

# Evaluating a Stochastic Programming Based Bidding Model for a Multireservoir System

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**Abstract**—Hydropower producers need to schedule when to release water from reservoirs, and participate in wholesale electricity markets where the day-ahead production is physically traded. A mixed-integer linear stochastic model for bid optimization and short-term production allocation is developed and tested through a simulation procedure implemented for a complex real-life river system. The stochastic bid model sees uncertainty in both spot market prices and inflow to the reservoirs. The same simulation procedure is also implemented for a practice-based deterministic heuristic method similar to what is currently used for bid determination in the industry, and the results are compared. The stochastic approach gives improvements in terms of higher obtained average price and higher total value than the deterministic alternative. It also performs well in terms of start-up costs. In the presence of river flow travel delay the practice-based method is even more outperformed by the stochastic model.

**Index Terms**—Bidding, Electricity markets, Hydro Scheduling, Price taker, Simulation, Stochastic programming, Reservoirs

## NOMENCLATURE

The notation used throughout the paper is stated below.

### Sets

$c \in C$	Index for all cuts $C$ for computation of the water value
$e \in E$	Index for different runs $E$ of the deterministic model
$h \in H$	Index for all hours in the short-term models and all weeks in the seasonal model, $H$
$i \in I$	Index for all bid points in $I$
$j \in J$	Index for all turbines $J$ in the system
$n \in N$	Index for known points on production curves with $N$ breakpoints
$r \in R$	Index for all reservoirs and belonging stations, $R$ , in the system
$s \in S$	Index for scenarios all $S$

### Parameters

$A_{rc}$	The dual variable for reservoir $r$ in cut $c$ [ $\text{€}/m^3$ ]
$C_{kr}^B$	Connection matrix for water courses for bypass between reservoir $k$ and $r$
$C_{kr}^P$	Connection matrix for water courses for production discharge between reservoir $k$ and $r$
$C_{kr}^S$	Connection matrix for water courses for spill between reservoir $k$ and $r$
$F_c$	Future profits for cut $c$ [ $\text{€}$ ]

$G$	Penalty for up and down ramping [ $\text{€}$ ]
$l_{rh}$	Inflow to reservoir $r$ in hour $h$ [ $m^3$ ]. For stochastic model: $l_{srh}$
$L_{rc}$	Initial storage level in reservoir $r$ used to generate cut $c$ [ $m^3$ ]
$L_r^{Min}$	The minimum reservoir level at reservoir $r$ [ $m^3$ ]
$L_r^{Max}$	The maximum reservoir level at reservoir $r$ [ $m^3$ ]
$P_h^{Avr}$	For seasonal model: Weekly average spot price for week $h$ [ $\text{€}$ ]
$P_i$	Price at bid point $i$ [ $\text{€}/\text{MWh}$ ]
$\pi_s$	For stochastic model: Probability of scenario $s$
$\rho_h$	Spot price realization in hour $h$ . For stochastic model: $\rho_{sh}$ [ $\text{€}/\text{MWh}$ ]
$S_j$	Start-up cost for turbine $j$ [ $\text{€}$ ]
$T_{kr}$	River flow travel time from reservoir $k$ to reservoir $r$ [Hours]
$V_r^{B,Min}$	The minimum amount of bypass from station $r$ [ $m^3$ ]
$V_r^{B,Max}$	The maximum amount of bypass from station $r$ [ $m^3$ ]
$V_{jn}$	Known point for discharge on the production curve for turbine $j$ [ $m^3$ ]
$V_j^{Min}$	The minimum discharge for turbine $j$ [MW]
$V_j^{Max}$	The maximum discharge for turbine $j$ [MW]
$W_{jn}$	Known point for power on the production curve for turbine $j$ [MW]
$W_j^{Min}$	The minimum production capacity for turbine $j$ [MW]
$W_j^{Max}$	The maximum production capacity for turbine $j$ [MW]

### Continuous variables

$l_{rh}$	Reservoir level at reservoir $r$ in hour $h$ [ $m^3$ ]
$m$	Water value [ $\text{€}$ ]
$v_{rh}$	Water discharged at station $r$ in hour $h$ [ $m^3$ ]
$v_{rh}^B$	Water discharged as bypass at station $r$ in hour $h$ [ $m^3$ ]
$v_{rh}^S$	Water discharged as spill at station $r$ in hour $h$ [ $m^3$ ]
$v_{jh}$	The water discharged from turbine $j$ in hour $h$ [ $m^3$ ]
$w_{rh}$	Power produced at station $r$ in hour $h$ [MW]
$w_{jh}$	The power produced by turbine $j$ in hour $h$ [MW]
$x_{ih}$	For stochastic model only: The cumulative bid volume for bid point $i$ in hour $h$ [MW]

$y_h$	Committed volume in hour $h$ [MW]
$z_h^+$	Up ramping in hour $h$ [MW]
$z_h^-$	Down ramping in hour $h$ [MW]

Binary variables

$d_{jh}$	If turbine $j$ has changed state in hour $h$
$u_{jh}$	On/off-state of turbine $j$ in hour $h$

## I. INTRODUCTION

**M**OST generating companies participate in wholesale electricity markets, of which the day-ahead market is the most important since it is here the largest volumes are traded. The day-ahead market facilitates trading of the next day's expected production via two-sided auctions in which producers and consumers submit their price-quantity bids. The determination of optimal bids in power markets is a complicated task that has to be undertaken every day. At the time of bidding, not only are the day-ahead prices uncertain, but also inflow to the reservoirs and the development of electricity prices over a longer time horizon. A system of hydro storage plants with multiple reservoirs has a complex and time-varying cost structure and requires a coordinated water release strategy as discharge from upstream reservoirs influences the downstream reservoir levels. The short-term hydropower scheduling problem therefore involves decisions on what to bid in the market for tomorrow and then how to allocate production to individual stations and turbines once the day-ahead market price and inflow is realized. The first of the aforementioned tasks is often referred to as the bidding problem, while the second is termed the problem of optimal dispatch or production allocation. The focus of this paper is on testing the performance of a stochastic-programming based approach for solving the bidding problem, and how this compares to methods currently being used in the industry. The implemented stochastic method extends [?], as both day-head prices and inflow to the reservoirs is considered stochastic.

What to bid in the market for tomorrow is currently determined by a multi scenario deterministic heuristic method by many producers. A forecasted price profile, i.e. a vector of 24 hourly prices, is scaled by a set of weights and the resulting price scenarios are used as input to a deterministic optimization model that determines the production volumes. For each bidding hour and scenario, the corresponding price and scenario-optimal production is used as the bid to be submitted to the market operator. In Scandinavia, the Short-term Hydro Operation Planning (SHOP) model [?], [?], is used as a decision support tool in the aforementioned way. In this paper, this method is referred to as the deterministic method or practice-based method, and represents the current industry standard for Nordic producers.

A number of authors have contributed to the state of knowledge about how to optimally bid in wholesale electricity auctions. Concentrating on situations without market power, we note that [?], [?] and [?] are early contributors from the point of view of thermal producers, retailers and hydropower producers, respectively. Based on the idea of fixing price points before bid optimization, introduced by [?], [?] develop a hydropower bidding model with stochastic prices. [?] also

use a stochastic programming approach with price uncertainty, considering flow travel delay effects. [?] show that statistical properties of the intermittent wind power are core elements in determining good bids for such generation sources. [?] consider bidding in a market-wide equilibrium analysis taking into account the combinatorial aspects that arise when matching failure-prone generation capacity to load duration curves. An important recent contribution is [?], who extend [?] using a two-scale multistage approach, allowing for the analysis of intraday market bidding as well as day-ahead bidding. Such multimarket aspects are also discussed in [?], [?], [?].

We add to this knowledge by an out of sample rolling horizon simulation-based evaluation of the two alternatives. We simulate the use of both methods in a setting that is made as close to the current operating situation as possible, in an attempt to showcase the practical value of stochastic programming. As a secondary contribution, the stochastic model is implemented for a complex river system with river flow delays, whose effect is also evaluated. Third, our approach is unique in that the scenarios we use for both spot prices and inflow come from fundamental (bottom up) models that are used in practice, as opposed to e.g. time series approaches employed in [?], [?]. Fourth, the interplay with longer-term scheduling in the form of water values is represented in a way that extends approaches in the bidding literature; we formulate and solve a separate optimization model for this purpose.

The layout of this paper is as follows; first, the three different optimization models used in the simulation procedure are given with relevant explanations in Section ??-??., namely the model for bid optimization, production allocation and seasonal scheduling. Then the flow chart of the entire test procedure is presented in Section ?. In the case study of Section ??, the simulation procedure is implemented for a complex Norwegian reservoir system owned by Agder Energi, and the results from this are shown in section ?. Finally, some concluding remarks are presented in Section ?.

## II. BID OPTIMIZATION

The formulation of the bid model is first presented in a deterministic formulation and then expanded to allow for uncertainty and other moderations necessary for the stochastic model. Both formulations have a horizon of up to seven days depending on how many days are left until the water values are updated, as will be further explained in section ?. The set of hours,  $H$ , hence contains up to 168 hours.

### A. Deterministic formulation

Hydropower production is constrained by the generation capacity and the efficiency of the installed turbines, and the amount and value of available water. In addition, the producer incurs losses due to start-up of turbines mainly because of wear on equipment. Capacity is bid into the day-ahead market, and when the market clears, producers are notified of their committed volumes. To cover the obligations from the spot market, a producer also has the option of participating in the intraday market. The intraday market facilitates trading closer to the operating hour and can hence be used to mitigate

some of the eventual imbalance between actual production and committed spot market volume. Participating in sequential electricity markets may be profitable for the producer [?], [?], but the transmission system operator declares a rule that expected generation is offered in the day-ahead market [?], so that the power system is planned in balance. In our formulation, the use of the balancing market is always penalized and not modeled in detail, as the optimization of the bids to the day-ahead market is the main emphasis.

For the production system, each turbine  $j$  has bounds on minimum and maximum capacity for volume produced and volume discharged, as stated by eq. (??) and (??).

$$u_{jh}W_j^{min} \leq w_{jh} \leq u_{jh}W_j^{max}, \quad j \in J, h \in H \quad (1)$$

$$u_{jh}V_j^{min} \leq v_{jh} \leq u_{jh}V_j^{max}, \quad j \in J, h \in H \quad (2)$$

The binary variable  $u_{jh}$  has value 1 if turbine  $j$  is running in hour  $h$ . The variables  $w_{jh}$  and  $v_{jh}$  are the production and discharge volume for turbine  $j$  in hour  $h$ . How much power a turbine generates,  $w_{jh}$ , from one unit of water discharged,  $v_{jh}$ , depends on the efficiency of the turbines and generators installed in the station. Generally, this is a non-linear relationship due to dependency on both water head and discharge level [?]. To keep the model linear, head effects are disregarded in the optimization, and the production functions are approximated by piecewise linear functions as in eq. (??).

$$w_{jh} \leq \frac{v_{jh} - V_{jn-1}u_{jh}}{V_{jn} - V_{jn-1}}(W_{jn} - W_{jn-1}) + W_{jn-1}u_{jh}, \quad j \in J, h \in H, n \in N \quad (3)$$

where for each turbine  $j$ ,  $V_{jn}$  and  $W_{jn}$  are known points for volume discharged and power produced and  $w_{jh}$  is the produced volume corresponding to the discharged volume  $v_{jh}$  in hour  $h$ . The volume of power produced is found by linear interpolation between the known points on the production curve for each turbine. To speed up calculations, the binary variable  $u_{jh}$  is included to make sure that the production functions are not valid when production is turned off. For each station  $r$ , the sum of the power produced from all turbines in the station is the total production for that station, and the sum of all water released through each turbine in a station is the total discharge for that station, as in eq. (??) and (??).

$$w_{rh} = \sum_{j \in J(R)} w_{jh}, \quad h \in H, r \in R \quad (4)$$

$$v_{rh} = \sum_{j \in J(R)} v_{jh}, \quad h \in H, r \in R \quad (5)$$

There are other forms of discharge from a reservoir than production discharge, such as bypass or spill. Bypass is controlled flow of water leaving the reservoir not used for production. Some reservoirs have bypass restrictions so that the river does not run dry or flood, as stated by eq. (??). Spill is uncontrolled water flow from a reservoir and happens when the reservoir is full. We formulate bypass and spill as separate variables as these may have different water courses and it is also of particular interest to see how the stochastic

and deterministic method compare to each other in terms of spillage.

$$V_r^{B,min} \leq v_{hr}^B \leq V_r^{B,max}, \quad h \in H, r \in R \quad (6)$$

The reservoirs  $r$  are modeled with bounds on minimum and maximum storage level as stated by eq. (??) and a reservoir balance connecting discharge and inflow to the change in storage level, eq. (??).

$$L_r^{min} \leq l_{rh} \leq L_r^{max}, \quad h \in H, r \in R \quad (7)$$

The storage level at the end of any hour is the reservoir level at the start of the hour minus the discharge used for production, bypass and spill, plus inflow and water released from upstream reservoirs flowing in to the reservoir. The water released from upstream reservoirs arrives at the current reservoir after a given river flow travel time. Travel time means that hours are more dependent on each other and hence the degree of freedom when making bid or production allocation decisions is reduced. At the end of the short-term horizon, water that is released from upstream reservoirs, but that has not reached the downstream reservoirs, is valued by water value cuts (see section ??) of the reservoir to which it is headed. Travel time in the watercourse from the upstream reservoir  $k$  to the downstream reservoir  $r$  is denoted by  $T_{kr}$ , but for reasons of clarity, the subscripts are neglected in eq (??). Inflow is denoted  $l_{rh}$ . The reservoir topology is represented by the connection matrices  $C_{kr}$  for production discharge, bypass and spill, and the entities in the matrices have value 1 if there is a direct water way between reservoir  $k$  and reservoir  $r$ .

$$l_{hr} - l_{h-1r} - l_{rh} + v_{hr} + v_{hr}^B + v_{hr}^S - \sum_{k \in R} (v_{h-Tk} * C_{kr}^P + v_{h-Tk}^B * C_{kr}^B + v_{h-Tk}^S * C_{kr}^S) = 0, \quad s \in S, r \in R, h \in H \quad (8)$$

Start-up costs are included and the binary variable  $u_{jh}$  for each turbine has value 1 if the turbine is running and zero otherwise. If a turbine is started up from one hour to the next, a binary variable,  $d_{jh}$  gets value 1 according to eq. (??).

$$d_{jh} \geq u_{jh} - u_{jh-1}, \quad j \in J, h \in H \quad (9)$$

The water value is the marginal opportunity cost of water in the reservoirs, and hence the resource cost of power generation. The water value is a non-linear function of future development depending on market prices and inflow, and has to be approximated to keep the formulation of the bid model linear. The water value function is thus constrained by linear cuts generated by information from a seasonal model covered in Section ???. The cuts are on the form of eq. (??) where  $F_c$  is the future profits for cut  $c$ ,  $A_{rc}$  is the dual variable for reservoir  $r$  in cut  $c$ ,  $L_{rc}$  is the storage level in reservoir  $r$  used in cut  $c$  and  $l_{rh}$  is the storage level at the end of the week when  $h = H$ . Finally,  $m$  is the value of the water left in the reservoirs at the end of the current week.

$$m \leq F_c - \sum_{r \in R} A_{rc}(L_{rc} - l_{rh}), \quad c \in C \quad (10)$$

The objective of the bid optimization model is to maximize the profit from selling power in the day-ahead market. The profit is the revenue from spot market sales less the start-up costs. We assume price-taking behaviour, since the producer in our case are relatively small, as are most hydropower producers in the Nordic area. The value of saving water for the future is taken into consideration by the water value. The objective function of the bid optimization model is shown in eq. (??).

$$\max \sum_{h \in H} \rho_h y_h + m - \sum_{h \in H} \sum_{j \in J} S_j d_{hj} \quad (11)$$

In the deterministic formulation, the committed volume  $y_h$  is simply the sum of the production volumes for all stations, as in eq. (??).

$$y_h = \sum_{r \in R} w_{rh}, \quad h \in H \quad (12)$$

The deterministic formulation presented so far is in fact a model for optimal dispatch, and does not have any determination of the market bids. To generate a bid matrix, the dispatch model is run for different input prices and the optimal production volumes for the first 24 hours of each run is combined into a matrix for the coming operating day. The input prices are created by weighting a forecasted price profile by a set of weights so that the expected uncertainty of future prices is covered. For an example of such weights, see Table ???. The bid matrix must adhere to the market rules which state that the bid volume must be increasing for increasing bid prices. Hence, a restriction is added to the consecutive runs of the deterministic model, imposing the bid volume of a run with a higher input price to be higher than the bid volume for the previous, lower-priced run. If the deterministic model is run with the input price profiles in increasing order, then the committed volume  $y_h$  in run  $e$  has to be larger than the committed volume in run  $e - 1$ , as stated by eq. (??), except for the first run where no previous value is known.

$$y_h^e \geq y_h^{e-1}, \quad e \in E, h \in H \quad (13)$$

This heuristic method for creating the bid matrix may lead to suboptimal results, since a non-physical restriction is imposed on the dispatch model.

### B. Stochastic formulation

The modeling of the production system in the stochastic model follows the same eq. (??)–(??) as the deterministic model, but the decisions on production and discharge are dependent on the uncertain market price and inflow,  $\rho_{sh}$  and  $\iota_{sh}$ . Hence every variable presented so far is in the stochastic formulation also dependent on scenario  $s$  in addition to whatever indices they may already be defined for. In the Nomenclature, all variables are presented for the deterministic setting for easier reading, but the stochastic model requires them to be indexed by  $s$  as well.

In addition, the stochastic model also has a representation of the bids and how the committed volume is calculated from the bids submitted to the market. Both the volumes to bid and

the prices at which to bid for could in fact be determined in an optimization, but this would lead to a non-linear formulation. To keep linearity, price points are fixed in advance and we optimize only the volume corresponding to each of the fixed prices as explained in [?]. After the market is cleared and the spot price  $\rho_{sh}$  is known, the committed volume,  $y_{sh}$ , is found by interpolation between neighboring points,  $(P_i, x_{ih})$  and  $(P_{i-1}, x_{i-1h})$ , as stated by eq. (??).

$$y_{sh} = \frac{\rho_{sh} - P_{i-1}}{P_i - P_{i-1}} x_{ih} + \frac{P_i - \rho_{sh}}{P_i - P_{i-1}} x_{i-1h},$$

$$P_{i-1} \leq \rho_{sh} \leq P_i, \quad h \in H, i \in I \quad (14)$$

Due to the rules of how to submit bids in the spot market, the bids have to represent monotone increasing curves, and an additional constraint on the bid volumes, eq. (??), is necessary.

$$x_{hi} \leq x_{hi+1}, \quad h \in H, i \in I \setminus |I| \quad (15)$$

The committed volume has to be covered by the producer's own generation or by use of the balancing market where the producer can sell or buy its imbalances, so that the volume balance eq. (??) is fulfilled. Use of the balancing market is penalized (see [?], [?]) by the stochastic model, as can be seen in the objective function in eq. (??), where  $R$  is the penalty cost. The value of the penalty cost is set higher than the highest price in the balancing market for the time period where the simulation in the case study period is performed, to avoid bidding strategies where capacity is withheld from the spot market in order to trade in the balancing market.

$$\sum_{r \in R} w_{srh} - y_{sh} + z_{sh}^+ - z_{sh}^- = 0, \quad s \in S, h \in H \quad (16)$$

The objective of the stochastic bid optimization is formulated as a probability weighted sum over all scenarios, stated by eq. (??).

$$\max \sum_{s \in S} \pi_s \left( \sum_{h \in H} \rho_{sh} y_{sh} + m_s - G \sum_{h \in H} (z_{sh}^+ + z_{sh}^-) - \sum_{h \in H} \sum_{j \in J} S_j d_{shj} \right) \quad (17)$$

The stochastic formulation determines the optimal bid volumes,  $x_{ih}$ , for every hour of the coming operating day for each of the fixed price points, and hence the bid matrix is a direct output from the model.

### III. PRODUCTION ALLOCATION

After the bid matrix is sent to the market operator and the market is cleared, the producers are notified of their committed volume for each hour of the following operating day. This production is allocated to individual stations and turbines by a production allocation optimization model. It is akin to SHOP or to [?], but a simplified version is implemented to fit our case study. Our production allocation model is a deterministic mixed integer linear program that takes in the now realized market price and the inflow forecast for the following day. The inflow is considered known in accordance with current

industry practice, although it in principle still uncertainty. The production allocation model has a time horizon of one day and is based on the same equations as the deterministic bid model, eq (??)–(??). The set of hours  $H$  thus contains the 24 hours of the coming operating day. The objective is to choose the unit commitment schedule that generates the committed volume with the minimum cost due to start-ups, use of balancing market and loss of water value.

The production allocation model uses the same linear representation of the production functions as the bid optimization, (??), and hence head effects are not accounted for. To validate the resulting production schedule, non-linear scheduling refinements (such as in [?]) are undertaken after the allocation of production for the stations where head effects are present. Thus, head effects are accounted for in the simulation, but not in the optimizations.

#### IV. SEASONAL SCHEDULING

A seasonal model is developed for the simulation procedure is used to find the value of water left in the reservoirs at the end of each week. According to current practice in the industry, the seasonal model has a time step of one week and hence the value of water is updated once a week. The value of water is a convex function of the current reservoir levels and also depends on the time of year and the expected future development of prices and inflow. In our linear formulation of the short-term models, i.e. the bidding problem and production allocation, this function has to be approximated by linear cuts. Information needed to generate these cuts is obtained from an optimization model that schedules the production of available resources over the seasonal time horizon of 6-18 months.

The seasonal model implemented for the simulation procedure is based on the same equations as the deterministic bid optimization model, eqs. (??) - (??), with modifications due to the longer time horizon and the fact that it is assumed that all power can be sold at the average spot market price. The time horizon is changed to 6 months, and the time step is now aggregated to one week, so every decision is made on a weekly basis, not hourly as for the short-term models.

The model involves no bidding and no balancing market, and the objective function is changed to

$$\max \sum_{h \in H} P_h^{Avr} y_h - \sum_{h \in H} \sum_{j \in J} S_j d_{hj} \quad (18)$$

where  $\rho_h^{Avr}$  is the weekly average electricity price. The seasonal model can hence be explained as finding the optimal production schedule on a weekly basis when generation is constrained by production functions, reservoir balances and generation capacities. The multireservoir system needs a coordinated scheduling strategy where water is saved and discharged from the different reservoirs to best utilize the resources over time, and the cuts generated reflect the cost of using the water in each reservoir given its interaction with the total system.

The cuts are based on the objective function value and the dual variables of the reservoir balances in the seasonal model, as in (??). The objective function is a measure of the potential

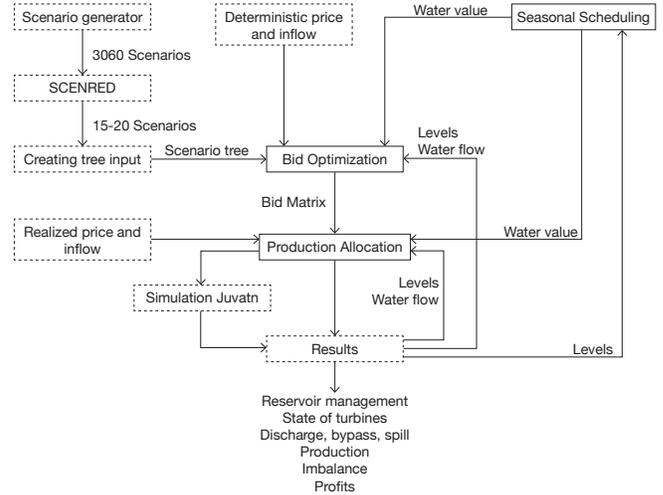


Fig. 1. Flow chart for the simulation procedure. The bid optimization is at core. Other optimization models include seasonal scheduling and production allocation

future profits over the seasonal time horizon based on the current reservoir level, and the dual variables are the marginal increase in profits if one more unit of water were available to the producer and hence represent the scarcity cost of water. The seasonal model is run for different values of weekly initial reservoir level and each run generates information for one cut. The reservoir levels are chosen based on different weekly production patterns and values for inflow. For instance, an extreme change in storage level over the week could stem from a production pattern where all units are run at maximum capacity and inflow is low, leaving a smaller reservoir level at the end of the week than at the start. Another extreme is found when all production is turned off, and inflow is large. We use 9 combinations of production patterns and inflow, so that the cuts span over all possible values of the storage level at the end of the current week.

#### V. SIMULATION PROCEDURE

The simulation procedure is presented in the flow chart in Fig. ???. For each day, the algorithm starts by generating suitable input in the form of a scenario tree for price and inflow for the stochastic bid model and a forecasted price profile and inflow expectation for the deterministic bid model. The stochastic input is generated by sampling of individual scenarios for price and inflow, which are then sent through a tree construction and reduction algorithm (Scenred) [?]. The price and inflow scenarios are obtained from power market analysts and from in-house forecasting processes. The deterministic method only uses one forecasted price profile as input, and this forecast is scaled to yield several individual scenarios for different runs of the deterministic model. The bid models also need the value of water as input, and this is obtained by using information on dual variables and objective function from the seasonal scheduling, which are used to generate the cuts shown in (??).

When all suitable input is generated, the bid optimization is run to generate bids for the following day. The stochastic bid

model has a time horizon of up to seven days, depending on how many days are left in the week before the seasonal model is run and the water values are updated. The deterministic model also has a seven-day horizon. From both the stochastic and deterministic model, only the bid decisions for the first day are used since this is the only information that is actually sent to the market operator. The bid models generate optimal decisions on what volumes to bid for each price point in each hour of the following operating day, which is sent to the market operator as a bid matrix. When the market clears, the committed volume for the producer is found by interpolation between the two neighboring price points on each side of the realized market price.

The committed volume has to be supplied by the producer's own generation or by buying power in the balancing market. Production is allocated according to the amount and cost of available water by the optimal schedule found by the production allocation model, which is run every day after the spot market is cleared and the price is revealed.

As a final step in the simulation procedure the resulting production plan is tested by detailed calculations and scheduling refinements that account for head effects at the reservoirs where these are present. This is done to make sure that the results are actually implementable for the real-life situation. The detailed calculations are named "Simulation Juvatn" in the flow chart in Fig. ??, since for the case study of section ??, head effects are only present at one reservoir, namely Juvatn. The reservoir levels and the amount of water in the watercourses at the end of each operating day are sent as input for the bid model for the next day, and the entire simulation procedure, from scenario generation to calculations for head effects, is repeated. Every seventh day the current reservoir levels are also sent to the seasonal model, and this is run in order to update the water values for the coming week.

## VI. CASE STUDY

The above simulation procedure is implemented for Mandalsvassdraget, which is a cascaded, Norwegian reservoir system located in the NO2-area and owned by Agder Energi that participates in the pool-based day-ahead market at Nord Pool Spot. The simulation is carried out over a period of seven weeks from 16 August 2012 to 30 September 2012. This specific period is chosen due to availability of data. Data for inflow is obtained from Agder Energi and data for price is obtained from SKM Market Predictor AS (a power market analysis company that develops and supplies price forecasts to the power industry). The inflow scenarios used in the stochastic model is Agder Energi's historical forecasted ensemble scenarios used at the time of operations, and the price scenarios are developed by SKM particularly for the purpose of this paper and are based on scenarios of fundamental events that affect the day-ahead market price such as electricity consumption or transmission to or from connected areas. The prices and inflow used by the seasonal model, which has a time horizon of 34 weeks, is taken as the

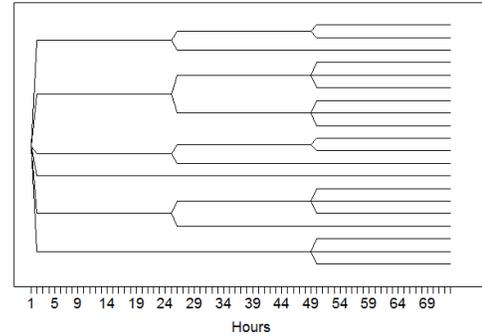


Fig. 2. Scenario tree structure used as input to the bid optimization. Spot prices and reservoir inflows are the stochastic variables

historical weekly average price and inflow<sup>1</sup> from 12 August 2012 to 1 April 2013 obtained from Nord Pool and Agder Energi, respectively. Hence the simulation period for use of the short-term model cover the first seven weeks of the 34 week seasonal horizon.

For the stochastic model, the input is generated by sampling 60 price scenarios and 51 inflow scenarios for each day. These are combined together in a tree structure that represents the information flow of the problem. The tree structure has hourly time steps and daily branching. The first three days are modeled with uncertain prices and inflow where the parameters gradually become known through the scenario tree. When the prices for the third day are revealed, all prices and inflow for the remaining days of the week also get known. Having three stages in the tree is a choice made due to the trade-off between accurate modeling of uncertainty and computational time. The effect of approximating the uncertainty with a three-stage tree is judged to be small, as it is only the first-stage decisions that are actually used and the producer has the opportunity of rescheduling for the following days of the week. The price and inflow for the remaining days of the week after the initial stochastic stages are chosen as a random realization of the forecasted price and inflow for the specific day. The stochastic bid optimization model has a variable horizon that accounts for the fact that the water values are updated once a week, and all the days from the current day and to the next update has to be represented in the tree. This is done so that the stochastic model is tailored to fit with current practice for the connection between short and longer-term scheduling. An example of the tree structure used in this paper is given in Fig. ??, where from the branching structure it is evident that new information is

<sup>1</sup>The seasonal scheduling developed for this simulation procedure is only needed to find the value of water to be used as an input to the short-term scheduling models, and hence we use simplified approach. In particular, the seasonal model uses historical data for price and inflow that was not available to the producer at the time of operations, and hence the water values are adapted to the actual development, which the producer has no way of foreseeing. The fit between the reservoir management strategy and the realized development of price and inflow is overestimated, and the management of the larger reservoirs is not directly comparable to the real-life situation. For the purpose of comparison, however, the seasonal model and the water values are adequate as both the stochastic and the deterministic model use the same water values.

TABLE I  
VALUES OF PRICE POINTS.

Number	1	2	3	4	5	6	7
Price	0	15	20	23	27	30	100

TABLE II  
WEIGHT FACTORS FOR THE DETERMINISTIC SCENARIOS.

Scenario	1	2	3	4	5	6	7	8	9
Weight	0.83	0.91	0.94	0.97	0	1.03	1.06	1.09	1.17

revealed when the market is cleared once a day. By the tree construction and reduction algorithm, [?], the tree is reduced to the smallest size that still adequately represents the statistical structure of uncertainty for price and inflow, resulting in 18-25 scenarios for each run of the stochastic model. This has consequences for the number of bid points, since according to [?], the number of scenarios,  $S$  has to satisfy (?). If there are too few scenarios compared to the number of price points, the optimization adapts "too well" to the information given.

$$S \geq 2I + 2 \tag{19}$$

According to eq. (?), the number of bid points  $I$  is chosen to be 7. The bid points are the same every day throughout the simulation procedure, and chosen in a way that covers the whole range of possible prices with more points in the areas where most of the prices will fall. The specific prices used can be seen in Table ??.

The practice-based approach uses a single forecasted value of price and inflow as input. The price forecast is scaled by different weights to yield individual scenarios that are used with the deterministic model to create a bid matrix. The weights are shown in Table ?? and are the same as currently used by Agder Energi and the price forecast is the base scenario from SKM. The method of simply scaling the scenarios means that all scenarios have the same time-profile as the forecasted price profile, and hence that the scenarios never intersect each other. This is a poor representation of the uncertainty in spot market prices.

A schematic of Mandalsvassdraget is shown in Fig. ?. Návavn and Juvavn have the largest capacities and also the highest degrees of regulation, while the other reservoirs are small and can be filled or emptied within the week. The discharge from upstream reservoirs is subject to river flow travel time in the watercourses between some of the reservoirs, and the longest time delay is 5 hours. Some characteristics of the reservoirs and stations in the system are shown in Table ?? and ??.

TABLE III  
STORAGE LIMITS FOR RESERVOIRS ( $Mm^3$ ).

Name	Min	Max
Návavn	6.5	206.2
Skjerkevavn	2.3	19.6
Juvavn	0	142.0
Lognavavn	1.4	10.0
Ørevavn	0	11.2
Tungesjø	0	2.2
Mannflávavn	0.8	2

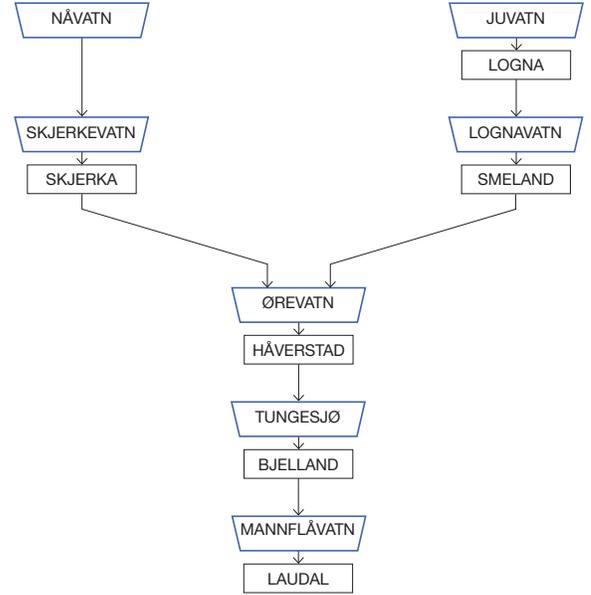


Fig. 3. Schematic of the topology of the river system in the case study, Mandalsvassdraet. Reservoirs are represented as trapezoids and power stations as rectangles

TABLE IV  
INSTALLED CAPACITY IN STATIONS (MW).

Name	Min	Max
Skjerka	26.4	102.7
Logna	6.0	14.6
Smeland	10.0	24.1
Håverstad	12.4	69.7
Bjelland	25.7	76.3
Laudal	38.5	103.2

For the specific case of Mandalsvassdraget, the solution time for one run of the stochastic model is under 50s, when implemented and solved in Xpress [?] on a 3.40 GHz Intel PC with 16.0 GB RAM. For 9 consecutive runs of the deterministic model to yield a bid matrix, the computational time is in fact a bit longer, about 100s.

## VII. RESULTS

### A. Obtained average price

The average price per MWh produced is a measure of performance of the short-term scheduling model and shows how improvements in modeling may increase the price at which a producer can sell their power. The obtained average price is the sum of all spot market revenues during the simulated period divided by the total produced volume over the same period. Even though use of the balancing market is penalized in the bid optimization model as explained in Section ??, the producer may make a profit by participating in the balancing market as explained in [?]. When revenue from the market is calculated, actual historical costs for up ramping and profits from down ramping is included in the measure. Table ?? shows the results for the obtained average price. The

TABLE V  
SIMULATION RESULTS FOR OBTAINED AVERAGE PRICE.

	Stochastic	Deterministic
Euro	23.084	22.925
Percent		-0.69

TABLE VI  
SIMULATION RESULTS FOR TOTAL VALUE.

	Stochastic	Deterministic
Euro	14442784	14354771
Percent		-0.61

TABLE VII  
SIMULATION RESULTS FOR THE NUMBER OF ODD STARTS.

	Stochastic	Deterministic
Hours	151	182
Percent		+17.0

improvement in average price with the stochastic method is 0.69% compared to the deterministic alternative. Such small numbers are to be expected given the flexibility the producers have to make corrective actions both in short-term markets and over time through its power plants. The difference in average price for the individual weeks can be found in Appendix ??.

*B. Total value*

The obtained average price may not be an adequate measure of performance, as it does not account for operational costs. The costs of hydropower are related to start-ups of units and the opportunity costs of lowering reservoirs. Hence another measure of performance is the sum of total obtained profits over the simulation period and the total value of the water left in the reservoirs at the end of the period, i.e. at the end of the first 7 weeks. This is a measure of the total value of water used in the simulation period and water saved for later use. Releasing more water and producing at high prices now means that less water will be available for future production, and a balance must be struck for scheduling the available water resources over time. Table ?? shows that the stochastic model has a total value that is 0.61% higher than the deterministic model. The stochastic approach uses more water than the deterministic model and hence has a lower value of the water left in the reservoirs, but achieves a higher total value.

*C. Odd starts*

To investigate the stability of the resulting production schedule, a measure called odd starts is developed where the number of times a turbine is turned on or off for just one or two hours at a time is recorded and added for all turbines in the system, and the result is shown in Table ?. A low number of odd starts is an indicator of a realistic and implementable production plan, as such frequent starts and stops are undesirable and often manually rescheduled by producers. The stochastic bid optimization results in a 17.0% decrease in odd starts. Odd starts often occur due to sudden changes in the spot price over few hours. When the stochastic model finds the bid matrix it takes into account scenarios with different price profiles and

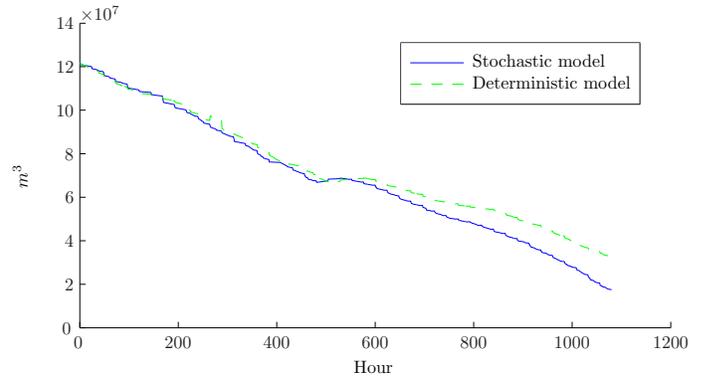


Fig. 4. Plot of reservoir level at Juvatn over the simulation period.

hence different scenarios for where the price peaks can occur. The practice-based method, on the other hand, sees the same profile scaled equally in all hours, resulting in a bid matrix that is not as robust to sudden changes in price.

*D. Reservoir management strategy*

The reservoir management strategy is important to give validity to the above results. Návatn and Juvatn have the capability to store water over seasons, and the water release strategy is controlled by the water value. The water value used in this case study is based on output from the seasonal model, which is possibly a rudimentary element of the simulation procedure in terms of representing real operating conditions, see Footnote 1 in Section ?. The stochastic model releases more water from the large reservoirs than the deterministic model to better utilize high price hours now instead of saving water for later. According to the water values used, this is the correct behavior since the prices during the simulation period are actually higher than towards the end of the seasonal horizon. A plot of the reservoir level at Juvatn is shown in Fig. ??, where it is evident that the reservoir level is quite low towards the end of the simulation period.

The management strategy for the smaller reservoirs shows significant differences between the stochastic and the deterministic model. Since inflow is uncertain in the stochastic model, the strategy is more conservative as the reservoir level does not tend as much towards the bounds of the reservoirs. When inflow is uncertain, the risk of spillage when reservoir levels are high is taken into account in the reservoir management strategy. In the simulation, the stochastic approach yields fewer hours with reservoir levels at the maximum level than the deterministic model, and hence a reduced risk of spillage. The simulated values for spill are also lower for the stochastic model. The percentage of time when reservoir levels are at maximum is presented in Table ??, the amount of spill in Table ?? and a plot of the reservoir level at Skjerkevattn in Fig. ??.

*E. Results without river flow travel time*

To investigate how the complexity of the river cascade affects the results, the simulation procedure is implemented

TABLE VIII  
NUMBER OF HOURS WHERE THE RESERVOIR LEVEL IS AT MAXIMUM.

		Stochastic	Deterministic
Skjerkevatn	Hours	135	181
	Percent		25%
Lognavatn	Hours	112	165
	Percent		32%

TABLE IX  
NUMBER OF HOURS AND THE AMOUNT OF SPILL.

		Stochastic	Deterministic
Skjerkevatn	Hours	4	6
	$m^3$	3156	5039
Lognavatn	Hours	63	95
	$m^3$	614651	1166413

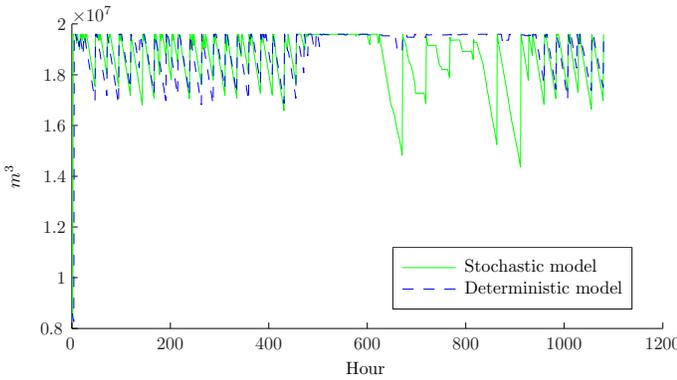


Fig. 5. Plot of reservoir level at Skjerkevatn over the simulation period. This is an intake-reservoir for one of the main power stations in the cascade and has a moderate risk of spill.

for the same reservoir topology as Mandalsvassdraget, but without river flow travel time in the watercourses between reservoirs. Having time delays means that hours are more strongly dependent on each other, because large discharged volumes from upstream reservoirs in high priced hours may force production in downstream reservoirs a few hours later when the price is lower. An extreme case would be that the downstream reservoirs are flooded and water is lost. These dependencies must be taken into consideration when the bids and the following production allocation is determined, as it is crucial that the bid decisions are flexible enough to account for possible adverse developments.

Tables ?? and ?? show the results for obtained average price and total value for the deterministic and stochastic approach without travel time. The improvement by the stochastic formulation is 0.61% for obtained average price and 0.57% for total value. Hence, the stochastic model still outperforms the practice-based method, but with a smaller difference than when travel time is present, as in Tables ?? and ?. This is taken to mean that the stochastic formulation has even larger potential for improvements for complex systems. Without river flow time delay, the difference between the stochastic and the deterministic model is smaller since the system is less constrained.

TABLE X  
OBTAINED AVERAGE PROFITS WITHOUT RIVER FLOW TRAVEL TIME.

	Stochastic	Deterministic
Euro	23.12	22.98
Percent		-0.61

TABLE XI  
SIMULATION RESULTS FOR TOTAL VALUE WITHOUT RIVER FLOW TRAVEL TIME.

	Stochastic	Deterministic
Euro	14467069	14384291
Percent		-0.57

## VIII. CONCLUSION

The results from the case study of Mandalsvassdraget show that the stochastic bid optimization model gives improvements in terms of a 0.69% higher obtained average price and a 0.61% higher total value than a heuristic method used in practice. Any small gain is considered interesting by producers, as this problem is solved every day. The reservoir management strategy is improved, as the stochastic model obtains a higher total value than the practice-based heuristics and hence utilizes the available water in a better way. This is due to the fact more water is scheduled for production now when prices are higher and not saved for later. Part of this extra production is a result of the increase in average price obtained by using the stochastic model. A higher price increases the profit from production and with identical water values a superior model will have a shift towards producing now instead of saving the water for later. In addition, the risk of spillage is also reduced, as the number of hours with full reservoirs is fewer when using the stochastic reservoir management strategy. The model is implemented for a complex real-life system, and the stochastic model uses best-practice forecasts that are, at a cost, currently available to the industry, namely price scenarios developed by market analysts or the power company itself. The algorithm is also fast enough to be used on a daily basis and is tailored to be included in the scheduling hierarchy used by most producers today.

The results from the specific case study of Mandalsvassdraget are positive on behalf of the stochastic model, and this can be seen as an indicator of the potential added value of implementing a stochastic method for the bidding problem. The simulation should be carried out over longer periods of time, other times of year and other reservoir systems before any general conclusions can be made. High reservoir levels and large inflows characterize the time of year when the simulation in this paper is performed. To get a good comparison between the stochastic and the deterministic model the simulation should cover a longer period of time with more variation in regards to the availability of water or market prices.

Our test framework could be used to investigate other approaches for short-term production scheduling. The framework is relatively easy to update to reflect potential changes in market rules, scenario input, or how the value of the water in the reservoirs at the end of the bidding horizon is calculated. A possible use of this framework is to evaluate the value of capacity changes or waterway improvements.

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APPENDIX

Table ?? summarizes the difference in obtained average price each week.

TABLE XII  
DIFFERENCE IN OBTAINED AVERAGE PRICE IN €/MW

Week	Stochastic average price	Deterministic average price	Difference
1	25.59	24.89	0.71
2	26.55	25.84	0.71
3	25.76	24.34	1.41
4	18.75	19.14	-0.39
5	17.60	16.93	0.68
6	17.96	17.50	0.46
7	25.67	24.45	1.22