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A sequential game analysis

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Optimal bailout during currency and financial crises: A sequential game analysis*

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ABSTRACT
We present a model that illustrates the close relationship between the possibility of a currency crisis and the amount of private-sector debt within a four-stage sequential game framework. The agents are the government and the private sector in a small economy. In the first stage, the government announces that a fixed exchange rate regime will be pursued, and all agents in the economy receive probabilistic information about a future shock that will occur in the last stage. This shock will affect unemployment and net returns on private sector investment. At this stage, the government can already commit to bailing out part of the private sector liabilities of outstanding stock in the event of a bad shock occurring. We also consider the case where the government waits until the last stage to make decisions about the optimal bailout. The private sector in stage 2 forms expectations about the future exchange rate and engages in risky investments. In stage 3, the government faces costs due to expectations of future devaluation and private-sector debt, anticipating the stochastic shock that will occur in stage 4. In this stage, the government may or may not find it optimal to pre-emptively abandon its fixed exchange rate policy. We find that these decisions to commit or not to commit have very important implications for the government’s optimal decision and expectations formation in the private sector. A commitment to bailing out provides a reconciliation of the multiple equilibria that result from self-fulfilling expectations. It is found that commitment to debt bailouts may discourage speculative attacks, in which case it will never be optimal to devalue, even in the case of losses in investment projects. Thus, the government may sometimes avert currency crises by committing to bailing out.

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1. Introduction

In light of the events that have occurred in East Asia and Latin America since 1997, many questions have been raised in about the causes of currency and financial crises. There is already a growing literature on both types of crisis. This study will mainly focus on the following two questions:

i) Can bailouts, a) effectively ameliorate the effects that costly liquidation associated with financial crisis can have on the fundamentals, and b) avoid the currency crises that usually follow a panic in the financial sector.

ii) Can financial and/or currency crises be caused by inconsistent and unsustainable macroeconomic problems, short-term domestic foreign currency debt, and/or self-fulfilling expectations.

With respect to the first question, it has been argued that a safety net may also increase the moral hazard incentives for excessive risk-taking on the part of banks (Schwartz (1998) and Bordo and Schwartz (2000). A regulatory and supervisory system is therefore also necessary to reduce excessive risk-taking in the financial system (Mishkin (1999)). In developed countries, the domestic central bank can control the supply of domestic currency without losing much of its credibility. In addition to a sound national currency, a history of price stability plays a very important role in increasing the incentive for counterparts to enter into contracts of sufficient duration. Under these conditions, it is easier for the financial sector to raise capital with debt denominated in domestic currency. In emerging economies, however, the situation is quite different. These economies are usually characterised by highly variable inflation causing the policymakers to have credibility problems. Thus, an expansionary monetary policy to promote recovery from a financial crisis is likely to lead to higher inflation and depreciation of the national currency. Consequently, the international financial sector has very few incentives to sign contracts denominated in the domestic currency and foreign lending, to a large extent, usually takes the form of foreign currency loans. Thus, central banks in these economies lose control over foreign claims on the economy because they are denominated in a foreign currency. We argue here that in such cases, a lender-of-last-resort, the domestic central bank and/or the IMF together, may be useful for shoring up balance sheets in order to ameliorate the effects of a financial crisis, should it occur. Mishkin (1995) and Freixas, Parigi and Rochet (1998) find that there could be cases where bailing out banks may be efficient. Most people would agree that if a lender-of-last-resort guarantees at least some of the outstanding stock of liabilities (i.e. a bailout), such guarantee should be of a short-run type, and for it to be effective, it has to be implemented very quickly. Less intervention may be required subsequently, once market participants realise that some necessary liquidity has been injected into the system, thus decreasing their uncertainty. One of the main results of this paper is that the government will generally have incentives to pre-commit itself to making such bailouts. Ex-ante commitments to optimal bailouts are “good” in that they facilitate the achievement of a more efficient unique equilibrium (with no devaluation) when the government has a potential problem of credibility regarding its exchange rate policy. These commitments thus serve as a strategic device for the government.

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1 On the second weekend of July 2000, finance ministers from the G-7 agreed a radical plan of reform for the lending facilities of the IMF. This is intended to improve the IMF’s effectiveness in tackling financial crises, while extracting it from long-term lending that could equally well be provided by the private sector. It was agreed to have to cut the cost of the Contingent Credit Line—a short-term facility intended for well-run countries affected by financial contagion that has so far not been used at all. The IMF hopes that more countries will be encouraged to use it to prevent crises developing and spreading. (Financial Times, Monday July 10th, 2000).
With respect to the second question, Allen and Gale (2000) argue that although banking crises typically precede currency crises, the common cause of both is usually a fall in asset values due to a recession or a weak economy. They find it difficult to explain the selection of one exchange rate equilibrium or another by “sunspots”, i.e. random events (shifting beliefs that are self-fulfilling) unrelated to the real economy. Recent empirical work has also attempted to study empirically the most likely causes of currency crises and their relationship with credit problems. Studies by the IMF (1998) and Kaminsky and Reinhart (1999) linking banking and currency crises, find that most currency crises are preceded by a buildup of private-sector debt, usually after a boom. Glick and Hutchison (1999) study the empirical relationship between currency crises and banking crises over the period 1975-97, and find a substantial correlation that is highest for the East Asian economies. Eichengreen and Rose (1998) and Rossi (1999) reach similar conclusions for subsets of developing countries. The consensus appears to be that, at least in some cases, financial crises may have “caused” exchange rate crises; in many cases, however, the twin phenomena are likely to be symptoms of underlying weaknesses of the economy, which may be manifested in various ways.2

The purpose of this paper is to study the interrelationship between exchange rate crises and debt crises. The model here shows that crises may be both belief-driven and fundamentals-based attacks. It is related to that of Mundaca and Strand (1999) which models a self-fulfilling exchange rate crisis. This paper is extended to include debt crises and the possibility of bailing out part of the private sector’s debt. It will be shown that ex-ante commitment to bailing out debt can prevent the multiple equilibria that result from changes in market beliefs. It is then possible to obtain a unique equilibrium outcome, with no currency crisis, even if the private sector ends up with investment losses.

The model is presented as a four-stage sequential game in which the players are the government and the private sector. Information about the probabilistic distribution of a forthcoming (good or bad) shock is given in the first stage. This shock will be realised in the fourth stage and will affect private-sector debt and unemployment. In this first stage, the government announces that it will pursue a fixed exchange rate regime and it may or may not already at this stage commit to an optimal bailout only if a bad shock occurs.3 It will also incur certain (fixed, constant and not further explained) costs when the fixed exchange rate regime is abandoned. In the second stage, private-sector agents appear as active traders in the exchange market, and may in addition undertake risky investment activities. The private sector’s debt is denominated in foreign currency and produces a good for export in the international market. Thus, revenues are obtained in foreign currency. In stage 3, the government may react by either retaining or abandoning the peg. It should be noted that defending the exchange rate regime against attack during this stage implies costs in the interim period for the government and the economy in the form of adverse shifts in the economic fundamentals. The latter is a result, not of a shock, but of the government’s policy to defend the fixed exchange rate regime against a speculative attack at this stage. In stage 3, the regime may be abandoned after the onset of the speculative attack, before it does further significant damage to the economy. This outcome, where a self-fulfilling speculative attack leads necessarily to devaluation, resembles the Krugman (1979) first-generation model, where a deterioration of the fundamentals (in addition to the market’s anticipation of a financial crisis) plays an important role in triggering crises. In the final stage, 4, the shock is revealed and the government decides on two things: the optimal bailout, if it has not committed already

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2 See also Miller (1996), who considers the opposite type of causation, from a currency attack to a banking crisis.

3 This policy does not need to be interpreted as strictly as a fixed exchange rate regime, but it can be more realistically thought of as having an exchange rate target.
in stage 1; and whether or not to abandon the fixed-rate regime (given that the fixed rate has
been retained up to this stage). Decisions at this stage depend on the state of the economy.

We show that when no commitment has been made to a specific bailout, the model may
yield multiple (2 or 3) equilibria. One of the equilibria involves always retaining the fixed
peg, even in the event of a bad shock and cumulative investment losses when there are no
speculative attacks. Another equilibrium involves abandoning the peg only in the worst state,
and the last one involves always abandoning the peg. The last two equilibria may occur when
there are speculative attacks. The last equilibrium is most likely to occur when the prior
probability of a bad shock to debt and unemployment is sufficiently high and/or if shocks are
quite serious when they do occur. Thus, a speculative attack is always triggered in stage 3.

It turns out, however, that a commitment to a bailout (only if a bad shock occurs) in stage
1, expressly announced before the market forms expectations, has very important implications
for the outcome of this sequential game. Specifically, we show that it is possible to obtain a
unique equilibrium without devaluation, in spite of the existence of cumulative investment
losses. Committing to a relatively bailout larger than the non-committed one (as it turns out to
be optimal to do) guarantees a less drastic impact on the economic fundamentals by
outstanding debt. The private sector takes this into account when forming expectations:
Commitment to a larger bailout will cause private sector debt to have more limited adverse
effects on the fundamentals. Therefore, devaluation is regarded by the private sector as less
likely to take place.

Thus, ex-ante commitments to optimal bailouts serve as a strategic device for the
government since they facilitate the implementation of a more efficient unique equilibrium
(without devaluation) when the government has a potential problem of credibility for its
exchange rate policy. Such an equilibrium is not a sunspot equilibrium, like the three multiple
equilibria mentioned above, when there was no ex-ante commitment to bailing out.

The paper is organised as follows: The next section highlights some of the many
contributions to the literature on currency and financial crises. Section 3 presents an overview
of the stages of the sequential game; section 4 presents the government’s problem, while
section 5 presents the representative investor’s problem. Section 6 aims to show the optimal
solutions in stage 4, both with and without speculative attacks and for both favourable and
unfavourable state of the economy. Section 7 shows the possible optimal decisions that can
be taken in stage 3, while section 8 presents the government’s problem when it decides to
commit to a bailout at stage 1. Finally, section 9 presents a conclusion.

2. Background to the literature

As noted, this paper proposes a new theoretical approach to several different strands of
literature, in particular to the “first-” and “second-generation” models of currency crises. It
also presents certain characteristics of both models. Henderson and Salant (1978), Krugman
(1979) and Flood and Garber (1984) represent the “first-generation” models, in which a
devaluation entails no reputational cost but a currency crisis arises as a necessary consequence
of adverse fundamentals. The model also presents an “endogenous-policy” model of currency
crises such as the ones pioneered by Obstfeld (1986, 1994, 1996), with additional
contributions from Bensaid and Jeanne (1993), Ozkan and Sutherland (1994), Davies and
Vines (1995). The government rationally chooses – on the basis of their assessment of costs
and benefits in terms of social welfare - whether or not to maintain a fixed exchange rate
regime. A crisis is driven by self-validating shifts in expectations where multiple equilibria
are possible.

There is one important feature of the model in this paper that makes it different from the
“second-generation” models. In these “second-generation” models, private sector expectations
are assumed to be formed at the "ex ante" stage, while the government's decision to retain or abandon the fixed exchange rate regime is made "ex post", after the realisation of some stochastic variable. Our approach, in contrast to these models, is one in which the government's own possible actions at the "ex ante" stage are also considered, which they ought to be if "ex ante" and "ex post" are separated in real time.

Morris and Shin (1998 a,b) have extended the basic results of Obstfeld's (1995) model to the case with less than common beliefs about the actual game played. By assuming that the agents in the economy may not have common knowledge of the underlying fundamentals, they show that such uncertainty about the beliefs of others regarding the fundamentals may yield a single course of action leading to uniqueness of equilibrium. Extensions of the present model incorporating uncertainty about beliefs should, however, be pursued in future work.

A growing literature has already attempted to explain the ERM 1992 crisis, the 1994 crisis in Mexico, or the recent (1997) financial and currency crisis in Southeast Asia, and analyses such phenomena by considering the simultaneous modelling of financial and currency crisis. Chang and Velasco (1998) consider the interaction between bank fragility, exchange rate and monetary regimes on the basis of the Diamond and Dybvig (1983) model of bank runs. They find that different exchange rate and monetary regimes induce different real consumption allocations and imply different degrees of financial fragility. They find that there may be a close relationship between financial and currency crises, in that maintaining a fixed exchange rate peg and stabilizing the banking sector may become mutually incompatible objectives. In their set-up, there are multiple equilibria, with the crisis brought on by a pure shift in expectations. Allen and Gale (2000) take a step further from the approach of Chang and Velasco (1998) and argue that it is problematic to obtain multiple equilibria because the selection between the good and the bad equilibrium is not modelled. They consider both the cases where countries can issue debt denominated in their own domestic currency (e.g. advanced industrial economies) and where they ought to issue bonds denominated in the foreign reserve currency (e.g. emerging economies). The latter occurs because the country lacks financial discipline and the lenders are then not willing to buy bonds denominated in the domestic currency. They show that when there is a combination of an appropriate exchange rate policy and borrowing and lending by banks in the international capital market, there will be optimal risk sharing, and inefficient liquidation of investment leading to bankruptcy can be prevented as well. However, this will be possible if and only if the debt is denominated in the domestic currency. In emerging economies, the authorities cannot adjust foreign claims on the domestic economy because the debt is in foreign currency. It is here that an international institution can play an important role; by providing liquidity to prevent inefficient liquidation of banks. The present paper does not consider the issue of optimal risk sharing, but suggests that bailouts provided by the government (in a industrialised economy) or a lender of last resort such as the IMF (in a emerging economy), could provide the necessary liquidity and prevent insolvency. This is important in a world where there are strong financial links between countries but each individual country has very few incentives to provide itself with liquidity, as Allen and Gale (2000) also suggest.

Burnside et al. (2000) also study the connection between banking crises and currency crises. In their study, government guarantees to repay bank’s foreign loans are contingent on devaluation occurring. This causes the investors to expose themselves to exchange rate risk and to declare insolvency when devaluation occurs. Such a guarantee scheme introduces the possibility of self-fulfilling currency crises, because the private sector is given incentives to expect devaluation, given that the government will finance banks’ insolvency with bailouts. By bailing banks out, the government validates the expectations of the private sector. Chari

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4 An early example of contribution to this literature was Velasco (1987).
and Kehoe (2000) take another approach by modelling investors as having herd-like behaviour. In their paper, investors have to choose between investing in a risky project in the emerging economy or in a safe project in their domestic economy. In each period a signal about the profitability of the risky project reaches the economy and is privately observed by one of the investors. Investors observe the aggregate amount of investment in each period and optimally decide whether to invest or to wait for more information. Waiting longer is costly. If the signals lead investors to be sufficiently optimistic, they choose to forgo the opportunity to acquire information and they all immediately invest in the emerging economy. If investors become sufficiently pessimistic they all invest in their home economy and capital flows to the emerging economy dry up completely. This is called herd behaviour.

Aghion et al. (2000) present a model that analyses what they define as “triple” crises, which involve currency, banking and output crises where multiple equilibria in the foreign exchange market are possible. In their dynamic model, prices are rigid in the short run and currency depreciation leads to an increase in the foreign currency debt repayment obligations of the firms that will consequently have reduced profits. Lower profits will in turn mean lower net worth, leading to less investment and lower output in the next period.

To our knowledge, this paper presents the first sequential-game model integrating the issues of exchange rate crisis and financial crisis. As such, it represents an advance, but we nevertheless recognise some weaknesses in the model that should be remedied in future work. First and foremost perhaps, the concept of private-sector debt is introduced in a somewhat rudimentary manner, simply as a factor contributing to a worsening of the unemployment problem. In particular, we do not consider the actual process of debt formation in the private sector. Second, the outstanding debt level is only considered (still in a rudimentary way) endogenous when we analyse the case in which the government wishes to have an ex-ante commitment to debt bailouts. Thirdly, we have no theory as to how and why speculators decide to attack the domestic currency. Last, we assume that the cost to the government of leaving the fixed-rate policy is a constant.

3. Stages of the sequential-game theoretical model

The model deals with a sequential game that consists of four stages and is represented by figure 1.

Stage 1
All the agents in the economy receive probabilistic information about a future shock that will occur in the last stage, affecting unemployment and the net returns on investments made by the private sector. There will be a bad shock, \( s_1 \), or a good shock, \( s_2 \), which will occur with probability \( \pi \) and \( (1-\pi) \), respectively. We assume that \( s_2 \) takes the value of zero. That is, if \( s_2 \) occurs, this shock will have no effect on the economic fundamentals. At this stage, the government announces that it will be pursuing a fixed exchange rate regime and may or may not commit to bailing out part of the outstanding debt of the private sector.
Stage 1: Government

Maintains regime
May commit to a bailout
(1)
R

Stage 2: Public forms expectations
Makes (risky) investments

No speculation
(2)
Speculation
(3)

Stage 3: Government

Abandons regime
(4)
Q
Maintains regime
(5)
R
Game ends
Abandons regime
(6)
Q
Maintains regime
(7)
R
Game ends

Stage 4: Government selects strategies

(Q,Q) (8)
(R,Q) (9)
(Q,R) (10)
(R,R) (11)
(Q,Q) (12)
(R,Q) (13)
(Q,R) (14)
(R,R) (15)

Q represents a decision to abandon the regime; R represents a decision to maintain the regime. (m,n), m=R or Q if shock of type one, s₁, occurs; and n=R or Q if shock of type two, s₂, occurs.
Stage 3
Only the government moves. It defends the fixed exchange regime against any speculative attack, or it may give up this regime already at this stage. We find that when there is no speculative attack, the equilibrium strategy of the government involves retaining the fixed exchange rate regime. However, as soon as speculative attacks start, the government incurs certain costs that may lead it to abandon the regime already at this stage. These costs take the form of first, expected adverse shifts in unemployment because of changes in the interest rate. The government uses interest rates to defend the exchange rate regime when there are speculative attacks. The other additional costs are the expected loss of credibility (given that devaluation may occur), and the expected net loss of investment, which could always worsen in the event of devaluation and an adverse shock. If the exchange rate is abandoned at some point in this stage, the government will still face these costs until it decides to abandon the regime during this stage.

Stage 4
A stochastic shock, either $s_1$ or $s_2$, occurs and affects employment and the net returns on investment. Also at this stage, the government decides on the value of the exchange rate and the proportion of the private liabilities of outstanding stock that will be bailed out if $s_1$ occurs and no ex-ante commitment to bailing out has been made in stage 1.5

Figure 1 is again the graphic representation of this sequential game and illustrates the basic decision structure with respect to the exchange rate. However, as noted, there are other decisions about investments and possible bailouts. In the case of government commitment to bailouts, such commitment enters the picture in stage 1. Private-sector investment decisions enter the picture in stage 2. Non-committed government debt bailouts enter the picture in stage 4.

A few additional introductory remarks are in order. First, we assume that whenever the government abandons the fixed exchange rate, we say that the game ”ends”; this applies to the game of setting the exchange rate, which is left entirely to competitive market forces from then on. Even in such cases, however, the government may still need to make decisions about bailouts in stage 4. We will come back to this below. Secondly, in stage 4 there are 8 possible end nodes for the game (provided that the game progresses to that stage). Few of these can be reached. We note that a rational expectations equilibrium can be compatible only with end nodes (11) (corresponding to maintaining the fixed rate in both states) and (14) (giving up the fixed rate in state 1, but not in state 2). Thirdly, the structure of payoffs in the game is such that end node (12) can also be counted out. The reason is that if the strategy combination $(Q,Q)$ were relevant in stage 4, the government would always anticipate this and quit preemptively already in stage 3, thus avoiding the costs of a speculative attack in stage 4 (i.e., one actually ends up in node 6). We will in the following disregard possible cases with the strategy combination $(Q,Q)$ and assume that the fixed rate will never be abandoned in stage 4 when a ”good” shock occurs in that stage.

4. The government’s problem

We assume that the government does not like unemployment, high bailouts and outstanding debt, and that it will try to minimise them, and thereby its total loss function. The
The government’s expected loss function when the fixed exchange rate is retained until stage 3 is the following:

$$V_j(s_j, Ex, x(s_j), \phi) = a_1(u_3 - u_n)^2 + a_2E(u_4 - u_n)^2 - E\phi(x(s_j)/\pi)(R_1(K) - K) + EC$$  \hspace{1cm} (1)

The loss function in stage 4 is:

$$V_4(s_j, Ex, x(s_j), \phi) = a_2(u_4 - u_n)^2 - \phi(x(s_j)/\pi)(R_1(K) - K) + \lambda C$$  \hspace{1cm} (2)

All parameters are greater than zero.

$E$ represents the conditional expectations operator.

$C$ represents the costs of abandoning the fixed exchange regime, for example through loss of credibility, while $\lambda$ is a dummy variable that takes the value of one when the fixed-rate policy is abandoned, and zero otherwise.

$u_j$ is the actual rate of unemployment in stage $j$ ($j=3$ or 4).

$u_n$ is its natural rate level.

$x$ is the (log) exchange rate (number of domestic currency units per unit of foreign currency).

$\pi$ is the (log) fixed exchange rate level and takes the value of one. We say that the fixed exchange rate policy is maintained as long as the government keeps $x(s_j) = \pi$.

$R_i(K)$ is the private sector’s revenue for investing $K$ if a shock of type $i$ ($i=1,2$) occurs.

$\phi$ is the fraction of the outstanding stock of liabilities that the government will bail out, and takes values between 0 and 1. In state 1, there will be investment losses such as $$(x(s_1)/\pi)(R_1(K) - K)<0.$$ In state 2, there will be no investment losses as $$(x(s_2)/\pi)(R_2(K) - K)>0$$, therefore no bailout is necessary. The absolute value of $\phi(x(s_1)/\pi)(R_1(K) - K)$ is the total bailout in state 1 while $(1-\phi)(x(s_1)/\pi)(R_1(K) - K)$ becomes the net loss for the private sector.

We assume that the unemployment rate in state $i$ in stage 4 is determined as follows:

$$u_4(s_j) \equiv u(x(s_j)) = -\alpha_1[x(s_j) - \pi] + u_n + \alpha_2r + s_j - \alpha_3[(1-\phi)(x(s_1)/\pi)(R_1(K) - K)];$$  \hspace{1cm} (3)

In (3), the first term implies that currency devaluation ex-post in stage 4 reduces unemployment. Private-sector debt problems also have consequences for employment in state 1 because debts lead to insolvency and thus loss of jobs. Moreover, the government may use high interest rates to stave off a speculative attack in the foreign exchange market, but increases in interest rates may slow down economic activity and therefore yield higher unemployment rates.

The following relationship is assumed:

$$r = g(Ex - \pi);$$  \hspace{1cm} (4)

where $g$ is a positive constant. $Ex - \pi$ represents the magnitude of the speculative attack that the government may try to fight against in stage 3, in stage 4 or in both stages by increasing interest rates. If $Ex > \pi$, $g$ will only take values greater than zero. Otherwise it equals zero.

5. The investors’ problem
The decisions of investors play an important role in the model only when the government decides to make an ex-ante commitment to bailout at stage one, that is, before the market forms expectations. As explained above, it is basically for the purpose of influencing these expectations that the government may find it worthwhile to make commitments on bailouts. Note that at the time (stage 2) when the private sector decides whether to speculate or not, it also decides on the level of risky investment it will engage in. Therefore, influencing expectations also implies influencing the level of risky investment that the private sector will make.

As already defined, $K$ denotes private-sector investment and $R_i(K)$ the gross return on these investments, in the final stage, 4, when returns are realised, and where $i$ denotes the state of the economy ($i=1, 2$). We make the following assumptions about the gross return functions:

$R_1(K) < K; R_1'(K)>0; R_1''(K)<0$ in the bad state 1, when $s_1$ occurs

$R_2(K) \geq K; R_2'(K)>0; R_2''(K)<0$ in the good state 2, when $s_2$ occurs

The decision problem of investors is to maximise expected net return with respect to the amount invested, taking into consideration that a fraction $\phi$ of the net losses will be bailed out by the government in state 1. The net return function, $NR$, can be written as follows:

$$E[NR] = \pi (1-\phi)(x(s_1)/x) [R_1(K) - K] + (1-\pi)(x(s_2)/x) [R_2(K)-K], \quad (5)$$

Maximising (5) with respect to $K$ yields the following first-order condition:

$$\frac{(R_1'(K)-1)}{(R_2'(K)-1)} = -\frac{(1-\pi)x(s_1)}{\pi(1-\phi)x(s_2)} \quad (6)$$

In (6), the left-hand side is the ratio of the difference between the marginal return on investment $K$ and the marginal cost of investment $K$ in the bad state to the same difference, but in the good state. For (6) to hold true, both the absolute and the marginal return on a given investment should be greater in state 2 than in state 1. Moreover, the marginal return must be smaller in state 1 and greater in state 2 than the marginal cost in each state at an optimal solution for private investors. $K$ will depend on the exchange rates and the bailout. We derive the following partial derivatives:

$$\frac{\partial K}{\partial \phi} = -\pi(1-R_1'(K)) \left[ x_1 - \frac{dx_1}{d\phi} (1-\phi) \right] > 0; \text{iff} \quad (7)$$

$$\frac{\partial K}{\partial x_1} = \frac{\pi(1-R_1'(K)) \left[ (1-\phi) - x_1 \frac{d\phi}{dx_1} \right]}{(1-\pi)R_2''(K) + \pi(1-\phi)R_1''(K)} < 0; \text{iff} \quad (8)$$
\[
(1 - \phi) - x_1 \frac{d\phi}{dx_1} > 0
\]

From (7), the anticipation of a greater fraction of bailouts, \( \phi \), in the "bad" future state (1), all else being equal, thus leads investors to invest more, and thus incur more debt, in stage 2. From (8), an increase in \( x_1 \), i.e. a stage 4 devaluation which is rationally anticipated by investors in stage 2, makes investors incur less debt. (7) indicates that the private sector would have a greater incentive to prefer high levels of risky investments, such as \( K \). Thus, the possibility of a safety net evidently causes a moral hazard problem. However, since the government will only provide a bailout in the case that a bad shock occurs, this may cause investors to choose a lower \( K \) because of the uncertainty as to what stage of the economy will prevail in stage 4. Note, however, that when depreciation is expected, there will be less risky investments. Moreover, (3) shows that bailouts reduce unemployment ex-post in stage 4, which is a potentially beneficial effect. We note that we are not modelling explicitly how the amount of investment is affected by \( \phi \) through expectations concerning the exchange rate. Nor are we modelling how different types of investment may be chosen, e.g. in terms of their riskiness, in response, for example, to \( \phi \).

6. Stage 4

6.1 There are no speculative attacks \( (Ex = \bar{x}) \) and the private sector chooses a certain level of risky investment, say \( \bar{K} \). The government makes no commitment to a specific bailout before stage 4.

**Proposition 1:** If there are no devaluation expectations, there will not be devaluation even if a “bad” shock occurs affecting the net returns. There will, however, be an unemployment gap because of the shock and the outstanding debt. The size of the unemployment gap is influenced by the bailout from the government or lender of last resort. The optimal proportion of debt that will be bailed out is:

\[
\bar{\phi} = 1 - \frac{2a_2s_1\alpha_3 - 1}{2a_2\alpha_3^2(R_1(\bar{K}) - \bar{K})}
\]

**Proof:** Note first that there will not be any bailout if:

i) \( 2a_2s_1\alpha_3 > 1 \); or

ii) \( (R_1 - \bar{K}) \geq \frac{2a_2s_1\alpha_3 - 1}{2a_2\alpha_3^2} \)

(9) is obtained from (2) and (3). The government then minimises the following loss function to find the optimal bailout:

\[
V_4(s_1, Ex = \bar{x}, x(s_1) = \bar{x}, \bar{\phi}, \bar{K}) = a_2[s_1 - \alpha_3(1 - \bar{\phi})(R_1(\bar{K}) - \bar{K})] - \bar{\phi}(R_1(\bar{K}) - \bar{K})
\]

Note that if the government decides instead to devalue in state 1 (say sets an exchange rate equal to \( \bar{x}(s_1) > \bar{x} \)), it needs to take as given \( \bar{K} \). This level of \( \bar{K} \) has been chosen when the private sector decided not to speculate. It is necessary, however, to choose optimally the
proportion of debt that will be bailed out. An optimal bailout ($\hat{\phi}$) is obtained by minimising
the following loss function with respect to $\phi$:

$$V_{4}(s_1, \bar{x}, \hat{x}(s_1) > \bar{x}, \hat{K}, C) = a_2[-\alpha_3(\hat{x}(s_1) - \bar{x}) + s_1 - \alpha_3(1-\hat{\phi})(\hat{x}(s_1)/\bar{x})(R_1(\hat{K}) - \tilde{K}))^2 - \hat{\phi}(\hat{x}(s_1)/\bar{x})(R_1(\hat{K}) - \tilde{K}) + C$$

(11)

The optimal $\phi$ will be:

$$\hat{\phi} = 1 - \frac{2a_2\alpha_3s_1 - 1 - [2a_2\alpha_3\alpha_1(\hat{x}(s_1) - \bar{x})]}{2a_2\alpha_3^2(\hat{x}(s_1)/\bar{x})(R_1(\hat{K}) - \tilde{K})}$$

(12)

The optimal devaluation $\hat{x}(s_1)$ will be:

$$\hat{x}(s_1) = \frac{2a_2\alpha_3^2\bar{x} + 2a_2\alpha_3\bar{x}(1-\hat{\phi})(R_1(\hat{K}) - \tilde{K}) + 2a_2s_1\alpha_3(1-\hat{\phi})(R_1(\hat{K}) - \tilde{K})}{2a_2[-\alpha_1 - (1-\hat{\phi})(R_1(\hat{K}) - \tilde{K})]^2} +$$

$$\frac{\phi(R_1(\hat{K}) - \tilde{K}) + 2a_2s_1\alpha_3}{2a_2[-\alpha_1 - (1-\hat{\phi})(R_1(\hat{K}) - \tilde{K})]^2}$$

(13)

Note also here that there will not be a bailout if:

i) $2a_2\alpha_3s_1 > 2a_2\alpha_3\alpha_1(\hat{x}(s_1) - \bar{x}) + 1$; or

ii) $(R_1(\hat{K}) - \tilde{K}) \geq \frac{2a_2\alpha_3s_1 - 2a_2\alpha_3\alpha_1(\hat{x}(s_1) - \bar{x}) + 1}{2a_2\alpha_3^2(\hat{x}(s_1)/\bar{x})}$

The comparison of the loss functions (10) and (11) indicates that the fixed rate should never
be abandoned in stage 4 when there are not speculative attacks, specially if the loss of
credibility, C, is sufficiently high. This should be so even if a "bad" shock occurs and there
are investment losses.

**Proposition 2** When there are no speculative attacks, the optimal bailout that the government
will give is larger when it decides not to devalue (even if a bad shock occurs) than the optimal
one when there is devaluation (and a bad shock occurs).

**Proof**: This can be seen by comparing (9) and (12). An intuitive explanation for this is that
unemployment, as assumed here, increases with the “bad” shock and outstanding debt (after a
bailout is received). However, devaluation alone also decreases unemployment. It then
becomes more likely that a larger bailout will be necessary when there is no devaluation than
when there is one, in order to mitigate the size of the outstanding debt and the “bad” shock on
unemployment.

**Proposition 3**: If there are no devaluation expectations and a “good” shock occurs, there
will be no devaluation and no unemployment gap.
Proof: By assumption, the shock and the outstanding debt are zero, therefore there will be no unemployment gap and no bailout will be then necessary. When \( x(s_2) = \bar{x} \), the loss associated with maintaining a fixed exchange rate is zero:

\[
V_4(s_2, \text{Ex} = \bar{x}, x(s_2) = \bar{x}) = 0, \tag{14}
\]

while the devaluation losses in state 2 will be \( C \):

\[
V_4(s_2, \text{Ex} = \bar{x}, x(s_2) > \bar{x}) = C. \tag{15}
\]

The government loss as a result of devaluing will be larger than if it does not do so.

Proposition 4: A rational expectations equilibrium can only be compatible with the end node (11), in figure 1. This node corresponds to the decision of maintaining the fixed exchange rate when either shock takes place. Node (11) implies that:

\[
x(s_1) = \bar{x};
\]

\[
x(s_2) = \bar{x}
\]

Proof: The structure of payoffs in the game is such that end nodes (8) and (9) in figure 1 must be counted out. The reason is that if the government is certain to abandon the regime when a “good” shock occurs, the private sector will always anticipate this and will always speculate. See also proposition 3. Because of proposition 1, node (10) will never reach either.

6.2 There are speculative attacks \( \text{Ex} > \bar{x} \), and the private sector chooses a certain level of risky investment, say \( \hat{K} \). The government makes no commitment to a specific bailout before stage 4.

Proposition 5: The amount of risky investment when there are no speculative attacks is likely to be larger than the amount of risky investment when there are speculative attacks (\( \tilde{K} > \hat{K} \)).

Proof: We have shown above that devaluation causes a decrease in risky investment (see equation (8)).

Proposition 6: If there are devaluation expectation, there will be devaluation (\( \hat{x}(s_1) > \bar{x} \)) when a “bad” shock occurs affecting the exchange rate and net returns. However, there will be some unemployment gap because of the shock and the outstanding debt. Here, too unemployment may decrease in proportion to the bailout given by the government or lender of last resort. The optimal bailout will be:

\[
\hat{v} = 1 - \frac{2a_2\alpha_3\alpha_1 - [2a_2\alpha_3\alpha_1(\hat{x}(s_1) - \bar{x}) + 1] + 2a_2\alpha_3\alpha_3\pi(\hat{x}(s_1) - \bar{x})}{2a_2\alpha_3^2(\hat{x}(s_1) - \bar{x})(R_1(\hat{K} - \tilde{K}))} \tag{16a}
\]

or

\[
\hat{v} = 1 - \frac{2a_2\alpha_3\alpha_1 - 1 - 2a_2\alpha_3(\hat{x}(s_1) - \bar{x})(\alpha_1 - \alpha_2\pi)}{2a_2\alpha_3^2(\hat{x}(s_1) - \bar{x})(R_1(\hat{K} - \tilde{K}))} \tag{16b}
\]

The optimal \( \hat{x}(s_1) \) will be:
\[
\hat{x}(s_1) = \frac{2a_2\alpha_3^2\bar{x} + 2a_2\alpha_3\bar{x}(1 - \hat{\phi})(R_i(\hat{K}) - \hat{K})(\alpha_i - \alpha_2 g\pi) + 2a_2s_1\alpha_3(1 - \hat{\phi})(R_i(\hat{K}) - \hat{K})}{2a_2[-\alpha_i - \alpha_3(1 - \hat{\phi})(R_i(\hat{K}) - \hat{K}) + \alpha_2 g\pi]^2} 
\]

(17)

\[
\frac{2a_2s_1\alpha_3g\pi(2\alpha_i - \alpha_2 gp) - 2a_2s_1(\alpha_i - \alpha_2 g\pi) - \phi(R_i(\hat{K}) - \hat{K})}{2a_2[-\alpha_i - \alpha_3(1 - \phi)(R_i(\hat{K}) - \hat{K}) + \alpha_2 g\pi]^2} 
\]

Proof: Note first that there will be no bailout if:

i) \[2a_2\alpha_3s_1 + 2a_2\alpha_3\alpha_3g\pi(x(s_1) - x) > [2a_2\alpha_3\alpha_1(x(s_1) - x) + 1]; \text{ or} \]

ii) \[(R_i(\hat{K}) - \hat{K}) \geq \frac{2a_2\alpha_3s_1 - [2a_2\alpha_3\alpha_1(\hat{x}(s_1) - \bar{x}) + 1] + 2a_2\alpha_2\alpha_3g\pi(\hat{x}(s_1) - \bar{x})}{2a_2\alpha_3^2(\hat{x}(s_1) / \bar{x})} \]

(16) is obtained by using (2)-(4). The government then minimises the following loss function to find the optimal bailout:

\[
V_4(s_1,\text{Ex} > \bar{x}, x(s_1) > \bar{x}, \hat{x}, \hat{K}, C) = a_2[-\alpha_i(\hat{x}(s_1) - \bar{x}) + \alpha_2 g\pi(\hat{x}(s_1) - \bar{x}) + s_1 
\]

\[-\alpha_i(1 - \hat{\phi})(\hat{x}(s_1) / \bar{x})(R_i(\hat{K}) - \hat{K})^2 - \hat{\phi}(\hat{x}(s_1) / \bar{x})(R_i(\hat{K}) - \hat{K}) + C \]

(18)

Here, we take into account that there are now speculative attacks in the foreign exchange market. The main difference from the case of no devaluation expectations is that equation (4) now comes into play. This equation represents the adverse effect on fundamentals caused by the speculative attack. Recall that the government raises domestic interest rates to stave off speculation but this policy shifts unemployment up. Considering that:

\[\text{Ex} = \pi x_1 + (1 - \pi)x_2; \]  \hspace{1cm} (19a)

it then turns out that after taking into account (3) and (4), we obtain:

\[\text{Ex} - \bar{x} = \pi(x_1 - \bar{x}) \] \hspace{1cm} (19b)

Now, if the government considers not devaluing, it will take as given \(\hat{K}\), the level of investment chosen when the private sector decides to speculate in the foreign exchange market. Another bailout should be then determined optimally under these conditions, say \(\hat{\phi}\). The loss function in this case is:

\[
V_4(s_1,\text{Ex} = \bar{x}, x(s_1) = \bar{x}, \hat{x}, \hat{K}) = a_2[s_1 - \alpha_3(1 - \hat{\phi})(R_i(\hat{K}) - \hat{K})]^2 - \hat{\phi}(R_i(\hat{K}) - \hat{K}) \]

(20)

The loss function (20) is similar to the one described by (10), which is the case with no speculative attacks and the government deciding not to devalue. They however differ in the amount of investment chosen. The optimal bailout, \(\hat{\phi}\), will be:

\[\hat{\phi} = 1 - \frac{2a_2\alpha_3s_1 - 1}{2a_2\alpha_3^2(R_i(\hat{K}) - \hat{K})} < \hat{\phi} \] \hspace{1cm} (21)
When the government decides not to devalue in the bad state, the optimal bailout will be larger in the absence than in the presence of speculative attacks: \( \bar{\phi} > \hat{\phi} \). This can be seen by comparing (16) and (21). There will be a more limited amount of risky investment and less investment losses when there are speculative attacks (follows from proposition 5). A smaller bailout will then be necessary.

To prove that devaluation in the bad state is optimal, the loss in (18) has to be smaller than the loss in (20). Nevertheless, it should be noted that, given the rational expectations assumption, if the private sector expects that the government will never devalue even if there is a “bad” shock, it will have no incentive to speculate. If so, the chosen level of risky investment will be \( \bar{K} \) (the amount of investment that is optimal for the private sector when it decides not to speculate in the foreign exchange market) and not \( \hat{K} \). Consequently, the optimal bailout will again be \( \bar{\phi} \). It then becomes established that the government will rationally never maintain the fixed exchange rate regime in the event of speculative attacks and a “bad” shock.

**Proposition 7** The bailout will be smaller when there are speculative attacks and the government devalues in the bad state, than in the case when there are no speculative attacks and no devaluation occurs in the bad state.

**Proof:** We just need to compare (9) and (16). An intuitive explanation for this is that only a smaller bailout will be necessary since devaluation compensates for the negative effects that the outstanding debt size and the “bad” shock have on unemployment. The only condition that needs to be fulfilled for this to be true is that the effect of devaluing has to have a stronger effect than the expected effect that a speculative attack on the exchange rate would have on unemployment (i.e. \( \alpha_1 - \alpha_2 g \pi > 0 \)).

**Proposition 8:** When the government decides to devalue, a larger bailout will always be necessary when there are speculative attacks.

**Proof.** Compare (12) and (16). The reason is that when the private sector decides to speculate in the foreign exchange market, the government needs to stave off speculation by raising interest rates (equation (4)). Higher interest rates may slow down economic activity and consequently increase unemployment. A larger bailout will make these negative effects on unemployment less severe.

**Proposition 9:** If there are devaluation expectations and a “good” shock occurs, there will be no devaluation. There will be an unemployment gap caused solely by the devaluation expectations.

**Proof.** By assumption, the shock and the outstanding debt are zero, making a bailout unnecessary. When \( x(s_2) = \bar{x} \), the loss resulting from keeping the fixed exchange rate is:

\[
V_4(s_2, Ex > \bar{x}, x(s_2) = \bar{x}) = a_2 [\alpha_2 r]^2
\]

while the loss due to devaluation in state 2 will be:

\[
V_4(s_2, Ex > \bar{x}, x(s_2) > \bar{x}, C) = a_2 [-\alpha_1 (x_2 - \bar{x}) + \alpha_2 r]^2 + C.
\]

It becomes evident that devaluing will be more costly for the government than not doing so.
Proposition 10: When there are speculative attacks, a rational expectations equilibrium is compatible only with end node (14), in figure 1, which corresponds to abandoning the fixed rate if a “bad” shock takes place but maintaining it if a “good” shock occurs. Node (14) implies that:

\[ x(s_1) > \bar{x}; \]
\[ x(s_2) = \bar{x} \]

Proof. The structure of payoffs in the game is such that end nodes (12) and (13) in figure 1 must be counted out. In proposition 9, it is shown that it can never be optimal for the government to abandon the exchange rate regime when a “good” shock occurs. Moreover, the node (12) will never be reached because if the regime is abandoned in both states, the government will have a lot of incentive to abandon the regime already in stage 3, reaching node (6). Node (15) can never be reached either, because if the government is certain to maintain the regime in both states, the private sector will not have any incentive to speculate. This is also shown in the proof of proposition 6.

7. Stage 3

It is important to notice that when there are no speculative attacks, it will never be optimal to abandon the regime in stage 3 because there are no costs for staving off speculative attacks. The government might as well wait until stage 4 to make decisions about the exchange rate and bailouts. If the fixed exchange regime is abandoned, the government will incur a cost \( C \), the loss of credibility. Interesting things happen, though, when there are speculative attacks in the exchange rate market, \( \text{Ex} > \bar{x} \).

We will still consider here the case where the government has made no commitment to a specific bailout before stage 4. Nevertheless, by rational expectations, the private sector can calculate the bailout that the government will optimally give in stage 4 if a “bad” shock occurs. This implies that at this third stage, agents in the economy know that in stage 4 there will be given a bailout \( \hat{\phi} \) (equation (16)), as well as that the amount of risky investments is \( \hat{K} \).

Notice that speculative attacks borne in stage 3 will move the unemployment rate by \( u_3(s_i) \). From (3) we make the following simplifying assumption, \( u_3 - \bar{u} = \alpha_2 g(\text{Ex} - \bar{x}) \), which allows us to write equation (1) as follows:

\[
V_3(s_i, \text{Ex} > \bar{x}, x(s_i) > \bar{x}, \hat{\phi}, \hat{K}) = a_2 E(u_4 - u_n)^2 + a_1 (\alpha_2 g)^2 (\text{Ex} - \bar{x})^2 - \\
E \hat{\phi}(x(s_i) / \bar{x})(R_1(\hat{K}) - \hat{K}) + EC \tag{24a}
\]

or

\[
V_3(s_i, \text{Ex} > \bar{x}, x(s_i) > \bar{x}, \hat{\phi}, \hat{K}) = a_2 \{\pi[u_4(s_1) - u_n]^2 + (1 - \pi)[u_4(s_2) - u_n]^2\} + \\
a_1 (\alpha_2 g)^2 (\text{Ex} - \bar{x})^2 - \pi \hat{\phi}(x(s_i) / \bar{x})(R_1(\hat{K}) - \hat{K}) + \pi C; \tag{24b}
\]

Note now that the government loss function at this third stage now includes the costs of excess unemployment originated by the speculative attacks at this stage. Now, when speculative attacks have been initiated, the government can make either of the following decisions:
i) To abandon the fixed exchange regime right after private sector’s expectations have formed, that is at the very beginning of stage 3.

ii) To stave off speculative attacks during stage 3 and face the costs of defending the exchange rate regime. If the government anticipates that the costs of continuing to defend the regime, not only during stage 3 but also in stage 4, will be unaffordable, it is possible that the exchange rate regime may be abandoned regardless of whether a “bad” or “good” shock occurs, i.e. \( x(s_1) > \bar{x} \) and \( x(s_2) > \bar{x} \). The government will need to evaluate whether or not to abandon the regime already at this stage.

iii) To maintain the exchange rate regime, bearing all the costs of defending it, and wait until stage 4 to make a decision on the exchange rate. Such a decision implies that the government may consider it worthwhile to wait until the shock to the economy occurs and to abandon the regime only in the bad state, i.e. \( x(s_1) > \bar{x} \) and \( x(s_2) = \bar{x} \).

In order to make the optimal decision, it is necessary to evaluate the loss functions resulting from each of the above alternatives and compare them. Note that the government will incur a cost equal to \( C \) if it chooses (i).

If the government decides to defend the exchange rate regime in stage 3 and maintain it until stage 4 only in the good state, that is \( x(s_1) > \bar{x} \) and \( x(s_2) = \bar{x} \), it will have the following loss function:

\[
V_3(Ex > \bar{x}, x(s_1) > \bar{x}, x(s_2) = \bar{x}, \hat{\phi}, \hat{K}) = a_2 \pi (u_4(s_1) - u_4)^2 + [a_1(\alpha_2 g)^2 + a_2 (1-\pi) g^2 \alpha_2^2 ](Ex - \bar{x})^2 - \pi \hat{\phi}(x(s_1) / \bar{x})(R_1(K) - K) + \pi C;
\]

which, after taking into consideration (19) \((Ex - \bar{x}) = \pi (x(s_1) - \bar{x})\), and (25a) becomes:

\[
V_3(Ex > \bar{x}, x(s_1) > \bar{x}, x(s_2) = \bar{x}, \hat{\phi}, \hat{K}) = a_2 \pi (-\alpha_1(x(s_1) - \bar{x}) + \alpha_2 g \pi (x(s_1) - \bar{x}) + s_1 - \alpha_3 (1-\hat{\phi})(x(s_1) / \bar{x})(R_1(K) - K)^2 + [a_1(\alpha_2 g)^2 + a_2 (1-\pi) g^2 \alpha_2^2 ]\pi^2 (x(s_1) - \bar{x})^2 \]

\[- \pi \hat{\phi}(x(s_1) / \bar{x})(R_1(K) - K) + \pi C \]

Note that one of the differences between (25) and (18), the government’s loss function in the fourth stage, is that (25) now includes additional costs from the initiation of the speculative attacks represented by \([a_1(\alpha_1 g)^2 + (1-\pi)a_2(g\alpha_2)^2]\pi^2(x-\bar{x})\). The other difference is that now we have the expected costs resulting from higher unemployment than the natural rate, net loss of investment and abandonment of the regime (i.e. another argument in (25) is \( \pi \), the probability that a bad shock will occur).

If the government believes that the regime will be abandoned in stage 4 in both states (that is \( x(s_1) > \bar{x} \) and \( x(s_2) > \bar{x} \)), it will have the following loss function:
\[ V_3(E x > \bar{x}, x(s_1) > \bar{x}, x(s_2) > \bar{x}, \hat{\phi}, \hat{K}) = a_2 [\pi (-\alpha_1(x(s_1) - \bar{x}) + \alpha_2 g \pi (x(s_1) - \bar{x}) + s_1 - \alpha_3 (1 - \phi)(x(s_1) / \bar{x})(R_1(K) - K)]^2 + (1 - \pi)[-\alpha_1(x(s_2) - \bar{x}) + \alpha_2 g \pi ^2 (x(s_1) - \bar{x})]^2 + \]
\[ a_1(\alpha_2 g)^2 \pi ^2 (x(s_1) - \bar{x})^2 - \pi \hat{\phi}(x(s_1) / \bar{x})(R_1(K) - K) + \pi C \]  

(26)

The decision to abandon the exchange rate regime at stage 3, whether or not it has already resulted in certain costs for defending the regime, is optimal if the value of the loss function (25) is larger than that of (26).

**Proposition 11:** If the government plans to abandon the exchange rate regime only if a “bad” shock occurs, with no commitment to a specific bailout before stage 4 and a very small probability, \( \pi \), it will never be optimal to abandon the exchange rate regime at the very beginning of stage 3.

**Proof:** If the government abandons the regime right after the private sector has decided to speculate in the foreign exchange market, the only cost it will incur is \( C \), the loss of credibility. Waiting until stage 4 and devaluing only in a bad state, the government will incur costs equal to \( \pi C \) (see 25b). This is also true for a very small \( \pi \). Note that the other terms in (25b) are multiplied by \( \pi \) and therefore they are close to zero. There is then a trigger value of \( \pi \) which makes devaluation a non-optimal decision at the start of stage 3. This value is derived from the following condition:

\[ \pi < \frac{C}{a_2(u_4(s_1) - u_4)^2 + [a_1(\alpha_2 g)^2 + a_2(1-\pi)(g\alpha_2)^2] \pi (x(s_1) - \bar{x})^2 - \hat{\phi}(x(s_1) / \bar{x})(R_1(K) - K) + C} \]

(27)

If the costs of maintaining the regime until stage 4 are very large, the probability that the economy will be in the bad state, \( \pi \), needs to be very small for the regime not to be abandoned in stage 3.

**Proposition 12:** There is a probability \( \pi \) that will make it optimal for the government to wait until stage 4 to decide on the optimal bailout and to abandon the exchange rate regime only if a “bad” shock occurs, instead of abandoning the regime already in stage 3. It will in the meanwhile have stave off the speculative attacks during stage 3.

**Proof.** We need to compare (25b) and (26). The threshold value of \( \pi \) is:

\[ \pi < \sqrt{\frac{\alpha_1}{2\alpha_2 g(x_1 - \bar{x})}} (x_2 - \bar{x}) \]

(28)

8. Bailout commitments

In this section we analyse a situation in which the government can commit to a certain bailout before the market forms expectations, that is at stage 1. From the point of view of the
government or a lender of last resort, it is important to find out whether committing to an optimal bailout will prevent an exchange rate crisis when there are investment losses. There are two questions that we would like to answer: What would then be the optimal bailout that the government can commit to before the private sector forms expectations, in order to avoid devaluation at stage 4? What will be the size of the bailout that the government could optimally commit to at stage 1 by comparison with the bailout given at stage 4, which is contingent on the state of the economy?

The government would in fact like to avoid not only the costs that it could incur already in stage 3 when staving off the speculative attacks, but, more important, those associated with abandoning the fixed exchange rate regime at stage 4 (after the shock occurs). This implies that the government would like to give the private sector incentives not to engage in speculative attacks. Optimal decisions are based on the assumption that speculative attacks are imminent and that devaluation will occur in the bad state (at stage 4) when no commitment to bailout has been made. In this case node (14) in figure 1 will be reached. (see proposition 10). The objective is precisely to avoid that, and try to reach at best node (11) in figure 1 (see proposition 4).

To find the optimal bailout that the government can commit to at stage 1 implies that analytically, the following should be taken into account:

1. The equilibrium exchange rate in the bad state, \( \hat{x}(s_1) \) (equation (17)). In the good state there will not be any devaluation so \( \hat{x}(s_2) = \bar{x} \).
2. The effect of the bailout, \( \phi \), first, on \( \hat{x}(s_1) \) (which can be obtained from (17)) and second, on the amount of the risky investment (equation (7)).
3. The effect of a depreciation on the amount of the risky investment (equation (8)).
4. The government’s loss function in stage 3, when it is expected that there will be devaluation only in the bad state (equation (25)). The government, as mentioned, would like to minimise all the costs that may already be incurred in stage 3 as a result of the speculative attack, with the perspective that a devaluation is likely in stage 4 if a bad shock occurs.

We let \( V_3 \) represent the government loss function at stage 3 when the fixed exchange regime is only expected to be abandoned in the bad state, i.e.:

\[
V_3 = V_3(E \bar{x} > \bar{x}, \hat{x}(s_1) > \bar{x}, x(s_2) = \bar{x}, \phi, K)
\]

The functional form of (29) is like (25). It is here the case that we will find, using (29) and other conditions, the committed \( \phi \).

Let \( V_1 \) represent the loss function to be minimised for obtaining the bailout committed ex-ante (say \( \phi^{\text{COMM}} \)). We can say that \( \phi^{\text{COMM}} \) can be obtained from the following expression:

\[
\frac{dV_1}{d\phi} = \frac{dV_3}{d\phi} + \frac{dV_3}{dK} \left[ \frac{\partial K}{\partial \phi} + \frac{\partial K}{\partial \hat{x}(s_1)} \cdot \frac{d\hat{x}(s_1)}{d\phi} \right] = 0
\]

\[6\] Note that the results above indicate that eliminating the risks of devaluing in the fourth stage should also imply avoiding devaluation in the third stage.

\[7\] Such a solution is one in which the government devalues optimally in the most adverse state and when the market has decided to speculate. Note that the optimal exchange rate (17) and the optimal bailout (16) determine the equilibrium exchange rate and bailout at the fourth stage, when the government has not made any commitment to do it before stage 4.
Where:
\[
\frac{dV_3}{d\phi} = \frac{\partial V_3 (Ex > \bar{x}, \hat{x}(s_1) > \bar{x}, x(s_2) = \bar{x}, \phi, K)}{\partial \hat{x}_1 (Ex > \bar{x})} \frac{d\hat{x}(s_1)}{d\phi} + \frac{\partial V_3 (Ex > \bar{x}, \hat{x}(s_1) > \bar{x}, x(s_2) = \bar{x}, \phi, K)}{\partial \phi}
\]

And:
\[
\frac{dV_3}{dK} = \pi(x(s_1) / \bar{x})(R_1'(K)-1)[2a2\alpha_3(1-\phi)[(-\alpha_1(x(s_1) - \bar{x}) + \alpha_2 g \pi(x(s_1) - \bar{x}) + s_1 - \\
\alpha_3(1-\hat{\phi})(x(s_1) / \bar{x})(R_1(K) - K)] - \phi] \\
\delta K/\delta \phi (the moral hazard effect) and \delta K/\delta \hat{x}(s_1) can be obtained from (7) and (8), (R_1'(K)-1) is the difference between the marginal revenue of investment and the marginal cost of investment (see (6)) while \frac{d\hat{x}(s_1)}{d\phi} in turn will be calculated from (17).

It is not possible to get an explicit solution for \phi^{COMM} from (30). However, by giving reasonable parameter values to (29), the size of \phi^{COMM} was found by solving (30) numerically. As noted in section 5, when we presented the investor’s problem, we did not present explicit functional forms. Since the solution of \phi^{COMM} is already complicated, we resolved to treat \delta K/\delta \phi and \delta K/\delta \hat{x}(s_1) as parameters and simulate different reasonable values of them to analyse how \phi^{COMM} may be affected by \delta K/\delta \phi and \delta K/\delta \hat{x}(s_1). Using functional forms for the returns on investment (R_1(K) and R_2(K)) will require that we make additional assumptions about the parameters of the functional form, since (30) cannot be solved explicitly anyway. We also parameterised (R_1'(K)-1), the difference between the marginal return on investment minus the marginal cost of investment.

The parameter values are as follows:
\[
\pi = 1, a_1=a_2=0.6, \alpha_1=2.2, \alpha_2=4, \alpha_3=1, g=0.71, p=0.5, s=0.25, \text{ and } R_1(K-K)=0.8, (R_1'(K)-1)=-0.5, \delta K/\delta \phi =1 \text{ and } \delta K/\delta \hat{x}(s_1) =-0.5.
\]

Recall that the government will have an incentive to commit to a bailout such as \phi^{COMM} if devaluation can be avoided. This outcome may in turn result if such a bailout is able to change the expectations of the private sector, even though investment losses are expected.

The size of the non-committed bailout contingent on the state of the economy in stage 4 is also obtained by solving numerically and simultaneously the solutions for \hat{\phi} and \hat{x}(s_1) (equations (16) and (17)) at stage 4. This will allow us to compare \phi^{COMM} with \hat{\phi} and, assuming the same parameter values, to answer the second question in this section. It is here taken into account assumed above. We find that the equilibrium bailout (\hat{\phi}) at stage 4 is:

\[\text{Note that we do not need the parameter values of } (R_1'(K)-1)=-0.5, \delta K/\delta \phi =1 \text{ and } \delta K/\delta \hat{x}(s_1) =0.5 \text{ when solving (16) and (17) simultaneously.}\]
This is represented by the following figure:

![Figure 2. Optimal bailout at the fourth stage with no commitment](image)

(31) implies that, with non-commitment, the larger the devaluation, the smaller the bailout the government will give to the private sector. Recall that devaluation will be used to compensate for the negative effects that a bad shock and outstanding debt have on unemployment. If the government finds it optimal to make a large devaluation in order to leave the economic fundamentals minimally affected, a smaller bailout will be needed. Important simulations had to be done to check that the solutions to $\hat{\phi}$ and $\hat{x}(s_1)$ were robust. Changes in the parameter values did cause shifts in the functions of $\hat{\phi}$ and $\hat{x}(s_1)$. The following were found:

1. The larger the investment losses, $R_1(K)-K$, the larger the bailout required for any given devaluation.
2. The larger the bad shock, $s_1$, the larger the bailout needed for any given devaluation.
3. The larger the government’s aversion to unemployment, measured by the parameter $a_2$, the larger the bailout needed for a given devaluation.
4. Proposition 7 above, shows that if the condition $\alpha_1 - \alpha_2 \pi > 0$ holds, the optimal bailout will be smaller when there are speculative attacks and devaluation than the optimal bailout.
when there are no speculative attacks and no devaluation. Note that such a condition indicates that unemployment is affected more by the ex-post devaluation than by the expected devaluation by the private sector. It turns out that the bailout would have to be larger for any given devaluation, irrespective of which parameter changed, as long as \((\alpha_1 - \alpha_2 g \pi)\) is negative.

Now, we turn to the original problem of this section, to find the optimal bailout that the government could commit to at stage 1, \(\phi^{\text{COMM}}\). We employ the parameter values assumed above when solving (29). Simulations of (29) with different parameter values and different (reasonable) exchange rates \(\hat{x}(s_1)\) were carried out. It is worthwhile repeating that the government would like to commit ex-ante (at stage 1) to give a certain bailout only in the bad state since it wishes to avoid speculative attacks that are likely to take place. Therefore, different devaluation levels may also induce the government to commit to different bailout levels.

We present the following table to illustrate the results of the various simulations:

<table>
<thead>
<tr>
<th>Table 1. Simulations for (\phi^{\text{COMM}}).</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x(s_1)=1.1)</td>
</tr>
<tr>
<td>(\phi^{\text{COMM}}) (initial p.v.)(^1)</td>
</tr>
<tr>
<td>(\phi(\hat{R}_1(K)-K=-0.8))(^2)</td>
</tr>
<tr>
<td>(\phi(\delta K/\delta \phi=2))</td>
</tr>
<tr>
<td>(\phi(\delta K/\delta \hat{x}(s_1)=-0.1))</td>
</tr>
<tr>
<td>(\phi(\hat{R}_1(K)-K=-0.1))(^3)</td>
</tr>
<tr>
<td>(\phi(s_1=0.5))</td>
</tr>
<tr>
<td>(\phi(a_2=0.9))</td>
</tr>
<tr>
<td>(\hat{\phi}) (initial p.v.)</td>
</tr>
</tbody>
</table>

\(^1\) The initial parameter values (p.v.) are used here. \(a_1=a_2=0.6\), \(\alpha_1=2.2\), \(\alpha_2=4\), \(\alpha_3=1\), \(g=0.71\), \(p=0.5\), \(s=0.25\), and \(R_1(K)-K=-0.8\), \(\hat{R}_1(K)-1=-0.5\), \(\delta K/\delta \phi =1\) and \(\delta K/\delta \hat{x}(s_1)=-0.5\). Notice that \(\phi\) in the table is actually \(\phi^{\text{COMM}}\), while \(\hat{\phi}\) is the equilibrium level of a bailout at stage 4 when no commitment has been made before this stage.

\(^2\) From the second row, the simulations with different values for the (most relevant) initial parameter values are presented. It is then shown how bailouts change with different devaluations at stage 4 \((x(s_1))\).

\(^3\) Simulations with \(R_1(K)-K<0.8\) yield complex numbers for the solution of \(\phi\).
Proposition 13: When the fixed exchange rate regime is expected to be abandoned in state 1 but not in state 2, the optimal bailout under commitment is greater than the optimal bailout with non-commitment when speculative attacks in the foreign exchange market are expected to take place.

Proof: The comparison between the first and last rows in table 1 shows the proposition. Such result has a very important consequence for the optimal final equilibrium. This is explained in the following proposition.

Proposition 14: When the government commits to a specific optimal bailout at stage 1, it will need to devalue less often or simply not to devalue. A commitment to bailing out can ameliorate the financial crisis and prevent a currency crisis.

Proof: A higher bailout decreases private sector net losses and consequently there will be a smaller deterioration of the fundamentals. It becomes less necessary to devalue. The private sector takes this into account when forming expectations about the exchange rate: devaluation is less likely to occur. A formal proof follows.

Keep in mind that we are in the situation where speculative attacks may occur. Let us consider the case when no commitment to a specific bailout has been made before stage 4. If speculative attacks and an adverse shock were to occur, the government will determine that the optimal exchange rate at stage 4 (\(\hat{x}(s_1)\)) will be, for example, 1.1 (10% devaluation when \(\bar{x} = 1\)) and the optimal bailout will be 23.45% of the investment losses (using equation (17) and the assumed parameter values, see also last row in table 1). The government now would like to avoid this expected 10% devaluation (if no devaluation were expected there will not be a need for commitment) to decide on the optimal bailout (\(\phi^{COMM}\)) that it can commit (ex-ante) to. From (30), \(\phi^{COMM}\) will be 63.3% (see second row of table 1).

If the government makes such commitment (\(\phi^{COMM}\)), private investors will rationally take this into account and find the optimal exchange rate that the government will decide on if they were to decide to speculate in the currency market\(^9\).

We can now make use of figure 2 that indicates that a higher bailout will correspond to a less depreciated currency. A 63.3% bailout will correspond to an equilibrium exchange rate (\(\hat{x}(s_1)\)) of 0.41. This is obtained from the equilibrium exchange rate at stage 4 (17) (\(\hat{x}(s_1)\)) (if we look at the analytical solution) or (30) (if we look at the numerical solution).\(^{10}\) By the time the private sector form expectations, they realise that with commitments, it will never be optimal for the government to devalue and therefore that it will be not rational to start a speculative attack. The node in the game tree (figure 1) that will be reached will be (11).

Proposition 15: Stronger moral hazard effects caused by government guarantees such as bailouts (large \(\delta K/\delta \phi\)) will cause the government to commit ex-ante to a smaller bailout. Which, nevertheless, is still higher than the optimal non-committed bailout (\(\hat{\phi}\)).

Proof: The comparison of the second row, fourth and last rows prove the proposition.\(^{11}\)

\(^9\) Recall again that when the private sector decides to speculate it also determines its level of risky investment such as \(\hat{K}\), as indicated above.

\(^{10}\) That we obtain a revaluation should not affect the main conclusions and arguments.

\(^{11}\) Recall that there is potential moral hazard effect of bailing out on the amount of risky investment and that higher depreciation decreases the amount of risky investment if the conditions \([x_i - (dx_i/\partial \phi)(1 - \phi)] > 0\) and \([(1 - \phi) - \phi_i (dx_i/\partial \phi)] > 0\) are satisfied. This happens to be the case with the assumed parameter values.
Other results are as follows:

1. When the private sector’s investment losses are expected not to be large, the government will commit to a smaller bailout. They will still be larger than those where there is no commitment, especially if the devaluation is expected to be relatively large.

2. If the exchange risk of investment is not very large \( \frac{\partial K}{\partial \hat{s}} \hat{s}_i \) is small), the committed bailout will be larger.

3. In the event of a greater adverse shock, \( s_1 \), and stronger government aversion to unemployment on the bailout level (in this case the committed one, \( \phi^{COMM} \)) we find that the bailout itself needs to be larger. This coincides with the results obtained for the effect of these variables on the non-committed bailout \( \hat{\phi} \).

4. It turns out that when the effect of devaluing \( \alpha = \alpha_1 - \alpha - \alpha_2 g \pi < 0 \) is weaker than the expected effect that the speculative attack on the currency has on unemployment, i.e. irrespective of which parameter change occurs to make that relation negative, the bailout needs to be larger for any given devaluation.

The most important results of this section are that if the government commits to a relatively large bailout (larger than the one when it does not commit before stage 4), it can persuade the private sector not to speculate in the foreign exchange market. A relatively larger ex-ante committed bailout will ameliorate the effect of investment losses on the fundamentals of the economy, making devaluation less likely.

9. Conclusions

We have studied a model for the analysing of the interrelationships between exchange rate crises and debt crises. It is shown that crises can be characterised by both belief-driven and fundamentals-driven attacks. The model is presented as a four-stage sequential game where the players are the government and the private sector. Information about the probabilistic distribution of a forthcoming (good or bad) shock is given in the first stage. This shock will be realised in the fourth stage and will affect private-sector debt and unemployment. The government may or may not wish to commit to a bailout for a specific amount of private debt before the market forms expectations, in stage 1. In stage 2, the private sector forms expectations and makes investment decisions. In stage 3, the government may react by either retaining or leaving the peg, depending on the costs that staving off speculative attacks at this stage and the expected costs that the government will incur in stage 4 if the peg is maintained. The regime may then be abandoned after the onset of the speculative attack, before it does further significant damage to the economy. This outcome, where self-fulfilling expectations of a speculative attack lead necessarily to devaluation, resembles the Krugman (1979) first-generation model, where a deterioration of the fundamentals (in addition to the market’s anticipation of a financial crisis) plays such an important role in triggering a crisis. In the final stage, 4, the shock occurs and the government decides on the optimal bailout, if it has not committed already in stage 1, and whether or not to abandon the fixed-rate regime (given that the fixed rate has been retained up to this stage). Decisions are here contingent on the prevailing state of the economy.

It turns out that when there is no commitment at the early stage of the game, the model yields multiple equilibria. However with an ex-ante commitment, the model is likely to yield a unique equilibrium where no devaluation occurs, even though the private sector ends up with investment losses. For this to happen, the proportion of private sector debt that the government or lender of last resort commits to bailing out has to be larger than the proportion
(non-committed) that is contingent on the state of the economy. A larger bailout reduces the private sector’s outstanding debt, leading to a less severe impact on the economic fundamentals. In consequence, the need for devaluation is not as great and the private sector takes this into account when forming expectations. There will be no incentive to initiate a speculative attack and the government will then avoid a currency crisis by committing ex-ante to bailing out part of the private sector’s debt. Thus, ex-ante commitment to optimal bailouts serves as a strategic device for the government when the government has a potential problem of credibility in its exchange rate policy, since it facilitates the implementation of a more efficient unique equilibrium (without devaluation). Such an equilibrium is not a “sunspot” equilibrium. This result becomes unambiguous even after the moral hazard effects of bailing out are taken into consideration. In the model, the government measures the moral hazard effect of this type of guarantee when it decides on the optimal bailout that it can commit to before the private sector forms expectations, and decides on the level of risky investments. The greater the effect of the moral hazard, however, the smaller the bailout that the government will commit to in stage 1.

Another result is that it is never optimal for the government to allow devaluation to take place when there are no speculative attacks even if a bad shock occurs and the private sector suffers investment losses. We also find that with no ex-ante commitment to a specific bailout before stage 4, a larger bailout will always be necessary when there are speculative attacks than when no attacks occur.

Otherwise, the larger the investment losses, the greater the shock in the bad state, the stronger the government’s aversion to unemployment and the greater the probability that the economy will end up in the bad state, the higher the equilibrium bailout level chosen by the government. This holds true whether the government commits ex-ante to a specific bailout or not.

This paper has some weaknesses that need to be taken up in future work. First, private-sector debt is introduced in a somewhat rudimentary manner, simply as a factor contributing to an exacerbation of the unemployment problem. In particular, we do not consider the actual process of debt formation in the private sector. In addition, the moral hazard effects are only considered in the case where the government commits ex-ante to debt bailouts. Nor do we model how different types of investment may be chosen, e.g. in terms of their riskiness, in response to the amount of (committed or non-committed) bailout that the government will give. Nevertheless, the model presented here, in which the moral hazard effects are balanced against expectational effects, has not been considered before in the relevant literature.

Finally, in countries where the liquidity supply is low or simply difficult for their own governments to provide because of lack of credibility, a lender of last resort such as the IMF may play an important role in providing such liquidity and preventing inefficient liquidation. Without such support, countries may find it difficult to sustain their exchange rate regime. If the currency is allowed to depreciate in an emergency, lenders may have more incentive to withdraw their money from such countries without losing anything, leaving behind insolvent firms. A further analysis of such issues is also an important topic for future work.

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KEYWORDS:

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