Observations seem to indicate that central banks adjust their key rates gradually. Uncertainty regarding economic relationships and measurement errors in the data may point to a need for key rates to be adjusted gradually. Uncertainty relating to possible shocks in exogenous variables on the other hand does not indicate the need for a gradualist approach to setting interest rates. The decisive factor regarding how the uncertainty inherent in monetary policy should be taken into account is whether the setting of interest rates in itself affects the degree of uncertainty and thereby the variance in the outcome of monetary policy. If the setting of interest rates affects both the expected outcome and the uncertainty of the outcome, it will be optimal to take both factors into account. Conclusions in the theoretical literature must be interpreted with caution. The models are highly simplified. Nevertheless, theory makes an important contribution to shedding light on the problems facing central banks in the conduct of monetary policy.

Introduction

Interest rates are the most important monetary policy instrument available to central banks. Experience shows that in many countries there is a tendency for central banks to take a gradualist approach to setting interest rates. It has been suggested that uncertainty in monetary policy is one possible explanation for this. Central banks have always had to take this uncertainty into account in the basis for monetary policy decisions, but it is only in recent years that there has been increased interest in this topic in the academic literature.

This article will review various types of uncertainty of relevance to monetary policy and the consequences for the setting of interest rates. We will begin by illustrating how gradual actual monetary policy has been in a number of countries, including Norway. Then we will go on to review various types of uncertainty and how they affect optimal monetary policy. We start by investigating whether gradual adjustments of monetary policy can be attributed to sudden, unexpected economic events, or shocks to the economy. We then discuss whether uncertainty in the parameters of the economic models offers an explanation of observed interest rate movements. Next, we consider the effect of measurement error on some variables in the models. Finally, we note that there is uncertainty associated with the models used by the central bank in its analyses. A simple model is used to illustrate the effects.

This article uses the terms gradual and cautious to describe monetary policy. Goodhart (1996) defines gradual monetary policy as the central bank adjusting interest rates in several small stages instead of a single jump when an inflation impetus arises which leads inflation away from the inflation target. A cautious monetary policy can be defined as a policy where interest rates react less to deviation from the inflation target than that which is optimal without uncertainty. Actual monetary policy tends to be both cautious and gradual, and we will discuss whether uncertainty is sufficient to explain the actual setting of interest rates. In many cases we will use the terms cautious and gradual monetary policy interchangeably.

Optimal monetary policy and actual monetary policy

In many countries, monetary policy is oriented towards low and stable inflation. It is often emphasised that the objective is to be achieved without incurring excessive real economic costs in the form of high unemployment and lost production. In economic theory it is common to use a loss function for the central bank, comprising inflation gap, measured as the deviation of actual inflation from its target, and output gap, ie the difference between actual output and potential output. By minimising the loss given the economic relationships, the central bank can derive an optimal path for interest rates.

It can be shown that in a static model, optimal monetary policy will mean that the probability that the next change in interest rates will be an increase is the same as the probability of a reduction, see Goodhart (1999). Assume, for instance, that random shocks continually occur in the economy, necessitating increases or decreases in interest rates. Over time it will be natural for these shocks to be evenly divided between those requiring tighter monetary policy and those requiring more expansionary monetary policy. Subsequent changes in interest rates will therefore be uncorrelated.

In a dynamic model, however, it may be optimal for an adjustment of interest rates to be carried out in several stages. This is because economic variables such as output and inflation change slowly, and because the effects of monetary policy are associated with relatively long lags.

1) Thanks to Farooq Qaisar Akram, Tom Bernhardsen, Kristin Gulbrandsen, Kåre Hageland, Kai Leitemo, Øistein Røisland, Karsten Stæhr and Ulf Söderström.

2) Martin (1999), however, uses the term conservative monetary policy. He reserves the term cautious monetary policy for situations in which the sum of consecutive changes in interest rates is less than in the case with no uncertainty. See Martin (1999) for an analytical discussion of gradual, conservative and cautious monetary policy.

3) Norges Bank’s objective is exchange rate stability. However, experience shows that price and cost stability are important preconditions for exchange rate stability in Norway, see Norges Bank (2000a).

4) See Lønning and Olsen (2000) for an introduction to optimal interest rate rules.
However, it is apparent that the dynamic structure of the economy does not provide an adequate explanation of the gradual setting of interest rates observed in many countries. Central banks adjust their key rates more gradually than indicated by optimal monetary policy. For this reason, some authors define the setting of interest rates as gradual if interest rates change less than can be explained by the dynamic structure of the economy, see for instance Sack (2000).

Table 1 shows actual interest rate setting in various countries. In the period August 1989 to March 1998, the US Federal Reserve lowered interest rates twice in succession on 22 occasions. During the same period, the Federal Reserve shifted from lowering interest rates to increasing them in only two cases. Interest rate changes have also been gradual in Norway. Since December 1992, Norges Bank has on 28 occasions lowered interest rates twice in succession, whereas it has only twice raised interest rates and then lowered them.

The following section discusses what economic theory says about how to take uncertainty in monetary policy into account, and whether various economic theories can explain why central banks adjust interest rates gradually and cautiously.

Additive uncertainty

We will illustrate various types of uncertainty using a simplified version of Svensson’s model (1997). This model uses a highly simple representation of economic relationships, but is nevertheless useful for illustrating the effect of various types of uncertainty on optimal monetary policy.

We assume that the inflation process can be modelled as a backward-looking, expectations-augmented Phillips curve:

\[ \pi_{t+1} = a\pi_t + b\delta_t + \epsilon_{t+1}, \quad a, b > 0 \]  

(1)

where \( \pi_t \) is the inflation rate and \( y_t \) is the output gap. The output gap provides an indication of pressures in the economy, \( \epsilon_t \) is a cost shock in the form of an additive residual, with zero expectation and constant variance, \( \sigma^2 \).

There are lags in inflation, which means that inflation from the previous period is part of the reason for inflation in the current period. These lags may be due to structural factors in price-setting and the fact that expectations regarding future inflation are backward-looking. A positive (negative) output gap will contribute to higher (lower) price inflation in the subsequent period.

The output gap depends solely on the nominal interest rate in the same period, \( i_t \):

\[ y_t = -\delta \pi_t, \quad \delta > 0 \]  

(2)

Potential production is normalised to zero, so that \( y_t \) denotes the output gap. As can be seen, higher interest rates in isolation will contribute to less pressure in the economy in the form of a lower output gap in the same period. By substituting (2) into (1) we obtain:

\[ \pi_{t+1} = a\pi_t + bi_t + \epsilon_{t+1}, \]  

(3)

where \( b = \beta \delta \). From equation (3) we can see that changes in interest rates have an effect on price inflation after one period. Let us assume that a period is equal to one year. There is full certainty regarding the effects of monetary policy on the economy. So far we have assumed that the uncertainty arises solely through the additive residual, \( \epsilon_t \).

A common assumption in the theoretical literature is that the central bank places emphasis on both production stability and inflation stability. However, here we shall assume that the central bank’s only objective is price stability, which simplifies the analysis. This will allow us nonetheless to shed light on the difference between adaptation with and without additive uncertainty. In formal terms, the expected squared differential between inflation and the inflation target, \( \pi_t^* \), is minimised:

\[ \text{Minimise } E_t (\pi_{t+1} - \pi_t^*)^2, \]  

(4)

where \( E_t \) is the expectation operator given the central bank’s information at the point in time \( t \). By minimising the objective function (4) given the equation (3), we can derive the optimal interest rate rule (see Annex A):

\[ i_t = -\frac{a}{b} \pi_t, \]  

(5)

where the inflation target is normalised to zero. By substituting the solution for the interest rate into (3) we obtain \( \pi_{t+1} = \epsilon_{t+1} \). The optimal interest rate is thereby given by:

\[ i_t = -\frac{a}{b} \epsilon_t. \]  

(6)
In the absence of economic shocks (ie $\varepsilon_t=0$), the interest rate should be set at its equilibrium level, in this case equal to zero owing to the normalisation. If a positive economic shock should occur, the interest rate should be raised in the same period, and then be reset at zero in the following period. Since monetary policy has a one-year lag, there will be a short-term deviation from the inflation target. However, monetary policy will succeed in achieving its inflation target in the following year. Equation (6) shows that monetary policy should respond immediately if shocks to the economy bring inflation above or below the inflation target. This shows that monetary policy is oriented in the same way irrespective of whether there is full certainty in the economy or whether uncertainty exists in the form of additive shocks. Interest rates must be set as if there were no shocks, and monetary policy should not respond until shocks have actually occurred. This is a variation on the certainty equivalence theory, see Theil (1956). The central bank knows exactly how much the interest rate must be adjusted in order to achieve the inflation target, and there is no reason for adopting a cautious approach to the setting of interest rates.

The additive uncertainty theory has important implications for central banks. The above model did not include exogenous variables, ie variables whose value is determined outside the model. In principle exogenous variables will be included in the same way as the additive residual. This means that an optimal monetary policy must be based on the expected value of the exogenous variables.

In practical monetary policy, however, there may be uncertainty as to the best estimates for future exogenous variables. In Norges Bank’s most important macro-economic model for projections, RIMINI, it is necessary to estimate expected future developments in fiscal policy, the exchange rate and commodity prices. If actual developments deviate considerably from the estimated values, economic trends may be very different from the estimates. For this reason, it is common practice for central banks to assess the risk outlook on the basis of various assumptions concerning developments in exogenous variables. However, theory indicates that interest rates should be adapted in relation to the expected future value of exogenous variables. Uncertainty regarding the estimates is therefore not a viable argument for gradual interest rate adjustment.

Let us illustrate the significance of uncertainty concerning the outcome of exogenous variables. One important exogenous variable in RIMINI is the price of oil, which is of considerable significance for developments in real variables and consumer price inflation. The projections in Norges Bank’s June 2000 Inflation Report are based on the assumption that the oil price will return to a normal level of around USD 18 per barrel in 2002. However, there is considerable uncertainty surrounding movements in the oil price, as recent years have shown. Signals from OPEC seem to indicate that the organisation is aiming for an oil price slightly above this average – a target price in the range of USD 22 to USD 28 has been indicated. The problem for the central bank lies in estimating the expected oil price. For instance, if Norges Bank were to use OPEC’s target in its projections, this could change the estimates for future price inflation. According to the calculations presented in the June 2000 Inflation Report, an assumption of an oil price of USD 26 per barrel will raise consumer price inflation by approximately ¼ percentage point over the next two years in relation to the baseline scenario. However, it is important to note that Norges Bank and most other central banks focus on second-round effects of an increase in the oil price, rather than the effect of the oil price on the consumer price index.

A further potential problem is that the probability distribution of the oil price is skewed. This may mean that the expected oil price and the most probable oil price (often referred to as the modus) are not the same. According to the additive uncertainty theory, the central bank should base its projections on expected oil price movements. If the probability distribution is skewed to the right (ie there is a greater risk of very high oil prices than of very low oil prices) the expected oil price will be higher than the modus. On the basis of option prices, we can calculate the risk distribution for the oil price in the period ahead, see the June 2000 Inflation Report.

Several empirical studies (including Rudebusch (1999) and Sack (2000) on US data and Goodhart (1999) on UK data) have attempted to compare the optimal interest rate under additive uncertainty with the historical interest rate which the central banks actually set. These empirical studies indicate that the optimal interest rate shows considerably greater fluctuations than central banks’ key rates. In other words, monetary policy appears to be adjusted more gradually when there are changes in economic variables than indicated by optimal policy when only additive uncertainty is taken into account. This may mean that caution in monetary policy is due to other types of uncertainty.

**Parameter uncertainty**

In the above model we assumed that the central bank has exact knowledge of economic relationships. This implies that the central bank is cognisant of the “true” model and the value of the parameters in this model. In this model an increase of 1 percentage point in the output gap as a result of shocks to the economy was consistent with an increase of $\beta$ percentage points in inflation the following year, see equation (1). The central bank could therefore raise interest rates today to curb the future inflation impetus. In practice, the central bank does not know the value of the parameters. This is because the
parameters are calculated on the basis of historical data in which noise leads to imprecise estimates for the parameters. In addition, structural changes in the economy, such as the removal of interest rate regulation in the 1980s in Norway, could change the value of the parameters.

Parameter uncertainty can be illustrated in the above model. Assume that the expected values of the parameters \( a \) and \( b \) are \( \bar{a} \) and \( \bar{b} \), with a variance \( \sigma_a^2 \) og \( \sigma_b^2 \), respectively. The parameters are assumed not to be correlated with each other or with the residual, \( \varepsilon_t \). We can write:

\[
E_t (\pi_{t+1}) = \bar{a}\pi_t + \bar{b}i_t
\]

(7)

If we minimise the loss function (4) given equation (7), we obtain (see Annex B):

\[
i_t = \frac{-a}{b + \sigma_b^2/b} \pi_t.
\]

(8)

If we compare (8) with the case of additive uncertainty, we see that the optimal interest rate should respond less to deviation in inflation. The greater the uncertainty in \( b \) (represented here by a higher \( \sigma_b^2/b \)), the smaller the interest rate adjustment required. This is because the more uncertain the parameters are, the greater the chance is that interest rates will be adjusted too much. If the relative uncertainty is small, however, the solution will deviate less from the solution with additive uncertainty.

If the central bank follows (8), the interest rate will be adjusted more cautiously than in the case of additive uncertainty. Consequently, it will take several periods to attain the inflation target. Blinder (1998) terms this “Brainard conservatism”. The reason why the central bank must proceed more cautiously in this case is that in the case of parameter uncertainty the interest rate affects not only the bias of inflation, but also the variance of future inflation (see Annex B for further discussion). Since the central bank has an instrument – the interest rate – for minimising both the bias and the variance of inflation, the central bank must weigh the two considerations. The more swiftly the interest rate is adjusted to achieve the inflation target, the greater the risk of variance in future inflation. Swift interest rate adjustment will therefore increase the uncertainty in inflation.

Several studies have attempted to calculate the relevance of parameter uncertainty. Sack (2000) demonstrates within a vector autoregressive model (VAR model) using US data that if there is parameter uncertainty, it is wise to adopt a more gradualist approach to setting interest rates. He concludes that parameter uncertainty may account for a considerable portion of actual movements in the Federal Reserve’s federal funds rate. Nevertheless, part of the interest rate movements remains unexplained. Batini et al (1999) carried out a VAR analysis on UK data. They also find that an economic shock arising from parameter uncertainty demands a more cautious, gradual interest rate response. In this study, the interest rate response to a shock after one quarter will be approximately \( \frac{2}{3} \) of the interest rate response in the case of additive uncertainty alone. However, the interest rate must be kept higher in the subsequent quarters. After approximately nine months, the cumulative interest rate adjustment is roughly the same. However, it is emphasised that these results are uncertain. Modelling the economy as a simple VAR model can be difficult since there have been many different monetary policy regimes in the UK in the period in question.

One simple way of shedding light on parameter uncertainty for the Norwegian economy is to look at the output gap coefficient on a Phillips curve. An empirical model constructed by Bårdsen et al (2000) calculates the output gap coefficient as 0.05, with a standard deviation of 0.01. On this basis, a one percentage point higher output gap would increase inflation by 0.05 percentage point. The uncertainty is fairly considerable. There is a 95 per cent probability that the coefficient will fall within the interval of 0.03-0.07.

Söderström (1999a) argues that in some cases of parameter uncertainty the optimal approach is to adopt a more aggressive monetary policy. This is due in part to the uncertainty associated with the value of the inflation coefficient in the previous period, \( a \) in equation (1). If there is a risk of a high value for this parameter, this means that the level of inflation in the previous period will have a considerable effect on the level in the current period. There is then a risk of inflation moving away from the inflation target. In order to prevent this, Söderström demonstrates that the optimal approach may be for the central bank to react more strongly in its monetary policy than without such uncertainty. Söderström (1999b) concludes, however, that the effect of parameter uncertainty regarding the previous period’s inflation is probably dominated by the uncertainty associated with the output gap and interest rate coefficients, so that all in all parameter uncertainty contributes to moderating interest rate adjustments.

**Measurement errors**

Our discussion so far has been based on the assumption that economic data can be precisely observed. This is a gross simplification. First, the statistical data are only available with a considerable lag. Second, the figures are often subject to extensive revision at a later date. Third, many important variables cannot be observed, but

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11) Söderström (1999a) obtains similar results.

12) However, other studies find parameter uncertainty to have less effect on US data, see for instance Rudebusch (1999). The difference between Rudebusch’s and Sack’s studies may be attributable to the fact that the latter used VAR analysis. VAR analysis may overestimate the effect of parameter uncertainty since a VAR model contains many variables which are often imprecisely estimated.

13) This coefficient is somewhat lower than for the US economy. Rudebusch (1999) calculates the output gap coefficient at 0.15, while Orphanides (1998) has a value of 0.18.

14) This applies provided that the central bank also has a production stability objective.
must be estimated using uncertain methodology. The output gap is vulnerable to measurement errors of all the above types.\(^\text{15}\) Orphanides (1998) has studied the difference between the output gap estimate available at the time the interest rates were set and the output gap figures calculated in 1994 for the period 1980-1992 for US data. He finds that the average output gap was –3.99 per cent of GDP in the preliminary figures, but just –1.64 per cent in the final figures. In addition, the variance of the preliminary output gap was considerably larger than the variance of the output gap based on the final figures.

The consequences of measurement errors for monetary policy will depend on how the measurement error is included. Assume that we can only observe a preliminary estimate of the output gap, \(y_{it}\), which is the sum of a measurement error, \(\gamma_t\), and actual output gap, \(y_t\):

\[
y_{it} = y_t + \gamma_t
\]

We assume that the measurement error is not systematic.\(^\text{16}\) By substituting (9) into (1) and (2), we obtain:

\[
\pi_{t+1} = \alpha\pi_t - \beta i_t + \epsilon_{t+1} + \beta\eta_{t+1}
\]

In formal terms, the measurement error will be included in the same way as the additive uncertainty residual. By minimising the loss function with respect to equation (10) instead of equation (3), it can be shown that the solution for the optimal interest rate will be the same as in the case of additive uncertainty. This result indicates that the central bank should not take possible measurement errors into consideration in its orientation of monetary policy.\(^\text{17}\)

Several studies have looked at the effects of measurement errors on optimal monetary policy when monetary policy is based on a simple rule for setting interest rates.\(^\text{18}\) A simple rule is one which expresses the interest rate as an explicit function of a limited number of economic variables such as inflation and output gap. These studies conclude that measurement errors will have consequences for optimal monetary policy. Smets (1998) discusses measurement errors in the output gap, and Orphanides (1998) and Rudebusch (1999) discuss measurement errors in both the output gap and inflation. All these studies are based on the assumption that monetary policy follows a Taylor rule. Taylor’s rule is a simple formula for the orientation of monetary policy, given that the role of monetary policy is to help stabilise the level of domestic activity and keep inflation low.\(^\text{19}\) In these studies, optimal coefficients are calculated for inflation and output gap in the Taylor rule in a model without measurement errors. Then the coefficients are calculated with measurement errors in the data. The studies conclude that uncertainty regarding the size of the output gap or inflation will contribute to reducing the optimal coefficients. Consequently, monetary policy should proceed more cautiously. Rudebusch finds that measurement errors in the output gap reduce the optimal weight on the output gap in the Taylor rule from a coefficient of 1.6 to 1.0.

In practice it can be difficult to ascertain whether an observation is subject to measurement errors or is a result of an economic shock. For instance, we can imagine a scenario where the economy is apparently hit by a positive demand shock which results in increased production and higher prices. If the economy was in balance prior to the shock, the Taylor rule states that the central bank must tighten monetary policy. But if it was in reality a supply shock, which led to higher prices at the same time as the output gap widened as a result of measurement errors, it is less certain whether the central bank should tighten its monetary policy stance. In this scenario, the central bank must weigh the probability that what it is observing is the result of a demand shock against the probability that it is due to a supply shock combined with measurement errors in the output gap.

Model uncertainty – uncertainty surrounding the functioning of the economy

In the above discussion the central bank was faced with uncertainty, but it was clear what type of uncertainty this was. It was also possible to specify a probability distribution for the uncertainty. In practice, it may be said that central banks face a more fundamental uncertainty. They cannot be sure which model expresses the economic relationships most accurately. All models of how the economy functions are by definition an extreme simplification of reality. Different models have different strengths and weaknesses. One example of model uncertainty is the relationship between interest rates and exchange rates. Changes in interest rates will have highly different effects on the exchange rate, depending on the model used. Another problem is structural breaks in the economy which are not captured by empirical models estimated on the basis of historical data. This may lead to sources of error in monetary policy.

It is not clear how model uncertainty should be dealt with. There are several approaches to this in the literature. Blinder (1998) was Vice-Chairman of the Federal Reserve when he advocated using a broad range of various models for analysing interest rate changes and then assessing the results reached using the different models. He believed that model uncertainty could be reduced in this way.

\(^{15}\) See for instance Frøyland and Nymoen (2000) for a review of the methodological problems associated with the output gap calculated using the Hodrick-Prescott filter and the production function method.

\(^{16}\) We assume that the expectation of the measurement error is zero, that the variance is constant and that there is no correlation between measurement errors and actual output gap.

\(^{17}\) The conclusions would be different in the event of systematic measurement errors, see for instance Orphanides (1998).

\(^{18}\) See for instance Lanning and Olsen (2000) for a discussion of simple rules and Taylor’s rule.

\(^{19}\) In a Taylor rule the interest rate is set as a function of the output gap and the deviation of inflation from its target. The Taylor rule can be expressed mathematically as:

\[
i = a\pi + b(y - \pi) + \epsilon
\]

where \(a\) and \(b\) are constants.
Given the model uncertainty factor, the use of simple instrument rules in monetary policy such as the Taylor rule has also been proposed, rather than more complex optimal rules. It has been shown that simple rules of this sort can work well in many different models, see Taylor (1999). Optimal rules are dependent on the model used. If the economy is misrepresented in the model used, there is a risk of making major mistakes in monetary policy.

Research also suggests a third approach to model uncertainty, with completely different implications, namely that central banks should seek to reduce model uncertainty through experimentation. This implies that central banks are in a position to learn more about how the economy functions when interest rates are changed, see Wieland (2000). Parameter uncertainty may also be reduced in this way. The need for this type of experimentation may arise as a result of structural changes in the economy which alter parameters and model structures, thereby increasing uncertainty. However, central bank economists will normally be extremely sceptical to this type of experimentation with the economy.

Conclusion

Observations seem to indicate that central banks adjust their key rates gradually. Uncertainty regarding the strength of various economic relationships and what the "true" model for these economic relationships is, together with measurement errors in data, may point to the need for a gradual and cautious approach to changes in key rates. Additive uncertainty, such as uncertainty associated with possible shocks in exogenous variables, is not, however, an argument for greater caution in setting interest rates.

The decisive factor regarding how to take uncertainty into account in monetary policy is whether the setting of interest rates in itself affects the degree of uncertainty and thereby the variance in the outcome of monetary policy. If the setting of interest rates affects both the expected outcome and the uncertainty of the outcome, it will be optimal to take both factors into account. In many cases this will necessitate caution in setting interest rates.

Research into the significance of uncertainty for optimal monetary policy is still in its nascent stages. The conclusions arrived at are based on analyses carried out using simple theoretical models. It is therefore important to exercise caution in drawing clear conclusions from the contents of this research. Blinder (1998) sums up a discussion on uncertainty by saying that in the real world gradual monetary policy is probably more common – and more sensible – than these models would suggest. The analyses do not provide a clear-cut answer as to the approach central banks should take in monetary policy. Nevertheless, recent theories make an important contribution to shedding light on the problems facing central banks in the conduct of monetary policy.

This article has not focussed on predictability, credibility and transparency of monetary policy. It has been said that central banks adjust interest rates gradually because they risk losing credibility if they adjust rates frequently, particularly if the adjustments take the form of reversals, see Goodhart (1999) and Dillén and Nilsson (1999). Market participants may perceive this as a sign that the central bank does not have control of the conduct of monetary policy. On the other hand, central banks are probably able to influence the uncertainty facing economic agents through an increasing degree of transparency as regards their own policy. A predictable, transparent monetary policy will enable market participants to monitor and interpret the central bank’s intentions. This will probably reduce speculation and uncertainty about the setting of interest rates and contribute to smoother developments in inflation expectations and long-term interest rates. This in itself may contribute to economic stability.

Annex A

By substituting (3) into (4), we obtain:

\[ E_t(\pi^2_{t+1}) = E_t(a\pi_t - b\pi_{t+1} + \epsilon_{t+1})^2 = \{ a^2\pi_t^2 + b^2\pi_{t+1}^2 - 2ab\pi_t\pi_{t+1} \} + \sigma^2_e \]  \hspace{1cm} (11)

By minimising (11) with respect to the interest rate and setting the first order condition at zero, we obtain the solution in (5).

Annex B

The objective function (4) can be rewritten as:

\[ E_t(\pi^2_{t+1} - \pi^2_t)^2 = E_t\{ E_t(\pi_{t+1})^2\} + E_t\{ E_t(\pi_{t+1})\} \cdot \sigma^2_t \]  \hspace{1cm} (12)

\[ E_t(\pi_{t+1})^2 \] is an expression of the (squared) bias of \( \pi_t \), i.e. the difference between expected inflation and the inflation target. The final component of (12) is an expression of inflation variance. Therefore, we can write:

\[ E_t(\pi^2_{t+1}) = (bias_t(\pi_{t+1}))^2 + var_t(\pi_{t+1}) \]  \hspace{1cm} (13)

In the case with parameter uncertainty we can use (7) and (13) to derive:

\[ E_t(\pi^2_{t+1}) = \left\{ \frac{\sigma^2_t}{a^2} + \frac{\sigma^2_e}{b^2} - 2ab\pi_t\pi_{t+1} \right\} + \{ \sigma^2_t + \sigma^2_e \} \]  \hspace{1cm} (14)

In this case both the bias and the variance of inflation will depend on the interest rate level, and thus both factors will depend on monetary policy. In the case of additive uncertainty, inflation variance will depend solely on the variance of the additive residual, see (11). By deriving (14) with respect to the interest rate and setting the derivative at zero, we obtain the solution in (8).

20) Goodhart (1999) raises the question of whether this is optimal monetary policy.

21) Dillén and Nilsson (1999) demonstrate that increased predictability – which in turn has a stabilising effect on inflation expectations – improves the relationship between production variability and inflation variability.

22) See Gjedrem (2000) for further discussion.
References


