Abstract—Both marine surface vehicles and underwater vehicles are often equipped with cranes, robotic manipulators or similar equipment. Much attention is given to modeling of both the dynamics of marine vehicles and the dynamics of manipulators, cranes and other equipment. However, less attention is given to the interconnected behaviour of the vehicle and the equipment, even though such equipment may have a considerable impact on the vehicle dynamic behaviour, and therefore risk, or conversely, the vehicle may have a considerable impact on the equipment dynamic behaviour. With main focus on ships equipped with cranes, this paper presents a framework for modeling the interconnected dynamics of rigid body systems, based on Lagrangian dynamics. The resulting equations of motion are implemented as a bond graph template to which any subsystem of interest, such as actuators, hydrodynamics, and controllers may be interfaced. An example on how this framework can be used in order to develop a high fidelity simulator of an offshore installation vessel with a heavy duty crane is presented. This work represents the first bond graph implementation of crane and vessel dynamics where the interconnections are modeled according to true physical rigid body principles without non-physical limitations such as diagonal mass-inertia matrix.

Index Terms—Dynamic Modeling, Marine Vehicles, Lagrange Dynamics, Quasi-coordinates, Marine Operations, Bond Graph

I. INTRODUCTION

The purpose of this paper is to develop a bond graph framework for mathematical modeling of the interconnected dynamics of a marine vehicle, such as a ship, a floating oil rig or a remotely operated vehicle (ROV), together with equipment such as cranes or robotic manipulators. There are numerous applications in which this kind of interconnected model can be useful. In a ship-crane context, such models can be used in order to develop and verify unified model based controllers for the interconnected dynamics, or to verify existing crane controllers during various conditions aboard a vessel. Similarly, it can be used in order to verify that installing a heavy crane on a vessel do not have adverse effects on vessel controllers. Improved offshore crane simulators can be designed in order to train personnel for high precision tasks, reducing the risk related to human error, or to investigate the vessel response in heavy lifting operations under various weather conditions. In this case an interconnected model can be used as a tool for determining in which configurations the crane safely can operate under given weather conditions. Hence, such a model may provide input to risk analyses for offshore lifting operations.

Even though interconnected models are developed for ROVs and manipulators in e.g. [1], [2] and for autonomous underwater vehicles (AUVs) and manipulators in [3], we believe that a bond graph approach to the problem will be useful because it allows model developers to rapidly connect different submodels from any physical domain, (e.g., mechanical, hydraulic, etc.), to the system, using well-defined and generic energy connectivity principles in order to enhance the basic model to the desired level of fidelity. As an example, hydrodynamic damping models and environmental loads such as wave excitations and current loads from e.g [4] and [5] can easily be developed and connected to both the vehicle and the equipment within the bond graph framework. The same is true for actuator systems such as marine thrusters and hydraulic motors.

Dynamic equations will be derived using the Lagrangian method with quasi-coordinates, as in [6]. Only the dynamics associated to the kinetic energy will be included in the Lagrange equations because the dynamics associated to the potential energy conveniently can be accounted for directly in the bond graph. The Lagrangian equations will be implemented through an IC-field in the bond graph modeling syntax, using the quasi-state momentum and the generalized displacements as states, as shown in [7].

A. Related work

A number of maritime simulator solutions have previously been developed for different applications. Two examples are the Marine Systems Simulator, MSS [8], [9], and the Marine Vessel and Power Plant Simulator [10], [11], [12]. The MSS simulator, a joining together of the GNC-toolbox [13], MCsim, [14], and DCMV [15], is an environment or a platform which allows for rapid formulation of dynamic equations for vessel, with special focus on maritime control systems. This framework is implemented in the Matlab/Simulink® software. However, it does not facilitate crane and manipulator extensions. The Marine Vessel and Power Plant Simulator is a Matlab/Simulink®-based extension of the MSS-simulator, that supports better thruster models and electric power plants. The main purpose of the simulator is to support development of advanced power system control and optimization methods [12]. As noted in [12], the disadvantage of using Simulink is that it is hard to model interconnections. It is noted that the system is hard to divide into the levels as required by the
subsystem architecture of Simulink. Bond graph theory, on the other hand, provides a unified description of physical systems across multiple energy domains [16], something that, in turn, makes interconnection of subsystems convenient.

Three references that do support interconnected crane and manipulator dynamics are [3], [17] and [18]. The former reference derives dynamic equations for AUVs using an Lagrangian approach, similar to what is done in this article. The main purposes of this approach in [3] is (i) to avoid the singularities that arise when using an Euler-angle representation, while at the same time keeping a minimal formulation (i.e., not using e.g. a unit quaternion-representation), by instead using quasi-coordinates, and (ii), to enable AUV-manipulator modelling. The main difference between this approach, and the approach presented in this article, is that our mathematical model is formulated as shown in [7] and [19], rather than in the traditional manner following the Lagrangian formulation as presented in e.g. [20], [6], thus enabling us to represent the model in bond graph. In addition to allowing for a bond graph representation, this is also advantageous because we avoid the task of time-differentiating the mass-inertia matrix. The second reference, [17], presents a modeling approach for heavy marine lifting operations based on the modeling and simulation software 20-sim, using the 20-sim 3D Mechanics toolbox [21]. This approach provides a bond graph interface to other subsystems. It is however a disadvantage that the approach is constrained to a particular software implementation and a particular toolbox within the software, something that in turn provides a number of limitations and restrictions. One example is that the 3D Mechanics toolbox will only allow for diagonal mass-inertia matrices and linear spring and damper relations. Our approach, on the other hand is not limited to any particular software implementation. Rather, the model is based on general bond graph theory, and can, as such be implemented in any software that supports scripting, since the system equations can easily be extracted from the bond graphs. However, it is convenient to use a software that directly supports bond graphs, as will be done in this work. The third reference, [18], does not depend on particular software, but is instead based on stiff spring connections between rigid bodies. This is done in order to resolve problems related to derivative causality appearing when connecting the rigid bodies. The disadvantage with this approach is that if a soft spring is used, the accuracy will be severely affected, while if a stiff spring is used, fast dynamics that will increase the simulation time will appear. As such, a compromise between slow simulations and large simulation errors must be made when deciding on the spring stiffness. In the framework presented in this article, the derivative causalities have been resolved algebraically, and no compliance between rigid bodies are necessary. The advantage with using the compliance-approach used in [18] is that it is easier to alter the structure of the crane, for example, by replacing a revolute crane joint with a linear one.

Our contribution then is to provide a framework that allows for effective simulator development based on the bond graph methodology, something which makes it well suited for multi energy-domain modeling and also arguably better suited for representing physical systems than, for example, block-diagrams [22], as seen both from a practical modelling-technical point of view, and from a pedagogical point of view [23]. The former is something we see as highly relevant for maritime vehicles as they, at least, will include components from the mechanical, electrical and hydraulic domain. Currently, there are a number of simulators specialized for various purposes, as pointed out earlier, but they lack the flexibility associated to bond graph modelling in terms of generic interfaces to subsystems, does not facilitate connection of cranes with true dynamic interconnections to the vessel, or are limited in terms of rigid body dynamics because they are based on very specialized toolboxes, (i.e., the 3D Mechanics toolbox for [17]) The framework presented in the following, on the other hand, retain all the advantages of bond graphs, while remaining independent of any particular software or toolboxes, and has no limitations or restrictions regarding the rigid body dynamics. As such, this is the first software independent bond graph framework for crane and vessel dynamics where the interconnections are modeled according to true physical rigid body principles without non-physical limitations such as diagonal mass-inertia matrix.

B. Structure of the article

In the following section we model the dynamics of the vehicle in a manner similar to that presented in [24], where the resulting equations are well suited for bond graph implementation. During this section, the purpose of the before mentioned quasi-coordinates will be made clear. In section III, we extend the model to include manipulator or crane dynamics in a similar manner as in [19], while keeping the convenient structure of the model. Note that in this discussion, the crane and manipulator equipment is assumed to comprise an open chain of linked bodies structure, with lower pair joints. This is not because it is problematic to use this approach on other kinds of structures, say parallel manipulators, or devices without lower pair joints, but rather because we seek to provide an unambiguous method for connecting the equipment to the vehicle. In section IV we proceed to implement the model as a bond graph template. Note also that a short introduction to bond graphs can be found in the Appendix, while more thorough treatments can be found in e.g. [7], [25], [26]. In section IV it is also briefly discussed how subsystems can be interfaced to the template. In section V, a case study is presented. This study is based on a simulator developed for an offshore installation vessel with a heavy-duty crane with three degrees of freedom, which exemplifies both why bond graphs are well suited for developing models of relevant subsystems and how these can be interfaced to the bond graph template. In addition to the interconnected dynamics of the vessel and the crane, vessel restoring and gravity forces, crane gravity forces, wave excitation forces, added mass, hydrodynamic damping, a thruster system, actuators for the crane, a crane wire based on [27], a crane control system, and a dynamic positioning system for the vessel are discussed. Note, however, that our purpose is not to present any state-of-the-art research regarding any of these subsystems, but rather to demonstrate the connectivity of the basic framework by providing examples on how it may
be used and interfaced. For example, while being aware that heavy-duty offshore crane vessels will be over-actuated, being equipped with for example eight thrusters, we are satisfied with equipping our case-study vessel with only three. We have also found that, in addition to serving as a demonstration of the connectivity of the framework, these subsystems are necessary in order to produce realistic simulation results that can be compared and discussed. The interfaces between all these subsystems are also emphasised.

II. MARINE VEHICLE DYNAMICS

In this section we seek to find equations of motion for the marine vehicle, using momentum and displacement as generalized states. A state space model expressed in terms of the body fixed reference frame is more easily derived from the Lagrangian equations than the traditional state space model for marine vehicles presented in e.g. [5], with displacements and displacement rates as states. This is mainly because we avoid the tedious task of differentiating the mass-inertia matrix when using momentum, as opposed to displacement rates. Figure 1 can be used as a reference for some of the variables in this section.

A. Kinematic Relations

Let the position and orientation of the vehicle be given relative to an inertial reference frame, denoted by 0. Attach a second reference frame to the vehicle body and denote it \( b \). The position of the vehicle is given by the vector \( r_{b/0} \), where the superscript indicate that the vector is expressed in terms of the inertial reference frame, while the subscript \( b/0 \) indicate that the vector give the position to the origin of the vehicle body fixed reference frame relative to the origin of the inertial reference frame. The orientation of the vehicle is given by the Euler angles \( \Theta = [\phi, \theta, \psi]^T \). In this paper, the Euler angles are defined such that if the vehicle is rotated an angle \( \phi \) about its x-axis, an angle \( \theta \) about the resulting y-axis, and finally an angle \( \psi \) about the resulting z-axis, then the body fixed coordinate frame have the same orientation as the inertial reference frame. Using this, we can find an expression for the angular velocity of the vehicle, expressed in terms of the body fixed reference frame as

\[
\omega_{b/0} = i_b \dot{\phi} + j_b \dot{\theta} + k_b \dot{\psi} = T_{\Theta}^{-1}(\Theta) \dot{\Theta}
\]  

(1)

where \( i_b \) is the unit normal vector along the x-axis of the vehicle body fixed reference frame, \( j_b \) is the unit normal vector along the y-axis of the reference frame resulting from the rotation \( \phi \), and \( k_b \) is the unit normal vector along the z-axis of the reference frame resulting from the rotation \( \theta \) about \( j_b \).

The \( 3 \times 3 \) matrix \( T_{\Theta}^{-1} \) is then defined as \( T_{\Theta}^{-1} = [i_b, j_b, k_b] \). Expressions for the unit normal vectors along the axes of the intermediate reference frames can be found by using the principal rotation matrices for the sequence of rotations described above. Consider first a coordinate \( c \) describing the location of point \( P \) in a local reference frame. Assume that this point now is observed from a reference frame rotated an angle \( \phi \) about the local reference frame, denoted \( c^x \). These coordinates can now be related as

\[
c^x = R_x(\phi)c
\]  

(2)

where \( R_x \) is a principal rotation matrix about the local x-axis. Because the rotation matrix is orthogonal, we can write the inverse of the matrix as \( R_x^{-1} = R_x^T \) [20]. Using this, an expression for the unit normal vector along the y-axis of the first intermediate reference frame, \( j'_b \) from (1), can be expressed in terms of the body fixed reference frame as

\[
j'_b = R_x^T(\phi)j_b
\]  

(3)

where \( j_b = [0, 1, 0]^T \) is the unit normal vector along the y-axis of the body fixed reference frame. Similarly, the coordinate \( c^y \), can be observed from a new reference frame, rotated an angle \( \theta \) about the previous reference frame, given in (2), denoted \( c^v \). The relation between \( c^x \) and \( c^y \) is then expressed as

\[
c^y = R_y(\theta)c^x
\]  

(4)

Using this expression, we find that the unit normal vector \( k''_b \) can be expressed as

\[
k''_b = R_y^T(\theta)k'_b = R_y^T R_x^T k_b
\]  

(5)

where \( k_b = [0, 0, 1]^T \). With these transformations defined, we can express the transformation matrix of (1) as

\[
T_{\Theta}^{-1}(\Theta) = \begin{bmatrix} i_b, & R_x^T j_b, & R_x^T R_y^T k_b \end{bmatrix}
\]  

(6)

The final principal rotation matrix \( R_z(\psi) \) can be used in order to transform a coordinate expressed in terms of the second intermediate reference frame, to be expressed in terms of the inertial reference frame. We can now design the rotation matrix transforming a coordinate representation from the vehicle body fixed reference frame to the inertial reference frame as

\[
R_{0/0}^b = R_z(\psi)R_y(\theta)R_x(\phi)
\]  

(7)
with

\[
R_z(\psi) = \begin{bmatrix}
    c_\psi & -s_\psi & 0 \\
    s_\psi & c_\psi & 0 \\
    0 & 0 & 1
\end{bmatrix},
R_y(\theta) = \begin{bmatrix}
    c_\theta & 0 & s_\theta \\
    0 & 1 & 0 \\
    -s_\theta & 0 & c_\theta
\end{bmatrix},
\]

\[
R_x(\phi) = \begin{bmatrix}
    1 & 0 & 0 \\
    0 & c_\phi & -s_\phi \\
    0 & s_\phi & c_\phi
\end{bmatrix}
\]

and \( s_x = \sin(x) \), and \( c_x = \cos(x) \). We can now write

\[
\dot{e}^0 = R_b^0 \dot{e}^b \tag{9}
\]

This rotation matrix, as with the principal rotation matrices, is orthogonal [20], such that

\[
(R_0^b)^{-1} = (R_0^b)^T = R_0^b \tag{10}
\]

With the kinematic relations in place, we can derive expressions for the kinetic energy of the vehicle.

B. Kinetic Energy of the Vehicle

The kinetic energy of the vehicle can be expressed as

\[
T = \frac{1}{2} \left( (v_{cg}^b)^T M v_{cg}^b + (\omega_{b}/0)^T I_{b} \omega_{b}/0 \right) \tag{11}
\]

where \( M = mI_{3x3} \), \( m \) is the mass of the vehicle, \( I_{3x3} \) is the identity matrix, \( I_{b} \) is the vehicle inertia tensor, and \( v_{cg}/0 \) is the linear velocity of the center of gravity relative to the inertial reference frame. However, using the Lagrangian approach, the kinetic energy should be expressed in terms of a set of generalized coordinates and their rates. The generalized coordinates are a set of coordinates that uniquely define the position and orientation of the vehicle, and are in this paper chosen as

\[
q = \left[ \dot{r}_{b}/0 \right]^T, \left[ \Theta^T \right]^T \tag{12}
\]

The linear velocity of the vehicle center of gravity can be expressed in terms of the generalized coordinates and corresponding rates as

\[
v_{cg}/0 = v_{b}/0 + \omega_{b}/0 \times r_{cg}/b = R_{0}(\Theta) \dot{r}_{b}/0 + T^{-1}(\Theta) \dot{\Theta} \times r_{cg}/b \tag{13}
\]

where \( v_{b}/0 \) is the velocity of the origin of the vehicle body fixed reference frame, and \( r_{cg}/b \) is the vector from the origin of the vehicle body fixed reference frame to the center of gravity. By substituting (1) and (13) in (11), the kinetic energy takes the form \( T(q, \dot{q}) \). We do however seek to replace the dependency on \( \dot{q} \) by the quasi coordinates given as

\[
\omega = \begin{bmatrix}
    \dot{v}_{b}/0 \\
    \dot{\omega}_{b}/0
\end{bmatrix} = \begin{bmatrix}
    R_{b}(\Theta) \dot{r}_{b}/0 \\
    0_{3x3} T_{b}^{-1}(\Theta)
\end{bmatrix} = \alpha^T \dot{q} \tag{14}
\]

because this will make the resulting equations of motion dependent on the linear and angular velocity of the body frame, rather than the linear velocity in terms of the inertial frame and the Euler angle rates. The inverse of (14) is

\[
\dot{q} = \beta \omega \tag{15}
\]

where

\[
\beta = (\alpha^T)^{-1} = \begin{bmatrix}
    R_{b}^0 & 0_{3x3} \\
    0_{3x3} & T_{b}
\end{bmatrix} \tag{16}
\]

Substituting (15) into the expression for \( T(q, \dot{q}) \), yields the expression \( T(q, \beta \omega) = \bar{T}(q, \omega) \), which can be found explicitly by finding the linear velocity of the vehicle center of gravity expressed in terms of the quasi-coordinates. This is recognized as the first expression in (13), and can be expressed compactly as

\[
v_{b}/0 = \begin{bmatrix}
    I_{3x3} & i_{b} \times r_{cg}/0 \\
    j_{b} \times r_{cg}/0 & k_{b} \times r_{cg}/0
\end{bmatrix} \omega = \sqrt{J_{b}} \omega \tag{17}
\]

where \( J_{b} \) is the geometric Jacobian matrix for the linear velocity of the center of gravity of the vehicle. More trivially, the angular velocity can be expressed in matrix form as

\[
\dot{\omega}_{b}/0 = \begin{bmatrix}
    0_{3x3} & I_{3x3}
\end{bmatrix} \omega = \sqrt{J_{b}} \dot{\omega} \tag{18}
\]

The vector \( v_{b} = [v_{cg}^b]^T, (\omega_{b}/0)^T \) collects the linear velocity of the center of gravity of the vehicle and the angular velocity of the body. This can be expressed as

\[
v_{b} = \begin{bmatrix}
    J_{b}^T \\
    J_{b}^T
\end{bmatrix} \omega = J_{b} \omega \tag{19}
\]

With this, we find the kinetic energy in terms of quasi coordinates as

\[
\bar{T}_{b}(q, \dot{q}) = \frac{1}{2} \omega^T J_{b}^T \begin{bmatrix}
    M & 0_{3x3} \\
    0_{3x3} & I_{b}
\end{bmatrix} J_{b} \omega \tag{20}
\]

where \( B_{b} \) is the symmetric and positive definite vehicle mass-inertia matrix. The equations of motion are found by inserting the kinetic energy expression into the Lagrange’s method.

C. Equations of Motion

In the traditional Lagrange method, in which the kinetic energy is expressed in terms of generalized coordinates and rates, as opposed to generalized coordinates and quasi-coordinates, the equations of motion takes the form

\[
\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}} \right) - \frac{\partial T}{\partial q} = \tau \tag{21}
\]

where \( \tau \) is the vector of generalized forces. Note that the potential energy of the system is not included here. Examples on how these effects can be included will be given in section V. When introducing quasi-coordinates, the chain rule must be used when differentiating because the quasi-coordinates are functions of the generalized coordinates and rates. From [6], we have that the quasi-equations of motion becomes

\[
\frac{d}{dt} \left( \frac{\partial \bar{T}}{\partial \dot{\omega}} \right) + \beta^T \gamma \frac{\partial \bar{T}}{\partial \omega} - \beta^T \frac{\partial \bar{T}}{\partial \dot{q}} = \beta^T \tau \tag{22}
\]
where the $n \times n$ matrix $\gamma$ of (22) is given as

$$\gamma = \begin{bmatrix} \xi_{11} & \cdots & \xi_{1n} \\ \vdots & \ddots & \vdots \\ \xi_{n1} & \cdots & \xi_{nn} \end{bmatrix} = \begin{bmatrix} \omega^T \beta^T \frac{\partial \alpha}{\partial q_1} \\ \vdots \\ \omega^T \beta^T \frac{\partial \alpha}{\partial q_n} \end{bmatrix}$$

(23)

and

$$\xi_{ij} = \omega^T \beta^T \frac{\partial \alpha_{ij}}{\partial q}$$

(24)

Note that $\frac{\partial \alpha}{\partial q_i}$ is a square matrix, in which each element $\alpha_{ij}$ are differentiated with respect to $q_i$, whereas $\frac{\partial \alpha}{\partial q}$ is a column vector in which the element $\alpha_{ij}$ is differentiated with respect to each of the generalized coordinates.

The kinetic energy differentiated with respect to the velocity constitutes the momentum of the system in question. Thus

$$\dot{p} = \frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}} \right)$$

(25)

where $p$ is the momentum of the quasi states, i.e., the momentum expressed in terms of the vehicle body fixed reference frame. Going back to (20), we find that

$$\frac{\partial T}{\partial \omega} = B^T \cdot \omega$$

(26)

Inverting (26) and substituting $p = \dot{\omega} \cdot \dot{\omega}$, yields

$$\omega = B^{-1} \cdot \dot{p}$$

(27)

We also find, by comparing (22) and (25), that

$$\dot{\omega} = \beta^T \cdot \omega + \frac{1}{2} \beta^T \cdot \dot{\omega} \cdot \frac{\partial B}{\partial q} + \beta^T \cdot r$$

(28)

$$\dot{\omega} = \triangle \cdot f(q, \omega) + \beta^T \cdot r$$

where we have used that

$$\frac{\partial T}{\partial \omega} = \frac{1}{2} \omega^T \cdot \frac{\partial B}{\partial q} \cdot \omega$$

(29)

and

$$\omega^T \cdot \frac{\partial B}{\partial q} \cdot \omega = \begin{bmatrix} \omega^T \frac{\partial B_{11}}{\partial q_1} \\ \vdots \\ \omega^T \frac{\partial B_{nn}}{\partial q_n} \end{bmatrix}$$

(30)

Note that, in the case of the marine vehicle, the system mass-inertia matrix is not a function of the generalized coordinates, so $\frac{\partial \dot{q}}{\partial \dot{q}} = 0$. We have however included the expression because this, in general, will not be the case when e.g. a crane is added to the system.

Combining (27) and (28) we obtain a state space model describing the basic dynamics of the vehicle, as

$$\omega = B^{-1} \cdot \dot{p}$$

(31a)

$$\dot{q} = \beta \cdot \omega$$

(31b)

$$\dot{p} = f_p(q, \omega) + \beta^T \cdot r$$

(31c)

We have now derived state space equations for the vehicle. In the following section this formulation is expanded to also include crane of manipulator dynamics.

With the state space equations for the vehicle derived, we can, in the following section expand these to include the crane or manipulator dynamics.

III. EXPANDING THE MODEL TO INCLUDE CRANE AND MANIPULATOR DYNAMICS

The system, now defined as the vehicle and the cranes or manipulators, move in $n = 6 + k$ degrees of freedom, where the vehicle move in 6 degrees of freedom, and the equipment in $k$. In case of lower pair jointed, open chain structured equipment, this means that the equipment have $k$ joints. For such equipment, the obvious choice for generalized coordinates are the joint displacements, denoted $\theta = [\theta_1, \theta_2, ..., \theta_k]^T$. The system vector of generalized coordinates are thus the $n \times 1$ vector $q = [(r_0^b_0)^T, \Theta^T, q_e^T]^T$. The quasi coordinates of the equipment are defined simply as the rate of the generalized coordinates of the equipment, such that the system vector of quasi-coordinates are $\omega = [(v_0^b_0)^T, (\omega_0^b_0)^T, q_e^T]^T$. With these augmented vectors of generalized coordinates and quasi-coordinates, it is necessary to augment the transformation matrices $\alpha^T$, and $\beta$. Recall that we had $\omega = \alpha^T \cdot \dot{\theta}$. Using the expression (14), together with the notation $\dot{q}_e = \dot{I}_k \cdot \dot{q}_e$, we find that the augmented $n \times n$ transformation matrix $\alpha^T$ is given as

$$\alpha^T(q) = \begin{bmatrix} R^b_0 & 0_{3 \times 3} & 0_{3 \times k} \\ 0_{3 \times 3} & T_{\Theta}^{-1} & 0_{3 \times k} \\ 0_{k \times 3} & 0_{k \times 3} & I_{k \times k} \end{bmatrix}$$

(32)

The augment inverse transformation matrix is then

$$\beta(q) = (\alpha^T)^{-1} = \begin{bmatrix} R^b_0 & 0_{3 \times 3} & 0_{3 \times k} \\ 0_{3 \times 3} & T_{\Theta} & 0_{3 \times k} \\ 0_{k \times 3} & 0_{k \times 3} & I_{k \times k} \end{bmatrix}$$

(33)

Before deriving the equations of motion, we shall investigate the kinematics of the system. In particular, we seek to find expressions for the velocity of the centre of gravity for each of the equipment bodies as functions of the generalized coordinates and the quasi-coordinates, in order to find an expression for the system kinetic energy. To this purpose it is necessary to find expressions for the coordinates of each of the bodies’ centre of gravity, relative to the preceding joints and the body fixed reference frame.

Figure 2 shows some equipment with an open chain structure, e.g. a robotic manipulator. In this case, there are two revolute joints and one prismatic joint. In each joint, there is a reference frame attached to the corresponding body such that body $i$ is attached to reference frame $i$. If joint $i$ is a revolute joint, body $i$ rotate about the vector $e_i$, and if joint $i$ is a prismatic joint, body $i$ displace along the vector $e_i$. For the sake of convenience, we place the reference frames such that the rotation or displacement of joint $i$ takes place about or along one of the principal axis of the local reference frame.

In the following, we assume that the location of the centre of gravity of each link relative to the link reference frame origin is known. For link $i$ these coordinates are denoted $r_{ci}$ of $i$. We also define the coordinates of joint $i + 1$ relative to joint $i$, in terms of reference frame $i$, as $r_{i+1,i}^i$. In the case when joint $i$ is a revolute joint, these coordinates are constant, and in the case of prismatic joints, the coordinates are dependent on the displacement $q_{e_{i+1}}$. In order to find the coordinates $r_{i+1,i}$, in this case, we define the coordinate $r_{e_{i+1},i}$ as the point where reference frame $i + 1$ is located for $q_{e_{i+1}} = 0$, relative to
A. Differential Kinematics

Both the linear and angular velocities of the various bodies of a chain of linked rigid bodies situated on a marine vehicle, are explicitly dependent on the velocity of the vehicle and the rates of the preceding joints. We define the contribution to the linear velocity of the centre of gravity of body $i$ from the linear velocity of the vehicle as

$$v_{cgi}^{(v_{b}/0)} = \dot{v}_{b}/0 = I_{3 \times 3} \dot{v}_{b}/0$$

where the superscript in parenthesis denotes from where the given contribution comes.

The contribution to the same velocity, by the angular velocity of the vehicle is defined as

$$v_{cgi}^{(\omega_{v}/0)} = \omega_{b}/0 \times r_{cgi}/b$$

where $r_{cgi}/b$ is the coordinate of the center of gravity of body $i$ relative to the origin of the vehicle body fixed reference frame. The contribution to the linear velocity of the $i$-th center of gravity from the rate of joint $p$ for $p \leq i$, depends on whether the joint is revolute or prismatic. We define

$$v_{cgi/p}^{(q_{ep})} = \mathcal{J}_{q_{ep}}^{v_{cgi}} \dot{q}_{ep}$$

where $r_{cgi/p}$ is the coordinate of the center of gravity of body $i$ relative to the origin of reference frame $p$, and $e_p$ is the vector about, or along, which body $p$ revolves or translates, respectively, in terms of the vehicle body fixed reference frame. In the case where $p > i$, the contribution is zero. Using (37) through (39), we can find the linear velocity of the center of gravity of link $i$, expressed in terms of the vehicle body fixed reference frame, as a function of the generalized coordinates and the quasi-coordinates, as

$$v_{cgi/0} = \mathcal{J}_{q_{ep}}^{v_{cgi}} \dot{q}_{ep}$$

where the dimensions of the zero matrix $0$ is $3 \times (k - i)$.

We now proceed to find the various contributions to the angular velocity of body $i$. There is no contribution to this velocity from the linear velocity of the vehicle. Thus, we can define

$$\omega_{i}^{(v_{b}/0)} = 0_{3 \times 3}$$

The contribution from the angular velocity of the vehicle can be formulated as

$$\omega_{i}^{(\omega_{v}/0)} = I_{3 \times 3} \omega_{b}/0$$

The contribution from the angular velocity of the vehicle can be formulated as

$$\omega_{i}^{(\omega_{v}/0)} = I_{3 \times 3} \omega_{b}/0$$
Finally, the contribution to the angular velocity from the joint displacement rate $\dot{q}_{ep}$, given that $p \leq i$, is

$$
\omega_i(\dot{q}_{ep}) \triangleq J_i^{\omega} \dot{q}_{ep}
$$

$$
\Rightarrow J_i^{\omega} = \begin{cases} e_p^b, & \text{for revolute} \\ 0_{3 \times 1}, & \text{for prismatic} \end{cases}
$$

The total angular velocity of body $i$ of the equipment, can be found by taking the sum of all contributions stated in (41) through (43) as

$$
\omega_i^{(\theta)} = \begin{bmatrix} J_{\omega_1}^{\omega}, & J_{\omega_2}^{\omega}, & J_{\omega_i}^{\omega}, & \ldots, & J_{\omega_{k_i}}^{\omega}, & 0 \end{bmatrix} \omega
$$

(44)

where the zero matrix is of dimension $3 \times (k - i)$. We define the $6 \times 1$ vector $v_i = [(v_{\omega_1}^b)^T, (\omega_i^{(\theta)})^T]^T$, where the linear and angular velocity of the center of gravity of body $i$ are collected. Furthermore, we define the $6 \times n$ geometric Jacobian matrix for the velocity of body $i$ as

$$
J_i(q) = \begin{bmatrix} J_i^{v}(q) \\ J_i^{\omega}(q) \end{bmatrix}
$$

(45)

Using this, a compact expression for the velocity of the center of gravity for body $i$ is

$$
v_i = J_i(q)\omega
$$

(46)

These kinetic relations can further be used in order to derive kinetic energy expressions for the system.

B. Kinetic Energy of System

The kinetic energy the system can be found by taking the sum of all contributions from each body in the system. In (20), the contribution to the total kinetic energy from the vehicle is found. It is however necessary to augment this expression, as $q$ and $\omega$ have been augmented. This is achieved by augmenting the geometric Jacobian matrix found in (19) to

$$
J_b = \begin{bmatrix} J_v^b & 0_{3 \times k} \\ J_\omega^b & 0_{4 \times k} \end{bmatrix}
$$

(47)

in order to make it compatible to the new vector of quasi-coordinates.

The kinetic energy of body $i$ of the crane or manipulator can, as for the vehicle, be found as

$$
\mathcal{T}_i(q, \omega) = \frac{1}{2} \omega^T J_i^{T}(q) \begin{bmatrix} M_i & 0_{3 \times 3} \\ 0_{3 \times 3} & I_i^b \end{bmatrix} J_i(q)\omega
$$

(48)

where $M_i = m_i I_{3 \times 3}$, $m_i$ is the mass of body $i$, and $I_i^b = R_i^b I_i R_i^b$ is the inertia tensor of body $i$, expressed in terms of the vehicle body fixed reference frame. The matrix $I_i$ is the locally expressed inertia tensor, and $B_i(q)$ is the equipment body $i$ mass-inertia matrix, which also is symmetric and positive definite.

To find the system kinetic energy, we take the sum of all contributions as

$$
\mathcal{T}(q, \omega) = \mathcal{T}_b(q, \omega) + \sum_{i=1}^{k} (\mathcal{T}_i(q, \omega))
$$

$$
= \frac{1}{2} \omega^T \left( B_b + \sum_{i=1}^{k} (B_i(q)) \right) \omega
$$

(49)

where the symmetric and positive definite system mass-inertia matrix $B(q)$ is the sum of the individual bodies mass-inertia matrices.

Using the equations (27) and (28), we find a state space model for the complete system as

$$
\begin{align*}
\dot{\omega} &= B^{-1} p \\
\dot{q} &= \beta \omega \\
\dot{p} &= f_p(q, \omega) + \beta^T \tau
\end{align*}
$$

(50)

The derived state space model describing the system can now be implemented in the bond graph framework.

IV. BOND GRAPH IMPLEMENTATION

We now have a set of equations describing the basic dynamics of the system, i.e., the dynamics of the system related to the kinetic energy of the system of bodies. This set of equations is well suited for implementation in the bond graph language. After creating a bond graph template of the system, i.e., implementing (50), interfaces between the template and subsystems are discussed in a general manner, before we, in the next section, introduce a case study, where examples of such subsystems and interfaces are demonstrated. At this point, gravity forces and restoring forces, along with other subsystems, are included.

A. Basic Model

The equations in (50) can be implemented in a bond graph as shown in figure 3. The set of equations is dependent on the generalized coordinates $q$, the quasi coordinates $\omega$, and the momentum $p$. The implementation to the left in figure 3, shows three vector power bonds sharing the same 1-junction. By letting the effort $e_1 = \dot{p}_1$, and the flow $f_2 = \omega_2$, be input ports to the IC-field, we seek to find expressions for the outputs $\dot{p}_2$ and $\omega_1$. Note that the subscript notation in this figure does not indicate certain elements of the vector, but the numbers assigned to the power bonds. As all three power bonds are connected to the same 1-junction, we have that $\omega_1 = \omega_2 = \omega$ and $\dot{p}_1 = \dot{p}_2 + \beta^T \tau = \dot{p}$. Thus, the constitutive relations for the IC-field are

$$
\begin{align*}
\omega_1 &= B^{-1}(q) \dot{p}_1 \\
\dot{p}_2 &= f_p(q, \omega_1)
\end{align*}
$$

(51)

where the vector of generalized coordinates is found by integrating the equation

$$
\dot{q} = \beta \omega
$$

(52)
In order to conveniently develop and interface extensions to this basic model, we partition the quasi-coordinate vector into the linear velocity of the vehicle, \( \omega_{b/0} \), the angular velocity of the vehicle, \( \omega_{v} \), and the joint rates of the equipment \( q_e \). Furthermore, it might be convenient to partition the vector of joint rates into \( k \) separate velocities \( \dot{q}_{e1}, \dot{q}_{e2}, \ldots, \dot{q}_{ek} \). We can now create separate 1-junctions, representing each of these velocity components, and connect each to the IC-field as shown to the right in figure 3.

B. Connectivity

As can be seen in figure 3, the basic template can be interfaced by a subsystem setting an effort expressed as a generalized force in terms of quasi-coordinates, i.e., \( \beta^T \tau \). The basic template then responds with a flow in terms of quasi-coordinates, i.e., \( \omega \). For most purposes however, the modeller does not need to consider this explicitly. Consider for example two subsystems, the first exerting a force \( F_p \) at the point \( p \), and the second exerting a torque \( T_k \) at a point \( k \) on the system.

The procedure for transforming this force and torque into the vectors \( \beta^T \tau_p \) and \( \beta^T \tau_k \) of generalized coordinates in terms of the quasi-coordinates is straightforward within the bond graph framework. This can be achieved by placing a 1-junction representing a linear velocity \( \nu_p \) for the force, and a one junction representing \( \omega_k \) for the torque, and then connecting the subsystems directly to the respective 1-junctions. The relations between the 1-junctions representing \( \nu_p \) and \( \omega_k \), and the quasi-coordinates can always be made by a modulated transformer as shown in figure 4. The constitutive relations for the modulated transformers are

\[
\nu_p = J_p(q) \omega \quad \beta^T \tau_p = J_p^T(q) F_p
\]

and

\[
\omega_k = J_k(q) \omega \quad \beta^T \tau_k = J_k^T(q) T_k
\]

where the matrices \( J_p(q) \) and \( J_k(q) \) can be found in a similar manner as have been done in section III-A. Figure 4 illustrates this concept. Gravity and buoyancy forces can be connected in this manner.

In the following, a case study, utilizing the bond graph template, along with other subsystems to demonstrate connectivity, will be presented.

V. CASE STUDY - OFFSHORE INSTALLATION VESSEL WITH CRANE

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>SHIP AND CRANE PARAMETERS.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td>Value</td>
</tr>
<tr>
<td>Length of ship</td>
<td>107 m</td>
</tr>
<tr>
<td>Width of ship</td>
<td>22 m</td>
</tr>
<tr>
<td>Height of ship</td>
<td>10 m</td>
</tr>
<tr>
<td>Mass of ship</td>
<td>2350 tons</td>
</tr>
<tr>
<td>Height of crane link 1</td>
<td>7 m</td>
</tr>
<tr>
<td>Mass of crane link 1</td>
<td>10 tons</td>
</tr>
<tr>
<td>Length of crane link 2</td>
<td>15 m</td>
</tr>
<tr>
<td>Mass of crane link 2</td>
<td>6 tons</td>
</tr>
<tr>
<td>Length of crane link 3</td>
<td>10 m</td>
</tr>
<tr>
<td>Mass of crane link 3</td>
<td>3 tons</td>
</tr>
</tbody>
</table>

In the previous sections of this paper a generic framework for bond graph implementation of the interconnected dynamics of marine vehicles and equipment consisting of multiple rigid bodies, such as manipulators and cranes, was derived. We have previously argued that one of the advantages of implementing this framework in bond graphs is that this facilitates well structured and well defined interfacing with models of relevant subsystems. In this section, a case-study is presented in order to demonstrate this. In particular, a simulation model of an offshore installation vessel with a three degrees of freedom heavy-duty crane mounted on the after-deck, as shown in figure 5, is presented. The intention is not to provide a state of
the art marine vessel simulator, but rather to demonstrate how a vessel simulator with interconnected vessel-crane dynamics can be built based on the frame-work. In other words, we seek to demonstrate how the basic rigid-body dynamics of the vessel can be placed in an environment, i.e., how environmental forces can be connected to the model, and how the vessel and the crane can be equipped with relevant equipment such as actuators and controllers. The following subsystems are modelled and interfaced to the basic rigid body dynamics: (i) gravitational -and buoyancy forces, (ii) environmental forces, (iii) added mass and hydrodynamic damping, (iv) a thruster system for the vessel with simple thruster controllers, (v) a DP-control system providing reference signals for the thruster system, also including a nonlinear passive observer and a reference model, (vi) a wire including a payload connected to the crane, and finally, (vii) simple actuators as well as a control system for the crane. An overview of the developed bond graph model is presented in figure 6.

The model is simulated using the 20-sim software [21]. We do, however, stress that the bond graph model can be simulated in any software supporting scripting since the bond graphs easily provides the state equations. One alternative, would for example be to extract equations of motion from figure 6 by hand and integrate them using Python, C, Matlab, or any other software capable of simulating a set of first order ordinary differential equations. Another alternative would be to transform figure 6 into a block diagram and use Matlab Simulink to simulate the system. An advantage with software that supports bond graphs, is that one avoids the tedious task of extracting the equations by hand, or transforming the bond graph into a block diagram.

For simplicity, the ship is modeled as a rectangular barge. The main dimensions for both the ship and the crane are summarized in table I.

A. Gravitational and Buoyancy Forces

Restoring forces are the forces and torques resulting from the the weight and buoyancy forces acting on the vessel. In other words, these are the forces and torques which would have been derived from the potential energy function, had it been included when deriving the Lagrangian equations. The linear restoring force, i.e., the restoring force associated to the linear motion of the vehicle, is the resulting force from the difference between the weight and the buoyancy force, while the torques appear when the centre of gravity and the centre of buoyancy are not aligned along a vertical line.

The ship is modelled as a rectangular barge, and the displaced volume is assumed to be given as $A_w z_d$, where $A_w$ is the waterline area and $z_d$ is the draught. Then, the buoyancy force can be expressed as

$$f_b^0 = A_w \rho_w g z_d$$

(55)

Note that $\rho_w$ is the density of the water and is used consistently during the whole case study. In this case, energy will be stored as a function of the vertical position of the vehicle relative to the water surface, and as such, a compliance element is the natural choice for bond graph implementation. The weight of the vessel is implemented as an effort source with the constant effort $e = [0, 0, mg]^T$. The linear restoring forces are expressed in terms of the inertial reference frame, while the rigid-body-system is expressed in terms of $b$. Therefore, a rotation transformation is made between the linear restoring and the 1-junction representing $v_{b,0}$, as seen in figure 6.

The restoring torques acting on the vehicle body are denoted $\tau^b_{R, b}$. These torques are expressed in terms of the vehicle body fixed reference frame and can in general be found as [5]

$$\tau^b_{R, b} = r^b_{cg/b} \times R^b_0 f^0_b + r^b_{cb/b} \times R^b_0 f^0_c$$

(56)

where $r^b_{cg/b}$ and $r^b_{cb/b}$ are the coordinates of the the vehicle centre of gravity and centre of buoyancy relative to the origin of the vehicle body fixed reference frame, and $f^0_b$ and $f^0_c$ are the weight and buoyancy of the vehicle. Energy will be stored as a function of the vehicle displacement due to the restoring torques, and the compliance element is thus a suitable implementation. In figure 6, the restoring torque of the vehicle body is represented by the C-element connected to the 1-junction representing the body fixed angular velocity.

In order to include weight to the crane links we place 1-junctions representing the linear velocity of the centre of gravity for each link. To the right in figure 6 it is shown how the quasi-coordinates can be used in order to find these velocities using the transformation given in (40). The gravity forces are modelled as the rightmost effort sources. We now proceed to include added mass in the model.

B. Added Mass and Hydrodynamic Damping for the Vessel

The added mass can be shown to be a function of excitation frequency [4]. In this case study however, it is assumed to be constant and frequency independent, which according to [13, ch. 6] is a good assumption in manoeuvring theory. The added mass can be shown to be a function of the vehicle relative to the water surface, and as such, a compliance element is the natural choice for bond graph implementation. The weight of the added mass can be shown to be a function of excitation frequency [5, p. 119], be further simplified
by assuming that the off-diagonal elements are negligible, such that

\[
A_{11} = \begin{bmatrix}
X_\dot{u} & 0 & 0 \\
0 & Y_\dot{v} & 0 \\
0 & 0 & Z_\dot{w}
\end{bmatrix}, \quad A_{12} = A_{21} = 0
\]

(59)

\[
A_{22} = \begin{bmatrix}
K_\dot{p} & 0 & 0 \\
0 & M_\dot{q} & 0 \\
0 & 0 & N_\dot{r}
\end{bmatrix}
\]

Here, \( X_\dot{u} \), is the added mass in surge due to motion in the surge direction, \( Y_\dot{v} \), added mass in sway due to motion in the sway direction and so forth. The Coriolis and centrifugal matrix due to added mass can be found as [5]

\[
C_A(\omega) = \begin{bmatrix}
0 & -S(A_{11}v_{b/0}^b + A_{12}\omega_{b/0}^b) & 0 \\
-S(A_{11}v_{b/0}^b + A_{12}\omega_{b/0}^b) & 0 & 0
\end{bmatrix}
\]

(60)

where \( 0 \) are \( 3 \times 3 \) matrices, and \( S(x) \) is the cross product operator.

The hydrodynamic damping force \( \tau_{f_\dot{u}}^b \), and torque \( \tau_{f_\dot{v}}^b \), acting on a marine vessel in terms of the body fixed reference frame can be expressed as

\[
\tau_{f_\dot{u}}^b = D_{NLv}(v_{b/c}^b) + D_{Lv}v_{b/c}^b
\]

\[
\tau_{f_\dot{v}}^b = D_{NLw}(\omega_{b/0}^b) + D_{Lw}\omega_{b/0}^b
\]

(61)

where \( v_{b/c}^b \) is the velocity of the vessel relative to the velocity of the water particles due to current, \( D_{NLv}(v_{b/c}^b) \) and \( D_{NLw}(\omega_{b/0}^b) \) are non-linear damping forces, and \( D_{Lv} \) and \( D_{Lw} \) are diagonal matrices of linear friction coefficients. Using the linear velocity of the vessel relative to the current velocity rather than the velocity relative to the inertial reference frame, automatically generates the linear forces due to current. The non-linear damping forces can comprise e.g. damping due to vortex shedding, radiation induced potential damping and wave drift damping [5]. In this case study, only non-linear vortex shedding forces and linear skin friction forces and torques are considered. The vortex shedding forces are assumed to be given as

\[
D_{NLv}(v_{b/c}^b) = \frac{1}{2}C_D\rho w \left( v_{b/c}^b \right)^T A_p |v_{b/c}^b|
\]

(62)

where \( C_D \) is drag coefficient and \( A_p = \text{diag}(A_u, A_v, A_w) \), and \( A_u, A_v, \) and \( A_w \) are the projected underwater areas in surge, sway and heave, respectively. The linear skin friction forces and torques are given as in (61), with diagonal coefficient matrices. The bond graph implementation of the friction forces acting on the linear velocity of the vessel can be seen as the R-element connected to \( v_{b/0}^b \) in figure 6, and the friction forces acting on the angular velocity can be seen in the same figure as the R-element connected to the 1-junction representing \( \omega_{b/0}^b \).

C. Wave Excitation Forces

A ship is excited by many different environmental forces, such as forces due to the dynamic pressure field generated by waves, radiation forces, diffraction forces, and second order effects due to irregular sea. In this case study potential wave theory is used to calculate the wave induced forces and torques acting on the ship [4].
In linear wave theory the wave potential for a sine wave propagating along the x-axis is given as

$$\Phi = \frac{g\zeta}{\omega} e^{kz} \cos(\omega t - kx + \epsilon) \quad (63)$$

where $\zeta$ is the wave amplitude found from the Jonsvap wave spectrum [4, Chapter 2, p. 25], $\omega$ is the wave frequency, $g$ is the acceleration of gravity, $k$ is the wave number, $x$ is the horizontal propagation of the wave, $z$ is the vertical distance relative to the surface with negative value below the surface and $\epsilon$ is a random phase angle. From the wave potential given in (63) the dynamic pressure field generated by a given wave component can be derived and expressed as

$$p_D = \rho_w \frac{\partial \Phi}{\partial t} = \rho_w g \zeta e^{kz} \sin(\omega t - kx + \epsilon) \quad (64)$$

where $\rho_w$ is the density of the water. A realistic sea state is irregular, containing a continuum of wave components with different frequencies. In this study all wave components are assumed to propagate from the north. By using superposition, the dynamic pressure field over the wet surface of the vessel can be derived and expressed as

$$p_D = \sum_{i=1}^{N} \rho_w g \zeta_{i,n} e^{k_{i,n}z} \sin(\omega_{i,n} t - k_{i,n} x + \epsilon_{i,n}) \quad (65)$$

In general, the excitation forces are derived by integrating the dynamic wave pressure field over the wet surface of the vessel and including diffraction forces. In this case study, only the bottom of the vessel is considered as wetted. The excitation forces in surge, sway and heave are in [4] given as

$$F_i = - \int_S \rho_w n_{s,i} n_{j} ds + A_{i,j} a_{i} + A_{i,2n_2} + A_{i,3n_3} \quad (66)$$

where $S$ is the wet surface of the ship, $n_{s,i}$ is an unit vector orthogonal to the surface for an excitation force in the $i$-direction, $A_{i,j}$ are the added mass terms and $a_{i}$ is the wave acceleration in the $i$-direction. Note that when the waves propagate with the x-axis, $a_{y} = 0$. Expressions for $a_{x}$ and $a_{z}$ can be derived based on the wave potential as done in [4]. The excitation torques roll, pitch and yaw are calculated based on the excitation forces, as will be seen shortly. However, it is not always easy to find algebraic expressions for the final excitation forces and torques. The geometry of the wetted surface is often complex, making it hard to find good integration limits, especially if the heading of the ship is not pointing in the same direction as the propagating waves. This would require a transformation of the integration limits dependent on the wave encounter angle. To avoid both problems, the pressure field can be integrated numerically over the wetted vessel surface. By dividing the wetted surface into small elements it is possible to find approximations of the excitation forces and torques as sums of contributions from each small element. Figure 7 shows how the wetted surface, (in this case the bottom area of the vessel), is divided. In the figure $\beta$ is the angle between the heading of the vessel and the propagating waves. From this division it is more or less straightforward to find an estimate of the excitation forces and torques acting on the ship. When neglecting the end effects, integrating numerically and by working in the body reference frame, the excitation forces and torques from the waves can be expressed as

$$F_{\text{surge,b,k}} = \frac{1}{n_i n_j} \begin{bmatrix} X_{\text{a}} & 0 & 0 \\ 0 & Y_{\text{o}} & 0 \\ 0 & 0 & Z_{\omega} \end{bmatrix} R_{b}^{b} \begin{bmatrix} a_{x}(x_{i}, z_{i}) \\ 0 \\ a_{z}(x_{i}, z_{i}) \end{bmatrix}$$

$$F_{\text{sway,b,k}} = \sum_{k} F_{\text{sway,b,k}}$$

$$F_{\text{heave,b,k}} = \sum_{k} F_{\text{heave,b,k}}$$

$$M_{\text{roll,b}} = \sum_{k} r_{k} x F_{\text{sway,b,k}}$$

$$M_{\text{pitch,b}} = \sum_{k} r_{k} y F_{\text{sway,b,k}}$$

$$M_{\text{yaw,b}} = \sum_{k} r_{k} z F_{\text{sway,b,k}}$$

where $n_i$ and $n_j$ are the numbers of wetted elements in the $i$ and $j$ direction, see figure 7, the subscript $k$ denotes a given wetted surface component, $x_{i}$, $y_{i}$ and $z_{i}$ are the coordinates for the position of each element $k$ related to the pivot centre of the ship. Note that the added mass coefficients are divided by the number of elements the wetted surface were divided into, since they are already given in section V-B. This is only valid if each element is assumed to contribute equally to the total added masses. The excitation forces and torques can be implemented as bond graphs, as shown in figure 6, through modulated effort sources, taking input from the integration algorithm.

Since multiple waves are used to form the sea state, second order effects may be included. In addition, second order mean
drift forces are included, and given as

\[ F_{2d,\text{surge}} = P_{\text{surge}} \sum_{i=1}^{N} \sum_{j=1}^{N} c_{a,i} c_{a,j} \cos((\omega_i - \omega_j)t + \epsilon_i - \epsilon_j) \sin(\psi) \]  
\[ F_{2d,\text{sway}} = P_{\text{sway}} \sum_{i=1}^{N} \sum_{j=1}^{N} c_{a,i} c_{a,j} \cos((\omega_i - \omega_j)t + \epsilon_i - \epsilon_j) \cos(\psi) \]  

(68a)

(68b)

where

\[ P_{\text{surge}} = \frac{1}{2} \rho_w g c_{\text{surge}} \]  
\[ P_{\text{sway}} = \frac{1}{2} \rho_w g c_{\text{sway}} \]  

(69a)

(69b)

are Newman’s approximation coefficients and \( c_{\text{surge}} \in [0, 1] \) and \( c_{\text{sway}} \in [0, 1] \) are approximated reflection coefficients. Note that only in-phase slowly varying drift forces are included since out-of-phase slowly varying drift forces are assumed small and negligible in this case study.

D. Thruster System

The vessel is actuated by two main thrusters in the stern and a tunnel thruster in the bow. The configuration of these are shown in figure 8. The main thrusters generate thrust along the body fixed x-axis, and the tunnel thruster along the body fixed y-axis.

In this section, the thruster models are presented. First, the dynamics of an individual thruster is discussed, before the bond graph connections between the thruster system and the bond graph template is presented.

The literature proposes a number of manners in which to model thrusters and propellers, e.g. [28], [29], [30]. In this case study, the two-state thruster model proposed by [31] is used because it is fairly easy to implement, while at the same time including the ambient flow velocity effect. First a motor delivers a torque \( Q \) to a propeller shaft, which responds with the angular velocity \( \omega_p \). The shaft, motor, and propeller have a moment of inertia denoted by \( J_p \). Furthermore, the total friction force of the propeller shaft bearings and the motor is assumed to be described as \( F_p = d_p \omega_p \). The angular velocity of the propeller transforms into a tangential velocity on the propeller blades, \( u_p = 0.7R \), according to convention [31], where \( R \) is the radius of the propeller. This velocity, together with the incoming velocity \( u_a \) due to the flow through the thruster duct combines into the fluid velocity \( v \) relative to the propeller blades as shown in figure 9. When a propeller blade propagates through the water with the velocity \( v \) relative to the water particles, a lift force \( L \) and a drag force \( D \) results. These are found as

\[ L = \frac{1}{2} \rho_w v^2 A C_L \sin(2\alpha) \]  
\[ D = \frac{1}{2} \rho_w v^2 A C_D (1 - \cos(2\alpha)) \]  

(70)

where \( A \) is the propeller duct cross section area, \( C_L \) and \( C_D \) are the lift coefficient and drag coefficient respectively, and \( \alpha \) is defined in figure 9. The lift force and drag force can in turn be used to find the thrust force \( T \) and the propeller shaft torque \( Q \) as

\[ T = L \cos(\theta) - D \sin(\theta) \]  
\[ Q = 0.7RF_p = 0.7R(L \sin(\theta) + D \cos(\theta)) \]  

(71)

where \( \theta = p - \alpha \), \( p \) is the propeller pitch at \( 0.7R \), and \( F_p \) is the force acting at \( 0.7R \) on the propeller, resulting in the torque \( Q \).

The thrust force acts to accelerate the fluid in the propeller duct and to create a friction force between the water and the thruster duct. The relative velocity between the thruster duct and the water is \( \bar{u}_a \), where \( \bar{u}_T \) is the velocity of the duct. The sum of the inertial force related to the acceleration of the fluid, and the friction force must be equal to the thrust force. Thus, we have

\[ m_w \ddot{u}_a + F_r(\bar{u}_a) = T \]  

(72)

where \( m_w \) is the mass of the fluid in the duct and \( F_r(\bar{u}_a) \) is the friction force. In this case study, it is assumed that the friction force is described as

\[ F_r(\bar{u}_a) = 2\rho_w A |\bar{u}_a| \bar{u}_a \]  

(73)

The above equations are described by the bond graph shown in figure 10, where all three thrusters are represented in the vector formulation. Note that the motors driving the propeller shafts are modelled simply as effort sources. For increased model fidelity, these effort sources could be replaced by variable frequency drive models, in turn powered by e.g., a diesel electric power system.

The forces and torques on the vessel due to the thrusters act in the body fixed reference frame as the thrusters are fixed to the body. As such, the thrust forces should be interfaced to the 1-junction representing \( v_{b/0} \), and the torques to the 1-junction representing \( \omega_{b/0} \). Then, it remains to make
connections between the individual thrust forces and the resulting forces and torques on the vessel. Consider the vector \( T = [T_1, T_2, T_3]^T \), representing the individual thrust forces from each thruster, and the vector \( \tau^b_T = [(\tau^b_{T1v})^T, (\tau^b_{T1\omega})^T]^T \), where \( \tau^b_{T1v} \) and \( \tau^b_{T1\omega} \) are the 3 \times 1 vectors of resulting thrust forces and thrust torques on the vessel respectively. Letting \( l_1 \) and \( l_2 \) be the distances in \( y_b \)-direction from the origin of the body fixed reference frame to the first and second thrusters, and \( l_3 \), the distance in \( x_b \) direction to the third thruster, and finally, \( l_4, l_5 \), and \( l_6 \), the distances in \( z_b \)-direction to the first, second and third thruster, we find that

\[
\begin{align*}
\tau^b_{T1v} &= \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = H_v T \\
\tau^b_{T1\omega} &= \begin{bmatrix} 0 & 0 & l_6 \\ l_4 & l_5 & 0 \\ l_1 & -l_2 & l_3 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = H_\omega T
\end{align*}
\]

Consider again the thruster velocity component \( u_{T1v} \), this time specified for the \( i \)-th thruster, in the direction in which the thruster in question produces thrust. There are two contributions to this velocity component; the contribution \( u_{T1v}^{(v)} \) from the linear velocity of the vessel, \( v^b_{0/0} \), and the contribution \( u_{T1v}^{(\omega)} \) from the angular velocity of the vessel, \( \omega^b_{0/0} \). Collecting the velocity components for each of the three thrusters in to a 3 \times 1 vector, we can write \( u_T = u_T^{(v)} + u_T^{(\omega)} \). The contribution from the linear velocity and the angular velocity of the vessel can be found as

\[
\begin{align*}
u_T^{(v)} &= (H_v)^T v^b_{0/0} \\
u_T^{(\omega)} &= (H_\omega)^T \omega^b_{0/0}
\end{align*}
\]

respectively. Thus, the thruster system and the vessel can be connected as shown in figure 6. In the next section, control of the thrusters are provided through a DP-control system.

E. DP-control system

The objective of the DP-control system is to provide reference signals for the thrusters such that the vessel is controlled in surge, sway and yaw. The DP-system consists of a second order reference model, smoothing the position and yaw angle set point into a reference signal, a position controller calculating a desired thrust vector, a thrust allocation algorithm using the desired thrust vector to allocate a desired thrust force for each thruster, local thruster controllers that realize the thrust commands, and a non-linear passive observer in order to filter out high frequency components of the position and angle measurements in addition to the corresponding rates. The overall control system layout for the ship is illustrated in figure 11. A detailed survey of DP-control systems and different state-of-the-art techniques for the different subsystems of the DP-control system can be found in [32].

The reference model is implemented as a second order filter with velocity saturation, which takes a reference position, \( x_r \), and provides the filtered position \( x_d \) and the velocity \( \dot{x}_d \) as input to the DP-controller. More details of this implementation can be found in [5]. The DP-controller then compares these reference states to corresponding states from the vessel observer, employing a PID control-law that calculates a desired thrust vector in terms of the inertial reference frame. This can be transformed into the body-fixed frame by using the rotation matrix \( R^0_b \). However, in the control system, we are only concerned with the position in the horizontal plane and the yaw angle, and therefore the transformation is simplified to \( R^0_b(\phi = 0, \theta = 0, \psi) = (R^0_b(\psi))^T \)

The control error is given as

\[ e = x_d - \hat{x} \]

and the integral of the error is given as

\[ e_I(t) = \int_{t_0}^{t} e(t) dt \]

where \( \hat{x} \) is the position and yaw angle estimates. The derivative of the error is given as

\[ e_D = \dot{x}_d - \dot{\hat{x}} \]

The control forces \( \tau^0_c \) given in the inertial reference frame is then

\[ \tau^0_c = K^0_p e + K^0_D e_D \]

where \( K^0_p \) and \( K^0_D \) are the control gain matrices.

The thrust forces given in the body reference frame is

\[ \tau^b_c = (R^0_b(\psi))^T \tau^0_c \]

This thrust vector command is provided as input to the thrust allocation in order to find a thrust force reference for each thruster. A survey of different methods for thrust allocation is provided in [33]. In this case-study, however, the problem is rather trivial because the vessel is not over-actuated, (i.e., there are three thrusters and three degrees of freedom that we seek to control). As such, the problem can be solved by multiplying the thrust command by a reduced version, \( \hat{H} \), of the thrust allocation matrix \( [(H_v)^T, (H_\omega)^T]^T \), where only the relevant degrees of freedom for control are extracted. This gives the relation

\[ \tau^b_c = \hat{H} \tau^0_c = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ l_1 & -l_2 & l_3 \end{bmatrix} \tau^0_c \]

The thrust can then be allocated by inverting (81).

The thrust command for each thruster is realized by local thruster controllers. These are controllers that in reality are hard to design because one generally does not have access to measurements of the thrust force. A number of references to recent literature on the local thruster control problem is provided in [32]. In this paper however, we have assumed...
that we have access to perfect measurements of the thrust. It is furthermore assumed that the electrical motors driving the propeller shaft follows perfectly a desired torque, \( Q_d \). Doing so, we can define the error between the commanded thrust and the measured thrust as

\[
e_T = T_d - T_m
\]

and then set the desired motor torques as

\[
Q_d = K_p e_T + K_i \int_0^t e_T dt
\]

The actual motor torque is given as

\[
Q = \text{sat}(Q_d, -Q_{\text{lim}}, Q_{\text{lim}})
\]

where

\[
Q_{\text{lim}} = \frac{P_{\text{max}}}{\omega_p}
\]

and \( P_{\text{max}} \) is the maximum power rating vector and \( \omega_p \) is the speed vector for the thrusters. Thus, \( Q_{\text{lim}} \) is the vector of torque limits for the thrusters. As the thrusters can be saturated, integrator anti-wind up algorithms are implemented for the controllers described in (79) and (83).

The non-linear passive observer is implemented according to [34]. The purpose of this observer is to estimate the high frequency components of the position and angle measurements, i.e., wave frequency and higher, and make sure they do not enter the controller feedback loop. If this is not done, the controller will seek to compensate for the motion induced by the first order wave forces. This however would require immense amounts of power, and is not desirable. Rather, the control system should compensate for only the slowly varying and constant disturbances such that the ship is free to oscillate with the wave frequency. This observer generates estimates of the north and east position, the yaw angle, the low frequency velocities and the bias force. This is achieved by running a simplified dynamic model, and then correcting for the difference in measurements and estimates, i.e., the estimation error. The dynamic model used is given in [34] as

\[
\begin{align*}
\dot{\xi} &= A \hat{\xi} + K_1 \tilde{y} \quad (86a) \\
\dot{r}_{b/0} &= R_z(\psi) \hat{v}_{b/0}^b + K_2 \tilde{y} \quad (86b) \\
\dot{\tilde{b}} &= -T^{-1} \tilde{b} + K_3 \tilde{y} \quad (86c) \\
M \dot{v}_{b/0}^b &= -D \dot{v}_{b/0} + (R_z(\psi))^T \tilde{b} + r_c^b + K_4 \tilde{y} \quad (86d) \\
\dot{\tilde{y}} &= \tilde{y} + C \omega \xi \quad (86e)
\end{align*}
\]

where \( \xi \) is the wave response estimate on position and heading, \( \tilde{y} \) is the position and heading estimate, \( \hat{y} = r_{b/0}^b - \tilde{y} \) is the position estimation error, \( \hat{v}_{b/0}^b \) and \( v_{b/0}^b \) are the estimates on the states \( r_{b/0}^b \) and \( v_{b/0}^b \). The matrices \( K_1 \in \mathcal{R}^{6 \times 3} \), \( K_2 \in \mathcal{R}^{3 \times 3} \), \( K_3 \in \mathcal{R}^{3 \times 3} \), and \( K_4 \in \mathcal{R}^{3 \times 3} \) are tuning parameters. The bias force estimate is denoted \( \tilde{b} \), and \( T \) is the time constant matrix of the bias force, and can also be considered as a tuning parameter. (86a) is a state space representation of the motion component of the ship due to wave forces, driven by the estimation error. \( M \) is the mass matrix of the ship, and \( D \) is in general the linear damping matrix. This damping matrix has been modified to also include nonlinear damping terms, such as \( D(\hat{v}_{b/0}^b) \). The linear wave spectra is characterized by the matrix \( A_w \) given as

\[
A_w = \begin{bmatrix}
0_{3 \times 3} & I_{3 \times 3} \\
-\text{diag}(\omega_0) & -2\text{diag}(\lambda_1 \omega_0)
\end{bmatrix}
\]

where \( \omega_0 \) is the peak frequency in the wave spectra, and \( \lambda_1 \) is a spectra tuning parameter. Finally, the matrix \( C_w = [0_{3 \times 3}, I_{3 \times 3}] \), such that the ship motions induced by the linear wave model is extracted.

**F. Wire model**

In [27] a lumped wire model in two degrees of freedom is presented. The wire is divided into smaller elements and connected like a mass spring damper system in series, as figure 12 shows.

![Fig. 12. Sketch of winch in operation.](image)

The same idea is used here and the only difference is that the wire model is updated to have three degrees of freedom in each node instead of two. The wire is also assumed to be directly connected to the crane and is not to be hoisted or lowered in this case study.

Starting with the wire dynamics, the stiffness of each wire element is given in [27] as

\[
k_w = \frac{EA_w}{L_{we}} = \frac{ED_w \pi}{4L_{we}}
\]

where \( E \) is the elasticity modulus, \( A_w \) is the cross section area of the wire, \( L_{we} \) is the length of each wire element and \( D_w \) is the diameter of the wire. The damping is found by assuming a constant damping ratio,

\[
\zeta = \frac{c_{\text{wire}}}{c_{cr}}
\]

where \( c_{\text{wire}} \) is the damping coefficient, \( c_{cr} \) is the critical damping coefficient and \( \zeta \) is the damping ratio. The wire is assumed to be over-damped, which means that \( \zeta >> 1 \). The critical damping coefficient is given as

\[
c_{cr} = 2m_{we} \sqrt{\frac{k_w}{m_{we}}} = 2 \sqrt{k_w m_{we}}
\]
where $m_{we}$ is the translational inertia of the wire element in air,

$$m_{we} = \rho_{wire}A_w L_{we} \quad (91)$$

where $\rho_{wire}$ is the density of the wire material. By inserting (90) into (89) the damping coefficient can be expressed as

$$c_{wire} = 2\zeta \sqrt{k_{w}m_{we}} \quad (92)$$

The elongation of one wire element is given as

$$r_{we} = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2} - L_0 \quad (93)$$

where $(x_0, y_0, z_0)$ and $(x_1, y_1, z_1)$ is the position of the top and bottom of the wire element, respectively, and $L_0$ is the initial length of the unstretched wire element. The derivative is given as

$$\dot{r}_{we} = \frac{(x_1 - x_0)\dot{x}_1 + (y_1 - y_0)\dot{y}_1 + (z_1 - z_0)\dot{z}_1}{\sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2}} - \frac{(x_1 - x_0)\dot{x}_0 + (y_1 - y_0)\dot{y}_0 + (z_1 - z_0)\dot{z}_0}{\sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2}}$$

$$= \frac{(\dot{x}_1 - \dot{x}_0)r_{t1} + (\dot{y}_1 - \dot{y}_0)r_{t2} + (\dot{z}_1 - \dot{z}_0)r_{t3}}{r_{we}} \quad (94)$$

where

$$r_{t1} = \frac{x_1 - x_0}{\sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2}} \quad (95a)$$

$$r_{t2} = \frac{y_1 - y_0}{\sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2}} \quad (95b)$$

$$r_{t3} = \frac{z_1 - z_0}{\sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2}} \quad (95c)$$

and is the tranformer modulus between the rates in three degrees of freedom and the rate of the wire elongation. By using Morrison's equation both the added mass and the drag forces can be found,

$$F_M = \rho_w A_w L_{we} \begin{bmatrix} C_{I, x} \dot{x} \\ C_{I, y} \dot{y} \\ C_{I, z} \dot{z} \end{bmatrix} + \frac{1}{2} \rho_w D_w L_{we} \begin{bmatrix} C_{d, x} \cos(\theta_1)(\dot{x} - V_{c, x})|\dot{x} - V_{c, x}| \\ C_{d, y} \cos(\theta_1)(\dot{y} - V_{c, y})|\dot{y} - V_{c, y}| \\ C_{d, z} \sin(\theta_3)(\dot{z} - V_{c, z})|\dot{z} - V_{c, z}| \end{bmatrix} \quad (96)$$

where $C_{I,i}$ is the added mass coefficients, $C_{d,i}$ is the drag coefficients, $V_{c,i}$ is the current velocity in each direction, and

$$\theta_1 = -\frac{y_1 - y_0}{z_1 - z_0} \quad (97a)$$

$$\theta_2 = -\frac{x_1 - x_0}{z_1 - z_0} \quad (97b)$$

$$\theta_3 = -\frac{\sqrt{x_1^2 + y_1^2} - \sqrt{x_0^2 + y_0^2}}{z_1 - z_0} \quad (97c)$$

The last contribution to the wire element dynamics are the buoyancy and the gravitational forces, which is given as

$$F_{BG} = [0, 0, (m_{we} - \rho_w A_w L_{we})g]^T \quad (98)$$

By implementing these equation in bond graphs, one wire element can be given as figure 13 shows. In addition the first and last wire elements must be slightly modified. The first wire element must have a connection to the tip of the crane, and the last wire element must have the payload characteristics included in the mass and the drag forces.

**G. Crane Control System and Actuators**

The crane is equipped with hydraulic actuators. In this case study however, these are simplified as effort sources, providing a torque which in turn is commanded from a crane joint position control system. The manner in which the effort
sources are connected to the model is shown in figure 14. Notice that a reaction torque is acting on the second link due to the actuator on the third joint. This is because the second and third link both rotate about the horizontal axis. Also here a reference model for generating a smooth and continuous reference signal is needed, and a similar filter as for the reference model of the vessel is used. Hence, the reference model is given as

\[
\dot{\hat{x}}_d = \text{sat}(\hat{x}_{m}^c, \hat{x}_{M}^c)
\]

where \( M_d \), \( D_d \), and \( K_d \) are tuning matrices for the reference model, \( \hat{x}_{m}^c \) and \( \hat{x}_{M}^c \) are the angular rate limits and \( \hat{x}_d \) is the reference signals given as input to the filter.

The control law for the crane is a Lyapunov stability based control design that enables cancellation of unwanted crane dynamics. The isolated crane dynamics, (i.e., the dynamics of the crane if we do not include the vessel), can, according to e.g. [35], be expressed as

\[
\dot{q}_c = \omega_c
\]

\[
\dot{\omega}_c = B_c^{-1}(q_c)(-C_c(q_c, \omega_c)\omega_c - g_c(q_c) + \tau_c)
\]

By defining the control error vector as

\[
e_{c1} = q_c - q_{cd}
\]

where \( q_c \) is the angle measurement vector and \( q_{cd} \) is the reference angle vector, and

\[
e_{c2} = \omega_c - \dot{q}_{cd}
\]

where \( \omega_c \) is the angular rate vector and \( \dot{q}_{cd} \) is the reference angular vector, it is possible to write the error dynamics as

\[
\dot{e}_{c1} = e_{c2}
\]

\[
\dot{e}_{c2} = B_c^{-1}(q_c)(-C_c(q_c, \omega_c)\omega_c - g_c(q_c) + \tau_c) - \dot{q}_{cd}
\]

Starting with a Lyapunov function candidate given as

\[
V = \frac{1}{2} e_{c2}^T B(q_c) e_{c2}
\]

which is positive definite for \( \forall e_{c2} \neq 0 \), gives

\[
\dot{V} = e_{c2}^T (-C_c(q_c, \omega_c)\omega_c - g_c(q_c) + \tau_c - B(q_c)\dot{q}_{cd})
\]

By choosing

\[
\tau_c = C_c(q_c, \omega_c)\omega_c + g_c(q_c) + B(q_c)\dot{q}_{cd} - K_d e_{c2} + u
\]

the derivative of the Lyapunov function becomes

\[
\dot{V} = -e_{c2}^T K_d e_{c2} + e_{c2}^T u
\]

where \( K_d \) is a positive diagonal matrix. This would stabilize \( e_2 \) when \( u = 0 \), and the new error dynamics become

\[
\dot{e}_{c1} = e_{c2}
\]

\[
B(q_c)\dot{e}_{c2} = -K_d e_{c2} + u
\]

Since it is known that \( e_2 \) is stabilized from the previous Lyapunov function, it is reasonable to believe that \( u \) can be chosen as a PI-controller that stabilizes \( e_1 \),

\[
u = -K_p e_{c1} - K_I \int_0^t e_{c1} dt
\]

This assumption should be verified through simulations. Hence the total control law is then given as

\[
\tau_c = C_c(q_c, \omega_c)\omega_c + g_c(q_c) + B(q_c)\dot{q}_{cd} - K_d e_{c2} - K_p e_{c1} - K_I \int_0^t e_{c1} dt
\]

This case study has been implemented as seen in figure 6 and simulation results comparing with and without crane load are shown in the following section.

VI. SIMULATION RESULTS

In this work the importance of proper modelling of heavy deck equipment closely coupled to the vessel in simulation models for maritime operations have been studied. In this section, we present simulation results from the case-study model derived in the previous section to emphasize the main results and to illustrate the importance of proper modeling. In particular, the same DP-manoeuvre, shown in figure 15 and 16, is performed both with and without a submerged load attached to the crane. In both cases, the reference position is first moved 80 meters northwards from the initial position, while the east and yaw reference is held constant. Then the reference position is moved 20 meters eastwards while the north and yaw reference still are held constant, at 0 and -25°, respectively, before finally, the yaw reference is changed to 25°. Note that all controllers and filters have been tuned to perform well in the case with no load attached to the crane, and the same tuning is used for the case with loaded crane.

The model parameters used are presented in table II.

In such a case study it is important that the wave filter and the control systems are tuned to be robust such that the vessel is able to perform good in both cases and keep its reference position and orientation. If this is achieved, there should not be large differences in the position and orientation when comparing the two cases, and it is believed that the second order mean drift wave forces are much larger than the environmental forces from the submerged wire and the load in this case study, resulting in small differences in power consumption as well. If this is the case, the main differences between the two cases would be reflected in the roll angle and in the wave filter.

Figure 15 shows that the vessel follows its references signals in both cases, and the results seem to overlap. The crane load also cause a static roll angle offset of about -2.5°, having oscillations with an amplitude of about 0.5°, about the same amplitude as in the unloaded crane case. Notice
that these roll oscillations are significantly reduced when the heading is $0^\circ$, and the reduction is largest in the unloaded crane case due to no static roll angle offset. From this figure one can conclude that the implemented controllers and filters introduced in section V seem to perform well. Notice also that the submerged crane load have a considerable effect on the yaw angle oscillations, $\psi$, as can be seen from the actual yaw rate in figure 16. One of the reasons for this is that the force acting on the vessel, due to the submerged load, create a torque about the body fixed $z$-axis because the crane is situated aft of the center of gravity. These results also argue for the use of proper rigid body models when testing control systems and tuning wave filters for vessels doing crane operations. The figure indicates that the filtered velocities and the yaw rate coincide with the measurements.

Figure 16 shows the filtered position- and orientation rates, that are fed to the DP-controller for calculating the derivative controller effects, compared with the actual rates. As can be seen in the figure the surge rates $u$ for the two different cases seem to converge and the wave filter filters out about the same amount of noise in the two cases. However, the same can not be said about the sway rate and the heading rate. The noise in the sway rate has a bit larger amplitude in the loaded case compared to the unloaded case, but it seems like the wave filter is able to perform equally in both cases. The last two

---

**TABLE II**

**Simulation Parameters.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vessel</td>
<td></td>
</tr>
<tr>
<td>Initial position and orientation</td>
<td>[0 m, 0 m, $-25^\circ$]</td>
</tr>
<tr>
<td>Power saturation main thrusters</td>
<td>3.5 MW</td>
</tr>
<tr>
<td>Power saturation tunnel thruster</td>
<td>3.5 MW</td>
</tr>
<tr>
<td>Sea state parameters</td>
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<td>Significant wave height</td>
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<td>Peak period</td>
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<tr>
<td>Northward current</td>
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<td>No. wave components</td>
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<tr>
<td>$\gamma$</td>
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</tr>
<tr>
<td>Lower wave spectra period</td>
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<tr>
<td>Upper wave spectra period</td>
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<tr>
<td>$\Delta_j$</td>
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</tr>
<tr>
<td>Crane and submerged load model</td>
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</tr>
<tr>
<td>Initial and reference joint angles</td>
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</tr>
<tr>
<td>Wire diameter</td>
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</tr>
<tr>
<td>Initial wire length</td>
<td>400 m</td>
</tr>
<tr>
<td>No. wire elements</td>
<td>5</td>
</tr>
<tr>
<td>Payload</td>
<td>100 tons</td>
</tr>
</tbody>
</table>
plots show the heading rate in the two different cases. As can be seen, the noise in the heading rate for the loaded case is significantly increased in comparison to the unloaded case. This, due to the submerged wire and the load. However, even though the wave filter is not able to filter out as much noise in the loaded case in comparison to the unloaded case, the performance of the filter is good and it filters out most of it. Due to these results, it is expected that at least the power consumption for the tunnel thruster, $P_3$, would contain more noise in the loaded case in comparison to the unloaded case.

The power consumption for the two cases is shown in figure 17, where the the total power consumption of each thruster, as well as the total power consumption of the thruster system and the total amount of consumed energy are shown. It is clear from this figure that the submerged load does not increase the energy consumption considerably, but affects the dynamics of the vessel, as will be shown later on. The figure also shows that the largest power peaks come from moving the ship and changing the heading. The case with the loaded crane consumes about 9.3 kWh more than the case without loaded crane, i.e., about 0.7 % difference, and about 1333.5 kWh in total. As expected, there are more oscillations in consumed power in the loaded case, especially for the tunnel thruster, in comparison to the unloaded case, due to the resulting forces from the submerged wire and load. However, the results show that the control laws and the wave filter are well tuned to be robust. Note that the power consumption for the crane system has not been included. It would be interesting in further work to look at energy regenerative control of the crane system in heavy compensation operations, which this model is well suited for.

To be able to pinpoint the differences in the simulation results even better the difference in crane joint angles, ship position and yaw- and roll angles are compared in figure 18. As can be seen in the figure there is a small oscillating difference between the crane joint angles, as one would expect. Note that the crane joint angle measurements for the crane control system have not been filtered before entering the control system. However, the largest differences can be seen in the ship position and orientations. The difference in heading oscillates with an amplitude of about 1 °, indicating that the loaded crane affects the heading of the ship significantly. The same can be said about the north-east position of the ship and the roll angle, as seen in the figure, although not as dramatically for the north-east position. The comparison between the roll angles for the two cases show that the loaded case gets an offset of about 2.5 ° as mentioned earlier. However, this is not the only result that can be obtained from the roll angle measurements. The roll angle is not controlled in any sense in the simulation, and is therefore not directly affected by the control law dynamics. This makes these measurements important when analysing the differences in the dynamics due to the submerged wire and the load. Figure 19 shows a FFT-analysis of the roll angle in the time range $t = [2500, 3000]$ for the two cases. As can be seen in the figure the responses seem to be equal for low frequencies. However, when the is in the range 0.1-0.5 Hz the results show that the roll angle for the unloaded vessel is more affected by the wave effects. This is not surprising since the submerged load in the loaded case acts as a mooring line and adds additional damping to the roll angle. However, the three major peaks that can
be seen in the unloaded case around \(0.12 \, Hz\), \(0.17 \, Hz\) and \(0.27 \, Hz\) can also be seen in the loaded case. These results also argue for that the submerged wire and load dynamics add significant contributions to the vessel dynamics, especially the roll dynamics that are uncontrolled.

It is not surprising that the submerged payload has such an impact on the motion of the vessel, which is also argued for by looking at the wire tension in figure 20, showing that the wire tension oscillates about 1028 kN with an amplitude of about 5 kN. One could perhaps expect that the wire tension would change more due to the change in north- and east position, but the ship moves quite slowly and does not affect the wire tension considerably. However, if the ship had moved faster the wire tension would have changed. Also note that the oscillations are lower when the heading of the ship is \(25^\circ\) compared to when the heading is \(-25^\circ\). This has to do with the orientation of the crane, which affects the roll angle more when the ship has a heading of \(25^\circ\), and since the roll is not controlled, the oscillations in the wire tension would decrease as seen in the figure.

Figure 21 shows the displacement of the submerged wire for a period of 15 seconds. Note that darker color denotes higher simulation time. The figure shows the wire displacement in the time interval when the vessel is moving northwards.

VII. CONCLUSION

In this paper we set out to make a template for developing simulation models of ships doing heavy lift operations using cranes. In order to do this, we formulated the dynamics of the marine vehicle in a compact manner, which allowed for connecting equipment such as cranes and manipulators in a true manner. If this were to be modelled directly without the Lagrangian formalism, challenges related to differential causality would arise. These are solved automatically when developing the Lagrangian equations. Alternative approaches for resolving the differential causalities are to employ so-called brute force techniques, which in general means to introduce some compliance between the rigid bodies. This does however introduce fast time constants which would affect the simulation time significantly. It should be mentioned that the cases presented in this paper were both solved faster than real time. Note however that the simulation speed is affected by the mesh size of the wetted surface in the numeric integration of the wave forces, and the number of wave components used to describe the irregular sea.
In the development of the Lagrangian equations of motion, the associated potential energy and the conservative forces were not included, but rather modeled directly in the bond graph implementation. The case study further illustrated how potential forces, as well as added mass, can be included in the bond graph model without going through the Lagrangian formalism.

Suggestions for how to interface a variety of different subsystems such as environmental forces, thruster models, wire-load model and crane actuators, were presented through
a case study. In addition, a DP-control system, a nonlinear wave filter, and a crane control system were implemented. The purpose of the control systems were to enable comparison of the two cases simulated. In addition, the nonlinear passive wave filter was included in order to get a realistic power consumption.

The simulation results indicated that the crane and wire-load model affected the performance of the ship. However, since the control laws and the wave filter were tuned to be robust, the results that show the effects the submerged wire and the load have on the ship is not as clear in the ship position and orientation. Although, these effects are significant in uncontrolled states such as the roll angle and the unfiltered position and orientation measurements in addition to the corresponding rates that are fed into the wave filter. Figure 17 showed that the total energy consumption of the thruster system was only slightly larger in the case with crane system as compared to the case without, mostly due to the good control laws and the wave filter. However, the results show clearly the importance of running such simulations for testing control laws and filters in various scenarios. Further work may include investigation of the effect of using these types of simulations as input to risk analyses in offshore operations that include heavy lifting.

**ACKNOWLEDGMENT**

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REFERENCES


APPENDIX

BASIC BOND GRAPH THEORY

The basic idea behind bond graph theory is that connected systems interacts between each other through exchange of power. Such a connection contains two signals, namely an effort and a flow. Note that the product between efforts and flows are always power, hence the name convention. For instance, a mass acted upon by a force $F$ sends the velocity $v$ in feedback. Here, the force is an effort and the velocity is a flow. Hence, the power $P$ is calculated as

$$ P = F \cdot v $$

In bond graph theory such systems are said to be connected by power bonds, as shown in figure 22.

![Fig. 22. Two systems connected through a power bond.](image-url)

Note that the orthogonal line in the power bond connecting the two systems denotes which way the effort goes and the half arrow denotes the direction of the positive power flow.

In bond graph theory different building blocks are used. These are summarized in table III. Both the $S_e$ and $S_f$ elements are sources, effort and flow respectively. If hydraulics are modeled then $S_e$ is a pressure source and $S_f$ is a fluid flow source. The $R$ element is describing energy dissipation like friction forces or viscous forces. The $C$ element describes the stored energy in the system, like a spring in a mechanical system or an accumulator in a hydraulic system. Inertia in a mechanical system or an inductor in an electrical circuit is given as an $I$ element. Transformation of efforts and flows between subsystems is usually done by using a $TF$ element. To sum different contributions of effort the $1$-junction is used and to sum different contributions of flow the $0$-junction is used. There is also one more basic element that is not included in the table. This is the gyrator element, $GY$ that transforms flows to efforts and vice versa. This element can be associated
with a generator that gets a rotational velocity, a flow, and transforms it to voltage, an effort.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Relation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_e$</td>
<td>$e = e(t)$, given</td>
</tr>
<tr>
<td>$S_f$</td>
<td>$f = f(t)$, given</td>
</tr>
<tr>
<td>$\rightarrow R$</td>
<td>$e = \Phi_R(f)$</td>
</tr>
<tr>
<td>$\rightarrow R$</td>
<td>$f = \Phi_R^{-1}(e)$</td>
</tr>
<tr>
<td>$\rightarrow C$</td>
<td>$e = \Phi_C^{-1}(\int f dt)$</td>
</tr>
<tr>
<td>$\rightarrow C$</td>
<td>$f = \frac{d}{dt}[\Phi_C(e)]$</td>
</tr>
<tr>
<td>$\rightarrow I$</td>
<td>$f = \Phi_I^{-1}(\int e dt)$</td>
</tr>
<tr>
<td>$\rightarrow I$</td>
<td>$e = \frac{d}{dt}[\Phi_I(f)]$</td>
</tr>
<tr>
<td>$\rightarrow TF \rightarrow 2$</td>
<td>$e_1 = me_2$</td>
</tr>
<tr>
<td>$\rightarrow TF \rightarrow 2$</td>
<td>$f_2 = mf_1$</td>
</tr>
<tr>
<td>$\rightarrow TF \rightarrow 2$</td>
<td>$e_2 = \frac{1}{m}e_1$</td>
</tr>
<tr>
<td>$\rightarrow TF \rightarrow 2$</td>
<td>$f_1 = \frac{1}{m}f_2$</td>
</tr>
<tr>
<td>$\rightarrow TF \rightarrow 2$</td>
<td>$e_1 - e_2 - e_3 = 0$</td>
</tr>
<tr>
<td>$\rightarrow TF \rightarrow 2$</td>
<td>$f_1 = f_2 = f_3$</td>
</tr>
</tbody>
</table>

Table III

Some basic bond graph elements used.

In bond graph theory only two of the basic elements are candidates for state generation. From physics we know that a one dimensional mass-damper-spring system has the position and the velocity as states if only first order differential equations are used. By modeling a mass-damper-spring system using bond graph theory a $C$, $I$ and a $R$ element are used together with 0-junctions, 1-junctions, $S_e$ and/or $S_f$ elements. To get the same states from a bond graph we know that one of the elements must describe the position and one must describe the velocity. Since the $R$-element only describes dissipation of energy it can not contribute to any states. This means that the $C$-and $I$-elements must give the system states. Then it is not surprising that the $C$ element gives the displacement and the $I$-element gives the momentum,

$$q = \int_0^t f dt \quad (112a)$$

$$p = \int_0^t e dt. \quad (112b)$$

For example, a mass damper spring system only affected by gravity is represented through the first order differential equations in bond graph theory as

$$\dot{q} = \frac{p}{m} \quad (113a)$$

$$\dot{p} = -kq - b\frac{p}{m} + mg \quad (113b)$$

where $q$ is the position, $p$ is the momentum, $m$ is the mass, $k$ is the spring stiffness, $b$ is the damping coefficient and $g$ is the acceleration of gravity. By defining $x_1 = q$ and $x_2 = \frac{p}{m}$ we can rewrite the differential equations as

$$\dot{x}_1 = x_2 \quad (114a)$$

$$m\dot{x}_2 = -kx_1 - bx_2 + mg \quad (114b)$$

which is the well known mass-damper-spring system. Figure 23 shows how the mass-damper-spring system would be implemented using bond graphs.

Fig. 23. Bond graph model of mass-damper-spring system.

The reader is referred to [7] for a thorough introduction to bond graphs.