
Upgrading hydropower plants with storage: Timing and capacity choice

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Abstract This paper presents an investment decision support framework for a hydropower producer with storage facilities that is considering upgrading the production capacity its power plant. A real options framework is proposed to support the investment decision, where both investment timing and capacity choice are considered. Using a case from a Norwegian hydropower producer, we employ the framework to evaluate investment opportunities in this company. Our main contribution is an approach that combines hydropower scheduling and real options valuation for already constructed power plants. Furthermore, the results from our case analysis suggest feasible investment strategies for the hydropower producer.

Keywords Real options · Electricity price uncertainty · Upgrading · Refurbishment · Timing · Capacity choice

1 Introduction

With increased utilization of renewable energy for electricity generation, there will be a higher requirement for balancing power with low response time. Hydropower plants with storage facilities meet these requirements and can play an important role in the transfer to a power supply based on renewable energy resources. In the EIA study IEO2009 [5], world energy consumption is projected to grow by 34 % in the period from 2010 to 2030 in the reference case. In the same reference case, world electricity generation is expected to double. With increased focus on how power generation affects the climate, most of this expansion should come from renewable energy resources. The European Union has ambitious goals for renewable energy through the Directive on renewable energy [6], where it states that by 2020, 20 % of the energy consumption in the EU should come from renewable sources. In 2009, the share was 11.6 % [7], signaling the necessity of strong growth in utilization of renewable energy resources the next decade.

The potential for capacity expansion through refurbishing and expanding existing hydropower plants in Norway is estimated to be approximately 16.5 GW [18]. Approximately 50 % of the storage capacity related to hydropower plants in Europe is located in Norway [18], and increased production capacity in the power plants connected to these reservoirs may ease the introduction of more renewable energy and in the long term improve the security of supply in Europe. But even though capacity expansion projects in the hydropower industry appear attractive, it seems that investors hesitate to realize projects unless profits are significant. In general, there are three main factors affecting the investment decisions. First,

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there is uncertainty regarding future cash flows. Second, the investments are irreversible, and third, the investors often have the opportunity to postpone investments. As a result, there is an opportunity cost related to realizing a project [4]. Thus, the reason for the hesitant behavior may be the high option value, i.e. the value of deferring the investment exceeds the value of investing immediately.

The purpose of this paper is to address uncertainty regarding future cash flows in hydropower upgrade projects and decide optimal investment time and capacity choice by using real options analysis. The main contribution is an approach that enables decision makers in hydropower companies with storage facilities to make better decisions regarding when they should upgrade their existing power plants, and which capacity to choose. The results from the analysis suggest investment strategies for the owner of such facilities.

Electricity generation and its timing lay the foundation for the cash flows generated by hydropower plants. The cost of water is in principle zero, but since it is a limited resource, it has a value in hydropower production. Producers can therefore calculate a so-called marginal water value (MWV), which can be regarded as an opportunity cost in order to determine a production schedule. The aim is to maximize the profit from the water released, plus the expected value of the water remaining in the reservoir [22]. The literature consists of different approaches for estimating the MWV. Yeh [23] reviews mathematical models developed for reservoir operations where linear programming, dynamic programming, nonlinear programming, and simulations are discussed, while Wallace and Fleten [22] give an overview of optimization models for energy, focusing mainly on stochastic models. Among these are Fosso et al. [10], who illustrate how uncertainty in electricity prices and inflow can be included using stochastic dynamic programming to find the water value for given reservoir levels and market prices. Gjelsvik et al. [12], who describe how stochastic dual dynamic programming can be used for long and medium term operations planning for hydro-dominated power systems, point out that computer time requirements pose an obstacle to using their models. Keppo and Näsäkkälä [14] have developed an approach for production planning quite different from traditional optimization models and is based on intuitive MWV calculations. They estimate a production threshold based on information from the electricity derivative markets, the reservoir level, on inflow and on time to maturity of a forward contract.¹ The parameters for the production threshold function are found by using simulations and are based on maximizing the lower bound on revenue the parameters give. In contrast to other optimization models, the method does not require the power producer to solve a complicated and often time-consuming optimization problem when deciding the production schedule.

Price fluctuations and volatility in prices within the day offer opportunities for hydropower producers with storage facilities in connection with their power plants. It is desirable to have a low capacity factor for the power plant, i.e. the ratio between annual production and output if it had operated at full capacity throughout the year. A low capacity factor enables the producers to allocate production to peak price periods, in contrast to producing at a constant rate continuously [13]. It is also desirable to save water in periods with low prices for use in periods with high prices. Some power plants built before electricity markets were deregulated have high capacity factors since they were adapted to provide base load power production. When upgrading existing power plants, the capacity factor can be reduced and given that the reservoir can contain the water, more water can be saved to periods with high prices. On the other hand, a lower capacity factor comes at the cost associated with higher capacity, so there is a tradeoff between costs and revenue in the capacity choice decision.

The option to invest in an upgrading project can be regarded an American call option [4]. McDonald and Siegel [17] study the optimal timing to invest and shows how uncertainty affects investment timing. This approach was expanded by Dangl and Wirl [2], who analyze an investment opportunity where a firm has to determine optimal investment timing and capacity choice at the same time, under conditions of irreversible investment expenditures and uncertainty. If an investor has the opportunity to choose between different capacities, Décamps et al. [3] show how the investment region may be dichotomous, and the investor should wait and see in which direction conditions evolve before making an investment decision. Fleten et al. [8] illustrate how this approach can be used when considering an investment alternative with capacity expansion opportunities.

Studies of real options and investments in hydropower production include Bøckman et al. [1], who study investment timing and optimal capacity choice for small hydropower projects. They show how an investment decision in a small hydropower plant will be postponed until a certain electricity price level is reached. Kjærland [15] estimates the value of investment opportunities in the hydropower sector in Norway using real options analysis and finds the electricity price trigger level for new investments. This paper contributes to the literature as it combines real options valuation with hydropower upgrading of

¹ Their approach is developed for option pricing purposes.

already existing power plants, indicating when investment in increased capacity should take place and which capacity choice is optimal.

In the following, Section 2 outlines the valuation framework. Section 3 introduces an investment case where a hydropower producer with storage facilities has power plants that are due for maintenance, and is considering capacity expansion in these. Section 4 presents empirical results while Section 5 concludes and gives final remarks.

2 Valuation Framework

In order to analyze the investment opportunity in upgrading existing power plants we create a valuation framework consisting of the following steps; production scheduling, real options valuation and Monte Carlo simulation. In the production scheduling, technical parameters and the electricity price dynamics for spot and forward prices are used to derive annual revenue for each investment alternative for a range of annual price levels. These revenues, together with the dynamics of the annual electricity price level, are used in the real options valuation to calculate the optimal decision rule for every year and annual price level. In the third step of the valuation framework, we extract the results by using Monte Carlo simulations and analyze how the optimal decision rules will materialize. Figure 1 gives an overview of the valuation framework. In the succeeding sections, the electricity price dynamics, the production scheduling, the real options valuation and the Monte Carlo simulations are presented in detail.²

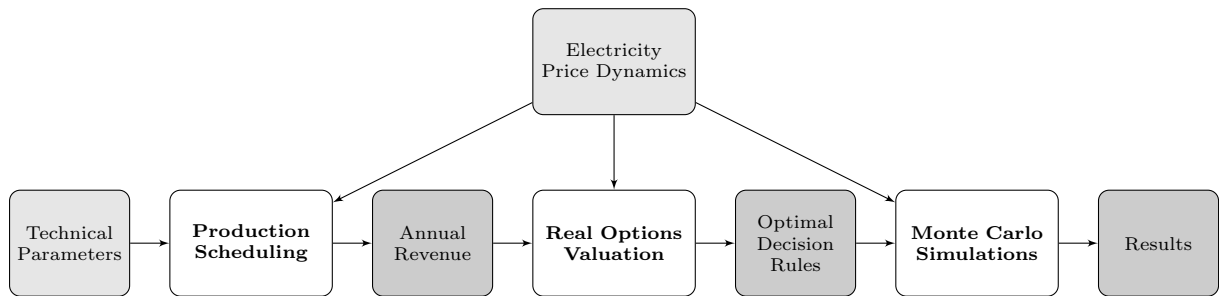


Fig. 1 Overview of valuation framework.

2.1 Electricity Price Dynamics

Since the case study presented in Section 3 and analyzed in Section 4 is taken from Norway, this section presents the dynamics of the electricity price in Norway. Within a year, we assume that the mean electricity price is at a given level and contains seasonality, while from one year to the next, we assume that the mean electricity price level changes. Seasonality in the electricity price within the year occurs due to how the demand for electricity is dependent on temperature. The assumption that the electricity price level varies from one year to the next is supported by the fact that 98 % of the power production in Norway comes from hydropower. The precipitation throughout the year affects the electricity price level to a great extent, and the precipitation varies from year to year. The production scheduling requires daily resolution of the electricity price dynamics while the real options valuation and Monte Carlo simulations require annual resolution for the electricity price dynamics. The seasonality is maintained in the production scheduling, while the characteristic of an annual price level is maintained in the real options valuation and the Monte Carlo simulations. Both price dynamics are presented in the following paragraphs.

2.1.1 Daily Electricity Price Dynamics

The production scheduling described in Section 2.2 requires specifications of daily spot and forward prices. Lucia and Swchwartz [16] have developed a framework for electricity prices suited for the Nordic

² Due to restrictions from the case provider, some technical parameters are not presented.

electricity market, and we represent spot and forward price dynamics similar to their one-factor model. Our electricity spot price model consists of three parts; a deterministic trend from an initial annual price level towards a long term price level³, a deterministic seasonal variation, and a stochastic part. The deterministic parts reflect the drift towards a long term price level and the seasonal changes in the price within the year, while the stochastic part represents the unforeseeable changes in prices. The electricity price dynamics are summarized in Eq. (1). The deterministic drift towards a long term price level is given by Eq. (2), the deterministic part containing seasonality is described in Eq. (3) and Eq. (4) presents the stochastic part.

$$S_t^l = a_t + f_t + h_t, \quad t \in T, \quad l \in L \quad (1)$$

$$\Delta a_t = \eta \cdot (a_t - \bar{Q}) \cdot \Delta t, \quad t \in T, \quad a_1 = l \quad (2)$$

$$f_t = \gamma \cdot \cos\left((t + \tau) \cdot \frac{2 \cdot \pi}{365}\right), \quad t \in T \quad (3)$$

$$\Delta h_t = -\kappa (S_t^l - (a_t + f_t)) \cdot \Delta t + \sigma \cdot \Delta Z, \quad t \in T, \quad l \in L \quad (4)$$

The spot price on day t for initial annual price level l is presented by S_t^l . The drift towards a long term price level is ensured by a_t where \bar{Q} is the long term mean of the electricity price and η is the long term mean reversion factor⁴, while the deterministic seasonality is presented by f_t , where γ and τ are constants. The stochastic part h_t has a mean of zero, κ denotes the rate of short term mean reversion, σ is the volatility of the process and ΔZ is the increment of a Wiener process. The time increment is one day, given by Δt . The set L contains all annual price levels, l , in the range $[Q_{\min}, Q_{\max}]$, while T is the set of all days in the production planning period, i.e. one year.

Given the spot price in Eq. (1), and assuming no risk premium, the forward price is given in Eq. (5):

$$F_{t,q}^l = a_{t+q} + f_{t+q} + (S_t^l - (a_t + f_t)) \cdot e^{-\kappa \cdot q}, \quad t \in T, \quad l \in L \quad (5)$$

where $F_{t,q}^l$ is the forward price for electricity delivered in q days at initial annual price level l . To ensure non-negative prices, we set $S_t^l = \max[S_t^l, 0]$ and $F_{t,q}^l = \max[F_{t,q}^l, 0]$ after finding both the spot price and the forward price. The production scheduling model requires estimates for the average future forward price, and these are found as follows:

$$\bar{F}_t^l = \frac{\sum_{i=1}^q F_{t,t+i}^l}{q}, \quad t \in T, \quad l \in L \quad (6)$$

where q is the time period over which the average future forward price is calculated. In the electricity spot price model, the price does not only change every day, it also changes each hour within the day. We let the electricity price variation within the day be deterministic and vary according to the average of historical prices for the given hour, where the degree of variation depends on the day of the week and season. This is illustrated in Figure 2.⁵

The parameters for the price model are estimated using a non-linear least square method and are based on price information from the Nordic electricity exchange Nord Pool. The parameter values and the corresponding t -statistics are presented in Table 6 in Appendix 1. Other case specific parameters for the valuation framework are presented in Table 7 in Appendix 1. All values are given in real terms.

2.1.2 Annual Electricity Price Dynamics

The annual electricity price dynamics serve as input to the real options valuation and the Monte Carlo simulations. The purpose of the electricity price dynamics is to determine at which level the mean electricity price is each year throughout the lifetime of the investment alternatives.

The annual electricity price level is determined by the mean-reverting price process in Eq. (7):

$$\Delta Q_y = \lambda \cdot (\bar{Q} - Q_y) \cdot \Delta y + \phi \cdot \Delta W, \quad y \in Y \quad (7)$$

where Q_y is the annual electricity price level in year y (where Q_y corresponds to a_1 in the daily electricity price dynamics), \bar{Q} is the long term electricity price level, and λ is the long term mean-reversion factor.

³ In the production scheduling, we find the present value of the annual revenue for each investment alternative for a range of annual price levels.

⁴ The daily long term mean reversion factor, η , corresponds to the annual long term mean reversion factor, λ , in Eq. (7).

⁵ We use spot price data from Nord Pool from the period 01.01.2005 to 31.12.2008.

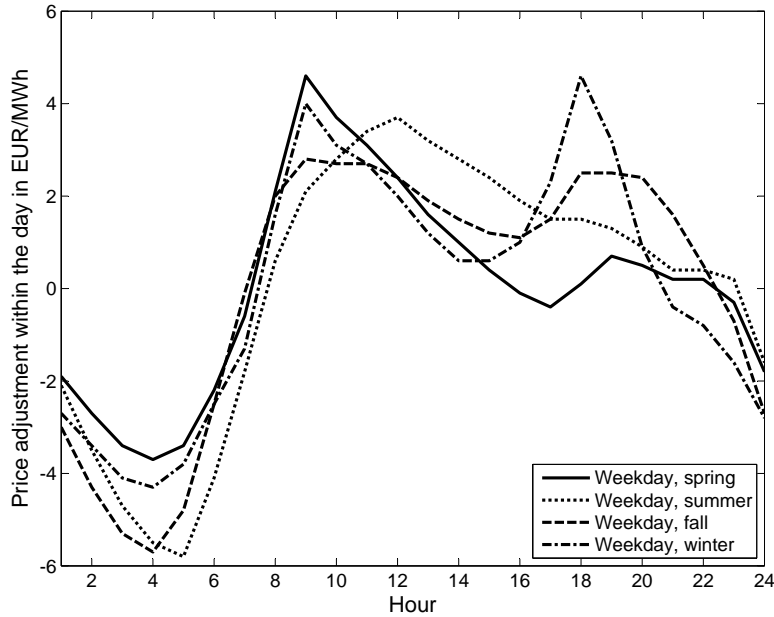


Fig. 2 Deterministic electricity price adjustment within the day.

The volatility of the process is given by ϕ , and ΔW is the increment of a Wiener process. The time increment, ΔW , is one year and Y is the set of all years in the lifetime of the power plant alternatives. We assume no correlation between the Wiener processes of the S_t^l price process (Eq. (4)) and the Q_y price process (Eq. (7)).

The parameters for the annual price level model are based on price information from Nord Pool and found by using an autoregressive process as suggested by Dixit and Pindyck [4]:

$$P_j - P_{j-1} = a + b \cdot P_{j-1} + \varepsilon_j \quad (8)$$

where P_j is the mean electricity price in historical year j , a and b are estimated parameters and ε_j is the error term. The parameters and their corresponding t -statistics are summarized in Table 8 in Appendix 1. The long term mean reversion factor, λ is found as $-b$ from the regression presented in Eq. (8) and the volatility of the process, ϕ , is found as the standard deviation of the residuals from the regression. The long term price level, \bar{Q} , is found as the average price of long term forward contracts.⁶ All values are given in real terms.

The overall price model has two stochastic factors; the short term dynamics are used in the production scheduling, and the long term dynamics in the investment analysis. What links the two is that the starting level of the annual price level, a_1 in Eq. (2), is the same as the annual price level Q_y in Eq. (7).

2.2 Production Scheduling

The aim of the production scheduling is to find the value of one year of power production for all power plant alternatives and all annual price levels (presented by a in Eq. (1)). In order to assess the water in the reservoir, we calculate a MWV. If the MWV is higher than the spot price, water is saved for later use, and if the spot price is higher than the MWV, the water is used for power production, given that reservoir constraints are met. When scheduling the production of the investment alternatives, we have chosen to use the framework suggested by Keppo and Näsäkkälä [14]. Our model differs from theirs in the following ways;

1. The MWV is a function of the storage level in terms of deviation from a median reservoir level. This intuition is supported in the work of Keppo and Näsäkkälä [14], but then in terms of spillage probability.

⁶ Specifically, we use contracts with a delivery period of one year, and three years to maturity. The data is taken from the period 01.01.2008 to 31.12.2008.

2. The production strategy is executed as a “bang-bang” strategy. That is, the power plant shifts instantaneously between using zero and full capacity. Consequently, we do not take minimum discharge and ramping constraints into account. This is a simplification, since the capacity used can be chosen to be between zero and the maximum capacity of the facility. This is illustrated in Zhao and Davison [24] where the release flow is allowed to vary between an upper and a lower boundary. When constraining the operation of the power plant the value of the plant will be reduced.
3. The minimum reservoir level requirement converges to 90 % of maximum reservoir level in the end of the planning horizon.⁷ This approach is supported by Fleten et al. [9] who proposes alternatives for avoiding end effects in the reservoir level, one of them being to choose the end of the planning horizon such that constraining the reservoir to be either full or empty in the end of the planning horizon is realistic. Keppo and Näsäkkälä [14] let the MWV be a function of time to maturity⁸ and let the MWV converge to zero towards the end of the planning period.

The parameterized mathematical form of the MWV is given by $MWV(\cdot)$ in Eq. (9):

$$MWV^{l,c} = \alpha_F^{l,c} \cdot \bar{F}_t^l \cdot e^{\left(\alpha_x^{l,c} \cdot \frac{\bar{x}_t - x_t}{\bar{x}_t} + \alpha_v^{l,c} \cdot \frac{\bar{v}_t - v_t}{\bar{v}_t}\right)}, \quad t \in T, \quad l \in L, \quad c \in C \quad (9)$$

The MWV on day t and initial annual price level l is a function of the average future forward price, \bar{F}_t^l , the reservoir level, x_t , and the inflow, v_t . The median reservoir level is given by \bar{x}_t and the median inflow is given by \bar{v}_t . c is a capacity choice and the set C contains all the capacity choice alternatives. The parameters $\alpha_F^{l,c}$, $\alpha_x^{l,c}$ and $\alpha_v^{l,c}$ denote the rate of change in the MWV as a function of the average future forward price, the reservoir level and the future inflow estimate, respectively. The parameters are estimated for the different capacity alternatives and annual price levels. In order to find the value of these parameters, we estimate the expected present value for the investment alternatives for a set of parameter combinations, and the parameter set chosen for each capacity alternative and annual price level is the one maximizing the lower bound of the present value, as suggested by Keppo and Näsäkkälä [14].

In addition to the spot price and the MWV, the production planning also depends on the reservoir level. The reservoir level is given in the following equation:

$$x_{t+1} = x_t + v_t - \sum_{h \in H} u_{h,t} - s_t, \quad t \in T \quad (10)$$

where $u_{h,t}$ is the discharge in hour h on day t . Eq. (10) states that the reservoir level the next day, x_{t+1} , is equal to the current reservoir level, x_t , plus inflow, v_t , minus discharge throughout the current day, $\sum_{h \in H} u_{h,t}$, minus spillage, s_t . That is; all inflow to the reservoir is either used for power production or spilled. H is the set of all hours within the day. The reservoir level is constrained by upper and lower bounds, x_{\max} and $x_{\min,t}$, respectively, and the lower bound depends on the season. The output is considered to be independent of head variation effect and this means that power generation is linear in discharge. This is a common assumption used in long-term generation planning [22].

The production strategy is executed as a bang-bang strategy, where production takes place if the spot price is higher than the MWV and the reservoir constraints are met. If the spot price is lower than the MWV, production will only take place at times when water will be spilled if no production occurs. A more thorough presentation of how the production strategy is modeled is outlined in Appendix 2.

The value of the power production is maximized by finding the MWV and producing according to the strategy outlined in the previous paragraph. The value of investment alternative c depends on the power plant capacity, ϖ^c , the plant efficiency, η^c , the discharge of power plant c in hour h of day t , $u_{h,t}^c$, the maintenance cost, m^c , as well as the spot price, $S_{h,t}^l$ and the discount rate, μ . The maintenance costs of each investment alternative depends on the production. This is a simplification made for modeling purposes. The present value of one year of operation of power plant c for annual electricity price level l can be summarized as in Eq. (11):

$$PV^{l,c} = \mathbb{E} \left[\varpi^c \cdot \eta^c \cdot \sum_{t \in T} \frac{1}{(1 + \mu)^{t/|T|}} \cdot \sum_{h \in H} (u_{h,t}^c \cdot (S_{h,t}^l - m^c)) | K^{l,c} \right], \quad l \in L, \quad c \in C \quad (11)$$

⁷ This approximately equals the historical median reservoir level.

⁸ The valuation procedure is developed for option pricing.

2.3 Real Options Valuation

The main idea of our approach for evaluating the real options is to estimate the optimal decision for each possible annual price level in each year. We assume that the the producer can only make an investment decision once each year, and in order to compare the investment alternatives, we annualize the construction costs for the production capacity expansion alternatives. By doing this, we can compare investment today with investment in the future, without expanding the planning horizon. Capital is not a limiting factor in our model. We also do not consider how the cash flow from the capacity expansion project affects the overall cash flows in the investing company. If the producer chooses to invest in a new power plant, the plant can be taken into use immediately.

The project value is maximized by solving Eq. (12):

$$\max_{a_y \in A_{g_y}} \left\{ \sum_{y \in Y} \frac{1}{(1 + \mu)^y} \cdot \mathbb{E} [AP_{g_y, a_y, Q_y}] \right\} \quad (12)$$

where AP_{g_y, a_y, Q_y} is annual profit and the discount factor is given by μ .⁹ The real option valuation is thus formulated as an optimal control problem that can be solved by dynamic programming. The annual profit from the investment depend on the power plant in use:

$$AP_{g_y, a_y, Q_y} = R_{g_y, Q_y} - I_{a_y}, \quad y \in Y \quad (13)$$

where the annual profit (AP_{g_y, a_y, Q_y}) depends on the power plant in use in year y , g_y , the action undertaken, a_y , the annual price level, Q_y , and the investment cost associated with action a_y , I_{a_y} . The annual revenue from power plant g_y at price level Q_y is given by R_{g_y, Q_y} . The power plant running in year $y + 1$ depends on the state of the system and its action in year y . The set of possible actions when the state is g_y is denoted by A_{g_y} . In the first year, the set of possible actions contain all the capacity expansion alternatives, $A_{g_1} = \{0, 1, 2, \dots, n\}$, where 0 means no action (keep existing facilities) and 1 through n stands for investing in one of n alternatives for a new production facility with increased production capacity. The state of the system in year $y + 1$ depends only on the state in year y , and on the action taken in the same year. As long as no investment in a new power plant has been undertaken, the possible actions remain the same. If an investment in a new power plant is undertaken in year y' , the possible action changes to $A_{g_y} = 0$ for $y > y'$. This is due to the following assumption: once a new power plant is installed, we cannot go back to use the old facilities or invest in another power plant.

The lifetime of the investment alternatives are $|Y|$ years. Let π_{y, g_y, Q_y} denote the value function and the terminal condition be given by Eq. (14):

$$\pi_{|Y|, g_{|Y|}, Q_{|Y|}} = 0 \quad (14)$$

This equation states that the salvage value of each investment alternative is zero. The main problem is then to determine the optimal investment strategies, g_y . The optimal action for each year and each annual price level, which corresponds to the optimal decision rules, can be found recursively by solving the Bellman equation which is stated in Eq. (15):

$$\pi_{y, g_y, Q_y} = \max_{a_y \in A_{g_y}} \left\{ AP_{g_y, a_y, Q_y} + \frac{1}{1 + \mu} \cdot \mathbb{E} [\pi_{y+1, g_{y+1}, Q_{y+1}} | Q_y] \right\}, \quad y \in Y \quad (15)$$

where AP_{g_y, a_y, Q_y} is the immediate profit the investor receives the first year after the investment decision, and $\frac{1}{1 + \mu} \cdot \mathbb{E}[\cdot]$ is the discounted continuation value. The continuation value is obtained by using the dynamic programming approach explained by Fuss et al. [11].

The optimal decision in year $y \leq |Y|$ is found for each possible annual price level in the range $[Q_{\min}, Q_{\max}]$. The optimal decision rules serve as input to the Monte Carlo simulations presented in the next section.

⁹ Note that this is the same discount factor as used in the production planning.

2.4 Monte Carlo Simulations

The results, that is, how the optimal decision rules found in the previous paragraphs will materialize in a certain price path, are extracted by conducting Monte Carlo simulations. For every year, the Monte Carlo approach provides an annual price level, and this price level is matched with the optimal policy for the given year and the given price level. The annual price levels are simulated using Eq. (7) in Section 2.1. This procedure yields the frequencies, that is, the number of times, investment in each of the investment alternatives will occur for each year throughout the lifetime of the power plant.

Summarizing, the analysis is conducted in three steps. First, we obtain the annual revenue from each power plant capacity alternative for a range of annual price levels in the production scheduling. Second, we find the optimal decision rules for each year and annual price level in the real options valuation. The third and final step is to use Monte Carlo simulations, where we simulate electricity price paths that we match with the optimal decision rule for the given year and the simulated annual price level. We do this a number of times and extract the frequencies for which each investment alternative is chosen.

3 Case Presentation

An electricity producer has a sequence of five hydraulically coupled power plants installed in a river system. These are old power plants that were designed to provide base load electricity supply to industries in the surrounding area. The existing configuration suffers from high response time and low efficiency. The majority of the inflow used for electricity production is accumulated and stored in a large reservoir. This reservoir has approximately 70 % degree of regulation, defined as reservoir size relative to annual inflow. That is, with no discharge and average inflow, an empty reservoir is refilled in approximately eight months. Due to the degree of regulation opportunities, it is regarded as a multi-seasonal reservoir. Considering the existing power plants, they have a capacity factor of close to 70 %, making it hard to exploit market fluctuations, even with a large reservoir. Also, requirements for maintenance of the existing power plants lead to large restoration costs.

Construction of a new power plant in order to replace existing ones often implies long outage times for existing plants, resulting in considerable revenue losses. One of the major costs when refurbishing and maintaining existing production facilities is the production lost due to the generators not being able to produce electricity. Refurbishing existing facilities will therefore to a certain extent imply the same costs as investment in a new production facility. Hence, investment in new production facilities should be considered when the existing facilities are due for restoration and maintenance.

In order to extract the value found in the standard deviation of electricity prices, the electricity producer considers upgrading the existing facilities and hence increasing the generation capacity in the river system. If the producer chooses to expand the capacity in the river system, three of the existing power plants will be replaced by one new power plant and the remaining two power plants will be upgraded. This paper investigates two alternatives for new total capacity in the river system in addition to maintenance of the existing power plants which leaves the installed capacity unchanged. In the following, the alternatives for upgrading the capacity in the river system are called the large size power plant alternative and the medium size power plant alternative.

Each new power plant has a lifetime of 50 years. The existing power plants will also have an extended lifetime of 50 years if the restoration and maintenance requirements are completed. The challenge at hand is to analyze whether the owner should refurbish the existing power plants, or invest in a new power plant. To comply with regulatory requirements, the existing power plants must undergo restoration and maintenance today and two times in the future. The costs associated with the restoration and maintenance of the existing power plants are given in Table 1:

Table 1 Costs associated with restoration and maintenance of existing facilities.

| Year | Cost [Million euro] |
|------|---------------------|
| 1 | 31 |
| 9 | 38 |
| 19 | 38 |

If the owner chooses to invest in a new power plant, the timing and capacity choice of the investment must be decided. It is important to note that if the owner chooses not to build a new power plant today,

the owner can decide to invest in a new power plant at a later time. That is, if the owner chooses to refurbish the existing power plants in year 1, the owner can still choose to invest in a new power plant later and thereby avoid the maintenance costs for the existing power in the succeeding years. The investment alternatives considered for a new power plant are mutually exclusive. All calculations are conducted in real terms, that is, in 2008 values.

4 Numerical Results

Under the base scenario with two investment alternatives in addition to keeping the existing facilities, the optimal action today is to invest in the large size power plant. This investment gives added net present value of 47.5 million euro, while the added value from the medium size power plant is 45 million euro. A power plant with larger capacity compared to the existing facilities has better opportunity to move water from periods with low prices to periods with high prices. The large size power plant gives an NPV that is 5.6 % higher than the medium size power plant. The large size power plant contributes to increase the average price per MWh produced by four euro, while the medium size power plant alternative increase the average price per MWh produced by almost three euro. Considering the effect of having two capacities to choose from instead of one in the upgrade project, the trigger level for when to invest is slightly increased. If no investment is undertaken in year one, the frequency for when investment in a new power plant in the second year is optimal decreases by 0.3 % when including the second capacity expansion alternative. The effect is small, but present.

The optimal policy for each year and each annual price level, given no prior investment in a new power plant, for the next 19 years are presented in Figure 3. The dark grey area presents the prices where investment in the large size power plant is optimal. Similarly, the light grey area presents prices where investment in the medium size power plant is optimal. The white area presents no action; which means keeping the existing production facilities. The black lines represent three annual price level simulations.

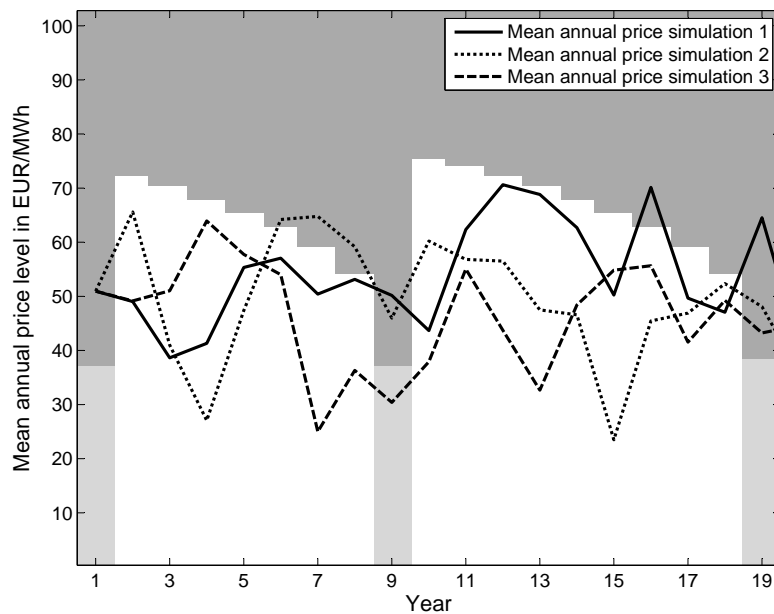


Fig. 3 Optimal policy for each year and each annual price level for the next 19 years, given no prior investment in a new power plant. The dark grey area is where investment in a large size power plant is optimal, the light grey area is where investment in a medium size power plant is optimal and the white area is where keeping and refurbishing the existing power plants is the optimal decision. The black lines represents three annual price simulations. The existing power plants are due for maintenance in year one, nine and 19.

Considering annual price simulation 1 in Figure 3: In year one, the optimal action is to invest in the large size power plant alternative. If no investment in a new power plant is undertaken in this year, the investor must pay the cost for refurbishment and maintenance of the existing power plants in this year, and the optimal decision for the next seven years is to keep the existing facilities. In the ninth year, when the existing power plants are due for the second round of maintenance, the optimal decision is to invest

in the large size power plant alternative. For annual price simulation 2, investment in the large size power plant is optimal in year one. If no investment is undertaken in this year, the optimal decision for years two through five is to keep the existing facilities. In the sixth year, the annual price level hits the trigger level for investment in the large size power plant, and investment in this power plant is the optimal decision. For annual price simulation 3, the optimal decision in year one is to invest in the large size power plant alternative. If the owner decides to refurbish the existing power plants, the optimal decision in years two to eight is to keep the existing facilities, while in the ninth year, the optimal decision is to invest in the medium size power plant.

Figure 3 shows that in the years when the existing power plants are due for maintenance (years one, nine and 19), investment in a new power plant is the optimal decision. Which capacity should be chosen depends on the price level in the given year. For the intermediate years, the trigger level is reduced each year, illustrating that high annual prices are exploited when the time approaches a new outlay for refurbishing existing power plants and investment in a new power plant should be undertaken in any case.

The timing and capacity choice for investing in a new power plant depends on many factors. In the following paragraphs, sensitivity regarding the deterministic adjustment of electricity prices within the day, the long term electricity price level, discount rate and investment cost is presented. The tables presenting the sensitivity analyses (Tables 2-5) contain the following: The left column gives the year considered and the first two rows give the parameter and values for which the sensitivity analysis is conducted. Like described in Section 2.4 we perform Monte Carlo simulations of the annual price level and match the price level with the optimal decision for the given price in the given year (also illustrated in Figure 3). The tables give how often (in percent) the trigger level for investing in either the medium size power plant (M) or the large size power plant (L) is reached in this year, given no prior investment.

With increased transmission capacity between Norway and continental Europe, both through direct high voltage direct current (HVDC) cables such as the NorNed cable which interconnects Norway and the Netherlands [20] and more indirect connections through Denmark and Sweden, such as Skagerak 4 which is under construction between Norway and Denmark [21], increased standard deviation of electricity prices within the day may be expected. This effect is strengthened by additional interconnecting transmission capacity, such as the Nord.Link HVDC cable, which is under planning and may interconnect Norway and Germany [19]. With more fluctuation, a low capacity factor is desirable. Table 2 presents how often the optimal decision is to invest in a new power plant, given no prior investment, for the first nine years of the investment opportunity with ± 15 and 30 % deterministic price adjustment compared to the base scenario, which employs historical values for the adjustment within the day (as illustrated in Figure 2). For all adjustment alternatives, investment in year zero is optimal, as illustrated in Table 2. If

Table 2 Sensitivity regarding price adjustment for electricity prices within the day. The table gives the percentages of times when investment in either the medium size power plant (M) or the large size power plant (L) occurs, given no prior investment to the given year.

| Year | Change in adjustment of the electricity price within the day | | | | | | | | | |
|------|--|----|-------|-----|-----|-----|-------|-----|-------|-----|
| | -30 % | | -15 % | | 0 % | | +15 % | | +30 % | |
| | M | L | M | L | M | L | M | L | M | L |
| 1 | 100 | 0 | 0 | 100 | 0 | 100 | 0 | 100 | 0 | 100 |
| 2 | 0 | 1 | 0 | 2 | 0 | 3 | 0 | 4 | 0 | 6 |
| 3 | 0 | 2 | 0 | 3 | 0 | 5 | 0 | 7 | 0 | 9 |
| 4 | 0 | 4 | 0 | 6 | 0 | 8 | 0 | 10 | 0 | 12 |
| 5 | 0 | 6 | 0 | 10 | 0 | 12 | 0 | 13 | 0 | 15 |
| 6 | 0 | 11 | 0 | 13 | 0 | 16 | 0 | 19 | 0 | 22 |
| 7 | 0 | 17 | 0 | 22 | 0 | 24 | 0 | 26 | 0 | 27 |
| 8 | 0 | 28 | 0 | 35 | 0 | 40 | 0 | 42 | 0 | 44 |
| 9 | 57 | 43 | 31 | 69 | 12 | 88 | 5 | 95 | 4 | 96 |

the standard deviation of prices within the day increases, the optimal decision remains being to invest in the large size power plant. If it is reduced by 30 % however, the medium size power plant is the optimal investment. This scenario is unlikely, and hence the optimal decision in year zero is robust to changes in the deterministic adjustment of electricity prices within the day. The table also illustrates how the trigger level for investment in a new power plant increases with less adjustment and decreases with a higher adjustment, as the percentages decrease and increase, respectively, in the years between maintenance costs of the existing power plants. If no investment is undertaken in year one, investment in a new power

plant will be the optimal decision for all adjustment alternatives no later than the time for the next scheduled maintenance of the existing power plants (year nine).

The long term electricity price is important when analyzing the profitability of a power plant. In the base scenario, the long term price is set to the average price of three year forward contracts. Table 3 presents how often the optimal decision is to invest in a new power plant, given no prior investment, for the first nine years of the investment opportunity when the long term electricity price is increased and decreased by 10 and 20 % compared to the base scenario. Table 3 shows that the optimal decision is

Table 3 Sensitivity regarding the long term electricity price. The table gives the percentages of times when investment in either the medium size power plant (M) or the large size power plant (L) occurs, given no prior investment to the given year.

| Year | Change in long term electricity price | | | | | | | | | |
|------|---------------------------------------|---|-------|----|-----|-----|-------|-----|-------|-----|
| | -20 % | | -10 % | | 0 % | | +10 % | | +20 % | |
| | M | L | M | L | M | L | M | L | M | L |
| 1 | 100 | 0 | 100 | 0 | 0 | 100 | 0 | 100 | 0 | 100 |
| 2 | 0 | 0 | 0 | 0 | 0 | 3 | 0 | 15 | 0 | 36 |
| 3 | 0 | 0 | 0 | 0 | 0 | 5 | 0 | 19 | 0 | 36 |
| 4 | 0 | 0 | 0 | 1 | 0 | 8 | 0 | 21 | 0 | 37 |
| 5 | 0 | 0 | 0 | 2 | 0 | 12 | 0 | 24 | 0 | 37 |
| 6 | 0 | 0 | 0 | 5 | 0 | 16 | 0 | 29 | 0 | 41 |
| 7 | 2 | 0 | 0 | 11 | 0 | 24 | 0 | 37 | 0 | 48 |
| 8 | 11 | 0 | 0 | 20 | 0 | 40 | 0 | 51 | 0 | 61 |
| 9 | 100 | 0 | 71 | 29 | 12 | 88 | 2 | 98 | 0 | 100 |

unchanged at investment in a new power plant in year one for all long term price level alternatives, while the capacity choice depends on the long term electricity price. If the long term price level decreases beyond the base scenario level, investment in the medium size power plant is optimal, while if the level increases, the optimal decision is unchanged at investment in the large size power plant. If the level increases or decreases by 10 or 20 % and no investment is undertaken in the first year, the optimal strategy is to invest in a new power plant no later than the next time when the existing power plants are due for maintenance. For the intermediate trigger levels, the trigger level increase when the long term electricity price decreases and decreases if the opposite occurs.

The discount rate affects the option value as higher discount rates value cash flows today higher compared to future cash flows and decrease the value of postponing investment, and a lower discount rate has the opposite effect. Table 4 presents how often the optimal decision is to invest in a new power plant, given no prior investment, for the first nine years of the investment opportunity for a range of discount rates, where a discount rate of 7 % is the base scenario.

Table 4 Discount rate sensitivity. The table gives the percentages of times when investment in either the medium size power plant (M) or the large size power plant (L) occurs, given no prior investment to the given year.

| Year | Discount rate | | | | | | | | | | | | | |
|------|---------------|-----|-----|-----|-----|-----|-----|-----|-----|----|-----|---|------|---|
| | 4 % | | 5 % | | 6 % | | 7 % | | 8 % | | 9 % | | 10 % | |
| | M | L | M | L | M | L | M | L | M | L | M | L | M | L |
| 1 | 0 | 100 | 0 | 100 | 0 | 100 | 0 | 100 | 100 | 0 | 100 | 0 | 0 | 0 |
| 2 | 0 | 89 | 0 | 69 | 0 | 40 | 0 | 3 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 88 | 0 | 69 | 0 | 40 | 0 | 5 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 88 | 0 | 69 | 0 | 41 | 0 | 8 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | 88 | 0 | 69 | 0 | 42 | 0 | 12 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 89 | 0 | 70 | 0 | 46 | 0 | 16 | 0 | 1 | 0 | 0 | 0 | 0 |
| 7 | 0 | 88 | 0 | 72 | 0 | 50 | 0 | 24 | 0 | 3 | 0 | 0 | 0 | 0 |
| 8 | 0 | 89 | 0 | 77 | 0 | 61 | 0 | 40 | 0 | 11 | 0 | 0 | 0 | 0 |
| 9 | 0 | 100 | 0 | 100 | 0 | 100 | 12 | 88 | 84 | 16 | 14 | 0 | 0 | 0 |

The optimal decision in the first year for discount rates of 7 % and lower is to invest in the large size power plant. If the discount rate is 8 or 9 %, however, the optimal decision in the first year is to invest in the medium size power plant. If no investment in a new power plant is undertaken in the first year, a discount rate lower than the base scenario discount rate (7 %) triggers investment at an earlier stage, while a higher discount rate triggers investment at a later stage. With discount rates of 8 % and less, investment in a new power plant will be the optimal decision no later than the next time the existing

power plants are due for maintenance. If the discount rate is 9 %, investment in a new production facility may never be optimal if not undertaken in the first year. If the discount rate is higher than 9 %, the optimal decision is to keep the existing facilities in all years. Both the investment decision on whether or not to invest in a new production facility in the first year, and the trigger level for the intermediate years between refurbishment costs for the existing power plants and the capacity choice is sensitive to the discount rate.

Investment costs for projects in the power sector are often uncertain. Table 5 presents how often the optimal decision is to invest in a new power plant, given no prior investment, for the first nine years of the investment opportunity with investment costs of ± 10 and 20 % compared to the base scenario.

Table 5 Investment cost sensitivity. The table gives the percentages of times when investment in either the medium size power plant (M) or the large size power plant (L) occurs, given no prior investment to the given year.

| Year | Change in investment cost of new projects | | | | | | | | | |
|------|---|-----|-------|-----|-----|-----|-------|----|-------|---|
| | -20 % | | -10 % | | 0 % | | +10 % | | +20 % | |
| | M | L | M | L | M | L | M | L | M | L |
| 1 | 0 | 100 | 0 | 100 | 0 | 100 | 100 | 0 | 100 | 0 |
| 2 | 0 | 65 | 0 | 36 | 0 | 3 | 0 | 0 | 0 | 0 |
| 3 | 0 | 65 | 0 | 36 | 0 | 5 | 0 | 0 | 0 | 0 |
| 4 | 0 | 66 | 0 | 37 | 0 | 8 | 0 | 0 | 0 | 0 |
| 5 | 0 | 67 | 0 | 40 | 0 | 12 | 0 | 0 | 0 | 0 |
| 6 | 0 | 68 | 0 | 42 | 0 | 16 | 0 | 1 | 0 | 0 |
| 7 | 0 | 68 | 0 | 48 | 0 | 24 | 0 | 3 | 0 | 0 |
| 8 | 0 | 74 | 0 | 59 | 0 | 40 | 2 | 11 | 0 | 0 |
| 9 | 0 | 100 | 1 | 99 | 12 | 88 | 84 | 16 | 4 | 0 |

Independent of an increase or decrease in investment costs of 10 or 20 % for the new production facilities, investment in a new power plant should be undertaken in year one. If the investment cost increases, the optimal capacity choice changes from being the large size power plant to being the medium size power plant. If no investment is undertaken in year one, a lower investment cost gives investment in the large size power plant no later than the time for the next scheduled refurbishment of the existing power plants. If the investment cost stay the same or increase, however, the optimal capacity may change from being the large size power plant to being the medium size power plant. If the investment cost increases by 20 % and no investment is undertaken in year one, the probability of reaching a price trigger level for investment in a new power plant is low.

5 Conclusion

The real options valuation and Monte Carlo simulations show that the optimal decision for the power producer is to invest in a new production facility immediately. Which power plant is the optimal choice, however, is not clear. The difference in added value from choosing the large size power plant alternative compared to choosing the medium size power plant alternative in the base scenario is 2.5 million euro. That is, the large size power plant gives an NPV that is 5.6 % higher than the medium size power plant. The sensitivity analysis shows that investment should be undertaken in year one, but the capacity choice varies with different assumptions for the long term electricity price level, discount rate and investment cost. A more thorough analysis of these parameters should be conducted by the power producer before the capacity choice of the upgrading project is made.

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Appendices

Appendix 1

Parameters for Valuation Framework

Table 6 Estimated parameters for the daily electricity price dynamics presented in Eq. (1)-Eq. (4).

| Parameter ^a | γ | τ | η^b | \bar{Q} | κ | σ |
|------------------------|----------|--------|----------|-----------|----------|----------|
| Value | 44.1 | 389.5 | 0.0036 | 408 | 0.014 | 18.7 |
| t-statistic | 2.5 | 16.9 | - | - | 410 | - |

^a We use spot price data from Nord Pool from the period 01.01.1993 to 31.12.2008.

^b η is the daily mean-reversion level equivalent to the annual λ in Eq. (7).

Table 7 Parameters and sets for numerical example.

| Parameter | Description | Value | Unit |
|------------|---|------------------|---------|
| $ Y $ | Lifetime of power plants | 50 | years |
| $ T $ | Production period | 365 | days |
| $ H $ | Hours in a day | 24 | hours |
| q | Length of forward curve | 180 | days |
| Q_{\min} | Lower bound for prices considered | 0.625 | EUR/MWh |
| Q_{\max} | Upper bound for prices considered | 102.5 | EUR/MWh |
| $ L $ | Annual price level, in the range $[Q_{\min}, Q_{\max}]$ | 164 ^a | EUR/MWh |
| μ | Discount rate | 7 ^b | % |

^a Increments of 0.625 euro. Starting at $Q_{\min} = 0.625$ we get $Q_{\max} = 164 * 0.625 = 102.5$.

^b In order to be consistent with common practice in electricity companies, we use a real before tax discount rate of 7 %.

Table 8 Estimated parameters for the autoregressive process presented in Eq (7).

| Parameter ^a | a | b | R^2 | std(residuals) ^b |
|------------------------|-------|-------|-------|-----------------------------|
| Value | 169.7 | -0.73 | 0.37 | 90.0 |
| t-statistic | -2.26 | 2.42 | - | - |

^a We use spot price data from Nord Pool from the period 01.01.1993 to 31.12.2008.

^b Standard deviation of residuals.

Appendix 2

Pseudocode for Production Scheduling

```

if  $x_t + v_t \leq x_{\max}$  &  $x_t + v_t - 24 \cdot u_{\max} \geq x_{\min,t}$  &  $S_{h,t} > K_t$  then
  Find the optimal production schedule for day  $t$ 
else if  $x_t + v_t > x_{\max}$  then
  Find the optimal production schedule and discharge ( $d$ ) for day  $t$ 
  if  $x_t + v_t - 24 \cdot u_{\max} > x_{\max}$  then
    Full production and discharge in any case
  else if  $x_t + v_t - d > x_{\max}$  then
    Find most profitable hours and produce so that requirement is met
  end if
else if  $x_t + v_t - 24 \cdot u_{\max} \geq x_{\min}$  &  $S_{h,t} > K_t$  then
  Find optimal production schedule and discharge ( $d$ ) for day  $t$ 
  if  $v_t + v_t < x_{\min}$  then
    No production during day  $t$ 
  else if  $x_t + v_t - d < x_{\min}$  then
    Find most profitable hours to produce
  end if
end if

```