Deforestation: Population or Market Driven?
Different Approaches in Modelling Agricultural Expansion

Arild Angelsen

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Summary:
The debate on causes of and remedies for tropical deforestation is often confused because the underlying assumptions of the arguments are not made explicit. This paper compares four different modelling approaches to agricultural expansion and deforestation, and explore the implications of different assumptions about the household objectives, the labour market, and the property rights regime. A major distinction is made between population and market driven approaches, and the labour market assumption is critical in this respect. Many of the popular policy prescriptions are based on the population approach. The paper shows that within a more realistic -- particularly for the long term effects -- market approach, well intentioned policies such as intensification programmes may boost deforestation. Many frontier agricultural systems are also characterized by open access where forest clearing gives farmers land rights. Deforestation therefore becomes an investment to the farmer and a title establishment strategy. In this situation, land titling and credit programmes may increase deforestation.

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Deforestation
Population
Property Rights

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1 Introduction

A major dividing line in the debate on the causes of tropical deforestation is between explanations emphasizing poverty and population growth as the driving forces (the population approach), and explanations emphasizing market factors such as prices, access costs and property rights (the market approach). Much confusion is created because the underlying assumptions are frequently not clearly described, and the arguments jump from one approach to the other. Moreover, some debates could be understood as differences in the approach applied, such as the debates on the effects of aid programmes targeted on poor farmers, of artificially low agricultural output prices, and of boycotts of tropical timber in Western countries.

The public and partly also professional debate on tropical deforestation is also bewildered because a number of distinguishable issues are pooled together. It is imperative to distinguish between different agents or sources of deforestation. This paper will focus on the share of tropical deforestation that relates to agricultural expansion, estimated at about 50 percent (UNEP, 1992; Myers, 1992). This is the result of decisions taken by numerous farming households in response to the prevailing economic environment. These decisions are generally beyond the direct control of governments, and are often referred to as "unplanned" deforestation. The paper will not consider the other main source of deforestation, that is, large-scale, (often) state-sponsored projects such as logging, plantations and infrastructural developments. This "planned" deforestation is commonly linked up with vested interests, and is more appropriately studied within a political economy perspective.

Another source of confusion relates to the framing of the decisions of agricultural households. This paper argues that decisions about agricultural expansion (deforestation) in many frontier areas should be modelled as an investment decision, because forest clearing commonly gives farmers rights to the forest. Deforestation is a title establishment strategy. This contrasts with a conventional framing of resource use decisions which emphasises the trade-off between different forest services, and the intertemporal aspect of these. It is critical to distinguish both in theoretical models and in policy analysis between land expansion (deforestation) and management of resources.

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2 The terms "unplanned" and "planned" deforestation that are frequently used by governments, may be misleading, partly because they represent a state-centred perspective which indicates that planned deforestation is socially desirable whereas unplanned is not. Others have coined the terms "the needy and the greedy" to describe the two main groups of actors in the deforestation process, a term which captures some characteristics, but the poetic appeal is stronger than the analytical.
which the farmers already control (e.g., soil erosion). The models presented in this paper only deal with the first set of decisions.

Closely related to this distinction, the paper challenges some conventionally held views on the causes of and remedies for tropical deforestation. In particular it shows how well intentional programmes for intensification, land titling or credit expansion may increase deforestation.

An aim of this paper is to contrast the assumptions, conclusions and policy implications of four different approaches in the modelling of agricultural expansion and deforestation. "The lack of empirical evidence [on the links between deforestation and government policies] magnifies the importance of using an explicit analytical framework when drawing conclusions about this important policy issue" (Deacon, 1995: 17).

The four approaches are defined by varying the assumptions related to three factors: the household's preferences, the labour market integration, and the property regime. A major reason for focusing on these factors is that these show great variation throughout developing countries, and the models should reflect this. Moreover, the policy implications may depend critically on the assumptions made. An additional aim is therefore to identify policies which are robust in the sense that the effect on deforestation does not depend on the modelling approach.

The outline of the paper is as follows: section 2 gives the basic assumptions and structure of the models. Section 3 discusses further the differing assumptions in the four models. Sections 4-7 present the four different models:

I. The subsistence or "full belly" approach: the households' objective is to reach a certain subsistence target, with the minimum labour input. No off-farm labour market exists.

II. The Chayanovian approach: a utility maximizing household, balancing consumption and leisure. Off-farm employment exists, but the household is constrained in the labour market.

III. The open economy, private property approach: unconstrained off-farm employment is available at a fixed wage, thus production decisions can be studied as land-rent maximization.

IV. The open economy, open access approach: the market assumption is as in 3, but forest clearance gives property rights.

We label the first two approaches population based explanations of deforestation, whereas the third and fourth are market based. Section 8 compares the four models, and discusses how the models can be extended, particularly to include general equilibrium effects. To illustrate the magnitude of the effect of exogenous changes and the differences between the models, a numerical illustration is given in section 9. Section 10 provides a further discussion of some policy implications, while section 11 suggests some extensions of the models. Section 12 gives some remarks on empirical testing of different approaches. The final section concludes.
2 Preliminaries

Farmers make a number of decisions that are potentially relevant for the management of natural resources: area of cultivation, crop choice, labour and other inputs, soil conservation investments, cropping and fallow periods, etc. In this paper we will focus exclusively on the determination of the area of cultivation and thereby the extent of deforestation. The simplification is justified on several grounds. First, we are able to derive explicit results and formulas for the extent of deforestation, which should also permit empirical testing. Second, the models illustrate the main differences between the approaches, and the approach chosen initially is normally more important than later refinements within the different approaches.

In addition to the option of being converted to agricultural land, forests provide a number of services, including protective functions (biodiversity, carbon sequestration, climatic and hydrological stabilization, etc.) and the provision of non-timber forest products. These are not included in the models. This does not, of course, imply that they are unimportant, but for the understanding of agricultural expansion they are less relevant. Most of these functions have strong public goods characteristics, and are therefore mostly ignored in individual decision making. It would be relevant to include these in a discussion of optimal rates of forest conversion and first best solutions with markets for environmental public goods, but this is beyond the scope of this paper.

Yield and area of cultivation

The yield or output per hectare (ha) of land \((x)\) represents the optimal crop (or crop mix), and is taken as exogenous in the models. Thus one important aspect not covered is the crop choice, which is a significant variable for the environmental effects of forest conversion. \(x\) also reflects the technological level. This formulation further implies that the elasticity of total production \((X)\) for a household with respect to land is one. This may be a strong assumption in models I and II where only family labour is used.

\[ X = xH \]

\(H\) is the total land area. We use the output price as numéraire in the models; thus \(x\) is also to be interpreted as the value of output per ha. We assume that \(x\) increases per time period at a rate \(g\), due to technical progress or higher output prices. As it turns out, this assumption is relevant only in model IV.

The models take the von Thünen (1826) approach: land is abundant and homogeneous, and the limit on expansion is costs related to accessibility (walking, transport, etc.), as measured by distance from a centre. The approach can be given both a micro (village) and a macro (regional) level interpretation. At the micro level, one may think of a village centre, where all people live, surrounded by forest of equal quality. The main

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3 Including more endogenous variables would complicate the models significantly, but not change the direction of the main conclusions when it comes to the effect on deforestation of various exogenous changes. See Angelsen (1994) for a discussion of the open economy case with labour input and fallow period as decision variables in addition to the area of cultivation.

4 This does not apply to model I, where transport costs are unimportant.
distance costs will be to walk back and forth to the fields. Locations beyond a certain point have too high costs to make cultivation worthwhile. A macro level, and more abstract, interpretation would be to rank all forest land according to accessibility. Under the micro-interpretation, total agricultural land will be a circle around the village;

\[ H^T = \pi (b_{\text{max}})^2 = \int_0^{b_{\text{max}}} 2\pi b \, db \]

\( b \) is distance from the village centre to the field, and \( b_{\text{max}} \) is the maximum distance at which cultivation takes place, that is, the agricultural frontier. In models I-III the focus is on a representative household, hence we assume that land is shared equally between the \( N \) village households, such that each household gets a \( 1/N \) fraction of the circle (see discussion on property regimes in next section). Land is not necessarily under continuous cultivation. We define \( m \) as the inverse of the share of land under cultivation; \( m \equiv \frac{C \cdot F}{C} \leq 1 \), where \( C \) and \( F \) are the length of the cropping and fallow periods, respectively. \( m \) is Boserup's (1965) land use intensity factor. Land area under cultivation by the household is then given by;

\[ H = \frac{\pi}{mN} (b_{\text{max}})^2 = \int_0^{b_{\text{max}}} h b \, db; \quad h = \frac{2\pi}{mN} \]

The assumptions reflected in (3) are important in models I-II, where the total agricultural income plays an important role. In an open access situation, it is more reasonably to assume that new migrants have to occupy land at the forest margin, and the focus should then be at the migration equilibrium under this assumption.

**Labour input and distance costs**

We include two types of labour cost in the models. First, there is on-the-field labour related to clearing and preparation of the field, weeding, pest control, harvesting, etc. Given the assumption of fixed yield, on-the-field labour is exogenous and set to one. Second, there are costs related to the location of the field, as measured by the distance from the village (\( b \)). These may be thought of as time spent on walking between the fields and the village, and is therefore also a kind of labour cost. A number of alternative formulations of the distance (transport) cost function is possible. We have chosen a specification which is both simple and has some intuitive appeal. It assumes distance costs to be proportional with both distance and time working on the field per unit land.\(^5\)

Total labour input per hectare (both walking and on-the-field) is;

\[ L = 1 + qb, \]

where \( q \) is labour time spent on walking or transport per km per unit labour on the field. Total labour input for a representative household is then;

\[ L = \int_0^{b_{\text{max}}} (1 + qb) h b \, db \]

Note that unlike for total production we cannot simply multiply total labour input per hectare (\( l \)) by total area (\( H \)), as \( l \) varies with distance.

\(^5\) The linearity assumption has could easily be modified in the models, but would not change the qualitative results. In the numerical simulation we test the implications of assuming that time spent on walking per km is convex in distance (section 9). See also Angelsen (1994) for a further discussion of this formulation.
The location costs are key elements in the models. In reality, there are several dimensions to such costs. Chomitz and Gray (1994) use both distance to the nearest road and travel time to the nearest market as determinants of the output and input prices. In our model, the cost related to a distant location from the village centre is included, whereas the disadvantage of a remote location of the village from a regional trading centre is not. This could appropriately be included by adjusting the output price. This would certainly be an important element in empirical models, but it adds little to our analytical model where the key feature is declining land rent as agricultural area expands.

3 Differing assumptions in the four models

The four models presented differ in three critical assumptions regarding (i) the household's preferences and objective function, (ii) the market assumptions -- in particular the labour market integration of the village economy, and (iii) the property regime. The assumptions are summarized in Table 1, and elaborated below.

<table>
<thead>
<tr>
<th>Model</th>
<th>Households' objective: utility maximization</th>
<th>Labour market</th>
<th>Property rights regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>I: Subsistence (&quot;full belly&quot;)</td>
<td>Minimize labour, given subsistence target (lexicographic)</td>
<td>No labour market</td>
<td>Private (or communal)</td>
</tr>
<tr>
<td>II: Chayanovian</td>
<td>Trade-off between consumption and leisure</td>
<td>Labour market exist, but households are quantity constrained</td>
<td>Private (or communal)</td>
</tr>
<tr>
<td>III: Open economy, private property</td>
<td>Maximize profit from production</td>
<td>Perfect labour market</td>
<td>Private (or communal)</td>
</tr>
<tr>
<td>IV: Open economy, open access</td>
<td>Maximize profit from production</td>
<td>Perfect labour market</td>
<td>Open access, property rights established by forest clearance</td>
</tr>
</tbody>
</table>

Table 1: Different assumptions in the four model (changes compared to previous model in bold).

Household's preferences and objective function

A widespread view is that people in "traditional" societies are less oriented towards material consumption than in "modern" ones. People are only interested in income as a means to cover their basic consumption needs; when these are covered they prefer more leisure to higher consumption. This implies lexicographic preferences; the households' objective is to reach a subsistence target with the minimum of labour effort. Stryker
(1976) and Dvorak (1992) are examples of rural economy models applying this objective function. Model I in the paper uses this assumption.

The standard formulation in the agricultural household literature is to assume that households maximize utility, allowing for substitution between consumption and leisure (see particularly Singh et al., 1986; Nakajima, 1986). The households face a trade-off between the drudgery or disutility of work, and the utility of consumption. The main tenet is that the household, even without being in touch with markets, will reach a "subjective equilibrium" (Nakajima, 1986), which resembles the equilibrium when facing competitive markets for labour and output. A subsistence level of consumption could be included in the utility function. Models II-IV are based on this approach, with the modification for models III and IV as given below.

**Market assumptions: labour market integration**

Peasant economies are characterized by their partial integration into imperfect markets (Ellis, 1988). We focus on the labour market assumption, which is the most critical one for the purpose of this paper. We employ three different assumptions: no off-farm labour market exists, and self-employment on the family farm is the only option (model I); a labour market exists, but the household is quantity constrained (model II); and a perfect labour market exists where the household can hire or sell the desired amount of labour at a fixed wage rate (models III and IV).

By assuming a perfect labour market, the production decisions of a utility maximizing household can be studied as a profit-maximizing problem. This is a key result in the agricultural household literature, see particularly Singh et al. (1986). The model becomes recursive in the sense that first the production decisions are made, then the consumption decisions. The production decisions are the relevant ones for the study of deforestation, thus we are allowed to ignore the consumption side. The households in models III and IV are as a result of the recursive property assumed to maximize profit, even though the preference structure could be identical to model II.

The logic of the recursive property is as follows. Assume the household is price-taker in all relevant markets, and farm and off-farm labour are perfect substitutes both in the production and utility functions. The production decision (including labour demand) then has no links to the consumption decision (including labour supply), except one: the household should maximize the surplus from agricultural production to be included in the household's budget. The recursive character of the decision-making process in the agricultural household significantly simplifies the model, but also carries some strong implications for the results, as seen below and discussed further in appendix 1.

In all models we assume for simplicity that the entire output is sold in a competitive market. An increase in \( x \) is therefore due to either increased productivity (physical

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6 According to Chayanov, the main factor influencing the trade-off between consumption and drudgery is the demographic structure of the household (the ratio between consumers and workers); it is "a demographic model of household decision-making" (Ellis, 1988: 106). We do not focus on this aspect in our model, even though this may be relevant for comparative studies at the household level. For more aggregate models of tropical deforestation, however, it seems less relevant and feasible to include it.
output) or increased output price. An alternative assumption would be to let output be consumed directly, hence changes in output would represent productivity changes only. If production is partly for consumption and partly for sale, the effect of a price change in a non-recursive model is more complicated as the change affects both output and a consumer good.

**Property regime**

The importance of property rights in resource management is well established. We consider three different property regimes: (i) private (or communal) property rights to both cultivated land and virgin forest, (ii) "pure" open access, and (iii) a version of open access where forest clearing gives the farmer property rights (homesteading).

In models I-III we assume, in line with (3), that each farm household has the right to expand their agricultural land in a \( 1/N \) fraction of the circle of land surrounding the village. These rights could be vested in a communal forest management regime (cf. Angelsen, 1996). It is commonly argued that such systems have a fairly egalitarian way of distributing resources, hence the assumption of equal sharing and identical households may be a reasonable one (cf. Stryker, 1976: 348).

Most tropical forests are de jure state property, but the remoteness and lack of institutional capacity to enforce government regulations often make forests de facto open access resources, unless some forms of communal management exist. The case of open access is discussed in model IV. An interesting result is that in its pure version (forest clearing does not give permanent land rights), open access yields the same level of deforestation as a regime with private or communal property rights.

A characteristic of many frontier systems is, furthermore, that forest clearing and cultivation give farmers some rights to the land. This way of gaining land rights is common in customary law throughout the tropics, and is also manifested in statutory law in some countries. In this situation farmers will not only look at the immediate benefits, but also at the future gains from having obtained rights to the land. This assumption is applied in model IV. Forest clearing is an investment, and should be modelled accordingly. This distinguishes the model from a large share of the literature on deforestation, and it produces some unconventional results.

Property rights are never certain. In order to account for uncertainty we introduce an exogenous probability for losing the land in each period (\( \lambda \)), for example, through appropriation by the state. This corresponds to a homogenous Poisson probability distribution. The level of \( \lambda \) will be affected by, *inter alia*, the protection provided by the state. Including risk has the same effect as adding \( \lambda \) to the discount rate, thus it is commonly termed *risk discounting* (e.g., Clark, 1990: 351). For simplicity we assume risk neutral farmers, which allows the use of expected values.

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7 The security of land rights is also influenced by decisions made by the farmers themselves. In Angelsen (1996) a simple model with endogenous property rights security is developed. The security is assumed to be a function of the intensity of production, and the enforcement efforts, for example, in obtaining a land certificate.
Given these assumptions, models I-III become static (one period) optimization problems. Optimal forest clearing is determined by considering the present period only; the farmers have full property rights to forest land, and there is no competition. Model IV, however, becomes dynamic because of the changed assumption about the property regime.

4 Model I: The subsistence or "full belly" economy

The subsistence models assume in the extreme case that no markets exist. The households produce only for their own consumption, with family labour as the only input besides land. The objective of the household is to minimize labour efforts (maximize leisure) given a subsistence target, implying that consumption beyond that level has no value. This is the "full belly" version of the subsistence model (e.g., Dvorak, 1992).

This approach can generally be formulated as a Lagrangian problem. In our case with only one decision variable, however, it can be presented in a simpler way. Given the assumption of fixed inputs, there is not much of a choice left to the household: the family has to expand the area of cultivation until the subsistence target (c_{min}) is met. Production for one household (x_H) must equal c_{min}, hence area of cultivation is simply given by:

\[ H = \frac{c_{min}}{x} \]

Combining this with (3) gives the agricultural frontier;

\[ b_{max} = \sqrt{\frac{mNc_{min}^2}{kx^2}} = \sqrt{\frac{2c_{min}^2}{hx^2}} \]

Under the subsistence approach the agricultural frontier is determined by productivity, population, and the subsistence requirement. Distance costs (q) are irrelevant for deforestation in this model.9 10

The effect on the agricultural frontier of changes in the exogenous variables is readily seen from (6), see also appendix 2. First, an increase in the value of production from one hectare (increased productivity or higher output price) will reduce the extent of deforestation. The subsistence income can be obtained from a smaller area. Second, given a subsistence requirement per capita, total area of cultivation is proportional to population.

8 The term subsistence is used in this context to refer to a situation where farmers' main objective is to reach a subsistence level of consumption, and not necessarily in the other, and frequently used, meaning of the term as an economy where the output is consumed directly and not sold in a market.

9 One aspect not included in the model is that the subsistence target, if defined in nutritional terms, will depend on the labour input. Higher distance costs will then actually increase deforestation.

10 The land use intensity (m) is also important for the agricultural frontier. A lower m will, however, affect x negatively -- an aspect not included in the model.
Third, an increase in the subsistence requirement will also expand the agricultural frontier. Whereas a basic subsistence requirement could be defined in nutritional terms, it also has strong cultural and social elements. It was evident in the field study area in Sumatra that there has been a change in what was considered "necessary expenses" over the last two decades, for example, in the form of higher pressure to send all the children to school and paying their fees (Angelsen, 1995). We may hypothesize that integration into a larger "modern" economy and national culture will increase the subsistence requirement, and therefore contribute to deforestation in the subsistence model.

5 Model II: The Chayanovian economy

Models in agricultural economics assuming household to maximize utility and not being integrated into perfect markets are commonly labelled Chayanovian (1966) models. Normally no off-farm labour market is assumed to exist, but we discuss the more general case when the household can sell a fixed amount of labour ($E$) in the labour market at a given wage ($w$). We assume that the labour market constraint is binding; the case when the constraint is not binding is model III. Only family labour work on the family fields.

The household maximization problem can then be written as;

$$\max_{b_{\text{max}}} \quad U(C, T) = U\left( \int_0^{b_{\text{max}}} x \cdot h \cdot b \cdot d b + wE, \int_0^{b_{\text{max}}} (1 + qb) \cdot h \cdot b \cdot d b + E \right)$$

$C$ is total consumption in real terms, and $T$ is total labour time. For analytical convenience we have formulated the problem as one of determining the agricultural frontier; as seen from (4) this is equivalent to maximizing with respect to agricultural labour input. The optimality condition is;

$$x = z(1 + qb_{\text{max}}); \quad z = \frac{U_T}{U_C}$$

In optimum, the production increase from clearing one more unit of land for cultivation ($x$) is equal to the labour inputs required at the agricultural frontier ($1 + qb_{\text{max}}$), multiplied by the household’s shadow or virtual wage rate ($z$). An alternative interpretation is that at the frontier the shadow wage should equal the output value per labour unit, including travelling time ($x/(1 + qb_{\text{max}})$). Rearranging (8) yields;

$$b_{\text{max}} = \frac{x - z}{q} = \frac{x}{q} - \frac{z}{q}$$

The frontier is determined by three factors: the value of yield ($x$), the travel efficiency ($q$), and the shadow wage rate ($z$). It is, however, critical to note that unlike in the three other models (equations (6), (13) and (15)), the expression on the right hand side of (9) depends on $b_{\text{max}}$: $z$ is a function of $b_{\text{max}}$ as $b_{\text{max}}$ affects both $X (C)$ and $L (T)$. Thus the effect of a change in, say $x$, cannot readily be seen from (9). This complicates the comparative statics, given in appendix 2.

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11 The name is taken from the Russian agricultural economist Chayanov who used this approach to study the adaptation of Russian farmers early this century.
To produce some more specific comparative statics results as well as for the numerical simulation in section 9, we introduce an additive utility function incorporating a subsistence level for consumption \((c^{\text{min}})\) and a maximum level of labour input, given by the total time available to the household \((T^{\text{max}})\).

\[
U(C, T) = (C - c^{\text{min}})^\alpha + \nu(T^{\text{max}} - T)^\beta
\]

\(\nu > 0; \alpha, \beta \in (0, 1); (C - c^{\text{min}}), (T^{\text{max}} - T) > 0;\)

\[
z = \frac{U_T}{U_C} = \frac{\nu\beta(C-c^{\text{min}})^{1-\alpha}}{\alpha(T^{\text{max}}-T)^{1-\beta}}
\]

This function yields positive and declining marginal utility of consumption, and increasing marginal disutility of labour, whereas the cross derivatives are zero. \(z\) is the marginal rate of substitution between consumption and labour, or the shadow wage rate or virtual price of labour. \(z\) increases in both consumption and labour, which ensures that the indifference curves are upward sloping and convex in the \(C-T\) space (see appendix 2). \(T^{\text{max}}\) determines an upper bound and \(c^{\text{min}}\) a lower bound on the area of cultivation. The formulation implies that the shadow wage becomes very low when consumption approaches the subsistence level, and very high when leisure approaches zero. We assume that the parameter values are such that the subsistence target can be met with the labour available.

This function may be seen as a combination of the standard (multiplicative) Stone-Geary utility function and the Houthakker additive function. By setting \(\alpha = \beta\) the function is, in fact, equivalent to using a CES function with subsistence level for consumption and maximum labour time. The chosen formulation yields some interesting results as seen below and further discussed in appendix 2. The appendix also briefly reviews the implications of using different functional forms.

**Comparative statics**

In discussing the comparative statics results, it is useful to distinguish between the dual role of agricultural households: the family farm is both a producer or a farm firm, and a labourer's and consumer's household (Nakajima, 1986). In its first role the household acts like a profit maximizing producer, whereas in its second role it acts as a consumer. This is elaborated further in appendix 1.

The full derivation of the comparative statics results is given in appendix 2. The most interesting case is when the output price or productivity \((x)\) increase. The total effect of a change in \(x\) on \(b^{\text{max}}\) can be split into two sub-effects, cf. (35). First, for given \(z\) it will be beneficial for the household to expand the area of cultivation. They get higher output per unit labour input, thus they will cultivate a larger area. We shall label this the farm firm effect. The second effect relates to the fact that production on land already under cultivation will increase. From (34) it follows that \(z\) will increase, that is, on the margin the cost of labour is now valued relatively higher than the benefit of more consumption. This effect will induce the household to reduce the area under cultivation. We shall refer to this as the subsistence effect, which is related to the mechanism at work in model I, cf. appendix 1. Note the parallel between the farm firm and subsistence effects, and the
textbook discussion on substitution and income effects, respectively, of a wage increase. The farm firm or substitution effects refer to the fact that leisure has become more expensive, thus farmers will work more. The subsistence or income effects refer to the increased consumption following the productivity increase, which makes the households wish to consume more leisure and work less.

The magnitude of the elasticity of $z$ with respect to $x$, $\frac{\partial z}{\partial x} z$, determines which effect will dominate. This logic of this is seen by considering (8). Given our specific utility function, this elasticity can be specified further, and as a key result in the Chayanovian model we get:

\[(35) \quad \frac{dE_{\text{max}}}{dx} = \mu \left[ 1 - \frac{\partial z}{\partial x} \right] = \mu \left[ 1 - \frac{X(1-\alpha)}{C-c_{\text{min}}} \right] \]

\[> 0 \iff X > \frac{c_{\text{min}} - wE}{\alpha}, \quad \mu > 0, \text{ see appendix 2.} \]

If the elasticity is larger than one, the subsistence effect dominates. This will be the situation when $X < (c_{\text{min}} - wE)/\alpha$. For levels of agricultural income above this limit the farm firm effect will dominate. This yields a backward-then-forward sloping or C-shaped labour supply curve, which is the inverse of the commonly assumed shape for rich economies.

The model produces an interesting and intuitive result which directly relates the sign of the net effect of an increase in $x$ to the consumption level (poverty). Consider the case with no off-farm employment; the condition is then $X < c_{\text{min}}/\alpha$. If the household is close to the subsistence level and working hard to survive, $z$ will be low. Higher income will have a relatively large impact on $z$; "it will allow them to relax a little bit more". In a wealthier economy, the subsistence considerations are less important, and the household behaves more like a farm firm.

Second, the condition can be related to the household's dependence on farm v. off-farm income: the lower farm income and the higher off-farm income are, the weaker the subsistence effect, ceteris paribus. The result is rather intuitive: when only a small proportion of the total income is from agriculture, the income increase and therefore the income (or subsistence) effect will also be small. If the off-farm income is higher than the subsistence requirement, we see that the farm firm effect will always dominate.

Third, the condition for the subsistence effect to dominate can also be related to an intuitive interpretation of $\alpha$. $\alpha(\alpha-1)$ is the elasticity of marginal utility with respect to surplus consumption ($C-c_{\text{min}}$). A low value of $\alpha$ means that the valuation of consumption above subsistence declines rapidly. This may be representative for societies without a strong materialistic orientation. In this case the subsistence effect will predominate: higher productivity will make the household cultivate a smaller area, as priority is given to increased leisure.\(^{12}\)

\(^{12}\) $(1-\alpha)$ is also the Arrow-Pratt measure for relative risk aversion, if defined only with respect to consumption. $\alpha$ can then be interpreted in terms of the household's attitude towards risk. A more intuitive interpretation of the household's risk preferences in the model is, however,
Furthermore, the integration of traditional agrarian societies into the larger economy and "modern" culture may imply that more emphasis is put on material consumption relative to leisure. This could be related to a higher value of $\alpha$. If the value of $\alpha$ is close to one, the farm firm effect will dominate even for consumption levels relatively close to subsistence. The model thus provides a possible explanation of how the integration into a "modern" society will make the household act more like a profit maximizing firm rather than a survival oriented household, in addition to the above argument about the growth in non-farm income.

The other comparative statics results are straightforward as the subsistence and farm firm effects -- when both are present -- pull in the same direction. The effect of higher transport costs ($q$) can be split into two sub-effects, cf. (36). For a given $z$ the net benefit from cultivation is reduced, thus the agricultural frontier will contract (farm firm effect). The subsistence effect is that higher $q$ increases the amount of labour required on land presently under cultivation, thus the shadow wage ($z$) increases.

An increase in the availability of off-farm employment ($E$) or the wage ($w$) will also unambiguously reduce deforestation. Higher off-farm income reduces the marginal utility of consumption, that is, $z$ increases. In the case of higher $E$ we also get an effect from increased disutility of work.

A population increase (lower $h$, cf. (3)) will lead to a lower $z$: for a given $b_{\text{max}}$ the area per household is reduced, and therefore also consumption and labour input, both effects which reduce $z$. A lower shadow wage augments the net benefit of cultivation on the margin, thus the agricultural frontier will expand. The relative increase in area under cultivation will be less than the population increase, because new land is brought into cultivation at increasing costs.

6 Model III: The small, open economy with private property

The small, open economy formulation takes all prices as exogenously given. In particular, we introduce a perfect labour market in the sense that any amount of labour can be sold or hired at a constant wage rate ($w$). The recursive property of the model (section 3) gives that the production problem then can be formulated as:

$$\text{Max } R = x - wL = x \int_0^{b_{\text{max}}} hb \, db - w \int_0^{b_{\text{max}}} (1 + qb)hb \, db$$

$R$ is total profit or land rent. The FOC is given by;

found by using the measure for absolute risk aversion, given by $(1-\alpha) / (C_{c_{\text{min}}})$: for consumption levels close to subsistence the farmers become very risk averse.

13 The open economy case of a shifting cultivation economy has been dealt with in a relatively comprehensive way in Angelsen (1994). In that model both the fallow period, labour input and the agricultural frontier are determined endogenously. In this paper we continue to limit the model to the case where only the agricultural frontier is endogenous. Note, however, that this simplification is more justifiable under the open economy (exogenous prices) assumption as the comprehensive model is recursive: the optimal fallow period and labour input are first determined, then the agricultural frontier (see Angelsen, 1994). This is not the case in the two previous models.
(12) \[ x = w(1 + qb_{\text{max}}) \]

or;

(13) \[ b_{\text{max}} = \frac{x - w}{q} = \frac{x}{q} - \frac{1}{q} \]

The interpretation of (12) and (13) is similar to (8) and (9) in model II. There is, however, a crucial difference: \( w \) is exogenous, unlike \( z \) in the Chayanovian model. As the subsistence effect in model II relates to changes in \( z \), the replacement of an endogenous \( z \) with an exogenous \( w \) implies that there is no subsistence effect in the small, open economy model; only the farm firm effect is present.

The effect of a change in \( x, w \) or \( q \) is readily seen (see also appendix 2). An increase in the productivity (\( x \)) will increase deforestation, as frontier farming becomes more profitable. The same effect would result from a decrease in transport costs (\( q \)), or in the real wage rate (\( w \)).

Figure 1 illustrates the rent gradient as a declining function of distance. In this static model (III) this is labelled annual land rent, to be distinguished from the discounted land rent of the next model. An increase in \( x \) will move the curve upwards, thus \( b_{\text{max}} \) expands. An increase in \( q \) or \( w \) will move the rent gradient downwards, and reduce \( b_{\text{max}} \).

\textbf{Figure 1: The determination of the agricultural frontier in the open economy cases.}

The new labour market assumption changes the underlying logic of the model dramatically compared with models I-II. The key variable for the determination of the agricultural frontier in the open economy model is the relative profitability of frontier farming, as determined by output price and productivity (\( x \)), roads and accessibility (\( q \)), and alternative employment opportunities (\( w \)). The initial size of the population has no effect; migration will level out any income differences, and labour in agriculture (or population) is thereby determined endogenously.
7 Model IV: The small, open economy with open access

In model IV the property rights assumption changes, and we assume a situation where no one has property rights to virgin forest. We discuss two versions of open access: first, we assume a shifting cultivation system where forest clearing does not give any permanent rights, the land goes back to the common pool of forest resources after the period of cultivation is over ("pure" open access).

Open access in combination with the perfect labour market assumption imply that labour will migrate to the forest frontier as long as there is a non-negative rent in frontier farming. The migration equilibrium is defined as a situation where land rent on the margin is zero. But this is exactly the solution in (12) of the previous open economy model. Thus, model III applies both to a situation with private (or communal) property rights as well as a static open access situation.14

The next case to consider is when forest clearing gives permanent land rights, which has a historical parallel to homesteading in the US. New settlers will cut forest as long as there exists a non-negative discounted rent, and the migration equilibrium and agricultural frontier are defined where the discounted rent is zero. The agricultural frontier is implicitly given by;15 16

\[ \int_0^\infty e^{-\lambda t} e^{-\delta t} [e^{\gamma t} x - w(1 + q b_{\text{max}})] dt = \frac{x}{\delta + \gamma} - w(1 + q b_{\text{max}}) = 0 \]

\( \lambda \) is the probability of losing the land in each period17, \( \delta \) is the discount rate, and \( g \) is the rate at which farmers expect the gross value of output (x) to increase per time period.18 We assume that \( \delta + \lambda > g \).19 We use (for simplicity) continuous time, infinite

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14 Generally, open access may also drive the overall resource rent to zero. This is impossible in this model as the output per ha is constant. In, for example, a model of endogenous labour and fallow period (Angelsen, 1994), the fallow period will be shorter for land inside the agricultural frontier in the open access model, making the resource rent zero for land at all distances.

15 Note that one could arrive at exactly the same condition by formulating the problem in a similar manner as in model III; each household maximizes discounted profit. It is, however, more problematic to use the assumption about land allocation underlying equations (3) and (4) in the open access case.

16 We ignore any possible capital gains which may result from future land sale, and assume that the land value (price) is determined only by future land rent. Self-fulfilling expectations of capital gains ("rational bubbles") could be important in situations when there is an active land market, as in some regions of Latin America. See Clark et al. (1993) for a theoretical discussion, and Kaimowitz (1995) for a discussion of the empirical evidence from Central America for such speculative forces of deforestation.

17 An empirically relevant modification would be that \( \lambda \) declines over time as the length of occupancy increases. This could be included in the model, see Angelsen (1994) for a case when \( \lambda = \lambda(t) \).

18 If soil degradation is expected to be important, \( g \) should be adjusted downwards. In this case model III will remain the relevant one.

19 If this condition is not met, we are back to model III.
time horizon, and let all relevant parameters (except $x$) be constant over time. Rearranging this expression yields:

\[
(15) \quad b_{\text{max}} = \frac{\theta \cdot w}{q - w} = \frac{\theta w}{q} \quad \text{for } q > 0
\]

Compared with the solution for the "pure" open access or the open economy with private property in (13), we have an additional element here - $\theta$. As this parameter is greater than one, given expectations about an increase in $x$, this model implies that the agricultural frontier will be pushed further away compared with model III, cf. Figure 1. Because $x$ increases over time, we also get that the agriculture frontier will steadily expand. Note that we would have got a similar story if farmers expected $q$ or $w$ to be reduced over time.

The intuition behind this result is simple. Forest is cleared even if it has a negative rent in the early years. This would have been even more intuitive if we included some initial costs of forest clearing and migration in the model. The initial loss will be outweighed through a positive land rent some time in the future. Early clearing is necessary to establish property rights; otherwise the land will be taken by others. This situation has been described as "the race for property rights" (Anderson and Hill, 1990). Such a land race is unproductive from a social viewpoint because it gives a negative contribution to overall production (as land rent will be negative for the first years), and is a kind of rent-seeking. The reason for the inefficiency is the link between resource use (forest clearing) and allocation of property rights.

The conclusion that the open access situation will lead to more agricultural encroachment than the private (or communal) property situation rests on two critical assumptions: (1) Expectations about increasing land rent per hectare, for example, an increase in $x$ as in our formulation. (2) Property rights are established by forest clearing. Open access in itself will not generate more deforestation than a situation with well defined property rights to all forest (model III). If there is no expectation about higher land rent in the future, the discounted rent curve in Figure 1 will intersect the x-axis in the same point as the annual rent curve.

The effect of changes in $x$, $q$ and $w$ is similar to model III, cf. appendix 2. The effect on deforestation of a higher discount rate is negative. This contradicts the conventional wisdom which holds that lower discount rates will help preserve the environment. The intuition behind our result is nevertheless simple: a lower discount rate implies giving relatively more weight to the future positive land rents. Farmers are willing to clear forest further away when the discount rate is lowered. Again, this result must be seen in the light of deforestation being an investment for the decision maker, and not an issue of conservation of a resource to which the agent has well established property rights.
It also follows that higher tenure security in terms of reduced probability of losing the land will increase deforestation. Again, this contradicts a conventional hypothesis on the impact of more secure property rights, as will be discussed further in section 9.

The effect of an increase in the expected productivity (or price) growth \((g)\) is as to be expected: higher \(g\) leads to more deforestation. The latter result highlights the role of expectations in the deforestation process. Expectations about increases in \(x\) or reductions in \(q\) or \(w\) can initiate a process of deforestation. Moving beyond the strict model assumptions, we could interpret expectations about increased land scarcity as a decline in \(w\). Angelsen (1995) in a case study from Sumatra suggests that such expectations have been important in initiating a self-reinforcing land race. It could be considered a kind of "self-fulfilling prophesies".

To summarize, anything that increases the expected rent will boost deforestation: improved technology, higher output prices, lower transport costs, lower opportunity costs of labour, more secure claims to cleared forest land, or lower discount rates.

8 Comparing the models

Summary of comparative statics results

The effects of changes in various parameters on deforestation in the four models are summarized in Table 2. We note in particular the different effects of a productivity or output price increase. Whereas higher \(x\) reduces deforestation in the subsistence and possibly also in the Chayanovian models, it increases deforestation in the two other models. For the discussion of this effect the Chayanovian model can be considered the most general model as it includes both the farm firm and subsistence effects.

The alternative employment opportunity is a critical variable in the open economy models, and to some extent also model II. Population growth has important effects on deforestation in the subsistence and Chayanovian models, but is endogenous and determined by relative profitability of frontier framing in models III and IV.

Whereas the subsistence model focuses exclusively on the agricultural sector, the open economy model draws the attention to the development in the rest of the economy. In particular, it underscores the role of alternative employment. The open economy models also highlight the (possible) counterproductive effect of intensification programmes which increase productivity in frontier agriculture, cf. section 10.
Table 2: The effect on deforestation of various factors in different models.

The realism of the models depends both on the characteristics of the economy in question and on the time horizon for the analysis. The population based models (I and II) may better reflect the short term response, and situations with high transaction costs and low mobility of the labour force. One possible interpretation of the small, open economy formulation in models III-IV is that the frontier agriculture sector is small compared with the rest of the economy. In particular, there exist sufficient outside employment opportunities (non-frontier agriculture and off-farm) which make it realistic to assume an exogenously given wage rate and households to be unconstrained in the labour market. For this reason, this model assumption may be considered more realistic for the study of long-term adaptations when migration is a real option. Migration tends to level out differences in the (expected) wage level between various sectors and regions.

A graphical illustration

Figure 2 gives a graphical illustration of the optimality conditions in the four models. The \( x(1+qb) \)-curve gives the output per unit labour input (including transport costs), whereas the other curves represent the (shadow) wage rate in the four models. In the

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1 If subsistence requirement depends on labour efforts.

---

20 Formulating model I as a Lagrangian problem, the FOC is \( x(1+qb^{\text{max}}) = 1/\gamma \), where \( \gamma \) is the Lagrange multiplier, that is, the labour input required to increase consumption by one unit.

21 We know that \( b^{\text{max}} \) is greater in model II than I, and greater in model IV than III, otherwise we cannot say if there is more or less deforestation in the market based than population based models.
comparative statics (with the exception for the first model), changes in the first curve refer to farm firm effects, whereas changes in the other refer to subsistence effects. For example, an increase in $x$ leads to an upward shift in the $x/(1+q_b)$-curve, which in models III and IV is the only effect, and therefore gives higher $b_{max}$. In the Chayanovian model (II) the $z$-curve also shifts northwest, and in a poor and agriculture dominated economy the shift is sufficiently large to reduce deforestation. In model I the shift to the left in the $1/\gamma$-curve will always make $b_{max}$ decline.

Figure 2: The adaptation in the four models.

**Household preferences**

One reason for including the "full belly" model was to highlight the rather rigid assumptions underlying some popular policy prescriptions to the deforestation problem. The subsistence approach dominates the thinking on the causes of and remedies for deforestation within the development aid community. The popularity of the "full belly" approach should be understood both in view of its simple and clear logic, and its policy implications. FAO (1992), which provides the most widely used estimates of tropical deforestation, employs a deforestation model to interpolate data where population density is the only explanatory variable. The Alternatives to Slash-and-Burn programme, initiated by a number of international development organizations, is based on the intensification hypothesis: increased productivity will reduce the pressure on forests. "For every hectare put into these sustainable soil management technologies by farmers, 5 to 10 hectares per year of tropical rainforest will be saved from the shifting cultivator's axe, because of their higher productivity" (ASB, 1994: 11). This hypothesis is discussed further in section 10.

The Chayanovian model provides a more general (and realistic) description of the household's preferences. The "full belly" model implies that the shadow wage rate is zero up to the subsistence level, and infinite above it. The most significant implication of the new assumption for household behaviour is to open up for the opposite effect on deforestation of an increase in productivity or output price: higher $x$ may lead to more
deforestation if the farm firm effect dominates the subsistence effect, which will be the case when the household is relatively rich, has a significant share of off-farm income, and/or have strong preferences towards consumption.

For poor, agriculture dependent and/or "leisure-oriented" economies the Chayanovian model gives the same qualitative result as the "full belly" model. In fact, model I could be seen as a special case of model II. As seen from (35), the subsistence effect will be very large and dominate completely when \( \alpha \) approaches zero, the household is agriculture dependent, and close to the subsistence requirement (which is implied by a low \( \alpha \)).

**Labour market integration**

There is a qualitative jump when moving from model II to model III, that is, when the household is not constrained any more in the labour market. The household model becomes recursive, and the subsistence effect disappears in the comparative statics. One should note that the critical factor is not whether an off-farm labour market exists or not, but whether the household is constrained or not in that market. When the household is constrained, the shadow wage rate is endogenous and changes in that rate represents the subsistence effect.

Economic development is associated with a gradual reduction of the agricultural sector's share of the total economy. Given this historical pattern, it is interesting to explore what happens when the availability of off-farm employment expands. When the household is constrained in the labour market, the shadow wage rate is by definition below the market wage: \( z < w \). If the employment opportunity (\( E \)) increases, \( z \) will also increase, as seen from (38) in Appendix 2. Hence the household's shadow wage rate will move closer to the market wage rate when off-farm employment increases.

Furthermore, as shown formally in Appendix 3, the subsistence effect gets weaker relative to the farm firm effect as \( E \) increases. In other words, the response in the Chayanovian model becomes more and more similar to the open economy model as the availability of off-farm employment increases. Thus, even if there is a qualitative jump between the models, the household's response is more gradual as the market integration increases.

**Property regime**

The shift from a private property regime to a "pure" open access situation does not have any implications on deforestation. This may be a surprising result in light of both the general resource economics literature and the popular environmental debate. This result is partly due to the fact that no benefits of forest conservation are included in the private property models. There may be some private benefits of standing forest, for example, various forest products. Most forest conservation benefits are, however, public environmental goods, either at the local, regional or global level. In order for private property to result in less deforestation than an open access regime, one must therefore argue that there are important conservation benefits which the decision-makers capture and therefore include in their decisions.
An argument for a private (or communal) property regime resulting in less deforestation is found when comparing the homesteading case of model IV with model III. An open access situation where forest clearing gives the farmers land rights may boost deforestation. This result premises that farmers expect higher land rent in the future. If they do not, even the homesteading case will not result in more deforestation than private property. The comparison of different models therefore clarifies some of the requirements for open access to result in more deforestation than other property regimes.

9 A numerical simulation

In order to get an indication on the magnitude of the different effects in the models, we present a simple numerical simulation. The values of the different variables are meant for illustrative purposes, but are as far as possible based on a household survey done in the Seberida district, Sumatra (Angelsen, 1995). The parameter values for the initial situation are given in appendix 4.

<table>
<thead>
<tr>
<th>Effect on b_{max} (km) of an exogenous change</th>
<th>Model:</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Subsistence (“full belly”)</td>
<td>II. Chayansvian</td>
</tr>
<tr>
<td>Initial situation</td>
<td>2.50</td>
</tr>
<tr>
<td>Population increased by 20 % (from 82 to 98 households)</td>
<td>2.74</td>
</tr>
<tr>
<td>Subsistence requirement increased by 20 % (c_{min} from 1200 to 1440)</td>
<td>2.74</td>
</tr>
<tr>
<td>Productivity increased by 20 % (x from 500 to 600)</td>
<td>2.28</td>
</tr>
<tr>
<td>Transport costs reduced by 20 % (q from 0.1 to 0.08)</td>
<td>2.74 / 2.79</td>
</tr>
<tr>
<td>Off-farm labour introduced (model II) (E from 0 to 0.5)</td>
<td>2.52 / 2.54</td>
</tr>
<tr>
<td>Real wage reduced by 20 % (w from 400 to 320)</td>
<td>5.63</td>
</tr>
<tr>
<td>Discount rate reduced from 20 to 10 %</td>
<td>2.89</td>
</tr>
<tr>
<td>Tenure security increased ((1-\lambda) from 70 % to 90 %)</td>
<td>3.00</td>
</tr>
<tr>
<td>Expectations about a 2 % points increase in x (g from 0.01 to 0.03)</td>
<td>3.52</td>
</tr>
</tbody>
</table>

Table 3: A numerical illustration of the effects of changes in the exogenous variables.

22 The simulation was done using a spreadsheet model in Lotus® 1-2-3®.
The numerical simulation is presented in Table 3. In model II two different situations are reported, depending on whether the subsistence or farm firm effects dominate, cf. appendix 4. These serve to illustrate the differences in the response to a productivity increase. In the first situation a 20 percent increase in $x$ will reduce the agricultural frontier from 2.7 to 2.6 km, in the second it will expand to 2.9 km. For changes in the other parameters there are only minor differences between the two situations.

The introduction of a fixed amount of off-farm employment in model II reduces the agricultural frontier from 2.72 to 2.52 and 2.54 km, respectively. Total labour input and consumption increase only slightly. In other words, in this simulation model providing alternative jobs directly replaces agricultural income rather than contributing to higher consumption.

More generally, we note that changes in the parameters which determine the agricultural frontier in the "full belly" and Chayanovian models have relatively modest effects on deforestation. This contrasts the open economy models (III and IV), where small changes in exogenous variables give quite dramatic effects on deforestation. Deforestation is very sensitive to changes in productivity or output price, and the wage level. A one time shift in $x$ of 20 percent will move the frontier from 2.5 km to 5.0 km in model III, and from 2.8 to 5.4 km in model IV. A drop in the wage rate by 20 percent has an even larger effect; the agricultural frontier moves to 5.6 and 6.0 km in the two models.

Expectations are important in model IV. An increase in the expected growth rate for the yield value ($x$) from 1 to 3 percent will move the agricultural frontier from 2.8 to 3.5 km away from the village centre. Changes in the discount rate and the land tenure security have relatively modest effects on deforestation in the model. This result is, however, sensitive to the relatively low initial expected growth in $x$. Combining the last three changes in Table 3 ($\delta = 0.1$, $\lambda = 0.1$ and $g = 0.03$) would move the agricultural frontier to 4.9 km. The results indicate, however, that the single most important factor to explain deforestation is the real wage rate, and that the main story is told in model III.

The results are not only sensitive to the parameter values, but also to the model specification. If we introduce convex distance costs, $l = (1 + qb^\beta)$, $\beta > 1$, in model III, the agricultural frontier is;

23 In interpreting the results, one should be aware of the fact that most economic models, including those presented here, assume friction free adaptations and no adjustment costs. Because such costs may be considerable, simple economic models tend to overestimate the effects, particularly for the short term responses.

24 The consumption level before and after the introduction of off-farm employment was 1415 and 1419 in the case when subsistence effect dominates ($\alpha = \beta = 0.2$), and 1413 and 1436 in the case when farm firm effects dominate ($\alpha = \beta = 0.9$).

25 One should note that the effects on the area of deforestation ($\text{km}^2$) are much larger than the effect on distance (km). For example, an increase in $b^\text{max}$ from 2.5 to 5.0 km implies an increase in the total area ($H^2$) from 1 968 to 7 854 hectares.
This will dampen the effects on deforestation of, say, an increase in the output price compared with the original model. The effect of a 20 percent increase in productivity with linear distance costs was to increase $b^{\text{max}}$ from 2.5 to 5.0 km. With quadratic distance costs ($\beta = 2$) the agricultural frontier will be extended to "only" 3.5 km.\footnote{The results are also sensitive to the assumption of uniform land quality. Assuming lower productivity (soil quality, slope, etc.) of new land would further dampen the effect of exogenous changes.}

10 Policy implications

Deforestation as an investment

In a situation where forest clearance gives land rights, deforestation should be modelled as an investment in land and a title establishment strategy. Much confusion is created in the literature and debate on agricultural expansion and deforestation because the issue of expansion (deforestation) is not clearly distinguished from the issue of management of resources which the farmers already control (e.g., soil conservation), as also argued in the case of logging by von Amsberg (1994). The general debate on environmental degradation in developing countries will be substantially clarified when this distinction is made, and more unambiguous results produced. Some of the unconventional results in the model on the effect of increased tenure security and reduced discount rate, and partly also technological progress, should be understood in this perspective.

Model IV should therefore be contrasted not only with the subsistence approach, but also with a conventional economic approach with the following framing of the deforestation problem: a social planner who maximizes the discounted benefits of various forest uses, in particular, between short and long term benefits, and -- related to this -- between productive and protective functions.\footnote{Examples of studies which use this approach include Strand (1992), Barbier and Rauscher (1994), Brazee and Southgate (1992) and Kahn and McDonald (1995). It should be added that some of the studies using this approach are intended for other purposes, for example, discussing optimal levels of deforestation; the argument is that for understanding the process of smallholder expansion this approach can be misleading.} Deacon (1994) provides an example of such an approach, where "deforestation is a form of disinvestment" (page 427). Whereas this may be true from a social planner's point of view, it may give a misleading description of the forces behind agricultural expansion and deforestation, and may yield conclusions and policy implications contrary those derived from model IV.

This point also concerns the micro level modelling of smallholder deforestation. Persson and Munasinghe (1995), in a CGE model for Costa Rica, have two versions of squatters' behaviour. When property rights are undefined (misleadingly called "common property" rather than open access), the model is similar to model III of this paper. In the second case "when property rights are well defined, there is a market for the forest" (page 282). The future value of forest is then included in the decision problem, and the conclusions follow conventional economic theory: higher discount rates and tenure insecurity lead to...
more deforestation. There are at least two objections to the relevance of this approach. First, markets in forest land are generally rare. Second, most values attached to standing forest has strong public goods characteristics, thus the incentive for individual farmers to include these in their decisions are very limited, even with secure property rights.

The intensification hypothesis

The subsistence approach focuses on high population growth, low agricultural productivity and people's struggle to reach the subsistence requirement (poverty) as the main causes of deforestation. A number of international reports are based on this framework, including the Brundtland report "Our Common Future" (WCED, 1987). The Alternatives to Slash-and-Burn (ASB) project headed by the International Centre for Research on Agroforestry (ICRAF) is based on the intensification hypothesis: "Intensifying land use as an alternative to slash-and-burn agriculture can help reduce deforestation (thereby conserving biodiversity and reducing greenhouse gas emissions) and alleviate poverty" (Tomich and van Noordwijk, 1995: 4; see also ASB-Indonesia, 1995: 119; ASB, 1994; Bandy et al., 1993). As noted in these reports the validity of the hypothesis depends on certain conditions, including effective monitoring and enforcement systems of the boundaries of the forest. This assumption is commonly violated. Thus we may have a very unpleasant conflict between reducing deforestation and increased agricultural production (poverty reduction), as Tomich and van Noordwijk (1995) also discuss.

An obvious response to the dilemma is to work at several levels to ensure that these necessary conditions for the intensification hypothesis to be valid are met. Establishing management and enforcement systems is, however, difficult and takes time. One option would then be to give priority to poverty reduction, and accept some deforestation. The argument could be that conversion to agroforestry preserves many of the natural forests' functions, which indeed is a valid argument (cf. Angelsen, 1995). Alternatively, priority is given to limiting deforestation. This implies that one should be very careful with implementing any projects (including agroforestry) which increase the benefits of frontier farming.

This potential trade-off is often ignored or not explicitly dealt with. Others are aware of the potential conflict, but it is downplayed, for example, by discussing how the extent of deforestation depends on farming methods, crop choices, farm investments, etc. (e.g., Deacon, 1992: 25ff; Panayotou and Sungsuwan, 1989). Whereas such factors may play a role, we still maintain as a general hypothesis that increases in productivity and/or output prices in frontier farming in the medium or long term lead to more forest being converted to agricultural land. This would also be in line with the general equilibrium model of Deacon (1995).

Land titling programmes

Possibly the most provocative and unconventional result of model IV is the potential counterproductive effects on deforestation of land titling programmes, which have as one of their prime objectives to increase tenure security. In a situation where rights to forest land are obtained through forest clearance, then facilitating more secure land
rights, for example, by offering cheaper land titles and more legal protection by the state, would increase the net present value of the deforestation investment.

Again, the result must be seen in light of the way smallholder deforestation is framed in our model. The conventional wisdom is that a secure property right is a prerequisite for taking long term environmental effects into account in the decision making. This is still relevant for the management of land already in production, or for the decision about forest conversion by an almighty, well informed social planner. Our argument is simply that neither of these approaches is appropriate for describing smallholder deforestation. Even with full tenure security there will be small incentives to include the effects on the protective functions of the forests.

How does this argument correspond with the empirical evidence? According to Kaimowitz (1995: 43) "recent land titling programmes initiated by Central American governments to provide secure rights for land occupants without legal title have fuelled the speculative drive for land".28 Other countries in Africa and Asia have had similar experiences. This should serve as a strong caution for initiating land titling programmes in situations where land rights can initially be obtained by forest clearing.

The fundamental problem is the unofficial rules of the game, where forest has to be cleared first in order to be eligible for titling. If it was possible to obtain a title without clearance, the farmers might consider retaining the forest, depending on the return of this option compared to alternative land uses. At least this would remove the incentive to cut the trees in order to stake an enforceable claim and capture a future positive land rent.

Discounting - credit markets

High discount rates are commonly blamed for environmental degradation, and this is confirmed within the social planner's approach to deforestation. In model IV, however, the effect of a lower discount rate is to augment the net present value of future land rents, and to boost deforestation. This result is analytically parallel to the discussion of tenure security. The empirical relevance of this is not yet fully explored. If farmers have no access to any credit market, the relevant discount rate will be the consumption rate of interest (CRI), reflecting pure time preferences, and the effect of diminishing marginal utility of consumption combined with (expectations about) consumption growth. CRI is likely to be positively related with the degree of poverty, which again could be seen as a reflection of alternative employment opportunities (w). It may not be unreasonable to assume that the latter effect on deforestation will predominate among poor people.

A discussion of the role of the discount rate seems more relevant in a situation of commercial agriculture, where the credit market interest rate may be the relevant discount rate. The effect of a change in the discount (interest) rate here follows the simple logic of investment theory. This also implies that getting access to credit, that is, replacing the CRI by a generally lower market interest rate as the relevant discount rate, may increase deforestation. Credit can be used to "finance deforestation", as many argue has been the case in Brazilian Amazon (e.g., Mahar and Scheider, 1994). Again, this contradicts conventional wisdom about improved access to credit being important to

28 See also Mahar and Scheider (1994: 163) for the Latin American evidence.
enhance environmental preservation. One modification of this result is in order. Credit
will often be important in the efforts to intensify cultivation on existing agricultural land
(the intensive margin), and could in this way reduce the pressure at the forest frontier.

**Alternative employment**

The opportunity for off-farm employment to (potential) frontier farmers is a variable of
critical importance to understand the process of deforestation. We found in the
numerical example deforestation to be very sensitive to changes in the wage rate. This
result is in line with Bluffstone (1995) in a study from Nepal. He concludes that the
availability of off-farm employment, albeit at a low wage rate, has prevented further
deforestation, and makes the agroforestry practised a stable system. The deforestation
record of Malaysia also illustrates the importance of providing alternative employment.
During recent decades, high economic growth and the creation of new jobs have
attracted a number of people from rural areas into the cities, thereby reducing the
pressure on forests from smallholder agriculture, although but not necessarily from other
agents of deforestation.

This situation should be contrasted with the Philippines, where economic crisis and
unequal land distribution in the lowlands have initiated a massive flow of migrants from
the lowlands to the uplands (World Bank, 1989). This push-migration has resulted in
significant pressure on forest resources in the uplands, including high rates of
deforestation.

**Accessibility (roads)**

Road construction is among the most indisputable and unambiguous factors in
promoting conversion of forest to other uses (e.g., Kaimowitz, 1995: 37; Chomitz and
Gray, 1994: 2; Schneider, 1995: 16; Mahar and Scheider, 1994: 161, Tomich and van
Noordwijk, 1995). Chomitz and Gray (1994), integrating the spatial dimension into an
economic model of land use in Belize, find strong support for market access and
distance to roads being key determinants of the type of land use, particularly for
commercial agriculture. Kummer and Sham (1994: 154) in a study from Philippines find
that 75 percent of the variation in forest cover is explained by road density. Yet, better
market access, including new roads, is often included in aid projects which aim to limit
deforestation.

This has led Schneider (1995) and others to suggest that road building should be
intensive rather than extensive. Decisions on road construction are normally not made
based on their impact on deforestation. Construction often takes place in relation to
logging operations, plantation developments, mining, or similar large-scale, (often) state
sponsored projects. In other cases, as in Northern Thailand and several Central
American countries, the rationale has been to facilitate military access and control of
areas influenced by opposition groups.

**11 Extensions**

The basic model could be extended in several directions. One possible extension would
be to endogenize the fallow period and/or the labour input. In models I-II this will
dampen the effects on deforestation of, say, a population increase. The need for increased production can be met by both higher labour input and shorter fallow period, as well as an area expansion. The former is in line with the Boserup (1965) and Ruthenberg (1980) hypothesis about increased population pressure leading to intensification of agricultural systems. The case with endogenous fallow period and labour input in the open economy models is discussed in Angelsen (1994).

The inclusion of other inputs would further highlight the fundamental difference between the population and market approaches. In the former lower input prices, e.g., on fertilizers, will reduce deforestation as fertilizers are substituted for land. Under the market approach it will augment the profitability of frontier framing and therefore increase deforestation, provided land and fertilizer are complementary inputs (see Angelsen et al., 1996).

The effects of including risk in frontier agriculture are also very different in the two modelling categories. In the open economy models increased risk makes risk-averse farmers reduce the scale of the risky activity (farming) and therefore reduce deforestation. In the subsistence case increased risk implies a larger area under cultivation as risk-averse farmers would aim to be on the safe side of the subsistence requirement (safety first models).

Another extension is to include the crop choice, and explore the effects of relative output prices on deforestation, depending on their land intensity. Moreover, it may be important to distinguish between annuals and perennials, particularly in model IV as perennials often would give higher tenure security than annuals (Angelsen, 1995; 1996). A further relevant disaggregation of the model is to split the agricultural sector into frontier (extensive margin) and non-frontier (intensive margin) agriculture. This could help clarify the deforestation effect of different policies, as it depends critically on the relative impact in the two sub-sectors.

**General equilibrium effects**

The four models presented are stylized descriptions, and real-life situations contain elements from all of them. One way of unifying the models would be through a general equilibrium approach, in particular, to endogenize the wage and output price (i.e., the real wage rate in models III-IV). In this way population could be included in models III-IV via effects in the labour and output markets: an increase in the labour supply would reduce the wage rate, and higher population would increase the prices for agricultural products. These effects have been found to be important in empirical studies (e.g., Kaimowitz, 1996).

29 This hypothesis is supported by, among others, Elnagheeb and Bromley (1994) in a study from Sudan.

30 See, for example, Roumasset (1976) for a more detailed discussion of safety first models.

31 See Deacon (1995) for an example of a general equilibrium model of deforestation, focusing particularly on tax instruments. The logic and conclusions of his model are similar to model III of this paper.

32 The Chayanov model resembles a general equilibrium (GE) model in the way that the
Including general equilibrium effects would modify some of the conclusions in models III-IV. Increased productivity in frontier agriculture will cause a downward pressure on domestic food prices, which will dampen the effects on agricultural expansion. Now it also becomes critical to distinguish between different sub-sectors within agriculture: a productivity increase in, for example, wet-rice production will limit deforestation through lower agricultural output prices. A general increase in agricultural productivity will boost deforestation if the market share of frontier farming is relatively small and/or food demand is price elastic.

Similarly, there may be important labour market feedbacks. Technological progress in frontier agriculture gives an upward pressure on wages, which also would dampen the effects on deforestation. The price elasticities in the output and labour markets will determine the net effects of, say, a technological jump. Yet, we could expect the qualitative results of the open economy models to remain valid.

Besides including general equilibrium feedback, another reason for extending the models would be to include more policy variables in them, and link a realistic description of micro level decisions to macro level and international economic variables. This represents a major challenge for future research.

12 Some remarks on testing of the models

The policy recommendations depend critically on the approach chosen. It is therefore important to know which approach gives the most realistic description of the frontier economy. Unfortunately, the discussion of this question in the literature is limited (López, 1992), but the above review of models provides some important lessons.

A major lesson is that the labour market assumption is vital. When unconstrained off-farm employment and migration are included in the model, the logic changes in a fundamental way. If alternative employment is available locally, or migration is an option, the open economy approach is more realistic. This depends, of course, on the time horizon: the open economy assumption is more relevant the longer the time horizon of the analysis. The review also suggests that for the purpose of analyzing deforestation, the usefulness of a conventional focus on the households' objectives for classifying different modelling approaches could be questioned.

The different models of deforestation should, in principle, allow for empirical testing and falsification/verification. There are at least two possible routes to pursue. Macro level statistical analysis has been attempted in a large number of studies.\footnote{See Brown and Pearce (1994) for the most comprehensive collection of such studies.} This approach suffers from some serious deficiencies. The quality of the data used seems to be very poor, in particular for rates of deforestation. The latter are commonly based on FAO (1992), which is derived from national forest inventories in 90 tropical countries. Only 18 of the countries have undertaken two or more inventories, which, in principle, makes it possible to estimate changes in forest cover. For the remaining 72 countries the deforestation in the period 1980-1990 is based on only one inventory (some as old as
from 1965). Forest cover and deforestation are then extrapolated by using a
deforestation model where population density is the only explanatory variable (FAO,
1992: 11). In effect, the estimates reflect changes in population density more than actual
changes in forest cover. Not surprisingly, population is the only variable that is
consistently significant in the regression models, cf. the summary in Brown and Pearce
(1994).

A regression model assumes implicitly that the independent variables have a similar
effect on the dependent variable in all countries, which is highly questionable. An
example is foreign debt, which is found to be very weakly correlated with deforestation
at the aggregate (global) level. At the same time, micro level studies show that debt may
have significant effects on deforestation in particular areas, in either direction (e.g.,
Reed, 1996). Partly related to the above, one should question the appropriateness of
focusing on macro level variables in cross country analysis. The agents of deforestation
make their decision based on a number of -- to them -- exogenous variables. The way in
which these variables are affected by macro level variables (such as GNP per capita,
economic growth, population growth, debt, etc.) varies significantly between countries.
Thus we should expect to find a much stronger correlation between deforestation and
the micro level decision parameters, than between deforestation and macro level
variables. A relatively rare example of a detailed analysis based on micro data is a
Mexican study by Deininger and Minton (1996).

A second line of inquiry would be to derive some general conclusions and policy
guidelines based on micro level field studies. Unfortunately, no comprehensive and
systematic aggregation of micro level studies has been attempted, thus the evidence is
scattered. The evidence from such relatively different settings as Brazil (Deacon, 1992;
Schneider, 1995), Central America (Kaimowitz, 1995), and Sumatra (Tomich and van
Noordwijk, 1995; Angelsen, 1995) seems, however, to support the explanatory power of
model IV. Studies on the effects of structural adjustment programmes suggest, however,
that the short term response of poor, subsistence farmers, particularly in African
countries, is more in line with the population approach (e.g., Reed, 1996: 329).

13 Summary and conclusions

The main purpose of this paper has been to illustrate some fundamental differences of
four different approaches to modelling agricultural expansion. The approaches differ
both with respect to which variables are important for deforestation, and in the case of
productivity or output price increases, the direction of the effect. Because these
differences are often not made explicit, the intention has been to highlight the
differences in assumptions and their consequences for the conclusions, rather than
exploring in detail what an empirical model of deforestation should look like.

The paper has also argued that the open economy, open access model (IV), possibly
extended to include general equilibrium effects in the labour and output markets, will
give the best description of many frontier economies, particularly when focusing on the
long term effects. Because forest clearing gives the farmers land rights, deforestation is
for them a title establishment strategy and an investment.
This framing of smallholder deforestation challenges not only the subsistence or population driven approach to deforestation, but also a conventional economic approach based on a social planner's perspective. Intensification, land titling and credit programmes -- commonly suggested remedies for reducing environmental degradation -- *may* enhance deforestation, although there are effects pushing in both directions. This approach also makes us face some potential and unpleasant conflicts, for example, between poverty reduction and reducing deforestation.

The paper has identified two sets of policies which are robust to a wide set of modelling assumptions. First, both this paper and empirical evidence suggest that lower access costs fuel deforestation. Second, provision of alternative employment and income opportunities reduce the pressure at the forest frontier. These results suggest a redirection of the focus, away from ambiguous intensification programmes and price policy reforms towards road building and the off-farm sector, in the efforts to reduce the rate of deforestation.
Appendix 1: Farm firm (substitution) and subsistence (income) effects in agricultural household models

The appendix aims to give a brief summary of some major points in the economic theory of agricultural households, discuss different assumptions and their implications, and relate this to the models of deforestation developed in the paper. Further, we want to link this theory to the standard theory of the consumer and the firm. Finally, some particular concepts (curvature effects and virtual prices) are discussed.

The distinction between farm firm and subsistence effects is central in model II of this paper. This refers to the dual role of an agricultural household as both a farm firm and a labourers' and consumers' household (Nakajima, 1986). In its first role it acts like a profit maximizing firm, in its second it acts like a utility maximizing consumer. The combination of these two roles is the key to the economic theory of agricultural households, see particularly Nakajima (1986) and Singh et al. (1986). A major result in this theory is that when the household is not constrained in the labour market (i.e., markets are competitive), the decision process is recursive. First, the production problem is solved, including the determination of labour demand. Then the profit from the agricultural production is carried over into the budget constraint, termed the profit effect, and the consumption and labour supply problem is solved. When the household is constrained, the household's labour demand on the farm must equal the household's labour supply, subtracted any exogenous off-farm sale of labour. The production and consumption problems must then be solved simultaneously.

Since labour demand is directly related to the extent of deforestation, our focus is on demand rather than supply. The aim of this appendix is to apply a simple model for an agricultural household, and clarify the effects of an output price increase on the labour demand under different assumptions. Not that the model is not identical to model II as the off-farm income (I) does not require any labour input, hence we get explicitly the income effect when I increases.

**The classical (primal) approach**

We consider the simple case with one commodity produced and sold in a competitive market at a price (p), using only family labour in production (L). There is no alternative use of labour, except leisure. The income consists of farm income and an exogenous income (I). All income is used for consumption (C). The price of consumption goods is set to unity (numéraire). We form the Lagrangian of the household's utility maximization problem;

\[ \Omega = U(C, L) - \gamma (pf(L) + I - C) \]

34 For simplicity we assume, unlike in model II, that there is no labour input attached to this exogenous income.

35 To simplify the study of price changes we have to think of the output as a cash crop which does not enter the consumption basket; or we interpret \( p \) to be a parameter reflecting the technological level in an autarkic economy where all output is consumed. See, for example, Singh et al. (1986) for models where the output is partly consumed.
The utility and production functions are assumed to be well behaved. The FOCs are:

\( U_L - \gamma pf_L = 0 \)  
\( U_C + \gamma = 0 \)  
\( pf(L) + I - C = 0 \)

The differential form of this system is:

\[
\begin{bmatrix}
U_{LL} - \gamma pf_{LL} & U_{LC} & -pf_L \\
U_{CL} & U_{CC} & 1 \\
\gamma pf_L & -1 & 0
\end{bmatrix}
\begin{bmatrix}
dL \\
dC \\
d\gamma
\end{bmatrix}
= \begin{bmatrix}
\gamma pf_L & 0 \\
0 & 0 \\
-f(L) & -1
\end{bmatrix}
\begin{bmatrix}
dp \\
dI
\end{bmatrix}
\]

This is the Fundamental Matrix Equation of Consumer Demand as applied to an agricultural household (see, for example, Barten and Böhm, 1982). For simplicity we shall assume an additive utility function \( U_{CL} = U_{LC} = 0 \). The effect on labour input of a change in exogenous income and in output price is then given by;

\[
\frac{\partial L}{\partial I} = -\frac{1}{D}[U_{CC} pf_L] < 0
\]

\[
D = U_{LL} + U_{CP} pf_{LL} + U_{CC} (pf_L)^2 < 0
\]

\[
\frac{\partial L}{\partial p} = \frac{1}{D}[-U_{CL} - U_{CC} f(L) pf_L]
\]

\( D \) is the determinant of the coefficient matrix (Hessian). The effect of higher income is negative, as it makes the household demand more leisure and therefore work less.

The sign of the effect of higher output price is ambiguous. The first term reflects the fact that value of marginal labour input has increased when the price has increased. This is a substitution effect, which is positive. We shall refer to it as the \emph{farm firm effect} for reasons elaborated below.

The second term reflects the effect of an income change, which is identical to the effect in (22), multiplied by the actual change in income \( -- f(L) \). Because of declining marginal utility of consumption this effect is negative. If the household is poor and close to its subsistence constraint, the net effect of reduced output price may be to increase its labour supply in order to survive, cf. appendix 2. Thus we label the second effect in (23) the \emph{subsistence effect}, keeping in mind that it is similar to the income effect.

\emph{Curvature effects}

Compared to the consumption-leisure choice of standard demand theory, where the consumer faces an exogenous wage rate, the main difference here is the element \(-pf_{LL}\) in the coefficient matrix. In their discussion of agricultural household models, Hymer and Resnick (1969) and Barnum and Squire (1979) refer to this as a \emph{curvature effect} (CE), which comes as a third effect in addition to the standard income effect (IE) and substitution effect (SE). The curvature effect is due to the non-linearity of the budget constraint. This may provide an intuitive explanation of the difference between the

The curvature effect is, however, not commonly used in the literature. One reason may be that the straightforward and explicit solution of the system is simpler. Further the curvature effect is already included in \( D \), and does in this way affect both the SE and IE even though its not included in the numerator. The curvature effect increases the absolute value of \( D \), hence the effect of a price increase is smaller compared to the standard theory with a linear budget constraint. The intuition is clear: suppose the farm firm effect dominates and \( L \) increases. The declining marginal productivity of labour will dampen the effect. Similarly, if the subsistence effect dominates (\( L \) is reduced), the reduction will be smaller because increasing marginal productivity of labour will restore the equilibrium faster as we move down on the production frontier. Thus the curvature effect is more logically seen as an effect which reduces the magnitude of both the IE and SE, rather than as a third effect. Finally, one should note that the curvature effect has no impact on the sign of (23).

**Profit maximizing**

Next we want to highlight the difference between the utility maximizing approach (endogenous shadow wage) and the profit maximizing approach (exogenous wage) on the labour input. Under the profit maximizing approach the agricultural household behaves like a farm firm. The simple profit maximizing problem is given by;

\[
(24) \quad \pi = pf(L) - wL
\]

The FOC is given by \( pf_L - w = 0 \). The response to an output price increase is;

\[
(25) \quad \frac{dL}{dp} = \frac{f_L}{pf_{LL}} > 0
\]

We see that this expression resembles the first term in (23), hence the name farm firm effect. In fact, the profit maximizing case can be viewed as a special case of the utility maximizing one, that is, when the marginal utility of consumption and labour is constant; in the case when \( U_c = 1 \) and \( U_L \) is constant, the expressions in (23) and (25) are identical.

**Full belly**

The full belly problem is rather trivial in the case with only one choice variable. The problem is simply to minimize labour efforts given the subsistence constraint: \( C = pf(L) + I \). The labour response to a price change is in this case given by;

\[
(26) \quad \frac{dL}{dp} = \frac{f_L}{pf_{LL}} < 0
\]

This effect resembles the income effect in the general utility maximizing case. Multiplying the expression by \( U_c pf_L \), we see that the numerator becomes identical to the income effect in (23), whereas the denominator lacks the first two elements, which implies that the absolute value is smaller. Thus even if there is some resemblance to the income effect, the change in labour supply will always be larger in the full belly case. As
a special case, when the marginal productivity of labour is constant (no curvature effect) and the disutility of labour is constant, the income effect in (23) is identical to (26).

**The Nakajima approach and virtual prices**

Following Nakajima (1986), the FOC of the agricultural household model can alternatively be written as;

\[
(27) \quad \frac{\partial f}{\partial z} = 0; \quad z = z(C, L) \equiv -\frac{U_L}{U_C} \cdot C = pf(L) + I
\]

\(z\) is the marginal rate of substitution between \(L\) and \(C\), or the subjective or shadow wage rate (Nakajima, 1986), or the virtual price of labour (Neary and Roberts, 1980). Virtual prices are defined as those prices which would induce an unrationed household to behave in the same manner as a rationed household (no access to an off-farm labour market). The use of virtual prices greatly facilitates the comparative statics analysis, as we can use a two step procedure; first we trace the effect of exogenous changes on virtual prices, then we study how the change in virtual prices affects the allocation (Schroyen, 1991).36 We shall therefore make heavy use of this in both this paper and chapters 6 and 7.

The response to a change in the exogenous income and the output price can then be written as;

\[
(28) \quad \frac{dL}{dp} = \frac{\partial L}{\partial p} = \frac{z_C}{pf_{LL} - pf_{LC}} \cdot z_C = \frac{U_L U_C}{U_C^2} > 0, z_L = -\frac{U_L}{U_C} > 0
\]

\[
(29) \quad \frac{dL}{dp} = \frac{f_L + \frac{\partial L}{\partial p}}{pf_{LL} - pf_{LC}} = \frac{-f_L + \frac{\partial L}{\partial p}}{pf_{LL} - pf_{LC}}
\]

Multiplying by \(U_C\) and using (27), it is straightforward to verify that this gives the same result as in (22) and (23). Compared to the latter, (29) gives a more intuitive interpretation of the effects: the first term in the numerator, the farm firm (substitution) effect, refers to the effect on \(L\) keeping \(z\) constant, whereas the second, the subsistence (income) effect, gives the effect on \(L\) of changes in \(z\) following a change in \(p\).

When comparing the above results with Singh et al. (1986: appendix), note that our concern is with the demand for labour, which is the relevant one for deforestation. We compare a situation where the household is constrained in the labour market (and therefore household supply equals demand) to a situation with a perfect labour market. The effect of an output price increase on the labour supply is in the latter case similar to the income effect in the constrained case (negative), whereas the effect on labour demand is similar to the substitution effect (positive).

Except for the curvature effects, the farm firm and subsistence effects are similar to the substitution and income effects in standard consumer theory. We shall therefore use the

---

36 For a general treatment of virtual prices and household behaviour under rationing, see Neary and Roberts (1980), using the dual approach, and Schroyen (1991), using the primal approach.
terms interchangeably.\textsuperscript{37} We find it, however, useful to introduce the terms farm firm and subsistence effects, as it directly refers to the dual role of agricultural households, and serves to highlight the importance of the labour market assumption. Moreover, as discussed above, they also include a curvature effect which is not included in standard consumer theory.

\textit{The elasticity of the shadow wage rate}

Using (27) the numerator in (29) can be reformulated to explore which effect will dominate when the output price changes. The condition for the subsistence effect to dominate, i.e., (29) being negative, is;

\begin{equation}
-f_L + f(L)z_C = -\frac{z}{z_f} + f(L)z_C > 0
\end{equation}

\[ \iff \frac{\partial x}{\partial p(L)} \frac{p(L)}{z} > 1 \]

The subsistence effect will dominate if the elasticity of the shadow wage rate or virtual price of labour with respect to farm income is larger than unity. With the specification of the utility function used in this paper we find that this will be the case for income levels sufficiently close to subsistence, cf. appendix 2.

\textsuperscript{37} See the appendix by Strauss in Singh et al. (1986) for a dual approach to the derivation of the income and substitution effects in the theory of agricultural households.
Appendix 2: Comparative statics

Model I

(31) \[ \frac{db_{\text{max}}}{dx} = \frac{b_{\text{max}}}{2c} < 0 \]

(32) \[ \frac{db_{\text{max}}}{dN} = \frac{b_{\text{max}}}{2N} > 0 \]

(33) \[ \frac{db_{\text{max}}}{dc_{\text{max}}} = \frac{b_{\text{max}}}{2c_{\text{min}}} > 0 \]

Model II

We use the following functional form for household utility:

(10) \[ U(C, T) = (C - c_{\text{min}})^\alpha + \nu(T_{\text{max}} - T)^\beta; \quad \nu > 0; \quad \alpha, \beta \in (0, 1); \quad (C - c_{\text{min}}), (T_{\text{max}} - T) > 0 \]

The first order condition, and the virtual price of labour marginal rate of substitution (z) are then given by:

(8) \[ x - z(1 + qb_{\text{max}}) = 0 \]

(34) \[ z = -\frac{U_T}{U_C} = \frac{\nu \beta (C - c_{\text{min}})^{1-\alpha}}{\alpha(T_{\text{max}} - T)^{1-\beta}} \]

\[ \frac{\partial z}{\partial C} = \frac{\nu \beta (1-\alpha)(C - c_{\text{min}})^{-\alpha}}{\alpha(T_{\text{max}} - T)^{1-\beta}} > 0 \]

\[ \frac{\partial z}{\partial T} = \frac{\nu \beta (\beta - 1)(C - c_{\text{min}})^{1-\alpha}}{\alpha(T_{\text{max}} - T)^{2-\beta}} > 0 \]

The effect of a change in the value of agricultural yield (x) is given by:

\[ \frac{db_{\text{max}}}{dx} = \frac{1}{\mu} \left[ 1 - (1 + qb_{\text{max}}) \frac{\partial z}{\partial x} \right] \]

\[ \mu = zq + (1 + qb_{\text{max}})hb_{\text{max}}[xz_c + (1 + qb_{\text{max}})z_T] > 0 \]

Using (8) and (34), and that \( \frac{\partial z}{\partial x} \equiv z_x = z_x H = z_c H \), we get;

(35) \[ \frac{db_{\text{max}}}{dx} = \frac{1}{\mu} \left[ 1 - \frac{\partial z}{\partial x} \right] = \frac{1}{\mu} \left[ 1 - \frac{X(1-\alpha)}{C_{-c_{\text{min}}}} \right] \]

\[ \begin{align*}
> 0 & \iff X > \frac{c_{\text{min}} - wE}{\alpha} \\
= 0 & \iff X = \frac{c_{\text{min}} - wE}{\alpha} \\
< 0 & \iff X < \frac{c_{\text{min}} - wE}{\alpha}
\end{align*} \]

The magnitude of the subsistence effect is determined by the elasticity of z with respect to x, as given by second element in the [ ]. The size of it depends on the level of agricultural income, off-farm income, the elasticity of marginal utility with respect to surplus consumption (\( \alpha - I \)), and the subsistence level. For values of X below \( (c_{\text{min}} - wE)/\alpha \), the elasticity is larger than one, and the subsistence effect will predominate the farm firm effect.
The implications of using different functional forms for utility should be noted. Consider the case with no off-farm employment: \( E = 0, \) and \( X = C, T = L. \) Using the standard (multiplicative) Stone-Geary utility function, \( (C - c_{min})^\alpha (T^{max} - T)^\beta, \) the expression in (35) becomes \( \frac{1}{\mu} \left[ 1 - \frac{x}{x_{min}} \right] < 0, \) thus the subsistence effect will always dominate. Reducing this further to a linear logarithmic (or Cobb-Douglas type) utility function, \( e_{min} = 0, \) gives that the net effect of an increase in \( x \) on \( b^{max} \) will be zero.

Using the Houthakker additive function implies setting \( e_{min} = 0 \) in our original formulation. It is readily seen that the elasticity is always less than one, and (35) simply becomes \( \frac{a}{\mu} > 0. \) With this functional form the farm firm effect will always dominate. In summary, the chosen functional form provides the most flexible form which allows the elasticity of \( z \) with respect to \( x \) to be either above (Stone-Geary) or below (Houthakker) unity. This clearly highlights what Stern (1986: 157) underscores in his review of labour supply functions: "inflexibility in the functional forms used may produce very misleading predictions and policies".

Finally we should note that the function used is equivalent to using a CES function if we assume \( \alpha = \beta, \) as done in the numerical simulation. The CES function with subsistence level and maximum labour input is given by \( \left[ \tau(C - c_{min})^{-\rho} + (1 - \tau)(T^{max} - T)^{-\rho} \right]^{-1/\rho}. \) Since positive monotone transformations are allowed, we find by assuming \( \alpha = \beta = -\rho, \) and setting \( \nu = (1 - \tau)/\tau \) that this is equivalent to the function used.

\[
\frac{\partial b_{max}}{\partial q} = -\frac{1}{\mu} \left[ zb_{max} + (1 + qb_{max}) \frac{\partial z}{\partial q} \right] < 0 \\
\frac{\partial z}{\partial q} = \int_0^{b_{max}} h b^2 db z_T > 0
\]

\[
\frac{db_{max}}{dh} = -\frac{1}{\mu} \left[ (1 + qb_{max}) \frac{\partial z}{\partial h} \right] < 0 \\
\frac{\partial z}{\partial h} = \int_0^{b_{max}} x b db z_C + \int_0^{b_{max}} (1 + qb)b db z_T > 0
\]

\[
\frac{db_{max}}{dE} = \frac{1}{\mu} \left[ -(1 + qb_{max}) \frac{\partial z}{\partial E} \right] < 0 \\
\frac{\partial z}{\partial E} = w z_C + z_T > 0
\]

\[
\frac{db_{max}}{dw} = \frac{1}{\mu} \left[ -(1 + qb_{max}) \frac{\partial z}{\partial w} \right] < 0 \\
\frac{\partial z}{\partial w} = E z_C > 0
\]

\[
\frac{db_{max}}{dc_{min}} = \frac{1}{\mu} \left[ -(1 + qb_{max}) \frac{\partial z}{\partial c_{min}} \right] > 0 \\
\frac{\partial z}{\partial c_{min}} = -\frac{\beta}{\alpha(T^{max} - T)} < 0
\]

\[
\frac{db_{max}}{dT_{max}} = \frac{1}{\mu} \left[ -(1 + qb_{max}) \frac{\partial z}{\partial T_{max}} \right] > 0 \\
\frac{\partial z}{\partial T_{max}} = -\frac{\beta}{\alpha(T^{max} - T)^2} < 0
\]
Model III

\[
\begin{align*}
\frac{d\ell_{\text{max}}}{dx} &= \frac{1}{wq} \frac{\dot{\rho}_{\text{max}}}{x-w} > 0 \\
\frac{d\ell_{\text{max}}}{dq} &= \frac{x-w}{wq^2} \frac{\dot{\rho}_{\text{max}}}{q} < 0 \\
\frac{d\ell_{\text{max}}}{dw} &= \frac{-x}{qw^2} \frac{1+q\dot{\rho}_{\text{max}}}{qw} < 0
\end{align*}
\]

Model IV

\[
\begin{align*}
\frac{d\ell_{\text{max}}}{dx} &= \frac{0}{wq} \frac{\partial \ell_{\text{max}}}{\partial x-w} > 0 \\
\frac{d\ell_{\text{max}}}{dq} &= \frac{-x-w}{wq^2} \frac{\dot{\rho}_{\text{max}}}{q} < 0 \\
\frac{d\ell_{\text{max}}}{dw} &= \frac{-x}{qw^2} \frac{1+q\dot{\rho}_{\text{max}}}{qw} < 0 \\
\frac{d\ell_{\text{max}}}{d(\delta+\lambda)} &= \frac{x}{qw} \frac{g}{(\delta+\lambda-g)^2} < 0 \\
\frac{d\ell_{\text{max}}}{dg} &= \frac{x}{qw} \frac{\delta^{1+\lambda}}{(\delta+\lambda-g)^2} > 0
\end{align*}
\]
Appendix 3: The weakening of the subsistence effect as the labour market integration increases in model II

The subsistence effect when the value of yield \( x \) increases is given from (35) as \( SE = \frac{x(1-a)}{X+wE-c^{min}} \), whereas the farm firm is equal to one, both divided by \( \mu > 0 \), which we ignore here as it is the strength of subsistence effect relative to the farm firm effect which is of interest. We want to prove that \( SE \) is reduced as \( E \) increases;

\[
\frac{\partial SE}{\partial E} = \frac{(1-\alpha)\left(\frac{X}{X+W-E-c^{min}}\right)(X+wE-c^{min})}{(X+wE-c^{min})^{2}} - \frac{(1-\alpha)\left(\frac{\partial X}{\partial E} + w\right)}{(X+wE-c^{min})^{2}} \left[ \frac{\partial X}{\partial E}(wE-c^{min}) - Xw \right] < 0
\]

\[\Leftrightarrow W/E - c^{min} - X \frac{W}{E} > 0; \quad \frac{\partial X}{\partial E} < 0 \text{ cf. (38)}\]

As the consumption cannot be lower than the subsistence level, \( X+W-E-c^{min} \geq 0 \), a sufficient condition for the inequality to hold is;

\[
\left| \frac{w}{\frac{\partial X}{\partial E}} \right| > 1 \Leftrightarrow -\frac{\partial X}{\partial E} < 1
\]

Using (1), (3) and (38) we get;

\[
\Leftrightarrow \frac{xh_{max}(1+q_{max})(w^{z}+\gamma)}{w^{z}q+w(1+q_{max})h_{max}[z+c+(1+q_{max})\gamma]} < 1
\]

\[\Leftrightarrow xh_{max}(1+q_{max})w^{z}c + z\gamma - wzq - w(1+q_{max})h_{max}[z+c+(1+q_{max})\gamma] < 0\]

Using the FOC and manipulating the expression yields;

\[
\Leftrightarrow xh_{max}(1+q_{max})w^{z}c + z\gamma - wzq - w(1+q_{max})xh_{max}(z+c + \frac{\gamma}{\gamma}) < 0
\]

\[\Leftrightarrow xh_{max}(1+q_{max})[\frac{z\gamma}{w} - \frac{\gamma z}{\gamma}] - zq < 0\]

When the household is constrained in the labour market, \( w \geq z \), thus the expression in [ ] is non-positive, and the inequality holds. This completes the proof.
### Appendix 4: Parameter values in the numerical example

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol</th>
<th>Initial value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population (households)</td>
<td>$N$</td>
<td>82</td>
</tr>
<tr>
<td>Land use intensity</td>
<td>$m$</td>
<td>10</td>
</tr>
<tr>
<td>Subsistence requirement</td>
<td>$c^\text{min}$</td>
<td>1200</td>
</tr>
<tr>
<td>Maximum labour input</td>
<td>$L^\text{max}$</td>
<td>5</td>
</tr>
<tr>
<td>Output per ha</td>
<td>$x$</td>
<td>500</td>
</tr>
<tr>
<td>Labour per ha</td>
<td>$l$</td>
<td>1</td>
</tr>
<tr>
<td>Distance costs</td>
<td>$q$</td>
<td>0.1</td>
</tr>
<tr>
<td>(parameter)</td>
<td>$\alpha$</td>
<td>0.2 / 0.9</td>
</tr>
<tr>
<td>(parameter)</td>
<td>$\beta$</td>
<td>0.2 / 0.9</td>
</tr>
<tr>
<td>(parameter)</td>
<td>$\nu$</td>
<td>8 / 242</td>
</tr>
<tr>
<td>Wage</td>
<td>$w$</td>
<td>400</td>
</tr>
<tr>
<td>Off-farm employment available in model II</td>
<td>$E$</td>
<td>0</td>
</tr>
<tr>
<td>Discount rate</td>
<td>$\delta$</td>
<td>0.2</td>
</tr>
<tr>
<td>Risk of loosing the land</td>
<td>$\lambda$</td>
<td>0.3</td>
</tr>
<tr>
<td>Expected growth in $x$</td>
<td>$g$</td>
<td>0.01</td>
</tr>
<tr>
<td>(parameter)</td>
<td>$\theta$</td>
<td>1.026</td>
</tr>
</tbody>
</table>

The two sets of parameter values for $\alpha$, $\beta$ and $\nu$ refer to two different situations in the Chayanovian model. In the first case the subsistence effect will dominate (low $\alpha$); in the second the farm firm effect will dominate.
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