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Title: *Optimization of design and operation of anaerobic digestion reactors*
Abstract:

The sustainable provision of bio-methane plays a key role in the future energy supply and is a promising environmentally friendly solution for waste processing. With rising number and size of biogas power plants process optimization is a vital task. Consequently, in this thesis a 6000 [L/d] swine waste feed into anaerobic digestion reactor will be degraded into biogas, which thesis aim is to optimize process with numerical models and programming skills to run multiple input variables in order to solve optimization problem.

Objective functions for optimization are economic revenue and net present value of process with optimization variables of reactor volume and temperature. Mathematical model for process is (Hill, 1983) method, which is a dynamic simulation model for anaerobic fermentation of biodegradable material. Also more mathematical model for temperature and volume has been made with physical rules such as energy balance and mass transfer. Economic analyses starts with cost estimation for equipment based on pilot reactor and market prices, and then time value of money has been included in calculation. The results contain measurement of payback ability for anaerobic digestion process in various financial criteria (such as internal rate of return); furthermore thesis results indicates an applicability of numerical algorithms in economic optimization performance in anaerobic digestion process where outcome of thesis could be a measurement standard for profitability of different process designs. Finally it is concluded that estimations for process equipment are playing a vital rule in economic performance; therefore cost estimation must be based on more precise data sources. General revenue and internal rate of return shows clear financial feasibility of process and optimization shows sensible added profits in cash flow diagrams.

Telemark University College accepts no responsibility for results and conclusions presented in this report.
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Abstract

The sustainable provision of biofuel plays a key role in the future energy supply and is a promising environmentally friendly solution for waste processing. With rising number and size of biogas power plants process optimization is a vital task. Consequently, in this thesis a 6000 [L/d] swine waste feed into anaerobic digestion reactor to be degraded into biogas, thesis aim is to optimize process with numerical models and programming skills to run multiple input variables in order to solve optimization problem.

Objective functions for optimization are economic revenue and net present value of process with optimization variables of reactor volume and temperature. Mathematical model for process is (Hill 1983) method, which is a dynamic simulation model for anaerobic fermentation of biodegradable material. Also more mathematical model for temperature and volume has been made with physical rules such as energy balance and mass transfer. Economic analyses starts with cost estimation for equipment based on pilot reactor and market prices, and then time value of money has been included in calculation. The results contain measurement of payback ability of anaerobic digestion process in various financial criteria (such as internal rate of return); furthermore thesis results indicates an applicability of numerical algorithms in economic optimization performance in anaerobic digestion process where outcome of thesis could be a measurement standard for profitability of different process designs. Finally it is concluded that estimations for process equipment are playing a vital rule in economic performance; therefore cost estimation must be based on more precise data sources. General revenue and internal rate of return shows clear financial feasibility of process and optimization shows sensible added profits in cash flow diagrams.
Preface

In this thesis economic potential of using manure as feedstock for anaerobic digestion reactor has been reviewed and the results been optimized to show ultimate profit availability of initial pilot design on Foss farm in Skien Norway. Analyses provide surplus energy that can be utilized for on-farm purposes or directly selling manufactured products. Prior to economic analyses and economic numerical optimization algorithm, it is important to construct precise mathematical models for predicting anaerobic digestion process, which has been done previously by Finn Haugen and in this thesis models are the same. Calculation of optimization algorithms and economic analyses uses computer programming such as Excel and Matlab, therefore prior knowledge about both optimization and economic methods in software is recommended. Matlab scripts and Excel sheets are available in appendixes, however to get same results as thesis, it is recommended to follow instructions in order to run programs correctly.

Finally I would like to thank all people in bioengineering team in Telemark University College who provide me useful information, and especially my gratitude goes to my supervisor Finn Haugen and Professor Rune Bakke for their assistance and valuable recommendations. I would also thank my sister for her unlimited support, encouragement and inspiration.

Porsgrunn, 30 May 2014
Arya Haddad
Nomenclature

GHG = Greenhouse gases.
IRR = Internal rate of return.
NPV = Net present value.
TVM = Time value of money.
CAPEX = Investment costs of process.
OPEX = Operational costs of process.
LHV\textsubscript{methane} = Low heating value of methane
P\textsubscript{el} = Electricity energy price
R = Annual discount rate
AD = Anaerobic digestion.
ADM1 = Anaerobic digestion model number 1.
BVS = Biodegradable volatile solids.
CSTR = Continuous stirred tank reactor.
UASB = Up flow anaerobic sludge blanket.
EGSB = Expanded granular sludge bed.
AHR = Anaerobic hybrid reactors.
VFA = Volatile fatty acids.
VS = Volatile solids.
HRT = Hydraulic retention time.
SRT = Solid retention time.
ODE = Ordinary differential equation.
CH4 = Methane.
CO2 = Carbon dioxide.
KWh = Kilowatt hour.
F\textsubscript{feed} [m3/d] = influent or feed flow, assumed equal to effluent flow (constant volume).
F\textsubscript{meth} [L CH4/d] = methane gas flow.
NH3 = Ammonia.
NH4 = Ammonium.
S\textsubscript{bvs} [g BVS/L] = concentration of BVS in reactor.
S\textsubscript{vfa} [g VFA/L] = concentration of VFA acids in reactor.
T\textsubscript{react} [°C] = reactor temperature.
T\textsubscript{feed} [°C] = feed flow temperature.
T\textsubscript{room} [°C] = ambient temperature.
X\textsubscript{acid} [g acidogens/L] = Concentration of acidogens.
X\textsubscript{meth} [g methanogens/L] = Concentration of methanogens.
1 Introduction

Advance processes are developing to improve energy removal in order to recover nutrients and utilize waste; generally this can be done by complex biological technics which is typically a combination of treatment reactors such as aerobic, anaerobic and anoxic. With increasing use of biological processes a need for optimized procedure for process design is crucial. In intelligent engineering design the optimum criteria of cost of profits is considered carefully, while the factors to later criteria are vast set of variables which need to be optimized. These variables can be equipment performance, techniques of processing procedure, arranging of process with optimum sequence and finally physical design condition in the process, besides operating condition is an important aspect that has been noticed for finest process optimization. To develop most cost effective design of process, all parts of plant and operation condition has to be investigated. A vital step into optimization is first step where optimization criteria are establishing based on objective functions for commercial plant and actual process conditions.

Manure management and nutrient recovery are important aspects of anaerobic digestion in cost effective diary process, but typical farm is designed to reduce the costs as much as possible therefore AD system which is considered to be complex process with large initial capital cost must boost economics of process to be practical. Economics of AD system will be enhanced if surplus energy could increase; therefore optimization optimal solution is to decrease the cost of digester system and operation energy cost for example by adding a heat exchanger for recovering energy.

In this thesis optimal design for AD reactor process has been established by using optimization technics for a combination of mathematical models that describe system. Mathematical models are consist of four different yet related subjects, these subjects are temperature model, dynamic AD model, heat exchanger model and finally economic performance model. To find optimum design an objective function has been made with all described models combine together, for optimization objective variables reactor temperature and volume; however in section 6.3, feed flow has been used as objective variable for optimization of methane income. Then optimization problem defined with mathematical expressions with known constraints of function based on realistic assumptions. In this thesis relation of mathematical models are investigated and effects of each variable on model has been examined by brute force optimization method to determine optimal design parameter that leads to most cost effective economic performance. Goal of his thesis is to define mathematical relation to use as objective function for optimization variables such as economic costs that gives results with maximum profits subject to problem constrains.
1.1 Environmental background

Sceptical environmentalists believe that no energy crises exist and there is sufficient energy to be used in future when every oil well is depleted, however these political discussions cannot deny the effect of burning carbon based energy sources on environment. In recent years, there has been an increasing desire for the use of renewable fuels such as biogas, not only to reduce dependence on petroleum fuels, but also to reduce the harm caused by burning fossil fuels to the environment through mitigation of greenhouse gases. The reduction of GHG by biogas can be achieved as it can provide alternative source of energy with low carbon emissions and recover methane that used to release into atmosphere particularly in dairy facilities.

One of the most interesting alternatives to fossil fuels is anaerobic digestion which is a process that transforms organic matter to useful yields such as methane and ethanol and it has been proved a promising technology to recover nutrition from vast variety of wastes. Benefits of AD process can be wide dependent on conditions of feedstock, on the other hand universal benefits could be more than just production of renewable energy for instance; useful fertilizer by-products, reduction in odour, decreasing total green gas emissions by KWh energy produced, eliminating cost of manure disposal and cleaner manure treatment process to reduce pathogen effects on environment (Jones, 1980). Bio-methane from anaerobic digestion process can also be upgraded to be used as a vehicle fuel.

Four types of conversion processes are used to break down biomass, including thermo-chemical, biological, chemical, and physical processes. When coupled with methods such as fermentation, combustion, and anaerobic digestion, the result is multiple alternative fuels such as biogas and ethanol, and precursors such as cellulose that can be further degraded to ethanol (Naik 2010). As any industrial process, anaerobic digestion needs to be economically profitable to be considered as a promising alternative solution. Production of bio fuels is costly and usually it receive governments assistant such as tax exemptions and low interest loans; therefore it is important challenge to define optimum conditions for process design and operational principals. Still there are some drawbacks of Biofuels such as size and time dependant energy yields which should be studied in details in order to obtain land availability, reactor size and operation conditions (Ajanovic, 2010).
1.2 Anaerobic Digestion

Biodegradable materials break down with assistance of microorganisms into biogas; this process which happens in absence of oxygen (presents of hydrogen gas) is anaerobic digestion. Bio-methanogenesis process (AD) is responsible for carbon cycle in many biological reactions mainly in wet lands and manure degradation.

Anaerobic digestion process opens the door to various new solutions to global energy crisis, fertilizer need, organic waste and pollution control. Biogas production process development is often slowed down by economic concerns and profitability uncertainties; while it is vastly used in sewage treatment. With anaerobic digestion nutrition return to soil by high quality fertilizer with high content of nitrogen, phosphorus and potassium, on the other hand by-product include high valuable gases or alcohols. Products comprises of biogas is consist of large portion of methane and carbon dioxide gas, also contain digestate which can be sold as a mineral rich fertilizer. Typically 30-60 % of input into anaerobic digester reactor converts to biogas remaining will be undigested products and water soluble solids, this percentage can be optimized by altering process conditions and input components (Demirbas, 2009).

Feasibility of anaerobic digestion can be examined based on financial approach where two side of problem are; end of the cycle products (methane, alcohol), byproducts (fertilizer, vermicompost) cost benefits and construction of plant and maintenance cost. Compared to traditional aerobic process, anaerobic digestion is more suitable solution for economic aspects; therefore it has been a popular alternative in many waste treatment plants especially in highly concentrated inputs such as animal waste treatment facilities. Generally anaerobic digestion is rather complicated phenomena and biochemical reactions are elaborate, but still there are quite powerful models to describe steps of degrading organic matter with microorganisms\(^1\).

\(^1\) More detailed information and applications are available in Hyeong-Seok, J. 2005. Analysis and application of ADM1 for anaerobic methane production. *Bioprocess Biosyst Eng.*, 27, 81-89.
In principal process of anaerobic digestion consists of four main steps (Palmisano 1996):

1. First organic structures break down through hydrolysis stage, which make product ready to undergo to next step.
2. Organic acids will be produced through Acidogenesis step; by product of this step can be hydrogen, carbon dioxide and various alcohol compounds.
3. Acetic acid will be produced through Acetogenesis step which use up most of produced organic acids.
4. In final step remaining acetic acid converts to methane with Methanogenesis process while produced hydrogen and carbon dioxide converts to methane.

Progression of each step in anaerobic digestion depends on different factors such as PH value and concentration of volatile fatty acids; though it is complicated to illustrate perfect balance for best condition for entire fermentation process. Figure 1-1 depicts steps related to anaerobic digestion process and simplifies biochemical degradation of various organic materials which finally lead to methane gas.

Figure 1-1 Anaerobic Digestion Process Description

Anaerobic digestion process can be optimized based on input substrate and total organic material though throughout this thesis it is assumed that substrate feed is identical on all times.
1.3 Thesis objectives

The purpose of thesis is to construct analysis for optimization conditions for various scenarios applicable in pilot anaerobic digestion reactor. In this thesis, optimum value for optimization variables for anaerobic digestion of waste manure will lead to:

- Increase in the production of biogas.
- Decrease reactor volume
- Increase energy efficiency and power surplus
- Increase financial revenue

The employed parameters of an anaerobic digesting reactor can be variable such as PH, alkalinity, and volatile fatty acid, and the content of feed, however in this particular case study, they are assumed to have no variation. The optimized production of biogas and economical aspects of such plant will led to efficient energy source allowing the waste treatment facility to reduce the need for outside resources of power and energy. This goal will be met through the following objectives:

- Complete energy balance on the anaerobic digesters at the waste treatment facility.
- Perform mathematical model for anaerobic digestion of manure.
- Calculate energy conversation and power generation.
- Operate chemical processes economic optimization methods.
- Optimization will be performed in Matlab program.
- Economic analyses and financial measurement will be provided in excel.

Finally objective functions lead to model predictions which can be used to design anaerobic digestion reactor with optimized operational parameters.
2 Operation principal

The basic design of biological plant consists of one or several storage tanks for organic matter based on the size of plant and scale of industrial operation which is connected to a fermentation tank and storage tank at the end of production line to collect fully digested input. As shown in Figure 2-1 the fermentation tank usually contains gas at top and liquid at the bottom where anaerobic digestion process digests organic material into biogas.

![Diagram of Simple Biological Treatment Plant Facility](image)

**Figure 2-1 Simple Biological Treatment Plant Facility**

Once anaerobic digester is set up, reactor fed with organic matter for treatment process. The sludge is then held in reactor based on solid retention time depending on process design and operational conditions. Produces methane and carbon dioxide goes to gas outlet, however it is possible gas contains portions of hydrogen sulphide.
Figure 2-2 shows a process flow diagram for pilot reactor design; however a shell and tube heat exchanger has been added to original design to recover some energy. A stream of 6000 [L/day] sludge from swine waste reservoir will be pumped into bioreactor to be digested. Feeding system comprises of four process equipment which are designed to feed system with lowest energy use as possible. Effluent which initially accumulates at the bottom of reactor will be send to nitrification reactor, where all nutrients will be recover and products contains valuable fertilizer. The process must be heated to increase the metabolic rate of the microorganisms to accelerate digestion biochemical process. Heat exchanger recovers heat from effluent and will heat up feed flow before entering reactor, therefore heater consumes less energy with lower duty.

![Process Flow Diagram](image)

*Figure 2-2 Process Flow Diagram (PFD) for Anaerobic Digestion Reactor Foss Farm Design with Heat Exchanger*
2.1 Optimization

Improving an existing design, system, process and situation is defined as optimization. Optimization problems can be solved with appropriate formulation and numerical algorithms in early stage of decision making process. Solution to optimization problem shows optimum values which process will perform in best economic conditions, these conditions may be minimum use of energy and maximum product with lowest initial costs. Technics and strategies are presented to define optimization problem and discover potential improvement to the initial design, then problem assumptions and objective function constrains must identified. Engineers will benefit from enhanced optimization algorithm to design physical description of process unit; though challenging part of optimization is how to appropriately show mathematical model for a system.

In general, optimization is about to find alternative pathways for an existed system. A good strategy for optimization will examine system from base design, however it is favorable to optimize less complicated problem. In order to manage uprising challenges in chemical industry there must be powerful tools to increase performance of existed technologies. Challenges such as rising energy prices, tightening environmental regulations and product compatibility with international rivals must be overcome by optimization methods. Aim of optimization is to determine the optimum solution among other possible process design or plant operation systems which can be obtain by numerical methods compiled with powerful computer and optimization software such as Matlab and Excel. Improvements in computers enable engineers to solve complex optimization problem, currently it is possible to implement various optimization scenarios and discover the optimal conditions based on multiple variable inputs. In this paper Matlab programming language has been used to compile existed mathematical models and discover optimum solutions (Ravindran, 2006).
Optimization of anaerobic digestion reactor begins with selecting an objective function to consider optimum answers to existing problem. Procedure for optimization is depicted in Figure 2-3 where input variables value will be determine process output, which lead calculation to optimum operating conditions. Optimization can be performed in three phases of management, design and operation conditions (Edgar 2001). In this thesis all three aspects have been taken into account, however design part of an anaerobic reactor has been studied in more details with assistance of material and energy balance for individual process variables.
2.2 Variables and Assumptions

In order to optimise any process, significant parameters must be identified and their effect on improving product yield and income must be known. As anaerobic digestion is a complex biochemical process simplifying assumptions are vital to model system and define optimization criteria. To show fundamental aspects of chemical process, physical bonds on the variables, relation between the different quantities and mathematical laws must be determined. Optimization of design in AD process is complex task, because there are great numbers of process variables and design parameters whose influence in process performance is challenging to quantify, therefore one must be very conscious when trying to define process assumptions.

- PH and alkalinity factor are important parameters in anaerobic digestion because they can affect methanogenic bacteria since it is very sensitive to acidity of solution, methane production inhibited in low PH conditions and acidic environment. Optimum value for PH is based on process procedure and reactor type and it is dependant to retention time. In each stage of anaerobic digestion PH value can alter, for instance acetogenesis step reduce PH, consequently an acid environment is accumulated. On the other hand methanogens increase PH value by increasing ammonia percentage in reactor. Therefore a constant value inside reactor is desirable. As it has proved that optimal PH value for maximum yield of methane is a number between 6.5 and 7.5, that varies based in influent of reactor and process selection (Cun-Fang, 2008).

- Feedstock content and load rate is important to consider in anaerobic digestion models and it is possible to consider feedstock as variable content and optimize model to describe different influent into reactor. To discuss alternative feed flow into AD reactor, more advanced AD models and combined processes need to be studied which is beyond the scope of thesis objectives. In this thesis it is assumed that livestock manure has constant load and volume throughout of process and substrate is swine waste, however to show influence of feed flow on production of methane, feed flow rate effect has been studied in chapter 6.
• Reaction temperature is important in anaerobic digestion reactor, as microbial growth is dependent on temperature. The optimum temperature is a variable in optimization criteria in this thesis. In fact higher temperatures need more energy demand for running process and heating increases operational cost but on the other increases methane production yield, this is why temperature effect must investigated in details Figure 2-4. Generally process temperature span is divided into two groups: Mesophilic (25–40°C) with higher tolerance due to harsh reactor environment which is easier to maintain, Thermophilic (50–65°C) suitable for higher loading rate with less retention time as it speed up reaction of degradation of substrate.

![Figure 2-4](image)

*Figure 2-4: Rate of AD process vs temperature (Ahn, 2002)*

• Resident time which called SRT (solid retention time) and HRT (hydraulic retention time) refers to time the substrate stays in anaerobic digestion reactor and can be modelled as: Retention Time = Reactor Volume [m³] / Flow rate [m³/day] and it is determined as time for degrading of substrate by calculating chemical oxygen demand (COD) and biological oxygen demand (BOD). In general the longer substrate stays in reactor the anaerobic digestion process will be more succeeded. As retention time is a function of temperature and solid content, the retention time ratio has to be defined as b = SRT/HRT. Throughout thesis the value for b is assumed to be 3.22, as the value for parameter b can be between 1 and 20, on the other hand diary waste has large energy potential and it needs more time to be in the reactor to degrade. After optimization of produced methane with b, it has shown that sensitivity ratio of process is not significant for higher values than 20 (Haugen, 2013b). In some cases of optimization the value of b is assumed to be equal to one, this assumption is made based on the fact that in CSTR reactors value of solid retention time is identical to value for hydraulic retention time.
• Reactor design and volume is an interesting variable to be optimized economically, because retention time reduced in higher volume reactors and production of biogas increases with enlargement of reactor size, though economic consideration suggests more material leads to higher cost of reactor (capital cost) and higher costs for heating (operational cost). Therefore reactor size is investigated in details and has been chosen to be optimization variable. In some cases reactor volume is assumed to be constant, this assumption is made based in the fact that market has shortage in providing different sizes for reactors.

• To obtain a decent reactor operation condition, reaction temperature has to be constant at desirable value during process. This can be possible with use of reactor heating system in combination of heat exchanger to use up deficit heat from effluent. Therefore heat exchanger coefficient and specific heat transfer coefficient are possible candidates to be optimized. In chapter 5.5 heat exchanger model discusses optimization variable and objective functions. In other parts of thesis the value for heat exchanger coefficient is assumed to be either perfect (highest efficiency) or no effect (zero efficiency).

• Financial parameters such as product cost increase, interest rates, discount factor, income tax and rate of return value are important variables in economic optimization. Consequently it is possible to optimize AD reactor system based on these objective variables, however throughout thesis these parameters are assumed to be constant and their effect on system performance has not been studied.
2.3 Reactor Design

Anaerobic digestion reactor must have special characteristics to perform degradation of high load influent and subsequently produce biogas in higher volumes. Reactor must have high thermal efficiency to avoid heat loss as well as good mixing. Primitive reactors have been made underground with rectangular shape with less effective digestion capacity and increasing maintenance time. Simpler design of reactor suggests batch process where feedstock remains in reactor for a period of time (identified as retention time) then it will be emptied. Today there are reactors that work under continuously fed process and new designs suggested multi-staged systems. Design of reactor is associated with feed flow contents and substrate organic material (Ward, 2008).

In order to improve the desired product yields in anaerobic digestion reactors, design technology must be revived, therefore in recent years sludge bed reactors developed as granular sludge based reactors. These reactors can be divided into main technologies of UASB (up flow sludge blanket), EGSB (expanded granular sludge bed) and AHR (anaerobic hybrid reactors). In order to get higher solid retention time; high rate reactors will be studied as they can be beneficial to obtain low hydraulic retention time, simple design features, efficient energy transfer, available in small scales and sufficient biogas generation.

In this thesis, design of UASB reactor has been studied because of various benefits such as shorter hydraulic retention time and extra energy surplus. In UASB reactor a layer of biomass is located in low stage of reactor and sludge blanket is mixed with produced gases on upper stage of reactor. Startup time for UASB reactors can reach up to several months in order to give optimum possible biogas production since it takes time to granular bed can be cultivated in reactor. Chemical oxygen demand of UASB reactors must be large which can be explained by sludge bed characteristics as suspended solid filter, high COD lead to shorter HRT and high organic loading rates such as 6 m$^3$/d diary waste can be digested in minimum time of 2-4 h with 80% conversion of COD into biogas (Demirbas, 2009).
2.4 Hill’s anaerobic digestion model

Waste treatment process is improving with anaerobic digestion technology, as it produces methane in higher rates with biological degradation of biomass. Typical anaerobic digestion is used to treat sewage sludge waste to reduce smell, recover energy and recirculate minerals for agriculture use by producing rich fertilizer. Animal waste has been interesting influent into anaerobic digestion reactors as it has high value of energy and can produce significant amount of biogas at the end of process. Biogas and particularly methane is important product that can replace typical fossil fuels and reduce greenhouse gas emissions (Magnusson 2012).

There are several approaches to mathematically describe anaerobic digestion process; a model is able to predict biogas production and methane percentage in produced gases in combination with prediction of temperature dependency in overall process. In selection of model for swine waste, simplicity of model is important for easy adaption and maintenance. A review of mathematical models applicable in anaerobic digestion swine waste diary can be found in (Haugen, 2013a). Common simple models are available in literature to describe process, but the fact that they are not precise in prediction of methane and temperature dependency of process makes them impractical, on the other hand complex models like ADM1 are precise in methane gas prediction but numerical challenges in simulation hinder use of them. Therefore nominated mathematical model is (Hill 1983) model which is suitable for swine waste diary process with significant anaerobic digestion steps included. This model has been validated by simulation and laboratory experiments which show it can predict biogas production in reasonable temperature span (20-60°C). A mathematical description of Hill’s model is discussed in chapter 5.3. It is beneficial to consider anaerobic digestion steps in the model. These steps are related to Hydrolytic enzymes, Acidogens and final step Methanogens which show procedure of degrading organic material to methane gas.
3 Literature review

To illustrate optimized model for anaerobic digestion reactor all variables must be studied and all possible routes of reducing cost and increasing product yield should be tested. Quite significant study has been done for optimization of modeling anaerobic digestion process and computer models are implemented in software to simulate degradation of organic matter with ADM1 model and optimize the operation of full scale industrial biogas production plants which examine profitability of such plant (Gaida, 2011). Scientific study to show economic benefits of AD process is not very popular subject, because it is usually done by companies that provide service to the costumer; on the other hand there are some regional bonuses to boost AD reactor green technology that might affect real life cost benefits estimation of such a process.

Generally engineering models cannot be judged with certain true or false statement, for each model it is important to illustrate constrains and relation to other scientific works and the outcome of mathematical modeling must be suitable to describe situation in objective process. Various researches through optimization of biogas plant and cost optimization analysis have been done to maximize benefits of this technology. Mathematical model based optimization is useful technique to discover optimum design of biogas plant based on factors which are constant throughout process. First step is always to design a mathematical model that can describe entire plant created with available physical rules such as energy and mass balance. Producing mathematic model advance by simulation is a powerfull method to unearth design parameters, in (Batstone, 2002) main focus is on producing separate mathematic model for each individual process in plant and optimise each individually.
3.1 Similar works

In (Rivas, 2007) a plant model has been designed for both steady state and dynamic assumptions. Influent substrate characterization, plant objectives and sizes are well-defined, to show optimum available plant design. In order to avoid long unnecessary calculations mathematic models have been simplified. With assumptions of steady state simulation, a objective function is formulated for chemical and physical processes, and with calculations in non-linear optimization algorithm the optimum solution is discovered. Selection of later optimization algorithm can be justified by the fact that it fits the problem characteristics. Analysing uncertainties of model is an important step to raise model accuracy and promote engineering design parameters. Dynamic simulation can give different optimization parameters especially for safety analysis a dynamic response must be studied; therefore a mathematical methodology has been implemented to cover dynamic model-based design. Optimization objectives have been solved with Microsoft Excel®, because it can use dynamic model simulator. In (Rivas, 2007) AD process has been optimized for two problems: At first problem, optimum total plant dimensions (particularly size of reactor) established with help of ASM1 mathematical model and HRT (hydraulic retention time) used as alternative design parameter to reactor size. In second problem long term process operation for influent flow has been discussed, however optimization does not include total operation cost of process.

In (Gillot 1999) objective function of optimization defined based on economic aspects that cover investment, fixed and operation costs. Various scenarios such as varying feeding load applied to discover optimized solutions. In this paper overall plant cost functions has been developed based on data given by specific sources and then accuracy for each data parameter has been validated (with maximum error of 25%) to show typical cost function for each equipment or process. Finally total cost of plant weighted out with net present worth method for both steady state simulation and dynamic simulation. The research concludes with design suggestions to be developed in initial process and payback period for each particular system has been calculated.

In (Fioresea 2008) cost benefits of anaerobic digestion plant has been optimized. The aim of report is to find optimum design criteria for AD reactor capacity, number and location. Optimization of economic objective function, results in a solution with net present value of plant for higher than 300 million Euros, nevertheless presented profits are just achievable if there is a public incentive for renewable biogas. In other words without government support economic objective function will end up with negative value.
4 Economic Analyses

Evaluation of economics for creation of chemical process will be performed by technical information from available data sources. Generally economics of project divided to construction part and operational part. The material of process cost analyses is based on estimation data for anaerobic digestion plant; all related cost from initial investment to final product sales will be discussed. Therefore cost analyses has been done for process equipment and facility building but legal cost, land for building and tax regulations has not been included in process estimation.

First capital cost for an anaerobic digestion plant is calculated with reliable estimation technics with cost index implementation based on pilot reactor design. Once capital cost and operating cost are calculated, it is necessary to combine data to show total economic performance of system. In this chapter focus is to modify all possible costs related to anaerobic digestion process with available data and estimation methods. The original cost data is given in pilot design of reactor, therefore relationship for equipment size and scaling must be calculated. Manufacturing cost analyses has not been included in the cost analyses; because it is assumed that operation design use same materials of equipment, while labour cost is included in all steps of calculation. Time value for investment discussed and profitability measures by net present value of operating process.
4.1 Cost estimation

Any process design must be economically reasonable, since net profitability of process is related to income minus all expenses, it is vital to consider all expenses of building and running a chemical process. In order to optimize economics of process, capital and operation cost must be considered thoroughly for life time of process; therefore to build a model all key variables such as size of reactor must be considered. In this thesis costs divided into two categories of capital costs and operation cost (maintenance included). Maintenance of equipment includes cost of periodic checks and necessary mechanical upgrades in order to change component parts to keep process conditions desirable. Cost of energy in terms of [kW/h] is fixed operation cost; also labor requirement has chosen to characterize as [hr/year]. It is possible to divide costs into fixed cost and variable cost; for example costs of energy to pump flow through heat exchanger considered to be variable cost.

In this thesis it is has been tried to find fair prices for each individual operation component, and usually estimations where based on data received from pilot plant, however new component where installed to process unit and therefore more calculation where needed to estimate those component prices. For example heat exchanger price is unknown, to determine price for heat exchanger based on market prices area must be known.

To determine area, energy balance is applied (Incropera, 2006):

\[
q = m \cdot c (T_{c\text{,out}} - T_{c\text{,in}}) = UAF \Delta T_{LM} \tag{4-1}
\]

Where \( \Delta T_{LM} \) and \( F_T \) is formulated as:

\[
\Delta T_{LM} = \left( \frac{T_{h\text{,in}} - T_{c\text{,o}}}{T_{h\text{,in}} - T_{c\text{,o}}} \right) = \left( \frac{T_{h\text{,o}} - T_{c\text{,i}}}{T_{h\text{,in}} - T_{c\text{,o}}} \right) = (25 - 17.4) - (17.5 - 10) = 7.5 \tag{4-2}
\]

\[
R = \left( \frac{T_{h\text{,in}} - T_{c\text{,in}}}{T_{h\text{,in}} - T_{c\text{,in}}} \right) = \left( \frac{25 - 15}{18 - 10} \right) = 1.25 \tag{4-3}
\]

\[
S = \left( \frac{T_{c\text{,out}} - T_{h\text{,out}}}{T_{c\text{,out}} - T_{c\text{,in}}} \right) = \left( \frac{25 - 15}{18 - 10} \right) = 0.53 \tag{4-4}
\]

The value of \( F_T \) will be found from shell and tube counter-current correction factor diagrams with respects to R and S values:

\( F_T = 1 \) also calculation for heat transfer coefficient leads to \( U = 1125 \text{ [W/m}^2\text{k]} \)
\[ A = \pi DL = \left( \frac{m^*_{\text{feedwater}}(T_{c_{\text{out}}}-T_{c_{\text{in}}})}{UF_T \Delta T_{LM}} \right) = \left( \frac{0.07 \times 4200 \times (17.5-10)}{1125 \times 1 \times 7.5} \right) = 0.32 \text{ } m^2 \]  

(4-5)

Once the area for heat exchanger is known, it is possible to estimate cost based on cost data sheets, using capacity factor method:

\[ \text{CostB} = (\text{CostA}) \times \left( \frac{\text{CapB}}{\text{CapA}} \right)^e \]  

(4-6)

Where \( e \) is proration factor (scale factor) obtained for heat exchanger shell and tube carbon steel to be 0.59 from (Perry, 1997).

A similar heat exchanger with same material (Stainless steel tube and carbon steel shell), duty range (P=5 kW), design pressure (15 bar), volume (0.913 Litter), diameter (51 mm), length (700 mm) and maximum flow (4 m³/h) suggests Cost of heat exchanger as 9700[NOK]. Detailed description of suggested heat exchanger is available in Appendix 2.

Capacity factor method reveals value for heat exchanger, however in order to be precise about calculation, cost proved by online cost estimator tools provided by Mc Graw Hill publication² (Peters, 2004). It is recommended to escalate estimation with other possible sources such as Aspen plant estimator tool.

To obtain an estimate for heat exchanger, available cost for previously purchased heat exchanger is used in equation (4-7) with capacity factor. Furthermore effect of time, material and currency unit for purchased equipment has been taken into account. The currency relation [EUR/NOK] value for update cost is (2014/2012)=8.14/7.6=1.07.

The estimated value is 9100 [NOK], However the most accurate estimate can be provided from suitable vendor.

\[ \text{Cost } H_{\text{ex}} = 1.07 \times (9700[\text{NOK}]) \times (0.32/0.357)^{0.59} = 9100[\text{NOK}] \]  

(4-7)

---

4.2 Investment cost (CAPEX)

Investment costs for a chemical process may be calculated as a function of equipment size such as production volume, area, flow rate and reactor size. In order to complete cost analyses economic factors must be added to all original costs along with pipes and instrumentations. Economical sources to compare cost data is hardly possible but there are some related literature that can be useful such as (Mort 1997), though it is possible to relate some estimation of investment cost based on literature with indexes related to region, country and year. To relate early stage economic analyses to real process cost data an economic function must be made; then accuracy of such a function can be investigated in order to obtain reliable cost estimation.

In defining of economic objective function for capital cost, there are different levels of design criteria and each level must be based on realistic data connected to process. By help of historical cost data general price flow sheet can be estimated, in this level of cost estimation, data obtained from charts and multiplying factors based on size difference, inflation and production year. A cost estimation function could be made based on process capacity as the size of equipment will alter economic outcomes. Still more detailed investigation of capital cost is needed for major equipment such as reactor, where more economic indexes applied for more precise estimation.

Table 4-1 Capital Cost Data

<table>
<thead>
<tr>
<th>CAPEX</th>
<th>Cost (pilot plant)</th>
<th>Cost index</th>
<th>NOK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Building civil cost</td>
<td>500000</td>
<td>1.00</td>
<td>500000</td>
</tr>
<tr>
<td>ADR cost</td>
<td>118107</td>
<td>7.57</td>
<td>894067</td>
</tr>
<tr>
<td>feed sieve</td>
<td>100000</td>
<td>7.57</td>
<td>757000</td>
</tr>
<tr>
<td>feed pump</td>
<td>30000</td>
<td>12.13</td>
<td>363900</td>
</tr>
<tr>
<td>supply screw pump</td>
<td>13000</td>
<td>23.63</td>
<td>307190</td>
</tr>
<tr>
<td>heat exchanger</td>
<td>9100</td>
<td>23.63</td>
<td>215033</td>
</tr>
<tr>
<td>Temp sensor (4x)</td>
<td>4000</td>
<td>1.00</td>
<td>4000</td>
</tr>
<tr>
<td>feed flow sensor</td>
<td>3000</td>
<td>1.00</td>
<td>3000</td>
</tr>
<tr>
<td>Gas flow sensor</td>
<td>13000</td>
<td>1.00</td>
<td>13000</td>
</tr>
<tr>
<td>Gas sensor (2x)</td>
<td>10000</td>
<td>1.00</td>
<td>10000</td>
</tr>
<tr>
<td>Computer</td>
<td>5000</td>
<td>1.00</td>
<td>5000</td>
</tr>
<tr>
<td>Monitoring</td>
<td>20000</td>
<td>1.00</td>
<td>20000</td>
</tr>
<tr>
<td>Control software</td>
<td>10000</td>
<td>1.00</td>
<td>10000</td>
</tr>
<tr>
<td>ADR civil labour</td>
<td>100000</td>
<td>1.00</td>
<td>100000</td>
</tr>
<tr>
<td>Operator labour</td>
<td>10000</td>
<td>1.00</td>
<td>10000</td>
</tr>
<tr>
<td>Fixed Capital Investment</td>
<td>945207</td>
<td></td>
<td>3212190</td>
</tr>
</tbody>
</table>
In thesis analyses and calculations has been done for installation cost, engineering cost, direct cost and administration cost. Furthermore value of contingency has been applied for some significant parts of process (mostly rotating equipment); detailed data are available in Table 4-2, however in certain analyses applied to CAPEX, estimation was excluding cost indexes the difference has been depicted in section 4.4.

Additional information for cost estimation will be obtained based on nature of process parts, as some equipment need special treatment for example; a reactor cost may differ based on utilities (water, steam, electrical, air) and special instrumentation and control systems that used on reactor.

Table 4-2 Economic Cost Index for CAPEX (Eldrup 2013)

<table>
<thead>
<tr>
<th>Cost index</th>
<th>Direct</th>
<th>Engineering</th>
<th>Administration</th>
<th>Commissioning</th>
<th>Contingency</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reactor AD</td>
<td>4.43</td>
<td>1.09</td>
<td>0.66</td>
<td>0.13</td>
<td>1.27</td>
<td>7.57</td>
</tr>
<tr>
<td>feed sieve</td>
<td>4.43</td>
<td>1.09</td>
<td>0.66</td>
<td>0.13</td>
<td>1.27</td>
<td>7.57</td>
</tr>
<tr>
<td>feed pump</td>
<td>6.53</td>
<td>2.11</td>
<td>1.21</td>
<td>0.26</td>
<td>2.02</td>
<td>12.13</td>
</tr>
<tr>
<td>supply screw pump</td>
<td>10.60</td>
<td>5.83</td>
<td>2.74</td>
<td>0.57</td>
<td>3.91</td>
<td>23.63</td>
</tr>
<tr>
<td>heat exchanger</td>
<td>10.60</td>
<td>5.83</td>
<td>2.74</td>
<td>0.57</td>
<td>3.91</td>
<td>23.63</td>
</tr>
</tbody>
</table>
4.3 Operation cost (OPEX)

In defining total cost of a process, economic evaluation of operating cost is a crucial step. This step includes labor cost for operation of plant, operation supplies used by process, maintenance and utilities cost as electricity. However there are still more data such as indirect costs which must be included in calculations. Indirect cost comprises depreciation, taxes, insurance, interest and general administrative overhead which may include distribution cost. Estimating cost for non-operating facility is done by rule of using existed cost categories, however in this thesis data has been used based on pilot operating process.

Discover related data for calculation operation cost and designing an accurate objective function need deep investigation of presented data, however in this thesis most of equipment costs has been provided from earlier stages of process design. In biogas plants there are diverse equipment used therefore a good calculation method must be provided for each part for example: in reactor design a complete cost function should consider size, design material and energy saving coefficients. All operating costs to maintain and operate anaerobic digestion reactor to remove energy from waste manure can be listed as:

- Raw material
- Personnel (professional and labour work)
- Process maintenance (mechanical and instrumentation)
- Operational costs (materials, services, lab supplies and office supplies)
- Electrical costs (pumps, electrical equipment and heating elements)
- Insurance, amortization, depreciation, taxes and interest
- Cost of R&D, monitoring and safety procedure
- Cost of distribution

The value of these parameters depends on operating conditions and initial investment, therefore good analyses required to optimize entire process at the same time. Cost of maintenance is dedicated to both civil works and equipment plus as it mentioned before it related largely on main process design (Haandel 2007).
The cost of manufacturing products directly depends on original design of process and capital investment and raw material price, however in anaerobic digestion process raw material has a low price; therefore cost of operating depends on labour and power use of equipment. A summary of operation cost is presented in Table 4-3; moreover detailed measurement procedure is available in Appendix 3.

### Table 4-3 Process Operation Cost Data

<table>
<thead>
<tr>
<th>OPEX</th>
<th>Multiplying Factor</th>
<th>Cost function</th>
<th>NOK/YEAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Insurance</td>
<td>Insurance Factor= IC</td>
<td>Total equipment cost * IC</td>
<td>39257</td>
</tr>
<tr>
<td>Amortization</td>
<td>Repayment multiplier=RM</td>
<td>Total equipment cost * RM</td>
<td>388462</td>
</tr>
<tr>
<td>Depreciation</td>
<td>Years of Depreciation=YD</td>
<td>Building<em>YD +Reactor</em>YD +Equipment*YD</td>
<td>333870</td>
</tr>
<tr>
<td>Labour</td>
<td>Labour Wage=LW</td>
<td>Man Hour*LW</td>
<td>100000</td>
</tr>
<tr>
<td>Maintenance</td>
<td>Fraction of Investment=FI</td>
<td>Equipment Cost * FI</td>
<td>75518</td>
</tr>
<tr>
<td>Heating Power</td>
<td>Electricity price (NOK/kWh)= P_{el}</td>
<td>Heating Power * P_{el}</td>
<td>6423</td>
</tr>
<tr>
<td>Sieve Power</td>
<td>Electricity price (NOK/kWh)= P_{el}</td>
<td>Sieve Power * P_{el}</td>
<td>3181</td>
</tr>
<tr>
<td>Screw Pump Power</td>
<td>Electricity price (NOK/kWh)= P_{el}</td>
<td>Screw Pump Power * P_{el}</td>
<td>3719</td>
</tr>
<tr>
<td>Lifting Feed Power</td>
<td>Electricity price (NOK/kWh)= P_{el}</td>
<td>Lifting Feed Power * P_{el}</td>
<td>2</td>
</tr>
<tr>
<td><strong>Total Operation Cost</strong></td>
<td></td>
<td></td>
<td><strong>1024234</strong></td>
</tr>
</tbody>
</table>

Depreciation is an uncommon charge that happens by wear, tear, corrosion and accidents; in other words it relates to process plant deterioration which means reduction in value of facility. In revenue calculation it is important to consider depreciation impact on process (Silla, 2003). In this thesis the technique to measure depreciation is straight-line method which assumes the plant value decreases linearly with time over recovery period. Insurance costs are annual payment which is assumed to be 1 percent of initial capital investment; however it is possible to reduce insurance costs by understanding legal regulation and available insurance type(Peters, 2004). Detailed calculations for both insurance and depreciation are considered in revenue calculations and they are available in Appendix 3.
4.4 Product cost and income (total plant cost)

Once all costs of process has been developed and sale of products produces in plan has been determined, the yearly generated revenue can be calculated. Gross annual cost benefit of product sale is the sum of quantity of each product multiplied by money value. Product prices has been obtained by market study, while for methane production, energy value has been established by calculating energy equivalent of methane converted to Kwh. Mathematical model for annual income can be shown as:

Annual Income [Nok/yr] = \sum (product quantity [L/yr]) (product money value [Nok/L])

<table>
<thead>
<tr>
<th>INCOME</th>
<th>NOK/YEAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Produced Biogas</td>
<td></td>
</tr>
<tr>
<td>Value Power(NOK/kWh)=VP</td>
<td>Power Biogas*VP</td>
</tr>
<tr>
<td>Produced Vermicompost</td>
<td></td>
</tr>
<tr>
<td>Value Vermicompost(NOK/L)=VV</td>
<td>Raw Feed<em>Sieve Fr</em>VV</td>
</tr>
<tr>
<td>Produced Fertilizer</td>
<td></td>
</tr>
<tr>
<td>Value Fertilizers(NOK/L)=VF</td>
<td>Feed Effluent*VF</td>
</tr>
<tr>
<td>Total Process Income</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The optimum condition is to design a process that delivers maximum rate of annual income in a way that OPEX and CAPEX remain logically low. Therefore production rates must be related to design capacity. It is important to consider the fact that production of vermicompost and fertilizer are not related to anaerobic digestion condition and production rates are assumed to relate to feed flow, however it is promising to relate fertilizer quantity to nitrification process.
Project evaluation is a method to show profitability of a process, however it was not primary desired to evaluate profits of anaerobic digestion reactor but interesting optimization objective function could be made by analyzing profitability. In order to visualize economic settings for anaerobic digestion reactor, data has been presented in two cost breakdown diagrams. Figure 4-1 estimate process cost based on direct cost of equipment in the market, in this process erection, engineering cost, administration, commissioning and contingency has been neglected.

![Design 1_ Market Price](image1)

**Figure 4-1 Process Economic Estimation Based on Market Prices**

Estimation in Figure 4-2 considers economic conditions that may change with time, it has been tried to include general estimate indexes to cover various economic uncertainties after process has evaluated. It is clear that capital cost will increase dramatically, however it is important to consider side factors such as contingency in calculations. Complete list of economic indexes are available in Table 4-2.

![Design 2_Cost Index included](image2)

**Figure 4-2 Economic Estimation with Cost Index for CAPEX**
4.4.1 Time value of money

Invested money makes more money; therefore value of initial costs that has been dedicated to the process erection increases as time goes by. In other words initial invested money for capital cost of process and equipment, will have more worth in future. In anaerobic digestion process due to large capital cost, it is important to consider process present worth.

Two main approaches to evaluate time value of process are net present value (time value of money) and internal rate of return. The magnitude of process is often determined using the Net Present Value, in this method all operating costs for each part of plant are converted into their corresponding present value then it will added to the investment cost of each equipment. It is also important to consider value of products as time goes by, for example; it is likely that the price of methane will be higher in 20 years from now.

A general principle to calculate Net Present Value is can be determined as:

\[
Net\_Present\_Value = \sum_{k=1}^{N} Investment\_Cost_k + \left( \frac{1 - (1+i)^{-n}}{i} \right) \sum_{k=1}^{N} Operation\_Cost_k \quad (4-8)
\]

Where interest rate (Bank rate consists risk, bank impatient and inflation) is represented as factor \((i)\), \(n\) is period of year that plant is working and \(N\) shows number of process units. A simplified method (single payment present worth factor) to calculate Net Present Value can be determined with adding up total plant cost including OPEX, CAPEX, and Income. Calculation method can be determined as (Turton 2012):

\[
Net\_Present\_Value = \sum_{k=1}^{N} \left( CF_n \ast \frac{1}{(1+p)^n} \right) \quad (4-9)
\]

Where \((n)\) represent number of years that process is operating or study period, \((p)\) is rate of return value which will be calculated due to economic situation based on risk factors and impatient of banking system and finally \((1/ (1+p)^n)\) is discount factor.

Another approach to measure capital budgeting of a process is internal rate of return (IRR), where a discount rate makes net present value of all costs to be equal to zero. In other words internal rate of return shows quality of initial investment, the bigger IRR values show higher efficiency of investment. For example in anaerobic digestion, the design with highest IRR would be best design and it will generate best product yield. Results for calculation of net present value and internal rate of return for AD process are represented in chapter 7; also calculations are included in Appendix 4.
5 Application of mathematical modeling

In order to describe necessary characteristics of a process, model must be defined based on requested criteria and objectives of problem. When designing an optimization algorithm for a process verbal aims are translated into logical mathematical forms. A mathematical model describes essential aspects of system that includes necessary limitation of process and constrains of system. Vital necessities to be taken care of are: simplification of model, analysis for sensitivity of variables and estimating various inputs.

There may be various models for one specific system, a decision must be made to choose the best fitted model in order to get desired outcome. In this thesis models are described with mathematical expressions, because it displays physical aspects of a system with measureable properties and it is possible to optimize with numerical methods. Variation of the process defined in mathematical terms, must be validated with real system numerical aspect or laboratory data. Obtaining a correct model is an important step to predict complete physical aspects of a system. Furthermore computer program can be helpful in simulation and optimization steps as well, which lead results to be more cost effective and easy to implement with diverse criteria (Buso 2011).

- Process information must be gathered and objectives must be known to define problem requirement. A model describes system based on process analyses and verbal and physical description, which determines independent variables. In this thesis to simplify model sub-models described separately, though it is possible to show relation of mathematical models in complete process. Another simplification is to introduce assumptions for physical aspects of system, which must be evaluated with uncertainty analyses.

- Translation into mathematical formulas can be achieved with known physical rules, for example mass balance formulates connections between inflow and outflow of the process. Model development can be achieved with complex rules like chemical balance inside reactor has been used to describe another aspect of process, generally mathematical models produced based on problem description and complexity of studying process. Mathematical description of process help system to be controlled and formulated with algebraic equations, while initial and boundary conditions must be selected properly. In conclusion more complex mathematical description lead to struggle to find appropriate solution for the initial problem, therefore model must be simplified based on required details.

- Once a model has been suggested to a process, now it must be solved to acquire a solution to initial problem which in this thesis is process optimization. The aim of this step is to find optimum design criteria which have been applied to designated mathematic model. This can be done with optimization methods and problem solving methodology described in chapter 6.

- After solving model and receiving initial answer, validation of outcomes should be checked with real process data to see if model is accurate. In this thesis a comparison between cost estimation results and real process cost, determines verification of economic models. As result of this step; mathematical model can be developed with more investigation.
5.1 Problem formulation

Significant elements of a system must be defined to start formulation of a verbal description of physics industrial design into mathematical form. In this paper there are four variables need to be identified to shod optimized solution: Reactor size (V), Energy surplus (P_{sur}), produced biogas (F_{meth}) and reaction temperature (T_{reac}); each of stated variables needs to be examined based on specific physical and empirical relation. General procedure into optimization of AD rectors can be shown as following steps:

- Process variable evaluation and characteristics of nominated variables, for example: when trying to optimize produced biogas, there might be several reactor temperature data for maximum production with various reactor volumes, in such situation definition of optimize criteria will decide the optimum solution.
- A performance model for optimized criteria which an objective function will use variables defined in previous step, for example in cost estimation there are two scenarios; increasing production of biogas and decreasing energy usage in heat exchangers, therefore objective function calculate each cost scenario coupled with general optimization criteria.
- A mathematical description of performance model which contains input-output variables, such a model can be obtained with help of material and energy balance. In this step degree of freedom must be defined in a valid mathematical expression, in this paper two degrees of freedom has been used and the other variables considered to be dependent and has been derived in objective function.
- Optimization method to find maximum amount of desired variable, a possible method can be using differentiate of objective function to discover trends; furthermore in chapter 5 it will be discussed that finding global optimization is always a challenge.
- Visualize answers to check if results of calculation can be logical to initial assumptions.

Formulation of desired function for optimization process is second step into any optimization procedure, after translation of verbal description into mathematical rules. When working on elaborate systems such as anaerobic digestion and complicated model such as Hill’s model that gives nonlinear objective functions, use of powerful optimization tools is crucial. Some areas that require multiple objective functions are treated as separate models to make problem more observable, though an advanced model will consider multiple objectives such as minimize cost and volume at the same time by suitable weighing.
5.2 Economical optimization model

A model to minimize capital cost of anaerobic digestion plant can be based on reactor design configuration, in such model best size of reactor which has most profitable output has to be optimized. Objective function for capital cost is based on value for volume and the mathematical formulation can be based on simplifying assumptions listed as:

The ends of reactor are circular and flat, whole body of reactor has same thickness and density and finally the cost of production is the same for all sides of reactor. Using this assumptions make it possible to define surface area of reactor as:

\[ A_{\text{AD_reactor}} = 2(\pi D^2/4) + \pi DL \]  \hspace{1cm} (5-1)

Where D is diameter of reactor and L is length. In order to make the system proportional to economic analyses, factors for weight determination (\( \rho \)) and cost of material per kg of reactor ($) must multiplied to equation (5-1).

\[ \text{Cost}_{\text{AD_reactor}} = \$ \times \rho [2(\pi D^2/4) + \pi DL] \]  \hspace{1cm} (5-2)

It is noticeable that values of independent variables are significant through objective functions and the density and cost variables can be added after optimization of reactor design. Final cost of AD reactor has been formulated with capacity factor method based on pilot reactor data; further equipment cost formulations are available in Appendix 3.

Economic revenue from anaerobic digestion process can be maximized by defining an objective function based on objective variable. The optimum economic performance can be depicted through net present value (NPV) method.

\[ OEP = \sum_{n=1}^{24} NPV_n = \left( -\frac{I_n}{(1+R)^n} + \sum_{n=1}^{24} \frac{CF_n}{(1+R)^n} \right) \]  \hspace{1cm} (5-3)

In equation (5-3) OEP is optimum economic performance objective function, \( n \) symbolizes year, with economic discount rate \( R \), \( I_n \) represents investment cost which in this process is provided in first year of process design, as it was mentioned before unit’s life period is assumed to be 24 years, which for year 1 there is no product yield as it is assumed to be plant erection time period. The value for cash flow (CF) is linked to variable costs (namely process income and operation costs) therefore \( CF_n \) is functioned as:

\[ CF_n = C_{\text{biogas}_n} + C_{\text{fertilizer}_n} + C_{\text{vermicompost}_n} - \sum_{n=2}^{24} C_{\text{operation}_n} \]  \hspace{1cm} (5-4)

Cash flow in equation (5-4) shows sum of cost benefits from produced resources of methane, fertilizer, and vermicompost minus all operational costs. Operational costs may contain large set of variables depending on constrains and assumptions, in section 4.3 more detailed calculations are available.
5.3 Anaerobic digestion model

Mathematical model for anaerobic digestion process can be produced based on selection of dynamic model. Models for anaerobic digestion analyze biochemical process and more precise models can define produced gas mixture which shows energy content of process outline, yet simplified model is profitable as it is easy to implement in simulation tools. Another aspect of a good anaerobic digestion model is prediction of temperature effect on total process. There is ongoing research into producing a model to describe best process simulation, in (Andrews, 1971) processes with any organic feed flow, going through chemical and biological reaction is simplified into conversion of substrate at methanogenesis step to biogas. In this model acidity and temperature inside reactor assumed to be constant. Model predicts biogas production (methane and carbon dioxide) with selecting several state variables such as: concentration of carbon dioxide, concentration of methanogen microorganism, and alkalinity inside reactor with concentration of organic substrate. This model cannot be useful to these thesis objectives as it is unable to predict temperature dependency of produced biogas.

Another approach for planning a dynamic model for anaerobic digestion is hill (Hill, 1977) where model is valid for animal waste substrate and hydrolysis step for anaerobic digestion has been included into calculation. Model predicts biogas production (methane, carbon dioxide) with state variable such as: concentration of acetate, acidogens, methanogenesis, ammonia, carbon dioxide and concentration of soluble organics. The model is complex and implementation of model into process needs more investigation.

In this thesis selected model for anaerobic digestion is hill’s model because problem criteria can be described in model and the overall model is simple and easy to implement into simulation which lead to practical solution for thesis objectives. Organic substrate can be animal waste and model verified with swine waste, yet model ignores hydrolysis step but still it contains acidogenesis and methanogenesis steps which predict temperature dependency of produced biogas (methane). The model has been modified further with help of general model (Batstone, 2002), where complex processes and reactions are described i.e. four stages of anaerobic digestion(hydrolysis, acidogenesis, Acetogenesis and methanogenesis. PH and Temperature dependency of produced biogas is provided in model; however complexity of model is drawback which leads to difficult calculation and special programing requirement. Therefore model used in thesis has been upgraded version of Hill’s model based on results obtained with ADM1 model and experimental parameters.
In model it is assumed that VFA consist of propionate, butyrate, valerate and acetate as main component. As some methane will be produced through hydrogenotrophic methanogenesis, $S_{vfa}$ shows total volatile fatty acids content. In (Haugen, 2013a) a mathematical description of model adopted on process based on material balance and results are following equations:

\[
S_{bvs} = (B_0S_{vbs} - S_{vbs}) \frac{F_{feed}}{V} - \mu k_1 X_{acid} \tag{5-5}
\]

\[
S_{vfa} = (A_f B_0S_{vbs} - S_{vfa}) \frac{F_{feed}}{V} + \mu k_2 X_{acid} - \mu c k_3 X_{meth} \tag{5-6}
\]

\[
\dot{X}_{acid} = (\mu - K_{dc} - \frac{F_{feed}/b}{V}) X_{acid} \tag{5-7}
\]

\[
\dot{X}_{meth} = (\mu_c - K_{dc} - \frac{F_{feed}/b}{V}) X_{meth} \tag{5-8}
\]

Produced methane gas can be calculated with following mathematical expression:

\[
F_{meth} = V\mu c K_s X_{meth} \tag{5-9}
\]

Reaction rates can be described with Monod kinetics, where maximum reaction rate is $\mu_{mc}$.

\[
\mu = \frac{\mu_m}{K_s + S_{bvs}} \tag{5-10}
\]

\[
\mu_c = \frac{\mu_m}{K_{ac} + S_{vfa}} \tag{5-11}
\]

For $20^\circ < T_{reac} < 60^\circ$ $\mu_{mc}$ formulated as:

\[
\mu_{mc} = \mu c = 0.013T_{reac} - 0.129 \tag{5-12}
\]
5.4 Reactor temperature model

As shown in equation, reaction rates increase in higher temperatures, these phenomena has been studied in Foss project which shows a good compatibility between experimental data and model prediction of biogas, results in Figure 5-1 displays a gradual change in $F_{\text{meth}}$ due to increase of $T_{\text{react}}$:

![Methane gas flow. Meas (red) and sim (blue)](image)

![Temperature in reactor](image)

**Figure 5-1 Produced Methane vs Reaction Temperature (Haugen 2014)**

In order to formulate temperature, energy balance for liquid phase inside reactor must be studied, to simplify model it is assumed that reactor is homogeneous and the content has thermal properties of plain water where temperature is the same in all points:

$$\frac{dT_{\text{react}}}{dt} = \frac{1}{c \rho V} \left[ P_{\text{heat}} + c \rho F_{\text{feed}} \left( T_{\text{feed}} - T_{\text{react}} \right) + G \left( T_{\text{room}} - T_{\text{react}} \right) \right]$$

(5-13)

For steady state assumption, model can be simplified and combined with heat exchanger model.

$$P_{\text{heat}} = c \rho F_{\text{feed}} \left( T_{\text{feed}} - T_{\text{react}} \right) + G \left( T_{\text{room}} - T_{\text{react}} \right)$$

(5-14)

Thermal conductivity $G$ assumed to be the same at all side of reactor and it can be estimated from experimental data; though this factor has been retrieved from similar approach in (Incropera, 2006).
5.5 Heat exchanger temperature model

A countercurrent shell and tube heat exchanger has been used to warm up feeding substrate into reactor with the heat of effluent stream. To generate a mathematical model for heat exchanger some assumptions has been made:

The flow of liquid inside pipes of heat exchanger assumed to be the same as substrate feed flow, furthermore the stream of material in and out of heat exchanger assumed to be the same therefore there are two homogeneous volumes.

In (Haugen, 2013b) a model has been obtained with energy balance:

\[ cpV_h T_{inf} = c_p \rho \dot{F}_{feed} (T_{feed} - T_{inf}) + G_{hx} (T_{hx,w} - T_{inf}) \]  
\[ (5-15) \]

\[ cpV_h T_{hx,w} = c_p \rho \dot{F}_{feed} (T_{reac} - T_{hx,w}) + G_{hx} (T_{inf} - T_{hx,w}) \]  
\[ (5-16) \]

Steady state that removes \( T_{hx\, out} \) model will be:

\[ T_{inf} = \frac{1 + g_{hx} T_{feed}}{1 + g_{hx}} + \frac{g_{hx}}{1 + 2g_{hx}} T_{reac} \]  
\[ (5-17) \]

When:

\[ g_{hx} = \frac{G_{hx}}{c_p \rho \dot{F}_{feed}} \]  
\[ (5-18) \]

With condition of no heat exchanger \( g_{hx} \) will be zero that means: \( T_{inf} = T_{feed} \)  
With ideal heat exchanger that has perfect efficiency: \( T_{inf} = 0.5(T_{feed} + T_{reac}) \)  
The amount of heat [J/d] which is transferred to feed flow can be shown as:

\[ P_{heat} = \frac{1 + g_{hx}}{1 + 2g_{hx}} c_p \rho \dot{F}_{feed} (T_{reac} - T_{feed}) + G(T_{reac} - T_{room}) \]  
\[ (5-19) \]
The sum of heat which has been saved due to use of heat exchanger can be calculated by the difference between no heat exchanger and perfect heat exchanger.

\[
\Delta P_{\text{heat}} = \frac{1}{2} c_p F_{\text{feed}} (T_{\text{react}} - T_{\text{feed}}) \tag{5-20}
\]

It can be seen from comparison between equations (5-23) and (5-21), that usage of heat exchanger leads to 50% reduction of energy use to heat up feed flow. In order to complete power calculation for entire process, all other source of power must be determined.

\[
P_{\text{methane}} = E_{\text{methane}} F_{\text{meth}} \text{[kWh/yr]} \tag{5-21}
\]

\[
P_{\text{sup ply}} = k_{\text{sup ply}} F_{\text{feed sup}} \text{[kWh/yr]} \tag{5-22}
\]

\[
P_{\text{seperator}} = k_{\text{seperator}} F_{\text{feed sup}} \text{[kWh/yr]} \tag{5-23}
\]

\[
P_{\text{feed}} = \rho g h F_{\text{feed}} \text{[J/d]} \tag{5-24}
\]

\[
P_{\text{agitator}} = k_{\text{agitator}} F_{\text{feed sup}} \text{[kWh/yr]} \tag{5-25}
\]

Then surplus power which has been generated by process can be shown as:

\[
P_{\text{surplus}} = P_{\text{methane}} - P_{\text{heat}} - P_{\text{agitator}} - P_{\text{sup ply}} - P_{\text{seperator}} - P_{\text{feed}} \ldots \text{[MWh/yr]} \tag{5-26}
\]
6 Optimization Technics

Various technics are available to find solution for optimization problem therefore it is important to use the best technic which satisfies problem criteria and optimum answer. A classical approach for optimization is well-defined in calculus maxima of a function, which can be obtained with derivation. Multivariable optimization can be done with calculus methods with help of Taylor series. It is observed that calculus method is limited only for functions that can be differentiating twice so it is not possible to use this rather simple technic to optimize anaerobic digestion reactor. Therefore more general methods such as Lagrange multiplier method must be used; however it may be difficult to find optimum point due to nonlinear simulation equations. Thanks to improvement of computers it is possible to consider numerical methods for wide range of problems, because iterative procedure can be done in very fast pace and the final optimum value is more reliable (Singiresu 2009).

Newton’s method (single variable), Brute force method, gradient decent approach, Monte Carlo simulation and combination of all methods are computation devices for optimization. In this chapter global and local optimum values will be defined and calculation technics for optimization will be discussed.
6.1 Implementation in Matlab

Computer calculation power help to solve complicated optimization problems, scientific software are designed in order to compile solutions to elaborate problems. Among various software programs Matlab has more developed toolbox dedicated to optimization tasks. Optimization toolbox contains premade simulators to solve maximization, minimization, least square curve fitting problem. These toolboxes contains optimization algorithms such as unconstrained nonlinear, constrained nonlinear, simple convex: LP, QP, least squares, binary integer programming and multi-objective algorithms.

To obtain a solution for known objective function, constraints of problem must be defined in Matlab; this can be done with writing m-files to define in separate scripts. Due to nature of objective functions for anaerobic digestion reactor which is constrained nonlinear programming, fmincon function can be a candidate to solve problem. Since problem criteria can be modified in more simplified methods like brute force, it has been decided to use numerical methods for optimization. Furthermore in chapter6 it is decided to use optimization method which guaranties to find optimum solution with a good accuracy. Accuracy of solution can be improved by increasing the grid resolution for optimization variables.
6.2 Global optimization

Best system performance in chemical processes can be completed by discovering finest answer to optimization problem; this can only be possible if there is method which can diverge away from local optima to global optima. Global optimum is a value where objective function has the maximum output for all input variables and there are no better possible solutions. When small changes in value of variables are unable to improve the output of objective function, local optimum will be determined. As it can be seen from Figure 6-1, a optimization problem may contain a bunch of solutions and therefore a reliable optimization method will ignore local optimums until the global optimum will be discovered. Alternative reason for choosing brute force method over fmincon Matlab premade function is methods ability to find maxima without lagging behind with local optimums.

![Figure 6-1 Global Optimization in Brute Force Method](image-url)
6.3 Brute Force Method

Brute force approach is a direct method that uses computation power to display value of objective function for each individual input variable, until it finds out acceptable solution. Benefits of brute force can be the fact that, all possible states will be checked with quite simple implementation, though there are some draw backs as well. This method discovers the optimum solution among all other calculated data, therefore it is hard to apply this method for problems with higher computational needs as calculation states rises exponentially with number of new dimensions. This unsophisticated method have problems with special system with non-continues variables which is impossible to converge to the optimum value or systems with multiple interrupted local optima, unbalanced and complex functions.

A random Brute force search can be used as reference method to discover possible answers then answers will be examined in more detailed calculation and more advanced methods, in other words brute force method find a point close to optimum solution which is usually uncertain to actually determine the best solution (global optimum). To overcome drawbacks it is possible that algorithm will be followed by Newton’s method; though in this particular case answers very close to optimum solution are acceptable.

A display of income from produced methane in reactor based on in feed flow rate is shown in Figure 6-2. In order to obtain maximum value of income from biogas production, global optimum must be determined and as it can be seen from simulated plot local optimum may be found before global optimum. However in brute force whole range of methane production will be considered and maximum value for entire function will be revealed.

![Income from Produced Methane vs Feed Flow](image-url)
7 Results

In thesis complete economic analyses applied to evaluate cost benefits of anaerobic digestion process. This is initial step to build optimization function for using numerical optimization method to increase process economic performance. Results of economic optimization for anaerobic digestion reactor are divided into sub groups. In each part, one or two optimization variable has been selected to uncover optimum reactor design and operation principal. Objective functions are based on produced methane and economic criteria (NPV and revenue). In economic model various costs; such as depreciation, amortization, OPEX, CAPEC and income are combined to a single function and the effects of optimization variables such as temperature and reactor volume are investigated. The method for optimization was brute force method that guaranties to find global optimum with reasonable accuracy. In order to show time value of money the optimization procedure has been divided into two parts: process revenue, and net present value.
7.1 Optimized revenue

Yearly revenue shows income of process in each year after plant has been erected, optimization procedure for revenue starts with choosing variables for numerical algorithms. In Table 7-1 system performance has been depicted as a function of feed flow, but as it can be seen from calculated data biogas production has no effect on yearly revenue of total process. The reason that feed flow has less effect on economic performance is related to income from vermicompost and fertilizers which are not related directly to AD reactor; therefor feed flow hasn’t been selected as optimization variable.

Based on mathematical model for anaerobic digestion process it is possible to design an objective function for revenue, where all costs related to process conditions are taken into account. In this objective function the values for anaerobic digestion volume and reaction temperature are being changed and the optimum revenue will be determined. Prior to implement changes to reactor volume and reaction temperature, their effect on other processes in the system must be recognized and mathematical form for revenue must consider their influence on further variables. For example reactor size (volume) has direct effect on insurance and maintenance cost; moreover reaction temperature disturbs both heating energy consumptions and produced biogas. These precautions lead to objective function described in section 5.2.

### Table 7-1 Total Economic Performance Based on Inflow Feed

<table>
<thead>
<tr>
<th>Feed flow [L/d]</th>
<th>1000</th>
<th>2000</th>
<th>3000</th>
<th>4000</th>
<th>5000</th>
<th>6000</th>
<th>7000</th>
<th>8000</th>
</tr>
</thead>
<tbody>
<tr>
<td>OPEX [NOK]</td>
<td>188435</td>
<td>190516</td>
<td>192597</td>
<td>194678</td>
<td>196759</td>
<td>198840</td>
<td>200921</td>
<td>203003</td>
</tr>
<tr>
<td>Income Biogas [NOK]</td>
<td>2800</td>
<td>5090</td>
<td>6862</td>
<td>7949</td>
<td>8013</td>
<td>6552</td>
<td>2742</td>
<td>39</td>
</tr>
<tr>
<td>Total Income [NOK]</td>
<td>216325</td>
<td>432140</td>
<td>647437</td>
<td>862049</td>
<td>1075638</td>
<td>1287702</td>
<td>1497417</td>
<td>1708239</td>
</tr>
<tr>
<td>Revenue [NOK]</td>
<td>-797504</td>
<td>-583770</td>
<td>-370554</td>
<td>-158023</td>
<td>53485</td>
<td>263467</td>
<td>471102</td>
<td>679842</td>
</tr>
</tbody>
</table>

The optimum values for objective variables could be revealed with brute force optimization method, where all possible answers to problem will be weighted out and maximum economic performance will be shown as optimum revenue. The complete code and detailed calculations are available in Appendix 3 implemented to a Matlab code. Running code will show value of 401195.21 [NOK] for yearly revenue after optimization which was originally calculated in initial Matlab script to be 263467.21 [NOK]. Simple calculation shows amount 137728 [NOK] extra profits could be improved by optimization of reactor size and temperature; however more detailed economic measurements are needed to describe system performance (section 7.3).
7.2 Optimized Net Present Value

Although optimizing yearly revenue gives a good insight for optimization of AD process, it is more beneficial to investigate economic performance of process in connection with time; therefore an optimization objective function is modeled for NPV. To maximize net present value of anaerobic digestion process, same optimization variables of reaction temperature and reactor volume has been selected. Simulation of AD reactor is based on Hill’s model with constant value for feed flow and other process variables such as b (SRT/HRT) is assumed to be constant, also same as revenue optimization heat exchanger is assumed to be perfect with highest efficiency to recover heat from effluent flow. NPV objective function has been formed based on economic model discussed in section 5.2, however NPV calculation is upgraded with methods discussed in section 4.4.1 and 10% value of rate of return. Result of NPV optimization is interesting when comparing to yearly revenue results.

Table 7-2 Economic Performance (NPV) Optimized

<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>4</th>
<th>7</th>
<th>10</th>
<th>13</th>
<th>17</th>
<th>21</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPV</td>
<td>-2975</td>
<td>-2380</td>
<td>-1932</td>
<td>-1597</td>
<td>-1344</td>
<td>-1102</td>
<td>-936</td>
<td>-849</td>
</tr>
<tr>
<td>Optimized value</td>
<td>-2282</td>
<td>-1659</td>
<td>-1191</td>
<td>-841</td>
<td>-577</td>
<td>-323</td>
<td>-150</td>
<td>-59</td>
</tr>
<tr>
<td>Recovered Cost [KKnOK]</td>
<td>693</td>
<td>720</td>
<td>741</td>
<td>756</td>
<td>768</td>
<td>779</td>
<td>786</td>
<td>790</td>
</tr>
</tbody>
</table>

As it can be seen from Table 7-2, optimization increases net present value as times goes by, in other words it shows how yearly income effect economic performance due time. Result of NPV optimization is interesting when comparing to yearly revenue results in section 7.1, where it shows that recovered benefits is not significant in distant prospect, due to reduction in value of money in future.
7.3 Economic evaluation

Once CAPEX, OPEX and revues are estimated, it is possible to provide information that can be used to achieve objective function in economic optimization problem. In process evaluation step, ways of contribution that depict net income of process will be discussed. Important criteria which are discussed in evaluation will be; cash flow diagrams, interest and rate of return. It is possible to include tax in the project evaluation, though in this thesis no tax factor has been taken into account.

7.3.1 Process revenue

By using already made mathematical model (Hill 1983) for AD reactor and logical initial condition for process shown in Table 7-3 where the values can be changed to other desired variables based on initial design criteria with implementation in Matlab code which is available in Appendix 3. The input values for anaerobic digestion are based on real time running processes, for example ratio of height to diameter could be equal to unity but it is not common to produce such a reactor.

Table 7-3 Process Inputs for Analysing Revenue

<table>
<thead>
<tr>
<th>Process inputs</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>CH4 fraction in biogas</td>
<td>CH4</td>
<td>0.72</td>
</tr>
<tr>
<td>Reactor size</td>
<td>m3</td>
<td>8.6</td>
</tr>
<tr>
<td>Feed flow</td>
<td>Litter/day</td>
<td>6000</td>
</tr>
<tr>
<td>Hydraulic retention time</td>
<td>day</td>
<td>2.05</td>
</tr>
<tr>
<td>Ratio height to diameter</td>
<td>m/m</td>
<td>2</td>
</tr>
<tr>
<td>Ambient temperature</td>
<td>°C</td>
<td>10</td>
</tr>
<tr>
<td>Feed flow temperature</td>
<td>°C</td>
<td>10</td>
</tr>
<tr>
<td>Reaction temperature</td>
<td>°C</td>
<td>35</td>
</tr>
</tbody>
</table>
Table 7-4 shows values that have been calculated to uncover yearly revenue of process, data has been presented in chapter 4 as a function of CAPEX, OPEX, income and cost of insurance (insurance factor=0.1). Presented data could be found in Appendix 3, these calculations are based on estimation indexes for equipment cost.

Table 7-4 Yearly Revenue of Anaerobic Digestion Reactor

<table>
<thead>
<tr>
<th>Measured variables</th>
<th>NOK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Produced biogas (L/d)</td>
<td>8616</td>
</tr>
<tr>
<td>Produced methane (L/d)</td>
<td>11966</td>
</tr>
<tr>
<td>Insurance</td>
<td>29996</td>
</tr>
<tr>
<td>Amortization</td>
<td>388462</td>
</tr>
<tr>
<td>Depreciation</td>
<td>333870</td>
</tr>
<tr>
<td>Labour cost</td>
<td>100000</td>
</tr>
<tr>
<td>Maintenance</td>
<td>74718</td>
</tr>
<tr>
<td>Heating power</td>
<td>6465</td>
</tr>
<tr>
<td>Rotating device power</td>
<td>6898</td>
</tr>
<tr>
<td>Methane</td>
<td>6552</td>
</tr>
<tr>
<td>Vermicompost</td>
<td>591300</td>
</tr>
<tr>
<td>Fertilizer</td>
<td>689850</td>
</tr>
<tr>
<td>Yearly revenue</td>
<td>347294.4</td>
</tr>
</tbody>
</table>

Cash flow shows the amount of funds into treasury of owner, as a result of process activity and income. Figure 7-1 depicts each year’s net profit minus depreciation charges for the particular year. Due to dependency of lifetime of the process, it is assumed that reactor will produce biogas for 24 years; also it is assumed that it takes 1 year to erect whole equipment and the process will yield products at second year.

![Cash Flow (no index)](image)

*Figure 7-1 Yearly Accumulated Cash Flow (Equipment Cost Estimation with no Economic Index Included)*
Once economic indexes are applied to equipment cost estimation data, cash flow diagram revised Figure 7-2. As it can be seen from diagram it takes up to 15 years until the revenue of process will be positive, it means just after 15 years the process give profit, yet these data must be converted to up to date equivalent as time value of money is vital concept.

![Economic index included](image1)

**Figure 7-2 Yearly Accumulated Cash Flow (Equipment Cost Estimation with Economic Index Included)**

The effect of applying optimization criteria to design and operation of AD reactor has been represented in Figure 7-3. By paying off investment cost the profits rise significantly by time, this growth get to 909,870 [NOK] money recoveries after process lifetime. This can be verification to defined optimal solution as it can improve process economic performance, although it cannot be claimed that it is most perfect process design because there might be more cost recovery by changing some assumptions in initial constrains.

![Cost Recovered by Optimization](image2)

**Figure 7-3 Recovered Income after Applying Optimization Criteria**
7.3.2 NPV, IRR and cash flow

Time value of money for AD process has been taken into account in three diverse yet related methods, as it has explained in section 4.4.1 net present value is powerful method to include interest rates with economic calculation. The NPV with highest positive value is more profitable than any other choice; therefore NPV can be an accuracy measurement for optimized process design. In Table 7-5 net present value shows high profits for optimized design, which is a good sign of accuracy in optimization procedure. The power of economic indexes are more magnified as an estimation alternative; therefore it can be seen the importance of uncertainties in economic estimation and sources which has been provided to calculate prices.

*Table 7-5 Net Present Value for Alternative Design and Assumptions*

<table>
<thead>
<tr>
<th>Year (n) (P=10%)</th>
<th>No index</th>
<th>Optimized no index</th>
<th>Economic index included</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAPEX</td>
<td>-945207</td>
<td>-945207</td>
<td>-2997157</td>
</tr>
<tr>
<td>OPEX</td>
<td>-1024234</td>
<td>-1024234</td>
<td>-1024234</td>
</tr>
<tr>
<td>INCOME</td>
<td>1287702</td>
<td>1302459</td>
<td>1281150</td>
</tr>
<tr>
<td>FACTOR</td>
<td>1/((1+p)^n)</td>
<td>1/((1+p)^n)</td>
<td>1/((1+p)^n)</td>
</tr>
<tr>
<td>NPV</td>
<td>-945207</td>
<td>-945207</td>
<td>-2997157</td>
</tr>
<tr>
<td>NPV(10 years)</td>
<td>673685</td>
<td>764365</td>
<td>-1662045</td>
</tr>
<tr>
<td>NPV(24 years)</td>
<td>1421979</td>
<td>1554573</td>
<td>-1100866</td>
</tr>
<tr>
<td>First Positive revenue (n)</td>
<td>53541 (n=5)</td>
<td>109484 (n=5)</td>
<td>n=NoN</td>
</tr>
</tbody>
</table>

Cash flow diagrams will not be accurate without present worth and time value of money consideration, therefore in Figure 7-4 cash flow patterns calculate profitability of process in numerically, it can be surprising to observe continuously negative value for cash flows in Figure 7-5 which means no economic benefits will be with this particular assumption and process design. This result might be more interesting as it is in apposition with other calculations such as Figure 7-2, but this phenomena can explained as; applying time value of money will decrease benefits in far future. Regarding to this data plant will lose up to 563,416 [NOK] after 24 years of running, which can be result in total failure.
Optimization of reactor design and operation leads to wide-ranging recovery of cash flow, the effect of optimization has not decreased by time value of money and after 24 years, unit owner will benefit 337120 [NOK]. Figure 7-6 simulate the optimized criteria’s effect on the process, therefore optimization algorithms can be verified to be correct, yet it is still possible to improve optimization with more investigation into assumptions and constraints. This will be discussed in chapter 8 in more details.
Another approach to show process design economic performance is to transmit costs with internal rate of return. Yield of initial investment can be seen in Figure 7-7 where it is possible to see how optimized condition will benefit economically. When considering loans and saving for the anaerobic digestion process it is vital to calculate internal rate of return, therefore it is wise to relate calculations based on authentic data sources as it can be seen in Figure 7-7 there is much different patterns based on economic analyses and sources.

![Internal Rate of Return](image)

*Figure 7-7 Internal Rate of Return for Discounted Cash Flow*
8 Discussion

Models for anaerobic digestion technology show a promising performance in producing valuable products from animal manure waste. Cost analyses for process design shows economic profits based on problem assumptions, in other words estimation data are referable just for same operation conditions. Economic model is established in financial routine for anaerobic digestion process involving CAPEX, OPEX and income of process unit. Economic analyses are based on pilot reactor cost data which needs to be adopted for actual plant to be successful in predicting total performance, though in cost estimation part in economic model, results has been approved with online plant design tools (Peters, 2004). To claim full accuracy of economic model, more detailed calculations are necessary, a suggestion is to adopt model in Aspen Process Economic Analyzer. Once economic model is established three more models are used in optimization of AD reactor. Three models are AD process dynamic model (modified Hill’s model), reactor temperature model (energy balance) and heat exchanger model (mass and energy balance for steady state assumption).

When relating optimization objective function to models it is important to consider assumption for model utilization, for example: modified Hill’s model is unable to take complicated reactor influent related constrains into account; therefore in actual process there might be some unexpected behavior of models, although model performance has been previously adopted to pilot plant successfully (Haugen, 2013a). Economic optimization shows increased profits in process performance, therefore for new design criteria and operating condition additional income will be achieved. Results of reactor design can be compared to (Ghafoori, 2007) where analyses shows no profits for surplus power in anaerobic digestion of animal manure and shows finish price of biogas from AD reactors is very higher than existing power prices.

Net present value displays time aspect economic profitability of AD process has measured to be not very significant and it is predicted that the process will fail to satisfy unit owner’s economical expectations. This decision has made based on economic criteria of scope analyses, which reveals high risk investment compiled with very high capital cost. In other words to justify anaerobic digestion unit it may be favorable to consider other positive side effects of process such as pathogen control and odor control.

In this thesis the model principally studies anaerobic digestion reactor design and operation condition, though to be more confident in showing economic performance of entire unit, more broad view is required. For example to discover fertilizer production rate and value, it is vital to consider nitrification reactor after anaerobic digestion reactor, furthermore adding data of nitrification process to economic analyses will unveil phosphate, nitrogen and potassium recovery from digestate. Entire process cost evaluation suggests more operational costs such as biogas cleaning cost, gas compression power cost, gas transportation and pipeline.
9 Conclusion and future work

This thesis finds out that if the feed flow is continuously available, anaerobic digestion process can be promising technology to produce sustainable energy and products, though in certain conditions the overall process is projected to lose money, and this is due to economic indexes and financial factors. For specific estimations this study shows poor financial performance including low methane production in low energy prices; on the other hand capital cost of this technology is noticeably high and it is not possible to reduce some essential costs through optimization.

Finally it is concluded that estimations for process equipment are playing a vital rule in economic performance; therefore cost estimation must be based on more precise source data. Total yearly revenue and internal rate of return shows clear financial feasibility of process and optimization shows sensible benefits in cash flow diagrams. Optimization methods estimated surplus yearly revenue at optimum will be 16875.06 [NOK] and total process will start to have positive revenue after year 5 on certain conditions with no extra economic indexes.

This thesis could be promoted to general biogas plant with verity of organic inflow feed, with more comprehensive model to predict biochemical processes; such mathematical model may consider more constraints and scenarios. A nominee future work is to define a mathematical model to describe nitrification step after AD reactor, since by modeling this step total process economic analyses would have more realistic input. Moreover it is possible to enhance optimum economic with combination of designated optimized values with commercial total plant simulator software. As a final point more precise economic analyses could be achieved by referring analyses on geographic economic bonuses such as: tax rebates and higher values for renewable energy.
References


Hill, D.T., Barth, C.L. 1977. A dynamic model for simulation of animal waste digestion. Water Pollution Control Federation, 49.


Magnusson, M. and Alvfors Per 2012. Biogas from mechanical pulping industry: Potential improvement for increased biomass vehicle fuels. In: Kth, School of Chemical Science and Engineering (Che) (ed.) The 25th international conferences on efficiency, cost, optimization, simulation and environmental impact of energy systems. Perugia Italy.


Appendix 1

Task Description:

Telemark University College
Faculty of Technology

FMH606 Master's Thesis

Title: Optimization of design and operation of anaerobic digestion reactors

TUC supervisor: Associate Professor Finn Haugen

External partner:

Task description:
The main aim of the thesis is to define useful optimization criteria for optimization of anaerobic digestion (AD) reactors, and to solve the optimization problems using numerical optimization algorithms. Alternative criteria can be defined, related to for example power surplus and economical surplus. An existing mathematical model which already has been adapted to a real pilot AD reactor, should be used to obtain the optimal solutions. The optimal solutions should be verified with simulations.

Task background:
AD reactors are used to produce biogas from organic substrates of various types. The main part of biogas is methane. If the methane concentration is above a certain limit, the biogas is combustible due to the methane contents. The combustion produces (releases) power. In general, power has an economical value. Since power is needed to operate the reactor, e.g. for operation of machinery, and to compensate for heat losses, etc., some optimal reactor design exist, and, for a given reactor design, some optimal reactor operation condition exists. It is of practical interest to identify the optimal design and optimal operation conditions.

Student category:
SCE or EET or PT student. If necessary, the student must acquire the necessary knowledge about optimization theory and methods during the first part of the thesis period.

Practical arrangements:
The thesis will be accomplished at TUC.

Signatures:
Student (date and signature):
Supervisor (date and signature):
Thesis code: SIV-22-44
Appendix 2

Heat Exchanger Purchase Data:

HEAT-CON AS
Professor Birkelands vei 24b, B4
1081 OSLO
Tlf.: +47 23141880  Fax.: +47 23141889
e-mail: heat-con@heat-con.no  Internet: www.heat-con.no

Til: TECHCONSULTANTS AS
Att: Ole J. Sivertsen
Dato: 11.06.2012
Vår ref.: T.10.234/03 rørveksler
Deres ref.: Oppvarming av dekantert avløp fra gris.
Vi takker for Deres forespørsel samt e-mail 20. september, og reviderer vårt tilbu iht. Orgalime 2000:
4 m³/h ~5kW
1 stk. XTube M-51/25-0,7-316L-316L-C monotube, med data som vist på vedlegte datablad
   Diameter 51 mmx Total lengde 700mm
Pris NOK 9.700,- /stk
For denne lille ytelsen vil det passe best med monotube løsning. Det vil si 1 stk. innerør i 1 stk. ytterrør
Prisen er å forstå Fra lager Oslo, eks. MVA.
Prisen bygger på dagens kurs EUR/NOK= 7,6  og avvik større enn +/- 3% gir rett til prisjustering
Leveringstid: 5 arbeidsuker fra fabrikk etter godkjent tegning.
Betaling: netto pr. 30 dager
Tilbudet er gyldig i 30 dager.
Vi håper at tilbudet er av interesse og at det kan lede til videre diskusjoner.
Med vennlig hilsen
HEAT-CON Varmeteknikk AS

Øyvind Hansen
oeyvind.hansen@heat-con.no
Dimension and Footprint of Heat Exchanger (Heat-con AS)
Appendix 3

Hill's AD Model in Matlab

%Calculations implemented in Matlab
%---------------------------------------------------
clear all
close all
format bank
format compact
%---------------------------------------------------
%MODEL PARAMS FOR HILL's MODEL:
b = 3.22;
K_s = 24.9;
K_sc = 3;
K_d = 0.02;
K_dc = 0.02;
K_i=9;
K_ic=11;
k1 = 3.3;
k2 = 0.12;
k3 = 31.7;
k5 = 34.1;
frac_ch4=0.72
B0=0.25;
Af=0.69;
%...........................
%Reactor volume:
Vol_ADR_m3=8.6
%...........................
V=Vol_ADR_m3*1000;
g=9.81;%Gravity
sieve_frac=0.7;
%---------------------------------------------------
%CALCULATION OF REACTOR DIMENSIONS:
%Calculation of area and volume for pilot reactor:
d_pilot=0.4;%m
L_pilot=2.2;%m
A_pilot=L_pilot*pi*d_pilot+2*pi*(d_pilot^2)/4;
V_pilot_m3=(pi/4)*(d_pilot^2)*L_pilot;
%Calculation of area and volume for actual reactor:
%optimization shows that Ratio_h_d=1 has best economic output but due to realistic design Ratio_h_d=2 has been selected :
d=power((Vol_ADR_m3/(pi*Ratio_h_d/4)),1/3);
h=Ratio_h_d*d;
\( A = h \pi d + 2 \pi (d + d)/4; \)

% AD REACTOR PROCESS
% REACTOR INPUTS:
T_room=10;
T_feed=10;
T_reac=35;
F_feed_raw_L_d=6000;
F_feed=F_feed_raw_L_d*sieve_frac;
F_feed_norm=F_feed/V;
HRT_days=V/F_feed
%---------------------------
% SIMULATION USING HILL's MODEL:
S_vs_in=30.4;
S_bvs_in=B0*S_vs_in;
S_vfa_in=Af*S_bvs_in;

S_bvs_init=6.0;
S_vfa_init=0.72;
X_acid_init=1.08;
X_meth_init=0.32;

S_bvs_k=S_bvs_init;
S_vfa_k=S_vfa_init;
X_acid_k=X_acid_init;
X_meth_k=X_meth_init;

Ts=0.05;
t_start=0;
t_stop=300;
N=(t_stop-t_start)/Ts;
t_k=t_start;
for k=1:N
  t_kp1=t_k+Ts;
  t(k)=t_k;
  t_k=t_kp1;

mu_m=0.013*T_reac-0.129;
mu_mc=mu_m;
mu=mu_m/(K_s/S_bvs_k+1+0*S_vfa_k/K_i);
mu_c=mu_mc/(K_sc/S_vfa_k+1+0*S_vfa_k/K_ic);

F_meth_k=V*k5*mu_c*X_meth_k;
F_biogas_k=F_meth_k/frac_ch4;

dS_bvs_dt_k=(S_bvs_in-S_bvs_k)*F_feed/V-mu*kl*X_acid_k;
\[ dS_{\text{vfa}}_{\text{dt}_k} = (S_{\text{vfa}} - S_{\text{vfa}}_k) * \frac{F_{\text{feed}}}{V} + \mu_k * k_2 * X_{\text{acid}}_k - \mu_k * k_3 * X_{\text{meth}}_k; \]
\[ dX_{\text{acid}}_{\text{dt}_k} = (\mu_k - K_d - (F_{\text{feed}}/b)/V) * X_{\text{acid}}_k; \]
\[ dX_{\text{meth}}_{\text{dt}_k} = (\mu_k - K_d - (F_{\text{feed}}/b)/V) * X_{\text{meth}}_k; \]

\[ S_{\text{bvs}}_{k+1} = S_{\text{bvs}}_k + T_s * dS_{\text{bvs}}_{\text{dt}_k}; \]
\[ S_{\text{vfa}}_{k+1} = S_{\text{vfa}}_k + T_s * dS_{\text{vfa}}_{\text{dt}_k}; \]
\[ X_{\text{acid}}_{k+1} = X_{\text{acid}}_k + T_s * dX_{\text{acid}}_{\text{dt}_k}; \]
\[ X_{\text{meth}}_{k+1} = X_{\text{meth}}_k + T_s * dX_{\text{meth}}_{\text{dt}_k}; \]

\[ S_{\text{bvs}}_{\text{sim}}(k) = S_{\text{bvs}}_k; \]
\[ S_{\text{vfa}}_{\text{sim}}(k) = S_{\text{vfa}}_k; \]
\[ X_{\text{acid}}_{\text{sim}}(k) = X_{\text{acid}}_k; \]
\[ X_{\text{meth}}_{\text{sim}}(k) = X_{\text{meth}}_k; \]
\[ F_{\text{meth}}_{\text{sim}}(k) = F_{\text{meth}}_k; \]

\[ S_{\text{bvs}}_k = S_{\text{bvs}}_{k+1}; \]
\[ S_{\text{vfa}}_k = S_{\text{vfa}}_{k+1}; \]
\[ X_{\text{acid}}_k = X_{\text{acid}}_{k+1}; \]
\[ X_{\text{meth}}_k = X_{\text{meth}}_{k+1}; \]

\textit{end}

\textit{Steady-state sim values:}
S_{\text{bvs}} = S_{\text{bvs}}_{\text{sim}}(N);
S_{\text{vfa}} = S_{\text{vfa}}_{\text{sim}}(N);
X_{\text{acid}} = X_{\text{acid}}_{\text{sim}}(N);
X_{\text{meth}} = X_{\text{meth}}_{\text{sim}}(N);
F_{\text{meth}}_{\text{L}_d} = F_{\text{meth}}_{\text{sim}}(N)

\textit{Various steady-state values:}
Gas\_prod\_eff = F_{\text{meth}}_{\text{L}_d}/F_{\text{feed}};
F_{\text{biogas}}_{\text{L}_d} = F_{\text{meth}}_{\text{L}_d}/\text{frac\_ch4};
F_{\text{meth}}_{\text{normalized}} = F_{\text{meth}}_{\text{L}_d}/V;
F_{\text{biogas}}_{\text{normalized}} = F_{\text{biogas}}_{\text{L}_d}/V;
CAPEX Model in Matlab:

%Calculations implemented in Matlab
%Economic analysis of AD reactor (ADR) with complete economic indexes
%based on data received from pilot reactor
%Financial unit is NOK.
%After discussion about NR reactor, it is decided to eliminate cost data
%related to this part of process
%To run this code correctly, it must proceed after running Hill’s Model
%code
%----------------------------------
clear all
close all
format bank

format compact
%---------------------------

%CAPEX COSTS (CAPITAL COST):
Cost_buildings=500000;
Cost_constr_pilot_ADR=14600;
Cost_constr_pilot_ADR_per_area_m2=Cost_constr_pilot_ADR/A_pilot;

%Ratio factors for estimating capital investment item are applies
%direct cost index = Equipment + erection + piping + electric + instrument + civil work +
% steel & Concrete + insulation
Direct_cost_index_AD_reactor = 4.43;
% engineering cost = process + mechanical + piping + electric + instrument + civil
%+ steel & concrete + insulation
Engineering_cost_AD_reactor = 1.09;
% administration cost = procurement + project control + site management +
project management
Administration_cost_index_AD_reactor = 0.66;
Contingency_cost_index_AD_reactor = 1.4;
Reactor_cost_index=...

   Direct_cost_index_AD_reactor...
+ Engineering_cost_AD_reactor...
+ Administration_cost_index_AD_reactor...
+ Contingency_cost_index_AD_reactor...

Cost_constr_ADR=Cost_constr_pilot_ADR_per_area_m2*A*Reactor_cost_index;
%Cost_constr_DNR=Cost_constr_ADR;

%direct cost index = Equipment + erection + piping + electric + instrument +
civil work +
% steel & Concrete + insulation
Direct_cost_index_feed_sieve = 4.43;
% engineering cost = process + mechanical + piping + electric + instrument +
civil
%+ steel & concrete + insulation
Engineering_cost_feed_sieve = 1.09;
% administration cost = procurement + project control + site management + project management
Administration_cost_index_feed_sieve = 0.66;
Contingency_cost_index_feed_sieve = 1.4;
feed_sieve_cost_index = ... 
  Direct_cost_index_feed_sieve...
  + Engineering_cost_feed_sieve...
  + Administration_cost_index_feed_sieve...
  + Contingency_cost_index_feed_sieve...

Cost_feed_sieve = 100000 * feed_sieve_cost_index;

% direct cost index = Equipment + erection + piping + electric + instrument + civil work +
% steel & Concrete + insulation
Direct_cost_index_feed_pump = 6.53;
% engineering cost = process + mechanical + piping + electric + instrument + civil
%+ steel & concrete + insulation
Engineering_cost_feed_pump = 2.11;
% administration cost = procurement + project control + site management + project management
Administration_cost_index_feed_pump = 1.21;
Contingency_cost_index_feed_pump = 2.28;
feed_pump_cost_index = ... 
  Direct_cost_index_feed_pump...
  + Engineering_cost_feed_pump...
  + Administration_cost_index_feed_pump...
  + Contingency_cost_index_feed_pump...

Cost_feed_pump = 30000 * feed_pump_cost_index;

% direct cost index = Equipment + erection + piping + electric + instrument + civil work +
% steel & Concrete + insulation
Direct_cost_index_screw_pump = 10.6;
% engineering cost = process + mechanical + piping + electric + instrument + civil
%+ steel & concrete + insulation
Engineering_cost_screw_pump = 5.83;
% administration cost = procurement + project control + site management + project management
Administration_cost_index_screw_pump = 2.74;
Contingency_cost_index_screw_pump = 4.48;
screw_pump_cost_index = ...
  Direct_cost_index_screw_pump...
  + Engineering_cost_screw_pump...
  + Administration_cost_index_screw_pump...
  + Contingency_cost_index_screw_pump;

Cost_supply_screw_pump = 13000 * screw_pump_cost_index;
%direct cost index = Equipment + erection + piping + electric + instrument + civil work + steel & Concrete + insulation

Direct_cost_index_heat_exchanger = 10.6;
% engineering cost = process + mechanical + piping + electric + instrument + civil + steel & concrete + insulation

Engineering_cost_heat_exchanger = 5.83;
% administration cost = procurement + project control + site management + project management

Administration_cost_index_heat_exchanger = 2.74;
Contingency_cost_index_heat_exchanger = 4.48;

heat_exchanger_cost_index= ...
   Direct_cost_index_heat_exchanger...
   + Engineering_cost_heat_exchanger...
   + Administration_cost_index_heat_exchanger...
   + Contingency_cost_index_heat_exchanger;

Cost_temp_sensor_per_sensor=1000;
   Num_of_temp_sensors=4;
   Cost_temp_sensors=Cost_temp_sensor_per_sensor*Num_of_temp_sensors;
Cost_feedflow_sensor=3000;
Cost_gasflow_sensor=13000;
   Cost_gas_conc_sensor_per_sensor=5000;
   Num_of_gas_conc_sensors=2;
   Cost_gas_conc_sensors=Cost_gas_conc_sensor_per_sensor*Num_of_gas_conc_sensors;
%Cost_pH_and_DO_sensor=15000;
Cost_computer=5000;
Cost_control_hardware=10000;
Cost_monitoring_and_control_software=20000;
Cost_labour_constr_ADR=100000;
%Cost_labour_constr_DNR=100000;
Cost_labour_monitoring_and_control_development=10000;

Cost_constr_reactor=Cost_labour_constr_ADR;

Total_cost_constr_and_equip= ...
   Cost_buildings...
   +Cost_constr_ADR...
   +Cost_feed_sieve...
   +Cost_feed_pump...
   +Cost_supply_screw_pump...
   +Cost_temp_sensors...
   +Cost_feedflow_sensor...
   +Cost_heat_exchanger...
   +Cost_gasflow_sensor...
% insurance costs per year:
Insurance_frac=0.01;
Cost_insurance_year=Insurance_frac*Total_cost_constr_and_equip

% amortization costs:
Time_amortization_years=10;
   n=Time_amortization_years;
Rate_of_interest=0.05;
   i=Rate_of_interest;
P=Total_cost_constr_and_equip;
r=(i*(1+i)^n)/((1+i)^n-1); \textit{r} = \textit{Repayment multiplier}=0.13
F=P*r;
Total_cost_amortization_year=F

% depreciation costs:
Time_depreciation_building_years=20;
Cost_depreciation_buildings_year=Cost_buildings/Time_depreciation_building_years;

Cost_depreciation_reactor_years=10;
Cost_depreciation_reactor_year=Cost_constr_reactor/Time_depreciation_reactor_years;

Cost_equipment=...
   Cost_feed_sieve...
   +Cost_feed_pump...
   +Cost_supply_screw_pump...
   +Cost_heat_exchanger...
   +Cost_temp_sensors...
   +Cost_feedflow_sensor...
   +Cost_gasflow_sensor...
   +Cost_gas_conc_sensors...
   +Cost_computer...
   +Cost_control_hardware...
   +Cost_monitoring_and_control_software
Time_depreciation_equipment_years=5;
Cost_depreciation_equipment_year=Cost_equipment/Time_depreciation_equipment_years;

Total_cost_depreciation_year=...
Cost_depreciation_buildings_year...
+Cost_depreciation_reactor_year...
+Cost_depreciation_equipment_year

Total_capital_cost_year=...
Total_cost_amortization_year...
+Total_cost_depreciation_year
OPEX Model in Matlab

%OPERATIONAL COSTS PER YEAR:
%Variable Costs calculations implemented in Matlab with combination of Heat
%exchanger model

%Economic analysis of AD reactor (ADR)
%based on data received from pilot reactor
%financial unit is NOK.
%after discussion about NR reactor, it is decided to eliminate cost data
%related to this part of process
%to run this code correctly, it must proceed after running CAPEX code
%Labour yearly cost:
Cost_labour_year=100000;

%Total maintenance is selected to be 0.05 (Turton 2012):
Equip_maintenance_frac_of_investment_year=0.05;
Cost_maintenance_year=Equip_maintenance_frac_of_investment_year*Cost_equipment;

%Thermal loss with perfect heat exchanger:
k_heat_ex=0; %Value 0 means ideal heat ex between liquid effluent and
influent assuming equal flows.
T_influent=((k_heat_ex+1)/(k_heat_ex+2))*T_feed+(1/(k_heat_ex+2))*T_reac;
c=4200;%J/(kg*K)
rho=1000;%kg/m3
G_J_K_d_pilot=170789;%(J/d)*K. Found from least square estimation on pilot
reactor
Gs=G_J_K_d_pilot/A_pilot; %Heat conductivity per m2 for pilot reactor
Gs_kWh_K_d=Gs/3.6e6;%=0.0157
Gs_J_d_K_d2=Gs_kWh_K_d*3.6e6; %Heat conductivity per m2 for full-scale
reactor
G_J_K_d=Gs_J_d_K_d2*A;
Power_heater_J_day=-c*rho*(F_feed/1000)*(T_influent-T_reac) -
G_J_K_d*(T_room-T_reac);
Power_heater_kWh_day=Power_heater_J_day/3.6e6;
Power_heater_kWh_year=Power_heater_kWh_day*365

%Sieve:
Power_sieve_kWh_ton=5.81;
Power_sieve_kWh_L=Power_sieve_kWh_ton/1000;
Power_sieve_kWh_day=Power_sieve_kWh_L*F_feed_raw_L_d;
Power_sieve_kWh_year=Power_sieve_kWh_day*365;

%Screw pump:
Power_screw_pump_kW_L_d=0.000283;
Power_screw_pump_kWh_year=Power_screw_pump_kW_L_d*F_feed_raw_L_d*8760;
%1kW = 8760 kWh per year

%Vacuum pump:
Power_vacuum_pump_kW_L_d=0;
Power_vacuum_pump_kWh_year=Power_vacuum_pump_kW_L_d*F_feed_raw_L_d*8760;
%1kW = 8760 kWh per year

%Lifting feed:
Height_lift_feed_meters=2;
Power_peristaltic_feed_pump_kWh_year=((F_feed/1000)*rho*g*Height_lift_feed_meters)*365/3.6e6;
%1kWh=3.6e6J

%DNR Venturi pump:
%Power_Venturi_kWh_year=(250/1000)*(6/7)*8760;%kW. 1kW = 8760 kWh per year
%Knut: 250 w 6min on 1 min off eksoterm like stor reaktor

%Total power consumption:
Total_power_consumption_kWh_year=...
  Power_heater_kWh_year...
  +Power_sieve_kWh_year...
  +Power_screw_pump_kWh_year...
  +Power_vacuum_pump_kWh_year...
  +Power_peristaltic_feed_pump_kWh_year...

%Value of total power consumption:
Value_power_NOK_per_kWh=0.25
Total_cost_power_consumption_year=...
Total_power_consumption_kWh_year*Value_power_NOK_per_kWh

Total_cost_operations_year=...
  +Cost_labour_year...
  +Cost_maintenance_year...
  +Total_cost_power_consumption_year
Yearly Income and Revenue

% INCOMES PER YEAR:
% Biogas:
F_biogas_m3_day=F_biogas_L_d/1000;
k_eff_biogas_energy=1;
k_energy_biogas=6;%[kWh/m3] = Biogas energy constant = 21.6 kJ/L
Power_biogas_kWh_day=F_biogas_m3_day*k_energy_biogas*k_eff_biogas_energy;
Power_biogas_J_day=Power_biogas_kWh_day*3.6e6;
Income_biogas_year=Value_power_NOK_per_kWh*Power_biogas_kWh_day*365;
% Vermicompost:
Value_vermicompost_per_L=900/1000;%900 NOK per L
Income_vermicompost_year=Value_vermicompost_per_L*F_feed_raw_L_d*(1-
sieve_frac)*365;
% Fertilizers:
Value_fertilizers_per_L=450/1000;%450 NOK per L
Income_fertilizers_year=Value_fertilizers_per_L*F_feed*365;
% Total income:
Total_income_year=Income_biogas_year+Income_vermicompost_year+Income_fertilizers_year

% REVENUE PER YEAR:
Revenue_year=
-Cost_insurance_year...
-Total_capital_cost_year...
-Total_cost_operations_year...
+Total_income_year

% CAPEX Cost analyses in Matlab
CAPEX =
Cost_buildings+Cost_constr_pilot_ADR_per_area_m2*A+Cost_feed_sieve+Cost_feed_d_pump+Cost_supply_screw_pump+...
Cost_temp_sensors+Cost_feedflow_sensor+Cost_gasflow_sensor+Cost_gas_conc_sensors+Cost_computer+...
Cost_monitoring_and_control_software+Cost_control_hardware+Cost_labour_cons
tr_ADR+Cost_labour_monitoring_and_control_development

% OPEX Cost analyses in Matlab
OPEX =
Cost_insurance_year+Total_cost_amortization_year+Total_cost_depreciation_year+...
-Cost_labour_year+Cost_maintenance_year+Total_cost_power_consumption_year

% INCOME Cost analyses in Matlab
INCOME =
Income_biogas_year+Income_vermicompost_year+Income_fertilizers_year
Optimization of Revenue

% Economic Optimization for AD Reactor
% Calculations implemented in Matlab
% Optimization technique is Brute Force method.

% Inbuilt objective function (f) for temperature (j) and volume (i) as objective variables:

f=@(Vol_ADR_m3,T_reac) (-Insurance_frac*(Cost_buildings... 
  + Cost_constr_pilot_ADR_per_area_m2*A*Reactor_cost_index)... 
  + Cost_feed_sieve... 
  + Cost_feed_pump... 
  + Cost_supply_screw_pump... 
  + Cost_heat_exchanger... 
  + Cost_temp_sensors... 
  + Cost_feedflow_sensor... 
  + Cost_gasflow_sensor... 
  + Cost_gas_conc_sensors... 
  + Cost_computer... 
  + Cost_control_hardware... 
  + Cost_monitoring_and_control_software... 
  + Cost_labour_constr_ADR... 
  + Cost_labour_monitoring_and_control_development)... 
  - ((r*(Cost_buildings... 
  + Cost_constr_pilot_ADR_per_area_m2*A*Reactor_cost_index)... 
  + Cost_feed_sieve... 
  + Cost_feed_pump... 
  + Cost_supply_screw_pump... 
  + Cost_heat_exchanger... 
  + Cost_temp_sensors... 
  + Cost_feedflow_sensor... 
  + Cost_gasflow_sensor... 
  + Cost_gas_conc_sensors... 
  + Cost_computer... 
  + Cost_control_hardware... 
  + Cost_monitoring_and_control_software... 
  + Cost_labour_constr_ADR... 
  + Cost_labour_monitoring_and_control_development)... 
  + (Cost_depreciation_buildings_year... 
  + Cost_depreciation_reactor_year... 
  + Cost_depreciation_equipment_year)... 
  - (Cost_labour_year... 
  + Cost_maintenance_year... 
  + ((((-c*rho*(F_feed/1000)*(T_influent-j)-\(G_J K_d\)*(T_room-j))/3.6e6)*365)... 
  + Power_sieve_kWh_year... 
  + Power_screw_pump_kWh_year... 
  + Power_vacuum_pump_kWh_year- 
  Power_peristaltic_feed_pump_kWh_year)*0.25)...
\[
+(0.25 \cdot \frac{F_{\text{meth} L_d}}{\text{frac}_\text{ch4}}} /1000 \cdot k_{\text{energy biogas}} \cdot k_{\text{eff biogas energy}} \cdot 365 ) + \text{Income vermicompost year} + \text{Income fertilizers year} \); \\
\text{vmax} = -10000 ; \\
N = 100; \\
M = 100; \\
\text{for } i = 1:N \\
\quad \text{for } j = 10:M \\
\quad \quad b = 3.22; \\
K_s = 24.9; \\
K_{sc} = 3; \\
K_d = 0.02; \\
K_{dc} = 0.02; \\
K_i = 9; \\
K_{ic} = 11; \\
k1 = 3.3; \\
k2 = 0.12; \\
k3 = 31.7; \\
k5 = 34.1; \\
\text{frac}_\text{ch4} = 0.72; \\
B0 = 0.25; \\
Af = 0.69; \\
\%
\%...........................
\%
\%...........................
\text{V} = i \cdot 1000; \\
g = 9.81; \text{Gravity} \\
\text{sieve}_\text{frac} = 0.7; \\
\%-------------------------
\%CALCULATION OF REACTOR DIMENSIONS: \\
%Calculation of area and volume for pilot reactor: \\
d_{\text{pilot}} = 0.4; \text{m} \\
L_{\text{pilot}} = 2.2; \text{m} \\
A_{\text{pilot}} = L_{\text{pilot}} \cdot \pi \cdot d_{\text{pilot}} + 2 \cdot \pi \cdot (d_{\text{pilot}}^2)/4; \\
V_{\text{pilot}} = \pi (d_{\text{pilot}}^2) \cdot L_{\text{pilot}}; \\
%Calculation of area and volume for actual reactor: \\
R_{\text{atio h d}} = 2; \\
%optimization shows that Ratio_h_d=1 has best economic output but due to realistic design Ratio_h_d=2 has been selected: \\
d = \text{power}((i/(\pi \cdot R_{\text{atio h d}}/4)), 1/3); \\
h = R_{\text{atio h d}} \cdot d; \\
A = h \cdot \pi \cdot d + 2 \cdot \pi \cdot (d + d)/4; \\
%AD REACTOR PROCESS \\
%REACTOR INPUTS: \\
T_{\text{room}} = 10; \\
T_{\text{feed}} = 10; \\
%T_{\text{reac}} = 35; \\
F_{\text{feed raw L d}} = 6000; \\
F_{\text{feed}} = F_{\text{feed raw L d}} \cdot \text{sieve}_\text{frac}; \\
F_{\text{feed norm}} = F_{\text{feed}} / V; \\
HRT_{\text{days}} = V / F_{\text{feed}}; \\
\%------------------------
\[
\begin{align*}
S\text{\_vs\_in} &= 30.4; \\
S\text{\_bvs\_in} &= B0\cdot S\text{\_vs\_in}; \\
S\text{\_vfa\_in} &= Af\cdot S\text{\_bvs\_in}; \\
S\text{\_bvs\_init} &= 6.0; \\
S\text{\_vfa\_ini} &= 0.72; \\
X\text{\_acid\_init} &= 1.08; \\
X\text{\_meth\_init} &= 0.32; \\
\end{align*}
\]

\[
\begin{align*}
S\text{\_bvs\_k} &= S\text{\_bvs\_init}; \\
S\text{\_vfa\_k} &= S\text{\_vfa\_init}; \\
X\text{\_acid\_k} &= X\text{\_acid\_init}; \\
X\text{\_meth\_k} &= X\text{\_meth\_init}; \\
\end{align*}
\]

\[
\begin{align*}
Ts &= 0.05; \\
t\text{\_start} &= 0; \\
t\text{\_stop} &= 300; \\
N &= (t\text{\_stop} - t\text{\_start})/Ts; \\
t\text{\_k} &= t\text{\_start}; \\
\text{\bf for} \ k = 1:N \\
& \quad t\text{\_k} = t\text{\_k} + Ts; \\
& \quad t(k) = t\text{\_k}; \\
& \quad t\text{\_k} = t\text{\_k} + Ts; \\
\end{align*}
\]

\[
\begin{align*}
mu\text{\_m} &= 0.013 \cdot j - 0.129; \\
mu\text{\_mc} &= mu\text{\_m}; \\
mu &= mu\text{\_m}/(K_s/S\text{\_bvs\_k} + 1 + 0 \cdot S\text{\_vfa\_k}/K_i); \\
mu\text{\_c} &= mu\text{\_mc}/(K_sc/S\text{\_vfa\_k} + 1 + 0 \cdot S\text{\_vfa\_k}/K_ic); \\
F\text{\_meth\_k} &= V \cdot k5 \cdot mu\text{\_c} \cdot X\text{\_meth\_k}; \\
F\text{\_biogas\_k} &= F\text{\_meth\_k}/\text{frac\_ch4}; \\
dS\text{\_bvs\_dt\_k} &= (S\text{\_bvs\_in} - S\text{\_bvs\_k}) \cdot F\text{\_feed}/V - mu \cdot k1 \cdot X\text{\_acid\_k}; \\
dS\text{\_vfa\_dt\_k} &= (S\text{\_vfa\_in} - S\text{\_vfa\_k}) \cdot F\text{\_feed}/V + mu \cdot k2 \cdot X\text{\_acid\_k} - mu \cdot c \cdot k3 \cdot X\text{\_meth\_k}; \\
dx\text{\_acid\_dt\_k} &= (\mu - K_d - (F\text{\_feed}/b)/V) \cdot X\text{\_acid\_k}; \\
dx\text{\_meth\_dt\_k} &= (\mu\text{\_c} - K_{dc} - (F\text{\_feed}/b)/V) \cdot X\text{\_meth\_k}; \\
S\text{\_bvs\_k}\text{\_p1} &= S\text{\_bvs\_k} + Ts \cdot dS\text{\_bvs\_dt\_k}; \\
S\text{\_vfa\_k}\text{\_p1} &= S\text{\_vfa\_k} + Ts \cdot dS\text{\_vfa\_dt\_k}; \\
X\text{\_acid\_k}\text{\_p1} &= X\text{\_acid\_k} + Ts \cdot dx\text{\_acid\_dt\_k}; \\
X\text{\_meth\_k}\text{\_p1} &= X\text{\_meth\_k} + Ts \cdot dx\text{\_meth\_dt\_k}; \\
S\text{\_bvs\_sim}(k) &= S\text{\_bvs\_k}; \\
S\text{\_vfa\_sim}(k) &= S\text{\_vfa\_k}; \\
X\text{\_acid\_sim}(k) &= X\text{\_acid\_k}; \\
X\text{\_meth\_sim}(k) &= X\text{\_meth\_k}; \\
F\text{\_meth\_sim}(k) &= F\text{\_meth\_k}; \\
\end{align*}
\]

\[
\begin{align*}
S\text{\_bvs\_k} &= S\text{\_bvs\_k}\text{\_p1}; \\
S\text{\_vfa\_k} &= S\text{\_vfa\_k}\text{\_p1}; \\
X\text{\_acid\_k} &= X\text{\_acid\_k}\text{\_p1}; \\
X\text{\_meth\_k} &= X\text{\_meth\_k}\text{\_p1}; \\
\text{vmax1} &= f (i, j); \\
\text{\bf if} \ \text{vmax1} >= \text{vmax} ,
\end{align*}
\]
vmax = vmax1;
end
end
end
disp ('Revenue_after_optimization=')
disp(vmax)
Total_cost_operations_year_After_Optimization=Total_income_year-vmax-
Cost_insurance_year-TOTAL Capital_cost_year

Income_after_optimization=Income_vermicompost_year+Income_fertilizers_year+
Value_power_NOK_per_kWh*((X_meth_k/frac_ch4)/1000*k_energy_biogas)*k_eff_bi
ogas_energy*3.6e6*365
Optimization of NPV

% Net present value with economic index included
% NPV = CAPEX + Accumulated rate of return *(Income - OPEX)
% Optimization technique is Brute Force method.
% Discount factor assumed to be calculated for lifetime number
% n=24,21,17,13,10,7,4 years, rate of return P=10%
% Discount factor r=1/ (1+p)^n
% To run this code correctly, it must proceed after running all model codes
Accumulated_discount_factor_24=9.98-1;
Accumulated_discount_factor_21=9.65-1;
Accumulated_discount_factor_17=9.02-1;
Accumulated_discount_factor_13=8.1-1;
Accumulated_discount_factor_10=7.14-1;
Accumulated_discount_factor_7=5.87-1;
Accumulated_discount_factor_4=4.17-1;
Accumulated_discount_factor_1=1.91-1;
CAPEX =
(Cost_buildings+Cost_constr_ADR+Cost_feed_sieve+Cost_feed_pump+Cost_heat_exchanger...
+Cost_supply_screw_pump+Cost_temp_sensors+Cost_feedflow_sensor+Cost_gasflow_sensor...
+Cost_gas_conc_sensors+Cost_computer+Cost_control_hardware+Cost_monitoring_and_control_software...
+Cost_labour_constr_ADR+Cost_labour_monitoring_and_control_development)
Total_income_year=Income_biogas_year+Income_vermicompost_year+Income_fertilizers_year
OPEX=Cost_labour_year+Cost_maintenance_year+Total_cost_power_consumption_year...
+Total_cost_depreciation_year+Total_cost_amortization_year
NPV_24_Years_No_Index=CAPEX+Accumulated_discount_factor_24*(Total_income_year- OPEX)
NPV_21_Years_No_Index=CAPEX+Accumulated_discount_factor_21*(Total_income_year- OPEX)
NPV_17_Years_No_Index=CAPEX+Accumulated_discount_factor_17*(Total_income_year- OPEX)
NPV_13_Years_No_Index=CAPEX+Accumulated_discount_factor_13*(Total_income_year- OPEX)
NPV_10_Years_No_Index=CAPEX+Accumulated_discount_factor_10*(Total_income_year- OPEX)
NPV_7_Years_No_Index=CAPEX+Accumulated_discount_factor_7*(Total_income_year- OPEX)
NPV_4_Years_No_Index=CAPEX+Accumulated_discount_factor_4*(Total_income_year- OPEX)
NPV_1_Years_No_Index=CAPEX+Accumulated_discount_factor_1*(Total_income_year- OPEX)
% Objective function for optimization of NPV
%f=@(Vol_ADR_m3,T_reac)(CAPEX+Accumulated_discount_factor_24*(Total_income_year- OPEX))
%A= Anew=(5*pi/2)*(power((2*i/pi),2/3)) where i represents volume
f=@(Vol_ADR_m3,T_reac)(- (Cost_buildings+(Cost_constr_pilot_ADR_per_area_m2* ((5*pi/2)*(power((2*(Vol_ADR_m3)/pi),2/3))))*Reactor_cost_index)...

77
+Cost_feed_sieve+Cost_feed_pump+Cost_heat_exchanger...
+Cost_supply_screw_pump+Cost_temp_sensors+Cost_feedflow_sensor+Cost_gasflow_sensor...
+Cost_gas_conc_sensors+Cost_computer+Cost_control_hardware+Cost_monitoring_and_control_software...
+Cost_labour_constr_ADR+Cost_labour_monitoring_and_control_development))...

Accumulated_discount_factor_24*(Total_income_year-(Cost_labour_year+Cost_maintenance_year...
+(((((-c*rho*(F_feed/1000)*(T_influent-T_reac)-G_J_K_d*(T_room-T_reac))/3.6e6)*365)
+Power_sieve_kWh_year...
+Power_screw_pump_kWh_year...
+Power_vacuum_pump_kWh_year-...

+Power_peristaltic_feed_pump_kWh_year)*Value_power_NOK_per_kWh)+Cost_insurance_year...
+Total_cost_depreciation_year+Total_cost_amortization_year));

vmax=-1000000000;
N=30; %REACTOR VOLUME i
M=85; %REACTION TEMPERATURE j
for i = 1:N
    for j = 10:M
        b = 3.22;
        K_s = 24.9;
        K_sc = 3;
        K_d = 0.02;
        K_dc = 0.02;
        K_i=9;
        K_ic=11;
        k1 = 3.3;
        k2 = 0.12;
        k3 = 31.7;
        k5 = 34.1;
        frac_ch4=0.72;
B0=0.25;
Af=0.69;
%...........................
%Reactor volume:
%Vol_ADR_m3=8.6; new values goes into each loop
%...........................
V=i*1000;
g=9.81;%Gravity
sieve_frac=0.7;
%-----------------------------
%CALCULATION OF REACTOR DIMENSIONS:
%Calculation of area and volume for pilot reactor:
d_pilot=0.4;%m
L_pilot=2.2;%m
A_pilot=L_pilot*pi*d_pilot+2*pi*(d_pilot^2)/4;
V_pilot_m3=(pi/4)*(d_pilot^2)*L_pilot;
%Calculation of area and volume for actual reactor:
Ratio_h_d=2;
%optimization shows that Ratio_h_d=1 has best economic output but due to realistic design Ratio_h_d=2 has been selected:
d=power((1/(pi*Ratio_h_d/4)),1/3);
h=Ratio_h_d*d;
A=h*pi*d+2*pi*(d*d)/4;
%AD REACTOR PROCESS
%REACTOR INPUTS:
T_room=10;
T_feed=10;
%T_reac=35; new values goes into each loop
F_feed_raw_L_d=6000;
F_feed=F_feed_raw_L_d*sieve_frac;
F_feed_norm=F_feed/V;
HRT_days=V/F_feed;
%--------------------------
%SIMULATION USING HILL's MODEL:
S_vs_in=30.4;
S_bvs_in=B0*S_vs_in;
S_vfa_in=Af*S_bvs_in;
S_bvs_init=6.0;
S_vfa_init=0.72;
X_acid_init=1.08;
X_meth_init=0.32;
S_bvs_k=S_bvs_init;
S_vfa_k=S_vfa_init;
X_acid_k=X_acid_init;
X_meth_k=X_meth_init;
Ts=0.05;
t_start=0;
t_stop=3000;
%N=(t_stop-t_start)/Ts;
N=100;
t_k=t_start;
for k=1:N
    t_kp1=t_k+Ts;
    t(k)=t_k;
    t_k=t_kp1;
    mu_m=0.013*j-0.129;
    mu_mc=mu_m;
    mu=mu_m/(K_s/S_bvs_k+1+0*S_vfa_k/K_i);
    mu_c=mu_mc/(K_sc/S_vfa_k+1+0*S_vfa_k/K_ic);
    F_meth_k=V*k5*mu_c*X_meth_k;
    F_bios_k=F_meth_k/frac_ch4;
    dS_bvs_dt_k=(S_bvs_in-S_bvs_k)*F_feed/V-mu*k1*X_acid_k;
    dS_vfa_dt_k=(S_vfa_in-S_vfa_k)*F_feed/V+mu*k2*X_acid_k-mu_c*k3*X_meth_k;
    dX_acid_dt_k=(mu-K_d-(F_feed/b)/V)*X_acid_k;
    dX_meth_dt_k=(mu_c-K_dc-(F_feed/b)/V)*X_meth_k;
    S_bvs_kp1=S_bvs_k+Ts*dS_bvs_dt_k;
S\_vfa\_kp1 = S\_vfa\_k + Ts \cdot dS\_vfa\_dt\_k \\
X\_acid\_kp1 = X\_acid\_k + Ts \cdot dX\_acid\_dt\_k \\
X\_meth\_kp1 = X\_meth\_k + Ts \cdot dX\_meth\_dt\_k \\
S\_bvs\_sim(k) = S\_bvs\_k \\
S\_vfa\_sim(k) = S\_vfa\_k \\
X\_acid\_sim(k) = X\_acid\_k \\
X\_meth\_sim(k) = X\_meth\_k \\
F\_meth\_sim(k) = F\_meth\_k \\
S\_bvs\_k = S\_bvs\_kp1 \\
S\_vfa\_k = S\_vfa\_kp1 \\
X\_acid\_k = X\_acid\_kp1 \\
X\_meth\_k = X\_meth\_kp1 \\
\quad \text{vmax1} = f(i, j); \\
\quad \text{if } \text{vmax1} \geq \text{vmax}, \\
\quad \text{vmax} = \text{vmax1}; \\
\end{end} \\
disp('OPTIMIZED NPV 24') \\
disp(\text{vmax})
Appendix 4

Net Present Value:

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<tr>
<th>No index included</th>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<td>0.62</td>
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</tr>
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<td>0.83</td>
<td>0.75</td>
<td>0.68</td>
<td>0.62</td>
<td>0.56</td>
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<td>-</td>
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