Report

REMES – a regional equilibrium model for Norway with focus on the energy system

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ABSTRACT
The RegPol project (Regional effects of energy Policy) studied interrelations between energy systems and the economy to improve the understanding of regional differences, needs and barriers for the move towards a more sustainable energy system. A quantitative framework was developed in the form of a hybrid modelling approach linking detailed energy system models with regional macro-economic models. Part of the latter, the REMES model represents a multi-regional multi-sectoral Computable General Equilibrium (CGE) model to analyze the impacts of different climate policy measures and the interactions between national and regional energy markets and the overall economy. The incorporation of fuel substitution allows to handle regional issues such as decentralized energy production or transmission needs or to investigate interactions between the local energy system and infrastructure and the broader economy, including labour and housing markets. Moreover, the model is flexible with respect to input structure such as number of regions, industry sectors and commodities and with respect to linking options with other models. This report discusses the background of the RegPol project, describes the data structure required for the model, documents preprocessing routines to enable a flexible data input and gives a general explanation of the REMES model and its implementation in the MPSGE extension to the GAMS system.

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<th>Description</th>
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<tbody>
<tr>
<td>CED</td>
<td>Constant Elasticity of Demand</td>
</tr>
<tr>
<td>CES</td>
<td>Constant Elasticity of Supply</td>
</tr>
<tr>
<td>CET</td>
<td>Constant Elasticity of Transformation</td>
</tr>
<tr>
<td>CGE</td>
<td>Computable General Equilibrium</td>
</tr>
<tr>
<td>CREEA</td>
<td>Compiling and Refining Environmental and Economic Accounts, creea.eu</td>
</tr>
<tr>
<td>GAMS</td>
<td>General Algebraic Modelling System</td>
</tr>
<tr>
<td>FNR</td>
<td>Regional accounts (“Fylkesfordelt NasjonalRegnskap”)</td>
</tr>
<tr>
<td>IDE</td>
<td>Integrated Development Environment</td>
</tr>
<tr>
<td>LCP</td>
<td>Linear Complementarity Problem</td>
</tr>
<tr>
<td>LP</td>
<td>Linear Programming Problem</td>
</tr>
<tr>
<td>MILP</td>
<td>Mixed Integer Linear Programming Problem</td>
</tr>
<tr>
<td>MPSGE</td>
<td>Mathematical Programming System for General Equilibrium analysis</td>
</tr>
<tr>
<td>NPISH</td>
<td>Non-Profit Institutions Serving Households</td>
</tr>
<tr>
<td>PANDA</td>
<td>Plan and analysis system for trade and industry, demography and labour market (“Plan- og Analysesystem for Næringsliv, Demografi og Arbeidsmarked”, panda-gruppen.no)</td>
</tr>
<tr>
<td>REMES</td>
<td>Regional Equilibrium Model for Norway with focus on the Energy System</td>
</tr>
<tr>
<td>ROW</td>
<td>Rest of World</td>
</tr>
<tr>
<td>SAM</td>
<td>Social Accounting Matrix</td>
</tr>
<tr>
<td>SCGE</td>
<td>Spatial Computable General Equilibrium</td>
</tr>
<tr>
<td>SSB</td>
<td>Statistics Norway (Statistisk SentralByrå, ssb.no)</td>
</tr>
<tr>
<td>SSB65</td>
<td>Statistics Norway standard aggregation of sectors (65 sectors)</td>
</tr>
<tr>
<td>TTM</td>
<td>Transport and Trade Margins</td>
</tr>
</tbody>
</table>
1 Introduction

1.1 The RegPol project

The primary objective of the project “Regional effects of energy policy (RegPol)” was to develop a hybrid regional model framework for Norway integrating technology-rich bottom-up models for the regional energy system and macro-type economic models for the regional economy. The main goal was to improve the understanding of interrelations between energy systems and the economy, in particular with respect to regional effects. This helped paving the way for deeper analyses of regional differences, needs and barriers on the way towards more sustainable energy systems. The RegPol project was a knowledge-building project for industry, funded by the Research Council of Norway under the RENERGIX program. Additionally, the user partners Enova, county authorities for Sør-Trøndelag, Nord-Trøndelag, Nordland and Møre og Romsdal counties, the Norwegian Water Resources and Energy Directorate (NVE), Nord-Trøndelag Elektrisitetsverk Holding AS (NTE), TrønderEnergi Kraft AS, TrønderEnergi Nett AS and TrønderEnergi AS contributed with funding.

Research activities in the RegPol project combined expertise in energy system and economic modelling and analysis from SINTEF, NTNU, IFE and TNO (Netherlands) and were tightly coordinated with corresponding activities in the Centre for Sustainable Energy Studies (CenSES). As one of the main deliveries in the project, a quantitative framework was developed in the form of a hybrid modelling approach linking detailed energy system models with regional macro-economic models. Part of the latter, the REMES model represents a multi-regional multi-sectoral Computable General Equilibrium (CGE) model.

The development of REMES was carried out in close collaboration with the project “Development of regional economic models and research on choice of residence – REGMODELL”, financed by the Ministry of Local Government and Modernisation through the Research Council of Norway. In particular, Olga Ivanova (formerly TNO, now PBL – Netherlands Environmental Assessment Agency) provided substantial contributions to the development of the CGE model and data preprocessing routines. REMES is at the core of further development efforts towards a spatial CGE (SCGE) model in the REGMODELL project.

Strengthening links between detailed energy system models and regional macro-economic type of models – and running such models together – helps to avoid sub-optimization as both areas can be taken into account in a more integrated manner. In order to achieve this, the project formulated a model approach for the regional level, combining a TIMES model of regional energy systems and the REMES model focussing on macroeconomic impacts. The project worked with different ways of linking the two models within a so-called hybrid modelling approach, spanning from “soft” (manual) to “hard” (fully integrated) linking (Helgesen et al., 2017). For a discussion and classification of hybrid modelling and linking between macro-economic models and energy system models, see Strachan and Kannan (2008), Schäfer and Jacoby (2005) or Böhringer and Rutherford (2007).

In addition to a detailed description of the regional economy and interregional trade (up to 64 industries and a free number of regions based on municipality aggregation), the REMES model contains regional details for various real economic variables such as production, intermediate consumption or gross value added as well as factors of production (labour and capital), trade, transport margins, taxes or subsidies. Expenditures and revenues are given for different economic sectors such as households and government at the regional level. Also fuel substitution in electric power generation, refineries, industry and demand sectors has been incorporated.

The project contributed to energy system modelling by improving the geographical representation of energy transport links, production and demand. It also demonstrated how the energy sector, and other related sectors, can be represented adequately in regional economic modelling. This allows, for example, a more efficient handling of potential regional issues such as decentralized energy production or transmission needs as well as investigation of interactions between the local energy system and infrastructure and the broader economy, including labour and housing markets. Hence, the model can be used for regional economic analysis with focus
on the energy system, for example, economic, energy and environmental policy analyses, including regional tax and subsidy differentiation. Lind and Rosenberg (2014) describe an application of a multi-region formulation of the TIMES energy system model while this report provides a summary of the development of the macroeconomic model. Within the scope of the RegPol project, the hybrid approach has been applied to cases suggested by the user partners, such as investigating regional effects of Green Certificates or effects on the energy system and the economy from implementing a green transport policy.

1.2 Computable General Equilibrium Models

Computable general equilibrium models are economic models that use actual data to describe relations between (groups of) the main actors in an economy and simulate how these actors and the whole economy will respond to changes in policy and to other exogenous changes. Quite often they are based on Social Accounting Matrices (SAMs, see section 2.1) and can, hence, be understood as models of National Accounts. The behaviour of the market actors is modelled according to microeconomic theory. Assuming that the economy is, in general, in an equilibrium state from which no actor has incentive to deviate unilaterally, CGE models can be used to estimate effects of external impacts such as tax changes, investments, technological changes, policies, but also more severe shocks such as price changes of important export goods, on the economy. As the various actors respond to these changes by adapting their consumption or production (or factors of production such as labour or capital), one can investigate how these responses will propagate through the economy and to other regions in the form of shifting supply and demand for goods – until the equilibrium in the economy is re-established. Examples of CGE models in a Norwegian or Nordic context are the various instances of the MSG model of Norway (starting with Johansen (1960), see also Heide et al. (2004)), SSB’s NORMEN model for the Nordic countries with focus on electricity (Aune et al., 1996) or GRACE with focus on environmental effects (albeit with a more global perspective, Aaheim and Rive (2005)).

Multi-regional CGE models have been developed for both the supra- and the sub-national level. These models emphasise interregional trade in addition to the common national/regional economic relations. There are several examples of global or international multiregional models, like the GTAP model. Multiregional CGE models at the sub-national level have been more rare, but have experienced an increasing interest during the last decades. An example for a long-lasting development of such models is the MONASH model for Australia and its successors. Lately, an interregional CGE model for the Brazilian economy has been developed, the ENERGY-BR model (Santos et al., 2013), incorporating energy substitution modelling from the MONASH variant MMRF-GREEN (Adams et al., 03). The model has been used for evaluating the long-run regional impacts of tariff policies on the Brazilian electric power sector.

Spatial CGE models (SCGE) (for the sub-national level) have gained much interest during the last two decades, especially in connection with the assessment of economic impacts of transport infrastructure investments. Examples are the PINGO model for Norway (Ivanova et al., 2006) and the STRAGO model for Sweden (Sundberg, 2009). A more comprehensive SCGE model incorporating elements from New Economic Geography theory and with dynamic features is the RAEM model for the Netherlands (Ivanova et al., 2007). An ambitious model development project which covers both the supra- and the sub-national level is the RHOMOLO model for the analysis of EU cohesion policy between EU regions (Brandsma et al., 2011, 2013). RHOMOLO incorporates semi-endogenous growth, endogenous location of firms and workers and integration of economic, social and environmental indicators.

As mentioned above, the core of all CGE models is in one or another way founded on input-output data from national accounts, supplemented with extended data for, at least, the household sector and labour market. Most countries also produce national accounts data at a regional level, such as the Regional accounts for Norway.

1https://www.gtap.agecon.purdue.edu/models/current.asp
However, the data lack specifications of regional import and export and of interregional trade across the regions. This is crucial for multiregional models, and much effort is spent on estimating interregional trade data. Moreover, due to lack of regional price data, the data from the NACs are only fixed-price data, so the data challenge is even more critical for multiregional models.

While many national CGE models, such as MSG, use econometrically estimated relations from time series for the behavioural relations, such data series are usually not available at the regional level. An alternative is then to calibrate the model on data that are complete and consistent for one base year. Therefore, multiregional models are typically based on a calibration of the SAMs, which are input-output tables extended with more detailed data for households, capital and labour markets.

As actual economic data of trade and other flows between the single regions in a country are notoriously missing or at best incomplete, they must be estimated, see p. 17. Also regional data on transfers between the market actors are often incomplete. However, little can be found in the literature on constructing complete regional data – beyond a wealth of work on creating regional input-output or supply-use tables, see, e.g., Lahr (2001). This is particularly true with respect to balancing the data and consistency of the regional SAM versions. For example, the RHOMOLO model (Brandsma et al., 2013, Potters et al., 2014) uses regionalized parameter values which are constructed as needed while building the model. Unlike REMES, complete regional SAMs are not created up front and the model only verifies satisfaction of trade and similar balances, either regionally or globally. One of few references constructing “full” regional SAMs including all elements is Bussolo et al. (2003). They arrange the regional SAMs side-by-side and check their consistency only combined. In other words, they are not concerned about consistency within each region, only about consistency overall, at national level. Kuhar et al. (2009) review various survey and non-survey methods to construct regional SAMs for several countries together (EU-27 and Western Balkan countries) with different sources and data availability for each country. Their work includes also a brief discussion of approaches to estimate interregional trade – which they identify as the main challenge when constructing regional SAMs. But also they touch only briefly upon the topic of regionalizing transfers between regional households, industrial sectors and various levels of government.

There are several examples of work linking national CGE models with global and national energy models, cf. Martinsen (2011). In contrast, energy models and economic models at the regional sub-national level are rarely combined. An approach linking MARKAL/TIMES energy system models and the REMI regional economic model is suggested in Loulou et al. (2005) and Goldstein (2006). Santos et al. (2008) provide an early-stage suggestion of modelling the regional economy and the energy system within a spatial context, while Santos et al. (2013) analyses long-run regional impacts of tariff policies of the Brazilian power sector using an interregional CGE model with a detailed description of the energy sectors. However, to our knowledge, linking of operational multiregional CGE models with regional optimization models for the energy system at the sub-national level has not been done previously.

1.3 The REMES model

For the REMES model, we chose to divide the creation of complete regional input matrices and the actual CGE model formulation into two stand-alone modules, see figure 1: Preprocessing routines (described in section 3) combine generally available statistic data such as a national SAM, data on county or municipality level and a choice of conversion tables to aggregate goods, sectors and regions with user-specified indicators for a particular case analysis. The routines produce fully-balanced regional SAMs and data on interregional trade and transport margins, supplemented by synthetically created data where necessary and tailored to the given case analysis. These data are then handed over to the REMES model to populate the CGE model described in section 4 with the necessary data and to solve it for the considered case analysis. For the basic CGE model implementation described here, results of the model runs are not written out to some specific file. However, for some model extensions, output of selected results to, e.g., .gdx or .xlsx files or a database can be implemented easily.
Apart from practicability reasons during the development and implementation phases, splitting the data preprocessing and the actual model allows for more transparency and flexibility during the analyses. It makes it easier to adapt the model formulation to various input data (e.g., a differently detailed SAM) and ways to aggregate or regionalize and, thus, specify different user cases. Hence, the system can respond quickly to user requests such as a different division of goods, sectors or regions without interfering with the model implementation. On the other hand, with this separation, there is no need to recalculate data in each model run (or to implement switches to turn off the data creation part). This comes in particularly handy for dynamic model implementations or for iterative model runs together with other energy system models: In these cases, the tailor-made regional input data are used just when starting the iterations while subsequent runs use results from a previous iteration or the other model. With a separate data preprocessing module, it is easier to build a wrapper around the standard model formulation to control the iterations. Also, the preprocessing routines may be used as standalone tool to create data for other models with similar requirements or – vice versa – the CGE model can use “any” other data as long as they are in a suitable structure. The development work during the RegPol project was conducted against a Norwegian background, describing the Norwegian energy system embedded in the Norwegian economy. A typical user case during this project consisted of six regions (the five Norwegian electricity tariff regions and the Norwegian Continental Shelf), 43 goods and 47 industry sectors.

Both the preprocessing routines and the CGE model were implemented in the General Algebraic Modelling System, or GAMS, a widely known high-level programming language and development environment (IDE). It allows one to write mathematical expressions in an expressive way, specifying symbols as parameters, variables, etc. which are used to describe equations (though inequalities are supported just as well). The user selects which of these equations to use to define one or several models, which are then solved as optimization or equilibrium problems. GAMS is not a solver, though, it merely acts as the front-end to a variety of commercial solvers which actually make the calculations and produce the solutions GAMS shows in its reports. Each supported solver can solve different kinds of mathematical programs, such as LPs, MILPs or LCPs.

GAMS can make it simpler to write and review mathematical expressions than matrix-based formats. However, since content can largely be entered into a CGE model in a standardized way, the Mathematical Programming System for Equilibrium Analysis (MPSGE) was developed to complement GAMS capabilities when formulating CGE models. This extension to the GAMS language takes information about the diverse entities of an equilibrium model in templates, which are later “translated” into GAMS code. Regular GAMS code and the MPSGE additions are then merged and compiled by GAMS into solver-readable data to be processed. The REMES model described in this report has been formulated in such a way as explained in section 4.
The remainder of this report follows the work flow structure outlined in figure 1: We start by discussing requirements on data sources and structures used as input during the development work, focussing on data available for Norway (section 2). This leads over to a presentation of the preprocessing routines in section 3 before section 4 describes the MPSGE implementation of the REMES CGE model. Finally, appendix A provides an overview of the notation (indices, sets, parameters and variables) used in this report.
2 Data

As mentioned in the introduction, the RegPol project set out to develop a hybrid modelling approach combining detailed energy system modelling and regional macro-economic models. A high degree of flexibility with respect to aggregation of regions, commodities and sectors allows a wide range of policy analyses. In principle, “any” data are suited as input to the analyses although the focus was on data available for the Norwegian economy. This section describes the structure and other details of potential input data, i.e., the leftmost part in figure 1, before the next section discusses how these are combined to tailored data sets for the CGE model.

Observe that terminology varies in the literature; throughout this report, we will use the term “goods” to describe commodities, production goods and services and “sectors” for sectors including industries and service-providing sectors. Regions are composed of counties or municipalities and are, hence, parts of a country or nation.

2.1 Social Accounting Matrix

A Social Accounting Matrix, or SAM, is a particular representation of the macro and meso economic accounts of a socio-economic system, which capture the transactions and transfers between all economic agents in the system taking place during an accounting period, usually one year. Alternatively, the SAM can be understood as a conceptual framework to explore the impact of exogenous changes in parameters such as exports, certain categories of government expenditures, and investment on the whole interdependent socioeconomic system. For detailed information about the development and characteristics of SAMs in general, we refer to, for example, Miller and Blair (2009, ch.11).

A SAM is (usually) a square matrix in which each transactor or account is represented twice; once as a row, showing receipts, and once as a column, showing payments. For example, if element \((i, j)\) is \(X\), we say that agent \(j\) is paying \(X\) monetary units to agent \(i\) in exchange for a good or service or as payment for a tax. Alternatively, we can say that agent \(i\) is selling \(X\) worth of goods, services, licenses or permissions to agent \(j\).

A square SAM should have the exact same labels (identifying the actors of the economy) in its columns as in its rows. It must also be balanced, i.e., the sum of all entries in row \(i\) must be equal to the sum of all entries in column \(i\). This implies that a given actor receives just as much money as it spends (that actor can of course save money, but this is modelled as an expenditure in a given “savings” good). If a SAM is unbalanced, an MPSGE model might still deliver a solution, but since the data in the SAM is used for calibration of the model, the results might have a different interpretation, if any, to those of a balanced SAM. Typically, a number of matrix elements are used to achieve a balanced SAM and we describe one approach in section 3.2.

Figure 2 shows the main elements of the SAMs used as input data for REMES and illustrates a division into nine submatrices used for the step-wise creation of the regional SAMs in the preprocessing routines.

2.2 REMES input data

The construction of a SAM and its regional versions demands quite detailed data on the economic transactions in an economy. The Norwegian statistics bureau, Statistics Norway (SSB), has a long tradition for producing national accounts SSB (2016), a type of economic statistics that traditionally is being used as the main source to create national SAMs. The REMES preprocessing routines and the CGE model were developed calibrating production, consumption, other economic activity and trade to data obtained from SSB and from the CREEA project (Compiling and Refining Environmental and Economic Accounts, creea.eu/). The latter data are based on the standard supply and use tables with 65 goods and sectors from SSB’s Norwegian National Account statistics (SSB65) but were disaggregated to a level of 163 sectors and 200 goods, with finer detail for the energy sectors. They are, hence, better suited for analyses with the REMES model.
Some further data are generated within the preprocessing routines due to incomplete information. This concerns, for example, interregional trade data. Also factors for distributing the various parameters between the single model regions are typically not known precisely and some judgment must be applied. Moreover, every data set is different and, despite the preprocessing being set up as general as possible, adaptations are necessary such that the data can be read and processed correctly. Most notably, these adaptations concern removing empty rows and columns for some sectors, splitting some sectors into subsectors to enable mapping against a finer detailed model sector structure when required and, finally, balancing the constructed SAM as described in section 3.2. Additionally, one may need to take care of inconsistencies such as missing or very small values; remedies may be to either aggregate these goods or sectors with suitable other goods / sectors or to delete them from the SAM. Also, for some sectors, exports appeared to exceed domestic production in a given year which may be due to, e.g., a long-term contract structure in these sectors and the SAM parameter values being given statically in monetary rather than volume units for a given year. This way, valuta fluctuations may not be reflected correctly in the SAM. In these cases, the SAM parameter values are adjusted by setting export equal to domestic production and adjusting import in these sectors accordingly. Then, the SAM is still balanced. Where this correction does not solve the problem due to insufficient import values, the sector may be aggregated with a suitable other sector.

To create regional versions with the same structure as the national SAM, register-based statistics data from Statistics Norway are employed, most notably Regional Accounts (“Fylkesfordelt Nasjonalregnskap”, FNR) data and employment statistics. We received these data with somewhat different sectoral structure, following that in the regional I-O model PANDA, such that a conversion is necessary during preprocessing. The PANDA structure of these data contains only 50 sectors compared to the 65 (or more) of the input SAM. It is, therefore, convenient to use this structure as common base for conversion to user-based goods and sector divisions.

However, as the representation level in the REMES model is flexible and fully parameterized with regard to regional, good and sector aggregation, any (similarly structured) data sources are suitable to construct input data for model analyses.
2.3 Input data and case specification

All necessary input data to the preprocessing routines (and, hence, ultimately all user-specified input data for an analysis using the REMES model) is assembled in an MS Excel file consisting of eleven sheets as follows:

- Sheets with input data and conversion tables:

  0. Sets List of municipalities and corresponding counties (if regions are built based on municipalities), explanations and additional information.
  1. SAMnasj SAM on national level with given goods, sectors and factors / final demands (e.g., SSB65 or CREEA classification).
  2.1 SAM_Ind_Map table to convert the sectors in the SAM from the original classification (e.g., SSB65 or CREEA) to the PANDA classification used in the regionalization data, $P_s$.
  2.2 SAM_Prod_Map table to convert the goods in the SAM from the original classification (e.g., SSB65 or CREEA) to the PANDA classification used in the regionalization data, $P_g$.
  2.3 SAM_Fin_Map table to convert factors, final demand, import and export in the original SAM to the internal classification $P_f$ using labels “131” – “149”.

- Sheet with data tables for regional disaggregation (based on PANDA classification):

  3.1. Keys_County Data to be used for disaggregation of the SAM from national to county level.
  3.2. Keys_Municip Data to be used for disaggregation of the SAM from county to municipality level.

- Sheets with conversion or aggregation tables:

  4.0 AggRegion tables to compose or select regions from counties or municipalities to be used in the analyses and case runs.
  4.1 AggIndustry tables to compose or select sectors.
  4.2 AggProducts tables to compose or select goods.
  4.3 AggFin tables to assign a classification for factors, final demand, import and export or to translate from the classification using the “131” – “149” labels to the classification used in the model code. By and large, this mapping is fixed to that given in table 1 as the REMES model implementation relies on it.

<table>
<thead>
<tr>
<th>Preprocessing</th>
<th>Model</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>131</td>
<td>Labour</td>
<td>Wages and salaries</td>
</tr>
<tr>
<td>134</td>
<td>tdirect</td>
<td>Personal income tax</td>
</tr>
<tr>
<td>135</td>
<td>Capital</td>
<td>Operational surplus, net</td>
</tr>
<tr>
<td>137</td>
<td>tax_sec</td>
<td>Net taxes on production</td>
</tr>
<tr>
<td>138</td>
<td>tax_com</td>
<td>Subsidies on goods</td>
</tr>
<tr>
<td>142</td>
<td>HOUS</td>
<td>Households and NPISH</td>
</tr>
<tr>
<td>143</td>
<td>GOVT</td>
<td>Government</td>
</tr>
<tr>
<td>144</td>
<td>INV</td>
<td>Gross fixed capital / investments</td>
</tr>
<tr>
<td>145</td>
<td>STOCKS</td>
<td>Changes in inventories</td>
</tr>
<tr>
<td>146</td>
<td>ttmarg</td>
<td>Trade and transport margins</td>
</tr>
</tbody>
</table>
| 149           | ROW (in national SAM) | Imports / exports (including inter-
|               | trade (in regionalized SAMs) | regional for regionalized SAMs) |

Table 1: Labels used for the fin vector in the preprocessing routines and the model formulation.
To enable the correct selection of conversion tables by the preprocessing routines, a number of MS Excel data ranges must be defined in this file with specific names ($r$, $i$, $p$ and $f$ denote index numbers). This way, e.g., several different ways to define regions can be given beforehand in the Excel file. One of these is then chosen for a specific case analysis by stating the corresponding index numbers in the CaseData.gms file. The definition of these ranges is, hence, part of the data preparation for a new user case specification.

**Counties and municipalities (sheet “0. Sets”)** which municipalities are located in which county$^6$ – required only if composing regions based on municipalities:

range Munic_Count, defines the mapping mapMunictoCount($m$, $c$) (i.e. mapMunictoCount($m$, $c$) = 1 if municipality $m$ is in county $c$ and 0 else)

**Regions (sheet “4.0 AggRegion”)** which counties or municipalities form which regions to be considered in the user case; defines the mapping mapCR($c$, $r$) (mapCR($c$, $r$) = 1 if county $c$ is part of region $r$ and 0 else)

Range RKeyr: the whole table (bar the top row) = conversion key

Range RLevelr: first “proper” element (county or municipality) in the table. This value is used to find out whether the regions consist of counties (value < 100) or municipalities (≥ 100).

Range RRegr: the right column in the table, specifying all regions to aggregate to in the considered case

**Sectors (sheet “4.1 AggIndustry”)** which (PANDA / internal classification) sectors form which sectors to be considered in the use case; defines the mapping mapPS($Ps$, $s$)

Range IKeyi: the whole table (bar the top row) = conversion key

Range ITInd: the right column in the table, specifying all sectors to aggregate to

**Products (sheet “4.2 AggProducts”)** which (PANDA / internal classification) goods form which goods to be considered in the use case; defines the mapping mapPG($Pg$, $g$)

Range PKeyp: the whole table (bar the top row) = conversion key

Range PProd: the right column in the table, specifying all goods to aggregate to

**Factors, final demand, import and export (sheet “4.3 AggFin”)** which factors etc. are translated to which factors etc. in the use case; defines the labels $F_{131}$, ..., $F_{149}$ (see table 1).

Range FKeyf: the whole table (bar the top row) = conversion key

Range FFinf: the right column in the table, specifying all factors etc. to aggregate to

The tables on sheets 2.1 – 2.3 are given and do not change with different user specifications for goods or sectors. Hence, they do not require a more general definition by way of data ranges.

In addition to this selection of generally valid input data, a (text) file CaseData.gms is needed. There, specifics for a certain analysis are stated such that the preprocessing routines can select and aggregate the input data from the MS Excel file to the user case at hand. More precisely, the file must contain at least the following information:

$SETGLOBAL input Excel_input
$SETGLOBAL RegKey r
$SETGLOBAL IndKey i
$SETGLOBAL ProdKey p
$SETGLOBAL FinKey f

with Excel_input the name of the MS Excel file to be used as input, structured as described above, and index numbers to select data ranges and, thus, tables for converting the finely disaggregated national SAM to the regional SAMs:

$^6$The correct county index number can be read from the first two figures of the municipality index number, but GAMS cannot perform this index algebra directly.
RegKey \( r \) ... select regionalization table no. \( r \)
IndKey \( i \) ... select sector aggregation table no. \( i \)
ProdKey \( p \) ... select goods aggregation table no. \( p \)
FinKey \( f \) ... select aggregation table no. \( f \) for factors, final demand, import, export etc.
3 Preprocessing routines

Using the national SAM and the selected disaggregation, conversion and aggregation tables, the preprocessing routines create a set of regional SAMs to be used in the REMES model implementation. First, the national SAM is mapped to an internal goods and sector structure, used as a common basis for further conversion to user-defined goods and sector structures. Then, the input data are broken down to the finest level (either county or municipality) and then aggregated to the user-defined regions. Figure 3 shows the work flow in the preprocessing routine.

Corresponding to this scheme, the main file RegPolPreprocG.gms starts with reading the user case specifications from CaseData.gms, selecting the corresponding data, sets and conversion / aggregation tables from the input MS Excel file and writing them to files AggKeys.gdx, AggCaseSets.gdx, DisAgg.gdx and DisAgg_m.gdx for further use. Then, procedures in SamRead+Map.gms, UserGSF.gms, TradeGravity.gms, CheckBalance.gms and write_msam.gms are called to carry out the single preprocessing steps, some of which are described in more detail in sections 3.1 – 3.2:

1. Read in the national SAM data and (dis)aggregation and conversion tables including goods, sectors and factors / final demands structure (SamRead+Map.gms).

2. Map this SAM to an internal good, sector, and factors / final demands structure (e.g., P ANDA classification) (SamRead+Map.gms). The SAM factors / final demands vector $F_{\text{in}}$ is mapped to a classification using labels “131” – “149” corresponding to that in the SSB65 data based SAM, see table 1. This gives the 9 submatrices $PGG$, $PGS$, $PGF$, $PSG$, $PSS$, $PSF$, $PFG$, $PFS$, $PFF$ with $PGG$ and $PSS$ being zero, see figure 2.

3. Convert to user-specified goods, sectors and factors / final demands, resulting in the submatrices $USGN$, $UGSN$, $UGFN$, $USFN$, $UFFN$, $UGFN$, and $UFSN$. (UserGSF.gms)

4. Read in tables and calculate factors for disaggregating the national SAM data to county and, possibly, to municipality level, convert to user-defined goods and sectors $g$ and $s$, normalize such that a given set of factors sums up to one over all counties and aggregate to regional levels (UserGSF.gms); section 3.1.1.

5. Check trade and investment balances on national level, read out parameters as used in the model (figure 4) and break down to regional levels using the disaggregation factors (TradeGravity.gms); section 3.1.1.

6. Estimate interregional trade and transport margins (TTMs) and trade flows by way of a gravity model (TradeGravity.gms); section 3.1.2.

7. Calculate the residual cells in $mSAM(r,\cdot,\cdot)$, ensuring that trade and investment balances are satisfied within each region (CheckBalance.gms); section 3.2.

8. Collect the calculated parameters, TTM and trade flows to regionalized SAMs, $mSAM(r,\cdot,\cdot)$, and matrices $mTradeMargin_{g,\cdot}$ and $mTradeData_{g,\cdot}$ (write_msam.gms).

9. Save the regional matrices $mSAM(r,\cdot,\cdot)$, $mTradeMargin_{g,\cdot}$ and $mTradeData_{g,\cdot}$, selected parameters not contained in these matrices and lists over the user-defined regions, goods and sectors to the msam_bal.gdx and msam_bal.xlsx files to be read by the model code.

3.1 Creating the regional matrices $mSAM$.

In order to distribute the parameter values in the national SAM to the user-specified regions, we use disaggregation tables on county and, if needed, municipality level. The values for these tables are given in the PANDA classification and we aggregate them first to the user-specified goods and sector classifications. Then, by way
1. Read SAMin, sheet 1. "SAMnasj"
1. Read mappings to internal structure, sheets 2.1, 2.2, 2.3
1. Read mappings internal → user defined, sheets 4.1, 4.2, 4.3
4. Read disaggreg. to county / municip., sheets 3.1, 3.2
4. Read aggregation county / municip. to regions, sheet 4.0

2. Map SAMin to internal structure, P.. matrices
3. Map P.. matr. to user defined U... matrices

4. Create distr... factors
4. Create indic... factors

5. Check / ensure balances

5. Read out and create parameters on regional level

6. Estimate interregional trade & transp. margins

7. Ensure balances on regional level

8. Assemble mSAM, mTradeData, mTradeMargin,

9. Save to msam_bal.gdx and msam_bal.xlsx

Figure 3: Work flow in the preprocessing step.
of the user-specified composition of regions from counties / municipalities, we calculate disaggregation factors on regional level. These are applied to the parameters extracted from the national SAM, which were already converted to the user-specified goods and sector classification, to create regional versions of these parameters. During the development of the REMES model, Norwegian Regional Accounts data as described in section 2.2 were employed but, obviously, any other suitable data sources can be utilized. As is typically the case, sufficiently precise and comprehensive data on TTMs and trade flows between the regions were not available at the time the model was developed. Instead, they are estimated through a gravity model, see section 3.1.2.

### 3.1.1 Distributing parameter values according to given shares

On county level, we read in FNR data from sheet “3.1 Keys_county” on the sectors’ production in each county (column “prod”) and aggregate from P ANDA classification to user-defined sectors, forming the parameter $distrPCountyProd_{c,s}$, and on the combined consumption by households, the government and the municipalities from each sector in each county (columns “konsum_hush”, “konsum_stat” and “konsum_komm”), forming $distrPCountyIncome$_sec$c,s$. Summing up over all counties in a given region and normalising such that the sum of the factors over all regions is one for each sector, we obtain the factors $distrPRegionalProdN_{r,s}$ and $distrPRegionalIncomeN_{r,s}$, respectively, for regional distribution of sector-based values.

If the regions are built from municipalities, additional data on municipality level are read in from sheet “3.2 Keys_Municip”. As sufficiently good economic data on municipality level (comparable to those on county levels) were not available, the disaggregation is based solely on column “Sum sysselsatte”, the total number of employees in each sector in each municipality. These data are then used to calculate versions of the county-wise shares $distrPCountyProd_{c,s}$ and $distrPCountyIncome$_sec$c,s$ on municipality level, resulting in $distrPMunicipalProd_{m,s}$ and $distrPMunicipalIncome$_sec$m,s$. Then, aggregating from PANDA sector classification to user-defined sectors, summing over the municipalities in the given regions and normalizing, we obtain $distrPRegionalProdN_{r,s}$ and $distrPRegionalIncomeN_{r,s}$.

If no data are available to build $distrPRegionalProdN_{r,s}$ or $distrPRegionalIncomeN_{r,s}$ for a sector $s$, we approximate by an even distribution over all regions, dividing the concerned parameter by the number of regions.

Using these (sector-wise) factors and the supply matrix $USGN$, we also create factors $distrGoodRegionalProdN_{r,g}$ and $distrGoodRegionalIncomeN_{r,g}$ describing the regional distribution of good-based values. These steps happen...
in UserGSF.gms. Then, in TradeGravity.gms, the four factor sets are assigned to individual regionalization indicators indic..., see table 2. Finally, these indicators are used to break down parameters read out from the national SAM to regional values as listed in section 3.1.3, table 3.

<table>
<thead>
<tr>
<th>Distribution factor</th>
<th>Indicator</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>distrPRegProdN_{r,s}</td>
<td>indicIO_{r,g,s}</td>
<td>regional production by good and sector</td>
</tr>
<tr>
<td></td>
<td>indicPROD_{r,s}</td>
<td>total regional production by sector</td>
</tr>
<tr>
<td></td>
<td>indicLAB_{r,s}</td>
<td>regional labour inputs by sector</td>
</tr>
<tr>
<td>distrsecRegincomeN_{r,s}</td>
<td>indicIMP_{r,s}</td>
<td>regional imports by sector</td>
</tr>
<tr>
<td>distrGoodRegProdN_{r,g}</td>
<td>indicEXP_{r,g}</td>
<td>regional exports by good</td>
</tr>
<tr>
<td></td>
<td>indicINV_{r,g}</td>
<td>regional investments / capital inputs by good</td>
</tr>
<tr>
<td>distrGoodRegincomeN_{r,g}</td>
<td>indicCONS_{r,g}</td>
<td>regional household consumption by good</td>
</tr>
<tr>
<td></td>
<td>indicCONSgov_{r,g}</td>
<td>regional governmental consumption by good</td>
</tr>
</tbody>
</table>

Table 2: Assignment of regional distribution factors to the indic... indicators.

### 3.1.2 Estimation of interregional trade flows

Trade between regions is typically not recorded statistically such that interregional trade and transport margins and trade flows must be estimated. How to determine how much of a sector’s demand is met by regional supply is a common problem. There exist many approaches such as quotient-based methods (e.g., Simple Location Quotient (SLQ), Flegg Location Quotient (FLQ) and their variants), commodity balance methods (e.g., Cross-Hauling Adjusted Regionalization Method (CHARM)) or gravity models (Vik, 2014). Also combinations such as using survey data completed by estimated data are conceivable. Sometimes, one can obtain information about the regional distribution of the sectors’ total demand or supply (i.e., column and row sums of the supply or use matrices), which is then distributed over all matrix elements by way of RAS or similar routines.

The former approaches are usually designed for estimating intra-regional trade flows in single-region models. Methods for (non-survey) estimation of interregional trade flows are (generally) developed as spatial interaction problems. Gravity, entropy-based, information theory, behaviour and programming models belong to this class of interregional trade estimation methods. Different methods for estimating interregional trade flows are also discussed in Sargento (2009) and Szabo (2015).

For REMES, a gravity model approach has been chosen, which, in essence, is a mathematical programming problem: Find values for each good’s trade flow between any pairs (r, f) of regions, including Rest of World (ROW) and within each region, such that total demand is matched by total supply (and find the “best” of all such value distributions).

We establish a new parameter for total initial domestic sales

\[
XZ_{r,g} = CZ_{r,g} + CGZ_{r,g} + IZ_{r,g} + SVZ_{r,g} + TMZX_{r,g} + \sum_s IOZ_{r,g,s}
\] (1)

and introduce variables \(XDDE_{oV_{g,r,f}}\), \(XDDE_{dV_{g,r,f}}\), \(SUM_{oV_{r,g}}\) and \(SUM_{dV_{r,g}}\) to satisfy the following constraints:
\[ \forall r, g \left| \sum_s XDZ_{r,s,g} \neq EROWZ_{r,g} : \right. \]
\[ \sum_r XDDE_{oV_{g,r,\tau}} = \sum_r XDZ_{r,s,g} - EROWZ_{r,g} \]  
(2a)

\[ \forall r, g \left| XZ_{r,g} \neq MROWZ_{r,g} : \right. \]
\[ \sum_r XDDE_{dV_{g,r,\tau}} = XZ_{r,g} - MROWZ_{r,g} \]  
(2b)

\[ \forall r, \tau, g \left| XDDE_{oV_{g,r,\tau}} \cdot XDDE_{dV_{g,r,\tau}} \neq 0 : \right. \]
\[ XDDE_{dV_{g,r,\tau}} = XDDE_{oV_{g,r,\tau}}(1 + transm_g + taxc_z_g) \]  
(2c)

\[ \forall r, g \left| SUM_{oV_{r,g}} \neq 0 : \right. \]
\[ SUM_{oV_{r,g}} = \sum_r XDDE_{oV_{g,r,\tau}} \]  
(2d)

\[ \forall r, g \left| SUM_{dV_{r,g}} \neq 0 : \right. \]
\[ SUM_{dV_{r,g}} = \sum_r XDDE_{dV_{g,r,\tau}} \]  
(2e)

Equ. (2a) calculates \( XDDE_{oV_{g,r,\tau}} \) as total domestic production which is available to be distributed to the regions in the nation (i.e., is not exported). Similarly, (2b) calculates all domestic demand which is not covered by imports from abroad, coming from production in the regions. Equ. (2c) couples both variables, requiring that domestic demand not covered by imports must be satisfied by domestic production exclusive exports, taking into account transport margins and taxes. Finally, (2d) and (2e) calculate the total flow into and out of each region.

As objective function we chose to minimize entropy:

\[
\text{entropy} = \sum_{r, \tau, g, r \neq 0} \left( XDDE_{oV_{g,r,\tau}} \log \left( \frac{XDDE_{oV_{g,r,\tau}}}{XDDE_{oV_{g,r,\tau}}} \right) \right) + \sum_{r, \tau, g, r \neq 0} \left( XDDE_{dV_{g,r,\tau}} \log \left( \frac{XDDE_{dV_{g,r,\tau}}}{XDDE_{dV_{g,r,\tau}}} \right) \right)
\]  
(3)

Based on the regionalized parameters determined according to table 3 and in equation (1), we estimate initial or start values for the variables using each region’s production and consumption shares.

\[ XDDE_{oV_{r,\tau}} = \left( \sum_s XDZ_{r,s,g} - EROWZ_{r,g} \right) \cdot \frac{XZ_{r,g} - MROWZ_{r,g}}{\sum_r (XZ_{r,g} - MROWZ_{r,g})} \]  
(4a)

\[ XDDE_{dV_{r,\tau}} = (XZ_{r,g} - MROWZ_{r,g}) \cdot \frac{\sum_s XDZ_{r,s,g} - EROWZ_{r,g}}{\sum_r (\sum_s XDZ_{r,s,g} - MROWZ_{r,g})} \]  
(4b)

\[ SUM_{oV_{r,\tau}} = \sum_r XDDE_{oV_{r,\tau}} \]  
(4c)

\[ SUM_{dV_{r,\tau}} = \sum_r XDDE_{dV_{r,\tau}} \]  
(4d)
Solving the optimization model (2)–(3), the interregional trade flow and transport margins can be estimated:

\[
\begin{align*}
\text{TRADEZ}_{g,r,\tau} &= XDDE_{oV}\_g,r,\tau \\
\text{TAXCZR}_{g,r,\tau} &= XDDE_{oV}\_g,r,\tau \cdot t\text{axc}_{g} \\
\text{TMCRZ}_{g,r,\tau} &= XDDE_{oV}\_g,r,\tau \cdot \text{transpm}_{g}
\end{align*}
\]  

(5a)  

(5b)  

(5c)

### 3.1.3 Assembling the regional SAM

Summarizing, table 3 describes how values for the single regional parameters are calculated from the national SAM, using several sets of disaggregation factors (section 3.1.1) and a gravity model (section 3.1.2). Parameters \(\text{INZ}_{r,s}, \text{KZ}_{r,s}, \text{SROWZ}_r, \text{TRHOWZ}_r\), and \(\text{TRROWZ}_r\), are calculated as residual elements such that certain balances are satisfied, see section 3.2.1, equations (6) – (9).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Disaggregation factor</th>
<th>Parameter</th>
<th>Disaggregation factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{CGZ}_{r,g})</td>
<td>indic\text{CONSgov}</td>
<td>(\text{TAXCZR}_{g,r,\tau})</td>
<td>gravity model</td>
</tr>
<tr>
<td>(\text{CZ}_{r,g})</td>
<td>indic\text{CONS}</td>
<td>(\text{TAXPZ}_{r,s})</td>
<td>(\text{XDZ}_{r,s}) (i.e. indic\text{PROD})</td>
</tr>
<tr>
<td>(\text{EROWZ}_{r,g})</td>
<td>indic\text{EXP}</td>
<td>(\text{TMCRZ}_{g,r,\tau})</td>
<td>gravity model</td>
</tr>
<tr>
<td>(\text{IOZ}_{r,g,s})</td>
<td>indic\text{IO}</td>
<td>(\text{TMXZ}_{r,g})</td>
<td>(\frac{\text{CZ}<em>{r,g}+\text{CGZ}</em>{r,g}+\sum_s \text{IOZ}_{r,g,s}}{r})</td>
</tr>
<tr>
<td>(\text{IZ}_{r,g})</td>
<td>indic\text{INV}</td>
<td>(\text{TRADEZ}_{g,r,\tau})</td>
<td>gravity model</td>
</tr>
<tr>
<td>(\text{LZ}_{r,s})</td>
<td>indic\text{PROD}</td>
<td>(\text{TRANSFZ}_{r})</td>
<td>calculated using (\text{CZ}<em>{r,g}, \text{LZ}</em>{r,s})</td>
</tr>
<tr>
<td>(\text{MROWZ}_{r,g})</td>
<td>(\frac{\text{ZX}<em>{r,g,s}/\sum_r \text{ZX}</em>{r,g}}{r}) (see (1))</td>
<td>(\text{KZ}_{r,s}, \text{TTYZ}_r) and (\text{SHZ}_r)</td>
<td></td>
</tr>
<tr>
<td>(\text{SGZ}_r)</td>
<td>(\sum_r \text{CGZ}<em>{r,g}/\sum_r \text{CGZ}</em>{r,g})</td>
<td>(\text{TTYZ}_r)</td>
<td>(\frac{\sum_r (\text{LZ}<em>{r,s}+\text{KZ}</em>{r,s})}{\sum_r (\text{LZ}<em>{r,s}+\text{KZ}</em>{r,s})})</td>
</tr>
<tr>
<td>(\text{SHZ}_r)</td>
<td>calculated using (\text{TTYZ}<em>r, \sum_r \text{KZ}</em>{r,s}) and (\sum_r \text{LZ}_{r,s})</td>
<td>(\text{XDDZ}_{r,g,s})</td>
<td>indic\text{PROD}</td>
</tr>
<tr>
<td>(\text{SVZ}_{r,g})</td>
<td>(\frac{\text{CZ}<em>{r,g}+\text{CGZ}</em>{r,g}+\sum_s \text{IOZ}_{r,g,s}}{r})</td>
<td>(\text{XDZ}_{r,s})</td>
<td>indic\text{PROD}</td>
</tr>
</tbody>
</table>

Table 3: Creating regional versions of the single parameters.

### 3.2 (Re)Balancing the regional SAMs

In a balanced SAM, the sum over any column is equal to the sum over the corresponding row. We presume that the input national SAM should already be balanced. This is verified when the matrix is read in from the .xlsx file. When breaking down this matrix to regionalized SAMs, a gravity model and several sets of disaggregation factors are employed as described in the previous section. Consequently, parameter values are distributed differently over the single regions and, hence, the resulting regional SAMs are most likely unbalanced.

Literature about balancing regional SAMs constructed from national values is scarce, not at least since there is typically not such a strict division into data preprocessing and equilibrium model routines as we follow here. Often, the national data are read in and any needed regional data are constructed directly when formulating the relations in the model. One such example is the RHOMOLO model (Brandsma et al. (2013), Potters et al. (2014)). On the other hand, Bussolo et al. (2003) describe an example where complete regional SAMs with all elements are built up-front. But also they verify balances at national level and not within each region.

In REMES, we consider the regions as rather self-contained entities (with the necessary national relations) and, thus, the regional SAMs as replicates of a national SAM. In order to ease generality and communication with the
equilibrium model implementation, we perform all necessary balance checks and parameter adjustments before these values are fed into the model.

### 3.2.1 Residual elements in the SAM

To balance the created regional matrices in the module `CheckBalance.gms`, we use similar procedures as for balancing the national SAM. This fills the remaining cells in the regionalized SAM, $mSAM(\cdot, \cdot, \cdot)$, see figure 4.

Households’ payments of income tax should be equal to government receipts of income tax.

$$mSAM(r, F_{134}, F_{142}) = mSAM(r, F_{143}, F_{134}) = TTYZ_r$$

The value of labour provided by households (wages received) should be equal to the value of the total labour inputs in all sectors.

$$mSAM(r, F_{142}, F_{131}) = \sum_s LZ_{r,s}$$

Operational surplus received by households should be equal to the total operational surplus paid out by all sectors (i.e., all operational surplus from the sectors should be received by households only).

$$mSAM(r, F_{142}, F_{135}) = \sum_s KZ_{r,s}$$

Net taxes on goods received by the government consist of all net taxes paid by the sectors.

$$mSAM(r, F_{143}, F_{137}) = \sum_s TAXPZ_{r,s}$$

Subsidies on goods paid out by the government should be equal to the sum of subsidies for all goods (i.e., these subsidies should go to goods only).

$$mSAM(r, F_{143}, F_{138}) = \sum_{r,g} TAXCZR_{g,r,r}$$

Changes in inventories of gross fixed capital or investments balance with total changes in stocks of all goods.

$$mSAM(r, F_{145}, F_{144}) = \sum_g SVZ_{r,g}$$

The parameter $capital_{r,s}$ is the sum of $KZ_{r,s}$ and $INVZ_{r,s}$ and balances the sector rows and columns.

$$\forall s, r : XDZ_{r,s} \neq 0 :$$

$$capital_{r,s} = XDZ_{r,s} - TAXPZ_{r,s} - \sum_g IOZ_{r,g,s} - LZ_{r,s}$$

(6)

For each sector $s$, $capital_{r,s}$ is then split into the regional parameters $KZ_{r,s}$ and $INVZ_{r,s}$ according to their ratio in the national input SAM.

The parameter $TRHROWZ_r$ is calculated as residual item balancing the “Household” ($F_{142}$) rows and columns.

$$TRHROWZ_r = \sum_s LZ_{r,s} + \sum_s KZ_{r,s} + TRANSFZ_r - \sum_g CGZ_{r,g} - TTYZ_r - SHZ_r$$

(7)

and, depending on the sign, is written to either the $F_{149}$ column or the $F_{149}$ row in the regional SAMs to indicate receipts or transfers.

$$mSAM(r, F_{149}, F_{142}) = TRHROWZ_r \quad \text{if } TRHROWZ_r > 0,$$

$$mSAM(r, F_{142}, F_{149}) = -TRHROWZ_r \quad \text{else}$$
Likewise, the parameter $TRROWZ_r$ is calculated as residual item balancing the “Government” ($F_{143}$) rows and columns in the national SAM,
\[ TRROWZ_r = \sum_g CGZ_{r,g} + TRANSFZ_r + SGZ_r - TTYZ_r - mSAM(r, F_{143}, F_{137}) - mSAM(r, F_{143}, F_{138}) \] (8)
and is written to either row or column $F_{149}$, depending on the sign.

Finally, the residual parameter $SROWZ_r$ is calculated to satisfy the investment balance (section 3.2.2). This balances also the “Investments” row and column ($F_{144}$).
\[ SROWZ_r = ITZ_r + mSAM(r, F_{145}, F_{144}) - \sum_s INVZ_{r,s} - SHZ_r - SGZ_r \] (9)

Here, $ITZ_r$ denotes total investments based on private investment demand, $ITZ_r = \sum_g IZ_{r,g}$. Again, depending on the sign, the calculated value is written to either row or column $F_{149}$:
\[ mSAM(r, F_{144}, F_{149}) = SROWZ_r \quad \text{if } SROWZ_r > 0, \]
\[ mSAM(r, F_{149}, F_{144}) = -SROWZ_r \quad \text{else} \]

The cells $mSAM(r, g, F_{149})$ include exports $EROWZ_{r,g}$ and outward interregional trade $\sum_g TRADEZ_{g,r}\tau$ while cells $mSAM(r, F_{149}, g)$ combine imports $MROWZ_{r,g}$ and inward interregional trade $\sum_g TRADEZ_{g,r}\tau$.

### 3.2.2 Other balances

Finally, the following balances must hold:

**Trade balance – sales.** For each good, sales in a region must equal domestic supply and exports to other regions and ROW.
\[ diXD_{rg} = \sum_s XDDZ_{r,s,g} - EROWZ_{r,g} - \sum_{\tau} TRADEZ_{g,r}\tau \overset{!}{=} 0 \]

**Trade balance – demands.** For each good, demand in a region must be satisfied by production in this region and imports from other regions and ROW (as goods or trade and transport margins).
\[ diXR_g = XZ_{r,g} - MROWZ_{r,g} - \sum_{\tau} (TRADEZ_{g,r}\tau + TMCRZ_{g,r}\tau) \overset{!}{=} 0 \]

**Investment balance.** Savings should balance with investments. Note that the element $SROWZ_r$ is calculated in (9) such that this balance is satisfied.
\[ investment\_bal_r = \sum_s INVZ_{r,s} + SHZ_r + SGZ_r + SROWZ_r - \sum_g IZ_{r,g} - \sum_g SVZ_{r,g} \overset{!}{=} 0 \]

**Global trade balance.** Incoming monetary flows (exports to other regions or ROW) must be balanced by outgoing monetary flows (imports from other regions or ROW).
\[ trade\_bal\_global_r = \sum_g EROWZ_{r,g} + \sum_{g,\tau} TRADEZ_{g,r}\tau - TRROWZ_r + TRROWZ_r + SROWZ_r \]
\[ - \sum_{g,\tau} TRADEZ_{g,r}\tau - \sum_g MROWZ_{r,g} \overset{!}{=} 0 \] (10)
3.3 Files created throughout the preprocessing steps

The single preprocessing steps create the following .gdx and .xlsx files. (X) marks content read in from the input .xlsx file while the content in the remaining files is created throughout the program run.

RegPolPreprocG.gms (main file)

(X) AggKeys.gdx Conversion tables to aggregate regions, goods, sectors, factors to user case specification.

(X) AggCaseSets.gdx Sets describing regions, goods, sectors, factors in the user case.

(X) DisAgg.gdx Sets describing counties and conversion table to distribute SAM to county levels.

(X) DisAgg_M.gdx Sets describing municipalities and conversion table with data on municipality level.

msam_bal.gdx and msam_bal.xlsx Regional SAMs, sets (domains) as used in the model, interregional trade & transport margins, interregional (and ROW) trade flows.

SAMread+maps.gms

(X) SAMSets Sets read in: national SAM and preprocessing internal classification.

(X) SAMmatrices Matrices read in: national SAM and mappings to internal classification.

SAM_Panda SAM submatrices on national level with internal classification.

UserGSF.gms

UserGSF.gdx Submatrices with user case aggregation of goods, sectors, and factors / final demands, on national level; nation-wide supply pattern of goods by sector.

regional_share.gdx Normalized factors to (dis)aggregate to regional levels.

3.4 Elasticities

In general, elasticities are dimensionless parameters that capture behavioural responses in an economy as functions of relative prices of production inputs. In a CGE, this concerns, for example, parameters defining production or demand functions, describing how various goods can be combined or substituted with each other. This includes also intermediates, thus describing a nesting structure between the various production factors.

Modelling a CGE using the MPSGE engine, all production and demand functions are assumed to have constant elasticities (CET, CES, CED). In other words, we assume that the elasticities in these functions are not dependent on the amount of goods involved and do not change over the course of a model run. These elasticities must be decided beforehand such that the resulting production and demand functions reflect reality as good as possible: An elasticity of 0 or 1 will make the CES and CET functions turn into their limit cases, Leontief and Cobb-Douglas, respectively. Any other feasible value will give a general CES or CET function. These functions can also be recursively nested inside other CES/CET functions, with a different elasticity of substitution/transformation for each nest.

Together with reference quantities which are read directly or indirectly from the SAM, elasticities are all which is needed to (internally) formulate the CES/CET production/cost functions and the calibration data for them.

Elasticities (and other model assumptions) represent, therefore, good candidates for inclusion through preprocessing routines. For the basic version of the REMES model implementation, however, these parameter values were considered to be beyond the control of the user and were hard coded directly in the model formulation.
3.5 Communication with CGE model

The files `msam_bal.gdx` and `msam_bal.xls` contain the complete use-case specific regional SAMs, $mSAM(r, \cdot, \cdot)$, interregional trade and transport margins, $mTradeMargin_{g, \cdot, \cdot}$ and interregional trade flows, $mTradeData_{g, \cdot, \cdot}$, to be used by the REMES model described in the subsequent section. For each region, the former matrix replicates the structure of the national SAM but takes into account the aggregation of sectors, goods and factors/final demands as well as interregional trade flows. The national SAM rows / columns “ROW (import)” and “ROW (export)” are replaced by rows / columns “trade” for the total trade flows which combine imports / exports from / to ROW and other regions.
This chapter explains the REMES model in general, focusing on its implementation in the MPSGE extension of the GAMS system. REMES’s mathematical formulation corresponds to an Arrow-Debreu macroeconomic model and will not be explicitly given in this report. The MPSGE formulation was developed from an original base model coded by Olga Ivanova, who later provided advice in adding some of REMES’ additional features. The model accounts for several regions (ad-hoc or realistically defined), one national government, one representative household per region, taxes on goods, services and consumption, interregional trade, one Rest-of-the-World (ROW) region which allows for international trade, transport and trade margins in domestic, interregional and international trade. The static version of the model takes a simplistic approach on stock changes and investments, though they are included in the model. A dynamic version takes a more nuanced approach at the involved types of capital, investment and production technologies. Figure 5 illustrates the relations, i.e., payment directions, between the elements of the model.

![Diagram of REMES model main elements.](image_url)

Figure 5: REMES model main elements.

All sector inputs and outputs, demands and endowments are determined from the input data used, in particular from the regional SAMs. If the original SAM is balanced, the parameters properly calibrated, and all flows are properly accounted for in the MPSGE model, the markets in the model will be cleared and a solution will be reported in which prices for goods are all one (according to the unitary price assumption of most CGEs). We assume that taxes, transport and trade margins have been properly accounted for in the prices of goods and services. Also, all coefficients in the CES/CET functions should have been properly identified.

Government actors can be local, national, or both. The investment system, labour and capital markets can be either local or national. One or more regions can be simplified, such that only interregional interactions with
this region are modelled, but nothing internally. Further, as mentioned before, the model can be either a static
version, with one period, or a dynamic one, in which investment from the first stage is transformed into capital
at the second stage. All these modalities are controlled with variables “SW_XXX” in the MPSGE code.

We will write, for each MPSGE block, the GAMS/MPSGE code along with a summarized explanation of the
dynamics contained in such a block. A producer, or an actor which produces something out of some inputs, is
identified with a “PRÔD” MPSGE block in which each input and output is assigned a price, and a representative
quantity. Optionally, taxes and nesting of production levels are possible, among other things. On the other
hand, an entity which has a budget and demands a good or service has a “DEMAND” block, in which budgets
and endowments are specified in a similar way to the inputs and outputs of the producers. In addition, custom
variables can be declared in “CONSTRAINT” blocks to express, for example, dynamically determined taxes or
scaling indexes.

An important part of any model development, and especially that of CGE models, is to properly calibrate the
model, i.e. to calculate or adjust relevant model parameters from benchmark values such as values given for a
reference year. One has to decide how to model each actor’s production and/or demand functions. Sometimes
a good can be substituted for another and sometimes production inputs are in fixed shares.

4.1 Households

There is one household actor per region, which represents the aggregated economies of all consumers in that
region. Households are characterized by both a PRÔD and a DEMAND function. The DEMAND block specifies the sources of financing for the household as endowments “E:”. Endowments include
income from labour and capital, transfers from the active level of government to the households, payments
from the ROW, as well as a “negative endowment” which represents the values from savings going into either
the local (SW_INVS = 1) or national (SW_INVS = 2) investment sector. All these transactions are put together
(elasticity equals one) to form a budget with which the households can demand goods, “D:”. Notice that gov-
ernmental transfers and savings have reference variables “R:”. These are defined in later blocks and serve as
indexes for those values, so that an increase in the reference is adequately scaled in the affected entry.

Labour, capital and investment are endowed according to the local/national status as specified in the model
settings. Notice that there are two groups of capital endowment, one for the static version, and one for the
dynamic. The static version can produce either locally traded capital or nationally traded capital. The dynamic
version owns “normal” capital, committed to a particular region, “new” capital, which can be traded freely into
any region, or “extant” capital, committed to one region and industry sector.

*Household Demand

$DEMAND:HOUS (cnt) $(R_S (cnt)) s:1
E:PL(cnt)$(SW_LMKT eq 1) Q:(LS.L(cnt)*(1-ty(cnt)))
E:PLN$(SW_LMKT eq 2) Q:(LS.L(cnt)*(1-ty(cnt)))
E:RKCN$(SW_KMKT eq 2 and (not SW_RDYN)) Q:(KS.L(cnt)*(1-ty(cnt)))
E:RK cheap 1 and (not SW_RDYN)) Q:(KS.L(cnt)*(1-ty(cnt)))
E:RK cheap 2 and (not SW_RDYN)) Q:(KS.L(cnt)*(1-ty(cnt)))
E:RK cheap 1 and SW_RDYN) Q:(KS_S(cnt, cntt)*(1-ty(cnt)))
E:RK cheap new 1 and SW_RDYN) Q:(KS_n(cnt)*(1-ty(cnt)))
E:RK cheap 1 and SW_RDYN) + Q:(KS_x(sec, cnt)*(1-ty(cnt)))
E:PS(cnt)$(SW_INVS eq 1) Q:(-SH.L(cnt)) R:R_SH(cnt)
The PROD block for the households represents the transformation of the households’ budget into consumption. The inputs are the worth of the goods purchased and the mark up on these goods from transport and trade margins (TTMs), both equally adjusted for consumption tax rates. Using these means, households “consume” just enough to spend a budget, which should match what they earn in the DEMAND block. The elasticity to combine consumption and TTMs is zero, while the elasticity to combine the goods has been set to one. Taxes on the inputs are assigned to the national government, if it is active, otherwise these go to the local government.

$PROD:U(cnt) s:1 s1:0$
0:PU(cnt) Q:CBUDZ(cnt)
I:P(cnt,com) Q:(CZ(cnt,com)+CZ(cnt,com)*trmz(cnt,com))
+ P:(1+taxcz(cnt,com) + ElCertTax("Hous", com, cnt))
+ A:GOVTN$(SW_GOVT ge 2) T:taxc(cnt,com)$(SW_GOVT ge 2)
+ A:GOVTL(cnt)$($(SW_GOVT eq 1) T:taxc(cnt,com)$($(SW_GOVT eq 1)
+ s1:

Linking the demand and the production block for the consumer we have the welfare index relationship,

$$HOU_{S cnt} = PU_{cnt} CBUDZ_{cnt} U_{cnt}.$$  (11)

### 4.2 Producers

Producers in the model consume intermediate goods available in the local market (marked up for taxes and the corresponding TTMs), employment of labour and capital, and enterprise investments. As output, they produce domestic goods. There is a complex nesting structure in which Capital and Savings occupy a 0-elasticity innermost nest which then integrates with labour at 1-elasticity. On the other hand, intermediates and their TTMs are nested also at 0-elasticity, the good of which is then combined into the final good of the sector with a 0-elasticity as well. Taxes on input goods, as well as taxes/subsidies on output goods are assigned to the national government, if active, and to the local level if there is no national government. Labour, capital and investment inputs come each from whichever market is active, either the local or the national one.

$PROD:XD(cnt,sec)$(XDZ(cnt,sec) and R_S(cnt) and (not SW_RDYN))) t:0 s:0
+ s1:1 s2:0 s3(s1):0
P:PD(cnt,sec,com) Q:XDDZ(cnt,sec,com) P:(1+taxpz(cnt,sec))
+ A:GOVTN$(SW_GOVT ge 2) T:taxp(cnt,sec)$(SW_GOVT ge 2)
+ A:GOVTL(cnt)$($(SW_GOVT eq 1) T:taxp(cnt,sec)$($(SW_GOVT eq 1)
+ s2:

I:P(cnt,com) Q:(IOZ(cnt,com,sec)+IOZ(cnt,com,sec)*trmz(cnt,com))
+ P:(1+taxcz(cnt,com) + ElCertTax(sec, com, cnt))
+ A:GOVTN$(SW_GOVT ge 2) T:taxc(cnt,com)$($(SW_GOVT ge 2)
+ A:GOVTL(cnt)$($(SW_GOVT eq 1) T:taxc(cnt,com)$($(SW_GOVT eq 1)
+ s2:

I:PL(cnt)$($(SW_LMKT eq 1) Q:LZ(cnt,sec) s1:
I:PLNS$(SW_LMKT eq 2) Q:LZ(cnt,sec) s1:
I:PS(cnt)$($(SW_INVS eq 1) Q:INVZ(cnt,sec) s3:
I:PSN$(SW_INVS eq 2) Q:INVZ(cnt,sec) s3:
The two lines at the end, which specify the demand of capital from the producer, apply only to the static version of the model, which uses capital of either local or national type. For the dynamic version, we have in fact two production blocks like the one above, one which applies to the producers using extant capital, and one for the producers using new capital. For the first group, the production function is like the one above, though with capital usage given as

$I:RKC\text{(sec, cnt)}$  
$Q:KZ\text{(cnt, sec)}$  
$s3:$

while for the producers using newly invested capital, the line is

$I:RKC\text{(cnt)}$  
$Q:KZ\text{(cnt, sec)}$  
$s3:$

This division of production in two separate blocks which produce the same output and use the same inputs, with the exception of capital, makes the dynamic model able to analyze changes in technology between phases. Elasticities of substitution, which differentiate one production technology from another, can and indeed should be different in the two production blocks, extant and new, for a given sector, with the block with new capital likely more flexible in its production function.

Once the domestic production is defined, the producers decide whether to sell the goods to the regional market, to ship them to other regions, or to export them to the rest of the world. Currently, these outputs have a CES of 1.4, while, if there was more than one producer of the same good, the CES to combine them would be 1.2.

4.3 Goods

There is one sector of the economy ensuring that all trade flows take the right direction, which implicitly represents the actions of a trader for each type of good, according to an Armington assumption. These are zero-profit entities and, unlike most other entities, they make no investments, but instead receive money from others’ investments in their corresponding goods. For goods produced locally, or in other regions, the traders add the value of the trade margin paid to the output price of the Armington good. Note that this does not happen with imports from the rest of the world.

First we have the aggregation of goods from all allowed sources into the composite good the trader will sell to the other entities in the region. The goods which the producers decided to sell locally are put together with the exports and the trade from all other regions. Because there is no real entity buying or selling, no taxes are paid nor mark ups are added to the reference prices or quantities. Goods from outside the region use an elasticity of 1.4 for their nest, and such Armington goods are aggregated with the local good with an elasticity of 1.2.

$\text{PROD:EXPORT(cnt, com)\$EZ(cnt, com) t:1.4 s:1.2}$

$\text{O:ER}$

$\text{Q:EROWZ(cnt, com)}$

$\text{O:PETRADE(com, cnt, cntt)\$(ord(cnt) ne ord(cntt)) Q:TRADEZ(com, cnt, cntt)}$

$\text{O:PDD(cnt, com) Q:XXDZ(cnt, com)}$

$\text{I:PD(cnt, sec, com) Q:XDDZ(cnt, sec, com)}$
There is also a transport sector. As indicated in its production block, it pays to the traders of the goods / services related to transport to produce a “transport service”, whose cost will then be aggregated (elasticity zero, i.e., fixed shares) as a mark up by the trader of that good or service in that region. This is seen in the I:PTM entries of the block.

However, as one would naturally assume, if one region sells more to other regions than it buys (or the converse), then this region would have a surplus compared to other regions and the corresponding regional SAM would be unbalanced. To compensate, we must charge or give the region an amount equal to the imbalance. This is done by transferring from each surplus region an amount $OutRegVol_{cnt}$ to an hypothetical account, which will in turn pay $InRegVol_{cnt}$ to each region with a trade deficit. Because these two are the only sinks/sources of PIT_{cnt}, the sum of $OutRegVol_{cnt}$ and $InRegVol_{cnt}$ should be equal, and if a region has a positive value in one, it should have zero in the other.

4.4 Government

The government can be modelled as either a national actor, a local actor, or as both, interacting. Regardless, each level acts on a regional level by collecting certain taxes, consuming goods and sometimes transferring money to individual actors. As expected, local governments are balanced in the local level, and the national government is balanced at the national level. If one level is not active in the model, the other level covers its tax and transfer functions entirely. If both levels are present, the national level might have to finance the local governments so that these are balanced.

Regardless of which level we talk about, the government, like the households, has both a producer and a demand block. The government’s demand block is characterized by a) a series of endowments, and b) taxes received from all entities; these go to form its budget. The government has labour, capital and money input from the ROW as possible positive endowments, whereas it sees subsidies to households and investments as negative endowment sources. Additionally, the government receives income from all relevant taxes paid in the model. This income does not appear in the block, but rather is automatically taken into account by MPSGE whenever we add a tax field and indicate the government as the collector agent.

For the local level, the government demand block is as follows,

$DEMAND:GOVTL(cnt)$(SW_GOVT 1 e 2) s:Govt_Dem_Elas
E:PGTR$(SW_GOVT eq 2) Q:(TRANSFGZ(cnt)) R:PGINDEX(cnt)
E:PL$(SW_GOVT eq 1) Q:(LS.L(cnt)*ty(cnt))
E:PLN$(SW_GOVT eq 2) Q:(LS.L(cnt)*ty(cnt))
E:RKC$(SW_GOVT eq 1) Q:(KS.L(cnt)*ty(cnt))
E:RKCN$(SW_GOVT eq 2) Q:(KS.L(cnt)*ty(cnt))
E:PS$(SW_INVS eq 1) Q:(-SGL.L(cnt))
E:PSN$(SW_INVS eq 2) Q:(-SGL.L(cnt)) R:PIINDEXN
E:ER$(SW_GOVT eq 1) Q:TRROW.L(cnt) R:PIINDEX
E:PTR$(SW_GOVT eq 1) Q:(-TRANSF.L(cnt)) R:PCINDEX(cnt)
D:PUGL(cnt) Q:CBUDGLZ(cnt)
where transfers are indexed and received from the national level only if both levels exist ($SW_{GOVT} = 2$), endowments from capital, labour and investments/savings come from the active market, and transfers to households are performed only if the national level is not present in the model. The corresponding block for the national level is analogous, with the money-transferring roles appropriately changed.

On the other hand, the production function or utility of the government actor(s) is similar to that of the households. The government consumes goods sold locally by the trader and pays a fixed share of transport costs, in order to produce a budget which should match that in the demand block. If there are taxes paid by the government in the SAM (neither level of government is exempt from consumption tax, for example), then the national government is the receiver of such tax income, unless inactive, in which case the same local government of that region receives the income. For the national level, the production block is given below, with the local level being analogous.

$$\text{PROD: UGN} \ (SW_{GOVT} \geq 2) \ s:1 \ s1:0$$

0: PUGN
  Q: CBUDGNZ
  I: P(cnt, com)
  Q: (CGNZ(cnt, com) + CGNZ(cnt, com) * trmz(cnt, com))
  + P: (1 + taxcz(cnt, com) + ElCertTax("GOVTN", com, cnt))
  + A: GOVTN
  T: taxc(cnt, com)
  + s1:

Linking the two blocks, we have the relationship

$$GOVT_{cnt} = CBUDGZ_{cnt} PUG_{cnt} UG_{cnt}. \quad (12)$$

4.5 **Investment**

The investment sector(s) in the model need(s) to address two things: the size of the payments from each actor and towards which good these payments go. A national system gathers and distributes the accounts across all regions, while a local level is balanced on each region, but otherwise work the same. As stated, only one investment system should be active at the same time.

The first is achieved through a demand block, which takes as endowments the savings of the local households and government acting in the region, properly scaled, as well as the investment send from abroad and the enterprise investments. Exogenous changes in stock act as negative endowments to this block, too. All this forms the investment budget, which will be distributed in the production block of the investment sector. For the local investment system, the demand block is as follows,

$$\text{DEMAND: INV} \ (SW_{INVS} = 1)$$

E: PS(cnt)
  Q: (SH.L(cnt))
  R: R_SH(cnt)$$(SW_{ELCERT} = 1)$$
E: PS(cnt)$$(SW_{GOVT} = 2)$$
  Q: (SGN.L(cnt))
  R: PIINDEX(cnt)
E: PS(cnt)$$(SW_{GOVT} = 1)$$
  Q: (SGL.L(cnt))
  R: PIINDEX(cnt)
E: ER
  Q: (SROW.L(cnt))
E: P(cnt, com)
  Q: (-SV.L(cnt, com))
  R: R阏t[cnt, com]
E: PS(cnt)
  Q: (sum (sec, INV.L(cnt, sec) * XD.L(cnt, sec)))
D: PUINVB(cnt)
  Q: ITZ(cnt)

The production block, quite simply, acts as though its “buying” goods (marked up by taxes and by proper TTMs) produce an utility which equals the budget in the demand sector. For the local case, the block is

$$\text{PROD: UINV} \ (SW_{INVS} = 1) s:1 s1:0$$

0: PUINVB(cnt)
  Q: ITZ(cnt)
I: P(cnt, com)
  Q: (IZ(cnt, com) + IZ(cnt, com) * trmz(cnt, com))
  + P: (1 + taxcz(cnt, com))
  + A: GOVTN$$(SW_{GOVT} = 2)$$
  T: taxc(cnt, com)$$(SW_{GOVT} = 2)$$
  + A: GOVTL(cnt)$$(SW_{GOVT} = 1)$$
  T: taxc(cnt, com)$$(SW_{GOVT} = 1)$$
  + s1:
Linking both blocks we have

\[ \text{INV}_V = P\text{UINV}_V \text{ITZ}_V \text{UINV}_V. \]  

(13)

Investment has a mere indicative role in the static model, balancing income flows. In the dynamic model, however, the investment from the first stage, or run, is calculated and assigned to different sectors and regions given predetermined rules, then converted into capital.

### 4.6 Capital modelling in the dynamic model

The dynamic model handles capital in a significantly different way compared to the static model. Whereas the latter can have either locally traded capital or nationally traded capital, the dynamic version has three different capital goods acting at a sectoral, regional and national level, all being active somehow at the same time.

As seen in the households demand block, three capital “goods” are endowed, namely normal capital, extant capital and new capital. Only two types of capital are used by the producers, either extant or normal. The new capital is transformed one to one into a region-specific normal capital by the VINTAGE production block:

\[ \text{PROD:VINTAGE(cnt)} \]  

O:RKC(cnt) q:1  
I:RKCnew q:1

Between stages, the volumes in which each capital type is present in the model are adjusted taking into account design parameters and the model’s endogenous responses:

- Extant capital for a sector and region can only be depreciated, which means that the volume of new capital endowed to a region decreases with the depreciation index \( \delta \):
  \[
  \text{KS}_X(\text{sec}, \text{cnt}) = (1-\delta) \times \text{KS}_X(\text{sec}, \text{cnt})
  \]

- Movable capital volumes in a region get a share of the capital which was newly invested in the region in the earlier stage, then depreciated:
  \[
  \text{KS}_S(\text{cnt}, \text{cntt}) = (1-\delta) \times (\text{KS}_S(\text{cnt}, \text{cntt}) + \theta_i(\text{cnt}) \times \text{KS}_n(\text{cntt}))
  \]

- New capital volumes are endowed according to investment and interests rates:
  \[
  \text{KS}_N(\text{cnt}) = (r_z + \delta) \times i.l(\text{cnt});
  \]

- Labour endowment is increased according to the population’s growth rate:
  \[
  \text{LS}_F(\text{cnt}) = \text{LS}_Z(\text{cnt}) \times (1+g_z);
  \]

The resulting capital volumes will make the model divert more capital to the sectors into which more investment was made, in the form of new capital. Depending on the calibration of the parameters, this could lead to an increased presence of a particular sector in a region, with the resulting systemic consequences, or a simple substitution of an old technology with a new one, without more alteration to the model’s other actors.

### 4.7 Simplified Regions

There might be the case that one or more regions in the model are not suitable to be completely modelled, either due to the lack of precise information on their internal processes or because the region in itself is not of interest for a particular analysis. In this case, the static version of the model allows for a simplified version of the region to be included, in which only the interactions of said region with others are modelled, while all the internal details are ignored. If this feature is switched on, and one or more regions have been marked as such, every production and demand block for this region will be replaced with a single, region-encompassing production block:
$PROD:REGION(cnt)$(not R_S(cnt)) s:0 t:0

*Replace TRANSPORTERS
O:PIT$OutRegVol(cnt) Q:OutRegVol(cnt)
I:PIT$InRegVol(cnt) Q:InRegVol(cnt)
O:PTM(cnt, cntt)#(com)$ (ord(cnt) ne ord(cntt))
+ Q:trademargins(com, cntt, cnt)

*REPLACE TRADERS
I:ER#(com) Q:MROWZ(cnt,com)
I:PETRADE(com,cntt,cnt)$(ord(cnt) ne ord(cntt)) Q:TRADEZ(com,cntt,cnt)
I:PTM(cnt, cntt)#(com)$ (ord(cnt) ne ord(cntt))
+ Q:trademargins(com, cntt, cnt)

*REPLACE EXPORTERS
O:PETRADE(com,cnt,cntt)$(ord(cnt) ne ord(cntt)) Q:TRADEZ(com,cnt,cntt)
O:ER#(com) Q:EROWZ(cnt,com)

*REPLACE CONSUMERS
O:ER Q:(TRHROW.L(cnt))$(TRHROW.L(cnt) ge 0)
I:ER Q:(-TRHROW.L(cnt))$(TRHROW.L(cnt) le 0)
O:ER Q:TRROW.L(cnt)
O:ER Q:(SROW.L(cnt))

The first group of lines guarantees that the interregional balance and trade margins are maintained. The second group replaces the trades which would normally buy or import goods from other regions and the rest of the world. The third group does a similar work replacing the export block the region would normally have. Lastly, the final group of entries replaces the interactions consumers (households, government) have with the actor outside the region.

All flows into and out of the region are taken into account and all volumes are the same as the original ones. Therefore, this block should completely replace a more detailed block for that region in the sense that, if the model is run with a shock, the results for all regions not simplified should be the same, regardless of whether one or more regions have been replaced by the simplified block.

### 4.8 Additional Constraints

We use additional constraints to round off the model. Note the $R_*$ fields in the Government, Investment and Households DEMAND blocks. This means that the endowments, positive and their corresponding negatives, are proportionally scaled to changes in the economy. This happens quite clearly with the consumer price index, $PCINDEX_{cnt}$, which changes as the amount of consumption in the region changes. The investment index, $PIINDEX_{cnt}$, and its national counterpart perform a similar function on the investment part. The same happens when the produced goods or the household budgets change, as they lead to changes in the reference for the stock changes and household savings, respectively.

$$\text{CONSTRAINT:PCINDEX(cnt)}$$
$$PCINDEX(cnt) = e = \frac{\text{sum}(\text{com}, (1+taxc(cnt,com)+ElCertTax("HOUS", com, cnt)) * P(cnt,com)\times CZ(cnt,com)\times(1+trmz(cnt,com)))}{\text{sum}(\text{com}, (1+taxcz(cnt,com)+ElCertTax("HOUS",com,cnt))*CZ(cnt,com)*(1+trmz(cnt,com)))};$$

$$\text{CONSTRAINT:PIINDEX(cnt)}$$
$$PIINDEX(cnt) = e = \frac{\text{sum}(\text{com}, (1+taxc(cnt,com)+ElCertTax("HOUS", com, cnt)) * P(cnt,com)\times IZ(cnt,com)\times(1+trmz(cnt,com)))}{\text{sum}(\text{com}, (1+taxcz(cnt,com)+ElCertTax("HOUS",com, nt))*IZ(cnt,com)*(1+trmz(cnt, com)))};$$

$$\text{CONSTRAINT:PIINDEXN}$$
$$PIINDEXN(cnt) = e = \frac{\text{sum}(\text{com}, (1+taxc(cnt,com)+ElCertTax("HOUS", com, cnt)) * P(cnt,com)\times IZ(cnt,com)\times(1+trmz(cnt,com)))}{\text{sum}(\text{com}, (1+taxcz(cnt,com)+ElCertTax("HOUS",com, nt))*IZ(cnt,com)*(1+trmz(cnt, com)))};$$
\[
\text{PIINDEXN} = e = \sum((\text{com, cnt}), (1+\text{taxc}(\text{cnt, com})+\text{ElCertTax}(\text{"HOUS", com, cnt})) * \\
\text{P}(\text{cnt, com})*\text{IZ}(\text{cnt, com})*(1+\text{trmz}(\text{cnt, com}))) / \\
\sum((\text{com, cnt}), (1+\text{taxcz}(\text{cnt, com})+\text{ElCertTax}(\text{"HOUS", com, cnt})) * \\
\text{IZ}(\text{cnt, com})*(1+\text{trmz}(\text{cnt, com})))
\]

\$\text{CONSTRAINT:R\_SV}(\text{cnt, com})$
\[
\text{R\_SV}(\text{cnt, com}) = e = X(\text{cnt, com});
\]

\$\text{CONSTRAINT:R\_SH}(\text{cnt})$(SW\_INVS eq 1)
\[
\text{R\_SH}(\text{cnt}) = e = \text{HOUS}(\text{cnt})/(\text{CBUDZ}(\text{cnt}) - \sum(\text{sec, NEGVZ}(\text{cnt, sec}))) ;
\]

\$\text{CONSTRAINT:R\_SHN}(\text{SW\_INVS eq 2})$
\[
\text{R\_SHN} = e = \sum(\text{cnt, HOUS}(\text{cnt}))/\sum(\text{cnt, CBUDZ}(\text{cnt}));
\]

Finally, the indexing amount for government-to-government transfers, active only when both levels are active, is

\$\text{CONSTRAINT:PGINDEX}(\text{cnt})$(SW\_GOVT eq 2)
\[
\text{PGINDEX}(\text{cnt}) = e = \sum(\text{com, (1+taxc(\text{cnt, com}))*P(\text{cnt, com}) ;}\]

\subsection{4.9 Supply balances}

When using MPSGE, it is not necessary to specify the way in which the market clears for any of the production factors or goods; these equations are generated automatically.

For labour, capital and investments, the supply balances are as follows.

\begin{align*}
\text{PL}_\text{cnt}\text{LZ}_\text{cnt,sec} &= a^L_{\text{cnt,sec}}\text{XD}_\text{cnt,sec}(\text{LZ}_\text{cnt,sec} + \text{KZ}_\text{cnt,sec} + \text{INVZ}_\text{cnt,sec}) \tag{14a} \\
\text{RK}_\text{cnt,sec}\text{KZ}_\text{cnt,sec} &= a^K_{\text{cnt,sec}}\text{XD}_\text{cnt,sec}(\text{LZ}_\text{cnt,sec} + \text{KZ}_\text{cnt,sec} + \text{INVZ}_\text{cnt,sec}) \tag{14b} \\
\text{PS}_\text{cnt}\text{INVZ}_\text{cnt,sec} &= a^I_{\text{cnt,sec}}\text{XD}_\text{cnt,sec}(\text{LZ}_\text{cnt,sec} + \text{KZ}_\text{cnt,sec} + \text{INVZ}_\text{cnt,sec}), \tag{14c}
\end{align*}

where \(a^L, a^K, a^I\) are the shares each factor has in the production process at each sector sec and region cnt, calculated by MPSGE according to the input in each production block involving them.

The supply balance for goods is divided into domestically produced goods, domestically consumed goods, Armington composite goods, regionally traded goods, import/export goods, and savings/investments goods. Equations for domestic production must reflect that all outputs go directly to the export block, which defines where each share of the production is sold, either locally, in other regions or the rest of the world. Conversely, the domestic availability of a good is equal to the domestic production for domestic consumption, regional imports and ROW imports.
References


A Notation

This section lists all indices, parameters, variables, and functions as used in the preprocessing routines and the model implementation. As the preprocessing routines and the model were developed separately and in parallel, different notation may have been utilized for the same or similar entities. Therefore, we mark in the following tables entities used in the preprocessing description (section 3) by P while M denotes entities used in the model description (section 4).

Parameters ending with 'Z' denote initial values, given through the model input data $mSAM(cnt, \cdot, \cdot), mTradeData_{g, \cdot}$ or $mTradeMargin_{g, \cdot}$ (or their original version on the national level). For these, there exist variable counterparts in the model without the 'Z' suffix, which are not listed here. Likewise, there exist counterparts of some variables on the local and / or national level. These are denoted by suffixes ‘L’ and ‘N’ (or ‘l’ and ‘n’, respectively) and are not mentioned here, either. For example, the parameters and variables $CGZ_{cnt,g}, CG, CGN$ and $CGNZ_{cnt,g}$ are all related to each other.

Indices

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<td>P Factors / final demands vector, mapped to user-defined (model) labels</td>
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<tr>
<td>$P_f$</td>
<td>P Factors / final demands vector, internal classification (labels “131”, ..., “149”)</td>
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<tr>
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<td>$Ps$</td>
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Parameters

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</tr>
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<td>$CBUDGZ_{cnt}$</td>
<td>M Governmental consumption budget</td>
</tr>
<tr>
<td>$CBUDZ_{cnt}$</td>
<td>M Households’ consumption budget</td>
</tr>
<tr>
<td>$CGZ_{cnt,com}$</td>
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</tr>
<tr>
<td>$CZ_{cnt,com}$</td>
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<tr>
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<td>M Depreciation index</td>
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<td>$EROWZ_{cnt,com}$</td>
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<tr>
<td>$EZ_{cntcom}$</td>
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<tr>
<td>$g_z$</td>
<td>M Population’s growth rate</td>
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<tr>
<td>$INVZ_{cnt,sec}$</td>
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<tr>
<td>$IOZ_{cnt,com,sec}$</td>
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<tr>
<td>$ITZ_{cnt}$</td>
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<td>$IZ_{cnt,com}$</td>
<td>P, M Investments</td>
</tr>
<tr>
<td>$KZ_{cnt}$</td>
<td>P, M Capital inputs</td>
</tr>
</tbody>
</table>
Parameter Description

- $\mathcal{L}Z_{cnt,sec}$: Labour inputs
- $\mathcal{M}ROWZ_{cnt,com}$: Imports
- $\mathcal{mSAM}(cnt,\cdot,\cdot)$: Complete SAM with regional dimension, mapped to user-specified goods, sectors and factors/final demand
- $\mathcal{mTradeData}_{g,cnt,\overline{cnt}}$: Interregional trade flows from region $cnt$ to region $\overline{cnt}$ (incl. ROW)
- $\mathcal{mTradeMargin}_{g,cnt,\overline{cnt}}$: Transport and trade margins for flow of good $g$ from region $cnt$ to region $\overline{cnt}$ (incl. ROW)
- $r_Z$: Investment rate
- $SGZ_{cnt}$: Governmental savings
- $SHZ_{cnt}$: Households savings
- $SROWZ_{cnt}$: Savings from ROW
- $SVZ_{cnt,com}$: Changes in stocks
- $SUM_{d,cnt,g}$: Initial value for variable $SUM_d V_{cnt,g}$; gravity model, equ. (2e)
- $SUM_{o,cnt,g}$: Initial value for variable $SUM_o V_{cnt,g}$; gravity model, equ. (2d)
- $\mathcal{TAXCZR}_{g,cnt,\overline{cnt},taxcZ_g}$: Net taxes on goods
- $\mathcal{TAXPZ}_{cnt,sec}$: Net taxes on production
- $\theta_a_{cnt}$: Share of capital newly invested in region $cnt$ at previous stage (dynamic model)
- $\mathcal{TMCRZ}_{g,cnt,\overline{cnt}}$: Transport and trade margins
- $\mathcal{TMXZ}_{cnt,com}$: Production of transport and trade margins
- $\mathcal{TRANSFZ}_{cnt}$: Governmental transfers to households
- $\mathcal{TRADEZ}_{g,cnt,\overline{cnt}}$: Trade flows
- $\mathcal{trade margins}_{com,cnt,ent}$: Trade margins as read from $\mathcal{mTradeMargin}_{g,cnt,\overline{cnt}}$
- $\mathcal{TRROWZ}_{cnt}$: Net transfers to households (closing trade balance)
- $\mathcal{trmz}_{cnt,com}$: Initial transport and trade margins, calculated in the model
- $\mathcal{TRROWZ}_{cnt}$: Net transfers to government (closing trade balance)
- $\mathcal{TTYZ}_{cnt,ty,cnt}$: Tax on income
- UGSN, UGFN, USGN, USFN, UFGN, UFSN, UFFN: Submatrices of the input SAM, mapped to user-specified goods, sectors and factors/final demands
- VINTAGE: Activity for transfer of “new” capital to normal region-specific capital (dynamic model, section 4.6)
- $\mathcal{XDDE}_d_{g,cnt,\overline{cnt}}$: Initial value for $XDDE_d V_{g,cnt,\overline{cnt}}$; gravity model, equ.s (2b), (2c)
- $\mathcal{XDDE}_o_{g,cnt,\overline{cnt}}$: Initial value for $XDDE_o V_{g,cnt,\overline{cnt}}$; gravity model, equ.s (2a), (2c)
- $\mathcal{XDDZ}_{cnt,sec,com}$: Detailed sectoral outputs (Armington composite)
- $\mathcal{XDZ}_{cnt,sec}$: Sectoral outputs
- $\mathcal{XXDZ}_{cnt,g}$: Domestic goods supply to domestic market
- $\mathcal{XZ}_{cnt,com}$: Total sales

Variables

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<tr>
<td>EXPORT$_{cnt,g}$</td>
<td>Export activity</td>
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<td>GOVT$_{cnt}$</td>
<td>Government activity</td>
</tr>
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</table>
### Variable Description

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<tr>
<th>Variable</th>
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<tr>
<td>$H_{OUS_{cnt}}$</td>
<td>M Representative household’s activity</td>
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<tr>
<td>$InRegVol_{cnt}$</td>
<td>M Contribution of negatively imbalanced regions (section 4.3)</td>
</tr>
<tr>
<td>$INV_{cnt}$</td>
<td>M Local investment agent activity</td>
</tr>
<tr>
<td>$OutRegVol_{cnt}$</td>
<td>M Contribution of positively imbalanced regions (section 4.3)</td>
</tr>
<tr>
<td>$P_{cnt,g}$</td>
<td>M Composite consumer price</td>
</tr>
<tr>
<td>$PCINDEX_{cnt}$</td>
<td>M Indexing amount for consumer prices</td>
</tr>
<tr>
<td>$PD_{cnt,sec,com}$</td>
<td>M Domestic output price</td>
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<tr>
<td>$PDD_{cnt,com}$</td>
<td>M Price of domestic goods provided to domestic market</td>
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<td>$PETRADE_{com,cnt,cntt}$</td>
<td>M Prices of traded goods</td>
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<td>$PGINDEX_{cnt}$</td>
<td>M Indexing amount for government-to-government transfers</td>
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<tr>
<td>$PDTR$</td>
<td>M Price of national government transfers to local government</td>
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<td>$PIINDEX_{cnt}$</td>
<td>M Indexing amount for investments</td>
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<td>M Terms of interregional trade</td>
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<td>$PS_{cnt}$</td>
<td>M Price of savings (artificial)</td>
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<tr>
<td>$PTM_{cnt,ent,entt}$</td>
<td>M Transport and trade margins</td>
</tr>
<tr>
<td>$PTR_{cnt}$</td>
<td>M Price of governmental transfers</td>
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<tr>
<td>$PU_{cnt}$</td>
<td>M Private consumption price index</td>
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<tr>
<td>$PUG_{cnt}$</td>
<td>M Governmental consumption price index</td>
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<tr>
<td>$PUINV_{cnt}$</td>
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<tr>
<td>$REGION_{cnt}$</td>
<td>M Artificial region’s activity</td>
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<tr>
<td>$RKC_{cnt,sec}$</td>
<td>M Price of (locally) traded capital</td>
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<tr>
<td>$RKC_{ext,sec,cnt}$</td>
<td>M Price of (locally) traded extant capital (dynamic version)</td>
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<tr>
<td>$RKC_{new,sec,cnt}$</td>
<td>M Price of (locally) traded new capital (dynamic version)</td>
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<tr>
<td>$R_{SH_{cnt}}$</td>
<td>M Multiplier for households savings</td>
</tr>
<tr>
<td>$R_{SV_{cnt,com}}$</td>
<td>M Multiplier for changes in stocks</td>
</tr>
<tr>
<td>$SUM_dV_{cnt,g}$</td>
<td>P Total of good $g$ delivered to region $cnt$; gravity model, equ. (2e)</td>
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<tr>
<td>$SUM_oV_{cnt,g}$</td>
<td>P Total of good $g$ delivered from region $cnt$; gravity model, equ. (2d)</td>
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<tr>
<td>$TRANSP_{cnt}$</td>
<td>M Transport services</td>
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<td>$UG_{cnt}$</td>
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<td>$UINV_{cnt}$</td>
<td>M Fixed capital investment activity – locally</td>
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<td>$U_{cnt}$</td>
<td>M Consumption by households</td>
</tr>
<tr>
<td>$XDDE_{dV_{g,cnt,ent}}$</td>
<td>P Destination – domestic production delivered to domestic regions (own and other); gravity model, equ.s (2b), (2c)</td>
</tr>
<tr>
<td>$XDDE_{oV_{g,cnt,ent}}$</td>
<td>P Origin – domestic production delivered to domestic regions (own and other); gravity model, equ.s (2a),(2c)</td>
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### Functions and mappings

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<td>$distrPCountyProd_{c,sec}$</td>
<td>P Factor sets for disaggregating national to county / regional parameters with sector dimension, based on production</td>
</tr>
<tr>
<td>$distrPRegProdN_{r,sec}$</td>
<td>P Factor sets for disaggregating national to county / regional parameters with sector dimension, based on consumption</td>
</tr>
<tr>
<td>$distrGoodRegProdN_{r,sec}$</td>
<td>P Factor sets for disaggregating national to regional parameters with goods dimension</td>
</tr>
<tr>
<td>Function</td>
<td>Description</td>
</tr>
<tr>
<td>------------------------------------------</td>
<td>-------------------------------------------------------</td>
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<tr>
<td>$indicIO_{r,g,sec}$</td>
<td>P Regional production by good and sector</td>
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<tr>
<td>$indicPROD_{r,sec}$</td>
<td>P Total regional production by sector</td>
</tr>
<tr>
<td>$indicLAB_{r,sec}$</td>
<td>P Regional labour inputs by sector</td>
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<tr>
<td>$indicIMP_{r,sec}$</td>
<td>P Regional imports by sector</td>
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<td>$indicEXP_{r,g}$</td>
<td>P Regional exports by good</td>
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<td>$indicINV_{r,g}$</td>
<td>P Regional investments / capital inputs by good</td>
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<td>$indicCONS_{r,g}$</td>
<td>P Regional household consumption by good</td>
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<tr>
<td>$indicCONSgov_{r,g}$</td>
<td>P Regional governmental consumption by good</td>
</tr>
<tr>
<td>$mapCR(c, r) \in {0, 1}$</td>
<td>P Mapping: to which region $r$ does county $c$ belong?</td>
</tr>
<tr>
<td>$mapMunicitoCount(m, c) \in {0, 1}$</td>
<td>P Mapping: to which county $c$ does municipality $m$ belong?</td>
</tr>
<tr>
<td>$mapPS(Ps, sec) \in {0, 1}$</td>
<td>P Mapping of sectors from “PANDA” to user-specified classification</td>
</tr>
<tr>
<td>$mapPG(Pg, g) \in {0, 1}$</td>
<td>P Mapping of goods from “PANDA” to user-specified classification</td>
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