Progress from forecast failure—
The Norwegian consumption function.

by

Øyvind Eitrheim, Eilev S. Jansen and Ragnar Nymoen
Arbeidsnotater fra Norges Bank kan bestilles over Internett:
www.norges-bank.no/publikasjoner
eller ved henvendelse til
Norges Bank, Abonnementsservice,
PB 1179 Sentrum, 0107 Oslo
Telefon 22 31 63 83, Telefaks 22 41 31 05

Norges Banks arbeidsnotater inneholder forskningsarbeider og utredninger som vanligvis ikke har fått sin endelige form.
Hensikten er blant annet at forfatteren kan motta kommentarer fra kolleger og andre interesserte.

Synspunkter og konklusjoner står for forfatterens regning.

Arbeidsnotater/working papers from Norges Bank can be ordered via Internet:
www.norges-bank.no/english/publications
or from Norges Bank, Subscription service,
P.O.Box. 1179 Sentrum, 0107 Oslo, Norway.
Tel. +47 22 31 63 83, Fax. +47 22 41 31 05

Norges Bank’s Working papers present research projects and reports (not usually in their final form), and are intended inter alia to enable the author to benefit from the comments of colleagues and other interested parties.

Views and conclusions expressed are the responsibility of the author alone.
Progress from forecast failure—The Norwegian consumption function.

Øyvind Eitrheim, Eilev S. Jansen and Ragnar Nymoen*


Abstract

After a forecast failure, a respecification is usually necessary to account for the data ex post, in which case there is a gain in knowledge as a result of the forecast failure. Using Norwegian consumption as an example, we show that the financial deregulation in the mid 1980s led to forecast failure both for consumption functions and Euler equations. Counter to widespread beliefs, we show analytically and empirically that this constellation of forecast failures is inconsistent with an underlying Euler equation. Instead, respecification led to a new consumption function where wealth plays a central role. That model is updated and is shown to have constant parameters despite huge changes in the income to wealth ratio over nine years of new data.

Keywords: Consumption functions, equilibrium correction models, Euler equations, financial deregulation, forecast failure, progressive research strategies, VAR models

JEL classifications: C51, C52, C53, E21, E27

*Paper presented at the workshop Macroeconomic Transmission Mechanisms: Empirical Applications and Econometric Methods in Copenhagen May 2000. Comments from Anders Rygh Swensen, David Hendry and Michael Clements are gratefully acknowledged. The first author is a head of research in Norges Bank (Central Bank of Norway), and the second author is a director of the Research Department in Norges Bank. The third author is a professor of economics at the University of Oslo and a special consultant in Norges Bank. The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting those of Norges Bank. Please address correspondence to the third author: Ragnar Nymoen, University of Oslo, Department of Economics. P.O. Box 1095 Blindern. N-0317 Oslo, Norway. Internet: ragnar.nymoen@econ.uio.no.
1 Introduction

Financial deregulation in the mid-1980s was followed by a strong rise in aggregate consumption relative to income in e.g., the UK and the Scandinavian countries, see e.g., Muellbauer and Murphy (1990), Berg (1994) and Lehmussaari (1990). The existing empirical macroeconometric consumption functions subsequently broke down — i.e., they failed in forecasting, and failed to explain the data ex post. These forecast failures permanently reduced the profession’s belief in the validity of “solved out” consumption functions—a belief that was already worn thin following the breakthrough of stochastic life cycle permanent income models led on by the seminal paper by Hall (1978), see Muellbauer and Lattimore (1995). Many economists have interpreted the forecast failure as a significant outcome of a “natural experiment” that corroborated the predictions of the rational expectations permanent income hypothesis: Spurred by financial deregulation, consumers revised their expected permanent income and their consumption, thus bringing about a huge change in the correlation between consumption and current income that underlies the conventional consumption function, see e.g. King (1990) and Pagano (1990). According to this view, it was a shift in to the (non-modelled) income expectation process that caused the structural break in the existing consumption functions. Thus, the breakdown was also seen as a confirmation of the Lucas-critique, (Lucas, 1976), i.e., that conditional consumption functions are invalid tools for policy analysis, see Blake and Westaway (1993).

However, in the Scandinavian countries it turned out to be respecified consumption functions that included broad measures of household wealth, not Euler equation models, that succeeded in accounting for the breakdown ex post, see Brodin and Nymoen (1989), (1992), Steffensen (1989), Brubakk (1994) and Berg and Bergström (1995). These developments illustrate that care must be taken when drawing model and policy implications from forecast failure, and confirm the empirical relevance of the conclusion in Clements and Hendry (1999a, Ch. 12.2): The single lesson that can be learned from forecast breakdown is that something unpredictable happened in the forecast period. By itself, an episode of forecast failure does not invalidate the underlying theory (the consumption function), nor does it validate any rivaling theory (the Euler equation). Instead, after a breakdown, a new model usually emerges from econometric modelling on a sample that include the breakdown period.

Viewing empirical modelling as a process, where forecast failures represent a potential for improvement, leads to a progressive research programme where models continuously become overtaken by new and more useful ones. By “useful” we understand models that are relatively invariant to structural changes elsewhere in the economy, i.e., they contain autonomous parameters, see Haavelmo (1944), Johansen (1977) and Aldrich (1989). Models with a high degree of autonomy can also be said to represent structure: They remain invariant to changes in economic policies and other shocks to the economic system. However, structure is partial in two respects: First, autonomy is a relative concept: An econometric model cannot be invariant to every imaginable shock (e.g., a war), but a consumption function estimated with data from a given country may be invariant to a range of “shocks” arising from decisionmaking in government and business (annual tax and budget changes, interest rate changes, unemployment rises and falls). Second, all parameters of an
economic model are unlikely to be equally invariant, and only the parameters with the highest degree of autonomy represent partial structure, see Hendry (1993) and Hendry (1995b). Since partial structure typically will be grafted into equations that also contains parameters with a lower degree of autonomy, forecast breakdown may frequently be caused by shifts in these non-structural parameters. A strategy that puts a lot of emphasis on forecast behaviour, without a careful evaluation of the causes of forecast failure ex post, runs a risk of discarding models that actually contain important elements of structure. Hence, for example Doornik and Hendry (1997) and Clements and Hendry (1999a, Ch. 3) show that the main source of forecast failure is deterministic shifts in equilibrium means (e.g., the equilibrium savings rate) and not shifts in the derivative coefficients (e.g., the propensity to consume) that are of primary interest for policy analysis.

In the following sections we elucidate several issues concerning model selection after a major forecast breakdown, using the modelling of private consumption as our example. In section 2 the consumption function (CF) and Euler equation (EE) approaches are set out as two contending mechanisms. Both models are consistent with cointegration between consumption and income. The discriminating feature is their implications for exogeneity. Section 3 derives the theoretical properties of forecasts based on the two models. When the CF-restriction holds, the EE can still have a smaller forecast error bias than the CF-based forecast. This possibility arises in a situation where there is a structural break in the underlying true data generating process prior to the forecast period, and provides an example of a non-causal model that outperforms the true model in a forecast contest, see Clements and Hendry (1999a, Ch. 3). The EE for consumption belongs to a class of models called differenced vector autoregressive systems, dVARs, that has been shown to be relatively immune to structural breaks that occur before the preparation of the forecast, see Clements and Hendry (1999a, Chapter 5) and Eitrheim et al. (1999). We also investigate the other state of nature, namely that the EE restrictions are true, and that the consumption function represents the misspecified model. We derive the new result that both sets of forecasts are immune to a shift in the equilibrium savings rate that occur after the forecast have been made. Failure in “before break” CF-forecasts is only (logically) possible if the consumption function restrictions hold within the sample.

Sections 4 and 5 illustrate the theory by reconsidering the breakdown of Norwegian consumption functions in the 1980s, and the respecified consumption function due to Brodin and Nymoen (1992), B&N hereafter. The B&N model is respecified on an extended sample that includes nine years of new quarterly data. Moreover, the historical series for income and consumption have been revised as a results of a new SNA for the National Accounts. Despite the extended sample and the change in the measurement system, important features of the B&N model are retrieved almost to perfection, showing the relevance of partial structure as an operational concept.

---

1See Muellbauer and Lattimore (1995) for a comprehensive discussion of other aspects of the two approaches.
2 Consumption functions and Euler equations

We assume that the variables are measured in logarithmic scale: $c_t$ is consumption in period $t$ and $y_t$ is income. Both series are assumed to be integrated of degree one, $I(1)$.

2.1 Cointegration

Assume a 1. order VAR:

\begin{align}
  c_t &= \kappa + \phi_{cc}c_{t-1} + \phi_{cy}y_{t-1} + \epsilon_{c,t}, \\
  y_t &= \varphi + \phi_{yc}c_{t-1} + \phi_{yy}y_{t-1} + \epsilon_{y,t},
\end{align}

where the disturbances $\epsilon_{c,t}$ and $\epsilon_{y,t}$ have a jointly normal distribution. Their variances are $\sigma^2_c$ and $\sigma^2_y$ respectively, and the correlation coefficient is denoted $\rho_{c,y}$.

Cointegration implies that the matrix of autoregressive coefficients $\Phi = [\phi_{ij}]$ has one unit root, and one stable root. The equilibrium correction (EqCM) representation is therefore

\begin{align}
  \Delta c_t &= \kappa - \alpha_c[c_{t-1} - \beta y_{t-1}] + \epsilon_{c,t}, \quad 0 \leq \alpha_c < 1, \\
  \Delta y_t &= \varphi + \alpha_y[c_{t-1} - \beta y_{t-1}] + \epsilon_{y,t}, \quad 0 \leq \alpha_y < 1,
\end{align}

where $\beta$ is the cointegration coefficient and $\alpha_c$ and $\alpha_y$ are the adjustment coefficients.

It is useful to reparameterize the system with mean-zero equilibrium correction terms. To achieve that, define $\eta_c = E[\Delta c_t], \eta_y = E[\Delta y_t]$ and $\mu = E[c_t - \beta y_t]$. Consequently the constant terms in (3) and (4) can be expressed as $\kappa = \eta_c + \alpha_c \mu$ and $\varphi = \eta_y - \alpha_y \mu$ respectively, and thus we can rewrite this system into

\begin{align}
  \Delta c_t &= \eta_c - \alpha_c[c_{t-1} - \beta y_{t-1} - \mu] + \epsilon_{c,t}, \quad 0 \leq \alpha_c < 1, \\
  \Delta y_t &= \eta_y + \alpha_y[c_{t-1} - \beta y_{t-1} - \mu] + \epsilon_{y,t}, \quad 0 \leq \alpha_y < 1.
\end{align}

In the case of $\beta = 1$, the savings rate is $I(0)$ and $\mu$ is the long-run mean of the savings rate. Cointegration represents the common ground between the consumption function which assumes a causal link from income to consumption, and the permanent-income/life-cycle theories which imply an Euler-equation for consumption.

2.2 Consumption function restrictions on the VAR

Underlying the consumption function approach is the idea that consumption is equilibrium correcting, i.e., $0 < \alpha_c < 1$. For the coefficient $\alpha_y$ there are two possibilities. First, the case of $0 < \alpha_y < 1$ which is consistent with hours worked etc. being “demand determined” and with $y_t$ adjusting to past disequilibria. In econometric terms there is mutual (Granger) causation between income and consumption, see Engle et al. (1983). The second possibility is that $\alpha_y = 0$, reflecting that income is
“supply-side” determined. In the context of the VAR, the restriction \( \alpha_y = 0 \) implies that income is weakly exogenous with respect to the long run elasticity \( \bar{\beta} \), Johansen (1992). Moreover, with the dynamics restricted to the 1.order case, there is one-way causation from income to consumption, so income is also strongly exogenous.

Interpretation is aided by writing the system (5)-(6) in model form

\[
\begin{align*}
\Delta c_t &= \eta_c + \gamma + \pi \Delta y_t - (\alpha_c + \pi \alpha_y)[c_{t-1} - \beta y_{t-1} - \mu] + \varepsilon_{c,t}, \\
\Delta y_t &= \eta_y + \alpha_y[c_{t-1} - \beta y_{t-1} - \mu] + e_{y,t},
\end{align*}
\]

where (7) is the conditional “consumption function” and (8) is the marginal income equation. From the properties of the normal distribution:

\[
\begin{align*}
\alpha_c &= (1 - \phi_{c\mu}) \\
\beta &= \frac{\phi_{c\mu}}{\alpha_c} \\
\pi &= \rho_{c,y} \frac{\sigma_c}{\sigma_y} \\
\gamma &= -\eta_y \pi, \\
\varepsilon_{c,t} &= e_{c,t} - \pi e_{y,t}.
\end{align*}
\]

Note that along a growth path characterized by \( E[c_{t-1} - \beta y_{t-1} - \mu] = 0 \), the growth rates of \( c_t \) and \( y_t \) are proportional, thus

\[
\eta_c = \beta \eta_y
\]

along a steady state growth path.

In the derivation of the forecast errors in section 3 we concentrate on the case where \( \alpha_y = 0 \). The model form of the system ((7)-(8) simplifies to

\[
\begin{align*}
\Delta c_t &= \eta_c + \gamma + \pi \Delta y_t - \alpha_c[c_{t-1} - \beta y_{t-1} - \mu] + \varepsilon_{c,t}, \\
\Delta y_t &= \eta_y + e_{y,t},
\end{align*}
\]

with \( \eta_y = \varphi \), since there is no equilibrium correction in income.

Equations (7) and (11) are conditional equilibrium correction equations for \( c_t \), see e.g., Hendry (1995a, Chapter 7), Davidson et al. (1978), Hendry and von Ungern-Sternberg (1981). However equation (7) is more general, and equation (11) rests on the property that causation is from income to consumption.

2.3 Euler equation approach

According to the permanent income/life cycle hypothesis the evolution of consumption is shaped by tastes and life cycle needs. Indeed, in the absence of uncertainty, there is no reason for consumption to track income. More generally, with uncertainty about future income, the proposition is that consumption growth \( \Delta c_t \) is not Granger-caused by lagged income levels, hence \( \alpha_c = 0 \) in equation (5). Consumption changes are orthogonal to \( c_{t-1} - \beta y_{t-1} - \mu \), the stationary linear combination of lagged consumption and lagged income (the orthogonality property).
Given $\alpha_c = 0$, cointegration implies that $0 < \alpha_y < 1$. The interpretation for the case of $\beta = 1$, due to Campbell (1987) and Campbell and Schiller (1987), is that growth in disposable income is negatively related to the lagged savings ratio because consumers have superior information about their income prospects. If savings are observed to be increasing “today”, this is because consumers expect income to decline in the future. Hence, after first observing a rise in the savings ratio, in the subsequent periods we will observe the fall in income.

The theoretical prediction that income equilibrium corrects carries over to less stylized situations: First, if a proportion of the consumers are subject to liquidity or lending constraints, we may find that aggregate income is Granger causing aggregate consumption, as in Campbell and Mankiw (1989). Still, as long as the remaining proportion of consumers adjust their consumption to expected permanent income, observed aggregate disposable income is negatively related to the aggregate savings ratio, so we would still find $\alpha_y > 0$. Second, the orthogonality condition may not hold if the measure of consumption expenditure includes purchases of durables, see e.g. Deaton (1992, p.99–103), but the implication that $\alpha_y > 0$ still holds. \(^2\) Finally, the basic implication of $\alpha_y > 0$ is unaffected by modifications of the basic Euler-equation, e.g., non-constant expected future interest rates, Haug (1996), and inclusion of demographic variables.

### 3 Consumption forecasts

In the case where the Euler equation restrictions hold, the best forecast of next period’s consumption is this period’s, and thus, consumption growth is unpredictable. In section 3.1 we ask how consumption forecasts are damaged by adding irrelevant causal variables in the form of a consumption function. Perhaps surprisingly, the answer is “not very much”. Even when parameter changes occur (i.e., regime shifts), the consumption function forecasts errors, though biased (over short forecast horizons), are unlikely to be significantly worse than the consumption function within sample errors. Thus if the Euler equation is indeed the true model, forecast breakdown is a very unlikely event—even in the case where the forecaster uses the wrong (i.e., a consumption function) model.

Section 3.2 then elucidates forecast errors for the alternative state of nature, namely that the consumption function restriction holds. In this case a regime shift does singularly damage the consumption function based forecast and does cause forecast failure. Euler-equation forecasts are also damaged by shifts that occur in the forecast period (i.e., after the forecast is made), but they are robust to shifts that have already occurred prior to the forecast. Robustness is due to the differencing aspect of the Euler equation, and we build on the analysis of the properties of forecasts from “models” that use differenced data by Clements and Hendry (1999a, Ch. 5).

\(^2\)It may even be strengthened by imperfections in the credit marked—credit rationed consumers are likely to cut back on purchases of durables when they anticipate a decline in disposable income.
3.1 Forecasting when the Euler equation restrictions hold

Forecasts for the periods $T+1, T+2, ..., T+H$ are made in period $T$. From the Euler equation, the mean and the variance of the consumption growth forecast errors are

\begin{align}
E[\Delta c_{T+h} - \Delta c_{T+h,EE} \mid \mathcal{I}_T] &= 0, \\
\text{Var}[\Delta c_{T+h} - \Delta c_{T+h,EE} \mid \mathcal{I}_T] &= \sigma_c^2
\end{align}

for $h = 1, 2, ..., H$. The subscript $EE$ is used to denote a Euler equation based forecast, and $\mathcal{I}_T$ denotes the information set that we condition on.

Using (7) and (8), the consumption function (CF) forecast is seen to be based on the following equation

\begin{equation}
\Delta c_t = \eta_c + \gamma + \pi \Delta y_t - \pi \alpha_y [c_{t-1} - \beta y_{t-1} - \mu] + \epsilon_{c,t}, \quad \alpha_y > 0 \tag{15}
\end{equation}

since $\alpha_c = 0$ in the true underlying system, and where $\eta_c = \kappa$. The forecast of income growth is derived from

\begin{equation}
\Delta y_t = \eta_y + \epsilon_{y,t} \tag{16}
\end{equation}

i.e., $\Delta y_{T+h} = \eta_y$, $h = 1, 2, ..., H$. Note that

\[\epsilon_{y,t} = \alpha_y [c_{t-1} - \beta y_{t-1} - \mu] + \epsilon_{y,t},\]

implying that the income forecast errors have a mean of zero even though the implied causality of the system is wrong.

The mean and variance of the 1-step consumption function based forecast (CF) are

\begin{align}
E[\Delta c_{T+1} - \Delta c_{T+1,CF} \mid \mathcal{I}_T] &= \alpha_y \pi [c_T - \beta y_T - \mu], \\
\text{Var}[\Delta c_{T+1} - \Delta c_{T+1,CF} \mid \mathcal{I}_T] &= \sigma_c^2. \tag{18}
\end{align}

Thus, the inclusion of an equilibrium correction term makes the conditional 1-step forecast errors biased because, $c_T - \beta y_T \neq E[c_T - \beta y_T] = \mu$ in general. However, the magnitude of the bias is small unless $c_T - \beta y_T$ is very far from its long run mean, which seems unlikely in a constant parameter world. Formally, the unconditional bias is zero, since taking expectations in (17) gives $\alpha_y \pi E[c_T - \beta y_T - \mu] = 0$.

For a $h$-period ahead CF based forecast we obtain

\begin{equation}
E[\Delta c_{T+h} - \Delta c_{T+h,CF} \mid \mathcal{I}_T] = b(h) + \alpha_y \pi (1 - \alpha_y \pi)^{h-1} [c_T - \beta y_T - \mu] \\
- \alpha_y \pi (h-1)(\beta \eta_y - \kappa). \tag{19}
\end{equation}

The term $b(h)$ is decreasing in $h$.\(^3\) The contribution of the “initial condition” $c_T - \beta y_T - \mu$ to the bias is also diminishing, so for a large $h$ a significant bias must be due to the last term in the expression. It is useful to rewrite (19) as

\begin{align}
E[\Delta c_{T+h} - \Delta c_{T+h,CF} \mid \mathcal{I}_T] &= \alpha_y \pi (1 - \alpha_y \pi)^{h-1} [(c_T - c_T^o) - \beta(y_T - y_T^o)] \\
- \alpha_y \pi (h-1)(\beta \eta_y - \kappa)
\end{align}

\(^3\)For $h = 4$ we obtain $b(4) = (\alpha_y \pi)^2 (3 - \alpha_y \pi)(\beta \eta_y - \kappa)$, and for higher $h$, the exponential is increasing.
for \( h = 1, 2, \ldots, H \). We have dropped the \( b(h) \) term, and

\[
c^0_T = \mu + \beta y^0_T,
\]
denotes the steady-state relationship between consumption and income. Along a steady state path \( \beta \eta_y = \kappa \). So we have that

\[
E[\Delta c_{T+h} - \Delta \hat{c}_{T+h,CF} \mid \mathcal{I}_T] = 0
\]

if consumption and income is growing along a steady-state path.\(^4\)

We have shown that unconditionally and for long-run forecasts, adding irrelevant explanatory variables in the form of a consumption function does not harm CF-based forecast, confirming the relevance of the analysis in Clements and Hendry (1998, Ch. 2.9) for our case. However, this result rests on the assumption that the underlying parameters are constant. Real economies are dynamic and evolving, so parameter constancy is unlikely to hold in practical forecasting situations. Thus it is important to investigate how the CF and EE consumption forecasts are affected by parameter changes.

Consider therefore a regime-shift in the form of a change in the long run equilibrium mean \( \mu \) (the equilibrium savings rate if \( \beta = 1 \)). Following Clements and Hendry (1999a, Ch. 3 and 4), such a “deterministic shift” is the singularly most likely cause of a forecast breakdown, see also Doornik and Hendry (1997). The exact timing of a shift is important. Consider first a shift in \( \mu \) to a new value \( \mu^* \), that occurs after the forecast is made, say in period \( T+1 \). Obviously, the analysis of these before break forecasts errors is identical to the analysis of the constant parameter case.\(^5\) In particular, the CF conditional forecast errors are unaffected, i.e., (17), (18) and (19), and the forecasts also remain unbiased unconditionally (because \( E[c_T - \beta y_T - \mu] = 0 \)). Next, assume that a change occurred before the preparation of the forecasts, say in period \( T \), and that we are studying post break forecast errors. The expressions in equations (17), (18) and (19) again applies, but the CF forecast is now unconditionally biased since, in (17) and (19) \( E[c_T - \beta y_T - \mu] = [\mu^* - \mu] \).

In sum, the Euler equation state of nature is inconsistent with observing consumption function forecast failure when the regime shift occurs in the forecast period. Forecast failure only emerges for the consumption function when the regime shift has occurred prior to the construct of the forecast and thus is contained in the information set, i.e., the implication is that the break should show up in post break CF forecasts.

### 3.2 Forecasting when the consumption function restrictions hold

As explained in section 2, we consider the case of (strong) exogeneity of income, so causality is one-way, from income to consumption, the direct opposite of the Euler equation case. Written in equilibrium correction form, the system becomes (11) and (12), with parameters (9).

\(^4\)\( \beta \eta_y - \kappa = 0 \), implies that \( b(h) = 0 \) for all \( h \).

\(^5\)The before break and post break (below) terminology is adopted from Clements and Hendry (1999a, Ch. 3.5).
The Euler equation (wrongly) imposes \( \alpha_c = 0 \), thus

\[
\Delta c_t = \eta_c + \varepsilon_{c,t},
\]

\[
\Delta y_t = \eta_y + \alpha_y[c_{t-1} - \beta y_{t-1} - \mu] + \varepsilon_{y,t}, \quad 0 \leq \alpha_y < 1
\]

where \( \varepsilon_{c,t} = \varepsilon_{c,t} - \alpha_c[c_{t-1} - \beta y_{t-1} - \mu] \).

Since the consumption function model coincides with the data generating process (DGP), its 1-step forecast

\[
\Delta \hat{c}_{T+1,CF} = \mathbb{E}[\Delta c_{T+1,CF} \mid \mathcal{I}_T] = \eta_c - \alpha_c[c_{t-1} - \beta y_{t-1} - \mu]
\]

is optimal in the sense that no other predictor conditional on information available at time \( T \) has smaller mean-square forecast error (MSFE), see Clements and Hendry (1998, Ch. 2.7.2).

Consider again the case of \( \mu \rightarrow \mu^* \) in period \( T + 1 \), i.e., the first period after the forecast is made. It is easy to show that both sets of forecast are damaged by this shift, and that the unconditional 1-step bias becomes \(-\alpha_c[\mu - \mu^*] \) for both the CF and the EE forecasts. Thus if the consumption function is the true model, we expect failure of both sets of before break forecasts.

Finally, consider the case where the forecast is made after the occurrence of the parameter change \( \mu \rightarrow \mu^* \). In equilibrium correction form, the true system in the forecast period is therefore

\[
\Delta c_{T+h} = \eta_c + \pi \Delta y_{T+h} - \alpha_c[c_{T+h-1} - \beta y_{T+h-1} - \mu^*] + \varepsilon_{c,T+h},
\]

\[
\Delta y_{T+h} = \varphi + \varepsilon_{y,T+h},
\]

for \( h = 0, 1, \ldots , H \). The consumption function forecasts are however derived from (11) and (12), i.e.,

\[
\Delta \hat{c}_{T+h} = \eta_c + \gamma + \pi \Delta y_{T+h} - \alpha_c[\hat{c}_{T+h-1} - \beta \hat{y}_{T+h-1} - \mu],
\]

\[
\Delta \hat{y}_{T+h} = \varphi.
\]

The EE forecasts are again generated from

\[
\Delta c_{T+h} = \eta_c.
\]

The post break forecast biases are

\[
\mathbb{E}[\Delta c_{T+1} - \Delta \hat{c}_{T+1,CF} \mid \mathcal{I}_T] = -\alpha_c[\mu - \mu^*],
\]

\[
\mathbb{E}[c_{T+1} - \hat{c}_{T+1,EE} \mid \mathcal{I}_T] = -\alpha_y \pi [c_T - \beta y_T - \mu^*].
\]

Manifestly, equation (28) shows that the post break CF 1-step forecast is biased. In fact the bias is identical to the before break bias, see Clements and Hendry (1999a, Ch. 3.5) for a generalization to vector equilibrium correction models. The CF forecast does indeed “equilibrium correct”, but unfortunately to the old equilibrium.

The Euler equation consumption forecast will be relatively immune in this case, since by taking expectations in (29) we obtain \( \mathbb{E}[c_T - \beta y_T - \mu^*] = 0 \). This robustness feature of the non-causal EE forecast is shared by all forecasting “model”
Table 1: 1-step forecast biases for CF and EE forecasts of consumption growth when there is a shift in the equilibrium mean, \((\mu \to \mu^*)\).

<table>
<thead>
<tr>
<th>State of nature: EE restrictions hold</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Before break</td>
<td>Post break</td>
</tr>
<tr>
<td>CF</td>
<td>0</td>
<td>(\alpha_y \pi [\mu^* - \mu] )</td>
</tr>
<tr>
<td>EE</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>State of nature: CF restrictions hold</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Before break</td>
<td>Post break</td>
</tr>
<tr>
<td>CF</td>
<td>(-\alpha_c [\mu - \mu^*] )</td>
<td>(-\alpha_c [\mu - \mu^*] )</td>
</tr>
<tr>
<td>EE</td>
<td>(-\alpha_c [\mu - \mu^*] )</td>
<td>0</td>
</tr>
</tbody>
</table>

that are cast in term of differences, see Clements and Hendry (1999a, Ch. 5). The implication for practice is that unless “consumption function” forecasters detect the parameter change and take appropriate action by (manual) intercept correction, they may find themselves losing to Euler-equation forecasters in forecast comparisons.

The expressions for the \(h\)-period post break forecast biases, take the form

\[
\begin{align*}
\text{E}[\Delta c_{T+h} - \Delta \hat{c}_{T+h,CF}] \mid I_T &= -\alpha_c (1 - \alpha_c)^{h-1}[(\mu - \mu^*)], \\
\text{E}[\Delta c_{T+h} - \Delta \hat{c}_{T+h,EE}] \mid I_T &= -b(h) + \alpha_c (h - 1)(\beta \eta_y - \kappa) \\
&\quad -\alpha_c (1 - \alpha_c)^{h-1}[(c_T - c_T^0) - \beta (y_T - y_T^0)]
\end{align*}
\]

for \(h = 1, 2, ..., H\). This shows that the CF-forecast remains biased also for the longer forecast horizons, although the bias dies away eventually. The unconditional CF-bias is seen to share the same property. The EE forecast bias is a linear function of the forecast horizon \(h\), i.e., the term \(\alpha_c (h - 1)(\beta \eta_y - \kappa)\). However, unconditionally, \(c_T = E[c_T] = c_T^0, y_T = E[y_T] = y_T^0\) and \(\beta \eta_y = \kappa\), implying that the unconditional EE forecast is unbiased for all \(h\).

We conclude that in a state of nature where the true underlying model fulfills the consumption functions restrictions, forecast failure is bound to arise if there is a shift in the equilibrium mean. The forecast of the “true model”, is damaged both before break and post break. The non-causal EE forecasts are robust to shifts that have already occurred prior to the preparation of the forecast, and the non-causal “model” will tend to win a forecast competition against a consumption function based on post break forecast errors. Finally, and for reference in the empirical section, Table 1 summarizes the discussion of consumption forecast failure in terms of the biases for the 1-period forecasts.

4 Modelling and forecasting Norwegian private consumption expenditure from 1968 to 1985.

In this section we show that a consumption function encompasses an Euler equation model on a sample that ends in 1984(4). The effects of financial deregulation was
already *en route*, following liberalization of the housing and credit markets earlier in the 1980s. We show how the consumption function loses to an (non causal) Euler equation in a forecast competition over the years 1985-1987. Finally, we report the respecified consumption function, based on a dataset that ends in 1989(4), see Brodin and Nymoen (1992). The results illustrate the relevance of the theory of forecast failure, and that respecification represents a progressive step in the modelling of consumption.

### 4.1 The breakdown in the mid 1980s

Table 2 shows an empirical consumption function model for (log of) total consumption expenditure ($c_t$) and disposable income ($y_t$). The first equation models the quarterly rate of change of consumption as a function of lagged rates of change ($\Delta c_{t-1}$ and $\Delta c_{t-4}$), the current rate of growth in income ($\Delta y_t$) and the lagged consumption to income ratio ($c_t - y_t$). The remaining explanatory variables are three centered seasonal ($CSj, j = 1, 2, 3$), a VAT dummy for 1969(4) and 1970(1), and finally a dummy ($STOP_t$ which is non-zero during a wage-price freeze that occurred in 1978, and is zero elsewhere, see Brodin and Nymoen (1992).

The second equation gives the rate of growth in income as an autoregression, augmented by deterministic terms. From the *Diagnostics* part of the table, note that the two equations imply 14 overidentifying restrictions on the unrestricted reduced form, and that these restrictions are jointly accepted. The omission of $(c - y)_{t-1}$ from the income equation in particular is statistically admissible—the incremental

<table>
<thead>
<tr>
<th>The consumption function</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta c_t = -0.302 \Delta c_{t-1} + 0.227 \Delta c_{t-4} + 0.471 \Delta y_t$</td>
</tr>
<tr>
<td>$-0.128 (c - y)_{t-1} - 0.352 STOP_t + 0.075 VAT_t$</td>
</tr>
<tr>
<td>$-0.13946 CS1_t - 0.088 CS2_t - 0.095 CS3_t$</td>
</tr>
<tr>
<td>$\hat{\sigma} = 15.3%$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>The income equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta y_t = 0.009 - 0.477 \Delta y_{t-1} + 0.311 \Delta y_{t-4}$</td>
</tr>
<tr>
<td>$-0.043 CS1_t - 0.040 CS2_t + 0.026 CS3_t$</td>
</tr>
<tr>
<td>$\hat{\sigma} = 1.60%$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Diagnostics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overidentification $\chi^2(14) = 18.78[0.17]$</td>
</tr>
<tr>
<td>$AR 1 - 5 F(20, 96) \chi^2(96) = 0.78[0.73]$</td>
</tr>
<tr>
<td>Normality $\chi^2(4) = 5.06[0.28]$</td>
</tr>
<tr>
<td>Heteroscedasticity $F(75, 93) = 0.67[0.96]$</td>
</tr>
</tbody>
</table>

FIML estimation. The sample is 1968(2) to 1984(4), 67 observations.
Likelihood-ratio test statistic for this single restrictions is $\chi^2(1) = 1.004[0.3164]$. Thus based on this evidence, causation runs from income to consumption, and not the other way round as implied by Campbell’s hypothesis, see section 2.2 and 2.3. The implication is that the first equation in Table 2 is indeed the empirical counterpart to the consumption function (11), not equation (15) as implied by the Euler equation. The other test statistics include tests of 5th order residual autocorrelation, residual non-normality and heteroscedasticity due to squares of the regressors. These statistics are explained in Doornik and Hendry (1996).\footnote{As indicated in the Table, the normality test is a Chi-square tests, the other tests are F-distributed under their respective null hypotheses.} They give no indication of residual misspecification of the model in Table 2.\footnote{The diagnostic tests for each equation are also insignificant.}

In sum, the Table 2 represents a congruent model on a sample ending in 1984(4). Following the discussion in section 3.2 we know that it will nevertheless forecast badly if there is a change in the equilibrium savings rate.

Table 3 shows the empirical model which is consistent with the Euler restrictions on the income-consumption system. The first equation is the Euler equation for consumption. We include a lagged growth rate of consumption growth, $\Delta c_{t-4}$, that capture significant effects of habit formation. This modification seems inconsequential, as the substance of the theoretical argument is that lagged changes in income are orthogonal to current consumption growth. In this model, the second
Table 3: Pre-break FIML Euler-equation estimates.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Coefficients</th>
<th>Standard Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Euler equation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta c_t = 0.0059 + 0.236 \Delta c_{t-4}$</td>
<td>(0.003)</td>
<td>(0.089)</td>
</tr>
<tr>
<td>$- 0.396 \text{STOP}_t + 0.097 \text{VAT}_t$</td>
<td>(0.102)</td>
<td>(0.016)</td>
</tr>
<tr>
<td>$- 0.184 \text{CS1}_t - 0.034 \text{CS2}_t - 0.070 \text{CS3}_t$</td>
<td>(0.025)</td>
<td>(0.009)</td>
</tr>
<tr>
<td>$\hat{\sigma} = 2.10%$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The savings equation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta y_t = 0.0144 - 0.308 \Delta y_{t-1} + 0.231 \Delta y_{t-4}$</td>
<td>(0.003)</td>
<td>(0.098)</td>
</tr>
<tr>
<td>$+ 0.158 (c - y)_{t-1} + 0.041 \text{VAT}$</td>
<td>(0.068)</td>
<td>(0.013)</td>
</tr>
<tr>
<td>$- 0.049 \text{CS1}_t - 0.023 \text{CS2}_t + 0.037 \text{CS3}_t$</td>
<td>(0.013)</td>
<td>(0.011)</td>
</tr>
<tr>
<td>$\hat{\sigma} = 1.62%$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Diagnostics</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Overidentifying restrictions $\chi^2(15) = 48.40[0.00]$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AR 1 - 5 $F(20, 98) = 1.55[0.08]$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Normality $\chi^2(4) = 7.43[0.11]$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heteroscedasticity $F(75, 96) = 1.15[0.26]$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The sample is 1968(1) to 1984(4), 68 observations.

equation has income growth equilibrium correcting to lagged savings, thus the model corroborates Campbell’s “rainy day” hypothesis. However, the test of the overidentifying restrictions now rejects (overwhelmingly), so the model is not congruent and it does not encompass the unrestricted system.8

Figure 1 shows three sequences of dynamic forecasts from the two estimated models. We show the forecasts of the annual growth rate of consumption, $\Delta c_t$, (D4cp in the graph). The thick line shows the actual development of $\Delta c_t$ over the period 1984(3)-1987(4). The first forecast period is 1985(1) and the dynamic forecasts for the period 1985(1)-1987(4) is shown as a thin line. The bars shows the corresponding 95% prediction intervals. The boxed line shows the forecast for the 8 quarters from 1986(1) to 1987(4), so here the forecasts are conditional on the 1985 data. Finally, the circled line shows the 4 period forecasts 1987(1)-1987(4). To avoid glutting the graphs, we do not report the prediction intervals for the 4- and 8-quarter dynamic forecasts—they will have the same typical magnitude as the first 4 periods (8 periods) of the 12 period forecast.

We see that the 1985(1) forecasts are biased for both the consumption function and the Euler equation. Moreover, the size of the bias is the same for the two set of forecasts, and they are also close for the rest of the forecast period. Given the appearance of $\Delta y_t$ in the consumption function, the CF forecast-errors could be a result of underprediction of income growth, but this is not the case: Forecasting

8Jaeger and Neusser (1987) is an early application that rejects the “rainy day” hypothesis on Austrian data.
Δ_4C_t in 1985(1) conditional on the correct growth in income also results in significant underprediction of consumption. Instead, the 1-step forecasts errors behave exactly as one would expect on the basis of the the algebra in the previous section: Given that there is a shift in the long run savings rate in 1985.1, one expects a bias in both the CF and EE before break forecast errors, cf. table 1, just as shown in figure 1.

The forecasts for 1986(1)-1987(4) can be seen to illustrate a situation where we are conditioning the forecast on a structural break that has occurred prior to the forecast period. We therefore expect the consumption function forecast to remain biased, but to see an element of error-correction in the Euler equation forecast, see equations (28) – (31). Indeed, the figure shows that the forecasts errors for 1986(1) and 1986(2) are much smaller for the Euler equation than for the consumption function. Again, this finding is entirely consistent with the forecasting theory in section 3, see the overview in table 1.

Interestingly, the box-line shows that 1986(3) and 1986(4) are much better forecasted by the consumption function than the algebra in section 3.2 suggests: This is brought out even clearer by the 4-period forecasts from 1987(1)-1987(4) where the consumption function has recovered completely. A possible explanation for this is that the savings rate might have changed back, toward its initial level so that the change in early 1985 was temporary. In turn this suggests that other factors than income might be acting on consumption (and the savings ratio), and that there may be ways of modelling those effects. We consider this possibility in the next section.

The analysis of the mid-1980 forecast failure illustrates that in forecasting, the consumption function and the Euler equation are complementary. Based on the evidence in Table 2 and 3 a modeller may feel confident that the consumption function model is closer to the underlying data generating process. Yet, the model cast in differences produces robust forecasts.

4.2 The reconstructed Norwegian consumption function.

Forecast failure invites modelling, but in itself does not provide any information about the re specification of the model, see Clements and Hendry (1999b) for a discussion. Thus, the response to the forecast failure investigated different routes, including measurement error (emphasizing income), see Moum (1991), Euler equations, Steffensen (1989) and finally model misspecification, see Brodin and Nymoen (1992) and . Among these, only B&N provided a model with constant parameters over the full sample, i.e., both over the pre-break sample and over a sample that contained the post-break period from 1985(1) to 1989(4).

B&N provides a model in which the equilibrium relationship is that the log of consumption (c_t) is determined by the log of disposable income (y_t) and the log of a real household wealth (w_t), made up of the net financial wealth and the value of residential housing. Hence, the implication of B&Ns model is that the forecast failure was due to model misspecification, revealed by changed behaviour of the omitted wealth variable.

In more detail, B&Ns results can be summarized in three points

1. Cointegration. Cointegration analysis of the three variables c (log consumption), y (log disposable income), and w (log net household wealth) establish
that the linear relationship

\[ c_t = \text{constant} + 0.56 y_t + 0.27 w_t, \]

is a cointegrating relationship.

2. **Weak Exogeneity.** Income and wealth were found to be weakly exogenous for the cointegration parameters. Hence, the respecified equilibrium correction model for \( \Delta c_t \) is a consumption function according to the definition in section 2.2, only augmented by wealth as a second conditioning variable.

3. **Invariance.** Estimation of the marginal models for income and wealth showed evidence of structural breaks. The joint occurrence of a stable conditional model (the consumption function) and unstable marginal models for the conditioning variables is evidence of within sample invariance of the coefficients of the conditional model and hence super exogenous conditioning variables (income and wealth). The result of invariance have been corroborated by Jansen and Teräsvirta (1996) using an alternative method based on smooth transition models.

Other researchers that modelled the B&N dataset replicated the long run relationship between \( c_t, y_t \) and \( w_t \), see Banerjee and Hendry (1992) and Franses (1992). Banerjee and Hendry suggested rewriting (32) in stylized form as

\[ (c - y) = \frac{1}{2} (w - y) - 0.2w, \]

in order to highlight the implied non-proportional consumption to income ratio. On the other hand, the short-run dynamics of the alternative models were different from B&N’s equilibrium correction model. For example, Franses’ model was basically static, with dynamics only in the form of a lag polynomial in \( \Delta(c - y)_t \). Irrespective of the merits of the three respecifications, we note that while forecast failure was endemic, several constant models were found on post-break data. The explanation seems to be that the introduction of the wealth variable helped capture the sudden jump in \( c - y \) during 1985. In other words, B&N developed a model of the change in the long-run mean \( \mu \), which was the prime source of forecast failure. Perturbations of the dynamics seem to have been of a much less importance.

5 **Stability of the Brodin-Nymoen model over an extended sample: 1990(1)-1998(4)**

5.1 **Data issues**

The revised and extended data series for consumption, income and wealth are shown in figure 2.\(^9\) A notable feature is that between 1990(1) and 1998(4), wealth first plunged almost down to its 1984-level, before a new steep rise begins in 1993(3). This development is not unlike that of the mid 1980’s, but this time around there

\(^9\)Details on the data are also briefly discussed in the appendix
was no dramatic plunge in the savings rate. So the question is whether the B&N model, which we have seen leaned heavily on the rise in wealth to explain the fall in savings after financial deregulation, has stable coefficients over the extended sample period.

Figure 2: Extended data for real consumption, income and wealth.

5.2 Cointegration

Table 4 reports Johansen tests for cointegration in a VAR model of the three variables $c_t$, $y_t$ and $w_t$, ($n = 3$), with five lags, ($k = 5$), see Johansen (1988), Johansen (1995). The full sample evidence 1968(3)-1998(4) is reported in the table. As in Brodin and Nymoen (1992), we find that the formal test statistics support at most one cointegrating vector, although taken at face value, the evidence is not very strong, cf. the reported $\lambda_{\text{trace}}$ test in table 4 and its small sample corrected counterpart, $\lambda_{\text{trace}, T-nk}$ (Reimers (1992)), in which the test is scaled down with a multiplicative factor $(T-nk)/T$. Strictly, the tests indicate that we cannot reject any of the hypotheses in the sequence of tests. Compared to B&N the long run elasticity of income and wealth are completed recovered from the sample ending in 1989(4), but as we can see from table 5 and figure 3 the income elasticity increase somewhat from 0.57 to 0.65 as we expand the sample and the same time the wealth elasticity decrease from 0.26 to 0.23, cf. figure 3 which shows the overall recursive stability of the two coefficients of the cointegrating vector over the longer period from 1978(2)-1998(4). The overall impression is that the long run relationship has been quite stable over this period in which the Norwegian economy experienced strong cyclical movements in the household savings and wealth to income ratios.
Table 4: Johansen tests for cointegration using revised data for consumption and income, full sample analysis 1968(3)–1998(4).

<table>
<thead>
<tr>
<th>VAR system of order: 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method: Johansen</td>
</tr>
<tr>
<td>Range: 1968(3) - 1998(4)</td>
</tr>
</tbody>
</table>

**Endogenous variables:** c y w
**Deterministic variables:** CPVAT AUDI Const D1 D2 D3
**Restriction type:** 0

<table>
<thead>
<tr>
<th>Eigenvalues</th>
<th>$\lambda_{trace}$</th>
<th>H0</th>
<th>H1</th>
<th>$\lambda_{trace,10}$</th>
<th>$\lambda_{trace}$</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1110</td>
<td></td>
<td>$r = 0$</td>
<td>$r \geq 1$</td>
<td>19.9681</td>
<td>24.1199</td>
<td>29.6800</td>
</tr>
<tr>
<td>0.0652</td>
<td></td>
<td>$r \leq 1$</td>
<td>$r \geq 2$</td>
<td>8.0846</td>
<td>9.7655</td>
<td>15.4100</td>
</tr>
<tr>
<td>0.0125</td>
<td></td>
<td>$r \leq 2$</td>
<td>$r \geq 3$</td>
<td>1.2710</td>
<td>1.5352</td>
<td>3.7620</td>
</tr>
</tbody>
</table>

95% fractiles are from Osterwald-Lenum(1992)

Table 5: Cointegration vectors, $\beta$, estimated on different subsamples, i.e. 1968(3)–1989(4), 1968(3)–1994(4) and 1968(3)–1998(4) - based on updated estimates of other short run parameters

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{c}$ = 0.5658 $y_t$ + 0.2604 $w_t$ + Const</td>
<td>$\beta_{c}$ = 0.5514 $y_t$ + 0.2257 $w_t$ + Const</td>
<td>$\beta_{c}$ = 0.6494 $y_t$ + 0.2255 $w_t$ + Const</td>
</tr>
<tr>
<td>(0.1134)</td>
<td>(0.1735)</td>
<td>(0.1736)</td>
</tr>
<tr>
<td>(0.0571)</td>
<td>(0.0706)</td>
<td>(0.0706)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{c}$ = 0.6302 $y_t$ + 0.2331 $w_t$ + Const</td>
<td>$\beta_{c}$ = 0.1244 $y_t$ + 0.2088 $w_t$ + Const</td>
<td>$\beta_{c}$ = 0.06302 $y_t$ + 0.2331 $w_t$ + Const</td>
</tr>
<tr>
<td>(0.4136)</td>
<td>(0.1057)</td>
<td>(0.3965)</td>
</tr>
<tr>
<td>(0.1568)</td>
<td>(0.1490)</td>
<td>(0.1516)</td>
</tr>
</tbody>
</table>

Multivariate cointegration analysis using av VAR-model with five lags sample.

Table 6: Estimated loading factors, $\alpha$, estimated on different subsamples, i.e. 1968(3)–1989(4), 1968(3)–1994(4) and 1968(3)–1998(4) - based on updated estimates of other short run parameters

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{c}$</td>
<td>- 0.8453</td>
<td>*</td>
<td>- 0.3965</td>
</tr>
<tr>
<td></td>
<td>(0.1604)</td>
<td></td>
<td>(0.1102)</td>
</tr>
<tr>
<td>$\alpha_{y}$</td>
<td>- 0.6302</td>
<td>*</td>
<td>- 0.1057</td>
</tr>
<tr>
<td></td>
<td>(0.2331)</td>
<td></td>
<td>(0.1568)</td>
</tr>
<tr>
<td>$\alpha_{w}$</td>
<td>- 0.1244</td>
<td></td>
<td>- 0.06302</td>
</tr>
<tr>
<td></td>
<td>(0.2088)</td>
<td></td>
<td>(0.1531)</td>
</tr>
</tbody>
</table>
Figure 3: The long run equilibrium relationship for consumption, recursive estimates (1978(2)-1998(4)) of the cointegration parameters $\hat{\beta}_y$ and $\hat{\beta}_w$. Based on full sample estimates of the short run parameters.

Figure 4: Estimated feedback coefficients $\hat{\alpha}_c$, $\hat{\alpha}_y$, $\hat{\alpha}_w$, recursive tests for weak exogeneity (1978(2)-1998(4)) - based on full sample estimates of other short run parameters.
Finally, table 6 shows the corresponding feedback coefficients (loading factors), and figure 4 shows the recursive stability. The reported ±2 standard error bands cover the zero-line for most subsamples, which indicate that income and wealth are weakly exogenous with respect to the cointegrating parameters.

5.3 Consumption, income and wealth

Conditional of the cointegration findings we next model the vector

\[ x_t = (\Delta c_t, \Delta y_t, \Delta w_t, \Delta rph_t)' \]

\( \Delta rph_t \) denotes the growth rate of the real price of residential housing, and is added to the system because of its importance for the valuation of the existing wealth. The system is

\[ x_t = \Gamma(L) \Delta x_{t-1} + \kappa D_t + \alpha EqCM_{t-1} + \varepsilon_t. \]

(33)

\( \Gamma(L) \) is a matrix polynomial. The polynomials for \( \Delta c_{t-1}, \Delta y_{t-1} \) and \( \Delta w_{t-1} \), are of 4. order. For \( \Delta rph_t \), a 2. order polynomial was found to be sufficient. \( D_t \) is a vector of deterministic terms, it includes an intercept, three (centered) seasonals and the dummies \( VATt \) and \( STOPt \) introduced earlier. \( \kappa \) is the corresponding matrix of coefficients. Finally, \( \alpha \) is the vector of equilibrium correction coefficients, and the equilibrium correction mechanism is specified as

\[ EqCM_t = c_t - 0.65y_{t-4} - 0.23w_t - 0.93, \]

with \( y_{t-4} \) rather than \( y_t \), since the implied parameterization of the income part of \( \Gamma(L) \) was easier to interpret. The number 0.93 is the mean of the long-run relationship over the period 1968(3)-1989(4), i.e., the sample used in section 5.2.

Estimation of the system in (33) on a sample from 1968(3) to 1994(4) yielded a residual vector \( \hat{\varepsilon}_t \) without any detectable autocorrelation, non-normality or heteroscedasticity, and we therefore sought to estimate a model that encompasses that system. The results are reported in Table 7. In the Diagnostics part of the table, \( \chi^2(63) = 77.74[0.10] \), shows that the overidentifying restrictions are jointly acceptable, so the VAR in (33) is indeed encompassed by the model.

The consumption function has the same autoregressive structure as on the pre-break data (cf. Table 2), but in addition there is now an effect of the average growth rate in income over four quarters, and the quarterly growth rate in wealth. Averaging real income growth means that the coefficients of \( \Delta y_t, ..., \Delta y_{t-4} \) have been restricted to 0.20. The test statistic for the joint restrictions yields \( \chi^2(14) = 11.95[0.61] \), and the results indicate that households smooth quarterly income growth to extract more permanent income changes.

The income equation is basically autoregressive, and a reparameterization allows it to be written with \( \Delta 4y_t \) on the left-hand side. In addition there are effects of lagged growth in wealth and of the real price of housing. There is no equilibrium correction term which is consistent with the weak exogeneity of income found in section 5.2 and in the B&N study. For wealth, the third equation in the model, the equilibrium correction term turned out to be not significant and was excluded, hence real wealth is basically a differenced-data equation, driven by the growth in
Table 7: Extended consumption function model (CF).

<table>
<thead>
<tr>
<th>Term</th>
<th>Coefficient</th>
<th>Std. Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta c_t$</td>
<td>$-0.2715$</td>
<td>$0.0593$</td>
</tr>
<tr>
<td>$\Delta c_{t-1}$</td>
<td>$0.4016$</td>
<td>$0.0514$</td>
</tr>
<tr>
<td>$\Delta c_{t-4}$</td>
<td>$0.2000$</td>
<td>$-0.0014$</td>
</tr>
<tr>
<td>$\Delta y_t$</td>
<td>$0.0783$</td>
<td>$-0.0091$</td>
</tr>
<tr>
<td>EqCM</td>
<td>$0.0676$</td>
<td>$0.0119$</td>
</tr>
<tr>
<td>$CS_1_t$</td>
<td>$-0.0709$</td>
<td>$-0.0089$</td>
</tr>
<tr>
<td>$CS_2_t$</td>
<td>$0.0401$</td>
<td>$0.0638$</td>
</tr>
<tr>
<td>$CS_3_t$</td>
<td>$0.0367$</td>
<td>$0.0054$</td>
</tr>
<tr>
<td>$VA T_t$</td>
<td>$0.0164$</td>
<td>$0.0031$</td>
</tr>
<tr>
<td>$c_{t-1}$</td>
<td>$-0.3431$</td>
<td>$-0.0054$</td>
</tr>
<tr>
<td>$c_{t-4}$</td>
<td>$-0.2374$</td>
<td>$-0.0014$</td>
</tr>
<tr>
<td>$r_{ph_t}$</td>
<td>$0.0956$</td>
<td>$0.0025$</td>
</tr>
<tr>
<td>$r_{ph_{t-1}}$</td>
<td>$0.0429$</td>
<td>$0.0025$</td>
</tr>
<tr>
<td>$CS_1_t$</td>
<td>$0.0308$</td>
<td>$0.0015$</td>
</tr>
<tr>
<td>$CS_2_t$</td>
<td>$0.0537$</td>
<td>$0.0047$</td>
</tr>
<tr>
<td>$CS_3_t$</td>
<td>$0.0593$</td>
<td>$0.0109$</td>
</tr>
<tr>
<td>$\hat{\sigma}$</td>
<td>$1.53%$</td>
<td></td>
</tr>
</tbody>
</table>

The income equation

<table>
<thead>
<tr>
<th>Term</th>
<th>Coefficient</th>
<th>Std. Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta y_t$</td>
<td>$0.0117$</td>
<td>$0.0028$</td>
</tr>
<tr>
<td>$\Delta y_{t-1}$</td>
<td>$-0.4082$</td>
<td>$0.0739$</td>
</tr>
<tr>
<td>$\Delta y_{t-2}$</td>
<td>$0.2491$</td>
<td>$0.0668$</td>
</tr>
<tr>
<td>$\Delta y_{t-4}$</td>
<td>$0.4755$</td>
<td>$0.0764$</td>
</tr>
<tr>
<td>$\Delta w_{t-1}$</td>
<td>$-0.3431$</td>
<td>$-0.1297$</td>
</tr>
<tr>
<td>$\Delta r_{ph_{t-1}}$</td>
<td>$0.0465$</td>
<td>$0.0170$</td>
</tr>
<tr>
<td>$VA T_t$</td>
<td>$0.0164$</td>
<td>$0.0065$</td>
</tr>
<tr>
<td>$CS_2_t$</td>
<td>$0.0031$</td>
<td>$0.0072$</td>
</tr>
<tr>
<td>$\hat{\sigma}$</td>
<td>$2.24%$</td>
<td></td>
</tr>
</tbody>
</table>

The wealth equation

<table>
<thead>
<tr>
<th>Term</th>
<th>Coefficient</th>
<th>Std. Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta w_t$</td>
<td>$0.0088$</td>
<td>$0.0015$</td>
</tr>
<tr>
<td>$\Delta c_{t-1}$</td>
<td>$0.1510$</td>
<td>$0.0460$</td>
</tr>
<tr>
<td>$\Delta w_{t-3}$</td>
<td>$-0.2374$</td>
<td>$-0.0428$</td>
</tr>
<tr>
<td>$\Delta r_{ph_t}$</td>
<td>$0.0949$</td>
<td>$0.0918$</td>
</tr>
<tr>
<td>$CS_1_t$</td>
<td>$0.0135$</td>
<td>$0.0047$</td>
</tr>
<tr>
<td>$CS_2_t$</td>
<td>$0.0308$</td>
<td>$0.0076$</td>
</tr>
<tr>
<td>$\hat{\sigma}$</td>
<td>$1.47%$</td>
<td></td>
</tr>
</tbody>
</table>

The house price equation

<table>
<thead>
<tr>
<th>Term</th>
<th>Coefficient</th>
<th>Std. Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta r_{ph_t}$</td>
<td>$-0.0007$</td>
<td>$0.00025$</td>
</tr>
<tr>
<td>$\Delta c_{t-4}$</td>
<td>$0.3443$</td>
<td>$0.0047$</td>
</tr>
<tr>
<td>$\Delta r_{ph_{t-4}}$</td>
<td>$-0.00007$</td>
<td>$-0.00025$</td>
</tr>
<tr>
<td>$\Delta r_{ph_{t-1}}$</td>
<td>$0.2473$</td>
<td>$0.0747$</td>
</tr>
<tr>
<td>$CS_1_t$</td>
<td>$0.0429$</td>
<td>$0.0133$</td>
</tr>
<tr>
<td>$CS_2_t$</td>
<td>$0.0537$</td>
<td>$0.0109$</td>
</tr>
<tr>
<td>$CS_3_t$</td>
<td>$0.0593$</td>
<td>$0.0147$</td>
</tr>
<tr>
<td>$\hat{\sigma}$</td>
<td>$2.02%$</td>
<td></td>
</tr>
</tbody>
</table>

Diagnostics

<table>
<thead>
<tr>
<th>Test</th>
<th>Value</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overidentification</td>
<td>$\chi^2(63)$</td>
<td>$77.7415$</td>
</tr>
<tr>
<td>Restrictions</td>
<td>$\chi^2(14)$</td>
<td>$11.9534$</td>
</tr>
<tr>
<td>AR 1 - 5</td>
<td>$F(80, 302)$</td>
<td>$1.2228$</td>
</tr>
<tr>
<td>Normality</td>
<td>$\chi^2(8)$</td>
<td>$9.7725$</td>
</tr>
<tr>
<td>Heteroscedasticity</td>
<td>$F(410, 496)$</td>
<td>$0.7754$</td>
</tr>
</tbody>
</table>

FIML estimation. The sample is 1968(3) to 1994(4), 106 observations.
real house prices along with some effect from lagged consumption growth. The last equation in the table shows that the growth rate of the house price index depends positively on its past values and on wealth and consumption growth.

Given the absence of labour market variables, government transfers and income taxes, the income equation in Table 7 has a low causal content. The same can be said of the other two marginal equations, for example the “housing price” equation does not include any of the variables that have been shown to be important econometric determinants of housing prices, see Eitrheim (1996). As a result, the estimated prediction intervals from our 4-equation model are certainly wider than those arising from a system that also incorporates sector models of income determination and the housing market. At least this must be true for the shorter forecast horizons, since the larger model would condition on a wider set of (relevant) variables. Another implication of the confluent nature of the marginal models is that they are potential sources of parameter non-constancy that may be harmful to the consumption forecasts.10

Figure 5 displays forecasts for the 16-quarters 1995(1)-1998(4). The quarterly growth rate of consumption (Dc in the figure) appears to be predictable, although to quite a large extent this is due to the predictability of the huge seasonal variations in total consumption. The forecasts for the annual growth rate (D4c in the graph)

---

10 The parameter constancy tests based on the 1-step forecasts errors of the estimated model for the period 1995(1)-1998(4), are $F(64, 96) = 1.63(0.015)$ (using $\Omega^2$) and $F(64, 96) = 1.44(0.053)$ (using $V(e)$), Doornik and Hendry (1996).
do not show much departure from the long run mean of 2.7% growth per annum. Hence, in the sense that the conditional mean of the model is almost the same as the sample mean, the annual growth rate of consumption is barely predictable from this model. The forecasts for the equilibrium correction terms (EqCM in the graph) are quite accurate, signifying that there is no serious changes in the long run mean of that relationship over the period 1995(1)-1998(4). The forecasted growth rates of real housing prices (Drph in the graph) are much too low in 1996(1) 1997(1) and 1998(1) and the same pattern in found in the forecasts for wealth (Dw), albeit in somewhat mitigated form. Given the information set used in the model, the forecasts for income growth (Dy) are actually quite good. In line with the statistical test of weak exogeneity, there is no gain in predictability of income from adding the EqCM term in the income equation.

6 Summary and conclusions

Financial deregulation in the mid-1980s led to a strong rise in aggregate consumption expenditure. Existing empirical macroeconometric consumption functions broke down—i.e., they failed in forecasting and failed to explain the data ex post. Predictability was lost, which to many was confirmation of the Lucas-critique of econometric consumption functions and support for an Euler equation specification, i.e., Hall’s original random walk hypothesis, or Campbell’s “saving for a rainy day” version of the permanent income hypothesis. The theoretical section of this paper showed that even in the case where the consumption function is the correct model, it will regularly loose in forecast contests with a random walk equation for consumption. This is a special case of Clements and Hendry’s finding that it cannot be proved that causal models will forecast more accurately than non-causal relationships. We also pointed out that failure of before break forecasts cannot be reconciled with an Euler equation. Thus, the deep-rooted intuition that failure of consumption function based forecasts supports the alternative rational expectations view is after all misleading.

An empirical re-appraisal of the breakdown in Norwegian consumption functions in 1985, showed the relevance of the theory. Over a sample ending in 1984(4), a consumption function encompasses a Campbell-type model. The forecasts of both models are hurt by the occurrences in 1985. However the Euler-equation forecast for 1986, conditional on 1985 is more accurate than the corresponding consumption function forecast. We next showed the results of an update of Brodin and Nymoen’s re-specified consumption function. That model contains a stable cointegrating relationship, despite major changes in the measurement system and nine years of new data, and the causal direction between income and consumption is also unchanged in the new model. Apparently, the Brodin and Nymoen model reinstalls predictability of consumption. However, the gains in predictability is limited by the fact that the wealth variable also has to be forecasted jointly with consumption and income.

Thus it is misleading to measure the gain from the re-specification after the 1985 forecast break only in terms of increased forecast success relative to the random walk. Instead the real merit of the respecified model is that it contains partial structure, and that it provided Norwegian policy agencies with a more realistic
model of the dynamics and causal links between income, consumption and wealth than they had available before the forecast failure in 1985.

A Data definitions

The data for total consumption expenditures, $C_t$, and income, $Y_t$, used in section 5 are collected from the annual and quarterly National accounts of Statistics Norway. They are in fixed 1996-prices. Nominal wealth is defined as

$$NW_t = (L_{t-1} + NL_{t-1} - CR_{t-1} + (PH/PC)_t \cdot nhf_t \cdot K_{t-1},$$

where:

$L_t$ = Household sector liquid assets (money stock and deposits).

$PC_t$ = Price deflator for total consumption expenditures.

$CR_t$ = Liabilities, loans and by banks and other financial institutions.

$K_t$ = Real values of residential housing stock, million 1996 NOK.

$NL_t$ = Non-liquid financial assets.

$nhf_t$ - Fraction of residential housing stock owned by households.

$PH_t$ = Housing price index. (1996=1).

In the model we use real-wealth, deflated by the implicit deflator of consumption (1996=1). Note that in the B&N data $NL_t$ was not included in the wealth measure, and (implicitly) $nhf_t$ was constant and equal to one.

There are two data sets used in the paper, the original data used by Brodin and Nymoen (1992), and a revised and extended version of that data. For a comparison of the two data sets we have matched the means and ranges of the consumption and income series in the overlapping period 1966(1)–1989(4). From figure 6(a) we see that the consumption data have only been subject to minor revisions, and although figure 6(b) reveal somewhat larger discrepancies between the two sets of income data, we conclude that the joint pair of income and consumption series match the old data rather closely. The level of household sector real income has been revised upwards inter alia since household wage income from domestic services production was revised upwards due to the implementation of new SNA. The household sector savings ratio in the early 1990s declined somewhat as a consequence of the SNA-revision. The historical real income series prior to 1991 have been adjusted for this shift in the saving ratio. Finally, figure 6(b) show that the old and new real wealth data match each other over the overlapping sample period.
Figure 6: Old and new data for real consumption, income and wealth - matching mean and ranges from 1966(1) to 1989(4)
References


KEYWORDS:

Consumption functions
Equilibrium correction models
Euler equations
Financial deregulation
Forecast failure
Progressive research strategies
VAR models