JOINT ENVIRONMENTAL DATA AT FIVE EUROPEAN OFFSHORE SITES FOR DESIGN OF COMBINED WIND AND WAVE ENERGY DEVICES

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ABSTRACT

The costs for an offshore wind farm, especially with bottom fixed foundations increase significantly with increasing water depth. If costs can be reduced to a competitive level, the potential for wind farms in deep water is huge. One way of reducing costs might be to combine offshore wind with wave energy facilities at sites where these resources are concentrated.

In order to design combined renewable energy concepts, it is important to choose sites where both wind and wave energy resources are substantial. Such facilities might be designed in ultimate limit states based on load effects corresponding to 50-year wind and wave conditions. This requires a long-term joint probabilistic model for the wind and wave parameters at potential sites.

In this paper, five European offshore sites are selected for analysis and comparison of combined renewable energy concepts developed in the EU FP7 project – MARINA Platform. The five sites cover both shallow water (<100m) and deep water (>200m), with three sites facing the Atlantic Ocean and the other two sites in the North Sea. The selection of the sites is carried out by considering average wind and wave energy resources, as well as extreme environmental conditions which indicate the cost of the system.

Long-term joint distributions of mean wind speed at 10-meter height ($U_{10}$), significant wave height ($H_s$) and spectral peak period ($T_p$) are presented for selected sites. Simultaneous hourly wind and wave hindcast data from 2001-2010 are used as a database, which are obtained from the National and Kapedistrian University of Athens.

The joint distributions are estimated by fitting analytical distributions to the hindcast data following a procedure suggested by Johannessen et al. (2001). The long-term joint distributions can be used to estimate the wind and wave power output from each combined concept, and to estimate the fatigue lifetime of the structure. For estimation of the wind and wave power separately, the marginal distributions of wind and wave are also provided.

Based on the joint distributions, contour surfaces are established for combined wind and wave parameters for which the probability of exceedance corresponds to a return period of 50 years. The design points on the 50-year contour surfaces are suggested for extreme response analysis of combined concepts. The analytical long-term distributions established could also be applied for design analysis of other offshore structures with similar environmental considerations of these sites.

INTRODUCTION

Offshore wind has an enormous potential for a long-term sustainable energy supply. However, as the installed capacity increases, suitable sites in shallow waters are becoming scarce – driving the technology into deeper waters. The exploitation of deeper waters comes with significant design, installation, grid connection, operation and maintenance challenges, which will increase the costs significantly. One possible way of reducing offshore wind costs is to exploit synergies with other renewable marine technologies. One choice is to combine offshore wind with wave energy at sites where both resources are concentrated. The EU project – MARINA Platform is dedicated to investigate the potential for combining offshore wind and wave energy devices [1]. Many new concepts with combinations of different wind turbines and wave energy converters are under analysis in this project.

For design of these combined concepts, 50-year extreme responses should be examined [2]. A consistent approach is to estimate the full long-term responses, in which the responses at all of the environmental conditions should be taken into account with the consideration of their probability of occurrence, so it
requires a large number of simulations. For a complicated response problem, the computational time is rather long, which makes the full long-term analysis impractical.

In this case, it is more convenient to apply the contour surface method (contour line method when two environmental variables are present) [3,4] to estimate the long-term extremes by exposing the structure to a short-term extreme sea state. Several sea states are chosen from the 50-year environmental contour surfaces, which can be extrapolated from a long-term joint distribution. The sea states that leads to the largest response should be chosen as the designed sea states. To account for the variability of the sea states, one derived extreme value needs to be taken considering a higher quantile of 75%-90% [5] rather than 50%. Alternatively, it can be obtained by multiplying a correction factor of 1.1-1.3 [6].

Therefore, in order to predict estimates for the 50-year response extremes, one in principle needs a long-term joint distribution model of important environmental parameters under consideration. Johannessen et al. [7] described a procedure to achieve joint distributions of wind and wave parameters and presented contour surfaces at one location in the Northern North Sea. To compare and analyze different combined wind and wave concepts, environmental information at more sites located at different areas should be applied.

The purpose of this paper is to provide a set of environmental conditions for the design of combined concepts. Five sites in both the Atlantic Ocean area and the North Sea area are considered. The sites are selected by comparing the factors indicating the energy outputs and the costs of the concepts. Long-term joint distributions of mean wind speed at 10-meter height ($U_w$), significant wave height ($H_s$) and spectral peak period ($T_p$) at these sites are acquired by fitting 10 years’ environmental hindcast data by using the procedure described by Johannessen et al [7]. For conditional distribution of $T_p$ for given $U_w$ and $H_s$, a simplified method is discussed. It is shown that the simplified method does not influence the determination of critical conditions for design purpose. Finally, the 50-year contour surfaces at the five sites are presented for the extreme response analysis.

**ENVIROMENTAL CONDITIONS AT FIVE SELECTED EUROPEAN OFFSHORE SITES**

**Data description**

The environmental data used in this study are generated by a numerical hindcast model from National and Kapodistrian University of Athens (NKUA). In order to select proper sites for the combined wind and wave energy concepts, a large database is needed for comparisons of site conditions, and observations can hardly satisfy the requirements. Hence, numerical hindcast data are more appropriate than observational records for site selection due to its wide spatial and temporal coverage and homogeneity.

The wave model uses wind input produced by the atmospheric model and the grid of the wave model covers the whole North Atlantic Ocean [8]. Assimilation techniques are applied for the correction of initial wind and wave conditions [9,10]. By comparing with measurements, the results from the numerical model are generally correlated to the measurements. However, the numerical model has some uncertainties in cases of shallow water or mixed seas (swell and wind sea), which require more calibrations [11].

**Selection of European sites**

**FIGURE 1. LOCATION OF EIGHTEEN POTENTIAL EUROPEAN OFFSHORE SITES**

Eighteen sites in the Atlantic Ocean and the North Sea area are considered as potential sites for the design of combined renewable energy concepts in the MARINA Platform project. General information and important statistics of these sites are shown in Table 1. The locations of these sites are pin-pointed on the map in Figure 1. Based on the information in the table, we can observe some characteristics of these sites:

1) The average water depth in the North Sea area is relatively small except for the sites close to Norway (site 14 and 16). For some of the sites which are very far from shore (site 15) the water depth are still very small. However, there exists a variation in water depth in the North Sea area. These sites with shallow water depth are suitable for bottom-fixed concepts.

2) Sites at Atlantic area have more wave energy resources on average compared to those at the North Sea area while the wind resources vary less from area to area except for sites 6, 7, 14 and 16 for which the available wind power is larger.

3) Two types of wave period distribution are envisaged. The average $T_p$ in the Atlantic Ocean is larger than that in the North Sea area. One important reason is that the area is relatively open in the Atlantic Ocean, so swell occurs more frequently which contributes to the large $T_p$ value while the waves in the North Sea area are mainly wind-generated waves, except for site 14 and 16 in the Northern North Sea which are actually open to the Atlantic Ocean.
After a study on the environmental data at these eighteen sites, it is noted that some of the sites have very similar wind and wave conditions. Therefore, it is concluded that five of these eighteen sites are representative for concept comparison of the combined wind and wave energy concepts. The following three important factors are considered for site selection:

1) Site geographic conditions (area, water depth and distance to shore);

2) Average wind and wave energy resources;

3) Extreme values of wind and wave conditions.

First, the selected sites should cover both Atlantic Ocean and the North Sea area, and include both deep water and shallow water for bottom-fixed and floating concepts respectively. Then, for combined wind and wave energy concepts, it is essential to require the specified sites to have both considerable wind and wave energy resources. Finally, the extreme wind speeds and significant wave heights should be examined to avoid significant increase to the costs.

Table 1 lists the factors mentioned above for the eighteen potential sites. For wind energy resources, wind power at 80 meters level which is the hub height of a typical offshore wind turbine is calculated. The average available wind and wave power density are the mean value of the power density at each hour in the 10 years' duration calculated from Eq. (1) based on the deep water assumption.

\[ P_{\text{wind}} = \frac{\rho \, U_{80}^3}{2} \quad P_{\text{wave}} = \frac{\rho \, H_{s}^2}{64 \pi} \quad T_{E} = \int_{0}^{\infty} \omega^2 S(\omega) d\omega \]  

(1)

where \( U_{80} \) is the wind speed at 80m height; \( H_{s} \) is the significant wave height; and \( T_{E} \) is the energy period given by

\[ T_{k} = \frac{2\pi}{\omega_{k}} = 2\pi \frac{m_{i}}{m_{0}} \quad m_{i} = \int_{0}^{\infty} \omega^2 S(\omega) d\omega \]  

(2)

Where \( m_{i} \) and \( m_{0} \) are the minus one and zero spectral moments; and \( S(\omega) \) is the wave spectrum.

The 50-year extreme values of wind speed and significant wave height in Table 1 are obtained by extrapolation of the marginal distributions of wind speed at 10 meters level and significant wave height. Two-parameter Weibull distributions are applied for both distributions. It will be shown later that Weibull distribution does not fit the marginal distribution of \( H_{s} \) perfectly for some sites and will underestimate the extreme values. However, in site selection process only two-parameter Weibull distribution is applied for simplicity. Other fitting methods will be introduced in the next section.

By considering the factors mentioned above, five sites are finally selected which are marked in Figure 1, with site information presented in Table 1. Moreover, a generic water depth (40m, 100m and 200m) will be considered instead of the actual water depth at each site for concept design. The site conditions are:

1) Site No. 01 with water depth of 40m
2) Site No. 05 with water depth of 40m
3) Site No. 15 with water depth of 40m
4) Site No. 03 with water depth of 200m
5) Site No. 14 with water depth of 200m

In the MARINA Platform project, analyses will be carried out at Site No. 01, 05 and 15 for bottom-fixed combined concepts, while for floating concepts analyses will be carried out at Site No. 03 and 14.

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**TABLE 1. GENERAL INFORMATION AND STATICS OF EIGTEEN EUROPEAN OFFSHORE SITES**

<table>
<thead>
<tr>
<th>Site No.</th>
<th>Area</th>
<th>Name</th>
<th>Water depth (m)</th>
<th>Distance to shore (km)</th>
<th>Average wind power density at 80m height (W/m²)</th>
<th>Average wave power density (kW/m)</th>
<th>50-year mean wind speed at 10m height (m/s)</th>
<th>50-year significant wave height (m)</th>
<th>Mean value of ( T_{p} ) (s)</th>
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<tr>
<td>01</td>
<td>Atlantic</td>
<td>Sem Rev</td>
<td>33</td>
<td>15</td>
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<td>8.15</td>
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<td>Buoy Estaca de Bares</td>
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<td>30</td>
<td>691.04</td>
<td>46.73</td>
<td>27.87</td>
<td>10.67</td>
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<td>42.72</td>
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<td>Wave Hub</td>
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<td>20</td>
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<td>31.79</td>
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<td>3</td>
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<td>37.1</td>
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<tr>
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<td>10.11</td>
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<tr>
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<td>FINO 3</td>
<td>22</td>
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<td>8.62</td>
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<tr>
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<td>Moray Firth</td>
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<td>25</td>
<td>659.87</td>
<td>6.05</td>
<td>26.50</td>
<td>5.94</td>
<td>6.77</td>
</tr>
</tbody>
</table>

* The 50-year mean wind speed at 10m height and the 50-year significant wave height are obtained from the marginal distributions.
PREDICTION OF LONG-TERM ENVIRONMENTAL CONDITIONS

The hindcast data has been sampled hourly for wind and wave and archived in a database from 2001 to 2010. The parameters used in the long-term joint distributions are:
- Mean wind speed at 10 meters height, $U_w$
- Significant wave height, $H_s$
- Wave spectral peak period, $T_p$

Marginal and joint distributions of wind and wave conditions are acquired by fitting analytical distributions to the raw data. The joint distribution of $U_w$, $H_s$ and $T_p$ can be applied to estimate the power output from the wind turbines and the wave energy converters in a combined concept. It should be noted that the mean wind speed considered in the joint distribution is at the height of 10 meters above the sea level. For power estimation of wind turbines, the mean wind speed at hub height is needed (e.g. the hub height of NREL 5MW wind turbine is 89m above the sea level) and can be obtained considering a wind speed profile. A preliminary study on wind speed at different levels indicates that a power law profile with the exponent $\alpha$ equal to 0.1 can be used for all of the five sites.

$$U(z) = U_{10}\left(\frac{z}{10}\right)^{\alpha} \quad (3)$$

where $z$ represents the height, $U_{10}$ is the mean wind speed at the reference height of 10 meters.

Usually it is more precise to use wind speed at higher levels if the wind profile is not stable. However, the wind speeds at all heights in the rotor plane (e.g. from 10m to 180m) of a wind turbine need to be considered for wind turbine load and response analysis. Therefore, for simplicity in present study the reference mean wind speeds at 10m height and a constant wind speed profile parameter $\alpha = 0.1$ are used to estimate wind speeds at higher levels.

For the case the wind and wave power is estimated separately, the marginal distributions of wind and wave are provided. The marginal distribution of $U_w$ (at hub height) can be used to estimate the wind power given the wind power curve of the wind turbine. Similarly, the joint distribution of $H_s$ and $T_p$ can be utilized to estimate the wave power given the power matrix of the wave energy converter.

As for ultimate limit state analysis, the environmental contour surface with a return period of 50 years is obtained. Extreme conditions with combinations of $U_w$, $H_s$ and $T_p$ can be selected along the contour surface.

Long-term distributions of wind and wave directions are not considered in this study and it is assumed that wind and waves are collinear and always have the same direction.

The fitting methods of marginal and joint distributions will be explained in details in the following subsections, and figures illustrating the fitting results at Site 14 are presented as examples to show the goodness of the fittings.

Marginal distribution of mean wind speed $U_w$

The raw data at five selected sites indicate that one-hour mean wind speed at 10m height ($U_w$) follows a two-parameter Weibull distribution and the probability density function is given in Eq. (4). Figure 2 shows the fitting curve of marginal distribution of $U_w$ at Site 14 on the Weibull probability paper. A good agreement between the raw data and the fitting is obtained. It should be mentioned that the maximum likelihood method is applied for the fittings of the distributions in this paper.

$$f_{U_w}(u) = \frac{\alpha_u}{\beta_u} \left( \frac{u}{\beta_u} \right)^{\alpha_u - 1} \exp \left[ - \left( \frac{u}{\beta_u} \right)^{\alpha_u} \right] \quad (4)$$

$\alpha_u$ and $\beta_u$ denote the shape and scale parameters, respectively. In this paper, $f()$ refers to the probability density function (PDF).

![Figure 2. Weibull plot of marginal distribution of $U_w$ at Site 14](image)

Joint distribution of $H_s$ and $T_p$

If only the wave data are considered, a joint PDF of $H_s$ and $T_p$ can be established. It consists of a marginal distribution of $H_s$ and a conditional distribution of $T_p$ for given $H_s$.

$$f_{H_s,T_p}(h,t) = f_{H_s}(h) \cdot f_{T_p|H_s}(t|h) \quad (5)$$

Regarding the marginal distribution of $H_s$, it seems that the main part of raw data follow a lognormal distribution while the data in the tail follow a Weibull distribution. Therefore, the hybrid lognormal and Weibull distribution - Lonowe model developed by Haver [12] is applied and the PDF is shown in Eq. (6).
\[ f_{H_s}(h) = \frac{1}{\sqrt{2\pi} \sigma_{LTC} h} \exp\left(-\frac{1}{2} \left(\frac{\ln(h) - \mu_{LTC}}{\sigma_{LTC}}\right)^2\right) h \leq h_0 \]

\[ f_{H_s}(h) = \frac{\alpha_{LTC} \left(\frac{h}{\beta_{LTC}}\right)^{\alpha_{LTC}-1}}{\beta_{LTC}^{\alpha_{LTC}}} \exp\left(-\left(\frac{h}{\beta_{LTC}}\right)^{\alpha_{LTC}}\right) h > h_0 \]  \hspace{1cm} (6)

\[ h_0 \] is the shifting point of \( H_s \) from the lognormal distribution to the Weibull distribution. \( \mu_{LTC} \) and \( \sigma_{LTC} \) are the parameters in the lognormal distribution, i.e. the mean value and the standard deviation of \( \ln(h) \), the natural logarithm function of \( h \). \( \alpha_{LTC} \) and \( \beta_{LTC} \) are the shape and scale parameters in the Weibull distribution, which are calculated by using the continuity condition of probability density function and cumulative density function at the shifting point. Figure 3 shows the fitting results of marginal distribution of \( H_s \) from a Lonowe model at site 14 on the Weibull probability paper. The fitted line from a pure Weibull model is also shown for comparison and this model does not fit the whole range of the data well from the figure.

For conditional distribution of \( T_p \) for given \( H_s \), the data seem to follow a lognormal distribution, which is also suggested by Johannessen et al [7].

\[ f_{T_p|H_s}(t|h) = \frac{1}{\sqrt{2\pi} \sigma_{LTC} t} \exp\left(-\frac{1}{2} \left(\frac{\ln(t) - \mu_{LTC}}{\sigma_{LTC}}\right)^2\right) \]  \hspace{1cm} (7)

\[ \mu_{LTC} = c_1 + c_2 h^{c_3} \]  \hspace{1cm} (8)

\[ \sigma_{LTC}^2 = d_1 + d_2 \exp(d_3 h) \]  \hspace{1cm} (9)

where \( \exp(\cdot) \) represents the exponential function. \( c_1, c_2, c_3, d_1, d_2 \) and \( d_3 \) are the parameters estimated from the raw data by nonlinear least-square curve fitting. The fittings are shown in Figure 5.

**FIGURE 5. FITTING OF LOGNORMAL PARAMETERS OF CONDITIONAL DISTRIBUTION OF \( T_p \) FOR GIVEN \( H_s \) (SITE 14)**

**Joint distribution of \( U_w, H_s \) and \( T_p \)**

The joint distribution of \( U_w, H_s \) and \( T_p \) consists of a marginal distribution of \( U_w \), a conditional distribution of \( H_s \) for given \( U_w \) and a conditional distribution of \( T_p \) for given both \( U_w \) and \( H_s \).

\[ f_{U_w,H_s,T_p}(u,h,t) = f_{U_w}(u) \cdot f_{H_s|U_w}(h|u) \cdot f_{T_p|U_w,H_s}(t|u,h) \]  \hspace{1cm} (10)

**FIGURE 4. LOGNORMAL PLOT OF CONDITIONAL DISTRIBUTION OF \( T_p \) FOR GIVEN \( H_s \) AT SITE 14 (a. 2.5m<H_s<3.0m; b. 7.0m<H_s<7.5m)**

The conditional distribution of \( T_p \) is estimated for different \( H_s \) classes with a bin size of 0.5m. Figure 4 shows the lognormal distribution fitting curves of \( T_p \) at two \( H_s \) classes. It can be seen that the lognormal distribution are reasonable for both low and high wind classes. In order to describe the conditionality of \( T_p \) on \( H_s \), the mean value \( \mu_{LTC} \) and variance \( \sigma_{LTC}^2 \) of \( \ln(t) \) are formed by smooth functions of \( H_s \).

**FIGURE 6. WEIBULL PLOT OF CONDITIONAL DISTRIBUTION OF \( H_s \) FOR GIVEN \( U_w \) AT SITE 14 (a. 4.0m<U_w<5.0m; b. 19.0m<U_w<20.0m)**
The marginal PDF of $U_w$ is the same as Eq. (4), while the conditional PDF of $H_s$ is given as two-parameter Weibull distribution,

$$f_{Hs|Uw}(h|u) = \frac{\alpha_{HC}}{\beta_{HC}} \left( \frac{h}{\beta_{HC}} \right)^{\alpha_{HC}-1} \exp \left( - \left( \frac{h}{\beta_{HC}} \right)^{\alpha_{HC}} \right)$$

(11)

where $\alpha_{HC}$ and $\beta_{HC}$ are the shape and scale parameters, respectively.

Figure 6 shows two examples of fitting curves of $H_s$ at two $U_w$ classes. The bin size of the wind data is 1 m/s. It seems that the raw data in low wind speed classes indicate a Lonewe model for conditional $H_s$ distribution, while for high wind speeds a Weibull model is more suitable. To get more accurate fitting for $H_s$ at high wind classes, only the Weibull model is considered. The shape and scale parameters are fitted as power functions of mean wind speed to express the conditionality:

$$\alpha_{HC} = a_1 + a_2 \mu^s$$

(12)

$$\beta_{HC} = b_1 + b_2 \mu^s$$

(13)

where $a_1, a_2, a_s, b_1, b_2$ and $b_3$ are the parameters estimated from the raw data by nonlinear curve fitting and the fitting curves are shown in Figure 7.

![FIGURE 7. FITTING OF WEIBULL PARAMETERS OF CONDITIONAL DISTRIBUTION OF $H_s$ FOR GIVEN $U_w$ (SITE 14)](image)

For the conditional distribution of $T_p$ for given $H_s$ and $U_w$, the data in each wind-wave class indicate a lognormal distribution.

$$f_{Tp|Hs,Uw}(t|u,h) = \frac{1}{\sqrt{2\pi \sigma_{\ln(T_p)}}} \exp \left( - \frac{1}{2} \left( \frac{\ln(t) - \mu_{\ln(T_p)}}{\sigma_{\ln(T_p)}} \right)^2 \right)$$

(14)

where $\mu_{\ln(T_p)}$ and $\sigma_{\ln(T_p)}$ are the parameters in the conditional lognormal distribution, i.e. the mean value and standard deviation of $\ln(t)$ at each combination of wave-wind class. Unlike in Eq. (7), now the parameters in the lognormal distribution are functions of both $H_s$ and $U_w$. According to the relationships:

$$\mu_{\ln(T_p)} = \ln \left( \frac{\mu_{T_p}}{1 + \nu_{T_p}^2} \right)$$

$$\sigma_{\ln(T_p)}^2 = \ln \left( \nu_{T_p}^2 + 1 \right)$$

(15)

$\mu_{T_p}$ and $\sigma_{T_p}$ are the mean value and standard deviation of $T_p$. $\nu_{T_p}$ is the coefficient of variance (COV). We need to fit $\mu_{T_p}$ and $\nu_{T_p}$ as functions of $U_w$ and $H_s$ to express the conditionality. Figure 8 shows the variation of the two parameters with $H_s$ and $U_w$. In the figure, the wave-wind classes with limited number of data were excluded from the analysis to avoid large uncertainties.

![FIGURE 8. MEAN AND COV OF $T_p$ FOR EACH WIND-WAVE CLASS (SITE 14)](image)

It can be observed that the two parameters are dependent mainly on $H_s$, while they shift with the variation of wind speed. Based on this, the parameterization of $T_p$ in the lognormal distribution could follow the methods suggested by Johannessen et al. [7]. The details of the method will not be given, but important equations are listed to explain the approach. The mean value of $T_p$ can be modelled by the following equation.

$$\mu_{T_p} = \overline{T_p}(u,h) = \overline{T_p}(h) \left[ 1 + \gamma \left( \frac{u - \overline{u}(h)}{\overline{u}(h)} \right) \right]$$

(16)

where $\overline{T_p}(h)$ and $\overline{u}(h)$ are the expected spectral peak period and mean wind speed for a given value of $H_s$. The two expected values are fitted as a function of $H_s$:

$$\overline{T_p}(h) = e_1 + e_2 \cdot h^b$$

(17)

$$\overline{u}(h) = f_1 + f_2 \cdot h^c$$

(18)

The term $1 + \gamma \left( \frac{u - \overline{u}(h)}{\overline{u}(h)} \right)^\gamma$ adjusts the expected $T_p$ according to whether the actual wind speed is above or below the expected wind speed for the particular significant wave height. In order to give reasonable values for $\gamma$ and $\overline{u}(h)$, Eq. (16) can be rewritten as,

$$\frac{\overline{T_p}(u,h) - \overline{T_p}(h)}{\overline{T_p}(h)} = \gamma \left( \frac{u - \overline{u}(h)}{\overline{u}(h)} \right)^\gamma$$

(19)
where \( \frac{\bar{T}_p(u,h) - T_p(h)}{T_p(h)} \) and \( \frac{u - \bar{u}(h)}{\bar{u}(h)} \) are the normalized period and wind speed, respectively. For each \( H_s \) class, the normalized period was plotted as a function of the normalized wind speed. Nearly linear relationships were observed for most \( H_s \) classes indicating \( y \) is close to 1. A mean value of \( \theta \) from all \( H_s \) classes is used for the slope in Eq. (16).

Moreover, the coefficient of variation can be assumed as a function of only \( H_s \) for simplification.

\[
\nu_{T_p}(h) = k_1 + k_2 \cdot \exp(hk_3) \tag{20}
\]

From above analysis, all the parameters for obtaining the lognormal distribution in Eq. (14) are calculated. Thus, the joint distribution modelled in Eq. (10) could be obtained.

It should be mentioned that the \( T_p \) data used in the joint distributions are the peak periods of the complete wave spectrums including both swell and wind sea components. \( T_p \) values from swell and wind sea are not analysed separately, which might not be physical when fitting conditional distribution of \( T_p \). One possible way to improve the fitting method is to fit the \( T_p \) values from swell and wind sea spectrums individually. In this case, a more complicated model is needed to fit the data.

**Distribution parameters and environmental contour surface**

By fitting the raw data with analytical distributions, the parameters in the long-term joint distributions are estimated for the five selected sites, which are presented in Tables 3, 4 and 5 in Appendix A. The associated equations are Eqs. (4)-(20).

Based on the joint distribution, a contour surface with a return period of 50 years can be obtained. This is done by transforming the joint distribution into a non-physical space consisting of three independent standard Gaussian variables using Rosenblatt transformation [13]. The three variables are reflecting the marginal variability of \( U_w \), conditional variability of \( H_s \) given \( U_w \) and conditional variability of \( T_p \) given \( U_w \) and \( H_s \), respectively. In this space, all 50-year combinations of the variables will be located on a sphere with a radius, \( r \), given by

\[
\Phi(r) = 1 - \frac{1}{N_{50}} \tag{21}
\]

\( N_{50} \) is the total number of one-hour sea state in 50 years. The 50-year contour surface of \( U_w, H_s \) and \( T_p \) can be obtained by transforming this sphere back to the physical parameter spaces.

The 3D 50-year contour surfaces for the five selected sites are shown in Figures 9-13 in Appendix B. In order to get readable values, the 2D contour lines of \( H_s \) and \( T_p \) for different levels of \( U_w \) are achieved from the contour surfaces and are also shown in the figures. For other values of \( U_w \), interpolation might be applied to obtain the \( H_s, T_p \) contour lines. Each figure contains eight sub-figures. The top-left sub-figure is the 3D contour surface and the top-right one shows the condition on the contour surface corresponding to the maximum mean wind speed, and the following six sub-figures are contour lines of \( H_s \) and \( T_p \) for different levels of mean wind speed.

Moreover, two particular extreme conditions from the contour surface are given in Table 2. The first one corresponds to the condition with the maximum mean wind speed and the second one corresponds to the condition with the maximum significant wave height. For structures that are not sensitive to wave periods, it might be sufficient to consider the extreme conditions in Table 2 for ultimate load state analysis.

**TABLE 2. ENVIRONMENTAL CONDITIONS ON THE 50-YEAR CONTOUR SURFACES WITH MAXIMUM \( U_w \) OR MAXIMUM \( H_s \)**

<table>
<thead>
<tr>
<th>Condition</th>
<th>Parameter</th>
<th>Site 01</th>
<th>Site 03</th>
<th>Site 05</th>
<th>Site 14</th>
<th>Site 15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Condition with maximum ( U_w )</td>
<td>( U_w ) (m/s)</td>
<td>23.7</td>
<td>28.3</td>
<td>27.5</td>
<td>33.6</td>
<td>27.2</td>
</tr>
<tr>
<td></td>
<td>( H_s ) (m)</td>
<td>8.0</td>
<td>8.8</td>
<td>11.4</td>
<td>13.4</td>
<td>8.1</td>
</tr>
<tr>
<td></td>
<td>( T_p ) (s)</td>
<td>12.2</td>
<td>11.9</td>
<td>13.5</td>
<td>13.1</td>
<td>10.0</td>
</tr>
<tr>
<td>Condition with maximum ( H_s )</td>
<td>( U_w ) (m/s)</td>
<td>21.4</td>
<td>24.3</td>
<td>25.1</td>
<td>31.2</td>
<td>25.3</td>
</tr>
<tr>
<td></td>
<td>( H_s ) (m)</td>
<td>10.2</td>
<td>12.1</td>
<td>14.0</td>
<td>15.6</td>
<td>9.5</td>
</tr>
<tr>
<td></td>
<td>( T_p ) (s)</td>
<td>13.8</td>
<td>13.8</td>
<td>15.11</td>
<td>14.5</td>
<td>12.3</td>
</tr>
</tbody>
</table>

**Simplified joint distribution of \( U_w, H_s \) and \( T_p \)**

The process of obtaining the distribution of \( T_p \) conditionally on both \( H_s \) and \( U_w \) is very complicated following the methods described by Johannessen et al. [7]. It is not straightforward to find a reasonable relationship between distribution parameters and \( U_w \). Moreover, the raw data indicate that the dependency of the distribution parameters for \( T_p \) on \( U_w \) is limited as seen from Figure 8. Hence, a simplified method is proposed that the distribution parameters for \( T_p \) are only dependent on \( H_s \), which simplifies the joint PDF of \( U_w, H_s \) and \( T_p \) as,

\[
f_{(Uw, Hs, Tp)}(u, h, t) \approx f_{Uw}(u) \cdot f_{Hs|Uw}(h|u) \cdot f_{Tp|Hs}(t|h) \tag{22}
\]

The marginal distribution of \( U_w \), conditional distribution of \( H_s \) for given \( U_w \) and the conditional distribution of \( T_p \) for given \( H_s \) are all examined in the analysis above. As the conditional distribution of \( T_p \) is only dependent on \( H_s \), the fitting of the lognormal parameters could be easily obtained. Following the simplified method, a contour surface can be also obtained and Figure 14 shows the contour surfaces from the simplified method for site 14. By comparing Figure 14 with the contour surface obtained from the complete methods in Figure 13, we can see that:

1. The \( T_p \) values at critical conditions which correspond to the largest \( H_s \) value for a given \( U_w \) level are almost the same from the two contours.
2. For larger \( T_p \) values, the corresponding \( H_s \) values are larger from the simplified method.
3. The shape of the contour lines for a given \( U_w \) level is skewed towards smaller \( T_p \) values for small \( H_s \) when taking \( U_w \) into consideration. The possible explanation is that the local wind increases the steepness of the wave, which results in smaller \( T_p \) value in the same \( H_s \) level. Because of this, the contours lines from the complete method cover broader range of
small $T_p$ values compared with those from the simplified method.

Therefore, for structures which are not sensitive to small $T_p$ values (normally less than 8s), the simplified method does not influence the determination of critical conditions for design, and gives larger $H_s$ values for larger $T_p$ value. However, if the structures are sensitive to $T_p$ value, the complete method should be applied to get more accurate contour lines.

CONCLUSIONS

1) Environmental conditions at eighteen European offshore sites are given in the MARINA Platform project. It is observed that sites at the Atlantic area have more wave energy resources compared with those at the North Sea area, while the variations in the wind energy resource from area to area are not significant as compared to the variations in the wave energy resource. The extreme values of $H_s$ are larger in the Atlantic area. During site selections, both energy resources and extreme conditions should be evaluated to maximize the power output and meanwhile reduce the potential costs of the combined wind and wave concepts. Five sites for concept design are selected from the eighteen sites considering these important factors as well as geographic conditions.

2) Long-term joint distributions of $U_w$, $H_s$ and $T_p$, marginal distributions of $U_w$, as well as joint distributions of $H_s$ and $T_p$ at five selected sites are presented. The long-term joint distribution can be applied to estimate the wind and wave power output from combined concepts, and to assess the fatigue damage. The marginal distributions of wind and wave can be used to estimate the wind and wave power separately. The parameters in the joint distributions are achieved by fitting hourly sampled data from 2001-2010 with analytical distributions. The data used in this paper are hindcast data generated from numerical model using assimilation techniques.

3) A simplified method is proposed to represent the long-term joint distributions of $U_w$, $H_s$ and $T_p$ by considering that the conditional distribution of $T_p$ only depends on $H_s$. Comparisons are made between the simplified and the complete methods. The simplified method does not influence the determination of critical conditions for design, and will lead to larger $H_s$ values for larger $T_p$ values. For structures sensitive to the $T_p$, the complete method is recommended. In addition, improvements in fitting the conditional distribution of $T_p$ could be made by examining wave periods from swell and wind sea components separately.

4) The 50-year contour surfaces for five selected sites are presented. From the contour surfaces, the 50-year extreme conditions could be selected. For different concepts, the most critical conditions may vary depending on the concept characteristics.

ACKNOWLEDGMENTS

The authors gratefully acknowledge the financial support from the European Commission through the 7th Framework Programme (MARINA Platform – Marine Renewable Integrated Application Platform, Grant Agreement 241402). The authors are also grateful to the Atmospheric Modeling and Weather Forecasting Group at NKUA for providing the hindcast data. The first author would like to thank Zhiyu Jiang from CeSOS, NTNU for valuable discussions.

REFERENCES

### APPENDIX A: DISTRIBUTION PARAMETERS AT FIVE SELECTED SITES

**TABLE 3. PARAMETERS FOR THE MARGINAL DISTRIBUTIONS OF $U_w$, $f_{U_w}(u)$**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Associated equation</th>
<th>Site No. 01</th>
<th>Site No. 03</th>
<th>Site No. 05</th>
<th>Site No. 14</th>
<th>Site No. 15</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{U}$</td>
<td>Eq. (4)</td>
<td>2.262</td>
<td>2.002</td>
<td>2.050</td>
<td>2.029</td>
<td>2.299</td>
</tr>
<tr>
<td>$\beta_{U}$</td>
<td>Eq. (4)</td>
<td>7.635</td>
<td>7.866</td>
<td>7.859</td>
<td>9.409</td>
<td>8.920</td>
</tr>
</tbody>
</table>

**TABLE 4. PARAMETERS FOR THE JOINT DISTRIBUTION OF $H_s$ AND $T_p$, $f_{H_s,T_p}(h,t) = f_{H_s}(h) \cdot f_{T_p|u}(t|h)$**

<table>
<thead>
<tr>
<th>Distributions</th>
<th>Parameter</th>
<th>Associated equation</th>
<th>Site No. 01</th>
<th>Site No. 03</th>
<th>Site No. 05</th>
<th>Site No. 14</th>
<th>Site No. 15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marginal distribution of $H_s$</td>
<td>$h_0$</td>
<td>Eq. (6)</td>
<td>3.5</td>
<td>5.0</td>
<td>5.0</td>
<td>5.0</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>$\mu_{HM}$</td>
<td>Eq. (6)</td>
<td>0.256</td>
<td>0.783</td>
<td>0.595</td>
<td>0.871</td>
<td>0.334</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{HM}$</td>
<td>Eq. (6)</td>
<td>0.583</td>
<td>0.493</td>
<td>0.557</td>
<td>0.506</td>
<td>0.615</td>
</tr>
<tr>
<td></td>
<td>$\alpha_{HM}$</td>
<td>Eq. (6)</td>
<td>1.160</td>
<td>1.385</td>
<td>1.179</td>
<td>1.433</td>
<td>1.369</td>
</tr>
<tr>
<td></td>
<td>$\beta_{HM}$</td>
<td>Eq. (6)</td>
<td>1.309</td>
<td>2.229</td>
<td>1.785</td>
<td>2.547</td>
<td>1.653</td>
</tr>
<tr>
<td>Conditional distribution of $T_p$ for given $H_s$</td>
<td>$c_1$</td>
<td>Eq. (8)</td>
<td>1.900</td>
<td>2.008</td>
<td>2.004</td>
<td>1.886</td>
<td>1.587</td>
</tr>
<tr>
<td></td>
<td>$c_2$</td>
<td>Eq. (8)</td>
<td>0.429</td>
<td>0.363</td>
<td>0.321</td>
<td>0.365</td>
<td>0.222</td>
</tr>
<tr>
<td></td>
<td>$c_3$</td>
<td>Eq. (8)</td>
<td>0.272</td>
<td>0.295</td>
<td>0.332</td>
<td>0.312</td>
<td>0.674</td>
</tr>
<tr>
<td></td>
<td>$c_4$</td>
<td>Eq. (8)</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.008</td>
</tr>
<tr>
<td></td>
<td>$d_2$</td>
<td>Eq. (9)</td>
<td>0.025</td>
<td>0.068</td>
<td>0.103</td>
<td>0.105</td>
<td>0.227</td>
</tr>
<tr>
<td></td>
<td>$d_3$</td>
<td>Eq. (9)</td>
<td>-0.487</td>
<td>-0.300</td>
<td>-0.285</td>
<td>-0.264</td>
<td>-0.956</td>
</tr>
</tbody>
</table>

**TABLE 5. PARAMETERS FOR THE JOINT DISTRIBUTION OF $U_w$, $H_s$ AND $T_p$, $f_{U_w,H_s,T_p}(u,h,t) = f_{U_w}(u) \cdot f_{H_s}(h|u) \cdot f_{T_p|u}(t|h)$**

<table>
<thead>
<tr>
<th>Distributions</th>
<th>Parameter</th>
<th>Associated equation</th>
<th>Site No. 01</th>
<th>Site No. 03</th>
<th>Site No. 05</th>
<th>Site No. 14</th>
<th>Site No. 15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marginal $U_w$</td>
<td>$\alpha_{U}$</td>
<td>Eq. (4)</td>
<td>2.262</td>
<td>2.002</td>
<td>2.050</td>
<td>2.029</td>
<td>2.299</td>
</tr>
<tr>
<td></td>
<td>$\beta_{U}$</td>
<td>Eq. (4)</td>
<td>7.635</td>
<td>7.866</td>
<td>7.859</td>
<td>9.409</td>
<td>8.920</td>
</tr>
<tr>
<td>Conditional $H_s$ for given $U_w$</td>
<td>$a_1$</td>
<td>Eq. (12)</td>
<td>1.894</td>
<td>1.643</td>
<td>2.044</td>
<td>2.136</td>
<td>1.755</td>
</tr>
<tr>
<td></td>
<td>$a_2$</td>
<td>Eq. (12)</td>
<td>0.012</td>
<td>0.093</td>
<td>0.034</td>
<td>0.013</td>
<td>0.184</td>
</tr>
<tr>
<td></td>
<td>$b_1$</td>
<td>Eq. (13)</td>
<td>1.741</td>
<td>1.000</td>
<td>1.375</td>
<td>1.709</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>$b_2$</td>
<td>Eq. (13)</td>
<td>0.929</td>
<td>1.969</td>
<td>1.323</td>
<td>1.816</td>
<td>0.534</td>
</tr>
<tr>
<td></td>
<td>$b_3$</td>
<td>Eq. (13)</td>
<td>0.024</td>
<td>0.031</td>
<td>0.032</td>
<td>0.024</td>
<td>0.070</td>
</tr>
<tr>
<td>Conditional $T_p$ for given $U_w$ and $H_s$</td>
<td>$\theta$</td>
<td>Eq. (16)</td>
<td>-0.268</td>
<td>-0.143</td>
<td>-0.233</td>
<td>-0.255</td>
<td>-0.477</td>
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<tr>
<td></td>
<td>$\gamma$</td>
<td>Eq. (16)</td>
<td>1.0</td>
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<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>$\epsilon_1$</td>
<td>Eq. (17)</td>
<td>5.0</td>
<td>5.0</td>
<td>8.0</td>
<td>8.0</td>
<td>5.563</td>
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<tr>
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<td>$\epsilon_2$</td>
<td>Eq. (17)</td>
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<td>5.970</td>
<td>2.600</td>
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<td>0.798</td>
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<td>$\epsilon_3$</td>
<td>Eq. (17)</td>
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<td>0.409</td>
<td>0.486</td>
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</tr>
<tr>
<td></td>
<td>$f_1$</td>
<td>Eq. (18)</td>
<td>2.0</td>
<td>1.0</td>
<td>1.8</td>
<td>2.5</td>
<td>3.5</td>
</tr>
<tr>
<td></td>
<td>$f_2$</td>
<td>Eq. (18)</td>
<td>3.947</td>
<td>4.055</td>
<td>3.478</td>
<td>3.001</td>
<td>3.592</td>
</tr>
<tr>
<td></td>
<td>$f_3$</td>
<td>Eq. (18)</td>
<td>0.620</td>
<td>0.466</td>
<td>0.667</td>
<td>0.745</td>
<td>0.735</td>
</tr>
<tr>
<td></td>
<td>$k_1$</td>
<td>Eq. (20)</td>
<td>-0.002</td>
<td>0.030</td>
<td>0.002</td>
<td>-0.001</td>
<td>0.050</td>
</tr>
<tr>
<td></td>
<td>$k_2$</td>
<td>Eq. (20)</td>
<td>0.341</td>
<td>0.234</td>
<td>0.298</td>
<td>0.316</td>
<td>0.388</td>
</tr>
<tr>
<td></td>
<td>$k_3$</td>
<td>Eq. (20)</td>
<td>-0.186</td>
<td>-0.221</td>
<td>-0.166</td>
<td>-0.145</td>
<td>-0.321</td>
</tr>
</tbody>
</table>
FIGURE 9. FIFTY-YEAR CONTOUR SURFACE FOR SITE 01 (COMPLETE METHOD)

FIGURE 10. FIFTY-YEAR CONTOUR SURFACE FOR SITE 03 (COMPLETE METHOD)
FIGURE 11. FIFTY-YEAR CONTOUR SURFACE FOR SITE 05 (COMPLETE METHOD)

FIGURE 12. FIFTY-YEAR CONTOUR SURFACE FOR SITE 15 (COMPLETE METHOD)
FIGURE 13. FIFTY-YEAR CONTOUR SURFACE FOR SITE 14 (COMPLETE METHOD)

FIGURE 14. FIFTY-YEAR CONTOUR SURFACE FOR SITE 14 (SIMPLIFIED METHOD)