

# **Modelling day-ahead Nord Pool forward-price Volatility: Realized Volatility Versus GARCH Models**

**Erik Haugom**

*Lillehammer University College, Norway Norwegian University of Science and Technology  
NTNU, Norway*

E-mail: erik.haugom@hil.no

Tel: (+47) 47 067 167

**Sjur Westgaard**

*Norwegian University of Science and Technology, NTNU, Norway*

E-mail: sjur.westgaard@iot.ntnu.no

**Per Bjarte Solibakke**

*Molde University College, Norway*

E-mail: per.b.solibakke@hiMolde.no

**Gudbrand Lien**

*Lillehammer University College, Norway*

E-mail: gudbrand.lien@hil.no

## **Abstract**

The argument that better volatility estimates can be obtained by using standard time-series techniques on non-parametric volatility measures constructed from high-frequency intradaily returns has been prevalent over the past decade. This study uses high-frequency data and the concept of realized volatility to make one-day-ahead predictions of Nord Pool forward-price volatility. We compare the predictions obtained from realized volatility using standard time-series techniques with the more traditional GARCH framework. Additionally, we examine whether different approaches of decomposing the total variation, and whether inclusion of exogenous effects, improves the accuracy or not. The main findings suggest that significant improvements in the one-day-ahead Nord Pool forward-price volatility predictions can be obtained by applying high-frequency data and the concept of realized volatility.

**Keywords:** Volatility Prediction, Realized Volatility, GARCH, Long-memory

## **1. Introduction**

The liberalization of electricity sectors around the world has led to a rapid growth in electricity-derivatives trading over the recent years. On the one hand, power producers wanting to hedge a certain quantity of electricity can do so by selling an amount equal to a portion of their total production on the forward market; on the other hand, big power consumers can buy the specific amount of the power they need at the same market place. In this way, producers and consumers can buy or sell electricity at

a fixed price today, with delivery over a specified period in the future. Additionally, this financial market is open to speculators who want to “bet” that the price of various derivatives will go one way or the other in the future. Good estimates of the short-term (day-ahead) volatility of these prices are crucial both for risk-management purposes (Value at Risk and Expected Shortfall) and as input for options pricing.

When operating in financial markets in general, asset holders are especially interested in the return volatility over the holding period and not necessarily over some historical period. In order to build an adequate model of the series under consideration, however, most researchers and practitioners use the available historical data to obtain estimates of future risk. Over the last two decades, the well known ARCH (autoregressive conditional heteroskedasticity model) approach and its various generalizations have been found to describe the non-linear features of financial returns satisfactorily (Enders, 2004; Laurent, 2009). Several researchers have also applied this framework to electricity volatility modelling and forecasting (e.g., see Chan and Park, 2007; Escribano et al., 2002; Garcia et al., 2005; Hadsell et al., 2004; Higgs and Worthington, 2005, 2008; Knittel and Roberts, 2005; Solibakke, 2002, 2006). Most of these studies have utilized electricity spot prices and returns as the unit of analyses, which exhibit several distinctive features that are not found in traditional commodity markets due to the non-storability of electric power. For power producers, big power consumers, traders, and other players in the electricity sector, the movements of various derivatives are perhaps of more interest than the underlying spot returns, since these are the prime tools for hedging energy-market risk.

Since 2005 the liquidity in the Nord Pool financial market has been growing rapidly. This development has made it possible to model intraday movements of derivatives in the market. With such tick-by-tick, high-frequency transaction data at hand, researchers and practitioners have availed themselves of new methods for modelling and making predictions of volatility. In particular, daily volatility estimates can be computed non-parametrically by a simple method formulated by Andersen and Bollerslev (1998) called *realized volatility*. By building on continuous-time theory, they show that this convenient quantity can also be used for process modelling by relatively simple time-series techniques. Chan et al. (2008) and Ullrich (2009) have applied this concept to electricity spot prices. However, since the concept is based on continuous-time theory, the practice of applying *Nord Pool* spot-price data – which is determined once a day for each hour the following day – is not quite in line with the underlying theory of realized volatility. Additionally, because electricity spot prices exhibit extreme movements on a day-to-day and intraday basis, data pre-processing must be carried out in order to remove any deterministic features in the series.<sup>1</sup> In contrast to the underlying electricity spot prices, electricity futures or forward prices exhibit far less variation. This lowered variation is especially the case for the longer contracts because less relevant information of the actual conditions at delivery is available as the horizon increases (also known as the Samuelson effect).

Wang et al. (2008) have utilized energy-futures data and the concepts of realized volatility and realized correlation for sweet crude oil and natural gas. This paper argues that many of the stylized properties of traditional financial assets also are found in the energy sector. These stylized properties include (1) a non-Gaussian distribution of unconditional daily returns and daily realized variance and (2) long memory in volatility. They also find evidence of asymmetric volatility in one of the examined contracts (natural gas). Overall, they conclude that the use of realized volatility and realized correlations is highly appropriate for energy data, and suggest that these concepts should be further examined within and between other energy and financial markets.

In this paper we briefly examine the stylized properties of financial electricity price volatility and suggest various models that can be used for one-day-ahead volatility modelling of two of the most liquid financial contracts traded at Nord Pool. In order to test the performance of the one-day-ahead estimates by means of standard time-series techniques on realized volatility, we compare the results

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<sup>1</sup> Chan et al. (2008) do this by demeaning the price changes in the Australian electricity spot prices before calculating the various volatility measures.

with volatility estimates produced with ARCH-type models using daily returns. We can then examine whether an increase in accuracy is observed when taking the step up to higher frequencies and the concept of realized volatility. We can summarize the aim of this study with the question: are the additional costs and complications in data gathering and data pre-processing advantageous with respect to more accurate volatility forecasts in financial electricity markets?

The results suggest that significant improvements in the one-day-ahead volatility predictions can be obtained by time-series techniques on current and historical values of realized volatility, which are simple to implement compared to the estimates obtained from the GARCH (generalized autoregressive conditional heteroskedasticity model) framework. Moreover, the inclusion of exogenous effects, such as intraweek seasonality and trading volume, significantly improve the accuracy. However, within the realized-volatility class of models, the various methods for separating the total variation into its continuous and jump components provide no clear indication of a superior model. Additionally, even though we find evidence of long memory in realized volatility in the preliminary analyses, there is no conclusive evidence that the forecasting ability of these models is better when compared to the simpler autoregressive models.

Next, we shall discuss theories on realized volatility and GARCH-type models, and the following section will describe the market, the data, and the preliminary analyses. Section 4 presents the results, and section 5 discusses the conclusions, implications, and suggestions for further research.

## 2. The ARCH Framework and Realized Volatility

### 2.1. The ARCH framework

Engle (1982) is the first to have introduced an autoregressive model in order to capture the conditional variance of a given time series. This model is based on the hypothesis that volatility as measured by the square of the mean-adjusted relative change in the dependent variable at time  $t$  is related to its values in previous periods,  $t - i$ , which can be expressed as equation 1:

$$\begin{aligned} \varepsilon_t &= z_t \sigma_t \\ z_t &\sim i.i.d.D(0,1) \\ \sigma_t^2 &= \omega_t + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2, \end{aligned} \quad (1)$$

where  $z_t$  is an independently and identically distributed (*i.i.d.*) with  $E(z_t) = 0$  and  $Var(z_t) = 1$ . The  $\sigma_t^2$  is the conditional variance (which may change over time),  $\omega_t$  is a constant, and  $\varepsilon_t$  is the innovation of the process.  $\omega$ , and  $\alpha$ , are parameters to be estimated.

By employing this model on quarterly inflationary data in the United Kingdom, Engle (1982) found a highly significant relationship, and thus dependency in volatility. In order to reduce the number of estimated parameters in the ARCH( $q$ ) model, Bollerslev (1986) has suggested the Generalized ARCH model:

$$\sigma_t^2 = \omega_t + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2, \quad (2)$$

where  $\omega$ ,  $\alpha$ , and  $\beta$  are parameters to be estimated.

With the GARCH, the conditional variance at time  $t$  depends not only on the squared-error term from previous periods (as in the ARCH( $q$ ) models), but also on its conditional variance in previous periods. For both the ARCH- and GARCH-model specification cases, the  $\sigma_t^2$  has to be positive for all time points,  $t$ . To ensure this,  $\omega_t > 0$  and  $\alpha_i \geq 0$  (for  $i = 1, \dots, q$ ) in equation 1 and  $\beta_j \geq 0$  (for  $j = 1, \dots, p$ ) in equation 2 provide the sufficient conditions. Exogenous variables can also be introduced in the conditional-variance equation in order to improve the forecasts. The positivity constraints are no longer valid, but the conditional variance must still be non-negative (Laurent, 2009). There have been several examinations of the GARCH model on a wide range of financial data, and some researchers

have claimed that even in its simplest form (GARCH 1,1) this model provides a good approximation to the observed temporal dependencies in daily data (Andersen and Bollerslev, 1998).

Since the introduction of the ARCH-model framework, there have been a vast number of generalizations put forward in the literature. There are mainly two properties of financial time series that have gained much attention in this matter: 1) volatility seems to react differently to big price increases than to big price drops, which is also referred to as a leverage effect, and 2) volatility exhibits a long-term dependence, or so called long memory (Poon and Granger, 2003). The former has, among other things, led to the development of the Exponential GARCH-model (EGARCH), originally introduced by Nelson (1991), and the Asymmetric Power ARCH-model (APARCH) by Ding et al. (1993). To capture the long memory in volatility, various Fractionally Integrated GARCH models (FIGARCH) have been proposed in the literature (e.g., Baille et al., 1996; Bollerslev and Mikkelsen, 1996; Tse, 1998).<sup>2</sup>

Even though it would be possible to run ARCH-type models at higher frequencies than one business day (see Racicot et al., 2008 for a comparison of such models with the concept of realized volatility), Dacorogna et al. (2001) point out that even when the intradaily seasonal patterns are accounted for, the aggregation properties of the ARCH model break down at these higher frequencies. Microstructure effects are also observed when intervals shorter than about 90 minutes are used. In order to avoid these practical problems, we used daily returns for the construction of volatility forecasts for Nord Pool forward prices.

## 2.2. Realized Volatility

In the GARCH models we use daily returns to provide volatility forecasts one day ahead. However, in the high-frequency data framework, this approach requires that we ignore all transactions between opening and closing of the given day. In order to utilize the available information provided in ticker data, Andersen and Bollerslev (1998) have proposed an ex-post measure of the volatility based on the cumulative intradaily squared returns, known as *realized variance*. This nonparametric measure provides a consistent estimate of the price variability, and can thus be used for process modelling. The daily realized variance at the end of day  $t$  is defined in the following way:

$$RV_{t+1}(\Delta) \equiv \sum_{j=1}^{\frac{\Delta}{\Delta}} r_{t+j\Delta, \Delta}^2, \quad (3)$$

where  $r_t$  is log price difference between  $t$  and  $t-1$  (returns at time  $t$ ).

Realized variation is defined as the sum of the intradaily squared returns, with  $\Delta$  equal to the return period (e.g., 30 minutes). When  $\Delta \rightarrow 0$  and in the absence of jumps,  $RV_{t+1}(\Delta)$  measures the latent integrated volatility ( $IV$ ) perfectly (Laurent, 2009):

$$RV_{t+1}(\Delta) \rightarrow \int_t^{t+1} \sigma^2(i) di, \quad (4)$$

In most practical circumstances, however, empirical studies have demonstrated that a simple continuous diffusion model has its limitations for attempts to explain some characteristics of asset returns (Barndorff-Nielsen and Shephard, 2001; Andersen et al., 2002). In other words, the total variation may better be described by a continuous component and a discontinuous jump component, as the following jump-diffusion process for the logarithmic price  $p(t)$  illustrates:

$$dp(t) = \mu(t)dt + \sigma(t)dW(t) + \kappa(t)dq(t), 0 \leq t \leq T, \quad (5)$$

where  $\mu(t)$  is the drift,  $\sigma(t)$  is a strictly positive and càglàd (left continuous with right limits) stochastic volatility process,  $W(t)$  is a standard Wiener process, and  $q(t)$  is a counting process that

<sup>2</sup> For a useful review of various GARCH models applied on financial data, see Poon and Granger (2003).

assumes the value 1 if a jump occurs at time  $t$  and 0 otherwise, with possibly time-varying intensity  $l(t)$  and size  $\kappa(t)$ .

In the presence of jumps, the realized variance measure may be specified as a continuous integrated variance component and a discontinuous jump component:

$$RV_{t+1}(\Delta) \rightarrow \int_t^{t+1} \sigma^2(i) di + \sum_{t < i \leq t+1} \kappa^2(i). \quad (6)$$

Barndorff-Nielsen and Shephard (2004) have shown that by decomposing the total quadratic variation into two components through the bi-power measure, we obtain a consistent estimator of the integrated variance under the presence of jumps. The bi-power variation measure can be defined thus:

$$BV_{t+1}(\Delta) \equiv \mu_1^{-2} \sum_{j=2}^{\frac{1}{\Delta}} |r_{t+j\Delta, \Delta}| |r_{t+(j-1)\Delta, \Delta}|, \quad (7)$$

where  $\mu_1 \equiv \sqrt{2/\pi} \approx 0.79788$ . Hence, in the presence of jumps and for  $\Delta \rightarrow 0$ , the probability of picking up discontinuities disappears in the limit:

$$BV_{t+1}(\Delta) \rightarrow \int_t^{t+1} \sigma^2(s) ds. \quad (8)$$

The discontinuous jump component can now consistently be estimated as the difference of the realized variance and bi-power variance measures:

$$RV_{t+1}(\Delta) - BV_{t+1}(\Delta) \rightarrow \sum_{t < i \leq t+1} \kappa^2(i). \quad (9)$$

However, since the bi-power measure in practice can exceed the realized variation measure, a positivity constrain should be imposed for the jump component:

$$J_{t+1}(\Delta) \equiv \max[RV_{t+1}(\Delta) - BV_{t+1}(\Delta), 0]. \quad (10)$$

In this paper, we apply a maxlog Z-test in order to determine whether a jump is present, as Barndorff-Nielsen and Shephard (2006) recommend. This test statistic is computed as follows:

$$\max \log Z_t \equiv \frac{\log(RV_t(\Delta)) - \log(\hat{IV}_t(\Delta))}{\sqrt{(\theta - 2) \frac{1}{M} \max\{1, \hat{TQ}_t(\Delta) \hat{IV}_t(\Delta)^{-2}\}}}. \quad (11)$$

Where we use the Tri-power quadraticity  $TQ_t(\Delta)$  to estimate integrated quarticity  $IQ_t$ , as suggested by Andersen et al. (2007), we employ the following expression:

$$TQ_t(\Delta) \equiv M \mu_{4/3}^{-3} \sum_{i=4}^M |r_{t,i}|^{4/3} |r_{t,i-1}|^{4/3} |r_{t,i-2}|^{4/3}, \quad (12)$$

$$\text{with } \mu_{4/3} \equiv 2^{2/3} \Gamma\left(\frac{7}{6}\right) \Gamma\left(\frac{1}{2}\right)^{-1}$$

However, the bi-power variation measure above has some drawbacks (Boudt et al., 2008; Andersen et al., 2009). First, for extremely short sampling intervals (small  $\Delta$ ), there is a small chance that jumps can affect two neighbouring returns, and the impact of jumps on the BV measure is almost non-existent. Moreover, the bi-power measure is sensitive to the presence of “zero” returns in the sample, which is highly probable in markets where liquidity is low, such as the Nord Pool Financial markets (even with sufficiently low sampling frequencies). One can find in the literature alternatives to the bi-power variation measure, however. The nearest-neighbour truncation estimator by Andersen et al. (2009), the range-based variance estimator by Bannouh et al. (2009), and the realized outlyingness weighted quadratic variation procedure (ROWVar) recently introduced by Boudt et al. (2008) and Laurent (2009) have been included as a nonparametric estimator for the integrated variance ( $IV(t)$ ). Here, we apply the last method and examine whether this way of constructing the continuous component can improve the one-day-ahead volatility predictions. This method downweights the returns that are local outliers relative to neighbouring returns. The more “jumpy” a local return window is, the lower weights it receives on the continuous outlyingness estimator. According to Laurent (2009) the

computation of the realized outlyingness weighted variance involves two steps. First, the outlyingness of return  $r_{t,i}$  is calculated as:

$$d_{t,i} = \left( \frac{r_{t,i}}{\hat{\sigma}_{t,i}} \right)^2, \tag{13}$$

which is the square of the robustly studentized return. Here,  $\hat{\sigma}_{t,i}$  is a robust estimate of the instantaneous volatility computed from all the returns belonging to the same local window as  $r_{t,i}$ . Owing to strong intradaily seasonality in volatility, Boudt et al. (2008) suggest the computation of  $d_{t,i}$  on filtered returns instead of raw returns. Accordingly, we also adjust for this periodicity in volatility and examine whether this adjustment can improve volatility predictions. The method of median absolute deviation (MAD) is applied and is defined as follows:

$$MAD = 1.486 \cdot \text{median}_i |y_i - \text{median}_j y_j|, \tag{14}$$

where 1.486 is a correction factor to make sure that the MAD is a consistent scale estimator of the normal distribution. The periodicity factor of  $r_{t,i}$  is thus equal to:

$$\hat{f}_{t,i}^{MAD} = \frac{MAD_{t,i}}{\sqrt{\frac{1}{M} \sum_{j=1}^M MAD_{t,j}^2}}. \tag{15}$$

The second step in this process involves choosing an appropriate weight function. Boudt et al. (2008) here recommend the use of the Soft Rejection Huber function defined as follows:

$$w_{SR}(d_{t,i}) = \min\{1, k / d_{t,i}\}, \tag{16}$$

where  $k$  is a tuning parameter to be selected. They also show that the outlyingness measure is asymptotically chi-squared distributed with one degree of freedom. Hence, the rejection threshold  $k$  can be set to  $\chi_1^2(\beta)$ , the  $\beta$  quantile of the  $\chi_1^2$  distribution function. In this regard,  $\beta = 95\%$  is suggested as a compromise between robustness and efficiency. Given the above, the ROWVar measure can be computed as follows:

$$ROWVar_t = c_w \frac{\sum_{i=1}^M w_{SR}(d_{t,i}) r_{t,i}^2}{\frac{1}{M} \sum_{i=1}^M w_{SR}(d_{t,i})}, \tag{17}$$

where  $c_w$  is a correction factor to ensure that ROWVar is consistent for the IVar under the Brownian Model.

A jump measure to test for significant jumps has also been proposed. It can be shown that  $\max_{i=1, \dots, M} \sqrt{d_{t,i}}$  follows a Gumbel distribution under the null of no jump during day  $t$ . Hence, we can reject the null hypothesis if

$$\max_{i=1, \dots, M} \sqrt{d_{t,i}} > G^{-1}(1 - \alpha) S_n + C_n, \tag{18}$$

where  $G^{-1}(1 - \alpha)$  is the  $1 - \alpha$  quantile of the standard Gumbel distribution,

$C_n = (2 \log n)^{0.5} - \frac{\log(\pi) + \log(\log n)}{2(2 \log n)^{0.5}}$  and  $S_n = \frac{1}{(2 \log n)^{0.5}}$ . Therefore, according to Laurent (2009), an alternative estimate of the realized jumps is:

$$J_{t,\alpha}(\Delta) = I_{t,\alpha}(\Delta) \square [RV_t(\Delta) - \overline{FV}_t(\Delta)], \tag{19}$$

where  $I_{t,\alpha}(\Delta)$  is a dummy variable assuming the value of one if a jump is present on day  $t$  (using the test in (18)).

Thus, in this study we shall use the various approaches for computing the continuous and jump components in realized volatility and examine whether one approach is able to outperform the other methods for predictions of one-day-ahead realized volatility for Nord Pool electricity forward prices.

### 3. The Nord Pool Financial Market and the Data

The Nord Pool power exchange has offered trade in financial contracts without physical delivery since 1994. Such financial contracts were designed to meet the needs of producers, retailers, and end-users who use the products for risk management. Additionally, the financial market is open to traders who profit on continuous price movements in the various contracts. High-quality ticker data for all the financial contracts traded at the Nord Pool exchange have been centrally recorded since 2005. From these data, it is clear that the market seems to prefer short-term futures and nearest-quarter and year-forward contracts. As the number of ticks is crucial in the construction of high-quality, high-frequency data, we shall examine nearest-quarter and year-forward contracts. The data range is from January 3, 2005 to May 29, 2009 for the yearly contracts and June 1, 2005 to May 29, 2009 for the quarterly contracts. The series of quarterly contracts had a different organization for the first quarter in 2005 (tertiary contracts), which is why the data range for these contracts is somewhat shorter than it is for the yearly contracts.

Financial trading at Nord Pool takes place on weekdays between 08:00 and 15:30. The average time between unique ticker observations over the business day in the sample is approximately 2 minutes and 28 seconds, and 6 minutes and 10 seconds for the quarterly and yearly contract series, respectively. The analysis concept require a strategy about sampling schemes, where in this study we use equally spaced price and return interval (calendar time sampling).<sup>3</sup> It will always be a balance between the accuracy of the continuous-record asymptotics underlying the construction of the realized volatility on the one hand, and the influences from market microstructure noise on the other (Andersen et al. 2003). As it is implausible to push the continuous record asymptotics beyond an average (or median) level of trade duration, equally spaced 30-minute intervals were constructed from the raw data, using closest tick interpolation (Dacarogna et al. 2001).<sup>4,5</sup>

In order to avoid the unwanted side-effect caused by unusually high returns (in absolute values) at contract roll-over, these are set to the expected value for a stationary series with a zero mean (i.e., 0). This is done for both the high-frequency observations and the daily-returns observations. Moreover, previous research has found significant differences between overnight and trading-hours returns, and that including these overnight returns introduces more noise than useful information (Martens, 2002). We shall, therefore, only use returns during trading hours in this study. The daily returns are calculated as the logarithmic difference of the mean daily prices as calculated of the 16 intradaily prices (constructed for the high-frequency analysis). As a starting point we present descriptive statistics and preliminary tests of the variables of interest. These results are presented in Table 1.

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<sup>3</sup> An overview and discussion of different sampling schemes, both in univariate and multivariate applications, are discussed in, e.g., McAleer and Medeiros (2008).

<sup>4</sup> In the highly liquid Deutschemark/Dollar and Yen/Dollar spot exchange rate markets Andersen *et al.* (2003) also used equally-spaced 30-minutes return strikes.

<sup>5</sup> For a thorough description of the almost identical sample data analyzed in this study, we refer to Haugom (2011) and Haugom et al. (2011).

**Table 1:** Descriptive statistics and various preliminary tests on the various variables.  $r$  = returns,  $r^2$  = squared returns,  $RV$  = Realized Volatility,  $CV$  = Continuous Volatility,  $JV$  = Jump Volatility,  $CVOW$  = Continuous Volatility based on outlyingness weighted procedure,  $JVOW$  = Jump Volatility based on outlyingness weighted procedure,  $CVOWa$  = Continuous Volatility based on outlyingness weighted procedure adjusted for intraday seasonality,  $JVOWa$  = Jump Volatility based on outlyingness weighted procedure adjusted for intraday seasonality.  $JB$  = Jarque-Berra test.  $Q5/Q20$  = The Box-Pierce statistic with 5 and 20 lags respectively.  $ADF$  = Augmented Dickey-Fuller test ( $H_0$ :  $I(1)$ ). The number of lagged difference terms included in the models is based on Akaike's Information Criterion and the following numbers were used for the quarterly-contract series:  $r = 1$ ,  $r^2 = 7$ ,  $RV = 25$ ,  $CV = 9$ ,  $JV = 9$ ,  $CVOW = 8$ ,  $JVOW = 9$ ,  $CVOWa = 4$ ,  $JVOWa = 2$ , and for the yearly-contract series:  $r = 18$ ,  $r^2 = 20$ ,  $RV = 19$ ,  $CV = 7$ ,  $JV = 8$ ,  $CVOW = 24$ ,  $JVOW = 35$ ,  $CVOWa = 11$ ,  $JVOWa = 6$ .  $KPSS$  = Kwiatkowski, Phillips, Schmidt, and Shin (1992) test ( $H_0$ :  $I(0)$ ). The same numbers as in  $ADF$  are used for bandwidth.  $LM$  1-2/1-10 is the  $LM$  ARCH test of Engle (1982) using 2 and 10 of its own lags, respectively ( $H_0$ : no ARCH effects).  $d^{GPH}$  is the long-memory test of Geweke and Porter-Hudak (1983). \*\*\* =  $p < 0.01$ , \*\* =  $p < 0.05$ , \* =  $p < 0.1$  (for the autocorrelation function \* = that the parameter is outside the 95%-confidence bands).

Quarterly	$r$	$r^2$	$RV$	$CV$	$JV$	$CVOW$	$JVOW$	$CVOWa$	$JVOWa$
#obs	995	995	995	995	492	995	275	995	86
Mean	-0.001	0.001	0.023	0.014	0.010	0.016	0.007	0.022	0.002
Min.	-0.128	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Max.	0.085	0.016	0.141	0.141	0.097	0.141	0.105	0.141	0.059
Std. Dev.	0.025	0.001	0.014	0.012	0.014	0.013	0.014	0.014	0.007
Skew.	-0.150*	5.183***	1.984***	2.233***	1.904***	1.906***	2.563***	1.952***	4.853***
Kurt.	2.080***	42.385***	7.523***	12.093***	4.743***	9.366***	7.877***	7.758***	26.883***
JB	184***	79332***	3013***	6924***	1541***	4260***	3679***	3142***	34037***
ACF Lag 1	0.225*	0.158*	0.492*	0.376*	0.039	0.395*	0.053	0.406*	0.071*
ACF Lag 2	0.006	0.057	0.419*	0.360*	0.018	0.362*	0.030	0.391*	0.074*
ACF Lag 5	0.007	0.128*	0.456*	0.268*	0.210*	0.258*	0.195*	0.402*	0.030
Q5	50***	68***	910***	471***	47***	532***	45***	713***	14**
Q20	98***	104***	1872***	961***	121***	853***	105***	1393***	32**
ADF	-20.11***	-8.65***	-3.24**	-5.50***	-7.69***	-6.50***	-7.64***	-6.74***	-16.06***
KPSS	0.17	1.11***	1.31***	2.17***	0.86***	1.41***	1.46***	3.99***	0.24
LM 1-2	13.27***	6.11***	66.02***	55.06***	3.83**	82.10***	3.44**	56.37***	3.33**
LM 1-10	6.00***	2.28**	21.17***	12.42***	9.74***	18.60***	8.61***	16.60***	0.94
$d^{GPH}$	0.15***	0.12***	0.29***	0.26***	0.07**	0.29***	0.07**	0.24***	0.11***
Yearly	$r$	$r^2$	$RV$	$CV$	$JV$	$CVOW$	$JVOW$	$CVOWa$	$JVOWa$
#obs	1100	1100	1100	1100	679	1100	263	1100	109
Mean	0.000	0.000	0.017	0.009	0.008	0.012	0.005	0.015	0.002
Min.	-0.110	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000
Max.	0.077	0.012	0.136	0.136	0.059	0.136	0.071	0.136	0.049
Std. Dev.	0.018	0.001	0.011	0.010	0.009	0.010	0.010	0.011	0.005
Skew.	-0.431***	7.100***	2.469***	3.322***	1.511***	3.013***	2.441***	2.412***	4.252***
Kurt.	3.099***	87.047***	13.980***	25.564***	2.861***	22.199***	6.570***	13.499***	21.040***
JB	474***	356530***	10085***	32006***	794***	24273***	3073***	9427***	23625***
ACF Lag 1	0.153*	0.230*	0.605*	0.361*	0.088*	0.361*	0.112*	0.544*	0.086*
ACF Lag 2	-0.034	0.158*	0.496*	0.349*	0.128*	0.320*	0.110*	0.435*	0.037
ACF Lag 5	0.002	0.168*	0.477*	0.297*	0.158*	0.241*	0.111*	0.427*	-0.007
Q5	29***	168***	1377***	521***	55***	453***	43***	1107***	16***
Q20	61***	474***	3481***	1125***	166***	1061***	110***	2767***	40***
ADF	-6.90***	-4.36***	-3.32**	-7.03***	-7.46***	-3.20**	-3.24**	-4.46***	-11.21***
KPSS	0.26	1.37**	1.76***	3.85***	0.86***	1.48***	0.78***	3.08***	0.56**
LM 1-2	37.82***	12.48***	138.05***	22.57***	21.31***	31.25***	12.95***	129.59***	4.31**
LM 1-10	15.98***	2.76***	32.57***	5.48***	12.45***	7.87***	5.03***	30.32***	1.74*
$d^{GPH}$	0.043	0.18***	0.44***	0.34***	0.21***	0.33***	0.16***	0.38***	0.13***

From Table 1 a number of interesting findings may be noted. First, consistent with findings from traditional financial markets, we find that none of the examined variables are normally distributed; that is, they are mostly right-skewed (except the returns) with high excess kurtosis (fat tails). Secondly, most of the examined variables exhibit significant autocorrelation for the first lag (except two of the jump-volatility measures). This result implies that when running GARCH models, we need to adjust the mean by running an AR(1) model. For the constructed realized-volatility measures, it means that current and recent past volatility can be used for estimating tomorrow's volatility. A weekly seasonality can also be observed as ACF estimates at lag five are significant for most variables (which is equal to one business week). Hence, this aspect needs to be accounted for in

the models. The Augmented Dickey-Fuller test (ADF-test) in all cases rejects the null hypothesis that a unit root exists (at the 5% significance level or lower). On the other hand, the KPSS test (which tests the opposite hypothesis, - I(0)) rejects stationarity for all variables (except for the raw returns and the adjusted outlyingness weighted jump measure for the quarterly-contract series). These results are consistent with findings in traditional financial-futures markets and provide evidence of long memory and thus hyperbolic decay of the autocorrelation function for the examined variables (Chen et al., 2006).

On the basis of this evidence, we calculate the fractionally integrated parameter,  $d$  with Geweke and Porter-Hudak's (1983) method. Nearly all the variables (with one exception) have a significant  $d$  parameter varying between 0.07 (JVOW) and 0.29 (RV). Surprisingly, even though the return variable for the quarterly-contract series is stationary based on the ADF test and conventionally stationary on the basis of the KPSS test, the  $d$  parameter is significantly different from zero (0.15) and higher than for the squared returns. A comparison of the autocorrelogram of the returns and squared returns for both the quarterly- and yearly-contract series does not however, indicate why the quarterly-contract returns exhibit a hyperbolic decay and why the yearly returns do not. In order to examine this aspect further, we also applied the long-memory test method proposed by Robinson and Henry (1999), but we arrived at the same conclusion. Either way, the significant  $d$  parameter in the series of quarterly-contract returns suggests that the mean should be adjusted with an ARFIMA (1, $d$ ,0) model before applying any ARCH models (alternatively, one could apply an ARFIMA-GARCH-type model). However, after running a simple ARFIMA model of the returns, we found no evidence of a significant fractionally integrated parameter, and, on the basis of this, the mean is corrected only for first-order autocorrelation. However, the significant  $d$  parameter for the squared returns is consistent with long memory in volatility (as squared returns is another proxy for volatility), and this suggests that a fractionally integrated GARCH (FIGARCH) model should describe the movements better than a GARCH model that does not account for long memory.

Both the Box-Pierce statistics and Engle's (1982) LM ARCH test clearly provide evidence of serial correlation in most of the examined series. Hence, when considering the returns, this suggests that volatility forecasting with the ARCH framework is suitable. For the various realized volatility measures, these results clearly show that standard autoregressive models (and fractionally integrated autoregressive models) can easily be applied in order to construct one-day-ahead volatility estimates for the Nord Pool forward prices.

Finally, one should note the effect of the various ways of calculating the continuous and jump components of realized volatility. As seen with the jump variables of interest, the number of detected jumps with the different approaches varies. The Z-test that uses the traditional measure of bi-power variation as an estimate of the integrated variance detects 492 and 679 jumps (49.4% and 61.7%) for the quarterly- and yearly-contract series, respectively. On the other hand, the results of the jump test with the outlyingness measure are 275 (27.6%) and 263 (23.9%) without controlling for intradaily seasonality, and 86 (8.6%) and 109 (9.9%) when taking this periodicity into account.

#### 4. Modelling One-Day-Ahead Volatility

In order to make one-day-ahead predictions of volatility for the quarterly and yearly Nord Pool forward prices, we compared the performance of RiskMetrics™ and a Fractionally Integrated GARCH (FIGARCH) model with the performance of standard time-series models that use lagged values of the constructed realized-volatility measures. Additionally, as the descriptive statistics suggest that realized volatility exhibits long memory, we employed models that included a fractional integrated parameter,  $d$ , to examine whether this can improve volatility predictions. The models that used for predicting one-day-ahead volatility are presented in (20) to (23) below:

$$\text{RiskMetrics}^{\text{TM}}: \sigma_{t+1}^2 = \omega_{t+1} + (1 - \lambda)\varepsilon_t^2 + \lambda\sigma_t^2 \quad (20)$$

$$\text{FIGARCH}: \sigma_{t+1}^2 = \sigma^2 + \lambda(L)(\varepsilon_{t+1}^2 - \sigma^2) \quad (21)$$

$$\text{HAR-CV-JV: } RV_{t+1} = \beta_0 + \beta_1 CV_t + \beta_2 CV_{t-5,t} + \beta_3 JV_t + \beta_4 JV_{t-5,t} \quad (22)$$

$$\text{HAR-CV-JV-d: } (1-L)^d RV_{t+1} = \beta_0 + \beta_1 CV_t + \beta_2 CV_{t-5,t} + \beta_3 JV_t + \beta_4 JV_{t-5,t} \quad (23)$$

The first model (20) is the internal-market risk-management methodology applied by J.P. Morgan. This model is actually a special case of an IGARCH(1,1) model with the ARCH  $(1-\lambda)$  and GARCH  $(\lambda)$  coefficients fixed at 0.06 and 0.94, respectively. The FIGARCH model (21) is the Fractionally Integrated GARCH process proposed by Chung (1999). Here,  $\sigma^2$  is the unconditional variance and L is the backshift operator. For a discussion on the technical details on this model, see Laurent (2009). In both the RiskMetrics™ model and the FIGARCH model, the mean is adjusted with an AR(1) model. The square root of the obtained conditional variance at time  $t + 1$  from the two GARCH models is used for comparisons of performance to the other models. The HAR-CV-JV Model in (22) is the heterogeneous autoregressive model of continuous and jump variation as proposed by Chan et al. (2008). Here,  $CV_{t-5,t} = \frac{1}{5} \sum_{s=1}^5 CV_{t-s}$  and  $JV_{t-5,t} = \frac{1}{5} \sum_{s=1}^5 JV_{t-s}$ , that is, the average of the continuous and jump volatility measures over the last five trading days. Model (23) provides regression estimates based on a fractionally differenced series in order to account for the long memory. Both models (22) and (23) use the different variations of continuous and jump volatility as discussed in section 2.

Moreover, previous research has found clear evidence both of higher volatility on Mondays (Giot et al., 2010) and of the impact of trading volume on volatility (Gallant et al., 1992). Thus, these variables will be entered to the various models in order to examine whether their inclusion can improve one-day-ahead volatility predictions.

In order to compare the ability of the various models to predict one-day-ahead volatility, we applied two different approaches. First, we used the  $R^2$  of the Mincer-Zarnowitz (1969) regression:

$$RV_{t+1} = \beta_0 + \beta_1 \hat{v}_{ModelX,t+1} + \varepsilon_{t+1}, \quad (24)$$

where,  $RV_{t+1}$  is the observed realized volatility at time  $t + 1$  constructed of the half-hourly squared returns as specified in section 2, and  $\beta_1 \hat{v}_{ModelX,t+1}$  is the volatility estimate obtained by one of the specified models. Thus, the model with the highest explained variance ( $R^2$ ) is the one that performs best. In addition to this method, we calculated the mean squared error (MSE), which, according to Patton (2010), is one of few loss measures that are robust to the noise in the proxy<sup>6</sup> when evaluating volatility predictions. The MSE is as follows:

$$MSE = \frac{1}{T} \sum_1^T (v_{t+1} - \hat{v}_{t+1})^2, \quad (25)$$

where T is the number of day-ahead predictions to be evaluated (i.e. 990 for the quarterly contracts series, and 1095 for the yearly contract series), and  $v_{t+1}$  and  $\hat{v}_{t+1}$  are the observed and predicted realized volatility at time  $t + 1$ , respectively. We examined the significant differences among the competing models based on the MSE loss function by applying the Diebold and Mariano (1995) test at the 5% significance level. In practice, this test can be easily calculated by regressing the difference (*dif*) in the squared errors on a constant and use a t-statistic with robust standard errors to determine whether the constant is statistically different from 0:

$$dif_{t+1} = \beta_0 + \varepsilon_{t+1}. \quad (26)$$

<sup>6</sup> I.e. the proxy of the unobservable/latent volatility – in our case; realized volatility as constructed from the intradaily returns.

If  $\beta_0$  is statistically different from zero at the chosen significance level, we can reject the null hypothesis of equal forecast accuracy.<sup>7</sup>

The results of the various methods' ability to predict one-day-ahead volatility are presented in Table 2.<sup>8</sup>

**Table 2:** The table reports results from the Mincer-Zarnowitz forecasting regression (24), and the robust loss function, mean square error (25). Significant differences between competing models are obtained by regressing the difference on a constant and using robust t-statistics (26). The sample period extends from January 3, 2005 to May 29, 2009 for the yearly-contract series, and from June 1, 2005 to May 29, 2009 for the quarterly-contract series. G1 = the RiskMetrics™ model (20), G2 = FIGARCH (21), RV1 = HAR-CV-JV with bi-power variation (22), RV2 = HAR-CV-JV with realized outlyingness weighted variance (22), RV3 = HAR-CV-JV with realized outlyingness weighted variance adjusted for intradaily periodicity (22), RV4 = HAR-CV-JV-d with bi-power variation (23), RV5 = HAR-CV-JV-d with realized outlyingness weighted variance, RV6 = HAR-CV-JV-d (23) with realized outlyingness weighted variance adjusted for intradaily periodicity (23). † = significant improvement in the forecasting performance with respect to G1 at the 5% significance level. ‡ = significant improvement in the forecasting performance with respect to G2 at the 5% significance level. § = significant improvement in the forecasting performance with respect to the poorest performing model within the RV class of models. • = significant improvement in the forecasting performance with respect to equivalent model without exogenous variables. MSE in the table = MSE × 1000. NC = the model did not converge.

Quarterly	Without exogenous variables			With exogenous variables		
	Adj. R2	MSE		Adj. R2	MSE	
G1	26.72 %	0.1486		27.60 %	0.1486	
G2	30.53 %	0.1430	†	34.46 %	0.1373	†•
RV1	<b>35.54 %</b>	<b>0.1307</b>	†‡§	42.97 %	0.1162	†‡§•
RV2	34.85 %	0.1322	†‡§	42.30 %	0.1176	†‡•
RV3	33.29 %	0.1355	†	40.84 %	0.1206	†‡•
RV4	35.50 %	0.1309	†‡§	<b>43.16 %</b>	<b>0.1159</b>	†‡§•
RV5	34.84 %	0.1323	†‡§	42.53 %	0.1172	†‡•
RV6	33.43 %	0.1353	†	41.33 %	0.1197	†‡•
Yearly	Adj. R2	MSE		Adj. R2	MSE	
G1	36.63 %	0.0803		NC	NC	
G2	39.99 %	0.0771	†	41.22 %	0.0766	
RV1	44.61 %	0.0695	†	47.99 %	0.0652	‡•
RV2	44.35 %	0.0698	†	48.46 %	0.0646	‡•
RV3	43.77 %	0.0705	†	47.77 %	0.0655	‡•
RV4	<b>45.02 %</b>	<b>0.0690</b>	†‡	47.97 %	0.0654	‡•
RV5	44.79 %	0.0693	†‡	<b>48.56 %</b>	<b>0.0645</b>	‡•
RV6	44.16 %	0.0701	†	47.91 %	0.0653	‡•

When comparing the models in the two left columns of the table (models without exogenous effects), we can observe that the various models utilizing current and past realized volatility measures to construct next-day volatility predictions in general are more accurate than the applied GARCH-models. In fact, all estimates obtained from the RV class of models are significantly better than the RiskMetrics™ model on the basis of (26) for both the quarterly- and yearly-contract series. Additionally, for the quarterly-contract series, four RV-type models (RV1, RV2, RV4, and RV5) produce significantly more accurate forecasts than the FIGARCH model, while RV4 and RV5 are

<sup>7</sup> Robust standard errors are important because the difference measure ( $d_{t+1}$ ) could be autocorrelated. The raw difference in the MSE cannot, then, be used to determine whether two competing models are significantly different (as Table 2 illustrates).

<sup>8</sup> Owing to the high number of models to be compared (32 models), this study does not report the parameter estimates, but they are available upon request.

significantly better than the FIGARCH model for the yearly contract series (both RV4 and RV5 are long-memory models). These results clearly indicate that the use of simple time-series techniques and volatility measures constructed from high-frequency data are useful.

Within the RV class of models, the differences are not as consistent; that is, for the quarterly-contract series, four models (RV1, RV2, RV4, and RV5) are significantly more accurate than the poorest performing model (RV3), and RV1 (HAR-CV-JV applying traditional bi-power variation) has the best performance overall. On the other hand, for the yearly-contract series, no significant differences among the six different RV models are found on the basis of the Diebold-Mariano test. Hence, the results indicate that no method of separating the total variation into a continuous and jump component works superior across the two data sets examined here. Additionally, accounting for the long memory initially found in the preliminary analyses does not result in significant improvements in the one-day-ahead forecasts.

The two right columns of Table 2 present the results of the models with exogenous effects. The major finding here is that the inclusion of exogenous effects (weekday dummies and current-day trading volume) in general improves the accuracy of the predictions of one-day-ahead volatility. For all models within the RV class, the differences in accuracy between equivalent models with and without exogenous variables are statistically significant at the 5% significance level. For the quarterly-contract series, this is also the case for the FIGARCH model. In contrast, the RiskMetrics™ methodology does not increase its accuracy significantly. In fact, for the yearly-contract series, the RiskMetrics™ model did not even converge, which illustrates the benefits of working with realized volatility and simple time-series techniques when modelling and forecasting volatility.

For the examination of the differences among the competing models when including exogenous variables, the results are consistent for the data analysed in this paper. All models within the RV framework predict one-day-ahead volatility significantly better than the two models representing the GARCH framework. On average, the models that use past values of realized volatility (RV1-RV6) explain 7.73% points more of the variation than the FIGARCH model, and 14.59% points more than the RiskMetrics™ model. These improvements in one-day-ahead volatility estimates represent a substantial opportunity for power producers, traders, and other market participants who need good volatility estimates on a daily basis in order to handle risk in financial electricity markets.

Within the RV class, the models utilizing the traditional bi-power variation measure to separate the total variation and the long-memory version of this (RV1 and RV4) ranks best for the quarterly-contract series. These two models perform significantly better than poorest performing RV model (RV3 – applying realized outlyingness weighted variance adjusted for intradaily periodicity). For the yearly-contract series, no significant differences among the competing RV models are found, but RV5 (the long-memory model applying realized outlyingness weighted variance without adjusting for intradaily periodicity) ranks best on the basis of both the  $R^2$  value and the  $MSE$ .

Overall, the results of this study provide important – and perhaps surprising – findings. First, by taking the step up to high-frequency data and realized volatility, we have shown that more accurate one-day-ahead volatility estimates for quarterly and yearly Nord Pool forward prices can be obtained compared the estimates of the GARCH framework. Second, the inclusion of exogenous effects, such as intraweek seasonality and trading volume, significantly improves the accuracy of volatility predictions. In this case, the differences between the non-parametric framework (RV class of models) and the parametric framework (GARCH models) become even more evident in favour of the realized framework. Third, even though a long-memory model ranked best for the yearly-contract series (both with and without exogenous effects), the differences from the models that do not account for this are small and not statistically significant. Fourth, the two new approaches for constructing the continuous and jump volatility examined in this paper (realized outlyingness weighted variance and realized outlyingness variance adjusted for intradaily seasonality) do not seem to increase the accuracy of volatility predictions. Therefore, even though reducing the number of jumps may be correct from a theoretical perspective, it does not seem to have any significant impact on the prediction performance for the Nord Pool forward-price volatilities.

## 5. Summary and Conclusion

In this study we have compared volatility predictions of Nord Pool forward prices for two contracts traded at the central Nord Pool exchange. The comparison has used a total of eight different models, where two have represented the GARCH framework, three have represented a traditional autoregressive framework, and the last three models are autoregressive models that account for long memory. We have also examined the usefulness of different approaches to calculate the continuous and jump variation in the constructed realized-volatility measure.

The results suggest that when comparing the one-day-ahead volatility predictions, the various models that use past values of the realized-volatility components outperform the traditional GARCH approach. The explained variance for these models is on average 5.95% points above the average explained variance of the two GARCH models for the quarterly-contract series and 6.14% points for the yearly-contract series when the exogenous effects are not included. When exogenous effects are taken into account, the differences between the two frameworks become even more apparent in favour of the realized-volatility models. These results clearly indicate improved accuracy in one-day-ahead volatility predictions when utilizing standard time-series techniques on the constructed realized volatility measures.

Another issue examined here is whether alternative methods of calculating the continuous and jump variation can improve the one-day-ahead volatility estimates for Nord Pool forward-price volatility. In general, our results provide no evidence of improved accuracy for the contracts' price volatility when separating the total variation by the concept of realized outlyingness weighted variance.

The descriptive statistics provide evidence of long memory in almost all the examined variables. These findings suggest that movements in volatility should be described by fractionally integrated models. However, the comparison of the results offered no conclusive evidence of more accurate predictions with these models.

A number of important areas require future research. First, as this study has mainly focused on modelling one-day-ahead volatility, a natural extension would be to estimate models and to use them for post-sample forecasting with various horizons and post-sample sizes. The univariate analyses carried out in this paper should be extended to the multivariate case. A comparison of a multivariate GARCH approach with realized covariance would be useful in this respect. The inclusion of other exogenous effects, like, for example, news announcements and weather forecasts, would also be fruitful topics for future research.

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