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Creative Content and Inclusive Classrooms to Transform Student Learning:
Liberating Structures for Mathematics Educators

by

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An Article in

I describe how K-12 teachers may significantly improve student learning, engagement, and retention of mathematical concepts by (1) creatively designing their course content around real-world problems, and (2) employing Liberating Structures (LS)—simple protocols to organize classroom interactions through different spatial arrangements, group configurations, distributing participation, and sequencing of steps (Lipmanowicz & McCandless, 2014).

As a teacher of 33 years, and an avid LS user for the past dozen, I believe it is possible to transform mathematics education—both through the creation of applied real-world curriculum, and an inviting, inclusive, and engaging classroom space (Singhal, 2014). I provide concrete examples of how STEM\textsuperscript{2} courses and lessons can be creatively designed to spark student interest, engagement, and curiosity and illustrate how K-12 teachers can significantly boost math learning by incorporating simple LS in classrooms.

Creatively Designed Content for Real World Problem-Solving

From a learners’ perspective, mathematics education privileges abstract thinking and esoteric symbols, and lacks a context-based, real world, problem-solving orientation (Garfunkel & Mumford, 2011). A simple illustration of mathematical abstractness is reflected in the manipulation of unknown quantities—the proverbial variable x in algebra. When a student does not see the relevance, salience, and applicability of mathematical concepts—whether in arithmetic, algebra, geometry, or calculus—fear, anxiety, and apathy toward the subject is natural (Drew, 2011; Schoenfeld, 2016). As the wise man pointed out: “You can’t fight something with nothing.”

What if the tables were turned? What if curricular practices in K-12 math education, were centered around concrete, applied, and real world problem-solving contexts? Let me illustrate.

A story on National Public Radio\textsuperscript{3}—about two engineering professors at the University of California Davis who teach a course titled Design for Coffee—grabbed my attention. Over 1,500 undergrads took this UC-Davis course in 2015—most of them non-engineering students. While STEM classes are generally feared by college students, why
is *Design for Coffee* so widely popular? Simply, the course brings coffee and chemistry together for college students in an engaging, hands-on manner. Student teams start with identical unroasted coffee beans, competing to produce the best-tasting brew. Along the way, discussions abound on the chemical reactions occurring while coffee beans are roasted, about “mass transfer” as hot water extracts oils and aromas from grounded beans, and about “fluid dynamics” as water drips and flows through undulating pipes.

Just as *Design for Coffee* represents a highly-creative way of designing a semester-long STEM offering, math educators can similarly design highly-engaging, practical, and user-relevant content.

**Pencils, Packaging, and Parallel Planes**

Consider a math session titled *Pencils, Packaging, and Parallel Planes*. The teacher’s first PowerPoint slide asks—*Why do pencils tend to have hexagonal rather than circular cross-sections?* The teacher may run through several slides to point out the attributes of hexagonal pencils:

- they are cheaper to produce i.e. a piece of wood that could make 8 round pencils can make 9 hexagonal ones,
- they are less likely to roll off a desk,
- they are easier to grasp and sharpen,
- they allow for easy printing of typeface on a flat surface.

The teacher’s concluding slide (Figure 1) may bring home the concept of parallel planes—i.e. an explanation of how, compared to round pencils, more hexagonal pencils can be packed in a box as the parallel planes of the flat edges allow for a snugger fit. A tighter fit means lower handling, packaging, and shipping costs.

![Figure 1. circular pencils versus hexagonal ones](image)

**Ham, Handling, and Hypotenuse**

Consider another class session titled *Ham, Handling, and Hypotenuse*. The teacher’s first PowerPoint slide asks—*Why are ham and cheese sandwiches mostly cut along the diagonal and not in the middle?* The teacher may use several slides to discuss the attributes of a diagonally-cut sandwich. For instance:
• the diagonal cut exposes more of the ham and cheese giving the illusion that the sandwich is bigger (as per Pythagoras Theorem, the hypotenuse of a right triangle is longer than either side),

• the angular corner allows for an easier bite and dipping,

• a longer surface along the diagonal allows the sandwich sides to stick better (i.e. more surface tension).

Figure 2. A diagonally-cut sandwich versus one cut in the middle

From a student’s learning perspective, curricula designed around every day, real world artifacts—like pencils and sandwiches—can make mathematical concepts more accessible, fun, and relevant, thereby aiding retention, sparking curiosity, and improving learning. Imagine what fun would it be if math teachers taught the fundamentals of differential calculus by having students plot and graph Usain Bolt’s 100M sprint—i.e. how his speed and acceleration changes with every step. Once a student can dissect Bolt’s sprint, they begin to grasp how differential calculus can help them predict the rates of chemical reactions when coffee beans, for instance, are roasted.

Transforming Classroom Interactions with Liberating Structures

In addition to creatively designing math content around real world problems, what if teachers could engage all students in generating solutions through carefully-designed interactions and distributing task, time, and responsibility? That is, by implementing liberating structures (LS).

To grasp what LS can do to further boost student learning, let us revisit our discussion on hexagonal pencils. After posing the question—Why do pencils tend to have hexagonal rather than circular cross-sections?, the standard practice would be to ask the students for a response, or for the teacher to answer the question lecture-style. If student input is solicited, the teacher may ask 2-3 students to share their thoughts, and then fill in
the missing elements. Most students would likely listen passively, and perhaps some would have likely zoned out.

Instead, a teacher may use a simple LS such as *Impromptu Networking* ([http://www.liberatingstructures.com/2-impromptu-networking/](http://www.liberatingstructures.com/2-impromptu-networking/)) or *1-2-4-All* ([http://www.liberatingstructures.com/1-1-2-4-all/](http://www.liberatingstructures.com/1-1-2-4-all/)). If *Impromptu Networking* is employed, the teacher would first ask the students to stand up, and pair up preferably with someone they didn’t know well (Lipmanowicz, Singhal, McCandless, & Wang, 2015). Each student would have 60 seconds (2 minutes in a pair) to answer—*Why do pencils tend to have hexagonal rather than circular cross-sections?* The teacher tells the student that after the first round a bell would ring and they would have to pair up with another student for another 2 minutes while addressing the same question. And then there would be a third round.

By distributing participation in three rounds of paired configurations for two minutes each, the teacher gets all students immediately engaged—first in sharing their response, and then listening to their partner (Figure 3). Three rounds of conversations mean three opportunities to reflect more deeply and learn from peers. At the end of the six minutes, the teacher can ask: “Who would like to share something you heard that you thought was valuable?” The teacher would let the sharing go on until it ended, effortlessly capturing all the student insights within a few minutes. If any gaps remain, the teacher can address them.

![Figure 3. All students engaged at the same time in an *Impromptu Networking* Session.](image-url)
This small example of adding LS to a creatively-designed lecture—whether on hexagonal pencils, diagonally-cut sandwiches, or Usain Bolt’s sprint—illustrates how LS-inspired interactions can make a classroom enjoyable, dynamic, and productive for all students (Lipmanowicz, Singhal, McCandless, & Wang, 2015). Each student is actively engaged in multiple interactions from start to finish, and has equal time to speak and be heard. These multiple peer interactions build connections and trust between students. Allowing the entire variety of contributions to emerge from the group enriches the conversations while leveling the playing field. The teacher becomes more of a facilitator, a partner in discovering solutions, as opposed to the sole dispenser of knowledge.

While Impromptu Networking is one of the simpler Liberating Structures, it is illustrative of what becomes possible to do in any classroom. My friends and LS co-founders and mentors, Henri Lipmanowicz and Keith McCandless, have systematized some three dozen LS and have made them freely available on their website (www.liberatingstructures.com) and also compiled them in a book (Lipmanowicz & McCandless, 2014). Table 1 lists some simple LS that math educators can use in their classrooms.

<table>
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<tr>
<th>Brief Description of Liberating Structures</th>
<th>Icon</th>
<th>Example of Classroom Use</th>
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<tbody>
<tr>
<td><strong>Impromptu Networking:</strong> Rapidly share ideas, expectations, and challenges building new connections.</td>
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<td>Invite participants to reflect on their first individual class presentation and ask what they would do differently.</td>
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<td><strong>1-2-4-All:</strong> Engage everyone simultaneously in generating questions, ideas, and suggestions.</td>
<td>🎉</td>
<td>Invite participants to generate the most vexing questions that they are struggling with, including prioritizing the ones the class should collectively tackle.</td>
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<td><strong>Conversation Café:</strong> Engage everyone in making sense of profound challenges.</td>
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<td>Invite participants to discuss how to tackle their most challenging questions by expanding and deepening the solution space.</td>
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<td><strong>User Experience Fishbowl:</strong> Share know how gained from experience with a larger community.</td>
<td>🐟</td>
<td>Invite groups to share their unique field experiences, insights, and struggles with the whole class.</td>
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*Note: More description of these liberating structures, including how and when to use them, can be found in (Lipmanowicz & McCandless, 2014) and at www.liberatingstructures.com*
For any educator—STEM or otherwise—creatively designed, user-relevant content and liberating structures can transform student learning. Table 2 summarizes the principles behind LS, and what is made possible by LS to transform student learning.

Table 2. What Liberating Structures Make Possible in a Classroom and How

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<tr>
<td>1. Include and Unleash Everyone:</td>
<td>Invite all students to share possible solutions.</td>
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<td>2. Practice Respect for Participants Past Experiences:</td>
<td>Engage the people doing the work (the students), letting go of the compulsion to control.</td>
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<td>3. Start with a Clear Purpose:</td>
<td>A teacher digs deep for what is important and meaningful to convey.</td>
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<td>4. Build Trust as You Go:</td>
<td>Cultivate a trusting climate in class by sifting ideas and utilizing input from everyone.</td>
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<td>5. Learn by Failing Forward:</td>
<td>Make it safe for students to speak up and to discover positive variation.</td>
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<td>6. Practice Self-Discovery Within a Group:</td>
<td>Engage students to discover solutions on their own. Increase diversity to spur creativity, and enrich peer-to-peer learning.</td>
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<td>7. Amplify Freedom and Responsibility:</td>
<td>Specify minimum constraints to invite student participation (e.g. in pairs, for two minutes, and three rounds). Value fast experiments and celebrate mistakes.</td>
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<tr>
<td>8. Emphasize Possibilities:</td>
<td>Teacher focuses on what is working well, and what is possible now.</td>
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<tr>
<td>9. Invite Creative Destruction to Enable Innovation:</td>
<td>Convene classroom conversations that make it easy for students to deal with fears and ask for help.</td>
</tr>
<tr>
<td>10. Engage in Seriously Playful Curiosity:</td>
<td>Stir things up—with levity and interesting questions—to spark a deep exploration of the world around us.</td>
</tr>
</tbody>
</table>

Source: Adapted from Lipmanowicz, Singhal, McCandless, and Wang (2015)

In Conclusion

K-12 math educators (or, for that matter, any educator) can significantly boost student learning by creatively designing their course content around practical, real-world problems, and by employing liberating structures—simple organizing protocols for classroom interactions that invite and include all students while simultaneously distributing task, time, and responsibility.

I invite you to try these strategies in your mathematics classes.

References


**LS Video Resources**

*Learning Nine Liberating Structures* with Lipmanowicz and Singhal.
http://vimeo.com/60843778 43”

*UnScripted: Liberating Structures* by Arvind Singhal
https://vimeo.com/51546509 10’ 20”
1 Singhal is the Samuel Shirley and Edna Holt Marston Professor of Communication at The University of Texas at El Paso and appointed Professor 2 at the Faculty of Business Administration, Hedmark University of Applied Sciences, Norway. He thanks his LS colleagues—Henri Lipmanowicz, Keith McCandless, Fisher Qua, and Lucia Dura—for their guidance and inputs.

2 STEM stands for Science, Technology, Engineering and Mathematics.


7 This description of *Impromptu Networking* is adapted from Lipmanowicz et al. (2015).