A comparison of fiscal rules for resource-rich economies

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Abstract

This paper produces a normative evaluation of fiscal rules for a resource-rich economy. Ad hoc fiscal rules might imply substantial welfare costs; the goal is to analyze the magnitude of these costs by quantitatively evaluating the relative welfare sub-optimality of these rules. I posit a closed form solution for the infinite horizon consumption problem of the planner of a resource-rich economy with resource price uncertainty and precautionary saving. The model is subsequently calibrated and simulated to provide a welfare-based comparison between the fiscal rules based on the Permanent Income Hypothesis and on the ad hoc Bird-in-Hand policy. The results of the simulation indicate the presence of a positive and substantial welfare loss suffered from switching to the Bird-in-Hand rule. This result is shown to be robust under different parameterizations.

JEL classification D9, H3, Q32, Q38.

Keywords Exhaustible resources, precautionary saving, fiscal rules.

1 Introduction

The motivation of this paper is to shed light on a particular aspect of the more general wealth management problem for resource-rich economies, namely, the design of a fiscal consumption rule in order to minimize welfare losses from risky resource income. In order to do so, this research produces a normative comparative analysis of fiscal rules in a resource-rich economy. Fiscal rules can be either a theoretical derivation or of the ad hoc type. Ad hoc rules might produce sub-optimal welfare results; the aim of this work is to analyze the magnitude of the welfare costs by evaluating the relative welfare optimality of two different benchmark rules.
The model builds on the analogy between the intertemporal consumption problem of an infinitely lived representative consumer who receives an uncertain labour income (Caballero, 1990; Bodie, Merton and Samuelson, 1992) and that of a social planner for a country that receives an uncertain income stream from its exhaustible resource stock. The extensive literature on life-cycle saving and permanent income (LC-PIH) started by the seminal work of Modigliani and Brumberg (1954) states that a consumer who receives such an income stream will simply spend the return of the present discounted value of his entire wealth. Holding the value of the income (stock) fixed, the actual timing of the income stream (flow) becomes irrelevant. This suggests that only the amount of the resource wealth might actually matter for the government that is behaving as a permanent income consumer.

Notwithstanding the increased attention to these topics in the literature of natural resource management economics, there is as yet no consensus on how governments of countries with substantial amounts of exhaustible natural resources should design their policies in order to optimally spend their resource revenues. As argued by van der Ploeg (2011), Frankel (2010) and Deacon (2011) in their surveys of the resource curse literature, governments have often overestimated revenues and dangerously relied on the inflated version of their budget constraints (consisting of overall fiscal surpluses), thereby incurring sustained budget deficits which could prove difficult to reverse once income from resources starts to become depleted. In order to avoid this result, some resource-rich countries have implemented more prudent ad hoc fiscal rules, in order to reduce discretion in spending rules and, in turn, the associated macroeconomic risks. The applicability of these rules is limited to countries in which domestic political authorities have full control over the resources and, in addition, accountability for rules governing the resources also is ensured. In order to be effective, fiscal rules need to be backed by a strong political will and complemented by efficient administration.

Spending behaviours in resource-rich economies have been extensively analyzed at the empirical level. Villafuerte and Lopez-Murphy (2010) documented fiscal policy behaviour in 31 oil-producing countries during the oil price cycle of 2000 – 2008. Through decomposition of the non-oil primary government balance into a cyclical and a structural component, they find that fiscal policy has been pro-cyclical during the boom period and has contributed to the volatility of business cycles. The degree of pro-cyclicality has been high for low-income countries and low for high-income countries. Villafuerte and Lopez-Murphy (2010) conducted a sustainability analysis estimating the effects of a sudden drop of the resource price on fiscal budgets. They conclude that financing these fiscal deficits might constitute a problem for those countries that did not precautionarily accumulate foreign assets and international
reserves during the boom period. Gelb and Grasemann (2010) also confirm empirically the finding that oil exporters alternate periods of booms with periods of declining GDP as a consequence of price cycles.

Turning to the theoretical literature, Bems and de Carvalho Filho (2011) develop a model with resource price uncertainty in order to compute the magnitude of the precautionary savings motive for a large sample of resource-rich economies. Their model is solved numerically and the results show the positive significance of the precautionary savings motive. Building on the stylized framework for oil-producing small open economies provided by Engel and Valdes (2000), Maliszewski (2009) has computed numerically the relative welfare gains of different fiscal rules. At first, he simulates random realizations of oil price series in order to obtain the paths for government expenditures under the various fiscal rules considered. Then, he ranks fiscal rules by comparing the values that these expenditure paths imply for the mean of the social welfare function. Another approach is that of Pieschacon (2009), which analyzes the effects of implementing different sustainable fiscal rules in a dynamic stochastic general equilibrium model with a deteriorating oil sector.

The present paper contributes to the existing literature in two ways. First, a closed-form analytical solution for the infinite horizon consumption problem of the social planner with resource price uncertainty and precautionary savings is presented. This makes it possible to draw clear theoretical implications by avoiding the black-box effect of numerical analysis. This result is made possible by the specific assumption of Constant Absolute Risk Aversion (CARA) utility for the representative agent. In addition, the model is calibrated and simulated to provide a welfare-based comparison between the fiscal rules based on the Permanent Income Hypothesis (PIH, hereafter) and on the ad hoc Bird-in-Hand (BiH, hereafter) policy. The results indicate the presence of a substantial welfare loss suffered from switching from the PIH rule to the ad hoc BiH rule. In addition, sensitivity tests prove the robustness of this result under different parameterizations.

The present model is built as a partial equilibrium framework in the sense that government policy decisions do not influence the behaviour of private agents in the economy; therefore, several macroeconomic variables will automatically be taken as exogenous. The appreciation of the real exchange rate, the diversion of capital and investment resources from the tradable productive sector into the resource sector and the possibility of rent seeking are all aspects of the economics of natural resources literature which are absent in this work.

The structure of the paper is organized as follows: Section 2 introduces the model, Section 3 and 4 present the PIH and the BiH rules, and Section 5 evaluates welfare under both rules, whilst Section 6 draws the conclusions.
2 The model

I model the intertemporal consumption problem of a representative agent economy which receives a stochastic resource windfall. In other words, I look at the consumption problem of the planner of an economy which lasts infinite periods, during which a strictly positive but uncertain exogenous resource income is received. The model is in discrete time. The planner’s objective is to choose the optimal level of consumption of the only (public) good in order to maximize the infinite sum of the agent’s discounted utility function. I use oil wealth as an example.

The motivation for the utility formulation used in this work comes directly from the microeconomics literature about intertemporal consumption, in which Caballero (1990) and Weil (1993) have shown that, for the intertemporal consumption problem of an agent with labor income uncertainty, a CARA instantaneous utility function makes it possible to obtain an analytical closed form solution with precautionary saving. As previously mentioned, the central role of precautionary saving is also justified by the quantitative results obtained by Bems and de Carvalho Filho (2011). The utility specification of the model is the following:

\[ W = E \left\{ \sum_{t=0}^{\infty} \beta^t [u(g_t)] \right\} \]  \hspace{1cm} (1)

\[ u(g_t) = -\left( \frac{1}{\alpha} \right) \exp (-\alpha g_t), \]  \hspace{1cm} (2)

where \( W \) is the social welfare function to be maximized, \( \beta \) represents the intertemporal discount rate parameter, and \( g_t \in R_+ \) is the government expenditure level at date \( t \) (i.e., the consumption of the public good). \( u : R_+ \rightarrow R \) is the CARA instantaneous utility function, where \( \alpha > 0 \) is the coefficient of absolute risk aversion. In addition to standard assumptions that utility is strictly increasing and that \( \lim_{g \to 0} u'(g) = \exp [-\alpha g] = +\infty \), we have that \( u''(g) > 0 \), which means strict convexity of marginal utility. In other words, with higher variability of income, the planner would choose to save more and consume less. I assume absence of non-resource income in the economy, and therefore exclude domestic supply-side and investment opportunities. Distributing only the resource wealth across generations might be

\footnote{As in Barnett and Ossowski (2003), the infinitely lived agent set-up can also be thought of as an infinite sequence of generations of households, each of them living just one period. The choice in the current paper of excluding the possibility of government transfers financed by tax revenues from non-resource GDP can then be justified by assuming that these non-resource revenues are only transferred within the generation bearing the specific tax burden.}
motivated by the fact that natural resources, as opposed to domestic non-resource GDP, are indeed an endowment of the whole country’s population and not the result of the effort of any specific generation of households. Moreover, I consider an economy on its balanced growth path (which constitutes the only realistic option for an infinite-horizon economy because it implies neither growing nor decreasing consumption paths) in which the domestic interest rate does not deviate from the world interest rate. In a model with non-resource income in which the returns from domestic projects and foreign assets can be different, domestic investments would provide an alternative diversification channel, in addition to purchasing of foreign assets, for the social planner\textsuperscript{2}. The planner’s infinite horizon constrained optimization problem is:

\[
\max_{(g_t)^\infty_{t=0}} \sum_{t=0}^\infty \beta^t \left[ -\left( \frac{1}{\alpha} \right) \exp \left[ -\alpha g_t \right] \right]
\]

subject to the constraints

\[
A_{t+1} = (A_t + Y_t - g_t)R, \quad t = 0, 1, 2, \ldots, \quad A_0 = 0
\]

\[
\lim_{t \rightarrow \infty} R^{-t} A_{t+1} = 0.
\]

Equation (4) represents the government’s flow budget constraint. I assume that purchasing foreign financial assets \( A_t \) allows the government to transfer wealth from one period to another. The initial financial wealth endowment of the government is \( A_0 = 0 \). By saving a fraction of the resource income revenues, the government starts holding foreign assets. \( Y_t \) is the exhaustible resource income, in other words, the only income source for the government. Because the private sector does not explicitly appear in the maximization problem, the government does not collect taxes. \( R = (1 + r) \) is the constant gross interest rate. In addition, I assume that \( \beta R = 1 \). In conclusion, the transversality condition (5) guarantees that the government is neither borrowing nor lending in the long run.

The next step is to solve forward the flow budget constraint given in (4) in order to obtain the government’s intertemporal lifetime budget constraint, creating a link between the present discounted value of consumption and the present discounted

\textsuperscript{2}In this respect, the assumption of \( r = r^* \) also rules out essential features of developing economies, in which capital scarcity results in higher returns on domestic spending than on saving abroad. A further diversification channel considered in the literature is that of hedging on financial markets (i.e., over-the-counter markets). However, Bems and de Carvalho Filho (2011) document that the total volume of exchange on those markets was estimated to be only 0.18% of proven oil reserves in 2009.
value of income (see Mathematical Appendix A.a for details):

\[
E_t \left[ \sum_{s=t}^{\infty} R^{t-s}(g_s) \right] = A_t + E_t \left[ \sum_{s=t}^{\infty} R^{t-s}(Y_s) \right].
\] (6)

This version of the intertemporal budget constraint states that the expected present discounted value of public consumption has to be equal in all periods to the total current public wealth plus the expected present discounted value of future uncertain resource revenues.

### 2.1 Modeling price uncertainty and income

Bems and de Carvalho Filho (2011) have shown that exhaustible resource prices are substantially more volatile than extraction quantities\(^3\). This empirical evidence motivates the following approach, which abstracts from resource extraction decisions and considers the resource price volatility as the one and only source of uncertainty. For simplicity, I assume oil income for the economy to be given in each period by the quantity of oil sold, \(X_t\), evaluated at real spot market prices:

\[
Y_t = P_t X_t.
\] (7)

As far as the stock of reserves is concerned, I assume that the peak of oil production has already been reached, and that no further discoveries of new fields are going to replace the depleting stock. Thus, the stock of oil is inevitably depleting until it vanishes. At that point in time, the model will become fully deterministic because the only source of uncertainty will disappear. I formalize the depletion dynamics of the stock in the following way, where the depletion rate is represented by

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\(^3\)For a large sample of oil-producing economies, price volatility has been \(2 - 3\) times higher than extraction volatility over the period from 1980 to 2007. In addition, Bems and de Carvalho Filho (2011) show that oil production had limited responses to the changes in the price of oil over the entire period 1999–2008, thus demonstrating a very small price elasticity of supply. An explanation for this can be that extraction capacity is costly and time-consuming, with the result that extraction plans do not respond rapidly to short-run changes in prices.
the exogenous parameter $\delta^4$:

$$X_t = (1 - \delta)X_{t-1}, \quad \delta > 0.$$  \hspace{1cm} (8)

Hence, the crucial role in the present model is played by the volatile price component. Before we discuss the empirical literature of resource prices, the time path for the world oil price 1987 – 2014 (Europe Brent Spot Price) is shown here in Figure 1:

\begin{center}
\includegraphics[width=0.5\textwidth]{figure1.png}
\end{center}

Figure 1 - Europe Brent Spot Price. Source: EIA (2014).

An important work in the empirical literature of resource prices is the study by Pindyck (1999), who builds a model in which oil prices are mean-reverting to a quadratic trend that fluctuates over time. The economic intuition behind the mean-reversion property is that, assuming that the resources are sold in a competitive market, their price will sooner or later revert to the long-run marginal cost. However, Pindyck (1999) concludes that, in the case where oil prices would rise substantially in the subsequent decade (which indeed happened, as shown in Figure 1), the multivariate stochastic process model proposed in his work would have not provided any better predictions than a simple model with mean-reversion to a fixed linear trend. More recently, Rogoff and Dvir (2009) argue that a very long-run perspective is necessary to understand the true stochastic process lying behind oil

\footnote{The parameter $\delta$ governing the speed and the path of resource depletion is not a control variable for the planner of the economy. Instead, the amount of resource income which is spent or saved will be endogenous. A study on the management of resource windfalls with high price volatility that takes $\delta$ as fully endogenous is van der Ploeg (2010), in which it is shown that it is optimal for prudent governments to extract oil more aggressively in order to cope with uncertain resource income.}
prices, because of the structurally different statistical behaviour of prices in different epochs. Hamilton (2009) has conducted a comprehensive statistical investigation of the properties of oil prices. He shows that changes in oil prices have always tended to be permanent, difficult to predict and governed by different stochastic regimes in different epochs. In conclusion, he claims that, although forecasts might turn out to be far from actual future values, actual current values provide the best available forecasts. In light of Hamilton (2009)’s result, this model assumes that the price of oil follows a random walk without drift of the following kind:

\[ P_t = P_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma^2_{\varepsilon}). \] (9)

I formulate the income process as a combination of a trend component represented by the depleting resource stock and a random walk deviation from the trend, represented by the price of oil:

\[ Y_t = (P_{t-1} + \varepsilon_t) [(1 - \delta) X_{t-1}] . \] (10)

3 The Permanent Income Policy

Let us proceed to derive the fiscal spending rule based on the PIH, in other words, the optimal consumption function of the maximization problem presented in Section 2. The value equation for the problem is given by:

\[ V(A_t) = \max_{\{g_t\}} \{ u(g_t) + \beta E_t V(A_{t+1}) \} . \] (11)

A standard solving procedure with the help of the envelope theorem gives the classic Euler equation for the marginal utilities of consumption (see Appendix A.b for details):

\[ u'(g_t) = \beta R E_t \left[ u'(g_{t+1}) \right] . \] (12)

Let us now observe how the introduction of income uncertainty (as a consequence of the resource price uncertainty \( \varepsilon_t \sim N(0, \sigma^2_{\varepsilon}) \)) triggers the presence of precautionary motives in the optimal consumption rule. The CARA utility specification implies that (12) becomes:

\[ \exp(-\alpha g_t) = \beta R E_t \exp[-\alpha g_{t+1}] . \] (13)

Because the income process has normally distributed innovations, I hypothesize and then verify (see Appendix A.c) that the consumption process will obey the
following dynamics:

\[ g_{t+1} = g_t + \log(\beta R)^{\frac{1}{2}} + \frac{\alpha}{2} \sigma^2 + \varepsilon_{t+1}. \]  

Based on these dynamics, we need to specify how future consumption levels are predicted. Conditioning the future unknown level of consumption on the current information gives (see A.d.):

\[ E_t(g_{t+1}) = g_t + \log(\beta R)^{\frac{1}{2}} + \frac{\alpha}{2} \sigma^2, \]  

\[ E_t(g_s) = g_t + (s - t) \left[ \log(\beta R)^{\frac{1}{2}} + \frac{\alpha}{2} \sigma^2 \right]. \]  

This result shows that more volatile resource income will trigger higher expected consumption growth, in other words, a steeper optimal consumption path. This is a consequence of precautionary motives which induce higher current savings in order to offset possible future adversities. As a result of this, the government’s consumption will be expected to grow faster from one period to another.

Now define \( \kappa = \log(\beta R)^{\frac{1}{2}} + \frac{\alpha}{2} \sigma^2 \) so that (16) becomes \( E_t(g_s) = g_t + (s - t)\kappa \). In order to proceed with the derivation of the optimal consumption function, we need to obtain the present discounted values to be inserted in the intertemporal budget constraint given by (6). The expected present discounted value of public consumption is obtained as follows:

\[ E_t \left[ \sum_{s=t}^{\infty} R^{t-s}(g_s) \right] = g_t \sum_{s=t}^{\infty} (1 + r)^{t-s} + \kappa \sum_{s=t}^{\infty} (1 + r)^{t-s}(s - t) \]

\[ = g_t \left( \frac{1 + r}{r} \right) + \kappa \left( \frac{1 + r}{r^2} \right). \]

Let us now turn to resource income. Given the income process described in (10), we have that (details in Appendix A.e.):

\[ E_t (Y_s) = (1 - \delta)^{s-t} Y_t. \]

Computing the present discounted value of income gives:

\[ E_t \left[ \sum_{s=t}^{\infty} R^{t-s}(Y_s) \right] = Y_t \sum_{s=t}^{\infty} (1 + r)^{t-s} (1 - \delta)^{s-t} = \left( \frac{1 + r}{r + \delta} \right) Y_t. \]
We can now substitute (18) and (20) into (6) to get:

\[ g_t^{PIH} \left( \frac{1 + r}{r} \right) = \left( \frac{1 + r}{r + \delta} \right) Y_t + A_t^{p_ih} - \kappa \left( \frac{1 + r}{r^2} \right) \]

and further solve for the optimal consumption function of the government:

\[ g_t^{PIH} = \left( \frac{r}{r + \delta} \right) Y_t + \left( \frac{r}{1 + r} \right) A_t^{p_ih} - \left( \frac{\alpha}{2r} \right) \sigma_\varepsilon^2. \]  

(22)

The first term on the right-hand side reflects the government’s direct consumption of the resource income. The propensity to consume directly out of the resource revenues is lower than unity because part of the resource revenue is invested by purchasing foreign assets. In other words, savings are accumulated in a sovereign wealth fund, especially during periods in which resource income is very high. The second term represents the annuity value of the financial wealth, i.e., the government consumes the return on its previously accumulated financial wealth. Finally, the last term (obtained by recalling that we assumed \( \beta R = 1 \)) indicates that uncertain future resource income makes it desirable for the government to precautionarily consume less and save more of its current total wealth. This result implies that, after resources have been depleted, both the direct consumption term \( \left( \frac{r}{r + \delta} \right) Y_t \) and the precautionary motives term \( \left( \frac{\alpha}{2r} \right) \sigma_\varepsilon^2 \) will disappear from the optimal consumption function; hence, the model becomes deterministic and the PIH rule will be given simply by \( g_t^{PIH} = \left( \frac{r}{1 + r} \right) A_t^{p_ih} \). In other words, when oil resources are depleted, the government will finance its expenditure exclusively by relying on the return on past savings from resources.

However, the formulation in (22) is not yet in reduced form. In fact, the value of the net foreign assets is determined endogenously in the current model and must therefore depend only on the model’s initial conditions, as well as on income shocks. Setting \( \phi = \left( \frac{\alpha}{2r} \right) \sigma_\varepsilon^2 \) and inserting the spending rule (22) into the dynamics of the budget constraint given in (4) implies:

\[ A_{t+1}^{p_ih} = (1 + r) \left[ A_t^{p_ih} + Y_t - \left( \frac{r}{r + \delta} \right) Y_t - \left( \frac{r}{1 + r} \right) A_t^{p_ih} + \phi \right], \]  

(23)

\[ A_{t+1}^{p_ih} = A_t^{p_ih} + \left[ \frac{\delta(1 + r)}{r + \delta} \right] Y_t + (1 + r)\phi. \]  

(24)
Solving this difference equation gives:

$$A_t^{pih} = A_0^{pih} + \left[ \frac{\delta(1+r)}{r+\delta} \right] \sum_{s=0}^{t-1} Y_s + t(1+r)\phi. \quad (25)$$

Inserting back into equation (22) finally allows us to obtain a reduced form PIH spending rule, as a function of only exogenous terms:

$$g_{t,PIH} = \left( \frac{r}{r+\delta} \right) (Y_t + \delta \sum_{s=0}^{t-1} Y_s) + \left( \frac{r}{1+r} \right) A_0^{pih} + (rt - 1)\phi. \quad (26)$$

In general, the intuition behind the PIH spending rule given in (22, 26) is explained as follows. This rule aims by definition to maintain a constant stock of wealth over the long run. This implies that the forward-looking government does not simply spend out of current resource and financial income, but instead spends out of permanent income or total wealth. In other words, the government optimally chooses a combination of consumption and savings that allows it to equalize the welfare of the agent over her entire lifetime horizon. A simulation of this rule in comparison with the BiH rule will be given in Section 5.

4 The Bird-in-Hand Policy

As opposed to the theoretical and forward-looking spending rule based on the PIH, a few countries have recently adopted ad hoc fiscal rules to govern the use of their resource income. These pragmatic and highly operational rules are intended to reduce the pro-cyclicality of fiscal policy and to direct the use of the resource revenues toward long-term sustainability objectives. The BiH rule is supposed to limit the macroeconomic impact of resource revenues by smoothing the spending path of these revenues.

This section presents a stylized formulation of the BiH rule, defined as:

$$g_{t,BIH} = \left( \frac{r}{1+r} \right) A_t^{bih}. \quad (27)$$

The BiH rule prescribes that the entire resource income shall be stored in a sovereign wealth fund. This implies that no intertemporal consumption problem arises for the government. When this rule is adopted, the stochastic process of the oil price becomes a negligible variable because the spending rule will no longer
directly react to it. Government consumption will be affected by the oil price shock only indirectly, through changes in accumulated financial assets rather than through variations in the present value of resource revenues. What does the rule imply for the dynamics of the budget constraint? Let us go back to equation (4) and substitute in the BiH rule (27) to get:

\[
A_{t+1}^{bih} = (1 + r) \left[ A_t^{bih} + Y_t - \left( \frac{r}{1 + r} \right) A_t^{bih} \right],
\]

\[
A_{t+1}^{bih} = A_t^{bih} + (1 + r) Y_t.
\]

This tells us that, after resource depletion, the amount of wealth which is saved in the fund stays at a constant level, because the oil income \( Y_t \) has been entirely depleted: \( A_{t+1}^{bih} = A_t^{bih} \) (the same will occur for the PIH rule because both rules become identical after depletion, net of the difference in accumulated assets). Solving the difference equation obtained in (29) gives:

\[
A_t^{bih} = A_0^{bih} + (1 + r) \sum_{s=0}^{t-1} Y_s.
\]

In turn, this allows us to express the BiH spending rule as a function of only exogenous terms and initial values:

\[
g_{t, BIH}^* = \frac{r}{1 + r} \left[ (1 + r) \sum_{s=0}^{t-1} Y_s + A_0^{bih} \right].
\]

The essence of the BiH rule given in (27, 31) is that it allows consumption exclusive of the resource revenues that already have been liquidated. The BiH rule therefore has a backward-looking nature as opposed to the forward-looking nature of the PIH. In each period, current spending depends only on the size of the sovereign wealth fund in the previous period, implying no direct link with the risky resource income. As stated above, the intention behind this approach is to reduce the impact of resource price volatility on current spending.

4.1 Discussion

When is the ad hoc BiH rule preferred over the fiscal consumption policy based on the PIH, and when is the reverse true? A possible disadvantage of the PIH rule is that it ignores future expenditure commitments related to population dynamics
and ageing. In light of this, does the BiH rule qualify as a more prudent spending rule? As anticipated above, less stylized forms of (27, 31) have recently appeared in several countries. An example of an ad hoc fiscal rule with a high degree of fiscal conservatism is that of Norway, one of the world largest producers of oil and gas. A concise but detailed presentation of the Norwegian experience with management of its petroleum resources is provided by Holden (2013).

As early as 1983, the "Tempo Committee" (Tempo Utvalg in Norwegian) recommended establishment of a fund in which oil and gas income could be deposited, in order to detach revenues from myopic public spending. The intention was to focus on the permanent income from resource wealth in order to smooth public government spending and to partly postpone the gains from hydrocarbon revenues to future generations of citizens, as the benchmark PIH rule prescribes. After a decade of debates, in 1996 Norwegian authorities established a sovereign wealth fund in which resource revenues are placed. It is debatable whether the wealth fund was administered according to the prescriptions of the PIH rule in the initial years or whether fully discretionary spending instead took place during that time. The fund is today called the Government Pension Fund Global (GPFG) and its size was 122% of GDP and 181% of non-resource GDP at the end of 2011 (NBIM, 2013).

In order to also deal with the volatility in present values of resource wealth (due to price volatility), in 2001 Norwegian authorities introduced an ad hoc spending rule based on a fixed level of interest spending, as in (27, 31): roughly 4% per annum of the fund wealth can be used for public consumption (although there are exceptions in case of recessions). Although the establishment of the fund in 1996 was inspired by the PIH, the subsequent adoption of the BiH rule in 2001 makes it possible to consider that the Norwegian spending rule incorporates features of both the PIH and the BiH spending rules. By estimating Norway’s fiscal reaction functions based on historical data, Harding and Van der Ploeg (2013) test this hypothesis and indeed confirm that Norwegian fiscal behaviour can be regarded as somewhere in between the two benchmark rules. However, it should not be taken for granted that the Norwegian spending rule will accommodate unexpected fiscal commitments in the long run. Harding and Van der Ploeg (2009, 2013) have also argued that neither the actual Norwegian spending rule nor the pure PIH-BiH benchmark rules will determine a level of foreign assets accumulation high enough to face the increasing future burden coming from the ageing population and related rising pension commitments (unless fiscal expenditure is tightened or the pension system reformed)\(^5\).

\(^5\)Jafarov and Leigh (2007) have also analyzed the long-run sustainability of Norway’s public finances under different fiscal rules. Their conclusion is that no rule dominates the others, and that under any reasonable rule Norway’s oil wealth will probably not be enough to cover the projected
Another interesting example is that of Chile, as mentioned in Frankel (2010). Chile managed to have a counter-cyclical fiscal policy due to a structural balance rule which allowed the government to run deficits larger than the target only in case of deep recessions and the price of the resource (copper) being lower than expected. The structural balance rule factors out the cyclical and random effects of GDP and of the copper price. The cyclical adjustment to the copper price is based on the gap between the actual export price and an estimated long-term moving average reference price.

5 The evaluation of fiscal rules

Let us proceed with the evaluation of fiscal rules. Cochrane (1989) pioneered the study of the utility costs of alternative decision rules. He pointed out that consumption patterns that deviate from the optimal permanent income consumption rules, but incur only in trivial utility costs, can be labeled as near-rational. In the current work, however, the two fiscal rules cannot be simply considered as deviations from each other.

The simplest approach to conduct a welfare comparison between the two fiscal rules would be to investigate the magnitude of the utility gap that would make the representative agent at least as well off under one fiscal rule as under the other. Lucas (2003) used a similar approach to identify the welfare gain from fully eliminating income uncertainty for a risk-averse consumer. In other words, when it comes to comparing two different rules, a fiscal rule would be logically preferred over the other if its contingency plan for consumption and asset accumulation yields a higher level of expected conditional welfare. Because the PIH benchmark represents the optimal rule under the model’s assumptions, the welfare measure in terms of consumption, which measures the cost of switching to the BiH rule, will have to be positive. The question is whether this welfare gap is substantial and robust to different parameterizations.

5.1 Calibration and simulation

This subsection calibrates and simulates a finite horizon version of the model using a few parameters and initial values for one generic oil exporting country, somewhat increase in future spending commitments.

The MATLAB code used for the simulation and the comparison of fiscal rules is available from the author on request.
resembling the economic profile of Norway\textsuperscript{7}.

The model is simulated for $T = 200$ periods, with one period assumed to be equivalent to one year. In the baseline simulation, the real rate of return is assumed to be $r = 0.04$ (therefore, the subjective discount rate is $\beta = 1/1.04$), whilst the coefficient of absolute risk aversion is set at $\alpha = 0.05$\textsuperscript{8}. The initial foreign assets level is set at $A_0 = 0$ without loss of generality. The amount of proven exhaustible oil reserves for Norway was estimated to be 8.7 billion barrels at the end of 2013 (BP Statistical Review of World Energy 2014). The depletion rate is set at $\delta = 0.03$, and the lifetime of oil reserves has been arbitrarily set at 100 years. The initial value for the real price of oil has been set at the European Brent spot price for January 2014, which was approximately $100 per barrel (EIA 2014). The rest of the oil price series is simulated according to (10). The variance for the price series has been initially set at $\sigma^2 = 30$.

Figure 2 below shows a (single) random realization of the price, stock and resource income dynamics:

\textsuperscript{7}Although the goal of the actual Norwegian spending rule is to smooth the combination of both domestic and resource income, in the current framework I abstract from the former, as explained in Section 2. This implies that the formulation of the BiH rule employed in this simulation is no more than a stylized form of the actual Norwegian spending rule. The rationale behind the two rules remains the same, however: trying to reduce uncertainty from resource depletion and income volatility in order to spread the benefits equally through generations and face future unexpected fiscal commitments.

\textsuperscript{8}A discussion of the range of possible Coefficients of Absolute Risk Aversion (CARA) is provided by Babcock et al. (1993).
Figure 2 - Price series, resource stock and income dynamics

The simulated series for the resource price, stock of resources, and resource revenues determine the consumption and asset accumulation prospects under the two fiscal rules in Figure 3, below. As anticipated in the previous sections, the PIH based rule (solid line) implies that a fraction of the resource revenues is accumulated in a fund; subsequently, its capital income is used to sustain the consumption of the representative agent after resource depletion. Therefore, the consumption series for this rule will look substantially flat, with only limited perturbations due to price volatility in the pre-depletion era, as can be seen in the upper series plotted in Figure 3:
Figure 3 - PIH and BiH, consumption and assets accumulation

The BiH rule (dashed line) implies instead a steeper asset accumulation path by prescribing that the entire resource income must be invested in foreign assets. As the bottom plot of Figure 3 clearly shows, the volume of assets accumulated in the fund turns out to be higher for the BiH rule throughout the time range of the simulation. The faster pace of foreign asset accumulation determines an initial lower consumption level for the BiH rule compared to the PIH. However, due to the higher amount of financial assets accumulated before depletion (for \( t \leq 100 \)), the BiH rule allows achievement of a sustained higher level of public consumption after depletion (for \( 100 < t \leq T \)). Notice that, after depletion, the two rules coincide and the consumption gap between them stays constant.
5.2 Comparing the fiscal rules

Let us now quantitatively evaluate the welfare profiles implied by the two fiscal rules. The first step is to define a constant welfare measure in terms of consumption for each of the two fiscal rules, a role that can be played by the so-called Certainty Equivalent. The Certainty Equivalent represents the values of constant consumption that generate exactly the same utility as does the consumption series from each fiscal rule. This implies that the fiscal rule with the greatest expected utility will also have the preferable Certainty Equivalent. Once the Certainty Equivalent measures $CE_{BH}$ and $CE_{PI}$ have been computed, the second and final step will be to estimate the gap between the two fiscal rules in terms of consumption.

Define $CE_{BH}$ and $CE_{PI}$, respectively, such that:

$$
\sum_{t=0}^{T} \beta^t [u(CE_{BH})] = E \left\{ \sum_{t=0}^{T} \beta^t [u(g_{t,BH}^*)] \right\} \quad (32)
$$

$$
\sum_{t=0}^{T} \beta^t [u(CE_{PI})] = E \left\{ \sum_{t=0}^{T} \beta^t [u(g_{t,PI}^*)] \right\} \quad (33)
$$

The Certainty Equivalent consumption levels $CE_{BH}$ and $CE_{PI}$, respectively, are given by:

$$
CE_{BH,PI} = u^{-1} \left\{ \frac{E \left[ \sum_{t=0}^{T} \beta^t [u(g_{t,PI}^*)] \right]}{\sum_{t=0}^{T} \beta^t} \right\} \quad (34)
$$

with $u^{-1} = -\frac{1}{\alpha} \ln \left[ -\alpha(\cdot) \right]$. By using the consumption levels $CE_{BH}$ and $CE_{PI}$, we can now proceed to estimate the value of the welfare measure $\Lambda$, expressing the constant change in Certainty Equivalent consumption necessary for the BiH rule to generate the same level of utility as the PIH rule:

$$
\Lambda = \frac{CE_{PI} - CE_{BH}}{CE_{BH}} > 0 \quad (35)
$$

The baseline simulation ($T = 200, \alpha = 0.05, r = 0.04, \delta = 0.03, \sigma^2 = 30$) estimates that $\Lambda = 4.7092$, confirming the theoretical model’s prediction that the rule based on the PIH provides a substantially higher welfare level for the representative agent of our economy. In other words, the jump in Certainty Equivalent consumption necessary for the BiH rule to generate the same level of utility as the PIH rule is estimated in the baseline simulation to be approximately 4.7092 times the level
expressed by the $CE_{BIH}$.

Now, let us conduct a sensitivity analysis to investigate how the result for $\Lambda$ varies with respect to changes in the baseline parameters ($\alpha; r; \delta; \sigma_\omega^2$). In order to focus on the comparative static effects, variation will be allowed for one parameter at a time$^9$. The interest rate, for example, will be $0.02 < r < 0.08$; the coefficient of absolute risk aversion, $0.02 < \alpha < 0.08$; the depletion rate, $0.03 < \delta < 0.1$; and the variance of the resource price, $20 < \sigma_\omega^2 < 100$. The results of this exercise are jointly plotted in Figure 4, as follows:

![Figure 4 - Sensitivity analysis](image)

At first, let us focus on the upper left part of Figure 4, in which $\Lambda$ varies with respect to changes in the CARA parameter $\alpha$. The series shows a clear increasing tendency for the welfare measure in terms of Certainty Equivalent consumption $\Lambda$, in response to a higher degree of risk aversion $\alpha$. Recall from (22) that a higher $\alpha$ implies stronger precautionary motives and thereby a more prudent consumption path for the PIH rule. This prediction is confirmed by observing that, from a level

$^9$The validity of this approach is confirmed by the set of Figures 5 a-b-c-d in Appendix B, showing that correlation between parameters is minor or absent. In more detail, each of the four figures shows the main dynamics for $\Lambda$ where only the chosen parameter varies, jointly plotted with the series for $\Lambda$ in which the chosen parameter varies and a shock occurs to each of the other parameters in turn.
of $CE_{PIH} = 437.85$ for $\alpha = 0.02$, the Certainty Equivalent consumption linearly decreases to $CE_{PIH} = 283.92$ for $\alpha = 0.08$. In addition, the impact of a higher $\alpha$ on the BiH rule determines an even stronger fall in the Certainty Equivalent consumption than is the case for the PIH rule, mostly due to the higher variability in the consumption series of the BiH rule. Specifically, from a level of $CE_{BIH} = 129.21$ for $\alpha = 0.02$, the Certainty Equivalent consumption decreases to $CE_{BIH} = 39.95$ for $\alpha = 0.08$. In other words, the effect of a higher CARA parameter $\alpha$ on $\Lambda$, through $CE_{BIH}$ and $CE_{PIH}$ jointly, turns to be positive and mostly linear, as shown in the upper left part of Figure 4.

Notice now that, from the formulation of the model in the previous sections, no straightforward conclusion can be drawn as regards the effects of a higher interest rate $r$ on $\Lambda$, the estimate of the welfare gap between the two fiscal rules. On one side, an increase in the interest rate $r$ increases the return from investing the resource income into financial assets, ceteris paribus, for both fiscal rules. As is clear from the lowest plot of Figure 3, which shows the different paths of assets accumulation, this gives the BiH rule the advantage over the PIH rule, apparently reducing the overall welfare gap between them. On the other side, (22, 26) predict that a higher interest rate translates into a higher share of the resource income being directly consumed and thereby higher consumption levels for the PIH rule, apparently increasing the welfare gap between the two rules. The lower left part of Figure 4, in which $\Lambda$ increases with respect to higher levels of $r^{10}$, shows that the latter effect of boosted consumption under the PIH rule plays a stronger role and determines on aggregate an increasing welfare gap between the two fiscal rules.

Let us focus now on how the results react in response to changes in the resource depletion parameter $\delta$. A higher $\delta$ implies a faster rate of depletion, although the total lifetime of oil reserves stays constant at 100 years. In other words, resource income decreases faster than is observed in the bottom plot of Figure 2 for a higher rate of depletion $\delta$, becoming negligible in the last years before depletion. The qualitative intuition goes as follows: the BiH rule, which prescribes faster accumulation of financial assets and postpones consumption, would be less penalized by the increased depletion rate. This prediction is confirmed by observing that the Certainty Equivalent consumption drops from a level of $CE_{PIH} = 352.72$ for $\delta = 0.03$, to $CE_{PIH} = 229.98$ for $\delta = 0.1$; whilst $CE_{BIH}$ slightly decreases from $CE_{BIH} = 61.40$ for $\delta = 0.03$ to $CE_{BIH} = 61.27$ for $\delta = 0.1$. Consequently, the upper right part of Figure 4 shows that the measure of the welfare gap $\Lambda$ shrinks when a positive shift occurs for the resource depletion rate $\delta$.

---

$^{10}$The simulation prescribes that, whenever the interest rate $r$ varies, $\beta$ changes accordingly to ensure that $\beta(1 + r) = 1$, as previously assumed.
Last but not least, let us analyze the sensitivity of the results with respect to the resource price uncertainty $\sigma_2^2$. On the one hand, the model of the current paper prescribes that higher volatility of resource income translates into stronger precautionary savings and thereby a lower consumption level under the PIH rule. On the other hand, as is shown in (27, 31), the ad hoc BiH rule does not directly react to changes in income volatility, although increased volatility gives higher consumption volatility under this rule as well. In other words, the theoretical prediction of the model is that of a lower welfare gap between the two fiscal rules for higher levels of $\sigma_2^2$. Notice that the lower right plot of Figure 4 clearly confirms this prediction by presenting a monotonically decreasing series for $\Lambda$.

To sum up, the evidence and results presented throughout this subsection and in Figure 4 show that the gap between the two fiscal rules estimated by the welfare measure in terms of consumption, $\Lambda$, is substantial and robust to variation in the different parameters. This confirms the intuition that, under the given calibration of the model of this paper, the BiH policy rule qualifies as the less preferred option in order to maximize the welfare of the representative agent of our economy.

6 Concluding remarks

The background motivation of this research was to focus on a particular aspect of the wealth management problem for governments of resource-rich economies. Resource wealth gives rise to uncertain income paths, creating the need to design fiscal consumption rules that minimize welfare losses. In order to expand the literature on the economics of natural resource management, this paper constructed a simple model of an economy endowed with a stochastic income from exhaustible natural resources. The stylized features of the specified model do not allow us to provide straightforward policy recommendations, because country-specific parameters often play a crucial role in determining the design of spending policies. However, the results provide a clear understanding of the mechanisms and properties of the two alternative fiscal rules under observation.

Specifically, I assumed that the planner of the economy decides to spend present and future resource income according to two benchmark fiscal rules - one rule derived from the planner’s infinite horizon intertemporal consumption problem (PIH) and the other an ad hoc rule (BiH). An additional contribution of the paper is to quantitatively evaluate the relative welfare-based optimality of these rules. After derivation of the closed-form spending rules for both the maximization-based PIH and the alternative ad hoc BiH policy, the model was calibrated and simulated in order to obtain random time series for the resource price, income dynamics and consumption
series.

The last step was to evaluate the welfare performance of the fiscal rules by computing a welfare measure in terms of consumption for each of them, namely the Certainty Equivalent. Based on these measures, the results indicate the presence of a substantial welfare loss suffered from switching from the PIH rule to the ad hoc BiH rule. In addition, sensitivity tests have proven the robustness of this result under different parameterizations.

More generally, this paper demonstrates that applying a life-cycle perspective to the public management of natural resource revenues can yield new insights. This paper focused specifically on one question related to the management of resource wealth, i.e., the optimal time profile for consumption of uncertain resource income. Additional research is therefore required in related aspects of how wealth management policies should be time-varying depending on different resource depletion paths and on the portfolio allocation of savings from resource income.

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A Mathematical Appendix

(a) From (4) to (6):

\[ A_{t+1} = (A_t + Y_t - g_t)(1 + r), \quad t = 0, 1, 2, \ldots, \quad (36) \]

\[ A_t = g_t - Y_t + \frac{A_{t+1}}{1 + r}, \quad (37) \]

\[ \frac{A_{t+1}}{1 + r} = \frac{g_{t+1} - Y_{t+1}}{1 + r} + \frac{A_{t+2}}{(1 + r)^2}. \quad (38) \]

Substituting this last equation back into the previous and repeating the exercise forward gives:

\[ A_t = g_t - Y_t + \frac{g_{t+1} - Y_{t+1}}{1 + r} + \frac{A_{t+2}}{(1 + r)^2} + \frac{A_{t+3}}{(1 + r)^3}, \quad (39) \]

\[ \frac{A_{t+2}}{(1 + r)^2} = \frac{g_{t+2} - Y_{t+2}}{(1 + r)^2} + \frac{A_{t+3}}{(1 + r)^3}, \quad (40) \]

\[ A_t = g_t - Y_t + \frac{g_{t+1} - Y_{t+1}}{1 + r} + \frac{g_{t+2} - Y_{t+2}}{(1 + r)^2} + \frac{A_{t+3}}{(1 + r)^3} + \frac{A_{t+4}}{(1 + r)^4}; \quad (41) \]

\[ A_t = \sum_{s=t}^{\infty} \left( \frac{1}{1 + r} \right)^{s-t} (g_s) = \sum_{s=t}^{\infty} \left( \frac{1}{1 + r} \right)^{s-t} (Y_s) + \lim_{t \to \infty} \left( \frac{1}{1 + r} \right)^t A_{t+1}. \quad (42) \]

Applying the transversality condition (5), we can rewrite this for the public expenditure term:

\[ \sum_{s=t}^{\infty} \left( \frac{1}{1 + r} \right)^{s-t} (g_s) = A_t + \sum_{s=t}^{\infty} \left( \frac{1}{1 + r} \right)^{s-t} (Y_s). \quad (43) \]

This budget constraint should hold exactly for all future dates \( s \). Thus it should hold in expectation terms at time \( t \):

\[ E_t \left[ \sum_{s=t}^{\infty} R^{t-s} (g_s) \right] = A_t + E_t \left[ \sum_{s=t}^{\infty} R^{t-s} (Y_s) \right], \quad (44) \]

which is the government’s intertemporal lifetime budget constraint as of (6).
(b) The first order condition of the Bellman equation given in (11) is given by

\[ u'(g_t) - \beta E_t V'(A_{t+1}) \frac{\partial A_{t+1}}{\partial g_t} = 0, \]  
\[ u'(g_t) = R \beta E_t V'(A_{t+1}). \]  

(45)  

Differentiating the Bellman equation and using the Envelope Theorem gives:

\[ V'(A_t) = \beta E_t V'(A_{t+1}) \frac{\partial A_{t+1}}{\partial A_t}, \]  
\[ V'(A_t) = \beta R E_t V'(A_{t+1}). \]  

(47)  

(48)

Equating to each other (46) and (48) (this result will hold at any date) gives

\[ u'(g_t) = V'(A_t). \]  

(49)

Substituting again (48) at \( t + 1 \) in (46) gives the Euler equation, describing optimization behaviour over time:

\[ u'(g_t) = R \beta E_t \left[ u'(g_{t+1}) \right]. \]  

(50)

(c) The "guess" on the consumption dynamics is:

\[ g_{t+1} = g_t + \log(R\beta)^{\frac{1}{2}} + \frac{\alpha}{2} \sigma^2 + \varepsilon_{t+1}. \]  

(51)

Let us verify whether this process specification works by inserting (51) into (13):

\[ 1 = R \beta E_t \exp[-\alpha \log(R\beta)^{\frac{1}{2}} + \frac{\alpha}{2} \sigma^2 + \varepsilon_{t+1} + g_t - g_t], \]  
\[ 1 = R \beta \exp[-\alpha \left( \frac{1}{\alpha} \log(R\beta) \right)] \exp(\frac{-\alpha^2}{2} \sigma^2) E_t \exp(-\alpha \varepsilon_{t+1}). \]  

(52)  

(53)

We know from the properties of the log-normal distribution function that, whenever \( X \sim N(\mu, \sigma^2) \), then \( E \exp(X) = \exp(\mu + \frac{1}{2} \sigma^2) \). Thus we can apply this result to the normally distributed innovations to obtain

\[ E \exp(-\alpha \varepsilon_{t+1}) = \exp[E_t(-\alpha \varepsilon_{t+1}) + \frac{1}{2} Var(-\alpha \varepsilon_{t+1})] = \exp(\frac{\alpha^2 \sigma^2}{2}) \]  

(54)
and simplify (53) as follows:

\[ 1 = R\beta \exp[-\alpha\left(\frac{1}{\alpha} \log(R\beta)\right)] \exp\left(-\frac{\alpha^2\sigma^2}{2}\right) \exp\left(\frac{\alpha^2\sigma^2}{2}\right), \]  
\[ 1 = R\beta \exp[\log(R\beta) - 1], \]  
\[ 1 = 1. \]  

(d) From (15) to (16):

\[ E_t(g_{t+1}) = g_t + \log(\beta R)^{\frac{1}{2}} + \frac{\alpha}{2} \sigma^2, \]  
\[ \Rightarrow E_t(g_s) = g_t + (s - t) \left[ \log(\beta R)^{\frac{1}{2}} + \frac{\alpha}{2} \sigma^2 \right]. \]  

(e) From equation (10) to (19):

\[ E_t(Y_{t+1} | Y_t) = E_t[(P_t + \varepsilon_{t+1})(1 - \delta) X_t], \]  
\[ = E_t[P_t(1 - \delta) \cdot E_t(X_t) = P_t(1 - \delta) X_t, \]  
\[ E_t(Y_{t+2} | Y_t) = E_t[E_{t+1}(Y_{t+2})], \]  
\[ = E_t[P_{t+1}(1 - \delta)^2 X_t] = (1 - \delta)^2 P_t X_t, \]  
\[ E_t(Y_s | Y_t) = (1 - \delta)^{s-t} P_t X_t = (1 - \delta)^{s-t} Y_t. \]  

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B Figures

Figures 5 a-b-c-d plot the "correlation check" for each of the varying parameters (\( \alpha; r; \delta; \sigma^2 \)). As explained in the paper, each of the four figures shows the main dynamics for \( \Lambda \) (called baseline in the legend) where only the chosen parameter (indicated on the horizontal axis) varies, jointly plotted with the series for \( \Lambda \) in which the chosen parameter varies \textit{and} a shock occurs to each of the other parameters in turn:

![Figure 5a](image-url)

![Figure 5b](image-url)
Figure 5c

Figure 5d
References


