated Manual Transmission (AMT) systems are one of these semi-automatic solutions, combining the best features from MT and AT systems, the high efficiency of MT and the comfort of AT. An AMT system consists of manual transmission with automatic transmission control, and can be used to employ automatic transmission divided by an automatic gear shift control system, or as a clutchless manual transmission exploiting drive-by-wire technology. As the manual transmission solutions.

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torque transmission is retained, AMTs can easily be added onto existing MT systems. This is an advantage over more conventional AT systems, and it is the main reason for the AMT systems growth in popularity the last decade, especially in the European market. Kongsberg Automotive ASA (KA) delivers AMT systems for heavy duty trucks. AMT systems are especially desired for such trucks, because conventional AT systems for the high torque transfer needed in these vehicles are expensive and have large loss of power. A great amount of research can be found on AMT systems, as the work by Montanari et al. (2004), Glicino et al. (2006), Lucente et al. (2007) and Viscio et al. (2010), often considering the task of automatic control of gear shifting as well.

The manual transmission of the AMT systems can be actuated by either hydraulic or pneumatic actuators. Pressurized air is already present in trucks, and pneumatics are therefore chosen over hydraulic actuation which is more common in clutch systems for cars. Pneumatic systems have cost advantages and are easier to maintain than hydraulic systems, and with pneumatics there are no risk for environmental damage in case of leakages. However, the pneumatic actuators are inherently more difficult to control due to nonlinearities mainly caused by the compressibility of air.

Cost is in general a crucial factor in the automotive industry, and this is no different for the heavy duty truck industry. The Scania test truck present at KA has both position and pressure sensors included in the clutch system, but the pressure sensor is unwanted in the finalized production clutch system. The lack of measurement of all desired parameters motivates adaptive observer designs. Cost, combined with space advantages and better robustness properties, has also lead to on/off solenoid valves being chosen as control valves over proportional valves, which is the other common option. The on/off solenoid valves' discrete behavior motivates for switched controller designs.

1.2 Background

The clutch plates transmit torque from the motor to the axle shaft. Smooth engagement of these is crucial, and the clutch is subject to satisfy small friction losses, fast lock-ups and driving comfort, Bonporta et al. (2001), which may be conflicting requirements. To obtain this, position control of the clutch actuator is the most important factor when designing AMT systems, Born et al. (2001).

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have advantages as cleanroom, low cost, high ratio of power to weight and easy maintenance, Ali et al. (2009), but they also hold highly nonlinear characteristics due to compressibility of air, stiction and high friction forces which complicate accurate position control. Several papers have been published on the topic of position control of pneumatic actuators, see Alm and Yokota (2005) and Grim et al. (2009), and references therein.

To control the flow of air to the pneumatic actuator, there are two common choices: servo valves and on/off solenoid valves. Servo valves are the conventional choice, while the much cheaper, but more challenging (from a control point of view) on/off solenoid valves are increasing in popularity. On/off solenoid valves also have advantages as robustness, higher power-to-mass ratio and small size compared to servo valves. The disadvantages are mainly the valve’s discrete nature, limitations in response time and a dynamic response which is hard to model accurately. Modeling of on/off solenoid valves are treated in details by several authors. The work by Topcu et al. (2006), Hiskia and Johansson (2007) and Taghizadeh et al. (2009) show that accurate modeling of such solenoid valves is a complex task. In order to get a complete model of the valve, electromechanical, mechanical movements and fluid flows need to be described in accurate details.

Thus, Pulse Width Modulation (PWM) is usually applied when on/off solenoid valves are used for position control of pneumatic actuators. This approach has been extensively used e.g. Varseveld and Bone (1997), Gentile et al. (2002), Ahn and Yokota (2005) and Messina et al. (2005). Exploiting the PWM technique, only a flow characteristic of the on/off solenoid valve is needed. It also provides the possibility of allowing control laws designed for servo valves to be used for on/off solenoid valves. During the last decade, sliding mode techniques have become a more common approach for designing controllers for pneumatic actuators, especially for systems with on/off solenoid valves since the valves discrete nature can be exploited, e.g. Paul et al. (1994), Shen et al. (2004), Nguyen et al. (2007).

All industrial systems suffer from trade-offs between cost of sensors and the need for real-time information. Hence adaptive observers which in addition to estimating the system states also provide online estimation of parameters, are attractive solutions and a vast number of papers are published on the subject. One of the earliest publications on adaptive observers for nonlinear systems are Bastin and Gross (1980) which present an adaptive observer for Single-Input Single-Output observable systems that can be transformed into a certain canonical form, followed by Marino (1990) which extended the results to Multiple-Input Multiple-Output observable systems with same properties. Marino and Tomei (1992) and (1995a) present adaptive observers for nonlinear single-output systems which are linear with respect to an unknown constant parameter vector and transformable through a filtered transformation. Under persistency of excitation (PE) conditions, convergence of the parameters is guaranteed. Adaptive observers for more general classes of nonlinear systems are treated in details by several authors in Rajamani and Hedrick (1995), Cho and Rajamani (1997) and Bousson (2000). Their papers treat a class of nonlinear systems which holds Lyapunov functions satisfying particular conditions. An general overview of observer tools for nonlinear systems is given by Bousson (2007). But there exist no general theory on designing adaptive observers which in addition to estimating the system states also provide online estimation of parameters, e.g. Paul et al. (1994), Shen et al. (2004), Nguyen et al. (2007).

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1.2. Background

have advantages as cleanroom, low cost, high ratio of power to weight and easy maintenance, Ali et al. (2009), but they also hold highly nonlinear characteristics due to compressibility of air, stiction and high friction forces which complicate accurate position control. Several papers have been published on the topic of position control of pneumatic actuators, see Alm and Yokota (2005) and Grim et al. (2009), and references therein.

To control the flow of air to the pneumatic actuator, there are two common choices: servo valves and on/off solenoid valves. Servo valves are the conventional choice, while the much cheaper, but more challenging (from a control point of view) on/off solenoid valves are increasing in popularity. On/off solenoid valves also have advantages as robustness, higher power-to-mass ratio and small size compared to servo valves. The disadvantages are mainly the valve’s discrete nature, limitations in response time and a dynamic response which is hard to model accurately. Modeling of on/off solenoid valves are treated in details by several authors. The work by Topcu et al. (2006), Hiskia and Johansson (2007) and Taghizadeh et al. (2009) show that accurate modeling of such solenoid valves is a complex task. In order to get a complete model of the valve, electromechanical, mechanical movements and fluid flows need to be described in accurate details.

Thus, Pulse Width Modulation (PWM) is usually applied when on/off solenoid valves are used for position control of pneumatic actuators. This approach has been extensively used e.g. Varseveld and Bone (1997), Gentile et al. (2002), Ahn and Yokota (2005) and Messina et al. (2005). Exploiting the PWM technique, only a flow characteristic of the on/off solenoid valve is needed. It also provides the possibility of allowing control laws designed for servo valves to be used for on/off solenoid valves. During the last decade, sliding mode techniques have become a more common approach for designing controllers for pneumatic actuators, especially for systems with on/off solenoid valves since the valves discrete nature can be exploited, e.g. Paul et al. (1994), Shen et al. (2004), Nguyen et al. (2007).

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observes for arbitrary nonlinear systems and observers are most often designed individually for a specific system, exploiting the systems characteristics. This is also the common approach for pneumatic actuator systems.

1.2.1 Other work on electropneumatic clutch actuator systems

A sketch giving an overview of the considered clutch actuator system is shown in Figure 1.2. The valve body contains a set of on/off solenoid valves and the actuator is connect to clutch plates through a clutch lever. Because of this, the clutch actuator system is called a Pneumatic External Actuated (PEA) clutch system. The thesis by Kaasa (2006), “Nonlinear output-feedback control applied to electropneumatic clutch actuation in heavy-duty trucks”, also considers position control of a electropneumatic clutch actuation system and obtains experimentally extremely high tracking performance in the test rig. This thesis considers a similar clutch actuator with the following main differences in mechanical design and control hardware.

- On/off solenoid valves are used instead of proportional valves, as discussed above.
- The clutch actuator is no longer concentric, which means that the actuator can be exchanged without dismounting the transmission.
- A mechanical alteration of the system is conducted to reduce the dead volume, with the sole purpose of reducing the hysteresis effect in the system.

While Kaasa (2006) in his thesis considers output-feedback control, including nonlinear observers designed for the system, this thesis considers switched control and provides adaptive observers for the system.

Work by Grancharova and Johansen, (Grancharova and Johansen (2009a), (2009b) and (2010)), consider Nonlinear Explicit Model Predictive Control for the clutch actuator system with on/off valves. Good quality reference tracking is obtained using this type of controller combined with PWM. The system has also been considered in master theses at NTNU, Knudsen (2005), Vallevik (2006), Løkken (2006), Gjone (2007) and Hegland (2008) with various topics within modeling, control and observer design.

There are not much research published on pneumatic clutch actuation systems, other than the work on the KA clutch actuator system. Pneumatic clutch systems have been treated in the work by Tanaka et al.; Shimazu et al. (1986) develop a direct control pneumatic clutch, and Tanaka and Wada (1995) deal with fuzzy control for clutch engagement. Some earlier work by Tanaka also consider pneumatic clutch systems, see references within the latter article. Xian and Wikan consider modeling and control of a pneumatic actuated truck clutch system in a Technical report, Xiang and Wihan (1997) included in the PhD thesis Xiang (2001). Recently, Sano et al. (2010) presented a linearization based observer for an electropneumatic clutch system actuated by on/off solenoid valves.

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Figure 1.2: An overview of the considered clutch actuator system. Courtesy of Kongsberg Automotive ASA.
KA is currently using a PD-controller, only considering position measurement. Note that some further mechanical improvements of the clutch actuator system have been implemented after our experimental tests.

1.3 Overview of the Thesis

The remaining chapters in this thesis are organized as follows:

- Chapter 2 - An overview of the electromechanical clutch actuator is given, outlining its operation and describing the major mechanical parts. Configuration of the system for experimental testing in a test truck is presented, and aspects concerning the experiments are discussed.

- Chapter 3 - This chapter considers modeling of the electromechanical clutch actuator, and provides a basis for the models used for control and observer designs, and simulations in the later chapters. Physical phenomena are presented, and assumptions valid for the electromechanical clutch actuator are discussed.

- Chapter 4 - This chapter deals with switched control with emphasis on position control. A 3rd order model of the clutch actuator is presented for the purpose of control design. Two switched state feedback controllers are developed and evaluated, along with a combined state feedback dual-mode switched controller, utilizing both these controllers’ individual strengths. Stability proofs of the controllers are derived, and experimental results for all three controllers are provided. The two individual controllers are presented in Sande et al. (2007) and Langjord et al. (2008b), respectively, while the combined dual-mode design is presented in Langjord et al. (2009) and Langjord and Johansen (2010).

- Chapter 5 - Adaptive nonlinear observers for the clutch actuator system are considered, for the purpose of state feedback for the switched controllers developed in Chapter 4. Both a reduced order observer and a full-order observer are presented, the latter for reduction of the noise impact. Persistence of excitation conditions for convergence of the estimated controller parameters of the adapted parameters are derived. Performance of the adaptive observers are evaluated compared to experimental results from the test truck. The reduced-order adaptive observer is treated in Langjord et al. (2010) and the full-order observer is presented in Langjord et al. (2011a).

- Chapter 6 - The dual-mode switched controller from Chapter 4 and the full-order adaptive observer from Chapter 5 are combined for position control of the actuator piston. A simulation model of the system is presented, validated and used to show performance of the resulting adaptive observer-based switched controller. This chapter contains simulation results, indicating that the pressure sensor can be eliminated by an adaptive observer used as a basis for a nonlinear state feedback control design. This work is presented in Langjord et al. (2011b).

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1.4. Contributions

The main challenge addressed in this thesis is position control of an electropneumatic clutch actuator system actuated by on/off solenoid valves. As sensors in the clutch load characteristics. This work is regarded as preparation for on-line estimation treated in Chapter 5, and the publications Langjord et al. (2010) and Langjord et al. (2013a). Note that the author’s surname has changed from Sande to Langjord during the period of the given publications.


1.4 Contributions

Chapter 7 – Conclusions on the work presented in the thesis are made, and recommendations for further work are given.

The following is the list of the author’s publications related to the work presented in this thesis. The only publication not directly included in the thesis is Langjord et al. (2008b) (included in the Appendix), which treats off-line estimation of the clutch load characteristics. This work is regarded as preparation for on-line estimation treated in Chapter 5, and the publications Langjord et al. (2010) and Langjord et al. (2013a). Note that the author’s surname has changed from Sande to Langjord during the period of the given publications.


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production system should be limited to a position sensor, estimates of other states of the system are needed. Parameter estimation is also needed, as models for accurate description of the system are complex and need to be simplified according to assumption on the system. In addition characteristics of the clutch actuator will change as it is used, due to wear and other external influences such as temperature. The main contributions of this thesis can be summarized as:

- The development of switched controllers presented in Chapter 4. The on/off solenoid valves discrete behavior is exploited to design simple nonlinear controllers that either switches the on/off solenoid valves between fully open and fully closed. Using such switching controllers avoids additional controllers that control the flow of the valves, as for operation no knowledge of the dynamics of the on/off solenoid valves are needed for accurate control. Controllers based on PWM provide similar control inputs as those switched controllers, but to be able to calculate the PWM control signal, an inverse characteristic of the flow through the valves is needed, hence an accurate flow model is needed also in this case. For analysis of performance of the switched controllers, an expression of maximum flow through the valves is needed and knowledge about response times. The simplicity of the switched controllers also make them easy implementable.

- The development of the adaptive observer-based switched control presented in Chapter 5. No other adaptive observer with derived sufficient conditions for convergence of the estimation errors, is found in literature for systems like the considered clutch actuator system. This would be systems that are characterized by strongly uncertain and time-varying clutch load characteristic, strong dynamic friction and with only position sensor available. This contribution is important for position control of the clutch actuator as the proposed switched controllers require state feedback.

- The experimental evaluations of the switched controllers in a full scale truck using production sensors. Experimental results are also used to validate the performance of the adaptive nonlinear observers. These experimental evaluations are important contributions as they confirm the theoretical analysis, and show that the designs are suitable for implementation in a heavy duty truck.

- The development of the adaptive observer-based switched control considered in Chapter 6. Although this chapter focuses on simulation results, and does not include mathematical stability and experimental performance proofs for the switched controller with feedback from the adaptive observer, it still is considered as a major contribution. Simulation results indicate that the combined design is well suited for position control of the clutch actuator system, and position control is considered as the most important task to solve in this thesis. The simulations also show that in this case the adaptive observer can replace the pressure sensor without any significant loss of performance of the controller.

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Chapter 2

Electropneumatic clutch actuator

This chapter gives an overview of the electropneumatic clutch actuator that is considered in this thesis. The main components of the system are described and the operation of the clutch is outlined. Important aspects about the setup for experimental testing in the test truck are discussed.

2.1 System overview

Figure 2.1 presents a schematic which gives an overview of the clutch actuator system. This clutch actuator system consists mainly of an Electronic Control Unit (ECU), a set of on/off solenoid valves, a pneumatic actuator, position and pressure sensors and a piston rod which connects the actuator to the clutch plates.

Pneumatics are chosen to drive this system, as pressurized air already is present in heavy duty trucks. The ECU calculates the control signals which are sent to the on/off solenoid valves. Based on these, the on/off solenoid valves control the flow to and from the actuator chamber. The piston position is a result of the acting forces, mainly the friction, the pressure, the actuator spring and the clutch compression springs, and this position determines the state of the clutch plates. The plates can either be engaged, slipping or disengaged. When engaged, the clutch transmits torque from the motor to the axle shaft. The clutch is of pull type, which means that the clutch plates are fully engaged at zero piston position. As the piston moves to the right (air added to the actuator chamber) the clutch plates are pulled apart, first they will be slipping and then fully disengaged. A lip seal is used to avoid leakage between the piston and the cylinder wall, and thereby avoid flow between the two chambers in the actuator.

The clutch load force is the lumped force of the clutch compression spring and the counteracting, much weaker actuator spring. The characteristic of the clutch compression spring changes due to wear, hence the clutch load force also changes. Both the magnitude of the force and the shape of the load characteristic curve

Experimental testing in the test truck are discussed.

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are subject to change. The initial piston position, defined as the piston position corresponding to no actuation and the chamber pressures equal to the ambient pressure, will be influenced. As the clutch gets worn, this initial piston position will move and result in a negative position measurement. To avoid having to deal with negative position as changes in the initial position happen, the measured zero piston position is adjusted such that it always correspond to the actual initial position. In the production clutch actuator system this will be automatically calibrated at start up of the vehicle.

On/off solenoid valves are chosen over proportional valves as control valves, because they are smaller, cheaper and more robust, even though the on/off valves dynamics are harder to model accurately. Figure 2.2 shows cross-section illustrations of the on/off solenoid valves. They consist mainly of an electromagnet (pink), a valve seat (blue), an armature (green) and a spring. The illustration on the left shows a closed on/off solenoid valve, while the right one shows a fully open on/off solenoid. High pressure is indicated in yellow, and low pressure by dark gray. When an electric force is applied to the electromagnet, this sets up a magnetic field which will attract the armature and try to open the valve. The spring and the friction acts as closing forces, in addition to the pressure force over the valve seat which in

Figure 2.1: Electropneumatic clutch actuator system.

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Figure 2.1: Electropneumatic clutch actuator system.
2.2 Experimental testing - system configuration

The experimental tests presented in this thesis have been conducted in a Scania test vehicle, Gamal, at Kongsberg Automotive ASA, see Figure 2.3. In this vehicle both position and pressure sensors are present, although only a position sensor will be available in the finalized production system. Position and pressure are measured at a rate of 1 ms. The controller algorithms are designed in a Matlab/Simulink environment and run on a dSpace MABX 1401 unit. The sampling interval, and the controller updating interval, are both set to 1 ms. Figure 2.4 shows how the test system is connected. A virtual instrument control panel was developed in a dSpace Control Desk program. Through this panel parameters can be adjusted, and variables and measurements can be plotted on-line. The reference position for the piston was set manually through this instrument panel in the experimental test. In normal operation, this reference position will either be set by the truck.

Figure 2.2: Cross-sections of an on/off solenoid valve, the left one illustrating a closed valve and the right illustrating a fully open valve.
2.2.1 Requirements for system performance

A benchmark clutch sequence for evaluating control system performance is shown in Figure 2.5. It is desired that the controller makes the system reach the reference point within 0.1 s, and with a steady state position error of less than 0.2 mm in the area where the clutch engage/disengage. This area is marked in Figure 2.5. Outside this area, the requirements can be somewhat relaxed. Using a somewhat smoother curve as reference clutch sequence might improve controller performance.

Figure 2.3: The test truck, "Gamal".

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Figure 2.3: The test truck, "Gamal".

driver through an electrical signal from the clutch pedal, or by calculations from an automated gear shift control system.

Experiments for testing the switched controller were conducted with different control valves (see section below and Chapter 4), in February 2008 and April 2009, and hence with different degree of wear of the clutch and temperature. After the tests in April 2009, the configuration of the clutch system were altered to an actuator with integrated control unit, and after this further testing was not available. For this reason, experimental data obtained from the test truck in November 2007 has been used for validation of the observer design in Chapter 5.
2.2. Experimental testing - system configuration

Figure 2.4: System configuration.

Figure 2.5: Clutch sequence used as position reference in the experiments to test control system performance.
in terms of less overshoot and reduced oscillation. The proposed clutch sequence is still used as it equals typical sequences used for testing of the clutch actuator system by KA.

### 2.2.2 Control valves

Two sets of on/off solenoid valves have been used in the experimental tests. Both sets are delivered by the Swedish company SO-Elektic, which has specialized in producing valves with short opening and closing times, that also maintain good response times even with large flow areas. A prototype SO-valveset was used in the tests in February 2008 and a prototype SO-valveset was used in the testing in April 2009. Their characteristics have been examined by testing at KA and are given in Table 2.1. The valvesets are quite similar, the main difference is the shape of the valvehead, which were altered for the prototype valveset mainly to reduce acoustic noise. The experimental tests for validation of the adaptive observer are conducted with the pre-prototype valveset present in the truck. Since different at the two testing times of the switched controller in Chapter 4. Figure

### Table 2.1: Valve characteristics.

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<tr>
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Both valvesets are prototypes, and suffering from overheated electronics, care had to be taken not to blow fuses or destroy the valveset electronic unit during testing. The clutch load characteristic changes with wear and temperature, and was different at the two testing times of the switched controller in Chapter 4. Figure

### 2.2.2 Clutch load

The clutch load characteristic changes with wear and temperature, and was different at the two testing times of the switched controller in Chapter 4. Figure

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</tr>
<tr>
<td>Maximum volumetric flow rate, exhaust</td>
<td>51 l/s</td>
<td>16 l/s</td>
</tr>
</tbody>
</table>

Table 2.1: Valve characteristics.
2.2.4 Sensors

The sensors are production quality sensors. These suffer from more noise than more expensive, higher quality sensors used in test rigs made for research and development, as the ones used by Kaasa (2006). Since the tests are performed in the actual truck, the measurements are also influenced by the motor vibrations of the test truck. These vibrations can make the clutch piston move, even with no vibrations from the test truck. These vibrations can make the clutch piston move, even with no vibrations from the clutch itself.

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actuation of the clutch actuator. Figure 2.7 shows position measurement provided by the position sensor in the test truck with no actuation, that is, no supply and no exhaust of air to/from the actuator chamber and reference position $y = 0$. The noise in these measurements correspond to variance $9.72 \times 10^{-3}$ mm$^2$ and mean $-0.0183$ mm. Similar position measurement noise is also detected with no actuation for the piston in other position.

Figure 2.7: Position measurement provided by the position sensor when no input is given, and reference position is $y = 0$, showing the noise caused by motor vibration and signal transmission in the position sensor.
Chapter 3
Modeling

This chapter treats modeling of the clutch actuator system. In principle, the same
clutch actuator system was also treated by Kaasa (2006) in his doctoral thesis, and
in general we refer to his thesis for more details on modeling of the system. The most
significant difference in the system here considered, is the change of control valve
from from a proportional valve to a set of on/off solenoid valves.

3.1 Motion dynamics

The motion dynamic of the clutch actuator piston is described by using Newton’s
second law

\[ M \ddot{p} = f_p - f_f \]  

where \( M \) is piston mass, \( \dot{p} \) is piston velocity, \( f_p \) describes the pressure force, \( f_f \)
describes the clutch load characteristic and \( f_f \) describes the friction force. Consi-

\[ f_p(p_a, p_B) = A_B p_B - A_A p_A - A_P l \]  

where \( A_A \) and \( A_B \) are the areas of chambers A and B, respectively, \( p_A \) and \( p_B \) are
the pressures in the chambers, \( A_P = A_A - A_B \) is the area of the piston rod and \( P_l \)
is the ambient pressure.

3.2 Friction

The friction force is a substantial force in the clutch actuator, and the elasticity
of the seal on the piston makes modeling of pre-sliding properties important for
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hysteresis to arise.
Pro-sliding friction force describes friction without movement of the piston. This is friction due to asperity junctions arising between the actuator piston and the cylinder wall when an external force is applied to the bodies. These junctions behave like springs, and result in an elastic deformation of the asperities, Englund and Gravdahl (2002). As a certain force the junctions will be broken and the piston will start to move, and this force is called the break-away force. This break-away force will also vary with the pressure in the cylinder, depending on how hard the seal is pressed against the cylinder. The friction force with a moving piston, called sliding friction, is dependent on the piston velocity. The simplest friction model is the viscous friction model

\[ f = Dv \]  
(3.3)

where the friction is proportional to velocity, and \(D\) is the viscous friction constant. This model is the best description of the friction force in cases where the bodies are separated by a lubricate building a full fluid film between the bodies, but can also be tuned to represent systems as damped rather well for other cases, Andersson et al. (2007). A common model is the Coulomb friction model, in its simplest form given as

\[ f = F_C \operatorname{sign}(v) \]  
(3.4)

where the Coulomb friction force \(F_C\) is given by

\[ F_C = \mu N \]  
(3.5)

where \(\mu\) is the friction coefficient and \(N\) is the load. The Coulomb friction is derived by assuming no contaminations on the surfaces, and is often referred to as dry friction, but is also used to model friction for boundary and lubricated contacts. The friction decreases with increased velocity for lubricated sliding contacts, until a full fluid film is built between the contacts. Similar behavior is also found for dry contacts, Andersson et al. (2007), and is called the Strubeck effect. A Strubeck friction model is

\[ f = f_s + (F_v - F_s)\exp\left(-\frac{|v|}{v_s}\right) \]  
(3.6)

where \(f_s\) is the Coulomb friction, \(F_v\) is the break-away force, \(v_s\) is called the Strubeck velocity and \(v\) is the piston velocity. This model gives a smooth description of the transition from the pre-sliding friction regime when combining pre-sliding friction and sliding friction models. The viscous, the Coulomb and the Strubeck friction models are all static models, and to account for the dynamic phenomena as the hysteresis discussed above, a dynamic model will be needed.

The LuGre model was proposed in Canudas de Wit et al. (1995) as a model well suited to describe friction forces for hydraulic and pneumatic control systems. This model is a model capturing most friction behaviors discussed above; viscous friction, Coulomb friction, the Strubeck effect, hysteresis and varying break-away forces can be tuned to represent systems as damped rather well for other cases, Andersson et al. (2007). A common model is the Coulomb friction model, in its simplest form given as

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all be included. This model is given as
\[ i = -\frac{1}{f_0} f z \]  
\[ j_f = K_d + D_d z + D v \]  
where \( f_0 \) is dry friction, \( K_d \) is deflection stiffness and \( D_d \) is the deflection damping coefficient. The state \( z \) is the friction state, and can here be described as the pre-sliding seal deflection.

3.3 Pressure dynamics
Air flow dynamics are described by both pressure and temperature dynamics. Kaasa (2006) presents a detailed derivation of a full model of the systems air flow dynamics based on elementary thermodynamics using conservation of energy. Here we propose a reduced isothermal model of the pressure dynamics. That is, we consider all temperatures in the system to be equal to the standardized atmospheric temperature (\( T_0 \)), which is done as the pressure dynamics sensitivity to temperature changes is found to be small for the clutch actuator system, Kaasa (2006).

The reduced model is obtained by taking the following assumptions, based upon the assumptions listed in chapter 4 in Kaasa (2006):
- At the attainable pressure, air behaves like an ideal gas obeying the ideal gas equation of state with negligible error.
- The thermodynamic properties are uniformly distributed (homogeneous) within the control volume, i.e., perfectly mixed. This is reasonable due to the small dimension of the system, and a complex distributed problem formulation is avoided.
- The energy change in the fluid due to elevation is negligible.
- The flow through pipes, valves and restrictions in the system is assumed to be isentropic. That is, frictionless flow is assumed, and the effect of heat transfer on the flow is disregarded (adiabatic flow). This is a common assumption when system dimensions are small, Bobrow and McDonell (1998).
- The kinetic energy within the chambers are negligible as the chambers can be considered to be reservoirs.
- We assume isothermal conditions, which means that the chamber temperatures are constant. Furthermore, we assume that all temperatures equal the standardized atmospheric reference condition (\( T_0 \)) given by the ISO standards. This gives constant specific heats \( c_p \) and \( c_v \) of air.
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- We have a constant supply pressure \( p_{in} \), and assume that the exhaust pressure \( p_{ex} \) equals a constant atmospheric pressure.
This gives us the following equations for the pressure dynamics,

\[ p_A = \frac{1}{V_A'} \left( V_A V_A'' \right) \frac{\partial h}{\partial t} \quad (3.9) \]
\[ p_B = \frac{1}{V_B'} \left( V_B V_B'' \right) \frac{\partial h}{\partial t} \quad (3.10) \]

where \( R \) is the ideal gas constant of air, \( \nu_A = \nu_{A,0} - \nu_{A,A} \) and \( \nu_B = \nu_{B,0} - \nu_{B,B} \) are the resulting flows into the chambers \( A \) and \( B \) through, respectively, the on/off solenoid valves and the outlet restriction. The variables \( V_A(p) \) and \( V_B(p) \) are the chamber volumes at a given position, \( V_A(p) = V_A + A p + A y \) and \( V_B(p) = V_B + A y \) where \( V_A \) and \( V_B \) are the chamber volumes at initial position, \( y = 0 \).

### 3.4 Valve flow dynamics

Flow through the on/off solenoid valves and the outlet restriction can be modeled as flow through a restriction. In ISO (1989) the standardized orifice flow equation is defined as

\[ w = \frac{p}{\sqrt{RT}} \left( \frac{D}{y} \right)^3 \frac{\rho}{y} \]

where \( C \) is conductance, \( \rho \) is density, \( r = \frac{\rho}{\rho_0} \) is the ratio between the low \( \rho \) and the high \( \rho_0 \) density at the sides of the orifice and \( w \) is a pressure ratio function which determines how the flow depends on the pressure rate. By assuming the the exhaust pressure equal to the ambient pressure and all temperatures equal to standardized atmospheric reference condition, \( \Omega_0 \) as in previous section, a simplified version of the standardized orifice flow equation (see also chapter 5.3 in Kaas (2006)) can be used to describe flow through a restriction

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\[ w = \frac{p}{\sqrt{RT}} \left( \frac{D}{y} \right)^3 \frac{\rho}{y} \]
and $R_0 = 0.528$ is the isentropic critical pressure ratio for air. The pressure ratio function (3.12) with the expressions in (3.14) and (3.15) can be characterized as approximately isentropic and incompressible laminar flow, respectively. The value of $R$ decides between the two flow types.

Experiments have shown that we can set $B = 0$, that is using the pressure ratio function
\[
\omega(r) = \Omega(r) = \begin{cases} 1 - r^2, & r \in [0, 1] \\ 0 & r > 1 \end{cases},
\]
and get a simplified model that still describes the flow through the on/off solenoid valves and the outlet restriction with sufficient accuracy for control purposes.

3.4.1 Flow of chamber A

Equation (3.11) with the pressure ratio function given in (3.16) is used to describe the maximum flow through the on/off solenoid valves. The resulting air flow to/from chamber A is
\[
w_s(p_A, u) = w_{u,s} - w_{c,s} = \rho A_w \left( \frac{P_A}{P_s} \right) P_{u,s} - \rho A_w \left( \frac{P_s}{P_A} \right) P_{c,s},
\]
where the subscripts "s" and "c" stand for supply and exhaust, respectively, and $w_s$ and $w_c$ are valve command inputs obtained from the input as shown in Table 3.1.

Table 3.1: Relations between the input to the system and the valves command input.

<table>
<thead>
<tr>
<th>$u$</th>
<th>$w_u$</th>
<th>$w_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Remark 3.4.1.

In Chapter 5, we compare our developed observer to experiments conducted in the test truck by KA. In these experiments PWM are used, and for describing flow through the on/off solenoid valves in this case, (3.17) needs to be rewritten to take the opening degree of the valves into consideration,
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w_{u,s}(p_A, u) = w_{u,s} - w_{c,s} = \rho A_w \left( \frac{P_A}{P_s} \right) P_{u,s} - \rho A_w \left( \frac{P_s}{P_A} \right) P_{c,s},
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3.4.2 Flow of chamber B

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<td>1</td>
<td>0</td>
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</tbody>
</table>
where 
\[ y_{in} = \omega^T \phi_{in} \]  
\[ y_{out} = \omega^T \phi_{out} \]  
(3.19)  
(3.20)  
describes the mean valve opening where \( w_i \) and \( w_o \) are the command inputs of the valves PMW input. \( R_{in} \) and \( R_{out} \) are the command inputs where the valves start to open and \( R_{in} \) and \( R_{out} \) are the command inputs for which the valves are fully open. In the rest of the thesis the opening degree of the on/off solenoid valves are considered as fully open or fully closed, corresponding to PWM design with only command inputs 0 and 1, respectively. As the set \([0, 1]\) is a subset of the input range \( w_{in} \), the equation (3.19) is still valid, and can be used in both cases.

### 3.4.2 Flow of chamber B

The resulting flow to/from chamber B is
\[ w_{in}(p_B) = w_{in} - w_{out} \]
\[ = \rho C_{in} \left( \frac{1}{R_{in}} \right) P_B - \rho C_{out} \left( \frac{1}{R_{out}} \right) p_B \]  
(3.21)  
where \( w_{in} \) represents the flow through the outlet restriction if \( P_B > p_B \) and \( w_{out} \) represents the flow if \( p_B > P_B \). 

These chamber flow functions satisfy
\[ \frac{\partial w_{in}}{\partial p} (p_B, u) < 0 \]  
\[ \frac{\partial w_{out}}{\partial p} (p_B, u) < 0 \]  
(3.22)  
(3.23)
as verified in Figures 3.1a and 3.1b.

### 3.5 Clutch load characteristic

The clutch load force describes the lumped force of the actuator spring and the clutch compression spring. The actuator spring is linear, while the compression spring is highly nonlinear and of a much higher magnitude than the actuator spring. The clutch load depends mainly on the piston position, and can generally be parameterized in the linear form
\[ f(y) = \varphi(y) \theta \]  
(3.24)
where \( \varphi(y) = (\varphi_1(y), \varphi_2(y), \ldots, \varphi_n(y))^T \) is a vector of basis functions and \( \theta \) is the corresponding weighting parameter vector. The linear parameterized form is well

\[ y_{in} = \omega^T \phi_{in} \]  
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The resulting flow to/from chamber B is
\[ w_{in}(p_B) = w_{in} - w_{out} \]
\[ = \rho C_{in} \left( \frac{1}{R_{in}} \right) P_B - \rho C_{out} \left( \frac{1}{R_{out}} \right) p_B \]  
(3.21)  
where \( w_{in} \) represents the flow through the outlet restriction if \( P_B > p_B \) and \( w_{out} \) represents the flow if \( p_B > P_B \). 

These chamber flow functions satisfy
\[ \frac{\partial w_{in}}{\partial p} (p_B, u) < 0 \]  
\[ \frac{\partial w_{out}}{\partial p} (p_B, u) < 0 \]  
(3.22)  
(3.23)
as verified in Figures 3.1a and 3.1b.

### 3.5 Clutch load characteristic

The clutch load force describes the lumped force of the actuator spring and the clutch compression spring. The actuator spring is linear, while the compression spring is highly nonlinear and of a much higher magnitude than the actuator spring. The clutch load depends mainly on the piston position, and can generally be parameterized in the linear form
\[ f(y) = \varphi(y) \theta \]  
(3.24)
where \( \varphi(y) = (\varphi_1(y), \varphi_2(y), \ldots, \varphi_n(y))^T \) is a vector of basis functions and \( \theta \) is the corresponding weighting parameter vector. The linear parameterized form is well

Figure 3.1: The flows $u_A(p_A, u)$ and $w_B(p_B)$ are monotonically decreasing in respectively $p_A, p_B$, for fixed $u$.

Figure 3.1: The flows $u_A(p_A, u)$ and $w_B(p_B)$ are monotonically decreasing in respectively $p_A, p_B$, for fixed $u$.
Figure 3.2: Clutch load characteristics model with $n = 2$, i.e. two B-spline basis functions.

(a) The basis functions used for clutch load modeling.

(b) Clutch load, modeled (dashed) and experimental (dashed).

The experimental data include not only the clutch load force but also the effect of dynamic friction (hysteresis).
suit for estimation, which is needed as the clutch load characteristics change with
temperature and wear of the clutch.
A model with \( n = 2 \) and the following B-splines are proposed. The spline basis
functions are built up by polynomials

\[
\phi_i(x) = \begin{cases} 
0, & x < t_i \\
\alpha_i x^3 + b_i x, & t_i \leq x < t_{i+1} \\
\alpha_{i+1} x^3 + b_{i+1}, & x \geq t_{i+1}
\end{cases} \quad (3.25)
\]

\[
\phi_i(x) = \begin{cases} 
0, & x < t_i \\
10(x - t_i), & t_i \leq x < t_{i+1} \\
10(x - t_{i+1}), & x \geq t_{i+1}
\end{cases} \quad (3.26)
\]

where the knots are set to \( t_1 = 0 \) mm, \( t_2 = 0.5 \) mm and \( t_3 = 8 \) mm. To be able to
find the spline coefficients from the positions of the knots we need some criteria:

- **\( \phi_1 \)**
  - Derivative in \( t_2 \) is to be equal to zero
  - The value is to be 400 in \( t_3 \)
- **\( \phi_2 \)**
  - Transition between the linear and the quadratic part is to be smooth
  - Derivative in \( t_3 \) is to be equal to zero

Mathematically written this gives

\[
\eta_i = A_i \beta_i, \quad i = 1, 2
\]

where

\[
A_1 = \begin{bmatrix} \frac{t_2}{2} & \frac{t_2}{2} & 0 \\ \frac{t_2}{2} & 1 & 0 \\ \frac{t_2}{2} & 0 & 1 \end{bmatrix}
\]

\[
A_2 = \begin{bmatrix} \frac{t_2}{2} & \frac{t_2}{2} & 0 \\ \frac{t_2}{2} & 1 & 0 \\ \frac{t_2}{2} & 0 & 1 \end{bmatrix}
\]

\[
B_1 = \begin{bmatrix} 400,0 \end{bmatrix}^T
\]

\[
B_2 = \begin{bmatrix} [t_2(t_2 - t_1), 10T, 0] \end{bmatrix}^T
\]

and

\[
\eta_1 = [a_1, b_1]^T
\]

\[
\eta_2 = [a_2, b_2]^T
\]
As an example, Figure 3.2 show the proposed splines and the resulting clutch load model using $\theta_0 = [4, 5]$ compared with a clutch load characteristic obtained from experimental data. To account for the clutch load curve moving significantly to the left/right due to wear and temperature, a Multiple Model Scheme can be used. Here a supervisory logic choosing the best set of basis functions from multiple models where $\phi_1$ deflects at different positions similar to approaches in Bakkeheim and Johansen (2006) and Narendra and George (2002). This is illustrated in Figure 3.3, where a set of $\phi_i$'s are given along with resulting clutch load models.
Figure 3.3: Clutch load characteristics using \( \theta = [4, 5] \). These are examples of models that could be used for a Multiple Model Scheme.
Chapter 4
Switched control

This chapter considers nonlinear state-feedback position control of the electropneumatic clutch actuator assuming a given nonlinear model. Two switched controllers are designed along with one dual-mode switched controller combining these. This chapter is mainly based on Sande et al. (2007), Langjord et al. (2008a), Langjord et al. (2009) and Langjord and Johansen (2010). Adaptive observer-based extensions are given in Chapter 5 and 6.

4.1 Introduction

On/off solenoid valves are used to control the airflow to/from the actuator, and accurate position control of the actuator piston depends on good control of these valves. The on/off valve can, in contrast to what the name implies, also be controlled to take positions in between fully open and fully closed. But as the on/off valves dynamics are highly nonlinear, and thus hard to model accurately, an interesting approach will be to consider the valve as only being able to hold one of the two positions; fully open or closed. This motivates for a switched controller design.

Switched systems have received lots of interest the last decade, with a vast number of publication in fields as control of mechatronic systems and automotive industry. Switched controllers used for continuous systems is one of two cases where switched systems arises, the other case being abrupt changes in the structure and the parameters of dynamic systems. In general, the main motivation for using a switched controller is to achieve better performance. The switched designs in this chapter are in addition motivated by the possibility of exploiting the on/off solenoid valves discrete behavior and thereby simplifying the modeling task for control design.

Dealing with discrete input devices, such as on/off solenoid valves, Pulse Width Modulation (PWM) is often employed. PWM may allow control laws made for proportional valves to be used for solenoid valves. Varseveld and Bøe (1997) show how fast, accurate and insensitive position control for pneumatic actuators can be obtained using on/off solenoid valves and PWM. Carducci et al. (2004) propose

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a vision feedback on a robotic arm where the on/off solenoid valves are driven by PWM techniques, while Alin and Yokota (2003) propose a modified PWM scheme to improve steady-state error often arising when using on/off valves.

During the last decades, sliding mode techniques have also become a common approach to design controllers for pneumatic actuators actuated by on/off solenoid valves. Many of the controllers developed by sliding mode are continuous and implemented by PWM, as in Shen et al. (2004) which create a sliding mode control with equivalent control and the use of PWM to produce the control signal. Some utilize the discrete nature of the on/off solenoid valves and the sliding mode technique. Paul et al. (1994) propose a position control algorithm based on sliding mode control for a pneumatic cylinder with on/off valves, and Nguyen et al. (2007) also use a sliding mode approach to construct a control signal which can be directly applied to solenoid valves.

One of the main advantages in designing controllers that can be applied without employing PWM, is that we do not need to know the on/off valves characteristics. The only knowledge needed are their response times and the maximum flow capacities for design, stability and performance analyses of the controllers. The switched controllers may also achieve faster closed loop response time since the controller is not limited by the PWM duty cycle.

As part of the switched controller designs in this chapter, the backstepping method is considered. Backstepping is in general used to produce stabilizing controllers through a recursively design, while also construct a Control Lyapunov function (CLF) for the system. This technique is not extensively used for pneumatic actuator systems. Rao and Bose (2008) propose a Multiple Input Multiple Output control law for a pneumatic actuator with four low-cost two-way proportional valves. Kaasa (2006) considers output-feedback control for the clutch actuator system with a proportional valve. A model-based nonlinear tracking controller is designed by a recursive observer-based backstepping approach, producing a robust controller design.

4.1.1 Model for control design

For control design a simple model of the electropneumatic clutch actuator is desired. Some assumptions are made, in addition to the aspects discussed in Chapter 3, to obtain a model adequate for control design:

- The pressure dynamics in chamber B are neglected, that is assuming \( p_B = 0 \).

This reduces the pressure dynamics to

\[
p = \frac{A_y}{V_y} \left( \frac{RT}{V_p} \right) \quad (4.1)
\]

where \( p = p_B + A \), \( A = A_y \) and \( V_y = V_y(p) \) for simplicity of notation.

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The clutch load characteristic is modeled by a nonlinear function
\[ f(q) = K(1 - e^{-q}) - M_{T}g \] (4.2)
where \( K, M_{T}, L_{d} \) and \( L_{s} \) are constants that are tuned manually.

Dynamic friction is neglected, leaving only viscous friction \( \dot{D}_{v} \).

The flow through the on/off solenoid valves are expressed by \( w = U_{max,th} \), where \( u \) is the control input and \( U_{max,th} = \mu_{CP}L_{th} \) is a simplified, constant expression for the maximum flow capacity through these valves.

The supply and exhaust capacities are in general different and are dependent on the pressure difference over the valves and the ambient temperature. The simplifying assumption on the flow capacity can be done by assuming choked flow conditions. In the most important operational region, the supply/diaphragm area, the pressure drop tends to be significant due to a supply pressure of 9.5 bar and a maximum cylinder pressure of less than 6 bar. Therefore, the region where the choked flow assumption is not valid is when the clutch is completely disengaged and the cylinder is being emptied from air. However, this modeling error is disregarded in this chapter as control accuracy is not as important in this region.

To simplify notation, we denote the state variables in this chapter by \( x = [x_{1}, x_{2}, x_{3}] \) where
\[ x_{1} = y \]
\[ x_{2} = v \]
\[ x_{3} = vM_{T} \]
that is, \( x_{1} \) is piston position \([m]\), \( x_{2} \) is velocity \([m/s]\) and \( x_{3} \) is accumulated air \([kg/m^{3}] \), which is proportional to the amount of air in the actuator chamber, and \( V_{A}(x) = V_{A} + A_{x}x_{3} \) describes the volume in chamber \( A \).

The resulting 3rd order model for control design is
\[ \dot{x}_{2} = \frac{1}{B_{v}} \left[ f(x_{1},x_{3}) - \frac{R_{T}T_{c}V_{A}}{B_{v}} \right] \]
\[ \dot{x}_{2} = \frac{1}{B_{v}} \left[ f(x_{1},x_{3}) - \frac{R_{T}T_{c}V_{A}}{B_{v}} \right] \] (4.4a)
\[ \dot{x}_{2} = \frac{1}{B_{v}} \left[ f(x_{1},x_{3}) - \frac{R_{T}T_{c}V_{A}}{B_{v}} \right] \] (4.4b)
where \( u \) is the valve control input. The state errors are written as \( \tilde{x}_{1} \), \( \tilde{x}_{2} = v \), \( \tilde{x}_{3} = vM_{T} \), and \( \tilde{x}_{4} = \) the reference piston position \( x_{4} \) is given, and \( \tilde{x}_{2} \) is given by the steady-state relation
\[ \tilde{x}_{2} = \frac{1}{B_{v}} \left( K(1 - e^{-\tilde{x}_{4}}) - M_{T}g \right) \] (4.5)

The theoretical region of operation for the considered clutch actuator is \( O = \{ x_{1} \in [0.0025, 0.1], x_{2} \in [0, 0.025], x_{3} \in [0, 0.05] \} \), where the position limits correspond to mechanical end stop. The only available inputs using one supply and one exhaust on/off valve are \( u \in [-1.0, 1.0] \). (4.6)
and Table 4.1 gives the relation between input and valve positions.

<table>
<thead>
<tr>
<th>Input</th>
<th>Supply valve</th>
<th>Exhaust valve</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Closed</td>
<td>Closed</td>
</tr>
<tr>
<td>1</td>
<td>Open</td>
<td>Closed</td>
</tr>
</tbody>
</table>

Table 4.1: Valve positions corresponding to the inputs.

In this chapter we assume full state feedback. The test truck provides position and pressure measurements, while velocity for the experiments in this chapter is numerically differentiated from the position measurement using a second order low pass filter with coincident poles in \(-40\) rad/s. Parametric values are given in Table 4.2.

### 4.1.2 Control strategy

Li et al. (2001) establish two major tasks that should be accomplished as a switched controller is designed. These are the design of basic controllers, and the definition of the switching law of the basic controllers. The input restrictions, limiting our controller choices to \( u \in \{-1, 0, 1\} \), reduce the piston position control problem to finding a switching law that can govern switching between these available inputs, in a way that stabilizes the system. We pose the following control strategy:

**Control strategy**: At each sampling instant, choose the available input which gives the most negative definite Lyapunov function time derivative with a well chosen control Lyapunov function (CLF).

Utilizing this strategy, a new task has to be solved, that is the design of appropriate CLFs. In this chapter we consider two different methods for this:

- **Using backstepping**
  - Based on knowledge of the system

and Table 4.1 gives the relation between input and valve positions.

<table>
<thead>
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<th>Supply valve</th>
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<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>( M )</td>
<td>10</td>
<td>kg</td>
</tr>
<tr>
<td>Actuator area</td>
<td>( A )</td>
<td>12.3 ( \times 10^{-3} )</td>
<td>m</td>
</tr>
<tr>
<td>Ambient pressure</td>
<td>( P_a )</td>
<td>1 ( \times 10^5 )</td>
<td>Pa</td>
</tr>
<tr>
<td>Supply pressure</td>
<td>( P_t )</td>
<td>9.5 ( \times 10^7 )</td>
<td>Pa</td>
</tr>
<tr>
<td>Temperature</td>
<td>( T )</td>
<td>293</td>
<td>K</td>
</tr>
<tr>
<td>Gas constant of air</td>
<td>( R )</td>
<td>288</td>
<td>( J/K )</td>
</tr>
<tr>
<td>Volume at ( y = 0 )</td>
<td>( V_0 )</td>
<td>0.148 ( \times 10^{-3} )</td>
<td>m(^3)</td>
</tr>
<tr>
<td>Viscous damping</td>
<td>( C )</td>
<td>267 ( \times 10^{-5} )</td>
<td>m(^2)/s/kg</td>
</tr>
<tr>
<td>Density</td>
<td>( \rho )</td>
<td>1.185</td>
<td>kg/m(^3)</td>
</tr>
</tbody>
</table>

Table 4.2: Parameters for the clutch actuator model.
By performing a full backstepping controller design, a Lyapunov function for the clutch actuator system is designed. Second we choose a Lyapunov function based on knowledge of the system. This is done by extending a Lyapunov function used to show stability of the second order reduced open loop system ($u = 0$, $x_t = x_2^*$), to yield for the 3rd order system.

4.2 Controller 1 - "Local controller"

Backstepping is a well known technique to design a state feedback control input and a Lyapunov function proving its stability in the same operation, see Kristic et al. (1995). If we assume no restrictions on the input, backstepping can be used to find a suitable control Lyapunov function for the system.

4.2.1 Backstepping

Step 1
First we define

$$\xi_1 = x_1 - x_1^*$$

(47)

which gives

$$\dot{\xi}_1 = x_2.$$  

(48)

We choose the virtual control $x_2 = \phi_0(\xi_1) = -k\xi_1,$ where $k$ is a positive constant. From the Lyapunov-like function

$$U_1(\xi_1) = \frac{\alpha_x}{24} \xi_1^4,$$

(4.9)

it is easy to show that this virtual control gives

$$\dot{U}_1 = -\alpha_x k^2 \xi_1^3.$$  

Step 2
The change of variables

$$\xi_2 = x_2 - \phi_0(\xi_1) = x_2 + k\xi_1,$$

(4.10)
	ransforms the system into

$$\dot{\xi}_2 = \frac{1}{4} \left( -K_1(1 - e^{-2\xi_2}) - D(\xi_2 - k\xi_2) + M_x x_1 + \frac{k}{\sqrt{V_{x_1}}} - A \right)$$

(4.11a)

and

$$K_2 - k^2 \xi_1.$$  

(4.11b)

We now choose $x_2$ as the virtual control $\phi_2(x_1, \xi_2, \xi_3)$ and

$$U_2(\xi_1, \xi_2) = U_1(\xi_1) + \frac{\alpha_x}{24} \xi_2^4,$$

(4.12)

By performing a full backstepping controller design, a Lyapunov function for the clutch actuator system is designed. Second we choose a Lyapunov function based on knowledge of the system. This is done by extending a Lyapunov function used to show stability of the second order reduced open loop system ($u = 0$, $x_t = x_2^*$), to yield for the 3rd order system.

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A new Lyapunov function is chosen to get

\[ U_2 = \alpha l \xi^2 - D^2 \xi^2 \]  

and by setting \( \alpha = \beta k^2 \)

we get

\[ E_2 = -\alpha l \xi^2 + \frac{D^2}{2} \xi^2 \]

We choose the virtual control as

\[ \phi(x, \xi) = \frac{V(x)}{4}(K_1(1 - e^{-\lambda \xi})) - M_1 x_1 - D \xi_1 + A P_1 + M \xi_2 \]  

Step 3

The change of variables

\[ \xi = x - \phi(x_1, \xi_1) \]

\[ = x - \frac{V(x)}{4}(K_1(1 - e^{-\lambda \xi})) - M_1 x_1 - D \xi_1 + A P_1 + M \xi_2 \]

transforms the system into

\[ \dot{\xi} = -\xi \xi + \frac{D}{2} \xi \]  

\[ \xi = \text{RE}_\text{switched control} \]  

\[ = \left( \xi - \phi(x) \right) (K_1(1 - e^{-\lambda \xi})) - M_1 x_1 - D \xi_1 + A P_1 + M \xi_2 \]

\[ - M(\xi - \xi_2) + D \xi_2 - M \left( \text{RE}_\text{switched control} - \dot{\xi}_1 \right) \]

A new Lyapunov function is chosen

\[ V_1(x) = U_1(x_1, \xi_1) + \frac{\lambda}{2} \xi_1^2 \]  

From

\[ U_2 = \alpha l \xi^2 - D^2 \xi^2 \]

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Step 3

The change of variables

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\[ \xi = \text{RE}_\text{switched control} \]  

\[ = \left( \xi - \phi(x) \right) (K_1(1 - e^{-\lambda \xi})) - M_1 x_1 - D \xi_1 + A P_1 + M \xi_2 \]

\[ - M(\xi - \xi_2) + D \xi_2 - M \left( \text{RE}_\text{switched control} - \dot{\xi}_1 \right) \]

A new Lyapunov function is chosen

\[ V_1(x) = U_1(x_1, \xi_1) + \frac{\lambda}{2} \xi_1^2 \]
and this leads to
\[ V_1 = -\alpha_1 k_x^2 - \beta \frac{D_i}{M} \frac{d}{dt} \frac{\partial V}{\partial x} + \lambda_2 \xi_x (RT e^{U_{max}} - u) \]
\[ - (\xi_x - k_x)(K_1(1 - e^{-\epsilon x}) - M \dot{x}_1 - DK \dot{x}_1 + AP_1 - MK \xi_x) \]
\[ - \frac{\partial V}{\partial x} \left( \xi_x - k_x \right) K_2 e^{-\epsilon x} - M \xi_x (\xi_x - k_x) + DK \dot{x}_1 \]
\[ - \frac{\partial V}{\partial x} \left( \xi_x - k_x \right) \]
\[ - MA \left( \xi_x - k_x \right) \]

By choosing the input as
\[ u = \frac{\partial V}{\partial x} \left( \xi_x - k_x \right) K_1(1 - e^{-\epsilon x}) - M \dot{x}_1 - DK \dot{x}_1 + AP_1 - MK \xi_x \]
\[ - \frac{\partial V}{\partial x} \left( \xi_x - k_x \right) K_2 e^{-\epsilon x} - M \xi_x (\xi_x - k_x) + DK \dot{x}_1 \]
\[ - \frac{\partial V}{\partial x} \left( \xi_x - k_x \right) \]
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\[ \delta = \alpha_1 k_x^2 - \beta \frac{D_i}{M} \frac{d}{dt} \frac{\partial V}{\partial x} + \lambda_2 \xi_x (RT e^{U_{max}} - u) \]

This shows exponential stability of the system reference equilibria with the backstep
controller, when there are no restrictions on the control input u.

4.2.2 Controller

The imposed input restrictions on the system make the input (4.18) not applicable.

The designed Lyapunov function
\[ V_1(x) = \sum \frac{\partial V}{\partial x} \left( \xi_x - k_x \right) K_1(1 - e^{-\epsilon x}) - M \dot{x}_1 - DK \dot{x}_1 + AP_1 - MK \xi_x \]
\[ - \frac{\partial V}{\partial x} \left( \xi_x - k_x \right) K_2 e^{-\epsilon x} - M \xi_x (\xi_x - k_x) + DK \dot{x}_1 \]
\[ - \frac{\partial V}{\partial x} \left( \xi_x - k_x \right) \]
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can still be used as a CLF. Recall the Lyapunov function time-derivative along the trajectories of (4.15) given by (4.17), and notice that the only term dependent on the input is
\[ \lambda_2 \xi_x (RT e^{U_{max}} - u) \]

such that minimizing \( V_1 \) is achieved by minimizing \( \lambda_2 \xi_x (RT e^{U_{max}} - u) \). Since \( R, T, U_{max} \) and \( \lambda_1 \) are constants, choosing the input which satisfies
\[ \text{sgn}(u) = -\text{sgn}(\xi_x) \]

will render the smallest \( V_1 \). The control input can then be written as
\[ u = \text{sgn}(\xi_x) \]
\[ 0 \quad \text{if} \quad \xi_x = 0 \]
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Every switch between \( s_1 = -1 \) and \( s_3 = 1 \) will be done when
\[
\zeta = 0 = \frac{x_3}{3} \frac{K_1 (1 - e^{-K_2 x_1})}{R_1 T \Omega_x} - M_3 x_1 - D_3 x_1 + \frac{A_1 - M_3 (x_2 + k(x_1 - x_2))}{1}
\]
which can be interpreted as the nonlinear switching surface.

Remark 4.2.1. Nonlinear switching surfaces are also treated in several papers where sliding mode controllers are designed, as in Lee et al. (1991) and Liberzon (2003). While we design a controller for a specific function and the switching surface is found from this, the sliding mode can define the surface first, and use this to design a controller that proves stability of the solution.

Proposition 4.2.2. The equilibrium point of the system (4.3) with the switched control input given by (4.19) is locally exponentially stable.

Proof. First we prove existence, uniqueness and continuity of the solution using Filippov solution theories as in Sekhar et al. (2005). The discontinuity surface can be described by
\[
S = \{ (s_1, z_1) : \} \quad (4.19)
\]
and this divides the solution domain \( \Omega \) into two regions: \( \Omega^+ = \{ (s_1, z_1) < 0 \} \) and \( \Omega^- = \{ (s_1, z_1) > 0 \} \). As the right hand side of (4.15) is defined everywhere in \( \Omega \) and is measurable and bounded for bounded states, the system (4.15) satisfies condition B of Filippov’s solution theory, Filippov (1960). According to Theorems 4 and 5 in the same reference, we then have local existence and continuity of a solution. Further, since the right hand side of (4.15) is continuous before and after the discontinuity surface, \( S \), and this surface is smooth and independent of time, conditions A, B, C of Filippov’s solution, Filippov (1979), are satisfied. Following the procedures introduced in Filippov (1980) the vector functions \( f^- \) and \( f^+ \) are defined as the limiting values of the right-hand sides of the state space equations in \( \Omega^- \) and \( \Omega^+ \):
\[
f^- = \frac{\zeta - k_3}{R T \Omega_x} - \phi'(s_1, \xi_3, z_3)
\]
\[
f^+ = \frac{\zeta + k_3}{R T \Omega_x} - \phi'(s_1, \xi_3, z_3)
\]
where
\[
\phi'(s_1, \xi_3, z_3) = (f_3 - k_3) (1 - e^{-k_2 x_1}) - M_3 x_1 - D_3 x_1 + \frac{A_1 - M_3 (x_2 + k(x_1 - x_2))}{1}
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\]
For all points on the discontinuity surface vector \( h \) is defined as
\[
h = f^+ - f^- = \begin{bmatrix} 0 \\ 0 \\ -2RT_{\text{dmax}} \end{bmatrix}
\]
which is along the normal of the discontinuity surface, \( N_h = (0, 0, 1)^T \). The scalar, \( h_N \), defined as the projection of \( h \) on \( N_h \), is
\[
h_N = h \cdot N_h = -2RT_{\text{dmax}} < 0
\]
and will always be negative. According to Lemma 7 in Filippov (1979), uniqueness of the Filippov solution is then guaranteed.

Second we consider stability of the solution. The Lyapunov function time derivative along the trajectories of (4.15) given in (4.17) can be rewritten as
\[
\dot{V}_l = -A\lambda h(RT_{\text{dmax}})^2 + \phi\varepsilon
\]
where
\[
a(\varepsilon) = -k_2 - \frac{h_N}{h(RT_{\text{dmax}})^2} + \frac{\varepsilon}{2} |\varepsilon| (1 - e^{-|\varepsilon|}) - M_{\text{d}} - D\varepsilon
+ A \phi = \frac{\varepsilon}{2} |\varepsilon| (1 - e^{-|\varepsilon|}) - M_{\text{d}} - D\varepsilon
+ D\varepsilon^2 - M_{\text{d}}(2 \varepsilon^2 + 3\lambda^2)
\]
and
\[
\sigma = 2 \min(k, D)
\]
Since \( u(0) = 0 \) and \( u(\varepsilon) \) is continuous there must exist an \( \delta > 0 \) such that for \( |\varepsilon| \leq \delta \) we have \( |u(\varepsilon)| \leq RT_{\text{dmax}} \). It follows that for \( |\varepsilon| \leq \delta \) we get
\[
\dot{V}_l = -A\lambda h(RT_{\text{dmax}})^2 < 0
\]
and the equilibrium point \( x^* \) is locally exponentially stable.

The region of attraction contains the invariant set
\[
\bar{\Omega}_l = \{ |\varepsilon| \leq \varepsilon_f \}
\]
where \( \varepsilon_f \) is the largest value such that \( \bar{\Omega}_l \subseteq \Omega_l \)
\[
\bar{\Omega}_l = \{ |\varepsilon| \leq \varepsilon_f \} \subseteq RT_{\text{dmax}}
\]

Remark 4.2.3. As the size of \( \varepsilon_f \) be will be decided from the area where the input is from (4.18) fails
\[
|u| < RT_{\text{dmax}}
\]
the region of attraction will be large in the context of local stability. \( \varepsilon \) depends on parameter values and tuning variables and are respectively 97.10\(^{-1}\) and 45.10\(^{-1}\) for the pre-prototype and the prototype subvectors considering the parameters given in Table 4.2 and the tuning variables given in Table 4.3.

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4.3 Controller 2 - "Global controller"

The lack of a stability proof in the whole operation region for Controller 1 motivates the design of another switched controller. We choose a Lyapunov function based on knowledge of the system. The proposed CLF is based upon a Lyapunov function which can be used to prove stability of the reduced order system defined by \( \dot{x}_2 = 0 \) and \( u = 0 \). First, we study this reduced order system.

4.3.1 Open loop stability of reduced order system

Consider the input \( u = 0 \) and assume \( x_2 = x^*_2 \). We can rewrite the system as a second order system

\[
\begin{align*}
\dot{z}_1 &= x_2 \\
\dot{x}_2 &= \frac{-K_l(1 - e^{-\alpha_1 x_2}) + M_2 \dot{x}_2 + \frac{4}{V(T)} - A\dot{x}_2 - D\dot{x}_2}{M_1} \\
\end{align*}
\]

which also can be expressed as

\[
\dot{z}_1 + \frac{D}{M_1} \dot{z}_1 + f(x_1, z_2) = 0
\]

where \( \dot{z}_1 = x_1 - x^*_1 \) and

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f(x_1, z_2) = \frac{1}{M_1} \left[ K_l(1 - e^{-\alpha_1 x_2}) - M_2 \dot{x}_2 - \frac{4}{V(T)} - A\dot{x}_2 \right]
\]

For this system we state the following result:

**Proposition 4.3.1.**

The function

\[
U_2(z_1, z_2) = \frac{1}{2} \left( f(x_1, z_2) \right) + \int_{0}^{t} \left( f(x_1, z_2) \right) dt
\]

which is the sum of potential and kinetic energy (except for a factor \( M \)) can be used as a Lyapunov function to show asymptotic stability of the equilibrium \( (z_1, 0) \) of the system (4.22) in \( \tilde{O} \), the largest invariant set in \( O \), under the condition

\[
M_1 \geq \frac{\delta(K_l(1 - e^{-\alpha_1 x_2}) + M_2 \dot{x}_2 + \frac{4}{V(T)} - A\dot{x}_2 - D\dot{x}_2)}{\epsilon_{\text{max}} + \epsilon_{\text{max}}}
\]

on the clutch load model parameters.

**Proof.**

The proposed Lyapunov function candidate is positive definite if for \( \dot{z}_2 \neq 0 \)

\[
\int_{0}^{t} \left( f(x_1, z_2) \right) dt > 0
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on the clutch load model parameters.

**Proof.**

The proposed Lyapunov function candidate is positive definite if for \( \dot{z}_2 \neq 0 \)

\[
\int_{0}^{t} \left( f(x_1, z_2) \right) dt > 0
\]
For $\hat{x}_1 > 0$ this is the true if
\[ f(y + x_1; x_2) > 0 \] for all $y \in [0, \hat{x}_1]$
and for $\hat{x}_1 < 0$ if
\[ f(y + x_1; x_2) < 0 \] for all $y \in [\hat{x}_1, 0)$
as \( \int_0^y f(x) \, dx > 0 \) if $f(x) > 0$ for all $0 < x < \hat{x}_1$. This can be summarized in the following condition for positive definiteness of (4.24),
\[ f(x_1, x_2) > 0 \] (4.25)
for all $x_1 \in [0, 0.025]$ such that $\hat{x}_1 = x_1 - x_2 \neq 0$ and $x_2$ as function of $x_1 \in [0, 0.025]$ as described in (4.5).
We have that
\[
\begin{align*}
 f(x_1, x_2^1) &= \frac{1}{M}(K_1 (1 - e^{-k_1 x_1}) - M_{x_1} - \frac{A_1}{\int_{a}^{b} (y) \, dy} + A_1) \\
 &= \frac{1}{M}(K_1 (1 - e^{-k_1 x_1}) - M_{x_1} + A_1 + A_1) \quad (4.26)
\end{align*}
\]
where $V(x) = Ax + Bx$ is a positive strictly increasing function in $x$ for $x \in [0, 0.025]$, and $x' = K_1 (1 - e^{-k_1 x_1}) - M_{x_1} + A_1$ and $P(x) = V(x)$. Furthermore, $P(x)$ is a positive strictly increasing function in $x$ for $x \in [0, 0.025]$, if $\int_{a}^{b} (y) \, dy > 0$ which leads to the condition
\[ M_1 < \frac{A(K_1 (1 - e^{-k_1 x_1}) + A_1) + V_{\max}(K_{1} (1 - e^{-k_1 x_1}) + A_1)}{\int_{a}^{b} (y) \, dy} \] (4.27)
Since (4.27) holds, for $\hat{x}_1 > 0$ we have
\[ \hat{x}_1 > 0 \implies x_1 > x_2 > P(x_1) > P(x_2) \implies f(x_1, x_2) > 0 \]
which gives
\[ f(x_1, x_2^1) > 0. \] (4.25)
For $\hat{x}_1 < 0$ we have
\[ \hat{x}_1 < 0 \implies x_1 < x_2 > P(x_1) > P(x_2) \implies f(x_1, x_2) < 0 \]
which gives
\[ f(x_1, x_2^1) > 0. \] (4.25)

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For $\hat{x}_1 > 0$ this is the true if
\[ f(y + x_1; x_2) > 0 \] for all $y \in [0, \hat{x}_1]$
and for $\hat{x}_1 < 0$ if
\[ f(y + x_1; x_2) < 0 \] for all $y \in [\hat{x}_1, 0)$
as \( \int_0^y f(x) \, dx > 0 \) if $f(x) > 0$ for all $0 < x < \hat{x}_1$. This can be summarized in the following condition for positive definiteness of (4.24),
\[ f(x_1, x_2) > 0 \] (4.25)
for all $x_1 \in [0, 0.025]$ such that $\hat{x}_1 = x_1 - x_2 \neq 0$ and $x_2$ as function of $x_1 \in [0, 0.025]$ as described in (4.5).
We have that
\[
\begin{align*}
 f(x_1, x_2^1) &= \frac{1}{M}(K_1 (1 - e^{-k_1 x_1}) - M_{x_1} + A_1 + A_1) \\
 &= \frac{1}{M}(K_1 (1 - e^{-k_1 x_1}) - M_{x_1} + A_1 + A_1) \quad (4.26)
\end{align*}
\]
where $V(x) = Ax + Bx$ is a positive strictly increasing function in $x$ for $x \in [0, 0.025]$, and $x' = K_1 (1 - e^{-k_1 x_1}) - M_{x_1} + A_1$ and $P(x) = V(x)$. Furthermore, $P(x)$ is a positive strictly increasing function in $x$ for $x \in [0, 0.025]$, if $\int_{a}^{b} (y) \, dy > 0$ which leads to the condition
\[ M_1 < \frac{A(K_1 (1 - e^{-k_1 x_1}) + A_1) + V_{\max}(K_{1} (1 - e^{-k_1 x_1}) + A_1)}{\int_{a}^{b} (y) \, dy} \] (4.27)
Since (4.27) holds, for $\hat{x}_1 > 0$ we have
\[ \hat{x}_1 > 0 \implies x_1 > x_2 > P(x_1) > P(x_2) \implies f(x_1, x_2) > 0 \]
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Then by the LaSalle-Krasovski’s theorem, Khalil (2000), the origin is proved asymptotically stable in $\mathcal{O}$.

4.3.2 Lyapunov function

We extend the Lyapunov function for the redneck open loop system and for the 3rd order closed loop system we propose the control Lyapunov function

\[ V_i(\hat{x}) = \alpha_3 \int_0^1 f(y + x_i, \hat{x}, x_i) dy + \frac{\lambda_2}{3} \hat{x}_i^2 + \frac{\lambda_3}{3} \hat{x}_i^2 \]  

(4.28)

where $\alpha_3$ and $\lambda_2$ are positive constants. First we show that the proposed control Lyapunov function is positive definite under a condition on the ratio $\frac{\lambda_3}{\lambda_2}$.

Positive definiteness of the Lyapunov function $V_i(\hat{x})$

Consider

\[ f(x_i, x_i) = f(x_i, \hat{x}_i) - \frac{\alpha_3}{\lambda_2} \frac{\partial f}{\partial x} \]  

(4.29)

and the Lyapunov function (4.28) can be rewritten as

\[ V_i(\hat{x}) = \alpha_3 \int_0^1 f(y + x_i, \hat{x}_i) dy + \frac{\lambda_2}{3} \hat{x}_i^2 + \frac{\lambda_3}{3} \hat{x}_i^2 \frac{\partial f}{\partial x} \]  

(4.30)

where we know that $\int_0^1 f(y + x_i, \hat{x}_i) dy$ is positive from the calculations above. If the last term is dominated by $\frac{\lambda_2}{3} \hat{x}_i^2$, i.e.

\[ \alpha_3 \int_0^1 f(y + x_i, \hat{x}_i) dy + \frac{\lambda_2}{3} \hat{x}_i^2 = \hat{x}_i \left( -\alpha_3 \frac{\partial f}{\partial x} + \frac{\lambda_2}{3} \right) > 0 \]

Then by the LaSalle-Krasovski’s theorem, Khalil (2000), the origin is proved asymptotically stable in $\mathcal{O}$.

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Then by the LaSalle-Krasovski’s theorem, Khalil (2000), the origin is proved asymptotically stable in $\mathcal{O}$.
we have $V_i(x) > 0$ for all $i \neq 0$, as the other terms of (4.30) are positive. This condition is satisfied if
\[
\frac{\lambda_3}{2} \|x_i\| > \frac{a_2}{M} \left( \frac{V_i(x_i)}{V_i(x_i)} \right),
\]

hence positive definiteness of (4.28) is established for $|x_i| > \frac{a_2}{M} \ln \left( \frac{V_i(x_i)}{V_i(x_i)} \right)$. For $|x_i| \leq \frac{a_2}{M} \ln \left( \frac{V_i(x_i)}{V_i(x_i)} \right)$, positive definiteness can be shown if the first term of (4.28) is shown to be positive. This is true if
\[
f(x_i, x_i) > 0
\]
for all $x_i, x_i^\prime \in [0, 0.025]$ such that $x_i = x_i^\prime \neq 0$ and $x_3 \in [0, \infty]$ such that $|x_i| = |x_i^\prime - x_i^\prime| \leq \frac{a_2}{M} \ln \left( \frac{V_i(x_i)}{V_i(x_i)} \right)$. By considering (4.29), the condition (4.31) can be rewritten as
\[
f(x_i, x_i) = f(x_i, x_i) \|x_i - \frac{a_2}{M} \ln \left( \frac{V_i(x_i)}{V_i(x_i)} \right)\|.
\]
From the analysis in the proof of Proposition 4.3.1, we have that $f(x_i, x_i) = |x_i| > 0$, and if $f(x_i, x_i)$ dominates the term $\frac{a_2}{M} \ln \left( \frac{V_i(x_i)}{V_i(x_i)} \right)$, then $\alpha_1 > 0$, which is the condition (4.31) is satisfied.

Figure 4.1 shows the value of the right-hand side of (4.32) as a function of $x_1$ with constant $\tilde{x}_1$’s for the parameters of the actuator clutch model. From this it is clear that a lower bound, dependent on the values of $\alpha_1$, $M$, and $L_2$, for the condition (4.32) can be found giving an upper bound on $\tilde{x}_1$.

4.3. Controller 2 - “Global controller”

we have $V_i(x) > 0$ for all $i \neq 0$, as the other terms of (4.30) are positive. This condition is satisfied if
\[
\frac{\lambda_3}{2} \|x_i\| > \frac{a_2}{M} \left( \frac{V_i(x_i)}{V_i(x_i)} \right),
\]

hence positive definiteness of (4.28) is established for $|x_i| > \frac{a_2}{M} \ln \left( \frac{V_i(x_i)}{V_i(x_i)} \right)$. For $|x_i| \leq \frac{a_2}{M} \ln \left( \frac{V_i(x_i)}{V_i(x_i)} \right)$, positive definiteness can be shown if the first term of (4.28) is shown to be positive. This is true if
\[
f(x_i, x_i) > 0
\]
for all $x_i, x_i^\prime \in [0, 0.025]$ such that $x_i = x_i^\prime \neq 0$ and $x_3 \in [0, \infty]$ such that $|x_i| = |x_i^\prime - x_i^\prime| \leq \frac{a_2}{M} \ln \left( \frac{V_i(x_i)}{V_i(x_i)} \right)$. By considering (4.29), the condition (4.31) can be rewritten as
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f(x_i, x_i) = f(x_i, x_i) \|x_i - \frac{a_2}{M} \ln \left( \frac{V_i(x_i)}{V_i(x_i)} \right)\|.
\]
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Figure 4.1 shows the value of the right-hand side of (4.32) as a function of $x_1$ with constant $\tilde{x}_1$’s for the parameters of the actuator clutch model. From this it is clear that a lower bound, dependent on the values of $\alpha_1$, $M$, and $L_2$, for the condition (4.32) can be found giving an upper bound on $\tilde{x}_1$.
Remark 4.3.2. The value of the right-hand side of (4.32) is not defined for \( x_1 = x_1^* \).

\[
\lim_{x_1 \to x_1^*} \frac{P(x_1) - P(x_1^*)}{\ln(\frac{x_1}{x_1^*})} = 0
\]

But from L'Hospital's Rule we have that \( \lim_{x_1 \to x_1^*} \frac{P(x_1) - P(x_1^*)}{\ln(\frac{x_1}{x_1^*})} = \frac{V(x_1^*)(\lambda P(x_1^*) + V(x_1)(K_L e^{-\lambda x_1^*}))}{A} \) which gives

\[
\lim_{x_1 \to x_1^*} \frac{P(x_1) - P(x_1^*)}{\ln(\frac{x_1}{x_1^*})} = \frac{V(x_1^*)(\lambda P(x_1^*) + V(x_1)(K_L e^{-\lambda x_1^*}))}{A}
\]

Hence it is shown that the Lyapunov function given by (4.28) is positive definite in \( \Omega \), given that the condition (4.32) on the ratio \( \frac{x_1}{x_1^*} \) is fulfilled. Summarized, we have the CLF

\[
V_2(t) = \sum_{i=1}^{n} \int_{y_i}^{x_i} f(y + t_1, x_i^* + \lambda_2 t_1^* + \lambda_3 t_2^* + \frac{\beta_1}{2} t_1^* + \frac{\lambda_1}{2} t_2^*) \, dt_1^* + \frac{\beta_2}{2} t_2^* + \frac{\lambda_2}{2} t_2^* \quad (4.33)
\]
for the 3rd order system
\[
\dot{x}_1 = \alpha_1 x_1 + u \quad \text{(4.34a)}
\]
\[
\dot{x}_2 = -f(x_1, x_2) + D_e \dot{x}_2 \quad \text{(4.34b)}
\]
\[
\dot{x}_3 = R E \dot{E}^{\text{max}} \quad \text{(4.34c)}
\]

The Lyapunov function time derivative along the trajectories of (4.34) is
\[
\dot{V}_r = \alpha_1 f(x_1, x_2) x_2 - \frac{\alpha_2}{\beta} \left( \frac{V(x)}{x^T x} \right) R E \dot{E}^{\text{max}}
\]
\[
- \alpha_2 f(x_1, x_2) + D_e \dot{x}_2 \right) + \frac{\alpha_2}{\beta} R E \dot{E}^{\text{max}}
\]
\[
= - \frac{\alpha_2}{\beta} D_e \dot{x}_2 + \left( \lambda_1 \dot{x}_1 - \frac{\alpha_2}{\beta} \left( \frac{V(x)}{x^T x} \right) \right) R E \dot{E}^{\text{max}}
\]
\[
= - \frac{\alpha_2}{\beta} D_e \dot{x}_2 + \frac{\lambda_1}{\beta} \left( \dot{x}_1 + \dot{x}_2 \right) R E \dot{E}^{\text{max}}
\]

where
\[
s(\dot{x}_1, \dot{x}_2) = \lambda_1 \dot{x}_1 - \frac{\alpha_2}{\beta} \left( \frac{V(x)}{x^T x} \right)
\]
\[
\dot{\lambda}_1 = 0 \quad \text{(4.36)}
\]

4.3.3 Controller

The input dependent term of the Lyapunov function time derivative given by (4.35) is
\[
s(\dot{x}_1, \dot{x}_2) R E \dot{E}^{\text{max}}
\]

and minimizing \( V_r \) is achieved by minimizing this term. \( R, T_0 \) and \( U_{\text{max}} \) are constants, such that choosing the input \( u \) which satisfies
\[
\text{argmin}_u \left( s(\dot{x}_1, \dot{x}_2) R E \dot{E}^{\text{max}} \right)
\]

will render the smallest \( V_r \) and ensure that \( V_r \) is non-positive. The switched control law can be written as
\[
u = \left\{ \begin{array}{ll}
- \text{sign}(\dot{x}_1, \dot{x}_2) & \text{if } s(\dot{x}_1, \dot{x}_2) \neq 0 \\
0 & \text{if } s(\dot{x}_1, \dot{x}_2) = 0
\end{array} \right.
\]

and we get a negative semidefinite Lyapunov function time derivative,
\[
\dot{V}_r = - \frac{\alpha_2}{\beta} D_e \dot{x}_2 + \left( \lambda_1 \dot{x}_1 - \frac{\alpha_2}{\beta} \left( \frac{V(x)}{x^T x} \right) \right) R E \dot{E}^{\text{max}} \leq 0
\]

where the variables \( \alpha_2 \) and \( \lambda_1 \) can be tuned to weight the parts of \( s(\dot{x}_1, \dot{x}_2) \).

Every switch between \( u_1 = -1 \) and \( u_1 = 1 \) will be done when
\[
s(\dot{x}_1, \dot{x}_2) = 0 \quad \text{if } \dot{x}_1 = 0
\]

which can be interpreted as the nonlinear switching surface.

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for the 3rd order system
\[
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\]
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\]
\[
= - \frac{\alpha_2}{\beta} D_e \dot{x}_2 + \left( \lambda_1 \dot{x}_1 - \frac{\alpha_2}{\beta} \left( \frac{V(x)}{x^T x} \right) \right) R E \dot{E}^{\text{max}}
\]
\[
= - \frac{\alpha_2}{\beta} D_e \dot{x}_2 + \frac{\lambda_1}{\beta} \left( \dot{x}_1 + \dot{x}_2 \right) R E \dot{E}^{\text{max}}
\]

where
\[
s(\dot{x}_1, \dot{x}_2) = \lambda_1 \dot{x}_1 - \frac{\alpha_2}{\beta} \left( \frac{V(x)}{x^T x} \right)
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\[
u = \left\{ \begin{array}{ll}
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Every switch between \( u_1 = -1 \) and \( u_1 = 1 \) will be done when
\[
s(\dot{x}_1, \dot{x}_2) = 0 \quad \text{if } \dot{x}_1 = 0
\]

which can be interpreted as the nonlinear switching surface.
The equilibrium point $x^*$ of the system (4.4) with the switched controller given in (4.35) is asymptotically stable in the largest invariant region in the region of operation $O$ if the condition (4.32) on $\mathbb{S}$ is satisfied.

Proof. First we prove existence, uniqueness and continuity of the solution using Filippov solution as in Sekhavat et al. (2005). The discontinuity surface is described by

$$ S := \{ x : x_1 \leq 0 \} $$

which divides the solution domain $\Omega$ into two regions

$$ \Omega^- := \{ x : x_1 < 0 \} $$
$$ \Omega^+ := \{ x : x_1 > 0 \} $$

The right hand side of (4.4) is defined everywhere in $\Omega$ and is measurable and bounded. This means that the system (4.4) satisfies condition B of Filippov's solution theory, Filippov (1960), and according to Theorems 4 and 5 in the same reference, we then have local existence and continuity of a solution. The right hand side of (4.4) is also continuous before and after the discontinuity surface, $S$, and this surface is smooth and independent of time.

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$$ f^- = \begin{cases} -f(x_1, x_2) - \frac{\lambda x_2}{RTU_{max}} \\ -f(x_1, x_2) + \frac{\lambda x_2}{RTU_{max}} \end{cases} $$

$$ f^+ = \begin{cases} -f(x_1, x_2) - \frac{\lambda x_2}{RTU_{max}} \\ -f(x_1, x_2) + \frac{\lambda x_2}{RTU_{max}} \end{cases} $$

The vector $h$, which is along the normal of the discontinuity surface,

$$ h = \begin{pmatrix} \alpha \frac{A}{2} \frac{1}{x_1 + x_2} \end{pmatrix} $$

is defined as

$$ h = f^+ - f^- = \begin{pmatrix} 0 \\ 0 \end{pmatrix} $$

for all points on the discontinuity surface. The scalar, $\lambda h$, defined as the projection of $h$ on $N_x$ is

$$ \lambda h = \frac{h}{ \langle h, N_x \rangle } $$

and will always be negative. According to Lemma 7 in Filippov (1979), the uniqueness of the Filippov solution is then guaranteed.

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$$ \lambda h = \frac{h}{ \langle h, N_x \rangle } $$

and will always be negative. According to Lemma 7 in Filippov (1979), the uniqueness of the Filippov solution is then guaranteed.
Next we consider the stability properties of the solution. From (4.3.3) we have that \( V_{2} ≤ 0 \), and we use Lasalle’s invariance principle, Khalil (2000), to prove asymptotical stability. From \( V_{2} = 0 \) we get \( \dot{x}_{2} = 0 \) \& \( [\dot{\alpha}(\tilde{x}) x] = 0 \).

From this it follows that \( \dot{x}_{2} = 0 \Rightarrow \dot{\tilde{x}}_{2} = 0 \Rightarrow \tilde{x}_{2} = c_{3} \Rightarrow \dot{\tilde{x}}_{2} = 0 \Rightarrow f(x_{1}, x_{2}) = 0 \) \& \( [\dot{\alpha}(\tilde{x}) x] = 0 \). This gives the solution \( (\tilde{x}_{1}, \tilde{x}_{2}, x) = (c_{1}, 0, c_{2}) \). It then remains to show that \( c_{1} = c_{2} = 0 \) is the only possible solution. In the equation

\[
[\dot{\alpha}(\tilde{x}) x] \leq 0
\]

is true if

\[
\tilde{z}_{1} = 0 \quad \& \quad \frac{\alpha_{2}}{M} \ln \frac{V(x_{1})}{V_{p}(x_{1})} \Rightarrow \tilde{z}_{1} \leq 0
\]

or

\[
\tilde{z}_{2} = \frac{\alpha_{2}}{M} \ln \frac{V(x_{1})}{V_{p}(x_{1})}
\]

(4.38)

The equation

\[
f(x_{1}, x_{2}) = f(x_{1}, x_{2}) = -\frac{\lambda_{1}}{M} x
\]

(4.39)

where \( f(x_{1}, x_{2}) = \frac{P(x_{1})(\tilde{x})}{MV_{p}(x_{1})} \) if \( z_{1} = 0 \), is true if

\[
f(x_{1}, x_{2}) = 0 \quad \& \quad \frac{\alpha_{2}}{M} \ln \frac{V(x_{1})}{V_{p}(x_{1})} = 0
\]

\[
\tilde{z}_{1} = 0 \quad \& \quad \tilde{z}_{2} = 0
\]

(4.39)

By combining (4.38) and (4.39)

\[
\frac{\alpha_{2}}{M} \ln \frac{V(x_{1})}{V_{p}(x_{1})} = \frac{\alpha_{2}}{M} \ln \frac{V(x_{1})}{V_{p}(x_{1})}
\]

and rearranging we get

\[
P(x_{1}) - P(x_{1}) = 0
\]

(4.40)

From the analysis of positive definiteness of the Lyapunov function we know that

\[
|P(x_{1}) - P(x_{1})| > \frac{2h_{0}}{M} \ln \frac{V(x_{1})}{V_{p}(x_{1})}
\]


4.3. Controller 2 "Global controller" 45

Next we consider the stability properties of the solution. From (4.3.1) we have that \( V_{2} ≤ 0 \), and we use Lasalle’s invariance principle, Khalil (2000), to prove asymptotical stability. From \( V_{2} = 0 \) we get \( \dot{x}_{2} = 0 \) \& \( [\dot{\alpha}(\tilde{x}) x] = 0 \).

From this it follows that \( \dot{x}_{2} = 0 \Rightarrow \dot{\tilde{x}}_{2} = 0 \Rightarrow \tilde{x}_{2} = c_{3} \Rightarrow \dot{\tilde{x}}_{2} = 0 \Rightarrow f(x_{1}, x_{2}) = 0 \) \& \( [\dot{\alpha}(\tilde{x}) x] = 0 \) \& \( \dot{\tilde{x}}_{3} = \tilde{x}_{2} = c_{2} \). This gives the solution \( (\tilde{x}_{1}, \tilde{x}_{2}, x) = (c_{1}, 0, c_{2}) \). It then remains to show that \( c_{1} = c_{2} = 0 \) is the only possible solution. In the equation

\[
[\dot{\alpha}(\tilde{x}) x] \leq 0
\]

is true if

\[
\tilde{z}_{1} = 0 \quad \& \quad \frac{\alpha_{2}}{M} \ln \frac{V(x_{1})}{V_{p}(x_{1})} \Rightarrow \tilde{z}_{1} \leq 0
\]

or

\[
\tilde{z}_{2} = \frac{\alpha_{2}}{M} \ln \frac{V(x_{1})}{V_{p}(x_{1})}
\]

(4.38)

The equation

\[
f(x_{1}, x_{2}) = f(x_{1}, x_{2}) = -\frac{\lambda_{1}}{M} x
\]

(4.39)

where \( f(x_{1}, x_{2}) = \frac{P(x_{1})(\tilde{x})}{MV_{p}(x_{1})} \) if \( z_{1} = 0 \), is true if

\[
f(x_{1}, x_{2}) = 0 \quad \& \quad \frac{\alpha_{2}}{M} \ln \frac{V(x_{1})}{V_{p}(x_{1})} = 0
\]

\[
\tilde{z}_{1} = 0 \quad \& \quad \tilde{z}_{2} = 0
\]

(4.39)

By combining (4.38) and (4.39)

\[
\frac{\alpha_{2}}{M} \ln \frac{V(x_{1})}{V_{p}(x_{1})} = \frac{\alpha_{2}}{M} \ln \frac{V(x_{1})}{V_{p}(x_{1})}
\]

and rearranging we get

\[
P(x_{1}) - P(x_{1}) = 0
\]

(4.40)

From the analysis of positive definiteness of the Lyapunov function we know that

\[
|P(x_{1}) - P(x_{1})| > \frac{2h_{0}}{M} \ln \frac{V(x_{1})}{V_{p}(x_{1})}
\]
has to be fulfilled. From this it follows that the only solution of (4.49) is $x_1 = 0$, hence the only solution which can stay identically in the set $O = \{ x \in \Omega : |V'(x)| < 0 \}$, where $O$ is the largest invariant set contained in $\Omega$, is the equilibrium point $(x_1, x_2, x_3) = 0$. By the LaSalle-Krasovsky's theorem, Khalil (2000), asymptoti-
cal stability of the equilibrium point $x^*$ is proved under the condition (4.42) on $\Omega$ in the region $O'$.

Remark 4.3.4. It is possible to characterize the largest invariant set in $O$, but we have chosen not to do so since the practical experiments show there are no practical
limit. For this reason, we refer to this controller as the "global" controller.

### 4.4 Controller 3 - "Dual-mode controller"

We design a third controller by combining the two former switched controllers into one that makes use of the individual controller's best qualities. The global controller is stable in the operation region of the clutch system, and can be used to bring the system close to the equilibrium point, and the local controller, whose control action depend more strongly on the position and error and has better robustness properties, is suitable for use close to the equilibrium point. Notice that $\tilde{x}$ is defined as the region of attraction for the local controller, and thereby also influencing the choice of switched controller, may be somewhat relaxed in the experimental testing.

The equilibrium point $x^*$ of the system (4.4) with the combined controller given in (4.41) is asymptotically stable in the largest invariant region in $O$.

Proof. In the region $\Omega_1$, $u_1$ guarantees local exponential stability of the system's equilibrium point. The equilibrium point lies inside $\Omega_1$, and if the system at the initial time $t_0$ is in the region $\Omega_1$, the exponential stability of the local controller and invariance of $\Omega_1$ will ensure that the region is not left and that the equilibrium point is asymptotically stable. If the system at $t_0$ is outside the region $\Omega_1$, but in $\Omega$, the asymptotic stability of the global controller ensures that the system will reach the region $\Omega_1$ after some finite time, and the switch to the local controller will bring the system to the equilibrium point. Hence the equilibrium point $x^*$ is asymptotically stable in the region $O$.

Remark 4.4.2. Experimental results indicate that the guaranteed region of attrac-
tion, $\Omega_1$, is rather conservatively estimated due to the sufficiency of the Lyapunov
function conditions, and that the local controller is stable also outside of this re-
gion.

Proposition 4.4.1. The equilibrium point $x^*$ of the system (4.4) with the combined controller given in (4.41) is asymptotically stable in the largest invariant region in $O$.

Proof. In the region $\Omega_1$, $u_1$ guarantees local exponential stability of the system's equilibrium point. The equilibrium point lies inside $\Omega_1$, and if the system at the initial time $t_0$ is in the region $\Omega_1$, the exponential stability of the local controller and invariance of $\Omega_1$ will ensure that the region is not left and that the equilibrium point is asymptotically stable. If the system at $t_0$ is outside the region $\Omega_1$, but in $\Omega$, the asymptotic stability of the global controller ensures that the system will reach the region $\Omega_1$ after some finite time, and the switch to the local controller will bring the system to the equilibrium point. Hence the equilibrium point $x^*$ is asymptotically stable in the region $O$.

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4.5 Experimental results

The performance of the proposed controllers has been tested in the KA test truck, with the two on/off solenoid valves as described in Chapter 2. The clutch load characteristic was determined manually before any experiments with the controllers were carried out. Those tuned parameters are given in Table 4.3 along with the control parameters used in the experiments.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Pre-prototype</th>
<th>Pro-prototype</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>4000</td>
<td>9000</td>
<td>$m$</td>
<td>Load characteristic term</td>
</tr>
<tr>
<td>$A_2$</td>
<td>5000</td>
<td>9000</td>
<td>$m$</td>
<td>Load characteristic term</td>
</tr>
<tr>
<td>$M_f$</td>
<td>20000</td>
<td>20000</td>
<td>N</td>
<td>Load characteristic term</td>
</tr>
<tr>
<td>$m_1$</td>
<td>400</td>
<td>400</td>
<td></td>
<td>Tuning parameters</td>
</tr>
<tr>
<td>$m_2$</td>
<td>400</td>
<td>400</td>
<td></td>
<td>Tuning parameters</td>
</tr>
<tr>
<td>$h_1$</td>
<td>0.01</td>
<td>0.00001</td>
<td></td>
<td>Tuning parameters</td>
</tr>
<tr>
<td>$h_2$</td>
<td>3.83</td>
<td>2.73</td>
<td></td>
<td>Tuning parameters</td>
</tr>
<tr>
<td>$h_3$</td>
<td>1</td>
<td>1</td>
<td></td>
<td>Tuning parameters</td>
</tr>
<tr>
<td>$h_4$</td>
<td>10</td>
<td>10</td>
<td></td>
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</tr>
</tbody>
</table>

Table 4.3: Load characteristics and control parameters.

4.5.1 Performance under nominal conditions

Figures 4.2, 4.3 and 4.4 show experimental results of the local, the global and the dual-mode controller under nominal conditions, respectively. The response times are given in Table 4.4, displaying both the response time it takes before the piston position first reaches the acceptable position error area, and the time elapsed before the piston position has settled within this area. The table also lists the number of switchings between the available inputs conducted for the respective controllers. This is an important performance factor as excessive use of the on/off solenoids valves leads to more rapid wear of the mechanical parts and should be avoided.

The local controller suffers from some chattering and oscillations. The position requirements are not met in the presence of oscillations, but for the cases not suffering from oscillations, see Figures 4.2 and 4.4. Although the theoretical analysis only managed to ensure local stability, the experimental results indicate that stability of the whole operation range is actually obtained. The high number of input switchings for the local controller originate from the chattering.

The requirements are almost fulfilled using the global controller with some excess input switching. The position requirements were defined by (4.32), limits the performance of the controller as it does not allow us to weight the position error as highly as desired. The error in accumulated air which is directly related to the pressure error, will be the main driving error of the controller, which leads to poor robustness properties of this controller since the calculation of $x_2$ is subject to model error.

The dual-mode controller is shown to be active close to the equilibrium point, while the global controller is active elsewhere.

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4.5 Experimental results

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<td>20000</td>
<td>20000</td>
<td>N</td>
<td>Load characteristic term</td>
</tr>
<tr>
<td>$m_1$</td>
<td>400</td>
<td>400</td>
<td></td>
<td>Tuning parameters</td>
</tr>
<tr>
<td>$m_2$</td>
<td>400</td>
<td>400</td>
<td></td>
<td>Tuning parameters</td>
</tr>
<tr>
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<td></td>
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<tr>
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<td></td>
<td>Tuning parameters</td>
</tr>
<tr>
<td>$h_3$</td>
<td>1</td>
<td>1</td>
<td></td>
<td>Tuning parameters</td>
</tr>
<tr>
<td>$h_4$</td>
<td>10</td>
<td>10</td>
<td></td>
<td>Tuning parameters</td>
</tr>
</tbody>
</table>

Table 4.3: Load characteristics and control parameters.
The exception is when the position reference is zero (clutch fully engaged), here the local controller is active as the restriction on position error is relaxed. The chattering experienced using the local controller alone is no longer present when the local controller is the active controller of the dual-mode controller. This is presumably because the local controller is used only close to the equilibrium point, where stability of the controller can be guaranteed. The improvement using the dual-mode compared to the local and the global controller alone is evident. Better accuracy, less oscillations and fewer switches between inputs are achieved.

### 4.5.2 Robustness considerations

Experiments where errors in the parameters of the clutch load characteristic model in the controller model are introduced, have been conducted. One lower and one higher clutch load characteristic is used to test robustness of the controller designs, obtained by setting \( K_1 = 5000 \) and \( K_2 = 6500 \), respectively, see Figure 4.5. The effect in terms of position error by including these errors in the modeled clutch load characteristic is shown in Figure 4.6 and Table 4.5. The dual-mode controller fails in terms of position error whenever the global controller is active, while the performance is much better whenever the local controller is active. The local controller is, as expected, more robust to model errors. This due to the fact that the switching surface depend on the errors of all three states, and that the parameters \( \alpha_1 \) and \( \lambda_1 \) can be tuned without restrictions. The global controller, on the other hand, relies only on position and accumulated air errors, and in addition the condition on \( \alpha_2 \) results in higher gain on the accumulated air error than the position error. Under nominal conditions a small accumulated air error correlates to a small position error, while with model errors introduced, the relation between \( \alpha_2 \) and \( x_2 \) described by (4.5) is no longer valid, and this relationship between these errors is not longer guaranteed.

### 4.5.3 Performance considering different valvesets

Figures 4.7 and 4.8 show the performance of the dual-mode controller under nominal conditions with the two different SO valvesets. In the region of engagement we see that this position requirement is only partly fulfilled. This is mainly because the switching surface depend on the errors of all three states, and that the parameters \( \alpha_1 \) and \( \lambda_1 \) can be tuned without restrictions. The global controller, on the other hand, relies only on position and accumulated air errors, and in addition the condition on \( \alpha_2 \) results in higher gain on the accumulated air error than the position error. Under nominal conditions a small accumulated air error correlates to a small position error, while with model errors introduced, the relation between \( \alpha_2 \) and \( x_2 \) described by (4.5) is no longer valid, and this relationship between these errors is not longer guaranteed.
Figure 4.2: Result from experimental testing showing the performance of local controller under nominal conditions. The gray curve for position is the reference position, while the dashed gray lines for position error are the requirement limits.
Figure 4.3: Result from experimental testing showing the performance of global controller under nominal conditions. The gray curve for position is the reference position, while the dashed gray lines for position error are the requirement limits.
Figure 4.4: Result from experimental testing showing the performance of dual-mode controller under nominal conditions. The gray curve for position is the reference position, while the dashed gray lines for position error are the requirement limits.
Table 4.5: Average absolute position errors with errors in the modeled clutch load characteristic.

<table>
<thead>
<tr>
<th>Controller</th>
<th>Lower clutch load [mm]</th>
<th>Higher clutch load [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Local</td>
<td>0.8 mm</td>
<td>1.1 mm</td>
</tr>
<tr>
<td>Global</td>
<td>4.7 mm</td>
<td>4.4 mm</td>
</tr>
<tr>
<td>Combined</td>
<td>3.9 mm</td>
<td>3.6 mm</td>
</tr>
</tbody>
</table>

Figure 4.5: Clutch load characteristics. Actual clutch load “measured” through $A(p - P_0)$ (also including dynamic friction) is shown together with three configurations of the clutch load characteristic model given by equation (4.2).
Figure 4.6: The position errors of experiments with local, global and dual-mode controller with error in the modeled clutch load characteristic, with higher load model in gray and lower load model in black.
Figure 4.7: Results of experimental testing showing dual-mode controller performance with the pre-prototype valveset present in the truck. The gray curve for position is the reference position, while the dashed gray lines for position error are the requirement limits.
Figure 4.8: Results of experimental testing showing dual-mode controller performance with the prototype valvset present in the truck. The gray curve for position is the reference position, while the dashed gray lines for position error are the requirement limits.
due to the fact that the controller is shut down whenever $|\epsilon| < 0.4$ mm something which is chosen for comparison as experiments with the pre-prototype valveset were conducted using this limit. The tuning of the controller has been improved between the experiments with the pre-prototype and the prototype valvesets, and the effect of this is clear comparing the performances. One improvement is that the chattering experienced in the first tests is not present anymore in the second test results. Also the switching between the local and the global controller, decided by the value of $\epsilon$, has been better tuned for the tests with the prototype valveset in the truck, resulting in fewer input switches, see Table 4.6. For the dual-mode controller also the number of switches between the controllers is reduced from 1210 for the pre-prototype experiment to 706 in the prototype experiment. These results show how important proper tuning of the controllers is.

It also seems like the prototype valves have larger capacities than the pre-prototype valves in those experiments, even though their maximum volumetric flow rates are the same. This comes from the fact that the capacities in general will vary with pressure and temperature.

### Table 4.6: Performance parameters of the local, global and dual-mode controllers employing pre-prototype and prototype valvesets.

<table>
<thead>
<tr>
<th>Controller</th>
<th>Valve type</th>
<th>Position</th>
<th>Input switches</th>
<th>Reach time</th>
<th>Settle time</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Local</td>
<td>Pre-proto</td>
<td>1.7 mm</td>
<td>366</td>
<td>0.10 ms</td>
<td>98.5 ms</td>
<td>299 mm</td>
</tr>
<tr>
<td></td>
<td>Proto</td>
<td>1.1 mm</td>
<td>231</td>
<td>0.12 ms</td>
<td>98.5 ms</td>
<td>299 mm</td>
</tr>
<tr>
<td>Global</td>
<td>Pre-proto</td>
<td>0.83 mm</td>
<td>375</td>
<td>0.13 ms</td>
<td>98.5 ms</td>
<td>375 mm</td>
</tr>
<tr>
<td></td>
<td>Proto</td>
<td>0.83 mm</td>
<td>245</td>
<td>0.13 ms</td>
<td>98.5 ms</td>
<td>245 mm</td>
</tr>
<tr>
<td>Combined</td>
<td>Pre-proto</td>
<td>0.74 mm</td>
<td>309</td>
<td>0.09 ms</td>
<td>138.0 ms</td>
<td>309 mm</td>
</tr>
<tr>
<td></td>
<td>Proto</td>
<td>0.60 mm</td>
<td>115</td>
<td>0.11 ms</td>
<td>156.8 ms</td>
<td>115 mm</td>
</tr>
</tbody>
</table>

4.6 Discussions

Switched controllers for an electro-pneumatic clutch actuator have been presented. The controllers are designed to switch between available inputs in a way that ensures that at each switching time the most negative Lyapunov time-derivative is obtained. The best results are obtained by the dual-mode switching controller. This controller is a combination of the two other switched controllers, and preserves the best individual properties of these controllers. Experimental results conducted in the test vehicle verify the stability properties of the controller, and show that it is well suited for the system.

The unmodeled delay of the on/off solenoid valve dynamics may lead to oscillations around the reference point, as experienced in the results from testing with the local controller. This happens as the dynamic response of the last command is obtained. The best results are obtained by the dual-mode switching controller. This controller is a combination of the two other switched controllers, and preserves the best individual properties of these controllers. Experimental results conducted in the test vehicle verify the stability properties of the controller, and show that it is well suited for the system.

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input signal is not finished before the next is commanded. Taking these delays into consideration, the chattering appearing in some of the experiments could be reduced. It is also evident from the robustness evaluations that the controller will benefit from online adaptation of the clutch load, as this is the main uncertainty in the system model. A practical implementation in a production system must therefore consider adaptation of the load characteristics and possibly other model parameters. All analyses are made under the assumption of full state feedback. In the experiments both a position and a pressure sensor have been used, with velocity calculated by numerical differentiation and filtering, described in Section 4.1.1. Adaptive observers that estimate velocity and pressure are treated in the next chapter.

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Chapter 5
Nonlinear adaptive observer design

In this chapter we present adaptive nonlinear observers for the electropneumatic clutch actuation system, making full-state feedback for the controller design in Chapter 4 possible. This chapter is mainly based on Langjord et al. (2010a) and Langjord et al. (2010).

5.1 Introduction

Only position measurement will be available in the finalised production clutch actuator system, and to be able to employ the controllers described in the previous chapter, a nonlinear observer must be designed to be able to provide estimates of the unmeasured states. The robustness study in Section 4.5.2 also shows that the control design will suffer in case of model errors, which also motivates for including adaptation in the observer design.

Extended Kalman filters (EKF) are the most widely used algorithm for nonlinear state estimations. This because they are easy to construct, and often produces good results even though analytical stability of the estimates is not necessarily guaranteed. EKFs suffer from high computational complexity, due to the recursive solution of the Riccati equation, and are therefore not convenient for application in the clutch actuator system. As previous discussed, general designs for nonlinear observers have been developed for particular classes of nonlinear systems. Wu et al. (2004) consider nonlinear observability analyses for a pneumatic actuator systems, and conclude that in general it is not feasible to guarantee a convergent pressure estimate from measurement of position only. Therefore, observers presented for pneumatic actuator systems are designed and analysed specifically. Bigras and Khayati (2002) present a nonlinear observer for estimation of the pressure in a pneumatic cylinder, ensuring exponential stability of the estimation error. Pas-dian et al. (2002) propose a Luenberger-type observer and a sliding mode observer to estimate pressure in a cylinder actuator, and Gulati and Barth (2005) present nonlinear adaptive observer design

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two LuGre-based pressure observers for a pneumatic actuator system. In our clutch actuator system, the clutch load is a position dependent and time-varying load while observers in the above reference states the electromechanical actuators with constant loads. Some load independent and varying load observers can be found, Taghizadeh et al. (2010) design a Kalman filter to estimate velocity for a pneumatic actuator with varying load, while Gulati and Barth (2009) present an energy-based observer which is load-independent. But Szabo et al. (2010) only consider a velocity observer, while Gulati and Barth (2009) use both position and velocity measurements and focus only on pressure estimation, those observers are not applicable to our system. In addition all the above references use a simpler friction model than required in our system, and have no adaptation to capture a time-varying clutch load. The theses by Knaus (2006) and Yalcinkaya (2006) consider observer designs for the same clutch actuator system as ours, but without rigorously deriving sufficient conditions for the convergence of estimation errors and considering adaptation. These authors also consider a three-way proportional valve as the control valve while we consider on/off solenoid valves. Szabo et al. (2010) propose a feedback linearization-based observer for an electropneumatic clutch system actuated by on/off solenoid valves, but do not consider adaptation. The presented observers in this chapter are deterministic observers with linear output-injections.

5.1.1 Model for observer design
We make some modification of the LuGre friction model (3.7-3.8) presented in Section 3.2, to obtain a model suited for observer design while still give a adequate description of the friction in the clutch actuator system. To get a simple static friction characteristic, the dry friction characteristic is chosen as the constant Coulomb friction, \( f_S = F \). In addition we remove the deflection damping term, \( Dz \), from the resulting friction force. To be more convenient for observer design we also rewrite the model by setting \( z = \frac{1}{K} \), where the new \( z \) state can be defined as the normalized pre-sliding deflection. This gives the friction force and the dynamics as

\[
\frac{F}{p} = -v - |v|z \quad (5.1)
\]

\[
f = F + Dv. \quad (5.2)
\]

For observer design it is also convenient to express the system dynamics with the mass of air, \( m_A \) and \( m_B \), rather than the pressures \( p_A \) and \( p_B \). The pressure in each chamber is then given as a function of the corresponding chamber volume and mass according to

\[
p_A (p_A, v) = \frac{RT}{V_A} m_A \quad (5.3)
\]

\[
p_B (p_B, v) = \frac{RT}{V_B} m_B. \quad (5.4)
\]
5.2 Reduced-order observer design

Assuming all model parameters are known, we first propose a reduced-order observer by not estimating position:

\[
M \dot{y} = - f(y) - f_1(y, z) + \frac{A_B R_T}{V_A(y)} y + \frac{A_B R_T}{V_B(y)} \hat{u} - A_B P_T
\]

where the flows \( w(y, p_A, u) \) and \( w_0(y) \) are described by (3.18) and (3.21), respectively. The clutch load force \( f(y) \) is described by (3.24) with the splines described in Section 3.5, and the friction force \( f_1(y, z) \) is described by (5.2). Parameter values are given in Table 5.1 where nominal values for the parameters are indicated.

Experimental data from the test truck provides position and pressure measurements, and the employed position reference is shown in Figure 5.1. The input signal, which in these experiments is constructed by PWM, is also available. The observer uses the input signal and the position measurement, while pressure measurement and a velocity signal filtered from the position measurement are used for validation of the observer performance. The observer is implemented in Simulink by using explicit Euler discretization with step size 0.1 ms. The position and pressure sensors are production quality sensors, providing data with a sampling interval of 1 ms, and are influenced by both noise and motor vibrations as discussed in Section 2.2.4.

5.2. Reduced-order observer design

where the volume of each chamber is given as a function of position according to

\[
V_A(y) = V_A - A_B y
\]

\[
V_B(y) = V_B - A_B y
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where \( V_{A, B} \) and \( V_{A, B}^{\hat{y}} \) describe the volumes in the chambers for \( y = 0 \).

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Table 5.1: Model parameters used in the testing of the observer performance.

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<th>Parameter</th>
<th>Annotation</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>$M$</td>
<td>10</td>
<td>kg</td>
</tr>
<tr>
<td>Area of chamber A</td>
<td>$A_A$</td>
<td>$12.3 \times 10^{-3}$</td>
<td>m$^2$</td>
</tr>
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<td>Area of chamber B</td>
<td>$A_B$</td>
<td>$11,848 \times 10^{-3}$</td>
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<tr>
<td>Area of piston rod</td>
<td>$A_P$</td>
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</tr>
<tr>
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<td>$P_a$</td>
<td>1 10$^{-4}$</td>
<td>Pa</td>
</tr>
<tr>
<td>Viscous damping</td>
<td>$D$</td>
<td>$2500$</td>
<td>Ns/m</td>
</tr>
<tr>
<td>Temperature</td>
<td>$T_0$</td>
<td>293</td>
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</tr>
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<td>J/kgK</td>
</tr>
<tr>
<td>Volume in chamber A at y=0</td>
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<td>$0.148 \times 10^{-3}$</td>
<td>m$^3$</td>
</tr>
<tr>
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Figure 5.1: Reference clutch sequence used in the experiments in this chapter.
where $\dot{z}_m$, $\dot{m}$, and $\dot{l}$ are observer injection gains.

Remark 5.2.1. Introducing $y$ in observer design requires careful attention, as differentiation of the measured signal will amplify any measurement noise. Note that the injection terms $\hat{y} - y$ are implementable with only $y$ measured, i.e., without using $y$ explicitly. Consider the velocity dynamics to demonstrate the principle:

$$\dot{M} = \dot{u}_s + l_s (\hat{y} - y),$$

where $u_s$ contains the remaining terms from (5.6a). Integration gives

$$\dot{M} = \int_0^t \left( \dot{u}_s + l_s (\hat{y} - y) \right) \, dt = \int_0^t \left( \dot{u}_s - l_s \dot{y} \right) \, dt + l_s \int_0^t \hat{y} \, dt = \int_0^t \left( \dot{u}_s - l_s \dot{y} \right) \, dt + l_s \left( \hat{y}(t) - y(0) \right),$$

and the estimate $\dot{y}$ can be implemented without differentiation as

$$\dot{y} = \frac{\dot{M}}{l_s}.$$ 

Using the same approach, the estimate for $\dot{m}_A$ can be implemented as

$$\dot{m}_A = \frac{\dot{m}_A}{l_s}.$$ 

This approach fails for avoiding $y$ in the implementation the estimate $\dot{y}$. But implementation in discrete-time of the estimate in the case of using zero-order hold on the measurements and Euler integration, simply becomes

$$\dot{z}_{m+1} = \frac{K \Delta t}{l_s} \hat{y} - \frac{K \Delta t}{l_s} \hat{y} + \frac{K \Delta t}{l_s} \dot{y},$$

where $\Delta t$ is the sampling period. The effects of measurement noise will be discussed later when considering experimental results, and in Section 5.4 when considering a full-order estimator.

The error dynamics are:

$$\dot{M} = -(D + l_s) \dot{z} - F + \frac{A_k}{V_m} \frac{\partial y}{\partial A_k} \dot{m}_A$$

for observer design.

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Stability of the error dynamics can be established using the Lyapunov function candidate

\[ U(v, l, \hat{m}, \eta) = \frac{1}{2} \| \dot{v} \|^2 + \frac{1}{2} \| \dot{l} \|^2 + \| A_\delta R_{\text{err}} \|^2 \| v \|^2 \]

The time-derivatives of \( U \) along the trajectory of the error dynamics are

\[ \dot{U} = -(D + l_0)^2 \| F \|^2 + A_\delta R_{\text{err}} \cdot \left( A_\delta R_{\text{err}} \cdot v + F \right) \]

which lead to

\[ \dot{U} \leq -(D + l_0)^2 \| F \|^2 + 1 \| A_\delta R_{\text{err}} \|_{\text{err}}^2 \| v \|^2 - \frac{1}{2} \frac{\| v \|^2}{V_{\text{err}}(v)} \]

where \( V_{\text{err}}(v) \) and \( V_{\text{err}}(v) \) are defined in (5.10)

**Proposition 5.2.2** (Persistently exciting (PE) signal). A signal \( u \) is persistently exciting if there exist \( T > 0 \) and \( \epsilon > 0 \) such that \( \int_0^T \| u(t) \|^2 \geq \epsilon \) for all \( t > 0 \).

Proof: Stability of the error dynamics can be established using the Lyapunov function candidate

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**Proposition 5.2.3.** The observer presented in (5.6), where \( \lambda \) is chosen to ensure that for any physically meaningful initial conditions and system trajectories

1. the error dynamics are stable and all estimates are bounded
2. \( \hat{v} \) and \( \hat{m} \) converge to zero
3. if \( v \) is PE then also \( \hat{\eta} \) and \( \hat{\epsilon} \) converge to zero

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5.2. Reduced-order observer design

Using Young’s inequality

\[ xy \leq \frac{x^2}{2} + \frac{y^2}{2} \]

with

\[ e = \frac{\|d \|_{V_{max}}}{D} \]

we obtain

\[ \frac{\dot{\|e\|}_{V_{max}}}{\|e\|_{V_{max}}} \leq \frac{D}{2} + \frac{1}{2} \left( \frac{\|d \|_{V_{max}}}{D} \right)^2 \]

This gives

\[ \dot{\|e\|}_{V_{max}} \leq \frac{D}{2} \|e\|_{V_{max}} - \|d \|_{V_{max}} \]

and proves stability of the error dynamics. From Barbalat’s lemma (Khalil (2000)) we have that \( \dot{\|e\|}_{V_{max}} \) converges to zero, thus in the limit

\[ F \dot{\|\hat{z}\|}_{V_{max}} + \|e\|_{V_{max}} = 0. \]

With FE of \( v \), \( U \) will be negative definite also in \( z \), which by Barbalat’s lemma implies that \( \dot{z} \) also converges to zero, thus from (5.11) \( \hat{m}_A \) must converge too.

**Remark 5.2.4.** For the special case \( v = 0 \), with \( \hat{v} = \hat{m}_A = 0 \), the \( \hat{z}, \hat{m}_A \)-dynamics are governed by

\[ \dot{\hat{z}} = 0 \]

\[ \dot{\hat{m}}_A = -a(t) \hat{m}_A. \]

Hence if \( \int_{t_0}^{t+} a(t) \, dt > 0 \), which is achieved with FE of \( v \), \( \hat{m}_A \) must converge to zero; hence also \( \hat{z} \). In the case that \( a(t) = 0 \), the observer may get stuck at \( \hat{m}_A \neq 0 \), \( \hat{z} \neq 0 \), with persistent errors in the estimates of pressure \( \hat{p}_A \) and the dynamic friction \( \hat{z} \). Since \( z, \hat{z} \in [-\frac{\tau}{2}, \frac{\tau}{2}] \), we have the following bound on the error

\[ \|e\|_{\max} \leq \frac{F \|v\|_{\max}}{A \|R\|_{V_{max}}} \frac{F}{N}. \]

1 The limit is a property of the LuGre friction model.

5.2. Reduced-order observer design

Using Young’s inequality

\[ xy \leq \frac{x^2}{2} + \frac{y^2}{2} \]

with

\[ e = \frac{\|d \|_{V_{max}}}{D} \]

we obtain

\[ \frac{\dot{\|e\|}_{V_{max}}}{\|e\|_{V_{max}}} \leq \frac{D}{2} + \frac{1}{2} \left( \frac{\|d \|_{V_{max}}}{D} \right)^2 \]

This gives

\[ \dot{\|e\|}_{V_{max}} \leq \frac{D}{2} \|e\|_{V_{max}} - \|d \|_{V_{max}} \]

and proves stability of the error dynamics. From Barbalat’s lemma (Khalil (2000)) we have that \( \dot{\|e\|}_{V_{max}} \) converges to zero, thus in the limit

\[ F \dot{\|\hat{z}\|}_{V_{max}} + \|e\|_{V_{max}} = 0. \]

With FE of \( v \), \( U \) will be negative definite also in \( z \), which by Barbalat’s lemma implies that \( \dot{z} \) also converges to zero, thus from (5.11) \( \hat{m}_A \) must converge too.

**Remark 5.2.4.** For the special case \( v = 0 \), with \( \hat{v} = \hat{m}_A = 0 \), the \( \hat{z}, \hat{m}_A \)-dynamics are governed by

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Hence if \( \int_{t_0}^{t+} a(t) \, dt > 0 \), which is achieved with FE of \( v \), \( \hat{m}_A \) must converge to zero; hence also \( \hat{z} \). In the case that \( a(t) = 0 \), the observer may get stuck at \( \hat{m}_A \neq 0 \), \( \hat{z} \neq 0 \), with persistent errors in the estimates of pressure \( \hat{p}_A \) and the dynamic friction \( \hat{z} \). Since \( z, \hat{z} \in [-\frac{\tau}{2}, \frac{\tau}{2}] \), we have the following bound on the error

\[ \|e\|_{\max} \leq \frac{F \|v\|_{\max}}{A \|R\|_{V_{max}}} \frac{F}{N}. \]

1 The limit is a property of the LuGre friction model.
5.2.1 Experimental results

Figure 5.2 shows results with of the open-loop observer, i.e. with zero gain. Nominal values of the friction and the clutch load parameter are used. We see that there are some errors in the estimated pressure in chamber A, especially in the engage/disengage area. The noise on the estimations of v and l are mostly due to sensor noise on y entering through f(y). Figure 5.3 shows results with the injection gains tuned to \( L_v = 2000 \) and \( L_l = 10^{13} \), and these gains have been used throughout the rest of the thesis whenever adaptation is introduced. We get a more accurate steady-state estimates for the pressure in chamber A, especially in the engage/disengage area. However, the injection gains also add some extra noise to the estimated velocity and the low pressure areas of the estimate of the pressure in chamber A. This is due to the use of the derivative of the position measurement, \( v \), in the observer design.

5.3 Adaptive nonlinear reduced-order observer

Due to temperature changes and wear, the friction and clutch load characteristic change during the operation and lifetime of the clutch. Therefore, adaptation of the load and friction characteristics are desired in order to achieve sufficient accuracy of the observer, and we propose adaptation laws for the clutch load parameter, \( \theta \), and the viscous damping, \( D \).

5.3.1 Adaptation of the clutch load characteristic

Since there are uncertain parameters in the clutch load characteristic, the observer (5.6) is implemented with the estimated parameter vector \( \hat{\theta} \) instead of \( \theta \). This results in the parametric uncertainty

\[
\hat{f}_l(v) = \phi^l(\hat{\theta} - \theta),
\]

which appear in the resulting \( \dot{\hat{\theta}} \)–dynamics as

\[
\dot{\hat{\theta}} = -\Gamma \hat{\theta} (v - \theta).
\]

We propose an adaptation law

\[
\dot{\hat{\theta}} = -\Gamma \hat{\theta} (v - \theta) \tag{5.14}
\]

5.3.2 Adaptation of the friction characteristic

Since there are uncertain parameters in the clutch load characteristic, the observer (5.6) is implemented with the estimated parameter vector \( \hat{\theta} \) instead of \( \theta \). This results in the parametric uncertainty

\[
\hat{f}_v(v) = \phi^v(\hat{\theta} - \theta),
\]

which appear in the resulting \( \dot{\hat{\theta}} \)–dynamics as

\[
\dot{\hat{\theta}} = -\Gamma \hat{\theta} (v - \theta).
\]

We propose an adaptation law

\[
\dot{\hat{\theta}} = -\Gamma \hat{\theta} (v - \theta) \tag{5.14}
\]
Figure 5.2: Performance of the open-loop observer. Observer states are dashed, and measurement from the truck are shown in solid gray. For velocity, the measurement curve is filtered from position measurement.
Figure 5.3: Performance of the observer with input injections \( l_v = 2000 \) and \( l_m = 10^{-5} \). Observer states are dashed, and measurement from the truck are shown in solid gray. For velocity, the measurement curve is filtered from position measurement.
Proposition 5.3.3. The observer presented in (5.6) implemented with $\hat{\theta}$ and the adaptation law (5.14), where $L_\nu, L_\tau \geq 0$ and $\Gamma = \Gamma^2 > 0$, ensures that for any physically meaningful initial conditions and system trajectories
1. the error dynamics are stable and all estimates are bounded
2. $\hat{\nu}$ and $\hat{n}_\nu$ converge to zero
3. if $u$ and $v$ are PE, then also $n_{\nu, \nu}$ and $\hat{\theta}$ converge to zero

Proof. Using the Lyapunov function candidate
\[ V(\hat{z}, \hat{\nu}, \hat{n}_\nu, \hat{\theta}) = U(\hat{z}, \hat{\nu}, \hat{n}_\nu) + \frac{1}{2} \| \hat{\theta} \|^2, \] (5.15)
the time-derivative along the trajectories of the error dynamics satisfies
\[ \dot{V} \leq -\left( \frac{D}{2} + L_\nu \right) \| \hat{\theta} - \Theta(\hat{z}, \hat{n}_\nu) \|^2 \]
\[ - \frac{1}{2} \left( \frac{1}{\tau_{\nu, \nu}} \right) \| \hat{\nu} - \hat{\nu}_0 \|^2 \] (5.16)
and proves stability of the error dynamics. Barbalat’s lemma gives that $\hat{z}$ and $\hat{n}_\nu$ converges to zero. Since the solution of the error dynamics are continuous and Lipschitz, this also implies that $\hat{\nu}$ converges to zero, thus in the limit
\[ -\| \hat{\theta} \|^2 \leq F(\hat{z}, \hat{n}_\nu, \hat{\theta}) + \frac{1}{2} \| \hat{\theta} \|^2 + \frac{1}{2} \| \hat{\nu} \|^2 \]
\[ = 0 \] (5.16)

With PE of $v$, $\nu$ will be negative definite also in $\nu$, which implies that $\hat{\nu}$ also converges to zero. Considering (5.13) which with PE of $u$ implies that also $n_{\nu, \nu}$ converges to zero. It then follows from (5.16) that $\hat{\theta}$ must converge to zero too.

Remark 5.3.4. Intuitively, PE of $v$ as result in PE of $u$ as motion of the system is a direct consequence of changes in the input signal $u$.

with some gain matrix $\Gamma = \Gamma^2 > 0$.

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and proves stability of the error dynamics. Barbalat’s lemma gives that $\hat{z}$ and $\hat{n}_\nu$ converges to zero. Since the solution of the error dynamics are continuous and Lipschitz, this also implies that $\hat{\nu}$ converges to zero, thus in the limit
\[ -\| \hat{\theta} \|^2 \leq F(\hat{z}, \hat{n}_\nu, \hat{\theta}) + \frac{1}{2} \| \hat{\theta} \|^2 + \frac{1}{2} \| \hat{\nu} \|^2 \]
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2. $\hat{\nu}$ and $\hat{n}_\nu$ converge to zero
3. if $u$ and $v$ are PE, then also $n_{\nu, \nu}$ and $\hat{\theta}$ converge to zero

Proof. Using the Lyapunov function candidate
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the time-derivative along the trajectories of the error dynamics satisfies
\[ \dot{V} \leq -\left( \frac{D}{2} + L_\nu \right) \| \hat{\theta} - \Theta(\hat{z}, \hat{n}_\nu) \|^2 \]
\[ - \frac{1}{2} \left( \frac{1}{\tau_{\nu, \nu}} \right) \| \hat{\nu} - \hat{\nu}_0 \|^2 \] (5.16)
and proves stability of the error dynamics. Barbalat’s lemma gives that $\hat{z}$ and $\hat{n}_\nu$ converges to zero. Since the solution of the error dynamics are continuous and Lipschitz, this also implies that $\hat{\nu}$ converges to zero, thus in the limit
\[ -\| \hat{\theta} \|^2 \leq F(\hat{z}, \hat{n}_\nu, \hat{\theta}) + \frac{1}{2} \| \hat{\theta} \|^2 + \frac{1}{2} \| \hat{\nu} \|^2 \]
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Remark 5.3.4. Intuitively, PE of $v$ as result in PE of $u$ as motion of the system is a direct consequence of changes in the input signal $u$. 

with some gain matrix $\Gamma = \Gamma^2 > 0$.
5.3.5 Adaptation of the viscous damping coefficient

With uncertainties in the viscous friction, the adaptive observer is implemented with estimated coefficient $\tilde{D}$ instead of $D$. Denoting $\tilde{D} = D - \hat{D}$ the resulting error becomes

$$\dot{D} = -\gamma_D (g - \hat{g})$$

which appears in the resulting $v$-dynamics as

$$\dot{v} = -\phi^T (y - (D + l_2) \hat{v} - \hat{D} - F) + \frac{A_D}{\tau_D} \frac{\mu_D}{\tau_D} \hat{v} \tilde{v} + \frac{A_D}{\tau_D} \frac{\mu_D}{\tau_D} \tilde{v} \hat{v}.$$ 

We propose the adaptation law

$$\dot{\hat{D}} = -\gamma_D (g - \hat{g}).$$

(5.17)

with $\gamma_D > 0$.

Proposition 5.3.5.

The observer presented in (5.6) implemented with $\theta, \hat{D}$ and the adaptation laws (5.14) and (5.17), where $l_1, l_2 \geq 0$ and $\gamma = \gamma_D > 0$, ensures that for any physically meaningful initial conditions and system trajectories:

1. the error dynamics are stable and all estimates are bounded
2. $\hat{v} \text{ and } \hat{\eta}_4 \text{ converge to zero}$
3. if $v$ and $u$ are PE then also $\eta_\theta$ and $\tilde{v}$ converge to zero

Proof. Using the Lyapunov function candidate

$$Z(\hat{v}, \hat{\theta}, \hat{\eta}_4, \hat{\eta}_5, \hat{D}) = V(\hat{v}, \hat{\theta}, \hat{\eta}_4, \hat{\eta}_5, \hat{D}) + \frac{1}{2} \tilde{v}^T \tilde{v},$$

(5.18)

the time-derivative along the trajectories of the error dynamics satisfies

$$\dot{Z} \leq -\frac{D}{\tau_D} \frac{\mu_D}{\tau_D} \tilde{v}^T \tilde{v}^2 + \frac{1}{2} \frac{A_D}{\tau_D} \frac{\mu_D}{\tau_D} \tilde{v}^T \tilde{v} \tilde{v}.$$ 

With the adaptation law (5.17) the $D$-dynamics satisfies

$$\dot{\hat{D}} = -\gamma_D \hat{D},$$

which would make $\hat{Z}$ negative semi-definite,

$$\dot{Z} \leq -\frac{D}{\tau_D} \frac{\mu_D}{\tau_D} \tilde{v}^T \tilde{v}^2 - \frac{1}{2} \frac{A_D}{\tau_D} \frac{\mu_D}{\tau_D} \hat{D}^2 \hat{D}$$

and proves stability of the error dynamics. Barbula's lemma gives that $\hat{v}$ and $\hat{\eta}_4$ converge to zero. With PE of $v$, $\hat{Z}$ will be negative definite also in $\hat{v}$, which implies that $\hat{Z}$ also converges to zero, and (5.18) implies that with PE of $u$ also $\eta_\theta$ converges to zero.
5.3.3 Experimental results

Figure 5.4 shows the state estimate results for the reduced-order adaptive observer. Values of the tuning parameters are given in Table 5.2. The initial values of the parameter estimates have been set far from the expected values to test the performance of the adaptation laws. Results from the two initial conditions 1) \( \theta_0 = [10, 10], l_y = 50 \) and 2) \( \theta_0 = [1, 1], l_y = 3000 \) are included in Figures 5.5, which shows the estimated parameter values, and in Figure 5.6, which shows the clutch load characteristic, to demonstrate the convergence of the estimated parameters. Note that such large changes in friction and clutch load characteristics as used for testing here will normally not occur during the normal operation of the clutch actuator system. We have used slow adaptation of the clutch load, as this is expected to improve the robustness of the approach and the uncertain parameters are expected to change slowly. The gain \( \Gamma \) is set 30 times higher in the region 3 – 6 mm as the clutch load characteristic is especially important in this area due to the steep curve, and since this region is visited only for short transient periods with a typical clutch sequence. The adaptation is shut down whenever the position is not changing, due to lack of PE that might lead to drift or divergence of the estimates due to noise. The estimate of the pressure in chamber \( A \) improves over time and we have a good estimate after approximately 150 s corresponding to adaptation in about 60 clutch sequences as seen in Figure 5.4. The estimates of the velocity and the pre-sliding deflection state suffer, as the observer without adaptation, from some noise. From Figure 5.6 it is clear that the adaptation of \( \hat{\theta} \) gives an accurate estimate of the clutch load characteristic. This indicates that the clutch sequence in Figure 5.1 provides sufficient excitation to estimate both load and friction parameters simultaneously.

Remark 5.3.6. Asymptotically we have \(-\phi'(y) - D\hat{v} = 0\) and with PE of \(\phi'(y)\) and \(\hat{v}\) it serves to estimate that \(\theta\) and \(\hat{\theta}\) will converge to zero. Intuitively, PE of \(\hat{\theta}\) will lead to motion of the actuator piston and PE of \(\hat{v}\) and \(\hat{v}\).

Comparison with the gradient method

As an alternative, we also adapt \( \theta \), \( F \) and \( D \) through slow adaptation not designed jointly with the observer, only updating estimates when the position changes. This

Table 5.2: Observer and adaptation gains.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l_y )</td>
<td>200</td>
</tr>
<tr>
<td>( l_m )</td>
<td>10^{-4}</td>
</tr>
<tr>
<td>( \Gamma )</td>
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Comparison with the gradient method

As an alternative, we also adapt \( \theta \), \( F \) and \( D \) through slow adaptation not designed jointly with the observer, only updating estimates when the position changes. This
Figure 5.4: Experiments with reduced-order adaptive observer. The curves show estimates after $t = 148$ where the parameter adaptation has already converged, after about 60 clutch sequences. Observer states are dashed, and measurement from the truck are shown in solid gray. For velocity, the measurement curve is filtered from position measurement.
5.3. Adaptive nonlinear reduced-order observer 73

Figure 5.5: Estimated parameters with the reduced-order adaptive observer during 150 s where 60 clutch sequences are executed. Results for $\theta = [10, 10]$, $D_0 = 50$ are shown in black and for $\theta = [1, 1]$, $D_0 = 5000$ in gray. $\theta_1$ are shown dotted and $\theta_2$ are shown solid in the upper plot.

Figure 5.6: Clutch load characteristics, estimated through $A_1(p_A - P)$ in solid, dot dashed in black for $\theta = [10, 10]$, $D_0 = 50$ and dashed in gray for $\theta = [1, 1]$, $D_0 = 5000$. The curves with the estimated $\hat{\theta}$ from the reduced-order adaptive observer at $t = 150$ (overlapping) are dotted in black and gray, respectively.
is done to evaluate the performance of the adaptation laws. The standard gradient method (Ioannou and Sun (1996)) is used:
\[ \dot{v} = -\frac{\partial J}{\partial v}(\theta(y)) \]
where \[ \theta = [\theta, D, F] \] and the cost function \( J \) can be the least squares error
\[ J(t) = \frac{1}{2} \bar{v}^2 = \frac{1}{2} (\theta - \hat{\theta})^2. \]
\( \hat{\theta} \) is a position prediction
\[ \hat{\theta} = \frac{1}{M} (-\dot{\theta}y_1^2 \theta + \dot{D}_y + F_{z1} + r \tau) \]
(5.19)
where
\[ \theta_1(y) = \frac{1}{1 + \lambda_2 \varepsilon_2} \theta(y) \]
\[ \dot{\theta}_1 = \frac{1}{1 + \lambda_2 \varepsilon_2} \dot{\theta}(y) \]
\[ \tau_1 = \frac{1}{1 + \lambda_2 \varepsilon_2} \tau \]
\[ r = \frac{A_k V \varepsilon_2}{A_k V \varepsilon_2} - A_k V \varepsilon_2 - A_k V \varepsilon_2 \]
and \( \dot{\tau}_1 \) is a low pass filter time constant and the signals \( \dot{\theta}, \dot{v} \) and \( r \) are given by the observer, see Figure 5.7. This results in the adaptive laws
\[ \dot{\theta} = \frac{\lambda_2 V \varepsilon_2}{\lambda_2 V \varepsilon_2} \theta_1(y) \]
(5.20)
\[ \dot{\theta}_1 = \frac{\lambda_2 V \varepsilon_2}{\lambda_2 V \varepsilon_2} \dot{\theta}_1(y) \]
(5.21)
\[ \dot{\tau}_1 = \frac{\lambda_2 V \varepsilon_2}{\lambda_2 V \varepsilon_2} \tau \]
(5.22)
with gains \( \lambda_1, \lambda_2, \lambda_3 > 0 \).

Figures 5.8, 5.9 and 5.10 show estimated states and parameters, and resulting clutch load characteristics from tests of the reduced order observer with this adaptation method. We see similar behavior for the estimates of all states, except the normalized pre-sliding deflection. This is due to the fact that also \( F \) is estimated and the normalized deflection depends on this. The comparison shows that the adaptation integrated in the observer provides better result than the alternative adaptation, even with one parameter less being estimated.

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\( \dot{\hat{\theta}} \) is a position prediction
\[ \dot{\hat{\theta}} = \frac{1}{2} (-\dot{\theta}y_1^2 \theta + \dot{D}_y + F_{z1} + r \tau) \]
(5.19)
where
\[ \theta_1(y) = \frac{1}{1 + \lambda_2 \varepsilon_2} \theta(y) \]
\[ \dot{\theta}_1 = \frac{1}{1 + \lambda_2 \varepsilon_2} \dot{\theta}(y) \]
\[ \tau_1 = \frac{1}{1 + \lambda_2 \varepsilon_2} \tau \]
\[ r = \frac{A_k V \varepsilon_2}{A_k V \varepsilon_2} - A_k V \varepsilon_2 - A_k V \varepsilon_2 \]
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5.4 Full-order adaptive observer

To reduce noise, we also propose a full-order adaptive observer where position is filtered:

\[\dot{\hat{y}} = c^T(y - \hat{y}) \]  
\[M\dot{\hat{v}} = -\phi^T(y)\theta - Dv - F\hat{v} + A_{\theta}R_{\theta}m_{\theta} \]  
\[\dot{\hat{l}} = -\phi^T(y)\nu - A_{\nu}R_{\nu}m_{\nu} + \nu \]  
\[\dot{\hat{\theta}}^{(5.23b)} = \frac{1}{\tau_{\theta}}[\nu - \phi^T(y)\theta] \]  
\[\dot{\hat{\nu}} = \nu_{\theta}(\hat{\theta}, \nu) \]  
\[\dot{\hat{v}} = \nu_{\nu}(\hat{\nu}) \]  

where \(l_\nu, l_{\theta}, l_{\nu} \geq 0\) are observer injection gains and the adaptation laws for \(\hat{\theta}\) and \(\hat{\nu}\) are given in (5.14) and (5.17).

**Proposition 5.4.1.** The observer presented in (5.23) with the adaptation laws (5.14) and (5.17), where \(l_\nu, l_{\theta}, l_{\nu} \geq 0\), \(\nu_{\theta} > 0\) and \(\Gamma = \Gamma^T > 0\), \(\nu_{\theta} > 0\), ensures that for any physically meaningful initial conditions and system trajectories

1. the error dynamics are stable and all estimates are bounded
2. \(\dot{\hat{y}}, \dot{\hat{\theta}}, \dot{\hat{\nu}}\) and \(\hat{y}\) converge to zero
Figure 5.8: Experiments with the reduced-order observer with alternative parameter adaptation. The curves show estimates after \( t = 148 \) where the parameter adaptation has already converged, after about 60 clutch sequences. Observer states are dashed, and measurement from the truck are shown in solid gray. For velocity, the measurement curve is filtered from position measurement.
Figure 5.9: Estimated parameters with the reduced-order observer with alternative parameter adaptation during 150 s where 60 clutch sequences are executed. Results for \( \theta_0 = [10, 10], D_0 = 50 \) are shown in black and for \( \theta_0 = [1, 1], D_0 = 5000 \) in gray. \( \theta_1 \) are shown dotted and \( \theta_2 \) are shown solid in the upper plot.
Proof. The error dynamics are given by
\[ \dot{\tilde{y}} = -L_0 \tilde{y} \]
\[ \dot{m}_\alpha = -u(t)m_\alpha - \frac{1}{2}\gamma m_\alpha \]
\[ \dot{m}_\beta = -b(t)m_\beta \]
\[ \theta = \Gamma_0 \tilde{y} \]
\[ \bar{D} = \gamma_{\alpha,\beta} \tilde{y} \]
Stability of the error dynamics can be established using the Lyapunov function candidate
\[ W = \frac{1}{2} \tilde{y}^T M_1 \tilde{y} + \frac{1}{2} \tilde{y}^T \tilde{y} + \frac{1}{2} \tilde{m}_\alpha^T \tilde{m}_\alpha \]
\[ + \frac{A_0 R T}{2m} \tilde{m}_\alpha^T \tilde{m}_\alpha + \frac{1}{2} \tilde{m}_\beta^T \tilde{m}_\beta \]
(5.24)

3. if \( v \) and \( u \) are persistently exciting (PE) then also \( m_\alpha \) and \( \tilde{y} \) converge to zero.

Proof. The error dynamics are given by
\[ \dot{\tilde{y}} = -L_0 \tilde{y} \]
\[ \dot{m}_\alpha = -u(t)m_\alpha - \frac{1}{2}\gamma m_\alpha \]
\[ \dot{m}_\beta = -b(t)m_\beta \]
\[ \theta = \Gamma_0 \tilde{y} \]
\[ \bar{D} = \gamma_{\alpha,\beta} \tilde{y} \]
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(5.24)
5.4. Full-order adaptive observer

Using Young's inequality that

Since

\[ W = \alpha_l \varepsilon_t^2 - \frac{1}{2} \left( \frac{A_{yy} R_{yy}}{V_{yy}^2} \alpha_l \right) \varepsilon_t^2 + \lambda \left( \frac{A_{yy} R_{yy}}{V_{yy}^2} \right) \varepsilon_t \]

The Mean Value Theorem gives

\[ \left| \alpha_l (\tilde{y}) - \bar{\alpha} (\tilde{y}) \right| \leq K \| \tilde{y} \| \]  

(5.25)

where \( K = \max \left( \frac{\| \tilde{y} \|}{\bar{\alpha} (\tilde{y})} \right) \). Using the bound (5.25), \( \theta_{\alpha, \bar{\alpha}} \geq \theta \) and (5.8), we see that \( W \) satisfies

\[ W \leq \alpha_l \varepsilon_t^2 - \frac{1}{2} \left( \frac{A_{yy} R_{yy}}{V_{yy}^2} \alpha_l \right) \varepsilon_t^2 + \lambda \left( \frac{A_{yy} R_{yy}}{V_{yy}^2} \right) \varepsilon_t \]

Using Young's inequality

\[ \varepsilon_t \leq \frac{\sqrt{A_{yy} R_{yy}}}{2 \lambda} \]

and

\[ \varepsilon_t \leq \frac{\theta_{\alpha, \bar{\alpha}} + \alpha}{\lambda} \]

we obtain

\[ A_{yy} R_{yy} \left| \varepsilon_t \right| \leq \frac{D}{2 \lambda} \left( A_{yy} R_{yy} \right)^2 \varepsilon_t^2 \]

and

\[ (K_{\theta_{\alpha, \bar{\alpha}}} + \alpha) \varepsilon_t \leq \frac{D}{2 \lambda} \left( (K_{\theta_{\alpha, \bar{\alpha}}} + \alpha)^2 \varepsilon_t^2 \right) \]

This gives

\[ W \leq \frac{D}{2 \lambda} \left( \frac{A_{yy} R_{yy}}{V_{yy}^2} \right) \varepsilon_t \]

Since

\[ t_\nu > \frac{(K_{\theta_{\alpha, \bar{\alpha}}} + \alpha) \varepsilon_t}{2 \lambda} \]

this proves stability of the error dynamics. Barbula's lemma, Khalil (2000), gives that \( \hat{\alpha}, \bar{\alpha}, \tilde{\alpha}_l, \tilde{\alpha}_m, \tilde{\alpha}_s \) converge to zero. With PE of \( u \) it follows by standard arguments that \( W \) will be negative definite also in \( \tilde{z} \), which implies that \( \nu \) converges to zero. PE of \( u \) implies that also \( \tilde{m}_A \) converges to zero. \( \square \)
5.4.1 Experimental results

State estimates from the full-order adaptive observer are shown Figure 5.11 with parameter estimates shown in Figure 5.12 and clutch load characteristics in Figure 5.13. The initial condition and the tuning are set just as for the testing of the reduced-order adaptive observer. Significant improvements in velocity and friction, which suffered from noise in the reduced order observer, are shown. As a result of this, also the estimate of the pressure in chamber A is improved, see Table 5.3 for average pressure errors.

<table>
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<tr>
<th>Adaptive observer</th>
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<tbody>
<tr>
<td>Full-order</td>
<td>41.94 kPa</td>
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Table 5.3: Maximum and average errors for pressure of chamber A for full- and reduced-order adaptive observers, calculated in the time interval 140 s ≤ t ≤ 150 s.

5.5 Discussions

Adaptive observer designs for estimation of unmeasured states of the electropneumatic clutch actuator are presented, with adaptation laws for parameter estimation of the clutch load characteristics and the viscous friction force. The state estimation errors are shown to be convergent under PE conditions, and from the experimental results these conditions seem to be fulfilled by a standard clutch position reference sequence. In some cases, such as a cold start of the truck, faster adaptation may be needed due to large temperature gradients. Certainly, this is possible if the clutch is used sufficiently and the piston position reference is changed often enough, i.e. PE of u and v are provided, something which usually will be done when starting to drive. The adaptation gains can also be increased for the first 10–20 gear shifts after a cold start, but care must be taken as testing show that the parameter estimates start to diverge if the gains are set too high.

The full-order adaptive observer which extends the reduced order adaptive observer with a filtered position estimate, shows improvement with respect to noise aspects. The estimated friction dynamic state and pressure in chamber B are in themselves not important, but good estimates of these states are important to obtain accurate estimates of the pressure in chamber A.

The experimental tests validate the adaptive observer designs, and good performance are shown. This may make the adaptive observer designs suitable for state feedback for the switched controllers developed in Chapter 4, and validation of such an adaptive observer-based switched control design is the topic of the next chapter.

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Figure 5.11: Experiments with full-order adaptive observer. The curves show estimates after \( t = 148 \) where the parameter adaptation has already converged, after about 60 clutch sequences. Observer states are dashed, and measurement from the truck are shown in solid gray. For velocity, the measurement curve is filtered from position measurement.
Figure 5.12: Estimated parameters with the full-order adaptive observer during 150 s where 60 clutch sequences are executed. Results for $\theta_0 = [10, 10]$, $D_0 = 50$ are shown in black and for $\theta_0 = [1, 1]$, $D_0 = 5000$ in gray. $\theta_1$ are shown dotted and $\theta_0$ are shown solid in the upper plot.

Figure 5.13: Clutch load characteristics, estimated through $A_x(p_k - P_k)$ in solid, dot dashed in black for $\theta_0 = [10, 10]$, $D_0 = 50$ and dashed in gray for $\theta_0 = [1, 1]$, $D_0 = 5000$. The curves with the estimated $\hat{\theta}$ from the full-order adaptive observer at $t = 150$ (overlapping) are dotted in black and gray, respectively.

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Chapter 6

Adaptive observer-based switched control

In this chapter we propose a combined design of the dual-mode switched controller from Section 4.4 and the full-order adaptive nonlinear observer design presented in Section 5.4. The focus is on validation of the output feedback with only position sensor, and this is done by simulation. This chapter is mainly based on Langjord et al. (2011b).

6.1 Introduction

In recent years, much work has been done on position control for pneumatic systems with on/off solenoid actuation, but not much is done on observers for such systems, Szabo et al. (2010), even though most proposed controllers require full state feedback. The main objective of the adaptive observer designs in the former chapter was to provide state feedback for control purposes, and particularly for the switched controller designs presented in Chapter 4. In this chapter we combine the previous designs into an adaptive observer-based switched controller validated with simulations, and discuss practical aspects for application to the electropneumatic clutch actuator system.

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At the time the adaptive observer design was finalized, experimental testing in the test truck at KA was unfortunately no longer possible. Hence, evaluation of the adaptive observer-based switched control has to be done by constructing and considering realistic simulations.

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6.1.1 Simulation model and assumptions for application

The 5th order observer design model (5.5) is also used as a basis for simulation, expressed with chamber pressures instead of masses of air,

\[ p = v \]

\[ v = \sum (A_{PA} - A_{Pb}) = A_{Pb} + f_f + f_f(v, x) \] (6.1a)

\[ p_A = \frac{A_{V}}{V} \frac{\Delta p}{A_{PA} - A_{Pb}} + \frac{B_{V}}{A_{V}} \frac{\Delta p}{A_{V}} \] (6.1b)

\[ p_A = \frac{A_{V}}{V} \frac{\Delta p}{A_{PA} - A_{Pb}} + \frac{B_{V}}{A_{V}} \frac{\Delta p}{A_{V}} \] (6.1c)

\[ \frac{F}{K_m} = v - |\dot{z}| \] (6.1d)

where \( f_f = \phi_f(y, x), f_f = F_v + D_r \) and the flows are given by (3.18) and (3.21). In

addition a “valve model” is included, which delays the input signals by 2 ms and 3.5 ms which correspond to the average response time of the on/off supply and exhaust solenoid valves, respectively, in the truck set up. This valve model is also added to the observer model. Figure 6.1 gives an overview of how the controller, the clutch actuator and the adaptive observer are connected for simulation. White noise corresponding to that measured in the system, see Figure 2.7 in Section 2.2.4, is added to the position output of the clutch actuator model, to capture the sensor noise present in measurements obtained in the test truck. The sampling rate of the position measurement in the actual truck and in the simulations is 1 ms, and the control signals to the on/off solenoid valves are set at the same rate. The position

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from the clutch actuator model, the calculated input signal and the estimated states are therefore generated by a sample-and-hold, to imitate this. The clutch actuator model and the adaptive nonlinear observer are implemented using explicit Euler discretization,

\[ y_k = y_{k-1} + \Delta \nu_{k-1} \]

\[ v_k = v_{k-1} + \Delta (p(\theta_v(k) - \theta_v(k-1)) - D v_{k-1}) - F v_{k-1} + A A p_{k-1} - A p_{k-1} \]

\[ z_k = z_{k-1} + \Delta (\theta(k) - \theta(k-1)) \]

\[ p_{k+1} = p_{k+1} + \Delta \left( \frac{\Delta v_{k+1}}{r(\theta_{v}(k+1))} + \frac{\Delta \theta_{v}(k+1)}{r(\theta_{v}(k+1))} \right) \]

and

\[ \hat{\theta}_k = \hat{\theta}_k + \Delta \theta_k \]

\[ \hat{v}_k = \hat{v}_k + \Delta \left( \hat{v}(k) - \hat{v}(k-1) \right) - \hat{F}_{K} \hat{v}(k-1) + A B p_{k-1} - A p_{k-1} \]

\[ \hat{z}_k = \hat{z}_k + \Delta \left( \hat{\theta}(k) - \hat{\theta}(k-1) \right) \]

\[ m_A = \frac{m_A}{1 + \Delta \gamma} \]

\[ \delta \hat{\theta}_k = \delta \Delta \theta_k \]

\[ D_k = D_k + \Delta \gamma \hat{\theta}_k^2 \]

where the Euler integration step, \( \Delta \), is set to 0.1 ms. Parameters are given in Table 6.1. The simulation model, including the valve delay, is validated against experimental data from the test track, see Figure 6.2, and is shown to capture the main behavior of the clutch actuator. The performance of the combined system is tested against the same clutch reference, see Figure 2.3, as the experimental tests, and the controller is switched off whenever the steady state requirement is fulfilled.

The observer and adaptation gains have been set based on the tuning in Chapter 5, see Table 5.2. The gain \( \Gamma \) is set 30 times higher in the region \( 3 - 6 \) mm as the clutch load characteristic is especially important in this area due to its steep curve, and since this region is visited only for short transient periods with a typical clutch sequence. The adaptation is switched off whenever the position is not changing, due to lack of PE which might lead to drift or divergence of the estimates.
Figure 6.2: Experimental measurements from truck (solid) and results from the simulation model (dashed in gray).
6.2 Simulation results

In this section we consider both simulation of the observer-based switched controller, and simulation where the pressure is obtained from the simulation model representing the actual system. This is done for comparison of the performance of the pressure estimate obtained from the observer against the estimate of the actual pressure measurement.

6.2.1 Performance of observer-based switched controller

Figure 6.1 shows simulation results for the dual-mode switched controller combined with full-order adaptive observer with nominal values of the pressure characteristic and the viscous friction, \( \theta_0 = [3.6] \) and \( D_0 = 2500 \). The controller makes the piston position meet the requirements, without too much unnecessary switching which could induce chattering, see Figure 6.4. Noise leads to a bias in the pressure estimate obtained from the observer, shown in Figure 6.5, while the clutch load characteristic

\[
\begin{align*}
\text{Table 6.1: Model parameters used in the simulations in this chapter.} \\
\text{Parameter} & \quad \text{Value} \\
\text{Mass} & \quad M = 10 \text{ kg} \\
\text{Area of chamber A} & \quad A_A = 12.3 \times 10^{-3} \text{ m}^2 \\
\text{Area of chamber B} & \quad A_B = 18.4 \times 10^{-3} \text{ m}^2 \\
\text{Area of piston rod} & \quad A_p = 4.52 \times 10^{-3} \text{ m}^2 \\
\text{Ambient pressure} & \quad P_0 = 1 \text{ bar} \\
\text{Supply pressure} & \quad P_S = 9.5 \times 10^5 \text{ Pa} \\
\text{Temperature} & \quad T_0 = 298 \text{ K} \\
\text{Gas constant of air} & \quad R = 288 \text{ J/kgK} \\
\text{Volume in chamber A at } y = 0 & \quad V_A0 = 0.148 \times 10^{-3} \text{ m}^3 \\
\text{Volume in chamber B at } y = 0 & \quad V_B0 = 0.032 \text{ m}^3 \\
\text{Viscous damping} & \quad D_0 = 2500 \text{ Ns/m} \\
\text{Asperity stiffness} & \quad K_s = 1 \times 10^8 \text{ N/m} \\
\text{Coulomb friction} & \quad F_c = 20 \text{ N} \\
\text{Density} & \quad \rho = 1.185 \text{ kg/m}^3 \\
\text{Conductance of outlet restriction} & \quad C_v = 17.1 \times 10^{-3} \text{ m}^2/\text{kg} \\
\text{Conductance for exhaust valve} & \quad C_e = 32.5 \times 10^{-3} \text{ m}^2/\text{kg} \\
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Figure 6.3 shows simulation results for the dual-mode switched controller combined with full-order adaptive observer with nominal values of the clutch load characteristic and the viscous friction, \( \theta_0 = [3.6] \) and \( D_0 = 2500 \). The controller makes the piston position meet the requirements, without too much unnecessary switching which could induce chattering, see Figure 6.4. Noise leads to a bias in the pressure estimate obtained from the observer, shown in Figure 6.5, while the clutch load characteristic

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\text{Conductance of outlet restriction} & \quad C_v = 17.1 \times 10^{-3} \text{ m}^2/\text{kg} \\
\text{Conductance for exhaust valve} & \quad C_e = 32.5 \times 10^{-3} \text{ m}^2/\text{kg} \\
\end{align*}
\]
6.2.2 Performance of switched controller with pressure measurement

Simulations with the dual-mode switched controller and

- position from the simulation model including noise
- velocity filtered from position
- pressure from the simulation model including noise

have been conducted for comparison and results are shown in Figure 6.9. The performance is similar to that shown in Figure 6.3; see Figures 6.10 and Table 6.2 which compares position results in more detail. Given that the simulation model captures the real system well enough, we can state that exchanging the pressure sensor with an observer does not result in any loss of performance.

Simulations with exactly the same state feedback sensors, filtering, and controller configuration, as the experimental tests shown in Section 4.5, have also been conducted. Comparing these results we get that they are quite similar, with a bit less chattering in the simulated results. This was expected as additional noise and disturbances are expected to be present in the truck due to motor vibration etc., and there may also be other unmodelled phenomena. The similarity in performance confirms that the simulation model captures the main behavior of the actual system.

Table 6.2: Performance parameters for simulations with pressure obtained from observer and measurement estimates. The position and pressure errors are average absolute values.

<table>
<thead>
<tr>
<th>Observer</th>
<th>Measurement</th>
<th>Pressure error</th>
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<th>Liquid switches</th>
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<td>Observer</td>
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<td>14.03 kPa</td>
<td>29</td>
<td>2163</td>
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<tr>
<td>Observer</td>
<td>Measurement</td>
<td>9.976 mm</td>
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Figure 6.3: Simulations of dual-mode controller and full-order adaptive observer with nominal values $\theta_0 = [3, 6]$ and $D_0 = 2500$. For position, the simulated state (dashed) and the reference (solid) are shown, and for pressure in chamber A, the estimated reference (solid) and the observer state (dashed) are shown.

Figure 6.3: Simulations of dual-mode controller and full-order adaptive observer with nominal values $\theta_0 = [3, 6]$ and $D_0 = 2500$. For position, the simulated state (dashed) and the reference (solid) are shown, and for pressure in chamber A, the estimated reference (solid) and the observer state (dashed) are shown.

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Figure 6.4: Controller signal and controller used in the simulations of dual-mode controller and full-order adaptive observer with nominal values $\theta_0 = [3, 6]$ and $D_0 = 2500$.

Figure 6.5: Estimated parameters of the simulations of dual-mode controller and full-order adaptive observer with nominal values $\theta_0 = [3, 6]$ and $D_0 = 2500$. 

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Figure 6.7: Controller signal and controller used in the simulations of dual-mode controller and full-order adaptive observer with nominal values $\theta_0 = [5, 9]$ and $D_0 = 4500$.

Figure 6.8: Estimated parameters of the simulations of dual-mode controller and full-order adaptive observer with nominal values $\theta_0 = [5, 9]$ and $D_0 = 4500$.
Figure 6.9: Simulations of controller with feedback from pressure measurement. For position simulated state (dashed) and reference (solid) are shown, and for pressure in ch. A solid estimated reference (solid) and observer state (dashed).
Figure 6.10: Segment of the resulting position for simulations of the controller with pressure feedback from measurement in dashed and from observer in solid.
6.3 Discussions

It is well know that combining individually designed observers and controllers are not straightforward from a theoretical point of view. The stability proofs of the controllers are not longer valid, as estimates from the observer are employed rather than true measurement which was assumed in the controller analysis. Still, no theoretical stability analysis of the combined system is included in this chapter.

The focus was set on validation by simulations, as the developed observer-based switched controller is intended for use in physical systems and the performance in the actual system may be more interesting than the theoretical performance analysis.

Position control for an electropneumatic clutch actuator is considered, using the dual-mode switched controller design and utilizing the full-order adaptive nonlinear observer. Tests using the validated simulation model show that the combined design makes the piston position follow the position reference with sufficient precision, even when starting with large offsets in friction and clutch load characteristics in the observer model. The simulations indicate that the pressure sensor can be eliminated by an adaptive nonlinear observer used as a basis for a nonlinear state feedback control design, and that similar accuracy as in the experimental results in Section 4.5 might be possible to achieve also without a pressure sensor.

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Chapter 7

Conclusions

7.1 Conclusions

The thesis focus on position control of an electropneumatic clutch actuator for heavy duty trucks. This has been achieved by developing appropriate switched controllers and adaptive nonlinear observers for applications to the clutch actuator system.

The mechanical configuration of the clutch actuator system greatly influences how the control task is approached. Some design choices, such as the reduction of the dead volume, hence reducing the effect of hysteresis in the actuator, are taken to simplify the control task. But most design choices are motivated primarily by cost, such as the choice of on/off solenoid valves and the choice to only provide a position sensor in the finalized production system. The decision on on/off solenoid valves makes switched controllers an appealing control strategy.

The main contribution in Chapter 4 is the dual-mode switched controller, combining the two switched controllers designed based on appropriate Control Lyapunov functions. Experimental testing shows that the controllers are well suited for application to the electropneumatic clutch actuator system. A strength of these switched controllers is that they are robust to noise. The feedback signals, which may be corrupted by noise, are for the switched controllers only used to decide on one of the inputs available for the system. Another quality of the switched controllers is their simple design. Combined with an adaptive observer, the controllers can be applied to clutch actuator system without knowledge on the detailed response and capacity of the on/off solenoid valves, and the degree of wear for the clutch compression spring.

The full-order adaptive observer presented in Chapter 5 produces estimates of the unmeasured states and calculates parameters for clutch load characteristics and viscous friction. Stability and boundedness of the state estimates are proved under PE conditions of the input and the velocity signals. The experimental testing indicates that the PE conditions are met under typical clutch sequences, and the convergence of the state estimates are shown. The pressure in chamber B seems to have little effect on the dynamics of the rest of the system, and could have been

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neglected to simplify the design. It was shown in robustness tests of the switched controllers, considering errors in the clutch load characteristics, that performance of the controllers diminish rapidly with the presence of such model errors. The parameter estimation provided by the proposed adaptive observer will therefore be an essential contribution for robust performance of the clutch actuator system to account for wear and variations in temperature.

The combination of the two main contributions into an adaptive observer-based switched controller for position control of the actuator piston is treated in Chapter 6. Simulation results indicate that the measurement from a pressure sensor can be replaced by the estimate from the nonlinear adaptive observer without loss of accuracy. A simple valve dynamic have been included in the simulation model used in this chapter to represent the clutch actuator system. By comparison of the position and the pressure estimates obtained from this simulation model and actual truck measurements, it is shown that the simulation model captures the main behavior of the clutch actuator system.

7.2 Recommendations for future work

The proposed adaptive observer-based switched controller shows promising performance from simulations. Experimental tests with this controller employed in the eletropneumatic clutch actuator in a heavy duty truck is clearly the most important topic for future work.

The work in this thesis has focused on showing that the idea of using switched controllers, is appealing for the clutch actuator system with on/off valves. For application, future adjustment will have to be done. For instance, too much focus has been on response time, and too little on avoiding overshoots. Such aspects must be considered to obtain a robust design for use in a production system.

The parameterization of the clutch load used is very simple, and improvement to this parameterization should be considered. Including method to account for the curve-shifting left/right, such as a Multiple Model Scheme, will make the design more adaptable to different clutches with different degree of wear.

To make the control design more independent of the mechanical structure of the clutch actuator system, the valve dynamics could be taken into account in a rigorous way. This would make it easier to tune the controller when valvesets are changed.

The observer design can be further improved by using $\hat{y}$ instead of $y$. This is likely to improve the observer performance as noise is no longer amplified through the differentiation of the position measurement.

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Bibliography
Appendix A

Estimation of electropneumatic clutch actuator load characteristics

This chapter includes the paper Langjord et al. (2008b).
Is not included due to copyright