Enhanced Fuzzy System for Student’s Academic Evaluation using Linguistic Hedges

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Abstract—In this study, the effect of concentration, intensification and dilation of three common linguistic hedges (LHs), namely, very, indeed, and more or less on the performance of a fuzzy system for evaluating student’s academic evaluation is presented. A LH may be viewed as an operator that acts on a fuzzy set representing the meaning of its operand. As an example, the operator very acts on the fuzzy meaning of the term high grade to have a secondary meaning of very high grade. This property changes the shape of the fuzzy sets and hence the amount of overlap between adjacent sets. It, in turn, improves the meaning of the fuzzy rules and hence the accuracy of the proposed fuzzy evaluation systems. The proposed LHs based fuzzy evaluator systems are compared with a standard fuzzy sets based fuzzy evaluator system using an example drawn from literature. Empirical results of the example presented in this paper show that concentration and dilation effect of LHs is not significant compared to standard fuzzy sets.

Keywords—Student’s evaluation; concentration; dilation; intensification, very; more or less; indeed; modifiers; uncertainty;

I. INTRODUCTION

The idea of fuzzy logic was first invented by L. A. Zadeh in 1965 in his seminar paper fuzzy sets [1]. The first practical application of fuzzy logic was presented by E. H. Mamdani ten years later in 1975 [2]. Fuzzy logic reflects how people think. It attempts to model our sense of words, our decision making and our common sense [3]. A fuzzy set, compared to classical set, is a class with unsharp boundaries where transition from membership to non-membership is gradual rather than abrupt [4]. A fuzzy set A defined on a universe of discourse U is characterized by a membership function \( \mu_A(x) \) which associates with each point x in U its grade of membership in A. \( \mu_A(x) \) is interpreted as the truth-value of the statement “x belongs to A,” and it is assumed to range in the interval \([0, 1]\), where 0 and 1 correspond to non-membership and full membership, respectively [4]. Classical set, on the other hand, has a sharp boundary, which means that statement “x belongs to A,” is either true or false. For students’ evaluation, for example, a question’s degree of difficulty can only be categorized into one subset: low, medium, or high with very clear boundary between each subset. In fuzzy sets, as it is shown in Fig. 1, these boundaries become vague or smooth. One question can be categorized into two or even more subsets simultaneously. For example, a question’s difficulty can be considered to belong to “low” to a certain degree, say 0.5 degree, but at the same time it can belong to “medium” to about 0.7 degree.

As an example, the Gaussian fuzzy sets, shown in Fig. 1, are used to represent the linguistic values “low”, “medium” and “high” of the fuzzy variable question’s difficulty in computer. These linguistic values are the basic building blocks used for representing vagueness and ambiguity which are basic concepts in the theory of interpretation and associated with how human judge things and inexact reasoning. For example, a percentage of students in one class might consider a question is of “medium” difficulty level, another percentage consider it of “high” difficulty level, while the rest cannot make a decision, they believe that it is at some where on the border line between “medium” and “high” and this is what is called vagueness. Another example, what is considered of “low” difficulty (i.e., easy) question in one place might be considered of “high” difficulty in some other place, etc. Also the term “high” difficulty itself might take different meaning from one place to another. So vagueness helps in solving conflicts from borderline cases while ambiguity gives flexibility by giving the word (i.e., linguistic value) more than one meaning.

Linguistic hedge (LH) is an operator or a modifier that can be used to change the fixed shape of the fuzzy sets and hence extending the primary meanings of its associated linguistic values to a secondary and more expressive meanings giving the
system more capabilities to mimic human linguistic reasoning to enhance artificial systems to better express and treat the more borderline and complex cases considering linguistic and cultural variations [4]. Zadeh [5] proposed LHs such as very, more or less, indeed, etc. to modify the membership functions of the fuzzy sets. Since the linguistic hedges proposed by Zadeh in 1973, only a small amount of literature dealing with these concepts has been published, for example [6]–[9].

Academic assessment can be defined as a systematic and ongoing method of gathering, analyzing and using information to improve student learning in terms of knowledge acquired, understanding developed, and skills and competencies gained [10]. Assessment methods probably have a greater influence on how and what students learn than any other factor. Conventional evaluation method, using the grading system (A, B, C, etc.), have been commonly practiced in spite of its drawbacks and there has been a lot of talk for the need for a new enhanced evaluation system in which can consider other factors which are impossible to consider in traditional approaches [11]. Weon and Kim [11] suggested a new fuzzy learning achievement evaluation strategy by assigning fuzzy approaches [11]. Assessment methods probably have a greater influence on understanding developed, and skills and competencies gained for the fuzzy set A results in a relatively small reduction in the magnitude of the membership degrees of those x with high degree of membership in A and relatively large reduction of those x with low degree of memership in A. Concentration operator is defined as:

$$\mu_{CON(A)}(x) = \left(\mu_A(x)\right)^\alpha; \alpha > 1$$

Based on the above definition, a few related hedge operations such as absolutely, very, much more, more and plus can be obtained for $\alpha = 4, 2, 1.75, 1.5$ and $1.25$ respectively.

2) Dialation: The effect of dialation is opposite to the effect of concentration. The hedge dialation is defined as:

$$\mu_{DIAL(A)}(x) = \left(\mu_A(x)\right)^\alpha; \alpha < 1$$

Similarly, some related hedge operations such as minus, more or less, and slightly can be obtained for $\alpha = 0.75, 0.5$, and 0.25 respectively.

3) Intensification: the hedge intensification is defined as:

$$\mu_{INT(A)}(x) = \begin{cases} 2\left(\mu_A(x)\right)^2 & \text{if } 0 \leq \mu_A(x) \leq 0.5 \\ 1-2(1-\mu_A(x))^2 & \text{if } 0.5 \leq \mu_A(x) \leq 1 \end{cases}$$

In this paper, the hedge very is used to stand for concentration; the hedge more or less is used to stand for dialation and hedge indeed is used to stand for intensification. The fuzzy set low, very low, more or less low and indeed low for the fuzzy variable difficulty are shown in Fig. 2.

In this paper, a fuzzy students’ academic evaluation system using LHs is presented. A problem from literature is used to compare results to existing systems. Fuzzy LHs are presented in Section II (A). In Section II (B), the basic configuration of fuzzy logic system/controller is presented. The fuzzy evaluation system is briefly introduced in Section II (C). In Section III, a demonstrative example and results are presented. Results are summarized and concluding remarks are drawn and presented in Section IV.

**Fig. 2.** Effect of fuzzy LHs “very”, “more or less”, and “indeed” on the linguistic value “very”.
In this figure, the membership function of the fuzzy sets “very low”, “more or less low”, and “indeed low” are generated by applying the hedge operators “very”, “more or less low”, and “indeed” to the fuzzy set “low”. It is obvious that the operator “very” tends to narrow the shape of the membership function and decrease the membership degree. On the contrary, the linguistic hedge “more or less” tends to widen the shape of the membership function and increase the membership degree. The linguistic hedge “indeed” tends to widen the membership function and increase membership degrees for the values of \( x \) of higher membership degrees while narrow the membership function and decrease further the membership degrees for the values of \( x \) of lower membership degrees.

B. Fuzzy logic system (FLS)

A FLS of two inputs and one output is the basic building block of the fuzzy evaluation system presented in this paper. Fig. 3 shows the block diagram of an FLS consisting of four principal units: the fuzzifier, fuzzy inference engine, knowledge base, and the defuzzifier. Since inputs, for example grades, are usually given in crisp form and since the data manipulation in an FLC is based on fuzzy set theory, fuzzification is necessary. Fuzzification is related to the vagueness and imprecision in a natural language, which translates the input crisp data into the fuzzy representation for further processing. In fuzzification, the numerical value of each crisp input is converted into a set of membership degrees in the membership functions of the linguistic values. A domain expert or an optimization algorithm usually determines the shape and the distribution of the membership functions on the universe of discourse. Fuzzification enables FLS to model the meaning of natural language expressions. A FLS is characterized by a set of linguistic statements derived by a domain expert to map inputs to outputs. Domain knowledge is usually represented in the form of a set of “IF–THEN” rules, known also as production rules, expressed as:

\[
\text{IF (a set of conditions are satisfied)} \quad \text{THEN (a set of actions can be inferred)} \tag{4}
\]

To deal with the fuzzy information described above, the fuzzy inference engine employs the fuzzy knowledge base to simulate human decision-making and infer outputs. Finally, the defuzzifier module is used to translate the processed fuzzy data into the crisp data suited to real world applications. Mamdani type fuzzy inference method and center of gravity (COG) defuzzification methods are adopted in this paper [2].

C. Fuzzy evaluation system (FES)

In this paper, the fuzzy evaluation system consists of three nodes, as it is shown in Fig. 4, where each node is an implementation of the FLS presented in Section II (C):

1) Difficulty node: It determines the difficulty of exam questions in terms of the accuracy rate matrix and answer-time matrix of all students. For each question, the average accuracy rate and average answer-time are obtained as results from time-based exams, normalized, and then fuzzified though five linguistic values; “low”, “more or less low”, “indeed medium”, “more or less high” and “high” as it is shown in Fig. 5. A rule base consists of 25 rules, shown in Table I, is used to infer difficulty in terms of accuracy rate and answer time rates using the above mentioned linguistic values where 1, 2, 3, 4 and 5 stands for “low”, “more or less low”, “indeed medium”, “more or less high” and “high”, respectively. Difficulty ratio of each question is then obtained, defuzzified and directed to the next node.

2) Answer-cost node: It determines the cost involved in solving a question in terms of the complexity and difficulty ratio of that question. Complexity on a scale from 0 to 1 is obtained from a domain expert such as teachers or students. Difficulty ratio is obtained from previous node. It is intuitively obvious that solving complex and difficult questions entails higher cost compared to simple and easy ones. 25 rules of the rule base, shown in Table II, is used to infer answer-cost in terms of difficulty ratio and complexity ratio. Answer-cost ratio is then obtained, defuzzified and directed to the next node, adjustment node.

3) Adjustment node: It determines, on a scale from -1 to +1, the necessary rate by which a question grade should change in order to fairly consider other factors such as difficulty, complexity and importance of exam questions in the total score of each student. Adjustment rate is determined in terms of importance rate and answer-cost of each question using the 25 rules of the rule base shown in Table II. Importance ratio of a question to a curriculum is determined by a domain expert, i.e., teachers or examiners while answer-cost ratio is obtained from previous node, i.e., adjustment node. It
is intuitively obvious that solving a costly and important question might entail increasing its grade and vice versa.

Fig. 5. Five linguistic values used to fuzzify/defuzzify all linguistic variables used in the system: “low”, “more or less low”, “indeed medium”, “more or less high” and “high”

<table>
<thead>
<tr>
<th>Accuracy</th>
<th>Time rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3 4 5 5 5</td>
</tr>
<tr>
<td>2</td>
<td>2 3 4 5 5</td>
</tr>
<tr>
<td>3</td>
<td>2 2 3 4 4</td>
</tr>
<tr>
<td>4</td>
<td>1 2 3 4 4</td>
</tr>
<tr>
<td>5</td>
<td>1 1 2 2 3</td>
</tr>
</tbody>
</table>

TABLE I. FUZZY RULE BASE TO INFER THE DIFFICULTY.

<table>
<thead>
<tr>
<th>Difficulty/Answer-cost</th>
<th>Complexity/Importance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 2 2 3 4</td>
</tr>
<tr>
<td>2</td>
<td>1 2 2 3 4</td>
</tr>
<tr>
<td>3</td>
<td>1 2 3 4 4</td>
</tr>
<tr>
<td>4</td>
<td>2 3 4 4 5</td>
</tr>
<tr>
<td>5</td>
<td>3 4 4 5 5</td>
</tr>
</tbody>
</table>

TABLE II. FUZZY RULE BASE TO INFER ANSWER-COST/ADJUSTMENT.

III. DEMONSTRATIVE EXAMPLE AND RESULTS

The capability and feasibility of the proposed LH based fuzzy evaluation system are demonstrated in this section. The focus of this work is to emphasize that using LHs can provide more objective and fair evaluation compared to traditional fuzzy sets.

A. Example

An example derived from [14]–[17] is used. Assume that we have $n=10$ students laid to an exam of $m=5$ questions where $n=10$ and $m=5$. The accuracy rate matrix, $A$, the time rate matrix, $T$, and the grade vector, $G$, are given as follows:

$$A = \begin{bmatrix} 0.59 & 0.5 & 0.66 & 0.11 & 0.08 & 0.84 & 0.23 & 0.04 & 0.24 \\ 0.01 & 0.27 & 0.14 & 0.04 & 0.88 & 0.16 & 0.04 & 0.22 & 0.81 \\ 0.77 & 0.69 & 0.97 & 0.71 & 0.17 & 0.86 & 0.87 & 0.42 & 0.91 \\ 0.73 & 0.72 & 0.18 & 0.16 & 0.5 & 0.02 & 0.32 & 0.92 & 0.9 \\ 0.93 & 0.49 & 0.08 & 0.81 & 0.65 & 0.93 & 0.39 & 0.51 & 0.97 \\ \end{bmatrix}$$

$$T = \begin{bmatrix} 0.7 & 0.4 & 0.1 & 1 & 0.7 & 0.2 & 0.7 & 0.6 & 0.4 & 0.9 \\ 1 & 0.9 & 0.3 & 1 & 0.3 & 0.2 & 0.8 & 0 & 0.3 \\ 0.2 & 0.1 & 0.1 & 1 & 1 & 0.3 & 0.4 & 0.8 & 0.7 & 0.5 \\ 0 & 0.1 & 1 & 1 & 0.6 & 1 & 0.8 & 0.2 & 0.8 & 0.2 \end{bmatrix}$$

$$G^T = [10 \ 15 \ 20 \ 25 \ 30]$$

Here, $A=[a_{ij}]$ and $T=[t_{ij}]$ are of $n \times m$ dimensions, where $a_{ij} \in [0, 1]$ denotes the accuracy rate of student $j$ on question $i$, $t_{ij} \in [0, 1]$ denotes the time rate of student $j$ on question $i$. $G^T$ denotes the transpose of $G$, where $G$ is of $m \times 1$ dimension, $G = [g_i]$, $g_i \in [1, 100]$, denotes the assigned maximum score to question $i$, where:

$$\sum_{i=1}^{m} g_{ij} = 100$$

Importance and complexity of each question, $I$ and $C$, are determined by a domain expert as follows:

$$I = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0.33 & 0.67 & 0 & 0 \\ 0 & 0 & 0.15 & 0.85 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0.07 & 0.93 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0.85 & 0.15 & 0 & 0 \\ 0 & 0 & 0.33 & 0.67 & 0 \\ 0 & 0 & 0 & 0.69 & 0.31 \\ 0.56 & 0.44 & 0 & 0 & 0 \\ 0 & 0 & 0.7 & 0.3 & 0 \end{bmatrix}$$

Matrices $I=[i_{mk}]$ and $C=[c_{ik}]$ are of dimension $m \times l$ where $i_{mk} \in [0, 1]$ denotes the membership value of question $i$ belonging to the importance level $k$, and $c_{ik} \in [0, 1]$ denotes the membership value of question $i$ belonging to the complexity level $k$, where:
\[
\sum_{k=1}^{n} i_{mk} = 1
\]
\[
\sum_{k=1}^{n} c_{ik} = 1
\]

**B. Classical evaluation**

The classical ranking is obtained as follows:

\[
\text{Score}_{\text{classical}} = \mathbf{G}^{T} \mathbf{A}
\]

Classical ranking method relies only on accuracy rate of each student’s in his/her exam questions. Although its simplicity, other factors such as complexity, difficulty and importance of question are considered. Classical scores using Eq. (7) is obtained as:

\[
S_{\text{classical}} = \begin{bmatrix}
9 & 1 & 2 & 8 & 10 & 4 & 5 & 6 & 7 & 3 \\
85.95 & 67.6 & 54.05 & 52.3 & 49.7 & 49.7 & 49.7 & 48.8 & 46.1 & 38.4
\end{bmatrix}
\]

from which we can see that students 4, 5 and 10 are of equal total score. Students ranked in a descending order are shown in Table III.

**C. Score adjustment using LHs based fuzzy evaluation system**

In this Section, LHs based fuzzy evaluation system presented in Section II C is used. The surface view of the rule bases shown in Tables I and II for the fuzzy sets shown in Fig. 5 are shown in Figs. 6 and 7, respectively.

Students scores using this approach is obtained as:

\[
S_{\text{LH}} = \begin{bmatrix}
9 & 1 & 2 & 8 & 10 & 4 & 5 & 6 & 7 & 3 \\
85.51 & 68.67 & 54.0 & 53.91 & 53.46 & 452.12 & 49.56 & 49.21 & 47.64 & 42.74
\end{bmatrix}
\]

from which we can see that the problem of students of equal total scores is overcome. The ranks of students in a descending order are shown in Table III.

**IV. Conclusions**

In this paper, a fuzzy evaluation system for students’ academic progress based on fuzzy linguistic hedges is
introduced. The system adjusts questions’ grades by incorporating other factors such importance, complexity and difficulty of exam questions in a way that can reflect deep learning and skills acquired during the course. Using linguistic hedges improved the system’s objectivity in differentiating between students who decided to go for complex and difficult questions with associated higher cost and risk. The developed system was compared to classical ranking approach and fuzzy approaches based on classical sets. Transparency of fuzzy systems and its ability to provide fairer and objective evaluation are expected to increase its social acceptance and convince stakeholders and educational community to adopt these approaches. These approaches are not limited to academic evaluation, it can be used in other decision support systems and other control applications where linguistic hedges can be used to adapt its performance.

REFERENCES