APPROACHES TO AVOID DOUBLE COUNTING IN HYBRID LIFE CYCLE INVENTORIES
Approaches to Avoid Double Counting in Hybrid Life Cycle Inventories

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Abstract

Hybrid life cycle analysis methodology offers a convenient way to combine traditional life cycle inventories and input output tables to construct inventory data that has both good detail and completeness. This combination easily leads to double counting, i.e. the accounting of the same production process in both LCA process and input-output requirements matrix. This paper presents some aspects of double counting in hybrid inventories. An algorithm for identifying double counting and four methods to adjust for this are developed. The first two have a low complexity of implementation but limitations to their applicability when performing a detailed assessment. The last two have a somewhat higher complexity in implementation but provide correct results for all types of structural inventory analysis. By the application of one of these, the quality of a hybrid life cycle inventory can be increased at a low effort.

Key words: Hybrid Life Cycle Assessment, Environmental Input-Output Analysis, Life Cycle Assessment, Double Counting

1 Introduction

Life cycle assessment has for quite some time been considered the favorite tool for analyzing the environmental impacts of products systems. The methodology has evolved and improved throughout the last decades. One of the strongest critiques against life-cycle assessment was directed against how the

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process inventories were established. Several independent studies have shown that life cycle assessment uses incomplete inventories. The processes left out account for up to 50% of the environmental pressures when assessed with input-output methods [1–4]. While environmental input-output analysis gives a good description of the total repercussions for a given purchase in the economy, life-cycle inventories provide a more detailed description of the flows and nodes thought to be the important ones in a product network. As a response the critique a combination of life-cycle assessment and input-output analysis has been found favorable. The methodological framework for this was brought forward by several researchers [1, 5–9], and is now commonly referred to as hybrid life-cycle assessment. In this paper we address the methodological challenge of double counting in hybrid LCA. We believe that this work will contribute to further mature the hybrid LCA framework.

To understand the problem of double counting in hybrid life-cycle inventories it is necessary to have a good understanding of the basic methodology. We here give a brief introduction to the subject. For a thorough introduction to hybrid LCA we recommend the articles [6, 8]. We start by giving a short introduction to the mathematical framework of input output analysis which hybrid LCA is based on. The core of an input-output model is the requirements, or coefficients, matrix $A$. The columns of this matrix describe the intermediate inputs an industry buys from itself and other industries, to produce one unit of output. In equation 1 the industry output vector, $x$, is the sum of the final demand vector, $y$, plus the industry activity required to supply input to the production, the intermediate demand, $Ax$. Then, solving for $x$ to find the resulting industry output for a given demand $y$;

$$Ax + y = x \iff x = (I - A)^{-1}y$$

$x$ represents the production of a set activities in all industrial sectors to satisfy a certain final demand $y$. $(I - A)^{-1}$ is known as the Leontief inverse. These equations will naturally work with $A$ describing, not only monetary flows, but also mass and energy flows as in LCA. The stressors matrix, $S$, contains the various emissions factors for each of the industries. Being able to calculate the resulting flows of a purchase and knowing the direct environmental stressors related each industry we can find the total environmental stress vector, $e$, as shown in equation 2

$$e = S(I - A)^{-1}y$$

Recommended background literature on input-output analysis includes [10–14]. For a thorough introduction to the mathematics of LCA see [15]. For our introduction to hybrid LCA we start by defining the dimensions of a hybrid
Table 1

<table>
<thead>
<tr>
<th>Indexes</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>f</td>
<td>Foreground processes. Processes defined in study</td>
</tr>
<tr>
<td>p</td>
<td>Background processes. Processes from LCA database</td>
</tr>
<tr>
<td>n</td>
<td>Background commodities. Commodities of the economy</td>
</tr>
</tbody>
</table>

\[
i, j \quad \{f, p, n\} \in \{i \wedge j\}
\]

requirements matrix. Here the requirements matrix, \(A_{i,j}\), contain a combination of monetary and physical units. We follow Suh et.al [8, 9], but expand the system so that we distinguish between the foreground and background processes. Those are respectively the processes defined in the study and base LCA database processes that are used to complete the inventory. The various indexes are described in table 1.

In equation 3, two types of hybrid LCA inventory matrices are shown. The nomenclature for the various sub-matrices are given in table 2. On the left is what we refer to as a integrated hybrid model [8, 9]. This approach aims to fully integrate the foreground and the background LCA matrices into the economy. That is, let \(A_{f,f}\) and \(A_{p,p}\) represent a detailed sub section of the economy. In the process of doing this Suh et.al [8, 9] adjust, at the make and use level, the \(A_{n,n}\) matrix so that the total amount of output from the combined \(A_{i,j}\) matrix is equal to the original \(A_{n,n}\) matrix. In other words the processes in \(A_{f,f}\) and \(A_{p,p}\) replace the corresponding input-output commodities and services linearly with the volume of production. Due to the downstream linkages, a integrated hybrid model provides an improved description of the "average commodity" since selected commodities within that commodity group have a more detailed inventory. The problem of double counting arises when we know that we do not purchase the average commodity.

\[
A_{i,j} = \begin{bmatrix} A_{f,f} & A_{f,p} & A_{f,n} \\ A_{p,f} & A_{p,p} & A_{p,n} \\ A_{n,f} & A_{n,p} & A_{n,n} \end{bmatrix} \quad A_{i,j} = \begin{bmatrix} A_{f,f} & 0 & 0 \\ A_{p,f} & A_{p,p} & 0 \\ A_{n,f} & 0 & A_{n,n} \end{bmatrix} \quad (3)
\]

An integrated hybrid model requires an substantial effort to establish all the data required. If the scope is to analyze environmental effects originating from all upstream activities, as in standard LCA, a tiered hybrid analysis is sufficient. The inventory matrix for tiered hybrid LCA is shown to the right in equation 3. As opposed to integrated hybrid analysis this approach does not aim to fully integrate the \(A_{f,f}\) and \(A_{p,p}\) matrices to be a subsection of the economy. In the tiered approach an improved LCA inventory, for a given set of foreground processes, is established using input from background LCA processes and input-output commodities. As can be seen in equation 3, no
Table 2
Hybrid Matrix Nomenclature

<table>
<thead>
<tr>
<th>Matrix Type</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{f,f}$</td>
<td>Foreground processes inter requirements matrix</td>
<td>(kg/kg)</td>
</tr>
<tr>
<td>$A_{p,p}$</td>
<td>Background processes inter requirements matrix</td>
<td>(kg/kg)</td>
</tr>
<tr>
<td>$A_{n,n}$</td>
<td>Background economy inter req. coefficients matrix</td>
<td>(£/£)</td>
</tr>
<tr>
<td>$A_{p,f}$</td>
<td>Upstream inputs of background processes to foreground processes</td>
<td>(kg/kg)</td>
</tr>
<tr>
<td>$A_{n,f}$</td>
<td>Upstream inputs of commodities to foreground system</td>
<td>(£/kg)</td>
</tr>
<tr>
<td>$A_{f,p}$</td>
<td>Downstream inputs of foreground processes to background processes</td>
<td>(kg/kg)</td>
</tr>
<tr>
<td>$A_{n,p}$</td>
<td>Upstream inputs of commodities to foreground system</td>
<td>(£/kg)</td>
</tr>
<tr>
<td>$A_{f,f}$</td>
<td>Downstream inputs of foreground processes to the background economy</td>
<td>(kg/£)</td>
</tr>
<tr>
<td>$A_{p,p}$</td>
<td>Upstream inputs of background processes to the background economy</td>
<td>(kg/£)</td>
</tr>
</tbody>
</table>

upstream feed of input-output commodities to the background LCA processes and no downstream inputs at are not accounted for here. Throughout this paper we do not use a integrated hybrid model rather, for simplicity, a tiered hybrid model. The algorithms presented are developed for tiered hybrid analysis, however in principle they can be adapted to a integrated hybrid model.

2 Double Counting in Hybrid Inventories

Life-cycle inventories have a very high level of detail for specific products. Input-output tables contain information on aggregated commodity groups. When combining these two the question arises on how to treat seemingly complementary information on products. All LCA processes, $f$ and $p$, can be said to belong to a commodity group, $n$, of the input-output tables. The problem of double counting arises when we know that the background LCA processes and input-output matrix unintentionally accounts for the same commodity. The double counting incident that we will focus on in this paper is perhaps most easily understood in Tiered-Hybrid analysis. It can be illustrated by the following example. In a hybrid analysis of the production of a pressurized vessel, $A_{f,f}(f^*, f^*)$, the steel usage is modelled with inputs, $A_{p,f}(p^*, f^*)$, of a LCA steel process $A_{p,p}(p^*, p^*)$. The transformation of the steel into a pressurized vessel is modelled by inputs, $A_{n,f}(n^*, f^*)$, of a service from the mechanical engineering sector, $A_{n,n}(n^*, n^*)$. We know that the inputs to the mechanical engineering sector also contains steel products $A_{n,n}(n^{**}, n^*)$. Since we already have accounted for the steel usage via the LCA data we have a double counting incident. More precisely, due to the aggregation level of the input-output data we are not able to specify the exact subset of mechanical engineering services we here actually require. The average commodity or service may therefor in-
Table 3
Aggregation related double counting

<table>
<thead>
<tr>
<th>$A_{f,f}(f^<em>, f^</em>)$</th>
<th>Production of: Pressurized Vessel</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{p,f}(p^<em>, f^</em>)$</td>
<td>Input of $\equiv A_{p,p}(p^<em>, p^</em>)$ Steel Products</td>
</tr>
<tr>
<td>$A_{n,f}(n^<em>, f^</em>)$</td>
<td>Input of $\equiv A_{n,n}(n^<em>, n^</em>)$ Mech. Engineering Services</td>
</tr>
<tr>
<td>$A_{n,n}(n^{**}, n^*)$</td>
<td>Input of $\equiv A_{n,n}(n^{<strong>}, n^{</strong>})$ Steel Commodities</td>
</tr>
</tbody>
</table>

Table 4
Assumptions and Adjustment principles

<table>
<thead>
<tr>
<th>Assumption:</th>
<th>Adjustment Principle:</th>
</tr>
</thead>
<tbody>
<tr>
<td>I $A_{p,p}(p^<em>, p^</em>) \equiv A_{n,n}(n^{<strong>}, n^*) \Rightarrow A_{n,n}(n^{</strong>}, n^*) = 0$</td>
<td></td>
</tr>
<tr>
<td>II $A_{p,p}(p^<em>, p^</em>) \in A_{n,n}(n^{<strong>}, n^*) \Rightarrow A_{n,n}(n^{</strong>}, n^*) = F(\triangle \mathcal{L})$</td>
<td></td>
</tr>
</tbody>
</table>

Exlude inputs we do not require. This type of incident we denote aggregation related double counting. Our example of this is summarized in table 3.

Given such an incident, the first and most intuitive approach to eliminate this error would be to simply set the $A_{n,n}(n^{**}, n^*)$ element to zero. This would eliminate the double counting, but it would simultaneously generate another problem. In would permanently change the input-structure of the commodity in question, and therefore erroneously alter all the other upstream activity from this commodity.

In section 3 we present four approaches that allow us to both avoid double counting and avoid disturbing the upstream processes. However, before presenting the technicalities of these we have to decide how we should adjust for the double counting. In the table 4 we have outlined two adjustment principles. In the first case, numbered I, we assume that the process described in vector $A_{p,p}(; p^*)$ and $A_{n,n}(; n^{**})$ are equivalent. This can be assumed if we, for example, know for certain that all steel inputs are accounted for by the process inputs. We can therefore set the purchase of this commodity in $A_{n,n}(n^{**}, n^*)$ to zero. Alternatively, in the second case we assume that the process $A_{p,p}(; p^*)$ is an element of $A_{n,n}(; n^*)$. This is often the case if the level aggregation in the input-output matrices and the LCA matrices are different. If the aggregation level is high there might not even be a single steel commodity group in the input-output matrices. Rather there would be a metals commodity group, to which steel belongs. Given this, our best approach would be to adjust for the already accounted steel. Taking into account the price and amount of steel already in the model, we are able to adjust the coefficient so that it only represents the volume of purchase of the average metals commodity equal to the value of the residual metals. This principle is numbered II in table one. For simplicity we have chosen to apply principle I in the work presented here.
The first step in our approach is to identify the various double counting incidents that occur in a given case. To do so we have developed a simple algorithm that analyzes the $A_{i,j}$ matrix, identifies and stores information on the double counting incidents in an array. We have called this algorithm Hybrid Inventory Double Counting Identification (HIDCI) Algorithm. The pseudo code for this is given in Algorithm 3.1. The algorithm first searches through all of the foreground processes and identifies all entries in the process and commodities upstream input matrices. It further branches on the entries in the commodities upstream input matrix, $A_{n,f}$, into the next tier in the background economy requirements matrix $A_{n,n}$. Then the algorithm checks, using an if sentence, whether the two entries, the one in the $A_{n,n}$ matrix and the one in the process upstream input matrix, $A_{p,f}$, are complementary. This is done by using a transformation matrix $T$ that contains information on which processes belong to the various commodities. If they are complimentary, the if sentence is true, and the counting variable, $d$, is increased by one, and all the information on the double counting incident is then stored in the $\theta$ array.

**Algorithm 3.1: HIDCI(Identification)**

```plaintext
for $f^* \leftarrow 1$ to $f_{max}$
for $p^* \leftarrow 1$ to $p_{max}$
for $n^* \leftarrow 1$ to $n_{max}$
for $n^{**} \leftarrow 1$ to $n_{max}$
do 
  if $T(n^{**}, p^*)A_{p,f}(p^*, f^*) > 0$ and $A_{n,f}(n^*, f^*)A_{n,n}(n^{**}, n^*) > 0$
  then 
    $d = d + 1$
    $\theta(d, 1..4) = [f^*, p^*, n^*, n^{**}]$
```

4 Adjusting

Having identified the double counting incident, we will here explore four methods that can be used to deal with this. To ensure that the core of the algorithms are conveyed without too much interferences by implementation technicalities, we have simplified the adjustment algorithms so that they only handle cases where there is one double counting incident for each $(n^*, n^{**})$ pair.
The first method, we denote the commodity requirements column vector negative transfer method (RCVT) and is an application of the negative by-product assumption. As explained in section 2, we cannot simply modify the $A_{n,n}(n^{**}, n^*)$ element because that would disturb the upstream paths for other activities. To avoid this a scaled and adjusted commodity requirements column vector of the commodity in question can simply be inserted into the column in the upstream commodity input matrix, $A_{n,f}$, belonging to the foreground process in question. This solves the double counting problem but imposes another problem. The stressors associated with the moved commodity are not accounted for. To solve this issue, one option is to have the stressor factors of the commodity in question scaled and added to the emission inventory of the foreground process. The generation of the adjusted upstream commodity input matrix and the adjustment of the stressor matrix using this approach is shown in algorithm 4.1.

Algorithm 4.1: RCVT($adjustment$)

\begin{algorithm}
\begin{align*}
&\text{for } d \leftarrow 1 \text{ to } d_{\text{max}} \\
&\quad \left[ f^*, p^*, n^*, n^{**} \right] = \theta(d, 1..4) \\
&\quad \text{for } n \leftarrow 1 \text{ to } n_{\text{max}} \\
&\quad\quad \text{if } n = n^* \\
&\quad\quad\quad \text{then } \tilde{A}_{n,f}(n, f^*) = A_{n,f}(n, f^*)A_{n,n}(n, n^*) \\
&\quad\quad\quad \text{else if } n = n^{**} \\
&\quad\quad\quad\quad \text{then } \tilde{A}_{n,f}(n, f^*) = A_{n,f}(n, f^*) \\
&\quad\quad\quad\quad\quad \text{else } \tilde{A}_{n,f}(n, f^*) = A_{n,f}(n, f^*) + (A_{n,f}(n^*, f^*)A_{n,n}(n, n^*)) \\
&\quad\quad \text{for } s \leftarrow 1 \text{ to } s_{\text{max}} \\
&\quad\quad\quad \tilde{S}_{s,f}(s, f^*) = S_{s,f}(s, f^*) + A_{n,f}(n^*, f^*)S_{s,n}(s, n^*)
\end{align*}
\end{algorithm}

We can use the case in table 3 to exemplify how this method works. To adjust the total, and ensure that the steel is not counted twice, this method copies the vector describing the mechanical engineering services to the upstream commodity input matrix. There the purchases from steel industries are set to zero. Since we skip the mechanical engineering services, we have to add the scaled and adjusted emissions from this sector to the emissions from the foreground process.

The second method is based on the positive by-product approach. As opposed to the previous method, the foreground process in question here generates a by-product that is identical to and the same amount of that is double counted.
The total will then be correct. If we go back to table 3 and the example listed there, this method adjust for double counting of steel by simply saying that the foreground process in question generates the same amount of steel that is purchased from the mechanical engineering services. We denote this method the single coefficient positive transfer method (SCPT). Formally this method is expressed in Algorithm 4.2.

\section*{Algorithm 4.2: SCPT (adjustment)}

\begin{verbatim}
for \( d \leftarrow 1 \) to \( d_{max} \)
\quad \{ \[ f^*, p^*, n^*, n^{**} \] = \theta(d, 1..4) \}
for \( n \leftarrow 1 \) to \( n_{max} \)
\quad \{ if \( n = n^{**} \)
\quad \quad \{ then \( \hat{A}_{n,f}(n, f^*) = A_{n,f}(n, f^*) - (A_{n,f}(n^*, f^*)A_{n,n}(n, n^*)) \)
\quad \quad \{ else \( \hat{A}_{n,f}(n, f^*) = A_{n,f}(n, f^*) \) \}
\quad \}
\}
\end{verbatim}

Both of these methods provide the correct total. The single coefficient positive transfer methods is the more elegant of the two. It has the advantage over the first method that no adjustment of the emissions matrix is necessary.

We now like to point to one major weakness of these two methods presented. The application of power series expansions of the inverse provides important insight on the structure of indirect flows and emissions generated [4]. When performing a power series expansions on hybrid matrices that have been adjusted for double counting by using one of the two methods above, the results will be flawed. The RCVT method literally skips one process instance and shifts the missions from this process instance into the previous process. For an analyst it will seem as if the foreground process in question generates a higher total fraction of the environmental impacts, both emissions and resource use, than it in reality does. To understand what happens it might be useful to think of the tree of process instances that is generated through the power series expansion. When applying the RCVT method two of the process instances are summed together and the whole three of the nodes following that process instance will be offset one tier early. The SCPT method generates a flawed power series expansion because of its positive byproduct approach. We remember that by adding a byproduct to the upstream input matrix the total will be correct since the amount of this product produced by the foreground process is equal to the amount consumed in the double counting incident. The problem of this method is simply that we add the positive byproduct one tier to early. This causes the flows initiated from the byproduct to balance out the double counting flows to appear offset one tier to early. As a consequence of
this the flows calculated for each tier upstream from the foreground process in question will be wrong. So if one is simply after the total, the SCPT method is adequate. If the results from a power series expansion of the inverse is to be used in an analysis, a better method is required.

The weakness of the SCPT method can be eliminated by splitting up the expansion of the inverse into two parallel calculations, one where no adjustments to the $A_{n,f}$ matrix are made and a second where only the positive by product is present. Having performed the parallel power series expansion the correct flows for each process instance in each tier can be found by summing together the vector from unadjusted calculation in tier $t$ with the vector from the positive by product calculation in tier $t+1$. We refer to this approach as the parallel expansion single coefficient positive transfer method (PE-SCPT). The pseudocode for the adjustment part of this method is given in Algorithm 4.3.

**Algorithm 4.3: PE-SCPT** (adjustment)

```plaintext
for $d \leftarrow 1$ to $d_{max}$
  \[ \begin{bmatrix} f^*, p^*, n^*, n^{**} \end{bmatrix} = \theta(d, 1..4) \]
  \[ \text{for } n \leftarrow 1 \text{ to } n_{max} \]
    do \[ \begin{cases} 
      \text{if } n = n^{**} \\
      \text{do } \begin{cases} 
        \text{then } \hat{A}_{n,f}(n, f^*) = -A_{n,f}(n^*, f^*)A_{n,n}(n, n^*) \\
        \text{else } \hat{A}_{n,f}(n, f^*) = 0 
      \end{cases} 
    \end{cases} \]

Having established the adjusted upstream input matrices the next step is to perform the calculation of the power series, see Algorithm 4.4. First the two matrices are assembled. The unadjusted, $\hat{A}_{i,j}$ matrix and the matrix only containing the double counting incidents $\hat{A}_{i,j}$. The tier count variable, $t$, is set to zero and $x$ is set equal to the demand $y$ in tier zero. A power series expansion of the inverse is then performed and the flows in each tier are stored in respectively $\hat{x}$ and $\hat{x}$ for the $\hat{A}_{i,j}$ and $\tilde{A}_{i,j}$ matrices. The correct flows in the next tier, $x(t+1)$ is then found by subtracting the double counting flows $\hat{x}(t+1)$ in the previous tier from the unadjusted $\hat{x}(t+1)$ in the next tier. The convergence parameter $\delta$ is calculated and while this is above the maximum allowed $\delta$, the calculation continues to the next tier. The convergence parameter defined here is very crude and obviously a more refined parameter can be introduced.
Algorithm 4.4: PE-SCPT (calculation)

\[
\tilde{A}_{i,j} = \begin{bmatrix}
A_{f,f} & 0 & 0 \\
A_{p,f} & A_{p,p} & 0 \\
A_{n,f} & 0 & A_{n,n}
\end{bmatrix}
\tilde{\hat{A}}_{i,j} = \begin{bmatrix}
A_{f,f} & 0 & 0 \\
0 & 0 & 0 \\
A_{n,f} & 0 & A_{n,n}
\end{bmatrix}
\]

\[
\tilde{x}(i, t) = \tilde{x}(i, t) = y(i, t) \quad \forall \quad i, t = 0
\]

while \( \delta > \delta_{\text{max}} \)

\[
\begin{aligned}
\tilde{x}(i, t + 1) &= \sum_j \tilde{A}_{i,j}(i, j) \cdot \tilde{x}(j, t) \quad \forall \quad i \\
\hat{x}(i, t + 1) &= \sum_j \hat{A}_{i,j}(i, j) \cdot \hat{x}(j, t) \quad \forall \quad i
\end{aligned}
\]

do

\[
\begin{aligned}
x(i, t + 1) &= \tilde{x}(i, t + 1) - \hat{x}(i, t) \quad \forall \quad i \\
\delta &= \sum_i x(i, t) \\
t &= t + 1
\end{aligned}
\]

The last method that we will present here deals with the double counting problems in a different manner than the three previous approaches. While the previously presented methods maintain the dimensions of the initial hybrid requirements matrix this last approach deals with double counting by allowing to resize and add altered commodities to the \( A_{n,n} \) matrix. Instead of moving the altered commodity requirements column vector to the \( A_{n,f} \) matrix as in the RCVT approach this method simply establishes a new commodity by resizing the \( A_{n,n} \) matrix. The element in the upstream input matrix must accordingly be moved. The emissions matrix must also be expanded as the altered commodities are added to the \( A_{n,n} \) matrix. In this process it is assumed that the altered commodities inherit the emission intensities of the original commodity. As for the altered commodity’s inter-connectivity in the economy it is assumed that it does not occur in any of the other commodities input-structure. The only column where the altered commodity occurs as an input is to the foreground process where the double counting incident originated. We refer to this as the altered commodity expansion (ACE) method. In Algorithm 4.5. the formal description of this method is shown. Including the adjustment of the stressors matrix \( S \).
Algorithm 4.5: ACE (adjustment)

\[ \tilde{A}_{d,d} = 0 \quad \forall \quad d, d \quad \tilde{A}_{n,d} = 0 \quad \forall \quad n, d \]
\[ \tilde{A}_{d,n} = 0 \quad \forall \quad d, n \quad \tilde{A}_{d,f} = 0 \quad \forall \quad d, f \quad \tilde{S}_{s,d} = 0 \quad \forall \quad s, d \]
\[ \tilde{A}_{n,f} = \begin{bmatrix} A_{n,f} \\ \tilde{A}_{d,f} \end{bmatrix} \quad \tilde{A}_{n,n} = \begin{bmatrix} A_{n,n} & \tilde{A}_{n,d} \\ \tilde{A}_{d,n} & \tilde{A}_{d,d} \end{bmatrix} \quad \tilde{S}_{s,n} = \begin{bmatrix} S_{s,n} & \tilde{S}_{s,d} \end{bmatrix} \]

for \( d \leftarrow 1 \) to \( d_{\text{max}} \)

\[ [f^*, p^*, n^*, n^{**}] = \theta(d, 1..4) \]

for \( n_i \leftarrow 1 \) to \( n_{\text{max}} \)

\[ \text{if } n_i = n^* \]
\[ \tilde{A}_{n,f}(n_{\text{max}} + d, f^*) = A_{n,f}(n^*, f^*) \]
\[ \tilde{A}_{n,f}(n^*, f^*) = 0 \]
\[ \tilde{A}_{n,n}(n_i, n_{\text{max}} + d) = A_n(n_i, n^*) \]
\[ \text{else if } n_i = n^{**} \]
\[ \tilde{A}_{n,n}(n^{**}, n_{\text{max}} + d) = 0 \]
\[ \text{else } \tilde{A}_{n,n}(n_i, n_{\text{max}} + d) = A_n(n_i, n^*) \]

for \( s \leftarrow 1 \) to \( s_{\text{max}} \)

\[ \tilde{S}_{s,n}(s, n_{\text{max}} + d) = S_{s,n}(s, n^*) \]

The first part of the ACE algorithm, simply copies the contents of the \( A_{n,n} \) and \( A_{n,f} \) matrix to respectively the \( \tilde{A}_{n,n} \) and \( \tilde{A}_{n,f} \) matrix and resizes them so there is room for the altered commodities. The main for loop of the ACE adjustment algorithm places the altered commodities in the resized \( \tilde{A}_{i,j} \) matrix. This it does by moving the element calling the original commodity in the upstream input matrix to the extended part so that it calls the altered commodity. Then the element initiating the double counting in the commodity upstream input matrix is set to zero. Further the altered commodity is moved and the element causing double counting is set to zero. Finally the stressor vector of the original commodity is copied to the position in the expanded stressor matrix belonging to the altered commodity.
5 Discussion and conclusion

Identification and characterization of double-counting incidents is obviously essential to eliminating them. We have, in this paper, focused on aggregation related double-counting that for us was the most urgent to deal with in our analysis. Further work should investigate this in more detail. The HIDCI algorithm that we have produced for identifying double counting incidents is simple and easy to implement in any LCA software.

As always with algorithms the outcome depends on the data input. In this case, the level of aggregation of the background processes and economic requirements matrix is of great importance. How well the various LCA processes and input-output commodity groups correspond has great influence on the result. Ideally the LCA processes would also have NACE sector codes. If this was so, generating the $T$ matrix would be much simpler and making the integration of input-output and LCA data in hybrid analysis much more robust. However, this is not so. Generally, a LCA matrix has a much higher level of resolution than a input-output matrix. This poses challenges when combining the two in a hybrid model. This can be illustrated by looking at steel products in input-output matrices and in life cycle process matrices. It is common in life cycle analysis practice to have different types of steels or at least have a high resolution on steel commodities. If the level of aggregation in the input output analysis is high, steel may not at all be listed as a single commodity. Steel might be grouped together in a metals commodity group.

The level of aggregation has implications for the algorithms presented here. Obviously a high degree of uniformity in the level of aggregation would directly relate to the precision of the transition matrix $T$. Naturally a one-to-one relation between commodity and process would eliminate any chance for the algorithm to misjudge a non double counting incident for a double counting incident. As the number of processes in the LCA requirements matrix, $A_{p,p}$, belonging to a commodity group increases, the danger of an erroneous adjustment also increases. We have not tried to make our algorithms deal with this, rather we suggest that this can be dealt with as the algorithms are incorporated in software. As double counting incidences are encountered, the user can be prompted to make the decision on if to make the adjustment or not.

The four methods we have presented here have different qualities which are important to take into consideration when choosing which one to use. The first two, RCVT and SCPT have a low complexity of implementation but has limitations to their usefulness in detailed analysis. The total will be correct but when for example performing a power series expansion of the inverse they will provide flawed results on the activity in each tier. Of the two we recommend the SCPT due to its simplicity since it does not involve any manipulation
of the emissions matrix. The PE-SCPT and ACE method both provide correct results for a power series expansion of the inverse. Even though these algorithms have some higher complexity than the first two they are relatively simple for skilled programmers to implement. We therefore strongly recommend to implement either the PE-SCPT and ACE. Which of these to implement is a matter of taste and obviously dependent on the structure of the code in which it shall fit so we cannot provide one conclusive answer to this. However by the application of one of these two methods the quality of a hybrid life cycle inventory can be increased at a low effort.

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The Industrial Ecology Programme (IndEcol) is a multidisciplinary university programme established at the Norwegian University of Science and Technology (NTNU) in 1998 for a period of minimum ten years. It includes a comprehensive educational curriculum launched in 1999 and a significant number of doctoral students as well as research projects geared towards Norwegian manufacturing, energy and building industries. The activities at IndEcol have a strong attention to interdisciplinary research and teaching, bridging technology, natural and social sciences in the search for sustainable solutions for production and consumption of energy and resources.