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Shear capacity of prestressed hollow-core slabs
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INTRODUCTION
This paper reports the results of a great number of full scale tests on factory-produced hollow-core slabs. The investigation was conducted at the Norwegian Building Research Institute. So far, very little research has been carried out to determine the shear capacity of the described type of slabs. Only few results from commissioned work ordered by industry in Europe and America are available.

BACKGROUND
There are no special code provisions for hollow-core slabs as far as the author knows. Normally the calculations are done in accordance with the general code for slabs and beams. According to the Norwegian Code [1] the smallest design shear capacity, without shear reinforcement and having a strand area as in present tests, is given by:

\[ V_d = f_r (bd + 75A_v) \]  

where: \( f_r \) = design shear stress \( (N/mm^2) \)  
\( b \) = the width (mm)  
\( A_v \) = total area of main reinforcement (mm²)

In a hollow-core slab \( b \) has normally been taken equal to the sum of the minimum rib widths. The effect of the prestressing force is normally neglected. Eq. (1) is based on the same principles as those on which the Model Code [2] has been based. In connection with the use of Eq. (1) for hollow-core slabs the following problems are important:

1. How strong is the anchorage of the strand?
2. What is the influence of the concrete extruding?
3. What is the influence of the geometry?
4. What is the correct value of \( f_r \)?

The special method of production (extruding) of these slabs makes the assembling of anchorage reinforcement and stirrups difficult if not impossible. The only way to increase the capacity is therefore to fill the cores near the support. Unfortunately this method is rather cumbersome and expensive. Still, the filling near the support often has to be done, because Eq. (1) gives low capacity.

The question of control of hollow-core slabs is of a considerable interest. Control criteria proposed by Anderson and Anderson [7] for flexural bond seems useful for the control of the shear capacity.

THE OBJECTIVE
The main objective of the present work is to establish:

1. Design shear capacity  
2. Behaviour and mechanical model  
3. Quality control criteria

An experimental investigation was the most convenient because of the many variables, and because the anchorage capacity seemed to be difficult to calculate.

TEST PROGRAMME
Procedure
The following is examined:

1. Dimensions  
2. Free end slip  
3. Concrete strength  
4. Deflection  
5. Strand deformation  
6. Slab stresses  
7. Cracks  
8. Failure

Test specimens
These consist of hollow-core slabs and beams grouped into a total of 7 different series (A to G). The beams are parts of the slabs. The cross section of Series A and E (Dy-core or Spanndekk as named in Norway) is shown in Fig. 1. Series B is sawn off from the slab. Series F (Spiroll) is shown in Fig. 2. Series C and D are wet casted and the strands were not prestressed. The section of Series G (Elematic) is roughly the same as Series F (Spiroll), but the production equipment is different.

Seires A, E, F and G have the following data:

- Section area: \( A_v = 173 \cdot 10^3 \text{ mm}^2 \)
- Concrete: extruded cube strength 55 N/mm²
- Reinforcement: 7-wire strands diameter 12,5 or 9,5 mm  
  02-limit 1750 N/mm²  
  total area \( A_v \) (bottom)  
  \( (A_v/A_n) \) = 0,23 - 0,58

The properties of concrete are given in Tab. 1.

<table>
<thead>
<tr>
<th>Materials</th>
<th>Series A, B and E</th>
<th>Series C, D and F</th>
<th>Series G</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand, mm</td>
<td>0–1</td>
<td>0–4</td>
<td>–</td>
</tr>
<tr>
<td>Gravel, mm</td>
<td>0–8</td>
<td>0–8</td>
<td>0–8</td>
</tr>
<tr>
<td>Crushed rock, mm</td>
<td>4–8</td>
<td>0–12</td>
<td>0–16</td>
</tr>
<tr>
<td>Cement/aggregate (weight)</td>
<td>1:5:6</td>
<td>1:7:3</td>
<td>1:5:0</td>
</tr>
<tr>
<td>W/C (weight)</td>
<td>0,32</td>
<td>0,32</td>
<td>0,29</td>
</tr>
<tr>
<td>Admixture</td>
<td>Betokem</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>
Summary of tests
The test setup and equipment are shown in Fig. 3 and 4.

Series A, E, F and G: 55 tests
" B, C and D: 59 "
Variables (Fig. 3): 
\[ s = 25 - 425 \text{ mm} \]
\[ a/d = 0.5 - 3 \]
Age of slabs: 22 – 317 days

TEST RESULTS
Dimensions
The measured values differed very little from the nominal ones. The average distance \( d_c \) (Fig. 3) to the centre of the strands was 32 mm for Series A, 34 for E, 38 for F and 27 for G.

Free end slip
The values as shown in Fig. 5 are small in spite of the fact that many of the elements were considered to be of second class quality. However, the slips within the same element varied considerably. The results do not indicate that the free end slip increases by time.

Concrete strength
Four cylinders were core-drilled vertically from the top of every element. Two of these were used to examine the compressive strength, the remaining two were used to examine the splitting tensile strength. The compressive strength is large with one exception, but varies considerably within the same element as shown in Fig. 6.

The ratio between the splitting tensile strength and the compressive strength is shown in Fig. 7. For Series E the splitting tensile strength is approximately 0.73 \( f_{ct} \), with \( f_c \) in N/mm².

Slab stresses
Strain gauges were mounted on only one element, as shown in Fig. 1 and 3. The concrete stresses indicate the effect of the prestressing and the load action, the results are shown in Fig. 8. The average of the compressive stresses from the prestressing force was 0.6 N/mm² in points 1 – 6 (Fig. 1). The ratio between the tensile stresses in points 3 and 3 d (Fig. 3) due to the load was:

0.21 with \( P = 50 \text{ kN} \)
0.31 " = 152 "
0.82 " = 240 "
(P = 242 kN at failure)

The stress was highest in the middle of the width of the slab.

Deflection
Except for a few tests there were no discontinuity in the deflection curves as shown in Fig. 9. This indicates that only few of the strands fail, if any, before the slab collapses.

Cracks
The observed cracks are caused by:
1. Diagonal splitting
2. Shear bending
3. Anchorage tension
4. Flexural tension
5. Shear
6. Diagonal compression
7. Instability of the arch

Fig. 9. Relationship between deflection \( \delta_c \) and load \( P \) for Series E, F and G. • Forholdet mellom nedbøyningen \( \delta_c \) og lasten \( P \) for serie E, F og G.

Fig. 8. Total stress at bottom of one slab with \( A_y = 400 \text{ mm}^2 \) of Series E. • Samlet spenning I underkant av en plate med \( A_y = 400 \text{ mm}^2 \) fra serie E.

Fig. 7. Ratio between splitting tensile strength \( f_{ct} \) and compressive strength \( f_c \) of concrete cylinders (73 x 73 mm).

Series A, E, F, G: 55 tests
B, C and D: 59 "
Variables (Fig. 3): 
\[ s = 25 - 425 \text{ mm} \]
\[ a/d = 0.5 - 3 \]
Age of slabs: 22 – 317 days

Fig. 6. Compressive concrete strength \( f'_c \) as measured on two cylinders (73 x 73 mm) from each slab.

Fig. 5. Free end slip \( \Delta_{ae} \) of strands in each test. • Glipp \( \Delta_{ae} \) i spenntau før belastning.

Fig. 4. Test in progress. • Provebelastning.

Fig. 3. Test setup. • Proveoppstilling.
The most frequent type of cracks are shown in Fig. 10-12. The ratio between the load \( P \) at the first visually observed crack and at failure was 0.8-1.0. The distance \( c \) from the edge of the support to the end of the element was in this case 50-100 mm.

If \( c \) is small, it is not necessary to calculate the crack width. If \( c > 400 \) mm the crack width must be controlled.

Flexural tension will cause cracks when the area of strands \( A_s \) is small. Shear may cause horizontal cracks between the pressure flange and the rib (especially the outer rib) if the compaction of the concrete is bad and/or the width is small. Diagonal compression and instability of the arch can only take place when both \( A_s \) and \( c \) are especially large.

Shear capacity at failure

Obviously both the geometry and the reinforcement are important, as shown in Table 2. It is assumed that the geometry of Series B and C cause about the same capacity. Based on Series B1 and C, as shown in Table 2, the shear capacity increases 31\% because of the prestressing force.

When the distance \( c \) is small the shear capacities are not much affected by the ratio of shear span \( a \) to effective depth \( d \) as shown in Fig. 13. It is assumed that \( a/d = 2, s = 25 \) mm and the support length 50 mm provide the smallest shear capacity. When \( s \) increases, the shear capacity increases too as shown in Table 3.

The ultimate shear capacities are shown in Fig. 14. The characteristic shear capacities \( V_{uk} \) for each series are given in Fig. 15. There is a probability of 25\% that more than 10\% of the slabs will have a lower value than \( V_{uk} \).

**TABLE 2.** Comparison of average shear capacity \( V_{um} \) at failure for Series B, C and D with \( a/d = 1.5 \) and \( s = 125 \) mm.

<table>
<thead>
<tr>
<th>Series</th>
<th>( n )</th>
<th>( A_s )</th>
<th>( V_{um} ) kN</th>
<th>( C/D )</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>6</td>
<td>100</td>
<td>36</td>
<td>C/D = 1.36</td>
</tr>
<tr>
<td>C</td>
<td>6</td>
<td>100</td>
<td>49</td>
<td>B1/C = 1.31</td>
</tr>
<tr>
<td>B2</td>
<td>2</td>
<td>200</td>
<td>24</td>
<td>B1/D = 1.78</td>
</tr>
<tr>
<td>B1</td>
<td>6</td>
<td>100</td>
<td>64</td>
<td>B1/B1 = 2.67</td>
</tr>
<tr>
<td>B2</td>
<td>2</td>
<td>200</td>
<td>95</td>
<td>B2/B1 = 1.48</td>
</tr>
</tbody>
</table>

\( n = \) number of tests  
\( R = \) ratio of capacities

**TABLE 3.** Relationship between average shear capacity \( V_{um} \) at failure and distance \( s \) for Series B2 with \( a/d = 1.0 \).

<table>
<thead>
<tr>
<th>( s ) mm</th>
<th>( n )</th>
<th>( V_{um} ) kN</th>
<th>( % )</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>4</td>
<td>08</td>
<td>100</td>
</tr>
<tr>
<td>125</td>
<td>2</td>
<td>99</td>
<td>113</td>
</tr>
<tr>
<td>225</td>
<td>2</td>
<td>123</td>
<td>140</td>
</tr>
<tr>
<td>425</td>
<td>2</td>
<td>189</td>
<td>214</td>
</tr>
</tbody>
</table>

\( n = \) number of tests
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The common capacity $V_k$ is estimated as a straight line with a reasonable degree of safety and given by the formula:

$$V_k = 80 \cdot 10^3 + 160 A_e$$  \hspace{1cm} (2)

where: $V_k$ = characteristic shear capacity (Newton)

400 $\leq A_e \leq 1000$ mm$^2$

The first term in Eq. (2) is the concrete contribution, while the second is from the strands. The influence on $V_k$ of the concrete compressive strength $f_c$ between 50 and 80 N/mm$^2$ is rather small, as shown in Fig. 16. When $f_c$ is lower than 40 N/mm$^2$ the influence may be considerable, as shown by Regan [6]. According to FIP [4] and to Anderson and Anderson [7], the anchorage capacity is not much influenced by $f_c$. Important is good consolidation of concrete around the strands.

It should be possible to expand Eq. (2) into a general formula following the same principle used in the Model Code [2], depending on:

1. Shape of cross-section
2. Effective depth
3. Rib width
4. Concrete strength
5. Area of strands
6. Prestressing

But it is doubtful to determine a general formula on basis of these tests, because the range of the variables in points 1 - 3 is too small and the concrete strength is too high.

**BEHAVIOUR AND MECHANICAL MODEL**

Before failure, the element carries the load through a combination of beam-and arch action. Diagonal tension and shear bending introduce the failure.

The mechanical model is shown in Fig. 17. The tension force $F_t$ consists of the concrete force $F_{ct}$ and/or the steel force $F_s$, depending on when and where the cracks occur. However, hollow-core slabs with a short support at the end of the element collapse because of anchorage failure. If the diagonal crack comes simultaneously or after the shear bending crack, the diagonal splitting force may work together with $F_t$, otherwise not. The width of the rib may also influence the shear capacity. But the slab does not collapse because of diagonal splitting as long as the shear bending and the anchorage capacity are larger. The dowel action is probably not important, because $A_d/A_e$ is small.

**DESIGN SHEAR CAPACITY**

**Line load**

On basis of the test results it is proposed that the design capacity is given by:

$$V_d = k_1 k_2 k_3 V_k = 0.55 V_k$$  \hspace{1cm} (3)

where: $k_1 = 0.91$ according to NS 3473 [1]

$k_2 = 0.80$ because of unknown distribution of load

$k_3 = 0.75$ because of unexpected impact load

The factors $k_1$ and $k_2$ are both conservatively estimated. A long term load reduces $V_d$ probably less than 25% if the concrete splitting stress, or the concrete compressive strength causes the failure.

The design shear capacity in Newton for one slab is given by Eqs. (2) and (3):

$$V_d = 0.55 \left( 80 \cdot 10^3 + 160 A_e \right)$$  \hspace{1cm} (4)

where: 400 $\leq A_e \leq 1000$ mm$^2$ and:

$V_d = 79 - 132$ kN

**Uniformly distributed load**

The shear capacities are similar for a line load and an uniformly distributed load when $a/d = 2$ according to Leonhardt [5]. Because the load distribution can be uneven, it seems to be adequate to use $k_2 = 0.80$ in this case too. Consequently the design shear force can be calculated by Eq. (4) which gives about twice as large capacity as Eq. (1).

**Fire resistance**

This depends on the anchorage capacity when the length $c$ is short. With the cover $d_1$ like 35 mm and fire for a period of 90 minutes, the following was found in the literature:

1. Temperature in the strands [10] = 490°C
2. Concrete compressive strength [9] = 60% *

- round steel = 48% *
  ($f_y = 35$ N/mm$^2$)
- shaped steel = 80% *
  ($100$% at 20°C).

The temperature 490°C corresponds very well with the temperature measured on Dy-core and Spirroll by Underwriters Laboratories in USA. On the other hand the temperature in the strands above the support may be lower if the support length is larger than 50 mm.

**The anchorage capacity of strands**

In tests similar to the tests by Reichel[11] would probably be 50 - 60% of the capacity without fire. According to FIP/CEB [9] concrete splitting tensile strength is 55% when the temperature is 490°C. However, these two references are dealing with wet casted concrete. It appears reasonable to assume that the anchorage capacity for hollow-core slabs as described in this paper, is at least 0.75 - 55% = 41%. Until fire tests concerning shear capacity are carried out, Eq. (4) may be applied provided the first term is set equal to zero and the second is reduced to 55%.

**Fatigue failure**

Tests by Anderson and Anderson [7] indicate a high capacity even though 50 cycles of each load were applied. According to Chang and Kesler [12] the diagonal splitting strength for wet casted concrete after one million cycles of load is 45 - 65% of the capacity without cycles. It appears reasonable to assume that the anchorage capacity for hollow-core slabs as described in this paper, is at least 0.75 - 45% = 34%. Until fatigue tests concerning shear capacity are carried out, Eq. (4)
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may be applied provided the first term is set equal to zero and the second is reduced to 45%.

QUALITY CONTROL CRITERIA
The free end slip is the best acceptance criterion concerning shear and anchorage. Following limits are proposed:

1. Moderate stresses:
   Max. 2,5 mm and highest average 2 mm
2. Very high stresses:
   Max. 2 mm and highest average 1,5 mm.

The slip is also an indication of the level of the concrete quality. It is not necessary to test the concrete compressive strength, because this strength is not decisive for the shear capacity. Important is good consolidation of concrete around the strands.

In addition the surface of the concrete near the end of the slab, must be inspected. Around the strands there should be no distinct cracks caused by the production method. But cracks limited to the surface of the concrete do not reduce the capacity to any important degree.

CONCLUSIONS
The scattering of the ultimate shear capacity is considerable, probably because of the production method of the slabs that may give very variable compaction around strands. Nevertheless, the number of tests is believed to be sufficient to give safe recommendations. Some of the capacities were low, because the elements were of second class quality.

The capacity is not the same for all types of hollow-core slabs which are reported in this paper. But, it is difficult to distinguish between the shear capacities of the different types, and it is unnecessary for practical design purposes.

Before failure, the element carries the load as a combination of beam- and arch action. Diagonal tension and/or shear bending introduce the failure. However, slabs with a short support at the end of the element collapse because of anchorage failure. The shear capacity consists of a concrete and reinforcement contribution. The latter is normally the largest.

These tests indicate that the examined hollow-core slabs have a very high shear capacity. Based on literature studies and the determined shear capacity formula, the fire resistance seems to be considerable for a period of 90 minutes. But the vertical end of the element must be protected against high temperature.

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SAMMENDRAG


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