X International Conference on Structural Dynamics, EURODYN 2017

Vortex-induced vibrations of a vertical riser with time-varying tension

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Abstract

Numerical simulations of a vertical tensioned riser in a sheared flow are performed, where the riser top end oscillates sinusoidally in the vertical direction. The oscillating top-end motion causes tension variations and changes in the natural frequencies of the riser. The flow around the structure causes vortex shedding, oscillating lift forces and vortex-induced vibrations (VIV). It is well-known that the vortex shedding is affected by structure motion, and may lock on to the riser’s natural frequencies. However, the vortex shedding frequency must remain close to the Strouhal frequency, and may therefore excite different modes of vibration as the riser tension changes. With this in mind, the overall aim of this paper is to investigate how tension variations affect the VIV response. The riser dynamics are simulated in time domain using a non-linear finite element structural model combined with an empirical hydrodynamic load model. The latter includes a synchronization model which simulates how the vortex shedding reacts to the structure motion to obtain lock-in. Simulations are run using different amplitudes and frequencies for the top-end motion, and the resulting cross-flow displacements and bending strains are studied. The results show that, when the riser top-end oscillates, the VIV response contains several modes, and the dominating mode may vary with time. The number of active modes are found to be strongly dependent on the period of the riser tension.

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Peer-review under responsibility of the organizing committee of EURODYN 2017.
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Keywords: Vortex-induced vibrations; Riser; Non-linear dynamic analysis;

1. Introduction

Vortex-induced vibrations (VIV) occur when an elastic cylinder is exposed to external fluid flow as a result of the oscillating lift and drag forces associated with the shedding of vortices [1–3]. Slender structures such as risers are widely used in the oil and gas industry and possibly also in future deep sea mining operations, and because of their flexibility, VIV may cause significant problems. The vibrations cause oscillating stresses, leading to crack growth and potentially fatigue failure.

When fluid flows past a stationary cylinder, vortex shedding occurs at a well-defined frequency, known as the Strouhal frequency, \( f_s = \frac{St U}{D} \), where \( St \) is the Strouhal number (around 0.2 for subcritical Reynolds numbers), \( U \) is the flow velocity and \( D \) the cylinder diameter. A characteristic feature of vortex shedding is however that it is affected
by cylinder motion, and may deviate from the frequency \(f_s\) and lock on to the frequency of vibration, \(f_{osc}\), if these are sufficiently close. For a riser, several natural frequencies may lie close to \(f_s\), and in such cases it is not straightforward to predict the resulting frequency of vibration. The problem is even more difficult if the current velocity varies along the riser length [4].

The natural frequencies of a tensioned riser is strongly influenced by the tension level. Considering the actual ocean environment, a riser suspended from a floating platform may experience tension variations caused by vertical and horizontal floater motions (although a heave compensation system can be used to minimize the tension variations). Assuming a constant vortex shedding frequency, tension variations might trigger changes in the mode of vibration, because the natural frequency closest to \(f_s\) changes over time. This can be important, because, although VIV occurs at a limited amplitude, the structural stresses will increase when the mode increases, because of the increased curvature. Therefore, changes in mode will influence the fatigue damage, even if the vibration amplitude remains the same.

The effect of tension variations on riser VIV has received limited attention in the past, which is perhaps linked to the fact that few empirical models are able to include non-linear structural effects. However, Silveira et al. [5] used a wake oscillator model to study this, which did predict mode switching in some cases. The coupling between top-end heave and riser VIV was also studied by Chen et al. [6], focusing on the instabilities caused by parametric excitation (Mathieu instability).

In the present paper, a non-linear finite element program is used to simulate riser dynamics in time domain. An empirical hydrodynamic force model is utilized, which has been under development for some years [7–9]. It has previously been shown that this model is able to predict non-stationary VIV, e.g. due to oscillating flow [10] or imposed structural motions [11]. It should therefore be well suited for the present purpose, which is to study the effect of tension variations on a vertical riser in a sheared current. A number of simulations are performed to shed light on the importance of the tension amplitude and period.

2. Numerical model

2.1. Hydrodynamic forces

At any point along the riser, the total hydrodynamic force is calculated as:

\[
F = C_M \rho \pi D^2 \ddot{u}_n - (C_M - 1) \rho \pi D^2 \ddot{x}_n + \frac{1}{2} \rho D C_D |v_n| v_n + \frac{1}{2} \rho D C_D |v_n| (j_3 \times v_n) \cos \phi_{exc}
\]

The three first terms make up Morison’s equation [12], while the final term represents the oscillating lift force due to vortex shedding [11]. \(C_M\) and \(C_D\) are the inertia and drag coefficients, while \(C_v\) determines the strength of the vortex shedding force. Furthermore, \(\ddot{u}_n\) is the normal component (i.e. perpendicular to the cylinder axis) of the fluid particle acceleration, \(\ddot{x}_n\) is the normal component of the cylinder acceleration and \(v_n\) is the normal component of the relative fluid velocity. The relative flow velocity is given as \(v = u - \dot{x}\), where \(u\) is the incoming flow velocity and \(\dot{x}\) is the velocity of the cylinder cross-section. \(j_3\) is a unit vector pointing in the direction of the cylinder axis, and \(\rho\) is the water density.

The oscillations of the lift force are described through the time-varying phase \(\phi_{exc}\). The evolution in time is given by equations (2), (3a) and (3b):

\[
\frac{d\phi_{exc}}{dt} = 2\pi \frac{\hat{f}_{exc}|v_n|}{D}
\]

\[
\hat{f}_{exc} = \begin{cases} \hat{f}_0 + (\hat{f}_{max} - \hat{f}_0) \sin \theta, & \theta \geq 0 \\ \hat{f}_0 + (\hat{f}_0 - \hat{f}_{min}) \sin \theta, & \theta < 0 \end{cases}
\]

Equation (2) gives the relationship between the dimensionless and the actual frequency, while equation (3a) and (3b) models the synchronization between the vortex shedding and the cylinder motion. Here, \(\theta\) is the instantaneous phase difference between the cylinder cross-flow velocity and the lift force, and the essential feature of the synchronization
model is that it makes it possible for the lift force to vary its instantaneous frequency between $\hat{f}_{\text{min}}$ and $\hat{f}_{\text{max}}$, and lock on to the frequency of vibration. For more details see [11]. The empirical parameters used in the present study are given in Table 1.

Table 1. Empirical parameters used in the hydrodynamic model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_M$</td>
<td>2.0</td>
</tr>
<tr>
<td>$C_D$</td>
<td>1.0</td>
</tr>
<tr>
<td>$C_v$</td>
<td>1.0</td>
</tr>
<tr>
<td>$\hat{f}_0$</td>
<td>0.17</td>
</tr>
<tr>
<td>$\hat{f}_{\text{min}}$</td>
<td>0.125</td>
</tr>
<tr>
<td>$\hat{f}_{\text{max}}$</td>
<td>0.3</td>
</tr>
</tbody>
</table>

2.2. Structural model and dynamic analysis

The structural analysis is performed by the finite element program RIFLEX [13], which is tailor made for dynamic analysis of slender marine structures. The riser is modeled using beam elements, formulated using a co-rotated reference frame to account for large displacements and rotations in 3D space. The non-linear dynamic analysis is based on the incremental form of the dynamic equilibrium equation, which is solved by the Newmark-\(\beta\) method and a Newton-Raphson iteration procedure.

A 500 meter long vertical riser, as illustrated in Fig. 1, is modeled as 500 beam elements. Key riser properties are given in Table 2. The external load is an incoming stationary flow, which varies linearly from zero at the bottom to $U_{\text{max}} = 1$ m/s at the top. The top tension is imposed by a prescribed vertical displacement of the top end, $z_{\text{top}} = z_0 + z_1 \sin(\omega t)$, where $T = 2\pi/\omega$ is the tension period. The mean top displacement $z_0$ is selected such that the riser pretension is 1000 kN before the riser weight, buoyancy and current is activated. When these are turned on, the mean top tension increases to approximately 1230 kN. The time step length is set to $\Delta t = 0.01$ s which gives more than 150 time steps per VIV vibration period for all the simulations. Structural damping is included through the proportional damping formulation, $C = a_1 M + a_2 K$ where $M$ and $K$ are the global mass and stiffness matrices. The values used for the damping parameters are $a_1 = 0$ and $a_2 = 7 \times 10^{-3}$, giving a damping ratio of approximately 1 %. Note that hydrodynamic damping, which is included through the drag term in Equation (1), comes in addition.
Table 2. Riser properties.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Riser length ($L$)</td>
<td>500 m</td>
</tr>
<tr>
<td>Outer diameter ($D$)</td>
<td>0.3 m</td>
</tr>
<tr>
<td>Axial stiffness ($EA$)</td>
<td>$1.91 \times 10^6$ N</td>
</tr>
<tr>
<td>Bending stiffness ($EI$)</td>
<td>$2.01 \times 10^7$ Nm$^2$</td>
</tr>
<tr>
<td>Torsional stiffness ($Gi_p$)</td>
<td>$1.34 \times 10^7$ Nm$^2$</td>
</tr>
<tr>
<td>Mass per unit length, including content ($m$)</td>
<td>140 kg/m</td>
</tr>
<tr>
<td>Density, seawater ($\rho$)</td>
<td>1025 kg/m$^3$</td>
</tr>
</tbody>
</table>

3. Results and discussion

3.1. Riser with time-varying tension

Figure 2 shows the cross-flow displacement of the riser when the tension oscillates with a period of 20 seconds. For comparison, the predicted VIV period is approximately 1.9 s. After completion of the simulation, modal analysis [14] is performed to identify the dominating modes for the cross-flow strain. This means expressing the strain as $\epsilon(z,t) = \sum_{i=1}^{N} \epsilon_i(t) \sin(i\pi z/L)$, and finding the modal weight factors $\epsilon_i(t)$ using the simulation results and the method of least squares. The modal weight factors are presented in Fig. 3a, and it is clear that many modes are active, and the dominating mode changes continuously. Typically, the lower modes (e.g. mode 6) increase in strength when the tension is high, while the higher modes (e.g. mode 9) dominate when the tension is low. Looking at the time series of strain (Fig. 3b), it is seen that the strain decreases when lower modes dominate. This occurs even though the displacement does not change significantly (Fig. 3c), which means that the variation in strain is caused primarily by mode shifting.
3.2. Effect of tension amplitude and period

Several simulations are performed with varying amplitude and period for the top end motion. For each case, some key results are evaluated. The standard deviation of the cross-flow displacement at the position where it is largest serves as an measure of the vibration amplitude. Similarly, the standard deviation of the cross-flow strain, also at the position where it is largest, gives an indication of the strain amplitude. The number of active modes are taken as the number of modes which has a maximum weight factor at least 0.5 times that of the dominating mode. The factor 0.5 was taken somewhat arbitrarily, as the purpose is simply to illustrate how the "bandwidth" of modes vary. The results are shown in Fig. 4, plotted as a function of the ratio between the standard deviation of the tension and the mean top tension.

Starting with the displacement (Fig. 4a) it is seen that for $T = 5$ s, the results vary little with tension amplitude. The exception is the case with the largest tension variation, where the riser experienced compression and buckling at the lower end. For $T = 10$ s and $T = 20$ s, the displacements are slightly reduced by the tension variation, compared to the case with constant tension. Moving on to the strain (Fig. 4b), this is also fairly constant for $T = 5$, while the tension variation reduces the strains slightly when $T = 10$ s and $T = 20$ s. Naturally, the strain increases suddenly for the highest tension amplitude, due to buckling at the bottom. The number of active modes (Fig. 4c) are seen to increase steadily together with the tension amplitude, but the importance of the period is also quite clear: When the tension changes rapidly there is not enough time for the riser to change mode. Therefore, the number of active modes is highly dependent on the period of the tension variation.

4. Conclusions

This paper has focused on how tension variations affect riser VIV, using a 500 m vertical riser in a sheared flow as an example. The vortex shedding forces are described by an empirical model which captures how the lift force frequency may vary with time to obtain lock-in. This hydrodynamic force model has been implemented into a development version of the non-linear finite element program, RIFLEX, which has been used in the numerical study. With this new tool, a number of simulations were performed, with different amplitudes and periods for the riser tension. As
expected, the results show that a riser undergoing VIV may experience mode changes due to the tension variations. This has implications for the strains, which were generally reduced when the tension was oscillating, compared to the constant tension case (except when the riser buckles in the lower end). By looking at the number of active modes, it is clear that the tension period is important, and if the period is too low, there is not enough time for the vibration mode to change.

Acknowledgements

This work was made possible through the Centre for Research based Innovation MOVE, financially supported by the Norwegian Research Council, NFR project no. 237929 and the consortium partners, http://www.ntnu.edu/move. The authors would also like to thank SINTEF Ocean, and particularly Andreas Amundsen, Pål Levold, Vegard Longva, Harald Ormberg and Elizabeth Passano for providing the RIFLEX code and for valuable assistance in making the software work.

References