Long-term analysis of gear loads in fixed offshore wind turbines considering ultimate operational loadings

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Abstract

The long-term extreme value analysis of gear transmitted load due to the main shaft torque is presented. Two methods, the multibody simulations (MBS) and a simplified method, are demonstrated for the gear transmitted load calculation. The simplified method is verified by the MBS results. The long-term extreme value of the gear transmitted load for wind speeds from the cut-in to the cut-out values is calculated by the simplified method from the long-term distribution of the main shaft torque. Three statistical methods for long-term extreme value analysis of the main shaft torque in the offshore wind turbines are presented. They are then used to predict the extreme value of the gear transmitted load. An alternative approach, the design state or the environmental contour method is proposed and verified by the full long-term results. The methods are exemplified by a 5 MW gearbox case study. The results of this paper are the basis for further work in Ultimate Limit State (ULS) gear design.

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1. Introduction

The wind industry is moving further offshore and deeper water exposing wind turbines and their components (such as drivetrains) to extreme loads and higher design uncertainties. With no doubt, current challenges with land based and fixed offshore wind turbine’s gearboxes needs to be well understood in order to limit uncertainties for future floating concepts. An overview of the published researches indicates

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that the design process may have the biggest contribution to the premature failures of wind turbine’s gearboxes [1]. There are very few publications addressing the detailed investigation of the wind turbine gear train failures – apparently due to the industrial confidentiality – but the replacement of gear trains in short time after installation is a strong evidence of design or perhaps manufacturing problems. According to Hau [2] “the cause of this problem is not the gearbox itself, but the right sizing of components with regards to the load spectrum”.

The current wind turbine gearbox design approach is based on IEC 61400-4 guideline [3] which refers to ISO 6336 series of standards [4] for the gear components. In these design codes, gears are designed by semi-probabilistic methods using safety factors or “application factors”. This method has several ambiguities with regards to the uncertainty sources that should be considered in the safety factors. In order to achieve a desired reliability level, safety factors shall be obtained from a systematic reliability analysis to reflect the uncertainties in estimating the load, load response and resistance in the wind turbine application. As an example, in the structural design of offshore structures, the partial safety factors are calibrated based on the reliability methods, meaning that a certain limit of reliability – annual target reliability – is expected if the calibrated factors are applied [5].

For the design with respect to ULS, the long-term extreme response needs to be estimated. The main objective of this paper is to establish a method for calculating the long-term distribution of the extreme values of the gear transmitted force. This is one step toward a rational reliability analysis of wind turbine gearbox based on the well-established structural reliability methods which will be presented in the future work.

2. Methods for gear transmitted load calculation

The decoupled analysis method is used to estimate the drivetrain dynamic response. The global analysis is performed using the aero-servo-elastic code HAWC2 [6] and then a local analysis of the drivetrain is carried out using the simplified method considering the main shaft torque from the HAWC2 analysis as input. In this section, the simplified method and its comparison with MBS method is presented.

2.1. MBS method

In multibody simulation (MBS), the drivetrain can be modeled as an interconnected system of rigid bodies connected with dampers and springs. In the MBS method, each component is modeled as a rigid or flexible body connected with appropriate joints or stiffness to the others. The application of the MBS model in wind turbine drivetrains is proven by comparison with the finite element method (FEM) and the experimental results [7-10]. Parker et al [11] has indicated that the elastic deformations of the gears are much smaller than the rigid body motions and thus they can be superposed. In a general form, if \( x \) represents the displacement vector of gears including tooth deformation, the motion equation is expressed as [11]:

\[
M \ddot{x} + C \dot{x} + K x = F
\]  

in which \( F \) is the internal and external force vector, including moments and torques and \( M \), \( C \) and \( K \) are inertial, damping and stiffness vectors respectively. They also include gear tooth stiffness and damping. The Newmark method can be used for time integration of the above equation [11-13] or MBS software such as SIMPACK [14] can be employed. In SIMPACK, the gear contact is modeled with consideration of tooth geometry and modifications. The advantage of this method is that the dynamic effect of components is inevitably included in the resulted response. Moreover the non-torque forces and moments
can be added to the input loadings. Nevertheless, the accuracy of the results is directly dependent to the precision of given stiffness, mass and damping values.

2.2. Simplified method

In the early design stage, the stiffness and damping parameters of the components are often unknown. The problem can be more simplified by assuming that the bodies are rigid and their interaction is through rigid contacts with zero damping. Since the natural frequency of the gearbox is much higher than the frequency contents of the input loads on the main shaft, the gearbox behaves quasi-statically. Thus, the internal gear dynamic effect can be neglected, and the interaction force between two gears is calculated directly from the external forces. Assuming $T_{ms}(t)$ is the time series of the torque on the main shaft, the transmitted force $F_i'(t)$ in the $i$ stage can be estimated from:

$$F_i'(t) = \left( \frac{2(m_{gi}m_{Gi(1-\frac{1}{N})}m_{Gi(\frac{1}{2})}m_{i(\frac{1}{2})})}{Nd_i} \right) T_{ms}(t) \tag{2}$$

where $m_{gi}$ is the stage gear ratio, $N$ is the number of planets in $i$ th stage, $d$ is the working pitch diameter and $n$ number of stages. In order to verify the simplified method, the gear transmitted force at cut-out wind speed is compared with MBS model for the 5 MW wind turbine gearbox described in Appendix A.

![Figure 1: MBS vs simplified approach comparison of gear transmitted force at cut-out wind speed for 5 MW drivetrain.](image)

Fig. 1 and Table 1 present the comparison results on the pitch circle. C.O.V in Table 1 represents the coefficient of variation. There exists a slight difference between the MBS method and the simplified method. The main reason of this difference is because of ignoring the internal dynamics in the gear system in the simplified model.

**Table 1**: MBS vs simplified (Simp.) method comparison of gear transmitted force at cut-out wind speed for 5 MW drivetrain.

<table>
<thead>
<tr>
<th></th>
<th>mean (kN)</th>
<th>standard deviation (kN)</th>
<th>C.O.V%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1\textsuperscript{st} stage</td>
<td>MBS 813.4</td>
<td>66.4</td>
<td>8.2</td>
</tr>
<tr>
<td></td>
<td>Simp. 816.1</td>
<td>66.0</td>
<td>8.1</td>
</tr>
<tr>
<td>2\textsuperscript{nd} stage</td>
<td>MBS 249.1</td>
<td>20.2</td>
<td>8.1</td>
</tr>
<tr>
<td></td>
<td>Simp. 249.4</td>
<td>20.2</td>
<td>8.1</td>
</tr>
<tr>
<td>3\textsuperscript{rd} stage</td>
<td>MBS 233.6</td>
<td>19.2</td>
<td>8.2</td>
</tr>
<tr>
<td></td>
<td>Simp. 237.5</td>
<td>19.2</td>
<td>8.1</td>
</tr>
</tbody>
</table>
3. Long-term environmental condition

The wind load is the only environmental load considered in this study. The cumulative distribution function \( F_u(u) \) of the 1-h mean wind speed \( u \) at 10 m above the average sea level is modeled by 2 parameters Weibull distribution [15,16]:

\[
F_u(u) = 1 - \exp \left[-\left(\frac{u}{\beta}\right)^\alpha\right]
\]

in which \( \alpha \) and \( \beta \) are the shape and scale parameters, for instance, 1.708 and 8.426 for the Northern North Sea respectively [15]. The wind speed at hub height \( u_{hub} \) is calculated from the power law, with the power value of 0.14 for offshore fields [17]:

\[
u_{hub} = u \left(\frac{h_{hub}}{10}\right)^{0.14}
\]

4. Methods for long-term extreme value analysis

In this section the most common extreme value analysis methods used in offshore structures are described. Such methods can also be used to predict the long-term extreme response in wind turbines. The extreme value methods are briefly introduced in Annex F of IEC 61400-1 [18], however, they are not comprehensive.

The idea behind the long-term extreme load analysis is to obtain an estimate of the extreme load with a given probability of exceedance by accounting the load from various short-term wind conditions. In general, two procedures for estimation of long-term extreme load can be considered:
- Method 1: by use of long-term extreme value analysis of independent wind conditions
- Method 2: by use of short-term extreme wind condition (i.e. the design state method or the environmental contour method)

The reference period for extreme response is selected as one year such that the result can be used directly for annual failure probability and reliability analysis [19]. However, the following methods can also be used to predict the life time (e.g. 20 years) extreme value, as required for Ultimate Limit State (ULS) design requirement.

4.1. Long-term extreme value analysis of independent wind conditions

There are three different approaches to estimate the long-term extreme values. These methods are based on (1) all peak values, (2) short-term extremes, or (3) the up-crossing rate [20].

4.1.1. Method of all peak values

For a given wind speed of \( u \), if \( F_{SP}^{ST}(s|u) \) is the short-term Cumulative Distribution Function (CDF) of the global peaks in a reference period of 1 hour, the unconditional long-term CDF of global peaks is expressed as:

\[
F_{SP}^{LT}(s) = \frac{1}{v_s^*} \int_{s}^{\infty} F_{SP}^{ST}(s|u) \cdot f(u) \cdot du
\]
value [21,22]. The \( f(u) \) is the probability density function of the 1-h mean wind speed at hub height modeled by the Weibull distribution – see section 3. The \( \bar{v}_s^+(m) \) denotes the long-term average up-crossing rate of the mean value given by:

\[
\bar{v}_s^+(m) = \int_{cut-out}^{cut-in} v_s^+(m | u) \cdot f(u) \cdot du
\]  

(6)

The wind load response time-domain simulation is often carried out in 1 meter per second wind span, thus equation (5) can be obtained numerically:

\[
F_{sp}^{LT}(s) = \frac{1}{\bar{v}_s^+(m)} \sum_{cut-out}^{cut-in} v_s^+(m | u_i) \cdot F_{sp}^{ST}(s | u_i) \cdot \left( f(u_i) \cdot \Delta u_i \right)
\]  

(7)

Equation (5) can be further simplified by excluding the effect of average up-crossing rate of the mean value; thus the reduced form can be:

\[
F_{sp}^{LT}(s) = \int_{cut-out}^{cut-in} F_{sp}^{ST}(s | u) \cdot f(u) \cdot du
\]  

(9)

The difference between the original method and the simplified approach is presented in the result section.

In the all peaks method, the distribution function of the annual extreme load can be approximated by:

\[
F_{LT \text{ year}}^{\text{all}}(s) = \left[ F_{sp}^{LT}(s) \right]^N
\]  

(10)

in which \( N = \bar{v}_s^+(m) \cdot T \) and \( T \) is one year in second. The annual extreme load distribution is then fitted by a Gumbel distribution, for instance by the moment method or the use of probability paper [23].

4.1.2. Method of short term extremes

Let \( F_{ST \text{ short}}^{\text{short}}(s) \) denotes the conditional CDF of the 1-h short-term extreme values, the long-term distribution of the 1-h extreme value is expressed by:

\[
F_{ST \text{ short}}^{LT}(s) = \int_{cut-out}^{cut-in} F_{ST \text{ short}}^{ST}(s | u) \cdot f(u) \cdot du
\]  

(11)

If \( N = 8760 \) 1-h realizations in one year return period are considered independent, the distribution function of the annual extreme value can be approximated by [24]:

\[
F_{LT \text{ year}}^{\text{short}}(s) = \left[ F_{ST \text{ short}}^{LT}(s) \right]^N
\]  

(12)

Similar as in the all peak method, a Gumbel distribution can be fitted for the annual extreme value distribution.
4.1.3. Up-crossing rate method

The CDF of global extreme value over a long-term period of $T$ - one year in this case - can be expressed directly as follow [20]:

$$F_{S_{\text{ext}}-\text{year}}^{ST}(s) = \exp \left\{ -T \int_{c_i}^{c_o} \nu_s^{ST} (s | u) \cdot f(u) \cdot du \right\}$$ (13)

in which $\nu_s^{ST} (s | u)$ is the average $s$-up crossing rate for the wind condition of $u$. This method is straightforward since no short-term distribution fitting is required. If the annual extreme value follows a Gumbel distribution, the Gumbel parameters can be directly obtained by the exponential curve fitting to the left side of the following equation:

$$T \int_{c_i}^{c_o} \nu_s^{ST} (s | u) \cdot f(u) \cdot du = \exp \left\{ -\left( \frac{s - \mu}{\alpha} \right) \right\}$$ (14)

where $\mu$ and $\alpha$ are the Gumbel parameters.

4.2. Conditional extreme event analysis or “design state”

In principle, the long-term extreme load analysis should account for the relative contribution from the different short-term wind conditions in a consistent manner. It is known that the eigen frequency of components – in particular gears and bearings – in wind turbine drivetrains are normally far above the excitation frequencies [7,25,26] even in very large wind turbines [27]. Therefore, the dynamic amplification in wind turbine drivetrain response due to the resonance under external excitations is unlikely to take place. The extreme load response in drivetrains will most likely occur at the above rated wind speeds when the extreme torque at the main shaft occurs. In the other words, the contribution from the short-term wind conditions near the cut-out wind speed is more than the other wind conditions in the long-term extreme load response analysis. This fact is demonstrated in Fig. 2 by presenting the exceedance probability $Q_{S_{\text{ext}}-\text{in}}^{ST} (s_0 | u) = 1 - F_{S_{\text{ext}}-\text{in}}^{ST} (s_0 | u)$ for $s_0 = 1.75 \times T_{\text{rated}}$ as a function of wind speeds from cut-in to cut-out. $s_0$ is the estimated mean value of the annual extreme and $T_{\text{rated}}$ is defined in equation (16) – see section 5. The ratio of $Q_{S_{\text{ext}}-\text{in}}^{ST} (s_0 | u) \cdot (f(u) \cdot \Delta u)$ is plotted in Fig. 2, in which $Q_{S_{\text{ext}}-\text{in}}^{ST} (s_0)$ is calculated from:

$$Q_{S_{\text{ext}}-\text{in}}^{ST} (s_0) = \sum_{c_i}^{c_o} Q_{S_{\text{ext}}-\text{in}}^{ST} (s_0 | u) \cdot (f(u) \cdot \Delta u)$$ (15)

Figure 2: Contribution of different wind speeds in the long-term exceedance probability of the expected annual extreme value (5 MW drivetrain).
The “design wind condition” is the wind condition that the expected largest conditional short-term response is equal to the largest response obtained from a full long-term analysis for a given return period with a proper correction factor [28]. This is also known as contour line (surface) method.

If design wind condition is known, one can run a drivetrain model test only under this wind speed and obtain a reasonable estimate of the long-term load response rather than a test under all the wind conditions. This approach can save a lot of time. For structures under wave loads, similar approach is often applied in model tests [19,28]. Nevertheless, the “design wind condition” should be selected by the long-term analysis in the first place. In the case study, the design wind condition is identified for the drivetrain of 5 MW wind turbine. There are many methods available for the short-term extreme value analysis based upon time series of responses from the time domain simulations [21,22,29]. The all peak method has been used in this paper for finding the design wind condition.

5. Results: 5 MW drivetrain case study

A gearbox for a 5 MW bottom-fixed offshore wind turbine, designed based on the wind turbine data from the NREL 5 MW reference turbine [30], is chosen as an example to demonstrate the proposed procedures for extreme response analysis. The wind turbine as well as the gearbox technical specifications are presented in appendix A.

The aerodynamic simulation of 5 MW case study wind turbine is carried by the HAWC2 version 11.3 [6]. For each wind speed, 15 simulations in 800 second is carried out but the first 200 second is removed to avoid transitional effects and numerical uncertainties. The IEC 61400-1 “B” wind class with reference turbulence intensity factor of 14% is considered. Fig. 3 shows the long-term annual extreme value CDF and PDF of the main shaft torque. The data is normalized by the rated torque:

$$T_{\text{rated}} = \frac{60P_{\text{rated}}}{2\pi n_{\text{rated}}} = \frac{60 \times 5000}{2\pi \times 12.1} = 3,946 \text{ KNm}$$

The Gumbel parameters in the form given below are listed in the Table 2 for various methods. Also the Most Probable Value (MPV) in a normalized form is listed.

$$F_{S}(s) = \exp \left\{ -\exp \left[ -\left( \frac{s - \mu}{\alpha} \right) \right] \right\}$$

Table 2: Gumbel parameters of the annual extreme value distribution of drivetrain input torque computed by different methods.

<table>
<thead>
<tr>
<th>Long-term analysis methods</th>
<th>$\mu$ (kNm)</th>
<th>$\alpha$ (kNm)</th>
<th>MPV</th>
</tr>
</thead>
<tbody>
<tr>
<td>All peaks – original</td>
<td>6996</td>
<td>197.2</td>
<td>1.78</td>
</tr>
<tr>
<td>All peaks – simplified</td>
<td>7137</td>
<td>190.3</td>
<td>1.82</td>
</tr>
<tr>
<td>Short term extremes</td>
<td>6850</td>
<td>198.0</td>
<td>1.74</td>
</tr>
<tr>
<td>Up-crossing rate</td>
<td>6760</td>
<td>185.0</td>
<td>1.72</td>
</tr>
</tbody>
</table>
In order to find the “design wind condition”, first the 1-h short-term extreme value CDF for the cut-out wind speed is calculated. The 20-year CDF is then extrapolated and the Most Probable Value (MPV) is obtained. It should be noted that the cut-out wind speed occurs more than once during the 20-year design life; therefore, the 20-year extrapolation is carried based on following equations:

\[
F_{ST,20\text{-year}}(s) = \left[F_{ST,1\text{-h}}(s)\right]^{\frac{20 \times 365}{N}}
\]  

(18)

where \(N\) is the return period (in days) of the cut-out wind speed in 20-year obtained from:

\[
-N \ln \left( \frac{1}{20 \times 24 \times N} \right) = \left( \frac{u_{hub}}{\beta} \right)^{\frac{1}{\alpha}}
\]

(19)

in which \(u_{hub}\) is the wind speed at hub height and \(\alpha\) and \(\beta\) are the Weibull parameters – see section 3. From above equation, the cut-out wind speed return period is calculated as 10.86 days. The \(F_{ST,20\text{-year}}(s)\) or 20-year short-term CDF at cut-out wind speed is very close to the 20-year long-term value. Therefore for similar drivetrains, one can calculate the expected 20-year extreme value by extrapolating the 1-h extreme value at cut-out wind speed to 20-year and measuring the Most Probable Value (MPV). This value is a good estimate for the 20-year MPV extreme value with a difference of about 3%, indicating a correction factor of 1.03 - Fig. 4.
presented in the Table 3. They are calculated based on the long-term CDF of the torque extreme values obtained from the original all-peaks method.

Table 3: Gumbel parameters of the annual extreme values of gear transmitted force.

<table>
<thead>
<tr>
<th>Stage</th>
<th>$F'$ ($kN$)</th>
<th>$\mu (kN)$</th>
<th>$\alpha (kN)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st stage</td>
<td>1365.18</td>
<td>38.48</td>
<td></td>
</tr>
<tr>
<td>2nd stage</td>
<td>417.14</td>
<td>11.76</td>
<td></td>
</tr>
<tr>
<td>3rd stage</td>
<td>397.27</td>
<td>11.20</td>
<td></td>
</tr>
</tbody>
</table>

6. Concluding remarks

The long-term extreme value of the main shaft torque has been estimated through 1) a full long-term extreme value analysis and 2) a conditional short-term estimate. Three methods for the long-term extreme value analysis (all peaks, up-crossing rate and short-term extremes methods) have been discussed. The difference of the results between the methods is found to be about 5-6% of mean value but with negligible difference in the standard deviation- the MPV of the annual extreme value differs from 1.72 to 1.82 in a normalized scale. The method “short-term extremes” is found as the simplest one for the long-term extreme value analysis. Alternatively the design state or contour method is demonstrated. It is shown that the short-term extreme value CDF of the wind speed of 25 m/sec – cut-out wind speed – extrapolated in 20 years is a good estimate of the MPV of the 20-year extremes.

Two methods, the MBS and a simplified method are presented for the gear transmitted load calculation from the main shaft torque. Good agreement is observed between the simplified model and MBS results for the case study 5 MW gearbox. The results of this paper are the basis for further work in Ultimate Limit State design (ULS) and reliability analysis.

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References

[7] Xing Y., Moan T., Multi-body modelling and analysis of a planet carrier in a wind turbine gearbox, Wind Energy, 2012 (DOI:


## Appendix A. 5 MW case study gearbox technical data

### Table A-1: 5 MW NREL reference wind turbine [30].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rating (MW)</td>
<td>5</td>
</tr>
<tr>
<td>Cut-in wind speed (m/sec)</td>
<td>3</td>
</tr>
<tr>
<td>Cut-out wind speed (m/sec)</td>
<td>25</td>
</tr>
<tr>
<td>Rated wind speed (m/sec)</td>
<td>11.4</td>
</tr>
<tr>
<td>Rated rotor speed (rpm)</td>
<td>12.1</td>
</tr>
<tr>
<td>Hub height (m)</td>
<td>90</td>
</tr>
<tr>
<td>Rotor diameter (m)</td>
<td>126</td>
</tr>
</tbody>
</table>

### Table A-2: 5 MW gearbox specifications.

<table>
<thead>
<tr>
<th>Type</th>
<th>Ratio</th>
<th>Designed power (MW)</th>
<th>Rated generator speed (rpm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2P+1H</td>
<td>1:97.20</td>
<td>5.00</td>
<td>1173.7</td>
</tr>
</tbody>
</table>

P: Planetary, H: Parallel helical

### Table A-3: Gear specifications.

<table>
<thead>
<tr>
<th>Stage</th>
<th>Type</th>
<th>1st stage</th>
<th>2nd stage</th>
<th>3rd stage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Planetary</td>
<td>1:5.17</td>
<td>1:5.80</td>
<td>1:3.24</td>
</tr>
<tr>
<td>Number of planets</td>
<td>3</td>
<td>3</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Normal Module (mm)</td>
<td>22</td>
<td>12</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>Normal Pressure angle (degree)</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>Helix angle (degree)</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>Face width (mm)</td>
<td>500</td>
<td>235</td>
<td>152</td>
<td></td>
</tr>
<tr>
<td>Centre distance (mm)</td>
<td>857</td>
<td>541</td>
<td>781</td>
<td></td>
</tr>
<tr>
<td>Number of teeth, sun</td>
<td>29</td>
<td>30</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Number of teeth, planet</td>
<td>46</td>
<td>57</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>Number of teeth, ring</td>
<td>121</td>
<td>144</td>
<td>81</td>
<td></td>
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</table>