Optimizing Jack-up Vessel Chartering Strategies for Offshore Wind Farms

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The purpose of this thesis is to study jack-up vessel chartering for maintenance operations on offshore wind farms. A tactical problem with a finite planning period is formulated using mathematical programming methods. Different solution methods for the models are evaluated and technical and economic analyses are conducted.
PREFACE

This Master’s Thesis is the concluding part of our Master of Science at the Norwegian University of Science and Technology. The degree specialization is Managerial Economics and Operations Research at the Department of Industrial Economics and Technology Management. This thesis examines cost effective jack-up vessel chartering strategies for offshore wind farms considering uncertainties in weather and turbine component failure occurrences and is a continuation of the Project Thesis written during the fall of 2015.

We would like to thank our supervisors Professor Marielle Christiansen (Department of Industrial Economics and Technology Management) and Associate Professor Magnus Stålhan (Department of Industrial Economics and Technology Management) for their excellent guidance and input throughout the semester.

Trondheim, June 7, 2016

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Offshore wind energy is an industry in rapid growth. However, it is still outperformed by conventional sources of energy in terms of costs. One way to cut costs is to make operations and maintenance (O&M) more efficient; O&M costs can constitute up to one third of overall lifetime costs for an offshore wind farm. Jack-up vessel chartering represents a significant part of these costs. This creates a demand for effective chartering strategies for the jack-up vessel.

Two stochastic models considering uncertainties in weather and turbine component failure occurrences are presented in this thesis. The models suggest chartering schedules for the jack-up vessel based on electricity prices, charter rates, mobilization costs and failure and weather scenarios. The schedules contain information about when and for how long the vessel should be chartered.

Due to the complexity of the studied problem, exact methods were insufficient and the Sample Average Approximation (SAA) method and heuristics are proposed. The SAA method solves a number of smaller problems in order to provide optimistic and pessimistic bounds for the optimal objective value of the true problem. The idea behind the heuristics is to create sequences of maintenance operations using a greedy approach. The most successful heuristic also has a random component.

When solving the jack-up chartering strategy problem, the combination of the SAA method and a greedy, randomized heuristic proved to be successful. The SAA method helped reduce computational complexity, and in combination with the heuristic, provided tight bounds for the optimal objective value.

It was found that for wind farm sizes ranging from 50 to 100 turbines, the costs of chartering the jack-up vessel dominate turbine downtime costs. This points towards an imbalance in the jack-up vessel charter market where demand is higher than supply, driving prices up. Analyses show that charter prices are at a level such that buying a vessel should be considered when wind farm size exceeds about 71 turbines.

Solving the stochastic jack-up vessel chartering strategy problem can provide useful insights for offshore wind farm operators and jack-up vessel owners. This is illustrated by analyses of whether chartering of buying a vessel is most economically sensible for different wind farm sizes. Moreover, the models can provide information about the value of having a more weather robust vessel.
Sammendrag

Offshore vindenergi er en bransje i sterk vekst, men kostnadsmessig ligger den langt bak konvensjonelle energikilder. En måte å kutte kostnader på, er å effektivisere drift og vedlikehold da disse kostnadene kan utgjøre opptil én tredel av totale levetidskostnader for en offshore vindpark. Inneie av jack-up fartøy utgjør en betydelig andel av vedlikeholds kostnadene, derfor er det behov for effektive inneleisingsstrategier for denne fartøytypen.

Denne studien presenterer to stokastiske modeller som tar hensyn til usikkerhet i værforhold og feilhendelser på turbinkomponenter. Modellene foreslår inneleiplaner for jack-up fartøy basert på elektrisitetspriser, inneiepriser, mobiliseringskostnader og feil- og værszenarier. Inneleiplanene inneholder informasjon om når og hvor lenge fartøyet bør leies inn.

Det studerte problemet er svært komplekst og kan derfor ikke løses med eksakte metoder innen rimelig tid. Heuristikker og Sample Average Approximation-metoden (SAA) introduseres for å løse problemer av realistisk størrelse. SAA-metoden løser et antall mindre problemer og gir optimistiske og pessimistiske grenser for den optimale objektivverdien til det faktiske problemet. Grunnideen bak heuristikkene er å lage sekvenser av fartøyets vedlikeholdsoppgaver ved hjelp av en grådig tilnærming. Den beste heuristikken hadde en tilfeldig komponent i tillegg.

Testing viser at SAA kombinert med den beste heuristikken gir en god løsningsmetode for det studerte problemet. SAA-metoden reduserte kompleksiteten til problemet og ga, sammen med heuristikken, stramme løsningsgrenser.

For vindparker med 50 til 100 turbiner, ble det konstateret at inneiekostnaden var mange ganger større enn nedetidskostnaden for de ødelagte turbinene. Dette tyder på en ubalanse i markedet for jack-up fartøy der etterspørselen er.mye høyere enn tilbudet, som driver prisene i været. Analyser viser at inneieprisene er på et slikt nivå at kjøp av et eget fartøy kan være økonomisk lønnsomt når vindparkstørrelsen overstiger omtrent 71 turbiner.

Å løse det stokastiske inneleistrategiproblemet for jack-up fartøy til offshore vindparker kan gi verdifull innsikt for vinparkoperatorer. Dette illustreres gjennom analyse av hvorvidt innkjøp eller inneie lønner seg for forskjellige vindparkstørrelser og verdien av et værmessig mer robust fartøy.

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INTRODUCTION

As the demand for electricity and green energy is increasing worldwide, offshore wind is an industry in rapid growth [74]. Between 2000 and 2012, world electricity consumption increased by about 48% [79]. Projections for the future show continued consumption growth. Currently, fossil fuels constitute the largest share of generation; about 67% of electricity generated worldwide hailed from fossil fuels in 2013 [4]. The numbers are approximately the same for Europe [2]. The EU goals for 2020 state that 20% of the energy consumption in EU countries should come from renewables [21], and this will further increase the focus on green energy.

Generation of renewable energy, and wind in particular, has experienced solid growth the last few years. In 2015, 419 new offshore wind turbines were installed in Europe and six new projects were under construction. Total grid-connected capacity at year end was about 11.0 GW. When completed, the six projects expand the capacity by 1.9 GW, thus reaching 12.9 GW, which gives an increase of 17% [73]. Figure 1 depicts cumulative and annual offshore wind power installation in Europe.

There are several reasons for wind turbines to be installed offshore rather than onshore, the most important being space. Offshore installation allows for wind farms with larger and more turbines being located where wind conditions are more stable, thus increasing production. Furthermore, noise and visual effects are minimized. However, currently, offshore wind is not financially competitive with lifetime levelized costs being 59% higher than onshore wind and about 127% higher than fossil fuels [78]. On the other hand, the load factor of an offshore wind turbine is higher than that of an onshore turbine and roughly the same as large scale hydropower [62]. Here, the load factor means the actual output as a percentage of theoretical output. This indicates reducing costs of offshore wind sufficiently can make it financially viable.

The elevated costs of offshore wind energy are caused by significant installation as well as operations and maintenance, hereinafter denoted O&M, costs. Currently, O&M costs constitute about one-third of lifetime costs [55]. This includes spare parts, transportation, technician salaries and costs of repair action as well as forfeit revenue due to turbines not producing. Rough weather does not only increase the failure rate, but also decreases accessibility. The harsher offshore conditions play a large part, with the availability of turbines plunging to 60-70% compared to 95-99% onshore [68]. With
the turbines situated far offshore, the opportunities for conducting maintenance are limited and expensive as extended periods of favorable weather are required. When conditions become worse in winter time, downtime for turbines can be extensive.

One way to make offshore wind more financially competitive, is to reduce O&M costs. Vessel chartering constitutes around 70% of lifetime O&M costs \cite{12} and decision supporting tools for this problem are limited. Thus, in order to lower costs of energy, there is a need for better vessel chartering strategies. The charter rate of a jack-up vessel vary between £45,000 and £287,000 (585,000 - 3,371,000 NOK) per day \cite{12}, while access vessels used for crew transfer, is about one fifth or one fourth of that \cite{13}. Because of the high charter rate, efficient use is required.

This study is based upon the work by Kirkeby and Mikkelsen \cite{39} who modelled the Jack-Up Vessel Chartering Strategy Problem. Much of the material presented in this thesis can also be found in their project thesis. Models presented in their thesis are further developed and solution methods for the problem are studied. The models presented in this study are stochastic and aim to suggest when to charter a jack-up vessel in order to minimize downtime costs and chartering costs and, by extension, O&M costs for an offshore wind farm. The models consider charter decisions for one wind farm only. As large stochastic problems are difficult to solve exactly, heuristics and the Sample Average Approximation method are introduced to facilitate solving the problem.

The organization of the thesis is as follows: background information needed to properly understand the studied problem is presented in \textit{chapter 2}. Following is a review
of offshore wind O&M optimization and vessel chartering strategy literature, and a re-
view of literature related to the solution methods used. Chapter 4 contains a thorough
description of the problem studied in this thesis and chapter 5 presents the basic math-
ematical model in developed. Following, in chapter 6 is an alternative formulation of
the mathematical model and a model considering multiple vessel types is presented
in chapter 7. Chapters 8, 9, and 10 describe solution methods, the implementation of
presented models and a scenario generator, and a computational study, respectively.
Chapter 11 provides concluding remarks and chapter 12 presents suggestions for fu-
ture research relevant to this thesis.
BACKGROUND

This chapter outlines background information on offshore wind required to understand the problem studied. A presentation of the current offshore wind market status and projected development is given in section 2.1. Section 2.2 describes different types of jack-up vessels, including functionalities and applications as well as charter market characteristics. In section 2.3, an elaboration on maintenance strategies and failure rates on offshore wind turbines is given. Section 2.4 presents aspects related to downtime costs of turbines and section 2.5 presents some background on different subsidy schemes.

2.1 OFFSHORE WIND MARKET

Current offshore wind capacity is about 11 GW in Europe, producing 40.6 TWh in a normal wind year, enough to cover 1.5% of EU’s total electricity consumption. The most significant contributors are the UK, Germany, and Denmark with 45.9%, 29.9% and 11.5% of installed capacity, respectively [74]. Figure 2 shows a map of current and planned offshore wind turbine capacity in Northern Europe.

It is expected that European offshore wind turbine capacity in 2020 will be about 23.5 GW. The growth is anticipated mainly in the UK, France, and the Netherlands. However, to achieve the projected growth, costs for the generated electricity must be reduced. EY reckon the levelized costs of energy to be competitive by 2023 at about 826 NOK/MWh. The introduction of turbines with higher capacity is expected to be the main driver of lowered costs along with increased effects of economies of scale, fiercer competition and optimization of supply chain and logistics [52].

For the future, projections suggest that offshore wind farms will be larger, move further from shore and be situated at greater water depths. Today, the average wind farm has a 337.9 MW capacity and is located 43.3 km from the shore at 27.1 m water depth. From Figure 3, it is apparent that the coming wind farms, marked by red and green dots, will increase both average water depth and distance from shore.
To sustain growth, access to financing must be improved. Currently, government subsidies are necessary to make offshore wind financially viable. The UK government is contemplating to gradually reduce subsidies, thus, the industry must look somewhere else for financing. Project finance has been an important tool in the industry. In 2014, project finance was integral in funding 41% of new capacity that reached a final investment decision during the course of the year. The equity side is, per 2014, dominated by power producers, as shown in Figure 4. For the future, EY describes offshore wind as an attractive investment opportunity due to low operating costs, a constant generation price, and great natural potential. However, high price volatility together with increased carbon taxes might make investments in fossil fuels a risky affair. A continued oil price decrease could magnify these effects. Offshore wind is labelled as a reliable technology appropriate for hedging against price volatilities.
2.1 Offshore Wind Market

Figure 3: Average water depth and distance to shore of online, under construction and consented wind farms [74]

Figure 4: Market segmentation of major equity investors in European offshore wind industry in 2014 [73]
2.2 JACK-UP VESSELS

A jack-up vessel is characterized by a number of extendable legs able to lift the vessel’s hull above the sea surface. When at the desired location, the hull is lifted using the seabed for support. When the maintenance operation is complete, the hull is lowered down to the sea surface and the legs are retracted to their default position. There are both self-propelled vessels and barges dependent upon tug boats for relocation. The jack-up vessel possesses a strong crane as well as generous deck space to accommodate spare parts which limits the need for sailing back to a depot. When jacked up, the far-reaching crane has a stable platform which facilitates heavy-lift maintenance that agrees with health and safety standards. Prior to the vessel arriving, technicians have prepared the turbine to ensure maintenance goes smoothly; for instance, electric and control cables are disconnected and oil is emptied [71]. Figure 5 depicts the jack-up vessel Pacific Orca.

![Figure 5: Wind turbine installation jack-up vessel Pacific Orca](image)

There are different types of jack-up vessels in terms of size, operating depth and weather capabilities, among other factors. Generally, a vessel that can operate in deeper waters and in rougher weather conditions is more expensive. It should be noted that not one vessel in the current market is able to access all offshore wind farms built or planned. Shenton and Mallett [71] compiled an extensive list of vessels in the market. Table 1 describes the ranges of specifications.

<table>
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<tr>
<td>Maximum Operating Depth (meters)</td>
<td>24 - 75</td>
</tr>
<tr>
<td>Deck Capacity ($m^2$)</td>
<td>430 - 4,300</td>
</tr>
<tr>
<td>Deck Capacity (tonnes)</td>
<td>492 - 8,400</td>
</tr>
<tr>
<td>Crane Load Capacity (tonnes)</td>
<td>30 - 1,500</td>
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Costs are affected by site and task specific, vessel related and market-related factors. Site and task-specific factors include location, transit time, the number of turbines requiring heavy-lift maintenance and all disruptions due to weather. The vessel size, specifications and capital cost or owners’ target utilization make up the vessel related factors. Market-related factors are type of charter, market demand, and seasonal variations [71].

Vessel chartering is strongly affected by weather and wind farm location. With maximum operating depths between 24 and 75 meters and the rule of thumb that charter rates increase along with operating depth, it is obvious that location will be an important factor. Furthermore, only a very limited number of vessels can operate when wave height exceeds 2.5 meters [52].

Chartering a jack-up vessel is expensive. Rates vary between £45,000 and £287,000 (585,000 - 3,371,000 NOK) per day and are volatile [12], [11]. Naturally, long-term charter will be cheaper per day than short-term spot charter, with studies estimating rates being up to 40% more expensive in the spot market [11]. Due to the volatility of rates, vessel chartering is viewed as a high-risk cost item for offshore wind farm operators, especially when long-term charter is not considered.

Charter rates are dictated by the availability of vessels. All vessels fitted for wind turbine installation are suited for heavy-lift maintenance. Most, however, exceed the required specifications and are thus unnecessarily expensive. The availability of these vessels might be limited in the future because of the significant expected growth in the industry. There are purpose-built vessels for offshore wind farm O&M that are cheaper to charter, if available. The vessels require severe alterations to be suited for other types of offshore operations and will therefore not be affected by any surge in oil and gas activity. The third category of jack-ups is multi-purpose vessels which can also be used in the oil and gas sector, for instance. The availability of these vessels is dependent upon activity in the oil and gas sector, thus highly volatile [71].

Offshore wind farms require extensive and varied maintenance throughout their lifespan. A significant portion of the maintenance, both corrective and preventive, can be done using regular Crew Transfer Vessels, technicians and the built-in turbine crane only [71]. Operators would want, because of the severe increase in charter costs, to limit the usage of a jack-up vessel to a minimum, therefore, it will only be chartered for major maintenance operations where the turbine’s built-in crane is not sufficient.

2.3 Corrective Maintenance and Failure Rates

Maintenance operations can be divided in two, namely corrective and preventive maintenance. Preventive maintenance is planned and precautionary operations to prevent failures. Jack-up vessels are usually not used for preventive maintenance [71]. Corrective maintenance is repairing or replacing already failed components.
2.4 DOWNTIME COSTS

Wind turbines are equipped with an alarm system notifying the operator of any failures. In the case of failure, the magnitude must be assessed to evaluate what kind of maintenance operation is needed. When an alarm is triggered, the wind turbine is shut down until the alarm is checked, thus not producing power. The time the turbine is not generating electricity is called downtime.

Estimates of how much corrective maintenance is needed on a wind turbine are based upon historical failure data. Realistically, failure rates follow a Weibull distribution over the component’s lifetime. Weibull distribution is probably the most widely used in reliability engineering because of its flexibility, simple interpretation of distribution parameters and the close relation to the bathtub curve concept. The bathtub curve is, by intuition, an appropriate way to present failure rates. A high initial probability reflects manufacturing defects and wrongful installation, for instance. The probability decreases rapidly, then less so. Near the end of the expected lifetime, the failure rate rises exponentially to account for normal wear and tear. Since offshore wind is a quite immature business, historical data is scarce.

2.4 DOWNTIME COSTS

When a major component fails, the turbine will stop generating electricity. Lost revenues due to generation disruptions are called downtime costs. The downtime costs of a failure is determined by how much electricity the turbine would have generated if not shut down, the electricity price plus subsidies, and how long the turbine is shut down. Together, these factors constitute forfeit revenue. Downtime costs can be calculated as:

\[
\text{Downtime Costs} = \text{Turbine Efficiency given Wind Speed} \times \text{Turbine Electricity Generation given Wind Speed} \times (\text{Electricity Price} + \text{Subsidies}) \times \text{Downtime}
\]

As indicated above, wind turbine efficiency and electricity generation are dependent upon wind speeds. This dependency is often presented as power and efficiency curves. Typical wind speed - generation/efficiency relations are depicted in Figure 6. Note that turbine efficiency is often referred to as the turbine’s power coefficient. Furthermore, wind turbines only operate within certain wind speed windows. The wind speed at which a turbine shuts down to prevent being damaged is called the cut-off speed. For the Enercon E-126 EP4 turbine presented, the cut-off wind speed is 25 m/s. At low wind speeds, the turbine electricity generation, naturally, is low while the power coefficient is higher.
2.4 DOWNTIME COSTS

The revenue obtained from generated electricity is dependent upon the electricity price and subsidies. The electricity price is dictated by the market the operator delivers to. Electricity prices are relatively predictable and tend to vary with the seasons. Subsidy schemes differ between countries and are discussed further in section 2.5.

The downtime includes time needed for identifying the failure, mobilization of required vessels and crew, waiting for spare parts and suitable weather windows in addition to the actual repair time. The chosen maintenance strategy is decisive for the downtime. When performing preventive maintenance tasks, the turbines are only shut down while maintenance takes place. Corrective maintenance, on the other hand, experience longer downtimes as there will be a time delay between failure occurrence and maintenance initiation. For maintenance requiring a jack-up vessel, downtime can be extensive because such a vessel is not necessarily available overnight.

To pinpoint some downtime cost challenges, climate and seasons are considered important factors; stable wind conditions and higher wind speeds at the site will cause downtime to be more expensive and higher charter frequency a possibly viable option. On the other hand, higher wind speeds mean fewer time windows viable for maintenance. Unpredictable weather also leads to more uncertainty in planning which in turn leads to higher downtime costs.

Figure 6: Power curve for Enercons E-126 EP4 wind turbine [19]
2.5 SUBSIDY SCHEMES FOR OFFSHORE WIND FARMS

The downtime costs are, as mentioned, dependent upon subsidy schemes. How subsidies are structured, varies from country to country and may or may not include tax refunds. A report by PwC compares subsidy schemes for offshore wind in six countries, namely Denmark, France, the UK, Belgium, the Netherlands, and Germany. As for revenue from generation, the two main types of subsidy schemes are feed-in tariffs and feed-in premiums, where the latter is the most common. In short, feed-in tariff contracts are long-term contracts based upon levelized costs of energy to offer producers of renewable energy some revenue stability. Feed-in premium contracts involve selling the generated power on the energy market and then receiving a premium on top, which can either be fixed or sliding based upon market price [54].

The UK utilizes a feed-in premium scheme; a budgetary pot is assigned for a given period and producers bid for funds. If the sum of all bids exceeds the budgetary pot, the highest bidder within budgetary constraints sets the strike price. This strike price applies to all bidders eligible for subsidies and the premium received is the difference between strike price and market price, and is thus sliding. The UK does not offer any tax schemes targeted specifically towards the offshore wind industry [54].

France uses feed-in tariff with a guaranteed price for 20 years. As mentioned, the feed-in tariff depends on levelized costs of energy, and in France the tariff ranges between €0.15/kWh and €0.22/kWh. In contrast to the other countries studied, levelized costs of energy includes development of grid connection [54].
This chapter contains a review of literature and theory relevant to the studied problem and applied solution methods. First, literature on offshore wind O&M optimization is presented in section 3.1. The same section includes a description of literature concerning jack-up vessel chartering for offshore wind farms. In section 3.2, a review of literature on vessel chartering strategies in other applications is presented since this is comparable to the jack-up vessel chartering strategy problem. Section 3.3 provides some insights on how to handle failure rates. Section 3.4 concerns stability of scenario generation methods used to solve stochastic problems. It also explains mathematical theory behind stability testing. Finally, some literature on Sample Average Approximation and GRASP, solution methods for stochastic problems, are provided in section 3.5 and section 3.6.

3.1 Offshore Wind O&M Optimization

An article by Shafiee [69] from 2014 structures and summarizes 137 articles on O&M operations for offshore wind. The article outlines a framework for classification of maintenance logistics based upon the length of the planning period. The framework divides maintenance in three echelons; operational, tactical and strategic. These echelons are covered in subsections 3.1.1, 3.1.2 and 3.1.3, respectively.

3.1.1 Strategic Echelon

The strategic echelon has a planning period close to the wind farm’s lifespan. Shafiee [69] identifies three studied logistic decisions within the strategic echelon.

The first decision concerns wind farm design. The two main design parameters are geographical placement and layout of the wind farm. These parameters heavily influence maintenance logistics and profitability. A wind farm with long distances between turbines experiences less wake effects, thus higher power output [57]. Wind farms far
from shore will face elevated power outputs because of higher and more stable wind speeds, but also increased logistics expenditures because of rough weather [69].

Marmidis et al. [47] optimize the placement and arrangement of wind turbines with the objective of maximizing energy production and minimizing installation cost, utilizing a Monte Carlo approach. Pookpunt and Ongsakul [57] optimize the geographical location of offshore wind farms using a binary particle swarm optimization model. Their objective is to maximize the power output while minimizing the investment cost. Pérez et al. [59] publish a similar approach, seeking to optimize expected power output while minimizing wake effects using mathematical programming techniques. A publication by Samorani [65] discusses how the wind farm layout can influence energy production and the cost related to maintenance. Chen and MacDonald [10] develop a cost-of-energy optimization model which optimizes turbine layout considering maintenance, replacement and overhaul costs. For further information, a thorough literature review of wind farm design optimization was published by González et al. [67] in 2014.

Selection of maintenance strategy has also been subject to optimization. Initially, one can separate between reactive and proactive strategies. Researchers seem to agree that reactive strategies are only feasible at wind farms close to shore and at locations where weather conditions are not too harsh. Project reports by van Bussel and Schöntag [76], and van Bussel and Henderson [75] support this attitude. They underline frequent stoppages, high repair costs and long delays as results of choosing a reactive strategy. Shafiee [69] divides the proactive strategies further into preventive and predictive strategies. Preventive maintenance operations are undertaken after a predefined period of time or at given power output levels. Predictive maintenance on the other hand, are sort of reactive in that it involves acting at specified system conditions, without those conditions meaning a component has failed. Such system conditions may be temperature, noise or corrosion.

Andrawus et al. [3] identify suitable condition based maintenance activities and assess their impact over the life cycle of wind turbines to maximize the return on investment in wind farms. Tian and Ding [15], [16] study opportunistic maintenance strategies of wind turbines. They make a comparative study with the widely used corrective maintenance policy which demonstrates the advantage of the proposed opportunistic maintenance methods, significantly reducing the maintenance cost. Rangel-Ramírez and Sørensen [61] propose risk based inspection planning as a methodology to identify the optimal maintenance strategy of offshore wind farms.

The third decision within the strategic echelon is whether to outsource repair services. To keep maintenance tasks in-house can be very expensive as it requires substantial investment in infrastructure and equipment as well as training of staff. Because of its expensive nature, there has been an increasing trend of outsourcing activities. It must also be mentioned that today, wind turbines are commonly delivered with a 2- to 5-year full-service contract [70]. These contracts cover maintenance, i.e. repair or replacement, of failed components and a rejuvenation action is carried out on critical
components in order to reduce probability of future failures. After this initial contract expires, wind farm owners can choose to further outsource maintenance.

The demand for maintenance services increases in the offshore wind market. As little research exist on the subject, the potential is considerable. Poore and Walford [58] propose outsourcing of maintenance, especially in early years, as a way to reduce O&M costs. A mathematical model which aims to reduce O&M costs under a performance based service contract is proposed by Jin et al. [34]. The purpose of such a contract is for the owner to define an availability goal which the service provider aims to satisfy.

### 3.1.2 Tactical Echelon

The tactical echelon, concerning planning periods from a few months to a couple of years, includes the problem described in this study. In [69], Shafiee describes the chartering of service vessels as a critical issue. This is further supported by Dalgic et al. [12] who state that vessel costs can constitute up to 73% of lifetime O&M costs. Several papers suggest optimization models for fleet size and mix for maintenance on offshore wind farms, e.g. Gundegjerde et al. [25], Gundegjerde and Halvorsen [24] and Halvorsen-Weare et al. [26]. The models assign different maintenance tasks to each type of vessel and output suggest the number of each vessel type to acquire within given budgetary constraints. Both deterministic and stochastic models are presented. Gundegjerde et al. [25] propose a stochastic three stage model considering uncertainty in vessel spot rates, weather conditions, electricity prices and failure rates. At stage one, the operator decides on what vessels and vessel bases are needed. At stage two, new information about charter rates is available and the operator can decide to charter now or wait. Stage 3 reveals true weather conditions, electricity prices, failure and charter rates. The model takes these uncertainties into account by scenario generation. The literature concerning jack-up vessel chartering strategy is severely limited. Dalgic et al. [12] exclaim the fact that jack-up vessel chartering is the largest contributor to lifetime O&M costs. Other key findings include rapid utilization drop for small sites as the charter period increases. Further, regional collaboration is hinted as being a solution towards optimized jack-up vessel costs. The authors suggest a Monte Carlo simulation approach which considers climate parameters, failure characteristics and vessel specifications. Thus, the article does not contain any mathematical optimization model of the problem; the results rely on repeated random sampling [27]. The sampling gives a distribution showcasing the uncertainties and a decision is made based upon this. It should be noted that in all simulated cases, the jack-up vessel chartering costs dominated the total O&M costs. For the studied 100-, 200- and 300-turbine farms, optimum short-term chartering times were determined as 3, 7 and 16 weeks respectively. Corresponding total O&M costs were found to be £39.2/MWh, £37.4/MWh and £36.2/MWh. Comparably, by utilizing long-term charter, the 100-, 200- and 300-turbine wind farms face total O&M costs of £67.6/MWh, £37.1/MWh and £27.1/MWh respectively. Thus, the article suggests long-term charter being more cost effective for
3.1 Offshore Wind O&M Optimization

200- and 300-turbine wind farms and for 300 turbines, significantly so. The paper concludes that total O&M costs can be reduced by 41.9%, 0.6% and 25.1% for 100-, 200- and 300-turbine wind farms, respectively, only by optimizing the jack-up vessel chartering strategy.

Dinwoodie and McMillan [17] investigate four operational strategies for heavy-lift operations at offshore wind farms using scenario simulation. The proposed strategies are fix on fail, batch repair, annual charter and purchase. The paper explores different failure rates to find key cost drivers and under which circumstances an operator would adopt what operational strategy. Not surprisingly, purchasing is a strong candidate when failure rates are high, and the strategy is not very sensitive to increasing failure rates. The optimum strategy is driven principally by the number of turbines in a wind farm as well as the number of failures requiring specialist vessels. Figure 7 shows the relationship between lifetime and levelized cost and the number of turbines in a wind farm. The costs only include direct vessel procurement cost and lost revenue due to downtime.

![Figure 7: Total lifetime cost and levelized cost of energy for different chartering strategies over a range of wind farm sizes](image)

For a 75-turbine wind farm, it is shown that the key cost drivers vary between the strategies. Batch repairing and purchasing, for instance, are driven by lost revenue and vessel costs respectively. It should be noted that an annual chartering strategy is between 20-30% more expensive than the others regardless of whether failure rates are high, medium or low. The authors identified strengths and weaknesses of the different strategies. Fix on failure is cost effective with low failure rates, it has no upfront costs and one can switch strategy without any penalty. However, the operator is exposed to volatile mobilization times as well as spot rates, and vessel costs increase dramatically
3.1 OFFSHORE WIND O&M OPTIMIZATION

if failure rates increase. The advantages of batch repair are similar to the those of fix on failure, but batch repair reduces exposure to vessel market price and mobilization time. An important problem in batch repair is to determine the optimum quantity. Adopting a wrong quantity in a strict batch approach may lead to some turbines having long downtimes as well as surpassing the favorable maintenance time windows during spring and summer. Additionally, if electricity prices increase, the performance of this approach deteriorates. In the case of purchasing, a significant capital investment is needed. In the case of lower than expected failure rates, purchasing is more expensive than any other approach and due to transaction-specific costs, the flexibility of the approach is low. However, availability is the highest, minimizing lost revenue costs and it is shown to be more expensive to underestimate failure rates and rely on the spot market than to overestimate failure rates and purchase a vessel. It should be noted that sub-chartering is not considered in this paper. For annual charter, the most significant advantages are consistent vessel costs and a guaranteed contract price, reducing exposure to spot rates.

3.1.3 Operational Echelon

The operational echelon which consists of short term planning, considers day to day operations. Aspects in the operational echelon include maintenance scheduling and routing of maintenance vessels [69]. Besnard et al. [7] propose a stochastic model for opportunistic maintenance planning for offshore wind farms. The model utilizes seven days wind production ensemble forecast and opportunities at corrective maintenance activities in order to perform service maintenance tasks at the lowest cost. The optimization is done on a daily basis to update maintenance planning. An interesting aspect of the model, is that it rewards performing preventive maintenance when downtime costs are low or the turbine is already out of service due to some failure requiring corrective maintenance. The model output is a set of corrective and preventive maintenance tasks advised to be performed during that day. In a real life example, the model saved 32% of production losses and transportation costs. The base case is performing maintenance during a fixed period. It should be noted that the model assumes a fixed-size maintenance staff and does not include helicopter use. The paper considers uncertainty in weather forecasts and power production, however, for the first time step, the uncertainty is negligible. Electricity prices and charter rates for the maintenance vessel are assumed constant, based upon historical figures.

Fonseca et al. [22] develop a method to schedule maintenance activities using genetic algorithms for both offshore and onshore wind farms. The paper compares performance of three methods: genetic algorithms, Djikstra’s algorithm and ant optimization when determining the cheapest path between wind farms for maintenance.
3.2 VESSEL CHARTERING IN OTHER APPLICATIONS

This section presents some literature on vessel chartering strategies in other applications. This is considered relevant to the studied problem since the literature on jack-up vessel chartering is severely limited.

Goulielmos and Goulielmos [23] consider the problem of timing in decisions to buy or charter a vessel and underline the lack of decision-making tools available in this area of interest. The article is geared towards vessels used in the offshore oil industry and uses Rescaled Range Analysis to analyze both the freight market and second hand ship market. When analyzing historical data, a strong seasonal effect is found in the 25th week in all three years presented. It is shown that the first half of the year, the prices are falling and from there to the year end, the prices are rising.

Meng and Wang [48] propose a scenario-based dynamic programming model for multi-period liner ship fleet planning. The model considers a liner shipping company facing a deterministic container shipping demand. To meet the demand, the model allows usage of own ships as well as chartering from the spot market or purchasing new or used ships. Sub-chartering is also incorporated. Each scenario is made up of vessel fleet size and mix options; for instance, one scenario is to keep and operate vessels 1 and 2, sublease vessel 3, charter vessel 4 and purchase vessel 5. The scenarios are suggested by experts before the model is run and solved for each scenario separately. Then, Dijkstra’s algorithm is used to find the combined maximum profit for the entire planning period given the solutions for every scenario. Output is purchases, sub-chartering, chartering and selling, route and cargo assignment and number of lay-up days for every vessel in every period. The computational example has a planning period of ten years; however, the model is flexible enough to consider planning periods as short as one year. The authors note that longer planning periods reward purchasing vessels rather than short-term charter.

Laake and Zhang [41] present an optimization model for strategic fleet planning in tramp shipping. It can be used to evaluate contracts against each other, thus finding the best mix of short and long term contracts given a fleet or find the optimal fleet size and mix for a set of contracts. This means, the model can be used to aid decisions in a fleet renewal program helping to find out when to buy, sell and charter vessels. The model is deterministic, which is different from Meng and Wang [48]. To handle end effects, an artificial period is added at the end of planning where only selling is allowed.

3.3 FAILURE RATES

Only the major components blade, gearbox, generator and transformer require a jack-up vessel [12]. Expected failure rates of the different components are of course an
integral part of maintenance planning. Dinwoodie and McMillan [17] show that different failure rates may strongly affect the cost-effectiveness of the chartering strategy selected. Dalgic et al. [12] propose a failure model using Weibull distribution based upon failure rates found by Lindqvist and Lundin [42]. Lindqvist and Lundin found that the most failure prone components are the electrical system, control system, hydraulics and sensors, see Figure 8. The electrical system includes the transformer, however, the paper does not investigate its failure rate separately. Furthermore, it is highlighted that the components causing longest downtimes are generator, gearbox and rotor/blades, which require a jack-up vessel, see Figure 9. Both Dalgic et al. [12] and Lindqvist and Lundin [42] highlight the limited knowledge about turbine failure rates, especially offshore. The most important reasons are the low number of offshore wind farms currently in operation and unwillingness of manufacturers to publish expected failure rates of their turbines.

![Figure 8: Average number of failures per year per wind turbine subsystem. Based upon [42](#)](image)

### 3.4 Stability of Scenario Generation Methods in Stochastic Programming

This section presents literature and theory about scenario generation methods for stochastic programming. Stochastic models cannot be solved with continuous distributions except for some trivial cases, thus the distributions of stochastic parameters must be discretized. Computational power often limit the number of possible outcomes. Kaut and Wallace [35] call the discretization scenario tree or event tree. The quality of the stochastic solution is directly tied to the quality of the scenario tree, i.e. how exact the scenario tree represents the uncertainties present. The article suggests to calculate the discretization error by finding the optimal solutions of the true and approximated problems. The error is then the difference between the value of the true objective function at the two solutions.
3.4 Stability of Scenario Generation Methods in Stochastic Programming

Kaut and Wallace [35] propose criteria which a scenario-generation method must satisfy. In-sample and Out-of-sample stability are the two such criteria and are presented in the following subsections.

3.4.1 In-Sample and Out-of-Sample Stability

Kaut and Wallace [35] state the requirement of in-sample stability as follows: if several scenario trees are generated with the same input data and the optimization problem is solved with these trees, the optimal objective values should be approximately the same.

Out-of-sample stability testing is described as solving the optimization problem with several scenario trees and then evaluate how the solutions compare when used in a reference tree. If solution performance is approximately equal, out-of-sample stability is present. The reference tree should ideally represent the real world quite closely and thus be large [35].

Having out-of-sample stability means that the performance of the solution is stable and does not depend on the scenario tree. However, without testing for in-sample stability, one can not know how strong the solution actually is compared to the optimal solution of the stochastic process. If in-sample stability is not present, then the solution value depends heavily on the scenario tree chosen, which is unfortunate [35].
3.4.2 Theory on Stability Testing

Kaut and Wallace [35] provide some mathematical theory on stability testing if scenario generation methods. $F(\cdot)$ represents a solution to a stochastic problem. The uncertain parameter is called $\xi$ and a large reference tree assumed to mimic real-life uncertainties quite closely, is called $\tilde{\xi}$. $K$ scenario trees $\xi_k$, of smaller size than $\tilde{\xi}$, are generated to represent the stochastic process and the optimization problem is solved with each of those trees, yielding optimal solutions $x^*_k, k = 1, .., K$. In-sample stability is then

$$F(x^*_k; \xi_l) \approx F(x^*_l; \xi_k)$$

$k, l \in 1...K$

Out-of-sample stability is described as

$$F(x^*_k; \tilde{\xi}) \approx F(x^*_l; \tilde{\xi})$$

$k, l \in 1...K$

Testing out-of-sample stability might be hard due to the nature of problem and the need to solve the large reference tree $\tilde{\xi}$ [35]. In cases where the distribution is unknown or solving the complete problem is impractical, a weaker out-of-sample stability test can be executed [38]. If the model is out-of-sample stable, then

$$F(x^*_k; \xi_l) \approx F(x^*_l; \xi_k)$$

$k, l \in 1...K$

A way to measure both in-sample and out-of-sample stability, is coefficient of variation, which is standard deviation divided by the arithmetic mean. It is often expressed as a percentage [8]. Evaluating stability of scenario generators is an used in solving stochastic problems. Kaut and Wallace [36] evaluated the in-sample and out-of-sample stability of a shape based scenario generator using copulas. Di Domenica et al. [14] investigate stability when they look at scenario generation in an information systems perspective.

3.5 The Sample Average Approximation Method

Sample Average Approximation is a solution method for stochastic optimization problems. Generally, the method utilizes statistical properties and solves many smaller problems to represent a larger problem. It was introduced by Kleywegt, Shapiro and Homem-De-Mello in 2001 [40]. The article proposes a Monte Carlo simulation-based approach to stochastic discrete optimization problems. A random sample of $N$ scenarios is generated and the expected value function is approximated using the corresponding sample average function. A solution is found for the sample average optimization
problem and the procedure is repeated a number of times until a stopping criteria is satisfied. The method allows for solution of stochastic problems where the feasible region is very large, for example when the probability distribution is continuous [40]. A theoretical outline of the SAA algorithm is provided in section 8.1.

3.5.1 Applications in Stochastic Problems

Sample Average Approximation is quite useful in solving a wide range of stochastic optimization problems. Schütz et al. [66] use the Sample Average Approximation method on a stochastic supply chain design problem. First stage decisions are strategic location decisions and the second stage consists of operational decisions. Kim, Pasupathy and Henderson [37] provide a guide to Sample Average Approximation and apply the method to a two-stage multi-dimensional news vendor problem. The article discusses when SAA is appropriate, listing two key principles: the approximating function must have some structure enabling application of an efficient deterministic optimization algorithm. Additionally, the limiting function subject to minimization shares that structure, so that the properties of the limiting function such as the location of local minima are similar to those of the approximating function. Royset and Senzhenmann [63] use SAA when optimizing an investment portfolio, combining it with the subgradient method, steepest descent method and Newton’s method. Verweij et al. [77] apply SAA together with branch-and-cut and decomposition to stochastic routing problems. Computational results indicate the method is successful in solving problems with up to $2^{1694}$ scenarios to within an estimated 1% of optimality. The number of optimality cuts required to solve the approximating problem to optimality does not significantly increase with the size of the sample. Chaisiri et al. [9] utilize SAA in solving a cloud resource provisioning problem where future consumer demand and providers’ resource prices are the stochastic parameters. They conclude that the SAA approach can be used to effectively achieve an estimated optimal solution, even when the number of scenarios is very large.

3.6 GRASP HEURISTICS

The Greedy Randomized Adaptive Search Procedure is a construction-based metaheuristic which usually has two phases. In the first phase, a feasible solution is created from scratch using some greedy algorithm combined with randomization. The constructed solution may then be modified using some local search, but this is not required [28].

A basic outline of the GRASP algorithm is provided in [28]. It is assumed that $V^*$ is the set of indices $j$ for variables which no value has been assigned and $D_j$ is the
domain for the variable $x_j$. $n$ is the total number of indices. Decisions made during construction are setting of a single variable.

```plaintext
while stopping criteria not met do
    $V^* := \{1, ..., n\}$;
    while $V^* \neq \emptyset$ do
        Define the set of possible assignments $L := \{(j, d) | j \in V^*, d \in D_j\}$;
        Find a reduced candidate list, $L_{RC} \subseteq L$, consisting of the best assignments according to a greedy criterion;
        Select randomly $(j, d) \in L_{RC}$;
        Set $x_j := d$ and $V^* := V^* \setminus \{j\}$;
    end
    Perform a local search starting from the constructed solution $x$;
    Update the incumbent solution $x^I$ if a new best solution is found;
end

Algorithm 1: GRASP pseudocode
```

At any construction step, there will be a set of possible assignments $L$. In a pure greedy algorithm, the best of these assignments is selected at each step, creating identical solutions every time. GRASP uses a restricted candidate list $L_{RC}$ limited for instance by number of assignments $[28]$.

### 3.6.1 Applications in Wind Energy Problems

GRASP can be adapted to a wide range of applications by tailoring the greedy criteria, the size of the reduced candidate list and how the local search is conducted. However, wind farm applications are scarce. Yin and Wang [82] combine GRASP with Variable Neighborhood Search (VNS) for placement of turbines in a wind farm. The wind farm is modelled as a grid where each grid point is a possible location for a wind turbine. First, the attributes of all grid points are calculated and a number of grid points are added to a restricted candidate list according to some greedy criteria. The decision variables of where to place a turbine are tied to grid points. After each placement, the grid point attributes are recalculated taking wake effects and the like into account. Then, a new candidate list is created, a new placement selected randomly and so on until a viable first solution is finished. The VNS attempts to improve the suggested solution. Computational results showed that the procedure is superior to genetic algorithms solving the same type of problems.
The objective of the jack-up vessel chartering strategy problem is to minimize chartering costs for the jack-up vessel and the expected downtime costs for turbines. The problem is based upon the existence of a jack-up vessel chartering market where prices are dependent upon factors such as seasonality. The problem addresses the scheduling of jack-up vessel chartering for offshore wind farms that do not possess their own jack-up vessel, and the option of buying one in the immediate future is not considered.

The jack-up vessel performs heavy-lift maintenance on offshore wind turbines. The components requiring a jack-up vessel are blades, gearbox, transformer, and generator. These components are too heavy for regular maintenance vessels. The vessel uses a number of extendable legs that elevates the hull above the sea surface for stabilization purposes and it makes the vessel suitable for heavy-lift maintenance operations.

For offshore wind farm operators, the issue of jack-up vessel chartering is well-known. The vessel pool is limited, on-site weather conditions are harsh and uncertain, and charter rates soar when conditions for maintenance are most favorable. Costs of chartering this vessel type make up a significant part of total O&M costs. For offshore wind to be a financially competitive option in the future, O&M costs must be reduced. Optimizing the chartering and usage of the jack-up vessel is an important step towards accomplishing this. The problem is, however, subject to several uncertain factors. The most notable ones are weather, failure rates, electricity prices and charter rates.

As mentioned in the background chapter, charter rates fluctuate based upon demand. For jack-up vessels, seasonality can be observed in historical data because of more favorable weather conditions during spring and summer. Furthermore, chartering comes with a mobilization cost, which is the costs related to sailing the vessel to the wind farm location.

Jack-up vessels are subject to weather restrictions both during the actual jack-up from the sea surface and when performing maintenance activities. This means the weather is of great importance when developing a chartering strategy. As the weather is unpredictable by nature, the operator is not necessarily able to utilize the vessel when it is chartered. Another weather aspect is seasonality, which affects the average number of
possible maintenance days. Weather is also important as it affects the potential power output of the wind turbines. In general, downtime costs increase at elevated wind speeds when potential power output is high.

Turbine components are subject to a failure rate which gives the probability that the component will fail at a certain time. Thus, there is a considerable uncertainty as to when a jack-up vessel is needed. Moreover, failure rates are difficult to estimate, as stated in the background chapter.

Electricity prices are another important parameter to consider. The electricity price and subsidies, together with the power output of the wind farm, decide the operator’s revenue. Electricity prices are continuously fluctuating and represent major uncertainties when developing a chartering strategy. No wind farm operator wants the turbines to be broken down when electricity prices are high.

Maintenance operations can be divided into two main categories; preventive maintenance and corrective maintenance. Preventive maintenance involves performing maintenance on components before they fail, prolonging the lifetime of the turbine. Corrective maintenance, on the other hand, is maintenance on already failed components. A maintenance strategy can focus on either preventive, corrective, or a combination of the two. For jack-up vessels, only corrective maintenance is considered. For preventive maintenance, other cheaper vessels are utilized.

To sum up, the jack-up vessel chartering strategy model supports the operator’s decision of when and for how long he/she is to charter a jack-up vessel. The strategy should account for uncertain factors such as failure rates of components, weather, electricity price and charter rate.
MATHEMATICAL MODEL

This chapter presents the basic stochastic model using a generalized set partitioning formulation. First, notes one how time is modelled is presented in section 5.1. All relevant assumptions are presented and their effects explained in section 5.2. This includes how the model handles all uncertainties in parameters. The basic mathematical formulation of the jack-up vessel chartering strategy problem including sets, indices, parameters and variables is presented in section 5.3.

5.1 NOTES ON TIME MODELLING

A finite time horizon is discretized in the model. How to handle end effects is a well-known challenge when modelling problems with a finite discrete time horizon [49]. The model presented handles this by connecting the last and the first time period. To illustrate, imagine that the ends of a straight timeline are connected, effectively creating a circular time line. As a result, failures occurring in the last time periods that are not repaired before the end of the planning period, are considered to be in a failed state at the start of planning. This approach avoids results that compensate for the fact that failures occurring in late time periods might be best left in a failed state for the remaining time periods. This gives the wind farm operator incentives to repair failures regardless of when they occur. This is considered a realistic approach for this study and was recommend by the supervisors.

5.2 MODEL ASSUMPTIONS

This section concerns problem uncertainty and assumptions relevant to the mathematical formulation proposed in section 5.3.
5.2 Model Assumptions

5.2.1 Repair Time

The model assumes the vessel requires a number of time periods to finish a maintenance operation. When the vessel commences maintenance, the vessel is considered occupied in a number of time periods after maintenance start. Realistically, the time needed to perform maintenance on large components would be longer in harsh weather conditions. For instance, it would be more difficult to fit a new turbine blade when winds are strong. In this study it is assumed that the number of time periods required to complete maintenance on a component is not affected by the weather. Time periods in which the vessel have to pause maintenance because of weather, however, can still occur. The turbine will start producing in the time period succeeding the finished maintenance operation.

It is assumed that spare components are always available and the operator knows what component type has failed immediately after failure and these factors do not affect repair time.

5.2.2 Weather and Failure Scenarios

A set of weather and failure scenarios is modelled and every scenario is subject to a probability. Weather conditions are assumed not to change within a time period or between turbines. Scenario inputs are given for the entire planning period, and the operator knows the scenario probability distribution. The weather component of the scenarios affects turbine generation, since wind speed is decisive when calculating turbine output. The jack-up vessel is subject to weather restrictions, both in terms of wave height and wind speed, due to health and safety standards. The input scenarios dictate whether the vessel is able to operate.

The scenarios also include a set of failures. The model assumes all turbines to be identical and that the failure rates of the different components are independent and known by the operator. It is assumed that each component can only fail once in each scenario. For tactical planning, which is from a few months to a few years, this is considered realistic.

The model does not incorporate costs for replaced components. These costs are assumed fixed, i.e. they are not dependent upon the choice of chartering strategy. One can argue that the choice of not replacing the component could be viable when the wind farm is close to decommissioning, but this is not considered in this model.
5.2 Model Assumptions

5.2.3 Charter Rates and Mobilization Cost

The model proposed in this study assumes deterministic charter rates for the planning period and a rate is given for every time period. An important assumption is that the vessel is always available as long as the operator is willing to pay, thus implying that if all vessels are chartered, a high price will convince other operators to sub-charter the vessel and postpone their own maintenance.

Mobilization costs are modelled deterministically and is constant throughout all time periods. It is assumed that the operator plans charter schedules sufficiently early so that a vessel can be chartered at any time during the planning period and is unaffected by mobilization times.

5.2.4 Minimum Length of Charter Period

It is assumed that if charter of a vessel is started, it must be chartered a minimum number of time periods in succession. There is, however, no maximum length of a charter period.

5.2.5 Electricity Prices

Electricity prices affect downtime costs for the wind farm and, by extension, the optimum chartering strategy. There are spot markets as NordPool giving hourly price fluctuations, however, in this model, the price is assumed constant throughout one time period. A price is generated for every time period in the planning period and, as charter rates, electricity prices are assumed deterministic and known by the operator.

5.2.6 Jack-Up, Jack-Down and Transit Time

In this model, one time period is assigned to transit and jacking operations. Transit times include travel from dock to the wind farm and between turbines. Supporting these assumptions, an EWEA report from January 2016 \[74\] states that the average distance from shore to operational wind farms was 43.3 km in 2015. Typical transit speed of a jack-up vessel is around 12 knots, or $\sim 22 \frac{\text{km}}{\text{h}}$ \[33\], \[31\], meaning transit time is about 2 hours on average. Typical elevation speed is $0.7 \frac{\text{m}}{\text{min}}$ \[71\] and with typical operating depths being between 24 and 75 meters, an estimated time needed for jacking is $\sim 70 \text{min}$. These simplifications are assumed not to affect the model significantly.
5.3 Mathematical Formulation

5.2.7 Unrepaired Components

It is assumed that the wind farm operator wants that no components are left unrepaired after the planning period. If components are left un repaired every year, the expected number of failures increases in later years. As the number of failed components accumulates, a large section of the wind farm will not be generating electricity. Therefore, costs are imposed to penalize unrepaired components. Penalty costs are divided into downtime and future charter costs.

5.3 Mathematical Formulation

This section describes the mathematical formulation, starting with all sets, indices, parameters and variables followed by the objective function and first and second stage constraints.

Definitions

Lower case letters represent variables and indices. Upper case letters represent sets, parameters and are used to distinguish parameters with the same name.

Sets

\[ P \] - Turbines  
\[ C \] - Components  
\[ T \] - Time periods  
\[ S \] - Scenarios  
\[ \mathcal{C}_s^B \] - Failed components on all turbines over the planning period in scenario \( s \)

Indices

\( p \) - Turbines  
\( c \) - Components  
\( t \) - Time periods  
\( \tau \) - Time periods  
\( s \) - Scenarios
5.3 Mathematical Formulation

Parameters

\[ P_s \] - Probability of scenario \( s \)
\[ T_{cts} \] - 1 if a vessel has to be chartered in time period \( t \) if maintenance is initiated on component \( c \) in time period \( t \) in scenario \( s \), 0 otherwise
\[ C_{pcts}^D \] - The total downtime costs at turbine \( p \) if the vessel jacks up at the failed component \( c \) in time period \( t \) given scenario \( s \)
\[ C_t^P \] - Jack-up vessel charter rate in time period \( t \)
\[ C^M \] - Mobilization costs for the jack-up vessel
\[ C^E_s \] - Total downtime costs associated with leaving a component unrepaired throughout the planning period in scenario \( s \)
\[ T^L \] - Minimum length of charter periods for the vessel
\[ C^C \] - Anticipated costs of one additional charter period beyond the planning period
\[ F \] - Factor indicating the maximum number of failed turbines not serviced during the planning period which can be handled by one additional charter period beyond the planning period

Decision Variables

\[ \delta_{pcts} \] - 1 if the vessel jacks up at turbine \( p \) to do maintenance on the failed component \( c \) in time period \( t \) given scenario \( s \), 0 otherwise
\[ x_{pcs} \] - 1 if the failed component \( c \) on turbine \( p \) is left unrepaired throughout the planning period in scenario \( s \), 0 otherwise
\[ v_t \] - 1 if a vessel charter period starts in time period \( t \), 0 otherwise
\[ y_t \] - 1 if a vessel is chartered and ready to jack up/down or perform maintenance operations in time period \( t \), 0 otherwise

Auxiliary Variables

\[ w \] - Integer representing the number of future charter periods required to service the expected number of unrepaired components
5.3 Mathematical Formulation

5.3.1 Objective Function

\[
\min z = \sum_{t \in T} C_P^l \cdot y_t + \sum_{t \in T} C_M \cdot v_t + \sum_{p \in P} \sum_{c \in C} \sum_{s \in S} P_s \cdot C_{pcts} \cdot \delta_{pcts} \\
+ \sum_{p \in P} \sum_{c \in C} \sum_{s \in S} P_s \cdot C_E \cdot x_{pcs} + P_s \cdot C \cdot w \\
\tag{5.1}
\]

The objective function aims to minimize jack-up vessel charter costs and expected downtime costs for a wind farm. Part a) gives the vessel charter costs, part b) is the corresponding mobilization costs while part c) represents the expected forfeit revenue caused by turbine downtime. Part d) represents the expected total downtime costs incurred by components being left unrepaired throughout the planning period. Moreover, if a number of failed components are left unrepaired, expected future charter costs will be incurred; this is represented by part e).

5.3.2 First Stage Constraints

First stage constraints concern the first stage decision of when and for how long to charter a vessel.

\[
y_t - v_t - y_{(t-1)} \leq 0 \quad t \in T \setminus \{1\} \quad (5.2)
\]

\[
y_1 - v_1 - y_{|T|} \leq 0 \quad (5.3)
\]

Constraints (5.2) and (5.3) ensure a vessel is available in time period \( t \) only if a charter period was started in time period \( t \) or if it was available in the preceding time period \( t-1 \), with constraint (5.3) handling end effects.

\[
T^L \cdot v_t - \sum_{i=t}^{t+T^L-1} y_i \leq 0 \quad t \in T \setminus \{|T|, |T| - 1, ..., |T| - (T^L - 2)| \} \quad (5.4)
\]

\[
T^L \cdot v_t - \sum_{i=t}^{|T|} y_i - \sum_{i=1}^{T^L - (|T| - t) - 1} y_i \leq 0 \quad t \in T \setminus \{1, 2, ..., |T| - (T^L - 1)| \} \quad (5.5)
\]

Constraints (5.4) ensure that if a vessel charter period starts in time period \( t \), the minimum charter time \( T^L \) must be satisfied in the time periods immediately following \( t \). Corresponding end effects are handled by constraints (5.5).
5.3 Mathematical Formulation

5.3.3 Second Stage Constraints

Second stage constraints concern the second stage decisions of which components to do maintenance on when a vessel is already chartered.

\[ \sum_{p \in P} \sum_{c \in C} \sum_{t \in T} T_{cts} \cdot \delta_{pcts} - y_t \leq 0 \quad \text{for } s \in S \quad \text{and } \tau \in T \quad (5.6) \]

Constraints (5.6) ensure that a vessel is chartered in the required time periods corresponding to selected maintenance operations. Furthermore, the constraints prevent more than one maintenance operation being executed at any time in a single scenario. Thus, no maintenance operation can be started before the previous one is finished.

\[ \sum_{t \in T} \delta_{pcts} + x_{pcs} = 1 \quad \text{for } (p, c) \in C^B \quad s \in S \quad (5.7) \]

\[ \sum_{s \in S} \sum_{(p,c) \in C^B_s} x_{pcs} - F \cdot w \leq 0 \quad (5.8) \]

Set partitioning constraints (5.7) force maintenance to be executed exactly once or not at all. The slack variable \( x_{pcs} \) is used to impose downtime costs of \( C^F_e \) in the objective function if a component is not repaired. The constraints are created for the set of failed components \( C^B \) only. Constraint (5.8) assures that if a number of failed components are left unrepaired, the variable \( w \) is increased accordingly. The increments of \( w \) happen stepwise due to the factor \( F \); for instance, if \( F = 3 \), then \( w = 1 \) for 1-3 unrepaired components and \( w = 2 \) for 4-6 unrepaired components, etc. As explained above, \( F \) describes the maximum number of failed turbines not serviced during the planning period which can be handled by one additional charter period beyond the planning period. \( w \) is used to add an anticipated future charter costs to the objective function (5.1).

5.3.4 Variable Restrictions

\[ y_t \in \{0, 1\} \quad t \in T \quad (5.9) \]
\[ v_t \in \{0, 1\} \quad t \in T \quad (5.10) \]
\[ \delta_{pcts} \in \{0, 1\} \quad p \in P \quad c \in C \quad t \in T \quad s \in S \quad (5.11) \]
\[ x_{pcs} \in \{0, 1\} \quad p \in P \quad c \in C \quad s \in S \quad (5.12) \]
\[ w \geq 0, \text{integer} \quad (5.13) \]

Constraints (5.10)-(5.13) provide variable restrictions.
5.3 MATHEMATICAL FORMULATION

5.3.5 Notes on Symmetry

Initially, the problem described might seem to include a lot of symmetry, and there is in the case where weather permits maintenance on several turbines where the same component type has failed. However, the model cannot be forced to service turbines of lower enumeration first, because this might cut the optimal solution. Figure 10 illustrates the problem arising if the weather does not permit repairing all failed components of the same type. Time periods a) and b) illustrates the incurred downtime if the failed component on turbine 1 or 2 is serviced, respectively. c) is the downtime periods corresponding to the failed component which is not repaired during the planning period. Regardless of which turbine is serviced, c) will be incurred. However, if one implements constraints saying that turbines with lower enumeration must be serviced first, then the total downtime will be a) + c), even if servicing turbine 2 is cheaper, ending at b) + c). There is, however, a special case where symmetry is present, namely if two components of the same type fail in the same time period in the same scenario and a) = b). This is considered very unlikely, thus implementing symmetry breaking constraints for this case is not considered to strengthen the formulation.

Figure 10: Explanation of how symmetry is handled in the model. The viable weather window is only sufficient to repair one of the components, with a) and b) showing the incurred downtime of turbine 1 and 2, respectively, and c) is the incurred downtime for a failed turbine which is not repaired during the planning period.
### 5.3 Mathematical Formulation

#### 5.3.6 Similarity With Set Partitioning Problems

As stated, the presented model is a generalized set partitioning problem, illustrated by Figure 11. This two-dimensional matrix represents the possible time periods in which maintenance can be initiated on the specific failures in a given scenario. The matrix element is 1 if maintenance can be initiated on failure \( p, c \) in time period \( t \), 0 otherwise. The number of failures in the scenario considered is five, each represented by a row. The number of time periods is 20, each represented by a column.

![Matrix Representation](image)

**Figure 11**: Illustration of the set partitioning component in the mathematical model

A solution covering all the failed components in this specific scenario \( s \) is to initiate maintenance on failures 1, 2, 3, 4 and 5 in time periods 2, 6, 16, 9 and 20, respectively. It is not desirable to force the model to repair all components as this can lead to infeasibility issues. Therefore, the variable \( x_{pcs} \) keeps track of components that are left unrepaiired throughout the planning period in each scenario.
This chapter presents an alternative way of formulating the jack-up vessel chartering strategy problem. Instead of indexing possible maintenance operations on \( p, c \) and \( t \), the concept of batches containing all maintenance information is introduced. The motivation for introducing this formulation is that it provides the possibility of bundling maintenance operations, i.e. including several maintenance operations in a single second stage decision. One can, however, choose to make one batch for each allowed maintenance operation, thus, the model will solve the exact same problem as the model presented in chapter 5. Selecting one batch is then equivalent to initiate maintenance on a given component in a given time period. The model assumptions are similar to those presented in chapter 5.

### 6.1 Mathematical Formulation

This section explains the alternative mathematical formulation, starting with sets, indices, parameters and variables followed by the objective function and constraints. Only those not earlier introduced in the basic model will be explained. Required sets, indices, parameters and variables used in the basic formulation are still valid.

**Sets**

- \( B_s \) - Batches in scenario \( s \)
- \( F_s \) - Failures in scenario \( s \). Contains information about turbine and component index for each failure
6.1 Mathematical Formulation

Indices

\( b \) - Batches
\( f \) - Failures

Parameters

\( T_{tbs} \) - 1 if a vessel has to be chartered in time period \( t \) when batch \( b \) is chosen in scenario \( s \), 0 otherwise
\( C^D_{bs} \) - Downtime costs for batch \( b \) in scenario \( s \)
\( A_{fbs} \) - 1 if maintenance on failure \( f \) is included in batch \( b \) in scenario \( s \), 0 otherwise

Decision Variables

\( \delta_{bs} \) - 1 if batch \( b \) in scenario \( s \) is selected, 0 otherwise
\( x_{fs} \) - 1 if failure \( f \) in scenario \( s \) is left unrepaired throughout the planning period, 0 otherwise

6.1.1 Objective Function

The objective function is changed slightly to accommodate the alternative decision variables.

\[
\min z = \sum_{t \in T} C^P_t \cdot y_t + \sum_{t \in T} C^M_t \cdot v_t + \sum_{s \in S} \sum_{b \in B_s} P_s \cdot C^D_{bs} \cdot \delta_{bs}^B \\
+ \sum_{s \in S} \sum_{f \in F_s} P_s \cdot C^E_f \cdot x_{fs} + P_s \cdot C^C \cdot w
\]  \hspace{1cm} (6.1)

As before, the objective function aims to minimize jack-up vessel charter costs and expected downtime costs for a wind farm. Part a) gives the vessel chartering costs, part b) is the corresponding mobilization costs while part c) represents the stochastic forfeit revenue caused by turbine downtime. Part d) represents the expected total downtime costs accrued if a failed turbine is not repaired during the planning horizon. It is expected that if a number of failed turbines are not repaired during the planning
6.1 Mathematical formulation

horizon, some expected future charter costs will be incurred; this is represented by part e).

6.1.2 First Stage Constraints

The first stage constraints (5.2)-(5.5) are not changed in the alternative formulation.

6.1.3 Second Stage Constraints

None of the second stage constraints from the basic formulation apply. They are replaced by constraints (6.2)-(6.4).

\[
\sum_{b \in B_s} T_{tsb} \cdot \delta_{bs} - y_t \leq 0 \quad t \in T \quad s \in S \quad (6.2)
\]

\[
\sum_{b \in B_s} A_{fsb} \cdot \delta_{bs} + x_{fs} = 1 \quad f \in F_s \quad s \in S \quad (6.3)
\]

\[
\sum_{s \in S} \sum_{f \in F_s} x_{fs} - F \cdot w \leq 0 \quad (6.4)
\]

Constraints (6.2) ensure that a vessel is chartered according to the required charter periods corresponding to the selected batches. Constraints (6.3) are the set partitioning constraints. Either a failed component is covered by one of the selected batches, or it is left unrepaired throughout the planning horizon. Constraints (6.4) handle future charter costs related to unrepaired components.

6.1.4 Variable Restrictions

The variable restrictions for \( \delta_{bs} \) and \( x_{fs} \) are given in (6.5)-(6.6).

\[
\delta_{bs} \in \{0,1\} \quad b \in B \quad s \in S \quad (6.5)
\]

\[
x_{fs} \in \{0,1\} \quad f \in F_s \quad s \in S \quad (6.6)
\]
INTRODUCING MULTIPLE VESSEL TYPES

This chapter presents an extension of the mathematical model presented in chapter 5 where different vessel types are available for charter. In section 7.1 the required alterations for this mathematical formulation of the jack-up vessel chartering strategy problem are presented.

The extended model allows the operator to hire different types of jack-up vessels to perform required maintenance. This provides flexibility, since cheaper vessels can be chartered in the more fair-weathered summer months and the more expensive and robust vessels can be reserved for winter use, for instance.

7.1 Mathematical Formulation

This section presents the mathematical formulation of this extension. Please note that the formulation resembles the basic formulation, bare vessel indices. Thus, to keep it brief, the entire model will be described mathematically, but in cases where the only difference is a $v$-index, no further explanation will be provided in text. New sets, indices, parameters and variables are presented before the objective function followed by first and second stage constraints. Sets, indices, parameters and variables used in the basic formulation are still valid. The model assumptions are similar to those presented in chapter 5.

Sets

$V$ - Vessels
7.1 MATHERNICAL FORMULATION

Indices

\( v \) - Vessels

Parameters

\( T_{cts}^{\tau v} \) - 1 if vessel \( v \) has to be chartered in time period \( \tau \) if maintenance is initiated on component \( c \) in time period \( t \) in scenario \( s \) using vessel \( v \), 0 otherwise

\( C_{pctsv}^D \) - The total downtime costs at turbine \( p \) if vessel \( v \) jacks up at the failed component \( c \) in time period \( t \) given scenario \( s \)

\( C_{tv}^P \) - Charter rate for vessel \( v \) in time period \( t \)

\( C_M^v \) - Mobilization costs for vessel \( v \)

Decision Variables

\( \delta_{pctsv} \) - 1 if vessel \( v \) jacks up at turbine \( p \) to do maintenance on the failed component \( c \) in time period \( t \) given scenario \( s \), 0 otherwise

\( v_{tv} \) - 1 if a charter period for vessel \( v \) starts in time period \( t \), 0 otherwise

\( y_{tv} \) - 1 if vessel \( v \) is chartered and ready to jack up/down or perform maintenance operations in time period \( t \), 0 otherwise

7.1.1 Objective Function

\[
\begin{align*}
\min \ z &= \sum_{t \in T} \sum_{v \in V} C_{tv}^P \cdot y_{tv} + \sum_{t \in T} \sum_{v \in V} C_M^v \cdot v_{tv} + \sum_{p \in P} \sum_{c \in C} \sum_{t \in T} \sum_{s \in S} \sum_{v \in V} P_s \cdot C_{pctsv}^D \cdot \delta_{pctsv} \\
&+ \sum_{p \in P} \sum_{c \in C} \sum_{s \in S} P_s \cdot C_E^s \cdot x_{pcs} + P_s \cdot C_C^v \cdot w \\
\end{align*}
\]  

(7.1)

The objective function aims to minimize jack-up vessel charter costs and expected downtime costs for a wind farm. Part a) gives the charter costs of the vessels, part b) is the corresponding mobilization costs for all vessels used while part c) represents the stochastic forfeit revenue caused by turbine downtime. Part d) represents the expected total downtime costs accrued if a failed turbine is not repaired during the planning
period. If a number of failed turbines are not repaired during the planning period, some expected future charter costs will be incurred; this is represented by part e).

### 7.1.2 First Stage Constraints

This section presents constraints directly tied to the first stage decision of when and how long to charter the vessel.

\[
\begin{align*}
y_{tv} - v_{tv} - y_{(t-1)v} &\leq 0 & t \in T \setminus \{1\} \ v \in V \quad (7.2) \\
y_{1v} - v_{1v} - y_{|T|v} &\leq 0 & v \in V \quad (7.3) \\
T^L \cdot v_{tv} - \sum_{t=1}^{T^L-1} y_{tv} &\leq 0 & t \in T \setminus \{|T| - 1, \ldots, |T| - (T^L - 2)\} \ v \in V \quad (7.4) \\
T^L \cdot v_{tv} - \sum_{t=1}^{|T|} y_{tv} - \sum_{t=1}^{T^L-(|T|-t)-1} y_{tv} &\leq 0 & t \in T \setminus \{1, 2, \ldots, |T| - (T^L - 1)\} \ v \in V \quad (7.5)
\end{align*}
\]

Constraints (7.2) - (7.5) are unchanged from the basic formulation apart from v indices.

Constraints (7.6) and (7.7) must be added. These constraints ensure that only one vessel is chartered at the time.

\[
\begin{align*}
\sum_{v \in V} v_{tv} &\leq 1 & t \in T \quad (7.6) \\
\sum_{v \in V} y_{tv} &\leq 1 & t \in T \quad (7.7)
\end{align*}
\]

### 7.1.3 Second Stage Constraints

This section presents constraints governing the second stage decisions of which components to do maintenance on when the vessel is already chartered. The only differences from the basic model are the v indices.

\[
\sum_{p \in P} \sum_{c \in C} \sum_{t \in T} T_{cts\tau v} \cdot \delta_{pc\tau sv} - y_{tv} \leq 0 & \quad s \in S \ t \in T \ v \in V \quad (7.8)
\]
7.1 Mathematical Formulation

\[ \sum_{t \in T} \sum_{v \in V} \delta_{pctsv}^U + x_{pcs} = 1 \quad (p, c) \in C^B_s, \ s \in S \quad (7.9) \]
\[ \sum_{s \in S} \sum_{(p, c) \in C^B_s} x_{pcs} - F \cdot w \leq 0 \quad (7.10) \]

7.1.4 Variable Restrictions

\[ y_{tv} \in \{0, 1\} \quad t \in T, \ v \in V \quad (7.11) \]
\[ v_{tv} \in \{0, 1\} \quad t \in T, \ v \in V \quad (7.12) \]
\[ \delta_{pctsv}^U \in \{0, 1\} \quad p \in P, \ c \in C, \ t \in T, \ s \in S, \ v \in V \quad (7.13) \]

Constraints (7.11)–(7.13) provide variable restrictions.
SOLUTION METHODS

The jack-up vessel chartering strategy problem considers uncertainty in weather and failure occurrences. These are parameters whose combined uncertainty is expected to require a high number of scenarios to represent properly, making the problem difficult to solve as shown by Kirkeby and Mikkelsen [39]. This chapter presents how the SAA method and heuristics can be utilized to solve the jack-up vessel chartering problem.

8.1 OUTLINE OF SAMPLE AVERAGE APPROXIMATION ALGORITHM

Schütz, Tomasgard and Ahmed [66] provide a thorough explanation of the SAA method for two-stage stochastic problems, however, the notation is altered to generalize the algorithm. The model encompasses the set of scenarios \( S \), \( c^T \) is first stage costs, \( y \) is the vector of first stage decision variables and \( Y \) is the set of feasible solutions. \( P_s \) is the probability of scenario \( s \) occurring and \( G(\bar{y}, \xi_s) \) is the solution to the second stage problem given the first stage solution \( \bar{y} \) and \( \xi_s \) as input. Realizations of the uncertainty vector \( \xi \) in scenario \( s \) are represented by \( \xi_s \). Note that the solution of the second stage problem includes second stage decision variables different from the first stage variables \( y \). This yields the two-stage stochastic problem.

\[
\min_{y \in Y} g(y) := c^T \cdot y + \sum_{s \in S} P_s \cdot G(\bar{y}, \xi_s) \quad (8.1)
\]

The first step is to convert Equation (8.1) into a sample average function. The sample size \( N \) denotes the number of realizations of the uncertain parameters considered. The result is shown in Equation (8.2)

\[
\min_{y \in Y} \hat{g}(y) := c^T \cdot y + \frac{1}{N} \sum_{n} G(\bar{y}, \xi_n) \quad (8.2)
\]
\( \xi_n \) is the \( n \)th realization of the random vector \( \xi \). The steps of the algorithm are as follows:

1. Generate \( M \) independent samples of \( N \) scenarios and solve the SAA problem in Equation (8.2). The optimal objective function value and the solution is denoted by \( v^N_m \) and \( \hat{y}^N_m, m = 1...M \) respectively.

2. Compute the average of all optimal objective function values from the SAA problems, \( \bar{v}^N_{M} \) and the variance of the sampling distribution of objective values, \( \sigma^2_{\bar{v}^N_{M}} \). The variance of the sampling distribution of objective values is equal to the sample variance divided by the number of samples and gives an estimate of how spread out the sample objective values are around the objective value of the true problem. As the sample size and/or number of samples grow, the properties approach the population properties and the variance of the sampling distribution of objective values decreases.

\[
\bar{v}^N_{M} = \frac{1}{M} \sum_{m=1}^{M} v^N_m \quad (8.3)
\]

\[
\sigma^2_{\bar{v}^N_{M}} = \frac{1}{M(M-1)} \sum_{m=1}^{M} (v^N_m - \bar{v}^N_{M})^2 \quad (8.4)
\]

The average objective function value \( \bar{v}^N_{M} \) provides an optimistic bound on the optimal objective function value for the original problem presented in Equation (8.1). For the extensive proof of this, readers are referred to Mak et al. [45] and Norkin et al. [50].

3. Pick a feasible first stage solution \( \bar{y} \in Y \) for the original problem, preferably one of the solutions \( \hat{y}^N_m \). The solution is then used to estimate the objective function value of the original problem using a reference sample of size \( N' \) as

\[
\bar{g}^{N'}(\bar{y}) := c^T \cdot \bar{y} + \frac{1}{N'} \sum_{n=1}^{N'} G(\bar{y}, \xi_n) \quad (8.5)
\]

4. The estimator \( \bar{g}^{N'}(\bar{y}) \) serves as a pessimistic bound on the optimal function value \( [45], [50] \). The reference sample of size \( N' \) is generated independently of the other \( M \) samples and since the first stage solution is fixed, \( N' \) can be greater than \( N \). This step requires solving \( N' \) independent second stage problems. The variance of the sample distribution of the objective values for \( \bar{g}^{N'}(\bar{y}) \) can be estimated:

\[
\sigma^2_{\bar{g}^{N'}(\bar{y})} = \frac{1}{N'(N'-1)} \sum_{n=1}^{N'} \left( c^T \cdot \bar{y} + G(\bar{y}, \xi_n) - \bar{g}^{N'}(\bar{y}) \right)^2 \quad (8.6)
\]
8.2 Batch Construction Heuristics

5. Compute the estimators for the optimality gap and its variance. Using the estimators calculated in steps 2 and 3, one gets a gap estimator and an estimation of the gap variance of the sampling distribution of the objective values

\[
\begin{align*}
gap_{N,M,N'}(\bar{y}) &= \tilde{g}_{N'}(\bar{y}) - \bar{v}_{N,M} \\
\sigma^2_{\text{gap}} &= \sigma^2_{N'}(\bar{y}) + \sigma^2_{\bar{v}_{N,M}}
\end{align*}
\] (8.7) (8.8)

The confidence interval for the optimality gap is then calculated as

\[
\tilde{g}_{N'}(\bar{y}) - \bar{v}_{N,M} \pm z_\alpha \left( \sigma^2_{N'}(\bar{y}) + \sigma^2_{\bar{v}_{N,M}} \right)^{\frac{1}{2}}
\] (8.9)

with \( z_\alpha := \Phi^{-1}(1 - \alpha) \), where \( \Phi(z) \) is the cumulative distribution function of the standard normal distribution.

8.1.1 Notes on the Number of Samples \( M \)

Determining \( M \) is not straightforward. Suppose that \( M \) samples of size \( N \) have been solved so far. If the distribution of the SAA problem \( \hat{g}(\bar{y}) \) is continuous, the probability that the \((M + 1)\)th SAA sample of size \( N \) will produce a better solution than all preceding solutions, is equal to \( \frac{1}{(M+1)} \). In the case of discrete distributions, the probability is less than or equal to \( \frac{1}{(M+1)} \). Thus, as \( \frac{1}{(M+1)} \) becomes sufficiently small, the additional SAA sample will provide little value and the procedure should be stopped or the sample size \( N \) increased [40].

8.2 Batch Construction Heuristics

This section presents the Greedy Maintenance Bundling Heuristic (GMBH), a greedy construction heuristic for the second stage problem, as well as the Greedy Randomized Adaptive Maintenance Bundling Heuristic (GRAMBH), which introduces adaptive randomness to the GMBH. The idea behind the heuristics is to extend the decision variable \( \delta_{bs} \), used in the Alternative Formulation presented in \textit{chapter 6}, to cover a sequence, or batch, of maintenance operations. Batches include information about which components are repaired if the batch is selected, the corresponding downtime costs and for which time periods the batch requires a vessel to be chartered. Below, outlines of algorithms proposed for generating batches are presented along with some required parameters. Parameters not listed are defined in the Alternative Formulation in \textit{chapter 6}. 

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8.2 Batch Construction Heuristics

**Heuristic Parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>Number of maintenance operations in a batch</td>
</tr>
<tr>
<td>$T^B$</td>
<td>Max number of time periods in batch</td>
</tr>
<tr>
<td>$T^M$</td>
<td>Max allowed waiting time between maintenance operations</td>
</tr>
<tr>
<td>$N^C$</td>
<td>Number of candidate components</td>
</tr>
<tr>
<td>$N^B$</td>
<td>Number of generated batches for each time period where at least one maintenance operation can be initiated</td>
</tr>
</tbody>
</table>

Batches are created during preprocessing, i.e. before optimization is initiated, and are not altered at a later stage. Two different approaches for defining the batch size are considered. One alternative is to specify the number of maintenance operations in each batch. The other alternative is to specify the length of a batch in terms of time periods. A detailed outline of the algorithm for the first alternative is presented below and a simplified flow chart is depicted in Figure 12.

8.2.1 GMBH 1 - Specifying the Number of Repairs in a Batch

1. Specify the batch size, $N$, and the max allowed waiting time between maintenance operations, $T^M$. Calculate $B_s$ and $F_s$ based upon the generated scenario data. The number of batches in a scenario is equal to the number of time periods in which at least one maintenance operation can be initiated. Failures are sorted by turbine and component index using lowest to highest, with turbine index having priority over component index. Further, initialize $C^D_{bs}$ and $A_{fbs}$ to 0, and set the time period in which batch $b$ in scenario $s$ starts to 1 in $T_{tbs}$. Proceed to step 2.

2. Starting at batch one in scenario one, set $t$ to the time period already set to 1 in $T_{tbs}$. Select the failure, $f$, with the lowest downtime costs at time period $t$ and add the costs to $C^D_{bs}$. Set the corresponding maintenance time periods to 1 in $T_{tbs}$ and set $A_{fbs}$ to 1. Go to step 3.

3. Starting with the time period after the preceding maintenance operation is completed, find the first time period, $t$, where at least one maintenance operation can be initiated, excluding all components already in the batch. If the waiting time, i.e. time periods in between maintenance operations, exceeds $T^M$, go to step 5. If not, set the waiting time periods to 1 in $T_{tbs}$ and proceed to step 4.

4. As before, find the failure, $f$, in time period $t$ with the lowest downtime costs and add the costs to $C^D_{bs}$. Set maintenance time periods to 1 in $T_{tbs}$ and set $A_{fbs}$ to 1. If the number of maintenance operations in the batch is $N$, go to 5. If not, repeat step 3 and 4.
5. The batch is complete and consists of accumulated downtime costs, a number of time periods in which a vessel must be chartered if the batch is selected and which components are covered by the batch. If \( b < |B_s| \), increment \( b \) and repeat step 2-5. If \( b = |B_s| \), set \( b \) to one, increment \( s \) and repeat step 2-5. Steps 2-5 are repeated until all batches in all scenarios are evaluated.

8.2.2 GMBH 2 - Specifying the Maximum Number of Time Periods in a Batch

As stated, an alternative to specifying the number of repairs in a batch \( N \), is to specify the length of a batch, \( T^B \). Following is a stepwise description of this approach.

1. Specify the length of a batch, \( T^B \), and the max allowed waiting time between maintenance operations, \( T^M \). Calculate \( B_s \) and \( F_s \) based upon the generated scenario data. Further, initialize \( C_{bs}^D \) and \( A_{fbs} \) to 0, and set the time period in which batch \( b \) in scenario \( s \) starts to 1 in \( T_{tbs} \). Proceed to step 2.

2. Starting at batch one in scenario one, set \( t \) to the time period already set to 1 in \( T_{tbs} \). Select the failure, \( f \), with the lowest downtime costs at time period \( t \) and add the costs to \( C_{bs}^D \). Set the corresponding maintenance time periods to 1 in \( T_{tbs} \) and set \( A_{fbs} \) to 1. Go to step 3.

3. Starting with the time period after the preceding maintenance operation is completed, find the first time period, \( t \), where at least one maintenance operation can be initiated, excluding all components already in the batch. If the waiting time exceeds \( T^M \), go to step 5. If not, set the waiting time periods to 1 in \( T_{tbs} \) and proceed to step 4.

4. As before, find the failure, \( f \), in time period \( t \) with the lowest downtime cost. If \( \sum_{t \in T} T_{tbs} \) plus maintenance time is lower than or equal to \( T^B \), add the costs to \( C_{bs}^D \) and set maintenance time periods to 1 in \( T_{tbs} \). Set \( A_{fbs} \) to 1. If \( \sum_{t \in T} T_{tbs} \geq T^B \), go to 5. If not, repeat step 3 and 4.

5. The batch is complete and consists of accumulated downtime costs, a number of time periods in which a vessel must be chartered if the batch is selected and which components are covered by the batch. If \( b < |B_s| \), increment \( b \) and repeat step 2-5. If \( b = |B_s| \), set \( b \) to one, increment \( s \) and repeat step 2-5. Steps 2-5 are repeated until all batches in all scenarios are evaluated.
8.2 Batch construction heuristics

Figure 12: GMBH 1 algorithm flowchart
8.2 Batch construction heuristics

8.2.3 GRAMBH - Greedy Randomized Adaptive Maintenance Bundling Heuristic

This section briefly explains how GMBH 1 and 2 can be extended to include a random component. The idea is that, in step 2 and 4 in the outlined algorithms, a maintenance operation is selected randomly from a list of candidates instead of always choosing the one with the lowest corresponding downtime cost. The candidate components are chosen based upon downtime cost, as to ensure a greedy approach. Furthermore, several batches are generated per allowed time period instead of just one. This reduces the risk of only generating batches which are good by themselves, but do not combine well.

1. Specify $N$ or $T^B$, $N^C$, $N^B$ and $T^M$. Calculate $B_s$ and $F_s$ based upon the generated scenario data. $|B_s|$ is equal to the number of time periods in which at least one maintenance operation can be initiated times $N^B$. Further, initialize $C_{bs}^D$ and $A_{fbs}$ to 0, and set the time period in which batch $b$ in scenario $s$ starts to 1 in $T_{lbs}$. Proceed to step 2.

2. Starting at batch one in scenario one, set $t$ to the time period already set to 1 in $T_{lbs}$. Select the $N^C$ failures with the lowest downtime costs at time period $t$ and add them to a candidate list. Randomly choose one of the candidates and add the corresponding costs to $C_{bs}^D$. Set the corresponding maintenance time periods to 1 in $T_{lbs}$ and set $A_{fbs}$ to 1. Go to step 3.

3. Starting with the time period after the preceding maintenance operation is completed, find the first time period, $t$, where at least one maintenance operation can be initiated, excluding all components already covered in the batch. If the waiting time exceeds $T^M$, go to step 5. If not, set the waiting time periods to 1 in $T_{lbs}$ and proceed to step 4.

4. As before, find the $N^C$ failures in time period $t$ with the lowest downtime costs and add them to a new candidate list. Randomly select one of the candidates in the list and add the corresponding costs to $C_{bs}^D$. Set maintenance time periods to 1 in $T_{lbs}$ and set $A_{fbs}$ to 1. If the number of maintenance operations in the batch is $N$ or $\sum_{t \in T} T_{lbs} \geq T^B$, go to 5. If not, repeat step 3 and 4.

5. The batch is complete and consists of accumulated downtime costs, a number of time periods in which a vessel must be chartered if the batch is selected and which components are covered by the batch. If $b < |B_s|$, increment $b$ and repeat step 2-5. If $b = |B_s|$, set $b$ to one, increment $s$ and repeat step 2-5. Steps 2-5 are repeated until all batches in all scenarios are evaluated.

Since batches are created for all time periods allowing to initiate maintenance in both the GMBH and the GRAMBH, it is expected that a high number of the batches will be equal in terms covered components and similar in terms of downtime costs and how many time periods are required. Thus, when the algorithms are complete, dominance
8.2 Batch Construction Heuristics

is used to reduce the number of batches in each scenario. A batch is considered dominant if it covers the same components, has a lower downtime cost, a lower number of required time periods where a vessel is chartered and the same last time period in $T_{tbs}$ as another batch. Hereinafter, a single time period where a vessel is chartered and ready to do maintenance is called a vessel period.

8.2.4 Batch Illustration

An example is presented in [Figure 13] to illustrate generalized set partitioning in the Alternative Formulation when the parameter $A_{tbs}$ is generated using GMBH or GRAMBH. This two-dimensional matrix represents the parameter for a given scenario, where the number of failures, $|F_s|$, is nine and the number of batches, $|B_s|$ is 20. Each column represents one batch and each row one failure. The number of repaired components in each batch is set to three. A solution covering all the failed components in this specific scenario $s$ is $\delta^B_{1s} = \delta^B_{8s} = \delta^B_{9s} = 1$. If batches 1-10 are removed, no combinations of batches 11-20 will cover all components. A solution covering eight failed components is $\delta^B_{12s} = \delta^B_{20s} = x_{7s} = x_{10s} = 1$. This solution leaves components 7 and 9 un repaired.

<table>
<thead>
<tr>
<th>Batch</th>
<th>Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0</td>
<td>0 0 0 0 0 0 0 1 1 1 1 0 0 0 0 0 0 0 0 0</td>
</tr>
<tr>
<td>0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0</td>
<td>1 1 1 1 1 1 1 0 0 0 0 0 1 1 1 1 1 1 1 0</td>
</tr>
<tr>
<td>1 1 1 0 0 0 0 0 0 0 0 0 1 1 0 0 1 1 1 1 0 1</td>
<td>0 0 1 0 0 0 0 1 0 0 0 0 0 0 1 0 0 0 1 0</td>
</tr>
<tr>
<td>0 0 0 1 1 0 0 0 1 1 1 1 0 0 0 0 1 0 0 0 0 0</td>
<td>0 0 0 1 1 0 0 0 1 1 1 1 0 0 0 0 0 1 1 0</td>
</tr>
<tr>
<td>0 0 0 0 1 1 0 0 0 1 1 1 1 0 0 0 0 0 0 0 0</td>
<td>0 0 0 0 1 1 0 0 0 1 1 1 1 0 0 0 0 0 0 0</td>
</tr>
<tr>
<td>0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 1 1 0 0 0 0</td>
<td>0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0</td>
</tr>
</tbody>
</table>

Figure 13: Example of $A_{fbs}$ parameter for given scenario with nine failures and 20 batches
IMPLEMENTATION

This chapter gives a thorough explanation of the implementation of the mathematical models. Section 9.1, section 9.2 and section 9.3 describe generation of weather and failure data, potential wind turbine generation and charter and electricity prices. Further, section 9.4 explains how downtime costs are calculated.

All mathematical formulations were implemented in FICO Xpress Optimization Suite and all scenario generation functions were implemented in MATLAB version R2015a. In the models, a scenario consists of two components; weather and failure data. Weather data encompasses both wave height and wind speed. Failure data provides information about which components fail and when. All input data generation scripts are intertwined, prompting the user to specify the length of the planning period, time resolution, the number of vessels, turbines, components, scenarios and expected number of failures desired. The output is the parameters presented in the mathematical models.

9.1 WEATHER AND FAILURE DATA

Weather data for a wind farm site is used as input, but must be altered to fit the mathematical model. A MATLAB script is used to adjust data to the desired time resolution using extreme values. For instance, if the model time resolution is four hours, the MATLAB program finds the highest wind speed and wave height value for every four hours and this is used as the input parameter. The weather parameters are utilized to find time windows for maintenance and calculate downtime costs as it affects potential production.

The user provides the size of the wind farm, the number of components, scenarios and time periods. The number of time periods is then used to calculate the probability of a component failing in any given time period in a scenario. A random number generator assigns a value to every all components on all turbines in every time period in every scenario from a uniform distribution. If the value assigned is lower than
its corresponding failure probability, the component is set to fail. As per assumptions, each component can only fail once in any given scenario for the entire planning period.

9.2 POTENTIAL WIND TURBINE GENERATION

Wind turbine generation is an important input parameter as it is combined with electricity prices to calculate downtime costs using available turbine power curves. As turbine electricity generation is dependent upon wind, the calculations are based upon the generated wind data. Fitting polynomials to a power curve is done using an online polynomial regression software \[50\]. The polynomials are then used directly in MATLAB to calculate wind turbine generation given wind data as input. For visual inspection of typical power and power coefficient curve, the reader is referred to Figure 6 in section 2.4. Note that polynomials are only fitted to the part of the curves where polynomial behavior can be observed. Otherwise, constant values are used. Furthermore, it should be noted that the power coefficient accounts for both the Betz’ factor and the efficiency.

The incurred future downtime costs for failed turbines not serviced are calculated as a multiple of the downtime costs for leaving the turbines unrepaird for the entire planning period. This is also calculated in this script.

9.3 CHARTER RATES AND ELECTRICITY PRICES

MATLAB scripts are written to generate charter rates and electricity prices for every time period and, if desired, every vessel type. Historical figures for both charter and electricity are used as input, they are deterministic and known by the operator, as per model assumptions. The user can, through input, decide whether subsidies on sold electricity should be considered or not. In the case of subsidized electricity prices, the prices are in reality deterministic, as the price for power delivered to the grid, is guaranteed.

9.4 DOWNTIME COST CALCULATIONS

This section presents implemented downtime cost calculation methods for the different models. First, a general description of downtime cost calculations is provided. Subsection 9.4.1, subsection 9.4.2 and subsection 9.4.3 describe three different approaches for calculating downtime costs.

Downtime costs are calculated as potential turbine electricity generation in downtime periods times the electricity price plus any subsidies. The downtime periods incurred
if a maintenance operation commences in a given time period is the time periods between failure and finish of maintenance operations. This includes the time period needed for jack-up and time periods in which maintenance is paused because of the weather. [Figure 14] and [Figure 15] illustrate calculations when unaffected and affected by end effects, respectively.

The downtime cost matrix serves two purposes, namely determining in which time periods maintenance is allowed to start and how much downtime costs are incurred if maintenance commences in a particular time period. When calculating the downtime matrix, the parameters describing when a vessel is required, $T_{cts}$, $T_{ctv}$ or $T_{tbs}$, are calculated depending on desired model. Three different approaches, hereinafter referred to as models, for generating the downtime cost matrix and vessel requirement matrix
have been evaluated. The three models differ in number of vessels and maintenance flexibility and are presented below.

When generating the downtime cost matrix, domination criteria are used. A maintenance operation is considered to be dominant over another if it repairs the same component, jacks up in a later time period and jacks down before or in the same time period.

9.4.1 Basic Model - Maintenance in Consecutive Time Periods

For maintenance to be commenced in this model, a time window where one can jack-up, complete maintenance, and jack down in consecutive time periods is required. These requirements are quite constraining in terms of when a maintenance operation can be initiated, especially if several time periods are required to complete maintenance. Furthermore, the number of time periods in which maintenance can be initiated will depend heavily upon the weather capability of the vessel considered. This is hereinafter referred to as the Basic Model.

9.4.2 Vessel Model - Multiple Vessel Types

The multiple vessel types model presented in chapter 7 requires downtime cost data for multiple vessels. The model is similar to the one used for the Basic Model in that no waiting time during maintenance is allowed. To accommodate multiple vessels types, one more dimension with length equal to the number of vessels types is added to the downtime matrix. The number of time windows in which maintenance can be initiated will depend upon each vessel’s weather capabilities. A more weather robust vessel will always be able to initiate maintenance as long as the less robust vessel is able to. On the contrary, if one vessel is more robust in terms of wind capabilities, while the other is more robust in terms of wave capabilities, the two vessels will be able to initiate maintenance in different time periods. This model is hereinafter referred to as the Vessel Model.

9.4.3 Extended Model - Opportunity to Wait

This model considers only one vessel, but allows for more flexibility than the Basic Model. It allows pausing maintenance operations while waiting for fairer weather. For instance, jack up in time period 1, wait for the wind to calm in time period 2, perform maintenance in time periods 3 – 5, wait for smaller waves in time period 7 and then jack down in time period 8. Flexibility can be constrained by specifying a maximum number of time periods allowed for the complete maintenance operation.
This model will see more time periods in which maintenance is allowed to start and can thus be considered a relaxation of the second stage decisions of the Basic Model. As a result, the model should produce a lower objective value. This is hereinafter referred to as the Extended Model.
This chapter presents a computational study conducted using the presented models. Section 10.1 presents data input. Stability testing and SAA calculations are presented in section 10.2. Sections 10.4 and 10.5 present testing of heuristics and economic analyses. As mentioned in section 9.4, the Basic Model is characterized by forcing maintenance operations to be conducted in consecutive time periods. The Vessel Model considers multiple vessel types, while the Extended Model allows for pausing of maintenance to wait for more favorable weather. Characteristics are summarized in Table 2.

<table>
<thead>
<tr>
<th>Model</th>
<th>Number of Vessels</th>
<th>Maintenance Flexibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic</td>
<td>1</td>
<td>No</td>
</tr>
<tr>
<td>Vessel</td>
<td>2</td>
<td>No</td>
</tr>
<tr>
<td>Extended</td>
<td>1</td>
<td>Yes</td>
</tr>
</tbody>
</table>

10.1 Data Input

This section provides explanation and justification of the chosen input parameters introduced in chapters 5 and 7. Most of the parameters were based upon external sources such as technical specifications or journal articles.

10.1.1 Scenario Probabilities

In all test cases, all scenarios were considered equally likely, thus, the probability of a scenario occurring $P_s = \frac{1}{|S|}$, where $|S|$ was the total number of scenarios considered.
10.1 DATA INPUT

10.1.2 Planning Period Length and Time Resolution

The planning period length for all testing was one year. This was considered reasonable for a tactical-level decision such as jack-up vessel chartering. Time resolution for all testing was 24 hours per time period, thus, the planning period consisted of 365 time periods.

10.1.3 Minimum Length of Charter Period

The minimum length of a charter period, imposed by constraints (5.4), (5.5), (7.4) and (7.5), was set to be two weeks. Supporting this choice is that the minimum length of a sub-charter period is considered to be 15 days by Dalgic [12].

10.1.4 Maintenance Time

The number of time periods required to complete maintenance on each component type was gathered from Dalgic [12]. Table 3 presents maintenance times for each component which were considered fixed during testing. It should be noted that these values are subject to some debate; for instance, values found in Dinwoodie et al. [18] tend to be lower. Hereinafter, the components will be referred to by their component number.

When generating downtime cost data for the Extended Model, the maximum time allowed for completion of maintenance operations was twice as long as the maintenance time of the corresponding components. Thus, on maximum, the vessel was allowed to wait for as many periods as it was actually conducting maintenance. Values are listed in the last column in Table 3.

Table 3: Number of time periods required to complete maintenance on each component type when length of one time period is 24 hours. Last column lists maximum allowed time periods for completing maintenance when using the Extended Model

<table>
<thead>
<tr>
<th>Component #</th>
<th>Component Type</th>
<th>Maintenance Time</th>
<th>Time Periods Allowed for Maintenance (Extended)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Blade</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>Generator</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>Gearbox</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>Transformer</td>
<td>6</td>
<td>12</td>
</tr>
</tbody>
</table>
10.1 DATA INPUT

10.1.5 Mobilization Cost

As explained in subsection 5.2.3, mobilization cost input in the models is a single value. Average value of mobilization costs given in Dalgic [12], is 5 MNOK, which was used as input. As the currency in the paper is GBP, an exchange rate of 13 NOK to 1 GBP was gathered from XE Currency [81] on 10.02.2016. These mobilization costs were used for both the primary and secondary vessel when the Vessel Model was tested.

10.1.6 Electricity Prices, Charter Rates, and Vessel Specifications

Subsidies were considered in all testing. A fixed electricity price of £150 /MWh (1,950 NOK/MWh) for the entire planning period was used. The price was gathered from The Department of Energy and Climate Change in the UK and their list of strike prices for offshore energy [30].

Charter rates were gathered from Dalgic [11] using Figure 16. These include fuel costs but not crew costs. The vessel with CAPEX (Capital Expenditure) of about £64 million was selected for single-vessel cases. For two-vessel cases, the one with CAPEX £102 million was chosen as secondary. Some of the cost factors described in section 2.2 were neglected; owner’s target utilization was not considered at all and the effects of varying demand and seasonal variations were simplified. Furthermore, only one type of charter was evaluated, namely spot charter. This came as a result of the applied planning period of one year. For such a short planning period, one year charter was regarded as infeasible.

It was assumed that one year consisted of four winter months and eight summer months, meaning there were only two different prices per vessel depending on the season. November-February was considered winter, the rest of the year was summer. This gave summer and winter charter rates for the primary vessel of 1.34 and 0.88 MNOK per time period, respectively. For the secondary vessel, the corresponding values were 2.23 and 1.40 MNOK using the 13 NOK to 1 GBP [81] exchange rate.
Vessel weather capabilities were gathered from Maples et al. [46]. These are presented in Table 4 together with seasonal charter rates. Useful data concerning wind restrictions during jacking and wave restrictions during maintenance was not found, but these vessel characteristics were not considered to be of great importance as they would be binding only in really extreme weather situations.
### Future Charter and Downtime Costs

Future charter costs were calculated as the 5 MNOK mobilization costs plus the costs of 14 days, the minimum length of a charter. To represent the uncertainty in future charter costs, the worst case scenario of 2.23 MNOK was used. An increment was added to account for the future charter to be a bit longer than 14 days, thus, future charter costs were set to 40 MNOK. This was considered reasonable, as the components must be repaired at some point in the future, incurring charter costs. The future charter factor was set to 3 since it was expected that, on average, one future charter of minimum length would be able to repair up to three failed components. Total downtime costs incurred for failed turbines not repaired during the planning period, was calculated as one and a half year’s worth of unsold electricity. This was to account for the extra downtime that runs into the next planning period.

Preliminary testing showed that with the current mathematical formulation, when setting $F = 3$ and $C^C = 40$ MNOK, optimal solutions for a wind farm with ten expected failures a year suggested leaving a number of components unrepaired. On average, this number was 5.6 and 2.2 for the Basic and Extended Models, respectively. These results did not agree with model assumptions, thus, the parameter $P_s$ was omitted from part e of the objective functions (5.1), (6.1) and (7.1). As a result, the accumulated number of unrepaired components in all scenarios, not the expected number, was penalized.

### Weather

Nine years of historical data with a time resolution of one hour was available. The weather data was gathered from the FINO 1 research platform in the North Sea between 2004 and 2012. In this study, each of the nine years represents a weather scenario, thus, if the user specified three scenarios, the scenario generator would pick three different years at random from the weather dataset. A similar approach for generating weather scenarios was used by Raknes and Ødeskaug in their master’s thesis [60]. To ensure that all scenarios started on January 1st and ended on December 31st, data for all occurrences of February 29th was removed. This was not considered to weaken the scenarios notably as the portion removed was extremely small compared to the size of the complete dataset. Dinwoodie et al. [18] aim to create reference cases
for offshore wind O&M simulation models and consider this set of weather data to be representative of central North Sea conditions.

The nine possible weather scenarios were closely evaluated to see whether any of them distinguished themselves as particularly harsh or gentle. If one of the weather scenarios were, and one of the scenario trees had abnormally many occurrences of that weather scenario, then this could affect the objective value. A harsher (gentler) year would have fewer (more) time windows for maintenance, thus affecting the number of unrepaired components. By studying the distributions of wave heights and wind speeds, they were found to be approximately equal and no year was particularly harsh or gentle.

10.1.9 Turbine Electricity Generation

Turbine electricity generation was implemented based upon the power curve and power coefficient curve of the Enercon E-126 EP4 turbine [19]. The curve provided by the manufacturer can be seen in Figure 17

![Calculated power curve for Enercon E-126 EP4 wind turbine](image-url)

Figure 17: Power curve for Enercons E-126 EP4 wind turbine [19]
10.1.10 Failure Data

For this study, it was feasible to assume that during the considered planning period of one year, the component failure rates were constant. This was because of the small intra-year changes in a 20-25 year Weibull distribution assumed to represent component failures. The concrete failure rates, i.e. the probability that a component fails in any given time period, were selected so that the expected number of failures in one year aligned with each component’s specific failure rate presented in figure 3 in Dalgic et al. \[12\]. Failure rates are presented in Table 5.

<table>
<thead>
<tr>
<th>Component</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.004/(Number of time periods)</td>
</tr>
<tr>
<td>2</td>
<td>0.044/(Number of time periods)</td>
</tr>
<tr>
<td>3</td>
<td>0.040/(Number of time periods)</td>
</tr>
<tr>
<td>4</td>
<td>0.012/(Number of time periods)</td>
</tr>
</tbody>
</table>

With the given probabilities, the expected number of failures for a 100-turbine wind farm across, say, 100 scenarios are 40, 440, 400 and 120 for components one, two, three and four, respectively. This gives a total of 1,000 expected failures.

10.1.11 Optimality Gap Tolerance

In Xpress, a tolerance of 1.0% optimality gap was implemented to prevent excessive solution times. Furthermore, first stage solutions are not expected to change significantly when closing the last percentage of the optimality gap.

10.2 Stability Testing and SAA

This section presents results and accompanying discussions from in-sample and out-of-sample stability testing, as well as SAA calculations for the different models.

Testing performed by Kirkeby and Mikkelsen \[39\] showed that solving the stochastic jack-up vessel chartering strategy problem for a large number of scenarios was time-consuming with the computational resources at hand. As a result, the SAA algorithm was utilized throughout this study. As explained in section 8.1 when using SAA, an appropriate sample size \( N \) is needed as well as a number of samples, or scenario trees, \( M \). \( M \) is chosen such that the probability of finding a better solution by adding another scenario tree, \( \frac{1}{M+1} \), is sufficiently small and computational times are tolerable. For this study, \( M \) was set to ten. The sample size \( N \) was then calibrated ensuring
that the $M$ generated scenario trees represented the considered uncertainty in a satisfying way. This was done by performing stability testing. In the following subsections, results from both in-sample and out-of-sample stability testing are presented and discussed. The chosen measurement of stability was the standard deviation divided by the arithmetic mean, or coefficient of variance.

Testing was conducted on scenario trees of different sizes. All scenario trees considered a 100-turbine wind farm akin to the British Thanet Wind Farm with 100 turbines \cite{44}. The number of scenarios in each scenario tree varied from 25 to 100. The different models were tested on the same scenario trees to better compare results.

### 10.2.1 In-Sample Stability

As explained in subsection 3.4.1, in-sample stability is a measure of variance in optimal objective value for different scenario trees. Figure 18 presents results from the in-sample stability testing for different number of scenarios in each scenario tree.

![In-Sample Stability](image)

**Figure 18:** Plot of in-sample stability for different scenario tree sizes and model evaluations. Coefficient of variation is the selected measurement for in-sample stability.

As the coefficients of variation presented in Figure 18 were higher than expected, especially for the Vessel Model where it increased with the number of scenarios, further investigation of results was conducted. The scenario trees of 100 scenarios proved the most promising for the Basic and Extended Models, and computational times for
these scenario trees were reasonable for all models. Thus, these scenario trees were chosen for further investigation. The improved stability of the 25-scenario case for the Extended Model was considered an outlier, and thus, it was not considered for further investigation. Furthermore, comparing results for the different models is more appropriate if the number of scenarios is the same. Therefore, the scenario tree considering 100 scenarios was also chosen for the Vessel Model. Table 6 presents objective values and expected number of failures for the scenario trees of 100 scenarios evaluated using the three different models. Table 7 lists correlations between objective values and the expected number of unrepaired components along with the average total number of maintenance windows across all scenarios for the different models.

Table 6: In-sample stability testing of the three presented models. An overview of objective values and expected number of unrepaired components in scenario trees 1-10 is provided. Each scenario tree considered 100 scenarios.

<table>
<thead>
<tr>
<th>Scenario Tree</th>
<th>Basic Model</th>
<th>Vessel Model</th>
<th>Extended Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Objective Value (’000)</td>
<td>Unrepaired Components</td>
<td>Objective Value (’000)</td>
</tr>
<tr>
<td>1</td>
<td>2,409,481</td>
<td>1.47</td>
<td>538,102</td>
</tr>
<tr>
<td>2</td>
<td>2,429,750</td>
<td>1.50</td>
<td>521,970</td>
</tr>
<tr>
<td>3</td>
<td>2,493,360</td>
<td>1.53</td>
<td>556,860</td>
</tr>
<tr>
<td>4</td>
<td>2,925,240</td>
<td>1.86</td>
<td>604,012</td>
</tr>
<tr>
<td>5</td>
<td>2,509,750</td>
<td>1.54</td>
<td>556,545</td>
</tr>
<tr>
<td>6</td>
<td>2,709,680</td>
<td>1.69</td>
<td>706,718</td>
</tr>
<tr>
<td>7</td>
<td>2,349,310</td>
<td>1.42</td>
<td>638,955</td>
</tr>
<tr>
<td>8</td>
<td>2,483,680</td>
<td>1.53</td>
<td>634,882</td>
</tr>
<tr>
<td>9</td>
<td>2,035,410</td>
<td>1.19</td>
<td>555,411</td>
</tr>
<tr>
<td>10</td>
<td>2,887,140</td>
<td>1.81</td>
<td>744,107</td>
</tr>
</tbody>
</table>

Table 7: Correlation between objective values and number of unrepaired components along with average total number of maintenance windows across all scenarios.

<table>
<thead>
<tr>
<th>Model</th>
<th>Correlation</th>
<th>Maintenance Windows</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic</td>
<td>0.999</td>
<td>26,239</td>
</tr>
<tr>
<td>Vessel</td>
<td>0.899</td>
<td>89,744</td>
</tr>
<tr>
<td>Extended</td>
<td>0.862</td>
<td>99,436</td>
</tr>
</tbody>
</table>

Components being left unrepaired throughout the planning period was a result of too few maintenance windows to complete service on all failed components in a scenario. Results presented in Table 6 shows that with the Basic Model, one vessel for a 100-turbine wind farm is not enough. The average expected number of unrepaired components was 1.55 per year, even with very high future charter costs. An early version of the Basic Model did not consider penalty costs but rather forced all failed components to be repaired; this lead to infeasibility. In conclusion, the primary vessel,
in general, was not able to repair all failed components on a 100-turbine wind farm placed in the North Sea when Basic Model assumptions apply.

It is evident that the Vessel, and Extended Models performed much better in terms of how many components were repaired, which was understandable when studying the average number of maintenance windows for the three models, listed in Table 7. This was also reflected in the objective values, which for the Basic Model were much higher because of the high future charter costs of unrepaired components. This was expected, since both the Vessel and Extended Models provided more flexibility in terms of when maintenance could be initiated, effectively being relaxations of the Basic Model. Note that the future charter costs were applied to all unrepaired components in all scenarios, see subsection 10.1.7. Furthermore, the significant differences in objective values between the models meant that comparing the coefficients of variation between the three models could give the wrong idea of which model was preferable. Naturally, variations would be larger for the smaller numerical values found for the Vessel and Extended Models even though the absolute numerical differences were similar to those of the Basic Model.

From examining Table 7, it is clear that the correlations between the expected number of unrepaired components and objective values were significant for all three models. For the Basic Model, the values can be considered perfectly correlated for all practical purposes. This indicates that the future charter costs for leaving components unrepaired throughout the planning period dictated objective values. Table 8 lists objective values for the different models when the future charter costs were subtracted, and the corresponding coefficients of variation. It is evident that the in-sample stability improved significantly as the new coefficients of variance were 3.84%, 6.69% and 4.34% for the Basic, Vessel and Extended Models, respectively. At this stage, the variations in objective values were considered to be as much a result of how the mathematical formulations handle unrepaired components as the scenario generator.

To better understand the variance in number of unrepaired components, the data produced by the scenario generator was investigated. The distribution of failed component types and the number of failures, especially on components 3 and 4, in each scenario, were considered possible causes for unrepaired components. However, as stated in section 9.1, no weather scenarios were significantly more gentle or harsh than others, thus, this possible explanation was not further investigated.
Table 8: Objective values without extra charter costs for future charter and the corresponding coefficients of variation

<table>
<thead>
<tr>
<th>Scenario Tree</th>
<th>Basic</th>
<th>Vessel Model</th>
<th>Extended Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>449,480</td>
<td>458,102</td>
<td>343,536</td>
</tr>
<tr>
<td>2</td>
<td>429,750</td>
<td>481,970</td>
<td>327,446</td>
</tr>
<tr>
<td>3</td>
<td>453,360</td>
<td>436,860</td>
<td>317,978</td>
</tr>
<tr>
<td>4</td>
<td>445,240</td>
<td>484,012</td>
<td>329,668</td>
</tr>
<tr>
<td>5</td>
<td>469,750</td>
<td>516,545</td>
<td>357,588</td>
</tr>
<tr>
<td>6</td>
<td>469,980</td>
<td>506,718</td>
<td>360,954</td>
</tr>
<tr>
<td>7</td>
<td>469,310</td>
<td>478,955</td>
<td>361,339</td>
</tr>
<tr>
<td>8</td>
<td>443,680</td>
<td>554,882</td>
<td>352,150</td>
</tr>
<tr>
<td>9</td>
<td>475,410</td>
<td>475,411</td>
<td>343,336</td>
</tr>
<tr>
<td>10</td>
<td>487,140</td>
<td>504,107</td>
<td>341,893</td>
</tr>
<tr>
<td>Coeff. of Variation</td>
<td>3.84%</td>
<td>6.69%</td>
<td>4.34%</td>
</tr>
</tbody>
</table>

Intuitively, having a high number of failures on component types 1 and 2 and a low number of failures on types 3 and 4 should be positive in terms of unrepaired components as there would be more time windows for performing maintenance. In combination with the high penalty costs for not repairing a component, this might partially explain the significant in-sample instability. Table 9 lists distribution of failed components in the scenario trees used to evaluate the different models and correlation with the number of unrepaired components.

Studying Table 9, it is noticeable that correlations between failures on components 1 or 2 and the number of unrepaired components were negative in all but one case. The reason was that these components required a lower number of time periods to repair than average, and were thus less likely to be left unrepaired. Moreover, the components with the highest number of average failures should in general be more influential, and thus correlate better with the number of unrepaired components. This was the case during testing, with the combined failure probability of component 3 and 4 being higher than both component 1 and 2 probabilities.
### 10.2 Stability Testing and SAA

Table 9: Overview of failure occurrences in all scenario trees and correlation between failure occurrences and number of unrepaired components for the different models

<table>
<thead>
<tr>
<th>Scenario Tree</th>
<th>Comp 1</th>
<th>Comp 2</th>
<th>Comp 3 &amp; 4</th>
<th>Unrepaired Components</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Basic Model</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>48</td>
<td>443</td>
<td>529</td>
<td>1.47</td>
</tr>
<tr>
<td>2</td>
<td>29</td>
<td>458</td>
<td>530</td>
<td>1.50</td>
</tr>
<tr>
<td>3</td>
<td>44</td>
<td>428</td>
<td>526</td>
<td>1.53</td>
</tr>
<tr>
<td>4</td>
<td>35</td>
<td>421</td>
<td>532</td>
<td>1.86</td>
</tr>
<tr>
<td>5</td>
<td>46</td>
<td>442</td>
<td>532</td>
<td>1.54</td>
</tr>
<tr>
<td>6</td>
<td>30</td>
<td>444</td>
<td>549</td>
<td>1.69</td>
</tr>
<tr>
<td>7</td>
<td>43</td>
<td>481</td>
<td>514</td>
<td>1.42</td>
</tr>
<tr>
<td>8</td>
<td>38</td>
<td>417</td>
<td>556</td>
<td>1.53</td>
</tr>
<tr>
<td>9</td>
<td>29</td>
<td>439</td>
<td>512</td>
<td>1.19</td>
</tr>
<tr>
<td>10</td>
<td>26</td>
<td>445</td>
<td>532</td>
<td>1.81</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td>37</td>
<td>442</td>
<td>531</td>
<td>1.55</td>
</tr>
<tr>
<td><strong>Correlation</strong></td>
<td>-2.18</td>
<td>-0.296</td>
<td>0.507</td>
<td></td>
</tr>
</tbody>
</table>

| **Vessel Model** |        |        |            |                       |
| 1             | 48     | 443    | 529        | 0.06                  |
| 2             | 29     | 458    | 530        | 0.03                  |
| 3             | 44     | 428    | 526        | 0.09                  |
| 4             | 35     | 421    | 532        | 0.09                  |
| 5             | 46     | 442    | 532        | 0.03                  |
| 6             | 30     | 444    | 549        | 0.15                  |
| 7             | 43     | 481    | 514        | 0.12                  |
| 8             | 38     | 417    | 556        | 0.06                  |
| 9             | 29     | 439    | 512        | 0.06                  |
| 10            | 26     | 445    | 532        | 0.18                  |
| **Average**   | 37     | 442    | 531        | 0.87                  |
| **Correlation** | -0.340 | 0.170  | 0.090      |                       |

| **Extended Model** |        |        |            |                       |
| 1             | 48     | 443    | 529        | 0.03                  |
| 2             | 29     | 458    | 530        | 0.03                  |
| 3             | 44     | 428    | 526        | 0.00                  |
| 4             | 35     | 421    | 532        | 0.03                  |
| 5             | 46     | 442    | 532        | 0.03                  |
| 6             | 30     | 444    | 549        | 0.00                  |
| 7             | 43     | 481    | 514        | 0.00                  |
| 8             | 38     | 417    | 556        | 0.03                  |
| 9             | 29     | 439    | 512        | 0.00                  |
| 10            | 26     | 445    | 532        | 0.06                  |
| **Average**   | 37     | 442    | 531        | 0.21                  |
| **Correlation** | -0.195 | -0.193 | 0.301      |                       |
Table 9 reveals that the number of failure occurrences on components 3 and 4 correlated well with the number of unrepaired components for the Basic Model. To exemplify, consider scenario trees 4 and 10, the scenario trees with the highest number of unrepaired components; the number of failures on components 3 and 4 was slightly above average in both scenario trees. For the Extended Model, the same correlation was less pronounced but still worth noting. The correlation between failures on components 1 and 2 and objective value was quite low for the Basic Model. Similarly, for the Extended Model, these correlations were less pronounced. The Vessel Model produced some unexpected results as the only notable correlation was between failures on component 1 and the number of unrepaired components. As this was the component with the lowest average number of failures, it was expected that this component should be the least influential.

Based upon the above reasoning, the Basic Model left components unrepaired mainly as a result of too few maintenance windows available for components 3 and 4, which required six time periods to be repaired. The Extended Model was less sensitive to the distribution of failed components, and the Vessel Model saw almost no correlation. Therefore, other plausible explanations were evaluated. As mentioned above, a possible reason is the number of failures, especially on components 3 and 4, in each scenario. As can be read from Table 9, scenario tree 10 had the highest number of unrepaired components for both the Vessel and Extended Models and the second highest number of failures for the Basic Model. Scenario tree 9 was one of the trees that had the fewest unrepaired components, on average. Figure 19 and Figure 20 present the failure distribution and total number of failures in each scenario in scenario tree 10 and 9, respectively.

First aspect evaluated was the total number of failures in the 100 scenarios. Scenario tree 10, presented in Figure 19, had one scenario with 20, one with 18 and three with 17 failures. Scenario tree 9, presented in Figure 20, saw at most 17 failures in a single scenario. Furthermore, this occurred in only two scenarios. The number of scenarios with 15 or more failures was nine and six for scenario tree 10 and scenario tree 9, respectively.

In terms of failures on components 3 and 4, even more pronounced differences between the two scenario trees were observed. The highest number of failures on components 3 and 4, ten, occurred in four scenarios in scenario tree 9. Scenario tree 10, on the other hand, had two scenarios with ten, three scenarios with eleven and one scenario with 15 failures on components 3 and 4. This indicates that scenarios with a high number of failures on components 3 and 4, more so than scenarios with just a high number of failures, dictated the number of unrepaired components. This can partially be explained by noting that almost every viable time window for maintenance on components 3 and 4, is also a viable time window for components 1 and 2. Intuitively, one might think that this should be the case for all viable time windows for components 3 and 4. However, this was not the case because of wave restrictions. It might be possible to perform maintenance in time periods where jacking up or down is not possible. With the added flexibility of the Extended Model, this effect is reduced.
Figure 19: Failure distribution and total number of failures in each scenario for scenario tree 10
Figure 20: Failure distribution and total number of failures in each scenario for scenario tree 9
Based upon the above reasoning, in-sample stability was considered satisfying at 100 scenarios for both the Basic and Extended Models. As shown, the initial instability encountered by both models was as much a result of how the mathematical formulation handled unrepaired components as the data produced by the scenario generation. The main reason for having a high number of unrepaired components was single scenarios with many failures on components 3 and 4. In reality this might happen, thus, the resulting instabilities were considered acceptable. The Vessel Model also saw satisfying stability in the 100-scenario case when future charter costs related to unrepaired components were subtracted. However, stability did not improve as the number of scenarios in each scenario tree increased. As a result, the Vessel Model was not considered very stable.

10.2.2 Out-of-Sample Stability

Out-of-sample stability is a measure of how well the scenario tree-specific solutions perform in the real problem. The real problem in this study, was represented by a scenario tree of 1,000 scenarios, hereinafter referred to as the reference tree, which was considered sufficiently large to represent the considered uncertainty in a satisfying way. Notes on the size of the reference tree are presented in subsection 10.2.3. Results from out-of-sample stability testing are presented in Figure 21. It should be stated that for the problem studied, where the actual first stage decision and not the objective value, is the most interesting, out-of-sample stability was considered more important than in-sample stability.

For the Basic Model, out-of-sample stability was very good when 75 or more scenarios were considered. The Vessel and Extended Models, on the other hand, produced more variable solutions. This can be explained by the increased flexibility provided by these models, allowing these models to better tailor the first- and second stage solutions to each scenario tree. However, both experienced a steady improvement in out-of-sample stability as the number of scenarios increased.

To better understand what caused out-of-sample instability and what characterized good first stage solutions, further analyses were conducted. Table 10 lists coefficients of variation for the number of vessel periods suggested by the models for different scenario tree sizes. Table 11 lists objective values and the number of vessel periods for the scenario trees considering 100 scenarios.

As the Vessel and Extended Models have more flexibility in terms of maintenance windows, second stage solutions, and thus first stage solutions, could be better tailored to each scenario tree. This increased variance. This claim is substantiated by the results listed in Table 10 where it is evident that the variance in number of vessel periods was higher for the Vessel and Extended Models than for the Basic Model.
### 10.2 STABILITY TESTING AND SAA

**Out-Of-Sample Stability**

![Plot of out-of-sample stability for different scenario tree sizes and model evaluations. Out-of-sample stability is measured in terms of coefficient of variation](image)

<table>
<thead>
<tr>
<th>Number of Scenarios in Scenario Tree</th>
<th>Coefficient of Variation [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>Basic Model: 6.07%</td>
</tr>
<tr>
<td></td>
<td>Vessel Model: 13.63%</td>
</tr>
<tr>
<td></td>
<td>Extended Model: 9.03%</td>
</tr>
<tr>
<td>50</td>
<td>Basic Model: 3.16%</td>
</tr>
<tr>
<td></td>
<td>Vessel Model: 8.99%</td>
</tr>
<tr>
<td></td>
<td>Extended Model: 9.36%</td>
</tr>
<tr>
<td>75</td>
<td>Basic Model: 3.19%</td>
</tr>
<tr>
<td></td>
<td>Vessel Model: 7.76%</td>
</tr>
<tr>
<td></td>
<td>Extended Model: 8.36%</td>
</tr>
<tr>
<td>100</td>
<td>Basic Model: 2.33%</td>
</tr>
<tr>
<td></td>
<td>Vessel Model: 8.95%</td>
</tr>
<tr>
<td></td>
<td>Extended Model: 4.73%</td>
</tr>
</tbody>
</table>

Figure 21: Plot of out-of-sample stability for different scenario tree sizes and model evaluations. Out-of-sample stability is measured in terms of coefficient of variation.

One finding from reading Table 11 is that the correlation between the reference tree objective values and the number of vessel periods suggested by the corresponding solution was notable for the Extended Model. The best solutions in terms of objective value were those suggesting the highest number of vessel periods, hence the negative correlation. For the Basic Model, the correlation was less pronounced and even hinted that more vessel periods was negative. This indicates that when having more maintenance flexibility, the number of vessel periods dictated how well the solution performed in the reference tree. This can be explained by the fact that with increased flexibility, the vessel utilization rate, i.e. time periods in which the vessel is used divided by the number of vessel periods, is higher. In other words, when maintenance flexibility is present, any first stage decision can be utilized more efficiently.

Table 10: Coefficients of variation for the number of vessel periods suggested by the ten scenario tree solutions for different scenario tree sizes

<table>
<thead>
<tr>
<th>Scenario Tree Size</th>
<th>Coefficient of Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Basic Model</td>
</tr>
<tr>
<td>25</td>
<td>6.07%</td>
</tr>
<tr>
<td>50</td>
<td>3.16%</td>
</tr>
<tr>
<td>75</td>
<td>3.19%</td>
</tr>
<tr>
<td>100</td>
<td>2.33%</td>
</tr>
</tbody>
</table>
Table 11: Scenario tree-specific solutions evaluated in the reference tree, number of vessel periods suggested by solutions and correlation between the two

<table>
<thead>
<tr>
<th>First Stage Solution #</th>
<th>Basic Model</th>
<th>Vessel Model</th>
<th>Extended Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>290</td>
<td>2,554,703</td>
<td>217</td>
</tr>
<tr>
<td>2</td>
<td>288</td>
<td>2,555,068</td>
<td>235</td>
</tr>
<tr>
<td>3</td>
<td>302</td>
<td>2,540,571</td>
<td>195</td>
</tr>
<tr>
<td>4</td>
<td>298</td>
<td>2,585,399</td>
<td>226</td>
</tr>
<tr>
<td>5</td>
<td>289</td>
<td>2,549,979</td>
<td>249</td>
</tr>
<tr>
<td>6</td>
<td>278</td>
<td>2,583,041</td>
<td>238</td>
</tr>
<tr>
<td>7</td>
<td>290</td>
<td>2,545,824</td>
<td>235</td>
</tr>
<tr>
<td>8</td>
<td>287</td>
<td>2,560,318</td>
<td>275</td>
</tr>
<tr>
<td>9</td>
<td>294</td>
<td>2,537,475</td>
<td>229</td>
</tr>
<tr>
<td>10</td>
<td>297</td>
<td>2,555,531</td>
<td>246</td>
</tr>
<tr>
<td>Correlation</td>
<td>0.177</td>
<td>-0.085</td>
<td>-0.478</td>
</tr>
</tbody>
</table>

10.2.3 Notes on Size of Reference Tree and Scenario Trees

In this study, it was assumed that 1,000 scenarios was sufficient to represent the considered uncertainty. As mentioned, the average number of vessel periods in the ten scenario trees increased as the number of scenarios in each tree increased. This was a result of solutions being less tailored to the scenarios considered. In general, it is more difficult to tailor solutions to accommodate 100 different failure outcomes than ten. This was further verified when evaluating two 1,000-scenario scenario trees which both suggested ~ 300 vessel periods.

As a result of the above reasoning, even though satisfying in-sample and out-of-sample stability was achieved when ten scenario trees of 100 scenarios were considered, the solutions might prove weak when evaluated on scenario trees of 1,000 scenarios.

10.2.4 SAA Results

The reader is referred to [section 8.1](#) for a detailed outline of the SAA algorithm. In this subsection, results from stability testing were used to calculate SAA values. Optimistic bounds and corresponding variances were given by the average objective values and variance from the ten scenario trees evaluated, listed in [Table 6](#). Pessimistic bounds and their variances were found by evaluating a feasible first stage solution in an independently generated reference tree of size $N' = 1,000$. The solutions selected were those that gave the best average objective value in all other trees during out-of-sample
stability testing. Table 12 presents bounds, the corresponding gap, gap standard deviation and the 90% confidence interval.

Table 12: Optimistic and pessimistic bounds, gap, standard deviation of gap and 90% confidence interval of the gap calculated in accordance with SAA

<table>
<thead>
<tr>
<th>Model</th>
<th>Optimistic Bound ('000)</th>
<th>Pessimistic Bound ('000)</th>
<th>Gap ('000)</th>
<th>St. Dev of Gap ('000)</th>
<th>90% Confidence Interval ('000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic</td>
<td>2,523,310</td>
<td>2,537,475</td>
<td>14,165</td>
<td>83,466</td>
<td>-92,802 - 121,132</td>
</tr>
<tr>
<td>Vessel</td>
<td>605,756</td>
<td>695,195</td>
<td>89,438</td>
<td>23,808</td>
<td>58,928 - 119,949</td>
</tr>
<tr>
<td>Extended</td>
<td>378,589</td>
<td>464,712</td>
<td>76,627</td>
<td>9,353</td>
<td>64,642 - 88,613</td>
</tr>
</tbody>
</table>

Noticeable from reading Table 12 is that the gap was considerably smaller for the Basic than for both the Vessel and Extended Models. However, as Basic Model objective values were so high, the variances corresponding to both bounds were also quite significant, resulting in the high standard deviation and the wide confidence interval. In combination with the small gap, this resulted in a negative lower limit for the gap, which is peculiar in an optimization setting. A negative gap limit meant that, when calculating bounds using the SAA method, one could get a pessimistic bound that was lower than the optimistic bound. These results substantiate the claim that unrepaird components are problematic in the Basic Model because of the impact these have on the objective values, see subsection 10.1.7, Table 6 and Table 8. This was further investigated by calculating SAA values disregarding charter costs related to unrepaired components. The bounds saw variances more in line with variances for the Extended Model bounds.

For the the Vessel and Extended Models, the confidence intervals were more satisfying. The results prove that as maintenance flexibility increased, either in terms of having more robust vessels or allowing to pause a maintenance operation, SAA bounds variance decreased. This was a result of fewer unrepaird components.

Wide confidence intervals, in general, indicate that sample sizes should be larger. However, because of limited computational resources, SAA parameters were chosen according to stability testing, thus, \( M = 10, N = 100 \) and \( N' = 1,000 \), and were used throughout the this computational study.

10.3 EVALUATING MODEL RESULTS

In this section, the results from the different models will be closely evaluated, compared and discussed. As for solutions, the focus is on the first stage decisions, as these are the decisions of interest and are of the same format for the Basic and Extended Models. The first stage solutions from the Vessel Model are not directly comparable and are more relevant in an economic analysis. Therefore, these solutions will not be paid much attention is this section.
10.3 Evaluating Model Results

Computational times were the first aspect to evaluate. Table 13 presents accumulated computational times for the three presented models. The times are divided into data generation, i.e. how long it takes for MATLAB to generate the required data for the ten scenario trees, and optimization time, i.e. how long Xpress takes to find the optimal solution. Optimization time is further divided into time required to load and build the model, and time needed to solve the problem.

Table 13: Accumulated computational times for ten scenario trees considering 100 scenarios

<table>
<thead>
<tr>
<th>Model</th>
<th>Total Time</th>
<th>Data Generation (MATLAB)</th>
<th>Building Problem (Xpress)</th>
<th>Solving Problem (Xpress)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic</td>
<td>4,774</td>
<td>1,020</td>
<td>3,729</td>
<td>25</td>
</tr>
<tr>
<td>Vessel</td>
<td>12,859</td>
<td>1,570</td>
<td>10,669</td>
<td>620</td>
</tr>
<tr>
<td>Extended</td>
<td>7,846</td>
<td>1,461</td>
<td>5,537</td>
<td>848</td>
</tr>
</tbody>
</table>

As explained in section 9.4, the Basic Model second stage decisions were more restricted than those of the Extended Model. This is verified from reading Table 7 where the average total number of maintenance windows is listed. This restriction resulted in a lower number of second stage variables, which reduced computational times as seen in Table 13. The computational times were not too extensive, and one could argue that more scenarios should have been evaluated to improve solution quality. However, with the limited computational power and time at hand for this study, increasing computational times further were not desirable.

The first stage solutions, in general, suggested a high number of vessel periods. For the Basic Model, the average number of vessel periods and charter periods for the ten scenario trees were 291 and 7.4, respectively. For the Extended Model, the numbers were 246 and 4.6. In terms of solution similarities, there was a pattern for the Basic Model in which time periods the vessel was chartered. However, since the total number of vessel periods was so high, it was more interesting to look at which time periods were not proposed for charter. No solutions suggested charter in time periods 14-25, 126-135 or 240-251. Eight out of ten solutions recommended not to charter in time periods 67-78. For the Extended Model, such patterns were less pronounced, but there was a tendency to suggest one or two longer charter periods in the months April-August. This indicates that weather was more influential than the elevated daily charter prices in March-October, see subsection 10.1.6, in terms of when to charter. This was further investigated by calculating the weighted daily charter in the winter and summer months. The time periods which were considered winter months were 1-60 and 306-365. For the Extended Model, the average number of vessel periods in these time periods was 61.6, resulting in an weighted daily charter of 0.513. For the summer months, the average number of vessel periods was 184.1, resulting in a weighted daily charter of 0.751. For the Basic Model, the weighted daily charter was 0.728 and 0.832 for winter and summer months, respectively. Another aspect worth mentioning is that first stage solutions from the Vessel Model stability testing suggested chartering of the
most robust vessel more so than the least robust vessel, even though the most robust vessel was more expensive. Both these findings further substantiate the above claim of weather being more influential than charter prices when generating an optimal charter schedule.

As the suggested charter periods were of the same format, how solutions from the Basic Model, characterized by shorter computational time, performed when tested in the Extended Model, was evaluated. Table 14 lists objective values when the ten first stage solutions from the Basic Model were tested in the Extended Model.

Table 14: Performance of Basic Model first stage solutions in the Extended Model. Pessimistic SAA bound is the one found for the Extended Model, see Table 12.

<table>
<thead>
<tr>
<th>First Stage Solution #</th>
<th>Objective Value in Reference Tree (‘000)</th>
<th>Deviation from Extended Model Pessimistic SAA Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>518,262</td>
<td>11.5%</td>
</tr>
<tr>
<td>2</td>
<td>612,518</td>
<td>31.8%</td>
</tr>
<tr>
<td>3</td>
<td>512,315</td>
<td>10.2%</td>
</tr>
<tr>
<td>4</td>
<td>613,264</td>
<td>32.0%</td>
</tr>
<tr>
<td>5</td>
<td>562,432</td>
<td>21.0%</td>
</tr>
<tr>
<td>6</td>
<td>661,363</td>
<td>42.3%</td>
</tr>
<tr>
<td>7</td>
<td>562,589</td>
<td>21.1%</td>
</tr>
<tr>
<td>8</td>
<td>576,776</td>
<td>21.4%</td>
</tr>
<tr>
<td>9</td>
<td>561,989</td>
<td>20.9%</td>
</tr>
<tr>
<td>10</td>
<td>571,846</td>
<td>23.1%</td>
</tr>
</tbody>
</table>

In general, the solutions did not perform well, as none of the objective values were within the Extended Model SAA gap listed in Table 12. Therefore, if the required computational power is available, one should use the Extended Model to ensure better charter suggestions. This is of course, only valid if the wind farm operator allows maintenance flexibility.

10.4 Testing of Heuristics

This section outlines testing of the heuristics presented in section 8.2. The idea behind the heuristics is to assign a sequence, or batch, of maintenance operations to a single second stage decision. As explained, two pure greedy heuristics were introduced, namely GMBH 1 and GMBH 2. The heuristic with a random component, the GRAMBH was also tested. To determine which was the most appropriate heuristic, testing was conducted. All testing was done using the Alternative Mathematical Formulation presented in chapter 6 and downtime costs were calculated in accordance with the Extended Model described in section 9.4.
10.4 Testing of Heuristics

10.4.1 Greedy Maintenance Bundling Heuristics

The purely greedy heuristics GMBH 1 and GMBH 2 were tested and optimistic SAA bounds are listed in Table 15 and Table 16 together with the number of vessel periods suggested. The Extended Model found a significantly cheaper optimistic bound of $379 \cdot 10^6$ found for the Extended Model, see Table 12. The number of vessel periods suggested by the heuristics were high; all were over 300 and most over 330 vessel periods per year. This was significantly higher than the same number for the Extended Model which was 246. Thus, results show the pure greedy proposed heuristics were weak. However, solving with the heuristics showed promising computational times.

<table>
<thead>
<tr>
<th>Repairs in Batch, $N$</th>
<th>Optimistic Bound (’000)</th>
<th>Vessel Periods</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>5,352,180</td>
<td>334</td>
</tr>
<tr>
<td>3</td>
<td>5,177,511</td>
<td>331</td>
</tr>
<tr>
<td>4</td>
<td>5,317,599</td>
<td>329</td>
</tr>
</tbody>
</table>

Comparing results for GMBH 1 listed in Table 15 with GMBH 2 results listed in Table 16 shows that GMBH 1 provided the most positive results in terms of optimistic bounds, thus this was selected for the GRAMBH; GMBH 2 was not further developed.

<table>
<thead>
<tr>
<th>Time Periods in Batch, $T^B$</th>
<th>Optimistic Bound (’000)</th>
<th>Vessel Periods</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>5,598,905</td>
<td>339</td>
</tr>
<tr>
<td>30</td>
<td>5,405,375</td>
<td>346</td>
</tr>
<tr>
<td>40</td>
<td>5,623,617</td>
<td>306</td>
</tr>
</tbody>
</table>

10.4.2 Introduction of Randomness

The GRAMBH was tested for different numbers of repairs in one batch and the corresponding SAA results are listed in Table 17 together with Extended Model results. For an elaborate explanation of the GRAMBH, the reader is referred to section 8.2. The reason for testing different values of $N$ was because it was assumed that this was most decisive for the obtained results. Ideally, all parameters should have been tested extensively, but this was not done due to time constraints. Preliminary testing suggested values for the parameters $T^M$, the maximum waiting time between operations, $N^C$, the size of the candidate lists and $N^B$, the number of batches to create in every time period in which maintenance could be initiated. The values used for $T^M$, $N^C$ and $N^B$ were 4, 5 and 10, respectively.
10.4 Testing of Heuristics

Table 17: Comparison of vessel periods and bounds provided by GRAMBH and the Extended Model for different number of repairs in batch, N

<table>
<thead>
<tr>
<th>Repairs in Batch, N</th>
<th>Vessel Periods</th>
<th>Optimistic Bound ('000)</th>
<th>Pessimistic Bound ('000)</th>
<th>Gap ('000)</th>
<th>St. Dev of Gap ('000)</th>
<th>90% Confidence Interval ('000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>263</td>
<td>396,204</td>
<td>440,837</td>
<td>44,633</td>
<td>10,376</td>
<td>31,336 - 57,929</td>
</tr>
<tr>
<td>3</td>
<td>259</td>
<td>404,408</td>
<td>425,454</td>
<td>21,047</td>
<td>13,818</td>
<td>3,338 - 38,755</td>
</tr>
<tr>
<td>4</td>
<td>273</td>
<td>423,426</td>
<td>425,573</td>
<td>2,151</td>
<td>10,034</td>
<td>10,709 - 15,010</td>
</tr>
<tr>
<td>Extended</td>
<td>246</td>
<td>371,589</td>
<td>464,712</td>
<td>93,535</td>
<td></td>
<td>64,642 - 88,613</td>
</tr>
</tbody>
</table>

It is evident that the GRAMBH gave tighter bounds than the Extended Model, and that the bounds became tighter as the number of repairs in a batch increased, but high variances yielded wide confidence intervals; four repairs even gave a negative lower bound for the gap confidence interval. The pessimistic bounds for the heuristic models are also pessimistic bounds for the true problem. This is a result of the heuristic models being restricted versions of the true problem. This means that the optimal objective value for the true problem lies somewhere between \(379 \cdot 10^6\) and \(425 \cdot 10^6\). The fact that the GRAMBH finds better pessimistic bounds than the Extended Model shows that the heuristic is useful and should be utilized accordingly. Probably, a reason for the GRAMBH finding better pessimistic bounds was because all first stage solutions were tested in the reference tree and the best bound was selected. For the Extended Model, just one first stage solution was tested in the independent reference tree. When comparing the values found with the GRAMBH to the values found with GMBH, it is obvious that the GRAMBH performed better. The optimistic bounds found by the GRAMBH for 2, 3 and 4 repairs were 6.6%, 8.8% and 14.0% higher than the bounds found by the Extended Model. This was expected since the GRAMBH is a restricted version of the true problem.

Compared to the GMBH, the numbers of vessel periods suggested by the GRAMBH were significantly smaller. With randomness, the numbers for 2, 3 and 4 repairs in a batch were 263, 259 and 273 respectively. These were quite close to the number suggested by the Extended Model, namely 246. Probably, the aspect which facilitated most of the improvements seen was creating several batches in every time period. This provides valuable flexibility whereas earlier heuristics only created one batch for every allowed time period. The GRAMBH also created batches in a less rigid way with randomness, thus limiting the risk of only generating good individual batches which did not combine well. To elaborate, consider the case where one batch is created for every allowed time period. When the optimization solver attempts to combine batches, it can only select one batch containing each maintenance operation; this may lead to weak combinations because of the likely large time spread of selected batches. When creating several batches for each allowed time period, the flexibility when optimizing batch combination increases since the batches contain different maintenance operations because of randomization.
10.4 Testing of Heuristics

10.4.3 Solution Time Comparison

Important motivation for implementing heuristics was to reduce solution time and thus enable solving larger problems. A comparison of total solution times, in seconds, with the different models are presented in Table 18.

Table 18: Comparisons of accumulated solution time for the Basic and Extended Model and the GRAMBH

<table>
<thead>
<tr>
<th>Model</th>
<th>Total Time (MATLAB)</th>
<th>Data Generation Time (MATLAB)</th>
<th>Building Problem Time (Xpress)</th>
<th>Solving Problem Time (Xpress)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extended</td>
<td>7,846</td>
<td>1,461</td>
<td>5,537</td>
<td>848</td>
</tr>
<tr>
<td>2 Repairs</td>
<td>6,780</td>
<td>4,970</td>
<td>60</td>
<td>1,040</td>
</tr>
<tr>
<td>3 Repairs</td>
<td>17,625</td>
<td>5,015</td>
<td>160</td>
<td>12,450</td>
</tr>
<tr>
<td>4 Repairs</td>
<td>19,442</td>
<td>4,940</td>
<td>160</td>
<td>14,342</td>
</tr>
</tbody>
</table>

First thing to notice from reading Table 18 is that the building the problem in Xpress is much faster when using the GRAMBH, while optimization times were longer. This was unexpected, but further investigations yielded no reasonable explanation.

It is evident that the optimization time in Xpress was longer when more repairs were added to each batch. This might be a result of the dominance criteria used. A batch was considered dominant if it covered the same components, had a lower downtime cost, a lower number of required vessel periods and the same last time period in $T_{tbs}$ as another batch. When the number of maintenance operations in a batch increased, the number of dominated batches likely decreased, thus, more options were available when the problem was sent to Xpress. This lead to a high number of variables and constraints. Studying different types of dominance criteria were not prioritized in this study, but is suggested for future research. In terms of total solution time, the heuristic proved to be ineffective when the number of repairs in a batch exceeded two.

Studying the results, it is observable that the amount of time required to generate scenario trees was higher for the heuristic models. This was expected, as the heuristic function came in addition to calculating downtime costs and other parameters required for the Extended Model. However, the solution time in Xpress decreased by 83% for the two-repair case. The solution time in Xpress for all heuristic cases largely consisted of solving the problem, not loading and building it. Moreover, scenario generation was time consuming. It should be noted that the programming experience of the authors is limited and it is acknowledged that the scenario generation and heuristics code could probably be rationalized.
10.5 ECONOMIC ANALYSES

This section presents economic analyses conducted with the proposed optimization models. Some economic aspects related to having a more robust primary vessel are discussed in subsection 10.5.1 and whether to charter or to buy a vessel is the best decision is analysed for different wind farm sizes in subsection 10.5.2. Some economic consequences of using Basic Model solutions for wind farms compliant with the Extended Model assumptions are discussed in subsection 10.5.3.

10.5.1 Having a More Robust Vessel as the Primary Vessel

Since the Vessel Model suggested a high number of vessel periods for the more robust vessel, it was investigated whether using it as the primary vessel in the Extended Model would be beneficial. Thus, secondary vessel charter rates and weather capabilities, see Table 4, were used as input and solved. Resulting costs and the number of vessel periods are shown in Table 19.

Table 19: Cost comparison for primary and secondary vessel using the Extended Model

<table>
<thead>
<tr>
<th>Vessel</th>
<th>Yearly Costs (MNOK)</th>
<th>Vessel Periods</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary</td>
<td>425.5</td>
<td>246</td>
</tr>
<tr>
<td>Secondary</td>
<td>561.5</td>
<td>241</td>
</tr>
</tbody>
</table>

It was observed that using the expensive and more robust vessel as the primary vessel increased yearly costs by approximately 32%. Comparing with the results from the Vessel Model which suggested that the secondary vessel was cheapest to charter, results are contradictory. However, the Vessel Model is significantly stricter in terms of allowed maintenance operations. Thus, when only a few time windows were available to begin with, opting for the robust vessel expanded those enough to make it the most economic decision. It should be noted here that the high future charter costs for un-repaired components were a contributor to these results. For the Vessel Model, it was cheaper to charter the robust vessel and get a lower number of un-repaired components. For the Extended Model, the number of un-repaired components was very low even with the primary vessel, thus, its weather capabilities were adequate. This claim is further substantiated by the small difference in the number of suggested vessel periods for the two vessels listed in Table 19.
10.5 Economic Analyses

10.5.2 The Decision of Buying or Chartering a Jack-Up Vessel

In the stability testing it was pointed out that for a 100-turbine wind farm, a high number of yearly vessel periods were suggested when solving the problem, see Table 11. As a result, it was decided to investigate for different wind farm sizes whether it would be preferable to buy a vessel and always having it available. Sizes studied were 50, 75 and 100 turbines. Real-life examples of wind farms with approximately 75 turbines are Lincs, Humber Gateway and Northwind in the UK [53]. 50-turbine wind farms are for instance Nordsee Ost and the under construction Nordsee One in Germany [64].

10.5.2.1 Methodology

Data gathered for calculations presented in the following is listed in Table 20. The best pessimistic bound was considered to represent the charter and downtime costs when using an optimized chartering strategy for each year in the wind farms' lifespan. For the 100-turbine wind farm, a pessimistic bound was already available, see Table 17. For the other wind farm sizes, the pessimistic bounds were found using the Extended Model. Ideally, the pessimistic bounds should be found using the GRAMBH, but due to time constraints, bounds from the Extended Model were used. Crew costs had to be added to get total yearly costs. Fuel costs were accounted for in the charter costs, see subsection 10.1.6. Crew salaries were found in [12] and considered a crew of 40 with a total salary of £2.8 million equalling 36.4 MNOK, annually [81]. When calculating crew costs, 36.4 MNOK was multiplied by $F_C$, see (10.1), where the number of vessel periods equalled the average suggested when solving the model. Yearly costs were then discounted to get present value and then summed to get total costs for the entire lifespan of the wind farm.

\[
F_C = \frac{\text{Number of vessel periods}}{365} \quad (10.1)
\]

To calculate downtime costs for the wind farm when owning a vessel, the Extended Model was solved with the vessel chartered at all times and no charter costs were incurred. Since there was only one first stage solution, having the vessel available at all times, no pessimistic bounds could be found using the SAA method. Thus, the average objective value in the reference tree was used to represent downtime costs. Vessel operating costs were added to get total yearly costs and these were discounted over the lifespan to get present value. The costs of buying a vessel similar to the primary vessel, see Table 4, was added at the start of the wind farm’s lifespan to get total costs and was considered a one-time expense. The salvage value of the vessel was considered to be zero. A new vessel acquired by A2Sea delivered in 2014 had a price tag of 890
10.5 Economic Analyses

MDKK, or 1,110 MNOK [1], using an exchange rate of 1.247 NOK per DKK gathered 01.06.2016 [81]. This was used as the input price. Annual operating costs of an owned vessel were gathered from Maples et al. [46] at $77,500 per day, equalling 233.6 MNOK per year using an exchange rate of 8.25 NOK per USD gathered 01.06.2016 [81].

When calculating lifetime costs, annual costs were discounted by a rate of 5% and the lifespan was considered to be 25 years [12]. Present values of costs were calculated using equation 10.2 for present value of an annuity [6], where \( C^Y \) is the yearly costs, \( i \) is the discount rate and \( n \) is the number of payments. All other input data than the number of wind turbines was unchanged.

\[
PV = C^Y \cdot \frac{1 - (1 + i)^{-n}}{i}
\]  

(10.2)

Table 20: Input data for calculation of total lifetime costs when buying or chartering a vessel

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price of Vessel (MNOK)</td>
<td>1,110.0</td>
</tr>
<tr>
<td>Discount Rate</td>
<td>5%</td>
</tr>
<tr>
<td>Lifespan (Years)</td>
<td>25</td>
</tr>
<tr>
<td>Yearly Operating Costs, Owned Vessel (MNOK)</td>
<td>233.6</td>
</tr>
<tr>
<td>Avg. Annual Crew Salary, Chartered Vessel (MNOK)</td>
<td>36.4</td>
</tr>
</tbody>
</table>

It was assumed that time since commissioning or time until decommissioning of the wind farm did not affect maintenance planning. This means that maintenance was executed at the same rate in the months right after commissioning and right before decommissioning as in the middle of the lifespan. This was a simplification, but since the vast majority of the lifetime sees approximately equal failure rates, it was considered feasible.

10.5.2.2 Results

Table 21 lists the obtained results for yearly and lifetime costs when chartering or buying a vessel for different wind farm sizes along with recommended strategy and cost differences between strategies. The yearly costs comprised of both downtime costs and charter costs when chartering a vessel, while when buying a vessel, the yearly costs consisted only of downtime costs. Vessel operating costs, the costs of purchasing a vessel and crew costs were added to the pessimistic bounds and discounted over the wind farm lifespan. As expected, yearly costs for both buying and charter were lower for 50- and 75- turbine wind farms; actually when chartering, costs seemed to
increase approximately linearly from 50 to 100 turbines. This comes as no surprise as the expected number of failures increases linearly with wind farm size.

Table 21: Yearly and lifetime costs when chartering or buying a vessel. Values in column 2-5 are quoted in MNOK

<table>
<thead>
<tr>
<th>Wind Farm Size</th>
<th>Charter Costs</th>
<th>Costs of Buying</th>
<th>Charter or Buy?</th>
<th>Cost Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Yearly</td>
<td>Lifetime</td>
<td>Yearly</td>
<td>Lifetime</td>
</tr>
<tr>
<td>50</td>
<td>246.4</td>
<td>3,473.0</td>
<td>238.6</td>
<td>4,472.4</td>
</tr>
<tr>
<td>75</td>
<td>335.8</td>
<td>4,733.4</td>
<td>242.0</td>
<td>4,520.3</td>
</tr>
<tr>
<td>100</td>
<td>451.7</td>
<td>6,365.6</td>
<td>246.8</td>
<td>4,588.6</td>
</tr>
</tbody>
</table>

As mentioned, the Extended Model suggested a high number of yearly vessel periods for a 100-turbine wind farm. The results listed in Table 21 verify that buying a vessel was 31% cheaper than chartering for a wind farm of 100 turbines. For the 75-turbine wind farm, the difference was less pronounced, but still favored buying a vessel. For the 50-turbine case, there was a significant difference in favor of the charter option which was 22.3% cheaper. Based upon these findings and linear interpolation, for wind farm sizes exceeding \( \sim \) 71 turbines, buying a vessel should be considered.

It can be seen from Table 21 that the lifetime costs did not depend heavily upon the wind farm size when buying a vessel. As downtime costs were the only variable costs, this was expected, since these were only a small part of total yearly costs when chartering. In fact, downtime costs proved to be between 2% and 5% of yearly costs for all wind farm sizes when chartering. Furthermore, the downtime costs did not increase linearly with expected failures, even when owning a vessel; this points towards a higher vessel utilization rate for larger wind farms. Utilization rate is thus a deciding parameter in terms of whether chartering or buying a vessel is the best decision. Because of the modelling method, full information about second stage decisions was cumbersome to retrieve, but an estimate of the utilization rate was calculated. It was assumed that the average number of time periods spent on maintenance was equal to the average of columns three and four in Table 3, thus, 1.5, 4.5, 9 and 9 for components 1, 2, 3 and 4, respectively. The average yearly number of failures on each component type for the different wind farm sizes were calculated using the failure probabilities from Table 5. Then, these numbers were multiplied by the expected required vessel periods to find the expected number of time periods the vessel was active. Resulting estimated expected number of active time periods and utilization rates are listed in Table 22.

The wind farm size for which lifetime costs were approximately the same for chartering and buying, was calculated to be \( \sim \) 71 turbines, and the corresponding utilization rate was 13.1%, or 47.8 days per year. The fact that the vessel only needed to be used 13.1% of the time to make buying an economically viable option reflects an imbalance in the jack-up vessel market. The demand is high compared to the supply, driving prices up, as stated by Dalgic et al. Assuming lifetime costs for buying are
Table 22: Estimated vessel utilization rate for different wind farm sizes in the case where a vessel was bought. Utilization rate equals number of time periods where the vessel was active divided by the total number of time periods.

<table>
<thead>
<tr>
<th>Wind Farm Size</th>
<th>Number of Failures</th>
<th>Active Time Periods (avg.)</th>
<th>Utilization Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Comp 1</td>
<td>Comp 2</td>
<td>Comp 3 &amp; 4</td>
</tr>
<tr>
<td>50</td>
<td>0.2</td>
<td>2.2</td>
<td>2.6</td>
</tr>
<tr>
<td>75</td>
<td>0.3</td>
<td>3.3</td>
<td>3.9</td>
</tr>
<tr>
<td>100</td>
<td>0.4</td>
<td>4.4</td>
<td>5.2</td>
</tr>
</tbody>
</table>

10.5 Economic Analyses

The 18.4% utilization rate for a 100-turbine wind farm was reasonably close to the 14.4% utilization rate found by Dalgic et al. [12] for long-term charter. Long-term charter is in this case comparable to buying since the time horizon for charter could be close to the wind farm’s lifespan. It should be noted that some assumptions and input data for instance weather and vessel capabilities, in the article were different from those used in this study. Further, Dalgic et al. utilize Monte Carlo simulation rather than optimization methods.

As the utilization rates were so low, it is expected that sub-chartering the vessel is a feasible option to acquire additional profits. However, sub-chartering will probably increase downtime costs as the vessel is periodically unavailable. Moreover, the time periods in which other wind farm operators would want to charter the vessel, are the probably most favorable for the operator’s own wind farm. Considering downtime costs were only a small part of expected yearly costs, significant increases can be tolerated if sub-charter rates are anywhere close to current charter rates.

10.5.3 Economic Consequences of Using Basic Model Solutions

Assume that an operator of a 100-turbine wind farm allowing maintenance flexibility in accordance with the Extended Model, uses the Basic Model to plan jack-up vessel chartering. Following is an analysis of the economic consequences of such a decision. Here, no crew costs were considered, only charter costs.

Costs and the number of vessel periods for Basic Model first stage solutions evaluated using the Extended Model for different wind farm sizes are listed and compared to Extended Model results in Table 23. The Basic Model solution selected for the 100-turbine wind farm was the one which gave the pessimistic bound for the Basic Model listed in Table 12. For the other wind farm sizes, the Basic Model was solved for ten scenario trees and those which performed best, on average, in the other scenario trees were selected.
Table 23: Comparison of yearly costs when using solutions for Basic and Extended Models for different wind farm sizes. Costs for the Basic Model were calculated as the Basic Model solution inserted in the Extended Model reference tree.

<table>
<thead>
<tr>
<th>Model</th>
<th>Wind Farm Size</th>
<th>Vessel Periods</th>
<th>Yearly Costs (MNOK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic</td>
<td>50</td>
<td>219</td>
<td>323.7</td>
</tr>
<tr>
<td></td>
<td>75</td>
<td>291</td>
<td>512.3</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>246</td>
<td>425.4</td>
</tr>
<tr>
<td>Extended</td>
<td>50</td>
<td>158</td>
<td>230.7</td>
</tr>
<tr>
<td></td>
<td>75</td>
<td>218</td>
<td>314.1</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>246</td>
<td>425.4</td>
</tr>
</tbody>
</table>

Examining Table 23, it is evident that if Extended Model assumptions apply, selecting the chartering strategy suggested by the Basic model increased costs by 86.9 MNOK annually compared to selecting the Extended Model chartering strategy, an increase of 20%. The increase in vessel periods was 18%, negatively affecting costs and yielding an unfavorable utilization rate. Since there are so few time windows with continuous favorable weather, the vessel periods must cover most of these to limit total penalty costs. This shows that the Extended Model is economically useful since the Basic Model solutions yielded unfavorable results.

The same analyses were conducted for 50 and 75 turbines with results listed in Table 23. For the 50-turbine case, yearly costs increased by 93.1 MNOK, 40.3%, and the number of vessel periods increased by 38.8%. For the 75-turbine wind farm, costs increased by 74.1 MNOK or 24% and the number of vessel periods increased by 19%. This shows that the percentage cost increase became higher for smaller wind farms. In absolute values, the cost increases were not that different when considering 50- and 100-turbine cases, although the increase was higher for the 50-turbine case. In the stability testing, it was shown that when solving the 100-turbine case with the Basic Model, the average expected number of unrepaired components were 1.55, see Table 6. For the 50- and 75-turbine wind farms, corresponding numbers were 0.29 and 0.79, respectively. Even if the number of turbines, and thus the expected numbers of failures, were 50% and 25% lower for the 50- and 75-turbine cases, the number of vessel periods suggested by the Basic Model decreased only by approximately 25% and 10%, respectively. Because of the future charter costs the Basic Model prioritized repairing more components rather than decreasing the number of vessel periods. This is a result of the severe future charter costs related to unrepaired components.
This thesis proposes two-stage stochastic optimization models for the jack-up chartering strategy problem for offshore wind farms considering uncertainty in weather and failure rates. The first stage solution suggests when and for how long to charter a vessel and the second stage decision suggests which turbines to service at what time. The models differ in that the Basic Model requires maintenance to be completed in consecutive time periods, while the Extended Model allows for weather-induced maintenance breaks. A model introducing more than one vessel type is also presented. The important output of the models is a suggested schedule for jack-up vessel chartering. As exact methods struggled to handle large instances, greedy construction heuristics and the Sample Average Approximation (SAA) method were applied to solve the problem.

Extensive stability testing of scenario generation methods was conducted to ensure that the underlying uncertainties were satisfactorily represented. With the implemented scenario generator, acceptable stability was found at around 100 input scenarios per scenario tree. However, stability testing indicated that a higher number of scenarios could potentially further improve solution quality. Moreover, results from stability testing were used to establish SAA parameters. In line with stability testing, acceptable SAA results were achieved when the number of samples was 10 and the sizes of sample and reference trees were 100 and 1,000 scenarios, respectively. Again, results indicated that a higher number of scenarios, or more samples, would be preferable. Uncertainty in failure occurrences was considered the most problematic as the number of possible combinations of failure occurrences is vast.

An important finding from the computational study was the difficulties of handling unrepaired components. It was not desired to leave components unrepaired, as there is no guarantee that these can be serviced at a later stage. However, as vessel charter costs dominated turbine downtime costs and weather is harsh in the North Sea, the models proposed to leave a high number of components unrepaired. In order to comply with model assumptions, significant future charter costs were imposed even if it was not considered to represent reality in a satisfying way.

It was shown that the SAA method in combination with a greedy, randomized heuristic was a suitable way of handling the complexity of the problem studied. While the SAA method reduced computational complexity, the heuristic found better pessimistic
concluding remarks

bounds than the exact methods. Thus, when applying SAA to solve the problem studied, the heuristic should be utilized to find pessimistic bounds. Compared to the Extended Model, solution times in Xpress were significantly lower when the number of repairs in each batch was limited.

In general, model solutions suggested a high number of vessel periods. Furthermore, in Basic Model solutions, an average of 15% of failed components were left unrepaired. This indicated that one vessel was not sufficient to service a 100-turbine wind farm subject to North Sea weather conditions when Basic Model assumptions applied. This was a result of too few time windows for performing maintenance, especially for the components which required long repair times.

Economic analyses were conducted using the models proposed. It was found that the wind farm size where lifetime costs of buying and chartering a vessel were equal, was around 71 turbines. For wind farms with 71 turbines or more, the option of buying a jack-up vessel should be considered. The accompanying approximate utilization rate making buying an economically sensible option was a mere 13.1%, or 47.8 days per year. When chartering, it was shown that vessel costs dominated turbine downtime costs with the latter constituting only 2-5% of total yearly costs. These results show an imbalance in supply and demand for jack-up vessels with the supply being low compared to demand, driving charter prices up.
FUTURE RESEARCH

This chapter suggests aspects of the problem that could be evaluated in future research. Section 12.1 suggests uncertainties that could be incorporated. Sections 12.2 and section 12.3 presents suggestions for sub-chartering and further developments for the heuristics, respectively. In sections section 12.4 and section 12.5, suggestions for how to handle unrepaired components and improve scenario generation are presented.

12.1 Uncertainty in Charter Rates and Electricity Prices

The models in this study assumed deterministic charter rates and electricity prices over the planning period. In reality, this was a bold assumption. As data for jack-up vessel charter rates is scarce, rates for dry bulk carriers can be investigated. The financial crisis in 2008 prompted a 93.5% collapse in dry bulk rates in seven months. This shows, even if it is an extreme case, that severe uncertainty is present, thus, the model should consider this in some manner.

Recently, the UK has contemplated reducing or even removing subsidies for offshore wind farms. Thus, revenue uncertainty for wind farm operators might increase drastically. This aspect should be incorporated in future models. One can argue that electricity prices carry less uncertainty than charter rates. At least in the Norwegian market, the production is somewhat flexible given enough water in hydropower reservoirs. Even so, a more realistic model should incorporate electricity price uncertainty.

If incorporating uncertainty in charter rates and electricity prices proves too time-consuming or difficult, sensitivity analysis can be conducted. For instance, low, medium and high prices/rates can be used as input and the solutions investigated to assess the effect.
12.2 SUB-CHARTERING

In the case where the jack-up vessel has been chartered and all failed components are repaired, the opportunity to sub-charter the vessel might limit any losses incurred by the wind farm operator. Because of the relatively long planning period, sub-chartering might mitigate some of the significant risk faced; if more uncertainties are added to the model, sub-charter provides valuable flexibility. In line with reality, a minimum sub-charter period should be imposed on the model.

12.3 FURTHER DEVELOPMENT OF HEURISTICS

As seen in chapter 10, the heuristics developed in this study has a potential for further development. For instance, improvement heuristics can be introduced together with tabu search to create better batches. A simple example is for every batch, to remove the most expensive or the most time-consuming maintenance operation and add another operation from a candidate list. Tabu criteria should be implemented to ensure maintenance operations previously in the batch are not added once more.

The authors suggest the dominance criteria utilized in this study to be investigated. The criteria were most likely too weak to erase a substantial number of batches when the number of repairs in a batch exceeded two. Improved heuristics might enable solving larger scenario trees and limit the need for the somewhat cumbersome SAA method. To facilitate usage of heuristics, the scenario generation and heuristics code written for this study should be rationalized. Probably, the highest potential for improvement is in the heuristics code which increased the time needed for scenario generation by approximately 240%.

12.4 HANDLING OF UNREPAIRED COMPONENTS

As described in subsection 10.1.7, the way the proposed models handle unrepaired components is problematic. The enforced future charter for leaving components unrepaired throughout the planning period does not reflect reality in a satisfying way since the penalty is incurred for every unrepaired component in every scenario. In this study, if the expected number of failures was used instead of the actual total, the vessel was seldom chartered at all, see subsection 10.1.7. The authors suggest that the way of handling penalty costs for unrepaired turbines is investigated and altered.

One way to handle unrepaired components is to introduce multiple objective functions and use Pareto fronts. The idea would be to have one objective function minimizing the number of unrepaired components and another minimizing costs without taking unrepaired components into account. A Pareto front can be found iteratively by first
fixing the maximum number of unrepaired components to a high number, then gradually decreasing it and solve the cost problem for every instance. This may better handle the unrepaired components and lead to more realistic solutions in terms of both first stage decisions and objective values.

In stochastic programming, representing uncertainty, especially uncertainty with multidimensional distributions, is difficult [29]. Failure uncertainties, in combination with weather uncertainties, are considered multidimensional as no significant correlation is present. Thus, it would require a huge number of scenarios to satisfactorily represent these uncertainties. In this study, the number of weather scenarios was nine. Therefore, with 1,000 scenarios in total, 111 failure occurrence scenarios were considered for each weather scenario, on average. Considering the vast number of possible combinations of failure occurrences, this might not be enough. Further, no means of reducing the number of scenarios required to represent these uncertainties were evaluated. This is, however, suggested for future research. If the number of scenarios required to represent, for example, failure uncertainties, is reduced, this could help improve computational times and allow for uncertainty in, for instance, electricity prices to be considered. One known method for scenario reduction is moment matching. The idea behind moment matching is to match the statistical properties, such as mean, variance and skewness of the population sample to the corresponding statistical properties of the entire population [29].
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