How suitable is the Fama-French five-factor model for describing German and Norwegian stock returns?

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Executive summary

In this thesis we investigate the suitability of the Fama and French (2015a) five-factor model for describing German and Norwegian average monthly stock returns in the period from 1991 to 2015. We do this by estimating factor exposures and risk premiums of test portfolios formed on three different double sorts on firm characteristics. Estimation is done by means of Fama and MacBeth (1973) regressions. To secure unbiased standard errors, we apply a GMM approach when estimating the risk premiums. We evaluate both absolute and relative model performance on the basis of the test statistic developed by Gibbons, Ross, and Shanken (1989). The three-factor model serves as benchmark when describing German stock returns and a two-factor model comprising the market and the size factor is the basis for comparison in the case of Norway.

With our thesis we make three contributions to existing literature. Firstly, we construct the Fama-French profitability and investment factors for the German and Norwegian stock markets. Although the market, size and value factors are provided by others, we rebuild them based on our model assumptions to secure internal model consistency. Secondly, by estimating risk premiums with a GMM approach, we introduce robust standard errors to the original estimation done by Fama and French. Thirdly, we show that the five-factor model does not outperform the more parsimonious benchmark models neither in describing German nor Norwegian stock returns within our sample period and that this result is unaffected by several changes in underlying assumptions. By this, we extend others’ findings about the German and the Norwegian stock markets with the conclusion that the inclusion of profitability and investment factors, at least in our setting, does not add value to already existing models.
Acknowledgements

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## Contents

Executive summary ........................................... I
Acknowledgements ............................................. II
List of Figures .................................................. III
List of Tables .................................................... III
List of R codes .................................................. IV
List of Abbreviations .......................................... V

1 Introduction ................................................... 1

2 Theory and literature review ............................. 3

3 Description of the stock markets ....................... 9
  3.1 The German and the Norwegian economies ............. 9
  3.2 Stock market developments ............................ 9

4 Method ......................................................... 12
  4.1 Portfolio construction .................................. 12
     4.1.1 Double sorting ..................................... 12
     4.1.2 Sorting variables .................................. 13
     4.1.3 Portfolio dynamics ................................ 14
     4.1.4 Test assets ......................................... 14
     4.1.5 Factor mimicking portfolios ....................... 16
  4.2 Testing procedures ..................................... 18
     4.2.1 Fama-MacBeth regressions ......................... 18
     4.2.2 GMM regressions ................................. 19
     4.2.3 Evaluating model performance .................... 22

5 Data .......................................................... 24
  5.1 Motivation for building factors from raw data .......... 24
  5.2 Sample construction .................................... 24
     5.2.1 Time period and compounding ..................... 25
     5.2.2 Operational vs financial firms .................... 29
     5.2.3 Exchange rates .................................... 30
     5.2.4 The risk-free rate ................................ 30
     5.2.5 Choice of stock exchanges ....................... 30
5.2.6 Common vs preferred stocks ........................................ 31
5.2.7 Penny stocks ......................................................... 31
5.2.8 Calculation of returns ................................................ 32
5.2.9 Negative book equity and book assets .......................... 35

6 Results ........................................................................... 36

6.1 Overview of the test portfolio returns ............................. 36
6.1.1 Simple sorts ............................................................ 36
6.1.2 Double sorts ............................................................ 36
6.2 Overview of the factor mimicking portfolio returns .......... 39
6.3 Factor exposures ........................................................... 42
6.4 GRS-test ................................................................. 43
6.5 Risk premiums .............................................................. 51
6.6 Robustness checks ....................................................... 52
6.6.1 Avoiding time gaps due to different fiscal year ends .. 52
6.6.2 Redefining penny stocks .......................................... 54
6.6.3 Better differentiation between LHS and RHS sorts ....... 54
6.6.4 Shorter time horizon ................................................ 55

7 Conclusion and outlook .................................................... 57

Appendices .......................................................................... 59

References ........................................................................ 88
List of Figures

1. GDP per sector .................................................. 10
2. Total market value of the German and Norwegian stock exchanges .......... 11
3. Illustration of portfolio construction ........................................ 15
4. Illustration of the Fama-MacBeth procedure ................................... 20
5. Number of companies in our sample .......................................... 25
6. Distribution of individual monthly stock returns ............................... 35
7. Spread between the two extreme portfolios ..................................... 37
8. Germany: Variation of the factor loadings over time .......................... 61
9. Norway: Variation of the factor loadings over time ........................... 62

List of Tables

1. Composition of the factor building blocks ..................................... 17
2. Composition of the four factor mimicking portfolios .......................... 17
3. Overview of the sample variables ............................................... 26
4. Germany: Number of sample observations ..................................... 27
5. Norway: Number of sample observations ....................................... 28
6. Descriptive statistics of the sample stock returns .............................. 34
7. Average monthly excess return per test portfolio ............................... 40
8. Summary statistics of monthly factor returns ................................... 41
9. Factor exposures of German size-BM portfolios ............................... 44
10. Factor exposures of German size-OP portfolios ............................... 45
11. Factor exposures of German size-Inv portfolios ............................... 46
12. Factor exposures of Norwegian size-BM portfolios .......................... 47
13. Factor exposures of Norwegian size-OP portfolios ............................ 48
14. Factor exposures of Norwegian size-Inv portfolios ............................ 49
15. GRS-test ............................................................................. 52
16. Estimated risk premiums .......................................................... 53
17. Average number of stocks per test portfolio ..................................... 59
18. Descriptive statistics of the test portfolios ...................................... 60
19. Summary statistics of the factor building blocks ............................... 63
20. Germany: Auxiliary regressions ................................................... 64
21. Norway: Auxiliary regressions ..................................................... 65
22. GRS-test based on alternative assumptions, part 1 ............................ 66
23. GRS-test based on alternative assumptions, part 2 ............................ 67
24. Risk premiums based on alternative model assumptions ...................... 68
List of R codes

1. Formatting exchange rates ........................................... 69
2. Risk-free rate ................................................................. 70
3. Formatting security data ............................................... 71
4. Return descriptive statistics and histograms ......................... 73
5. Formatting accounting data ............................................ 75
6. Sorting variable construction ........................................... 76
7. Form test portfolios ....................................................... 77
8. Test portfolio characteristics ............................................ 78
9. Double sorting as basis of factor construction ....................... 79
10. Construction of the SMB factor ......................................... 80
11. Construction of the HML, RMW and CMA factors .................. 80
12. Construction of the market factor ....................................... 81
13. Figure extreme portfolio spread ...................................... 81
14. Summary statistics for factor returns .................................. 82
15. Auxiliary regressions ..................................................... 82
16. Stepwise regressions ...................................................... 83
17. Estimate factor loadings by Fama-MacBeth first-step regressions . 83
18. Rolling window regressions to assess the factor loadings’ time variation . 85
19. Estimating risk premiums with GMM ................................. 86
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADR</td>
<td>American Depository Receipt</td>
</tr>
<tr>
<td>APT</td>
<td>Arbitrage Pricing Theory</td>
</tr>
<tr>
<td>at</td>
<td>total assets</td>
</tr>
<tr>
<td>AUD</td>
<td>Australian Dollars</td>
</tr>
<tr>
<td>B/M</td>
<td>book-to-market</td>
</tr>
<tr>
<td>BE</td>
<td>book equity</td>
</tr>
<tr>
<td>CAPM</td>
<td>Capital Asset Pricing Model</td>
</tr>
<tr>
<td>CMA</td>
<td>conservative-minus-aggressive</td>
</tr>
<tr>
<td>cogs</td>
<td>Costs of goods sold</td>
</tr>
<tr>
<td>CRSP</td>
<td>Center for Research of Security Prices</td>
</tr>
<tr>
<td>DAX</td>
<td>Deutscher Aktienindex</td>
</tr>
<tr>
<td>DEM</td>
<td>Deutsche Mark</td>
</tr>
<tr>
<td>e.g.</td>
<td>example given</td>
</tr>
<tr>
<td>EU</td>
<td>European Union</td>
</tr>
<tr>
<td>EUR</td>
<td>Euro</td>
</tr>
<tr>
<td>FF3F</td>
<td>Fama French three-factor model</td>
</tr>
<tr>
<td>FF5F</td>
<td>Fama French five-factor model</td>
</tr>
<tr>
<td>GBP</td>
<td>Great Britain Pounds</td>
</tr>
<tr>
<td>GDP</td>
<td>Gross domestic product</td>
</tr>
<tr>
<td>GMM</td>
<td>generalized method of moments</td>
</tr>
<tr>
<td>HML</td>
<td>high-minus-low</td>
</tr>
<tr>
<td>i.e.</td>
<td>in explanation</td>
</tr>
<tr>
<td>ICAPM</td>
<td>Intertemporal Capital Asset Pricing Model</td>
</tr>
<tr>
<td>Inv</td>
<td>investment</td>
</tr>
<tr>
<td>IPO</td>
<td>initial public offering</td>
</tr>
<tr>
<td>LHS</td>
<td>left-hand-side</td>
</tr>
<tr>
<td>ME</td>
<td>market equity</td>
</tr>
<tr>
<td>Mkt</td>
<td>market factor</td>
</tr>
<tr>
<td>MSCI</td>
<td>Morgan Stanley Capital International</td>
</tr>
<tr>
<td>NOK</td>
<td>Norwegian Kroners</td>
</tr>
<tr>
<td>OLS</td>
<td>ordinary least squares</td>
</tr>
<tr>
<td>OP</td>
<td>operating profitability</td>
</tr>
<tr>
<td>OSE</td>
<td>Oslo Stock Exchange</td>
</tr>
<tr>
<td>OTC</td>
<td>over-the-counter</td>
</tr>
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<td>p.</td>
<td>page</td>
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<tr>
<td>P/E</td>
<td>price-earnings</td>
</tr>
<tr>
<td>RHS</td>
<td>right-hand-side</td>
</tr>
<tr>
<td>RMW</td>
<td>robust-minus-weak</td>
</tr>
<tr>
<td>SIC</td>
<td>standard industrial classification</td>
</tr>
<tr>
<td>SMB</td>
<td>small-minus-big</td>
</tr>
<tr>
<td>U.S.</td>
<td>United States</td>
</tr>
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USD ............. United States Dollar
xsga ............. selling, general and administrative expenses
ZAR ............. South African Rand
1 Introduction

According to asset pricing theory, assets earn risk premiums when they are exposed to underlying systematic risk factors. It is however still an unanswered question what these risk factors are. The research around this topic can be split into two groups. First, there are theoretical approaches trying to find economic explanations for systematic risk. Second – and this is by far the largest group – there are approaches that model systematic risk by variables that empirically seem to correlate with asset returns. Among these empirical approaches, the three-factor model introduced by Fama and French (1993) is probably best-known. This model explains asset returns by market movement, firm size and firm value. Although there is no theory that justifies the latter two variables as source of systematic risk, a lot of research shows that they have explanatory power in models that describe asset returns. Some see in this a connection between size or value and yet undetected actual sources of risk. Others regard it as pure coincidence given the lack of theoretical foundation.

Whoever is right, since the rise of the Fama and French three-factor model, a lot of research has been done on finding further variables that persistently identify patterns in stock returns. One of the latest developed models is the five-factor model by Fama and French (2015a). In this approach, the authors extend their original three-factor model by the influence of a firm’s operating profitability, as well as its investment behavior. Fama and French (2015a) test this model on the U.S. stock market and find that including the two new factors persistently leads to enhanced model performance, relative to the three-factor model. Besides, they show that this model extension causes a redundant value-factor and thus argue that a four-factor model containing market, size, profitability and investment factors is most adequate in describing asset returns. Nevertheless, Fama and French (2015a) admit that these findings might be sample-specific and thus call for further research to verify the model.

One way of testing whether a newly-developed factor model is useful to describe asset returns or if it merely detects sample-specific effects, is to apply the model to other capital markets. Under the assumption of globally integrated financial markets, fundamental findings should hold for assets in any country, though derived from the U.S. stock market alone.

Against this background, Fama and French (2015b) test their five-factor model on four regional markets – North America, Europe, Japan, and Asia Pacific – as well as on one global market composed of these four submarkets. They concede that a global version of the five-factor model does not succeed in explaining international stock return patterns. For the regional markets, augmenting the three-factor model leads though to better model performance in their analysis. Fama and French (2015b) find that a four factor model excluding the investment factor suits best to all regions outside of North America. This ambiguity relative to the findings for pure U.S. data leaves us still uncertain about the universality of the model.

Griffin (2002, p.798) finds that international Fama and French three-factor models fail to explain stock returns, and that “cost-of-capital calculations, performance measurement, and
risk analysis using Fama and French-style models are best done on a within-country basis”. If this also holds for the latest five-factor approach, more model evidence might be found by going further than Fama and French (2015b), and test the model on individual country stock markets outside of the U.S. There are many candidate countries for such an analysis. Among them, we chose Germany and Norway for several reasons:

1. As far as we know, there are at the moment of writing this thesis no studies applying the Fama French five-factor model to the German and Norwegian stock markets.

2. These markets have quite different characteristics, the first being a large economy with heavy export orientation and a EU member, the latter being a little open economy and a non-EU member that scarcely affects market prices with its actions. We argue that more evidence is found for the model if it fits to both markets.

3. Due to our own background we have better understanding of the German and Norwegian stock markets than we do have for many foreign markets.

In this thesis we hence scrutinize the applicability of the Fama-French five-factor model by testing its suitability for describing stock returns on the German and Norwegian markets. To do so we construct the model factors as well as test portfolios by using stock market and accounting data. Our test assets are portfolios based on three kinds of double sorts: size-book-to-market, size-operating profitability and size-investment behavior. To evaluate the relative performance of the five-factor model, we compare its performance relative to the performance of the Fama-French three-factor model in the case of Germany and a two-factor model comprising the market and size factor in the case of Norway. We find that the neither the profitability nor the investment factors explain cross-sectional variation in stock returns on the two markets. In several robustness checks we show that these results are unaffected by changes in underlying assumptions.

This thesis is structured as follows. In Section 2 we present theoretical frameworks and findings by others that form the basis for our analysis. In Section 3 we provide the reader with a country-specific overview of the German and the Norwegian stock market. A description of the methods we use is given in Section 4. This part is followed by Section 5 in which we substantiate why and how we construct our data set. In Section 6 we present the results of our analysis and go through several robustness checks. Section 7 concludes and gives an outlook on further research possibilities.


2 Theory and literature review

Factor pricing models try to explain risk premiums that can be observed in the market. They originate from the assumption of rational investors whose utility is increasing in consumption, but with a decreasing rate\(^1\). In bad times the investor has a higher marginal benefit of consumption than in good times, when his wealth level is already high. Companies that do well in bad times, are thus highly valued by the rational investor who seeks opportunities to increase his wealth level and consumption. The high demand of assets that give a high return in bad times (low beta assets), drives up the prices of those assets. In the same manner, prices of assets that have low returns in bad times (high beta assets) are driven downwards. These dynamics explain the creation of risk premiums, which compensate the investor for the risk he takes when investing in high beta assets.

At each point in time the investor is confronted with the trade-off between instant consumption and investing to increase future consumption. Formally this can be expressed as

\[
U(c_t, c_{t+1}) = u(c_t) + \beta E_t[u(c_{t+1})]
\]

where \(u(c_t)\) and \(u(c_{t+1})\) are the utility of consumption at time \(t\) and \(t + 1\), respectively and beta is a subjective discount factor that captures the investor’s impatience. The investor then chooses the optimal consumption and investment level by maximizing Equation (1) subject to the budget constraint that in order to consume more today, he has to reduce consumption tomorrow and vice versa. The solution of this maximization problem is

\[
p_t = E_t \left[ \beta \frac{u'(c_{t+1})}{u'(c_t)} x_{t+1} \right]
\]

where \(p_t\) is today’s price of an asset and \(x_{t+1}\) is the assets future payoff. Equation (2) is the fundamental asset pricing formula. It is often expressed in a more general way, by defining the stochastic discount factor

\[
m_{t+1} = \beta \frac{u'(c_{t+1})}{u'(c_t)}
\]

So that the pricing equation (2) simplifies to

\[
p_t = E_t[m_{t+1} x_{t+1}]
\]

Assuming rational investors, one can transform this equation into the Euler equations

\[
1 = E_t[m_{t+1} r_{t+1}]
\]

\(^1\)The part about the consumption-based model is based on Cochrane, 2005, chapter 1.
Using the formula for the covariance and simplified notation, the Euler equations can be rewritten as expected return-beta representation

\[ E(r^i) = \frac{1}{E(m)} - \frac{\text{cov}(r^i, m)}{E(m)} \quad (6) \]
\[ \Leftrightarrow E(r^i) = \alpha + \left( \frac{\text{cov}(r^i, m)}{\text{var}(m)} \right) \left( \frac{-\text{var}(m)}{E(m)} \right) \quad (7) \]
\[ \Leftrightarrow E(r^i) = \alpha + \beta_{i,m} \lambda_m \quad (8) \]

where \( \beta_{i,m} \) can be interpreted as the quantity of risk in each asset and \( \lambda_m \) is the risk premium.

In theory, there is one single stochastic discount factor, which prices all assets in the investment universe.

Factor pricing models assume that the stochastic discount factor takes the linear form

\[ m_{t+1} = a + b f_{t+1} \quad (9) \]

where \( f_{t+1} \) is a set of observable factors. In line with Equation (8) this can alternatively be expressed as the multi-beta model

\[ E(r_{t+1}) = \alpha + \beta' \lambda \quad (10) \]

From Equation (3) and (9) follows directly that factors should be good proxies for the growth of the marginal utility of consumption:

\[ \beta \left( \frac{u'(c_{t+1})}{u'(c_t)} \right) \approx a + b' f_{t+1} \quad (11) \]

Depending on the realization of the factors, they thereby represent states that let the investor suffer from lower levels of consumption ("bad times") and favorable states that provide the investor with increased consumption levels ("good times"). There is a wide range of literature that deals with identifying such factors. In the following we give the reader an overview about the main findings from others – both of theoretical and of empirical nature – which ultimately led to the development of the Fama and French five-factor model that is the main subject of this thesis.

The most basic factor model is the Capital Asset Pricing Model (CAPM) which was developed out of the works of Treynor (1962), Sharpe (1964), Lintner (1965) and Mossin (1966). Building on the findings on mean-variance preferences and portfolio diversification by Markowitz (1952) the CAPM is an attempt to give a theoretical explanation for risk premiums. According to the CAPM, there is a linear relationship between asset returns and market risk of the form

\[ E(r^i) = r_f + \beta (E(r_M) - r_f) \quad (12) \]

---

\(^3\)see Cochrane, 2005, p.149.
where \( r_f \) is the risk-free rate, \( E(r_M) \) is the expected return of a market portfolio and beta is the asset’s covariation with this market portfolio. The CAPM is however tied to very strong assumptions and is often called an “empirical failure” (e.g. Fama & French, 2015b, p.23) as many empirical studies find stock return patterns, so-called anomalies, that cannot be explained by the simple linear relationship assumed by CAPM.

There are two main theoretical approaches that were developed to overcome some of the limitations of the CAPM. The first is the Intertemporal Capital Asset Pricing Model (ICAPM) by Merton (1973). It is a multi-period model based on the assumption that the investor’s utility does not only depend on his wealth level, but also on which state of the world occurs in future time periods. Merton (1973) argues that it is unrealistic to assume constant investment opportunities over time and therefore introduces the concept of state variables which “describe changes in the opportunity set”. He extends Tobin’s two-fund separation (Tobin, 1958) which is underlying CAPM to a three-fund theorem. According to this theorem, the investor’s optimal investment choice is a linear combination of the risk-free asset and a risky asset – which give the investor the optimal risk-return combination today – and a third asset that hedges against intertemporal changes of the investment opportunity set. In this sense, asset excess returns do not only reflect market risk, but also risk caused by state variables.

The second theoretical alternative to CAPM is the Arbitrage Pricing Theory (APT) by Ross (1976). According to this model, asset returns are a linear combination of the returns of multiple systematic risk factors and an asset-specific return. Ross (1976) shows that the idiosyncratic risk can be diversified away by holding portfolios instead of single assets and that returns thus should only incorporate the asset’s exposure to factor risk. He argues that then, in the absence of arbitrage, an asset’s excess return is the sum of the factor risk premiums, weighted with the degree to which it covaries with the respective factor:

\[
E_i - r_f \approx \beta_{i,1}(E^1 - r_f) + \ldots + \beta_{i,k}(E^k - r_f)
\]

where \( r_f \) is the return of the risk-free asset and \( \beta \) is the asset’s factor exposure. In opposite to CAPM, this model does not require equilibrium and opens up for more explanatory factors than just the market factor.

The drawback of both ICAPM and APT is that the state variables, which define systematic risk and thereby risk premiums, are unknown. Breeden (1979) develops a setting in which the unspecified state variables in the ICAPM can be reduced to one specific explanatory variable. He inter alia shows that state variables can be replaced by portfolios which correlate highly with the same state variables. Grinblatt and Titman (1987) show that using “proxy portfolios” as factor estimates is in line with APT. Huberman, Kandel, and Stambaugh (1987) examine the attributes and framework of these portfolios, that are mostly called factor mimicking portfolios in financial literature. By using mimicking portfolios that hedge state variable risk, one can identify risk premiums despite the fact that the “true” risk factors are unobservable (Ferson, Siegel, & Xu, 2006).
Empirical asset pricing models then explain risk premiums by variables that empirically appear to detect persistent asset return patterns. The rationale behind this is the assumption that the model factors or factor mimicking portfolios have explanatory power because they correlate with the actual underlying state variable risk. The foundation of this relationship is though usually not evaluated any further but taken as given and only explained ex post.

The first anomaly whose presence was observed in many studies is the so-called size effect. Reinganum (1981) finds that returns of portfolios formed on firm size are not well-described by the CAPM. Banz (1981) observes that stocks of low market value firms persistently show higher returns than those of large firms.

Value strategies, where investors finance the acquisition of "inexpensive" assets by short selling "expensive" assets, were first proposed by Graham and Dodd (1934). When a company’s book equity value is high relative to its market price, the purchasing investor gains a high proportion of book assets relative to a marginal dollar spent on the firm. Countless of papers written later on, have shown that such strategies generate profits. Basu (1977, 1983) for example uses price-earnings (P/E) as a measure of value and finds that stocks with low price relative to their earnings perform better than those with a high P/E-ratio. He thus shows that value strategies produce both absolute and risk-adjusted average excess returns in the U.S. market.

In their three-factor model Fama and French (1993) integrate the findings of size and value anomalies with the explanatory power of the market factor. They use book-to-market ratios instead of P/E as value measure, because P/E is shown to be redundant in the multivariate regression analysis conducted by Fama and French (1992). This results from both measures being scaled versions of an asset’s price and hence explaining the same variation in cross-sectional returns.

Fama and French (2006) reason that further factors are implied by the valuation relation of Miller and Modigliani (1961), here divided by time $t$ book equity

$$\frac{M_t}{B_t} = \frac{\sum E(Y_{t+\tau} - dB_{t+\tau})/(1 + r)^\tau}{B_t}$$

(14)

where $M_t$ is the market value of a firms stocks at time $t$, $B_t$ is the book equity at time $t$, $dB_{t+\tau}$ is the change in book equity, $dB_t = B_t - B_{t-1}$, $Y_{t+\tau}$ is equity earnings in period $t + \tau$, and $r$ is the long-term average stock return. This equation comes from combining the dividend discount model with the clean surplus accounting relationship and contains three firm characteristics: Book-to-market equity as a measure of company value, earnings relative to book equity as a measure of profitability and book equity growth as a measure of investment behavior. Fama and French (2006) argue that Equation (14) implies that if one controls for two of the three factors, the third factor should capture all variation in expected stock returns. They therefore propose profitability and investment behavior as additional factors.

The reasoning above has the following implications about the relation between the three anomaly variables and expected stock return:
Firstly, given constant expectations about future cash flows, a raise in book-to-market equity will increase the future expected stock return, i.e. a value premium exists.

Secondly, holding the B/M-ratio and the change in book equity constant, higher profitability leads to a higher expected stock return. Haugen and Baker (1996), Cohen, Gompers, and Vuolteenaho (2002), and Fama and French (2006) all document this effect for U.S. data. Novy-Marx (2013) questions though the way profitability is defined in previous papers. He argues that “Gross profits is the cleanest accounting measure of true economic profitability” (Novy-Marx, 2013, p.2), and finds that gross profits-to-assets absorbs asset return patterns arising from earnings differences. Fama and French (2015a) extend their discussion of the valuation relation, and use the findings of Novy-Marx (2013) to introduce the profitability measure as an additional factor when presenting their five-factor model.

Thirdly, for fixed levels of the B/M-ratio and profitability, increasing book equity by investing, results in lower expected future stock returns. An economic intuition for this may be the investors’ willingness to increase investments when their equity cost of capital, i.e. the long-term measure of return, is low. This phenomenon is called the q-theory of optimal investment hypothesis (Liu, Whited, & Zhang, 2009). An alternative explanation is the overinvestment hypothesis (Titman, Wei, & Xie, 2004). According to this theory managers aim at building a huge empire to seem successful and gain bonuses, instead of focusing on what is actually best for the firm’s shareholders at the moment in time. Doing so, they create a negative relation between asset growth and stock returns. As predicted by the valuation equation, Fairfield, Whisenant, and Yohn (2003) and Richardson and Sloan (2003) find the existence of a negative return/investment relation for U.S. companies. Fama and French (2006) however, find an insignificant and positive relationship between the two variables when testing the model on a per share level. Later on, Aharoni, Grundy, and Zeng (2013) show that the valuation relation holds on all metrics when replicating the study of Fama and French (2006) using measures at the firm level. When Fama and French (2015a) include investment in the five-factor model they adopt this firm level view and find that investment in fact is negatively correlated to the cross section of average U.S. stock returns. Instead of book equity growth, Fama and French (2015a) use asset growth as a measure of investment behavior, since a robustness check shows that results are not affected by the choice between the two measures.

Under the assumption of globally integrated financial markets, fundamental findings should hold for assets in any country, though derived from the U.S. stock market alone. Applying models that are recognized for the U.S. market to a global context, turns however out to lead to inconsistencies and ambiguities with regard to risk factors and their premiums. Ferson and Harvey (1993) find that international risk factors produce loadings that vary through time when using portfolios constructed with data from different countries. Others such as Dumas and Solnik (1995) find that stock returns around the world price the exchange rate risk from different markets. Based on these and others’ findings, Fama and French (1998) argue that a complete description of global stock returns requires an asset-pricing model that includes
several dimensions of risk in addition to time-varying risk loadings. Nevertheless they assume integrated capital markets, non-discrepant purchasing power parity, and time-constant risk factors in their model. Many researchers come to the conclusion that an adequate global factor model is hard to find. Griffin (2002), Hou, Karolyi, and Kho (2011), and Fama and French (2012, 2015b) even find that local models perform better than their global counterparts.

An early documentation of international factor premiums is presented by Heston, Rouwenhorst, and Wessels (1995), who find the size effect among stocks in the U.S. and twelve European countries. In their studies they observe return differences between small and big stocks both for German and Norwegian firms in the period from 1978 to 1990. Amel-Zadeh (2011) investigates German companies more extensively from 1996 to 2006 and observes the same effect during this later time period. Næs, Skjeltorp, and Ødegaard (2009) examine the Oslo Stock Exchange in the period from 1980 to 2006 and observe that firm size provides risk compensation.

Capaul, Rowley, and Sharpe (1993) use data from 1981 to 1992 and find that international stock returns, among others at the German stock market, are inflated by persistent value premiums, and hence that value stocks outperform growth stocks. Fama and French (1998) scrutinize the time period from 1975 to 1995 and confirm the global value premium. They use a global portfolio consisting of twelve countries from Europe, Australia, and “Far East” in addition to the U.S. sample. They discover a pervasive value premium both for the global portfolio and for most individual countries, including Germany. Ziegler, Eberts, Schröder, Schulz, and Stehle (2003) specifically analyse the German stock market and observe the presence of a value effect in their data. Hou et al. (2011) however do not find evidence that supports a value effect during 1981 to 2003, when using cross-sectional Fama-Macbeth regressions on a global portfolio composed of 49 countries. Næs et al. (2009) neither observe a significant value premium at the Norwegian stock market.

Novy-Marx (2013) finds that profitability is positively related to average stock returns from international portfolios composed of companies in developed markets outside of North America, including Germany and Norway. He uses a data sample covering the years 1990 to 2009, a period extended to 2015 by Fama and French (2015b). The latter test their five-factor model in an international context, which supports the profitability effect found by Novy-Marx (2013), for most regions including Europe.

International evidence suggests that the investment growth effect occurs in most developed countries, but that the power of the effect varies a lot across countries (Titman, Wei, & Xie, 2013). During the period of 1982 to 2010 Titman et al. (2013) find that the highest asset growth quintile in Germany achieves an equal-weighted size-adjusted monthly return of 0.298% less than the lowest quintile. In Norway during 1988 to 2010 the equivalent was found to be 0.288%, making this effect in both countries less severe than in the U.S. (0.953%). Fama and French (2015b) conclude that dropping investment as a factor does not affect the five-factor models’ capability in describing international average stock returns.
3 Description of the stock markets

3.1 The German and the Norwegian economies

Figure 1 shows the German and Norwegian GDP by sector. Apart from the fact that both countries show the typical pattern of industrialised countries that most value is created in the service sector, the two economies differ clearly in nature. Germany is a large open economy that due to its large industry sector is strongly export-oriented. At the same time its lack of natural resources, especially in the energy sector, makes it also dependent on imports. Important industries are machinery, automobile manufacturing, technology, etc. Furthermore, Germany is one of the founding members of the European Union (EU) and has ever since played a central role in European decision making. With the introduction of the Euro in January 1999, Germany additionally became a key nation for guidelines on the monetary policy in the Eurozone. The German stock market consists today of eight stock exchanges, of which Frankfurt Stock Exchange is by far the most important one. Traditionally companies are first listed at a local exchange. When they succeed they reach for several listings, often by separating stock types (i.e. preferred or common stocks etc.) at different exchanges. Today 90% of all stock trading are done through the electronic trading platform Xetra (Xetra, 2016).

The Norwegian economy is a small open economy that has little or even no impact on international stock prices (Norman & Orvedal, 2010). A high proportion of the Norwegian stock market consists of companies within the Energy sector, a sector often comprising around 50% of Oslo Stock Exchange’s total market value. This is mainly due to the two large companies Statoil and Norsk Hydro (Næs et al., 2009). As a result, the Norwegian economy is strongly affected by the oil price and especially by sudden changes in supply and demand of oil. Another distinctive feature of the Norwegian stock market is the composition and distribution of market value between large and small companies. At Oslo Stock Exchange (OSE) the three largest companies (Telenor in addition to the two already mentioned) account for more than half of the total exchange value.

3.2 Stock market developments

Figure 2 shows the total market value of the German and the Norwegian stock exchanges in the period 1991-2015.

After the resolution of the former Soviet Union and the fall of the Berlin Wall in 1989, the economic reunification of the two German states started in July 1990 (Bundesministerium für Wirtschaft und Energie, 2016). Around this time the households and the economy in general were characterized by huge disparities between the former East and West. The new economy faced large costs when trying to rebuild a unified nation and investing heavily in infrastructure. We see from Figure 2 that the value of the German stock market was relatively low in the beginning of the 1990s, but that total market capitalization increased steadily from 1994 until
the recession caused by the burst of the dot-com bubble around the millennium.

In the mid-nineties the internet began seeing the light of the day. Companies staking this new technology received increasing attention from investors, who sensed lucrative investment opportunities. In 1999 a majority of U.S. IPOs came from such companies with a big proportion doubling their value the first day of trading (The Economist, 2012). Investors grabbed every opportunity from investing in technology-based companies, without looking into the likelihood of future returns. When these returns never came, many companies got bankrupt and stock market growth fell quickly (Doms, 2004).

At that moment Germany faced a balance sheet recession, while the Eurozone was in need of individual monetary policy (Schnabl, 2013). To boost the German economy The European Central Bank reduced interest rates, with solely marginal effect. Instead, other member countries got in trouble and stopped demanding German products. This reinforced the recession in Germany, so that steady stock market growth did not return until late 2003 and forward.

The dot-com boom did not hit the Norwegian economy as hard as the German one, even though as of February 2003 the value of the Oslo Stock Exchange was market down to 1996-levels (Oslo Børs, 2016a).

Figure 2 shows that total market capitalization increased fast in both countries during the preceding years. Between 2003 and 2008 the exchange value more than tripled in Germany and grew around 6.5 times in Norway. After these years of growth, the Financial Crisis hit the world economy in autumn 2007. This crisis was mainly caused by securitized banking, i.e. banks using short loans (Repos) to finance their customers’ mortgages, i.e. long-term debt. When house prices began to fall in the U.S. and the most risky mortgage holders stopped paying
their bills, the concerned banks found themselves in an extreme need of liquidity. This problem spread to non-financial industries and exceeded the scope of the U.S. stock market. We see from Figure 2 that the German and Norwegian stock markets got heavily affected as well, collapsing at about the same point in time.

Around 2010 these markets started to grow again, but rates remained highly volatile in the aftermath of the global crisis.

![German Stock Exchanges](chart1)

![Oslo Stock Exchange](chart2)

**Figure 2**

Monthly total market value of the German and Norwegian domestic stock exchanges, July 1990-December 2015. Market capitalization is calculated as a company’s shares outstanding multiplied with its respective price, summed over all companies at the end of each month. For a company which is listed at several exchanges its total market value is calculated as the weighted mean of all stock prices, using the respective number of shares outstanding as weights. Values are denoted in billion USD.
4 Method

In this study, we test to which extent German and Norwegian stock returns can be described by the Fama French five-factor model

\[ r_{it} - r_{Ft} = a_i + b_i(r_{Mt} - r_{Ft}) + s_iSMB_t + h_iHML_t + r_iRMW_t + c_iCMA_t + \epsilon_{it} \]  

where \( r_{it} \) is the return on test asset \( i \) for month \( t \), \( r_{Ft} \) is the risk free rate of return, \( a_i \) is the pricing error, \( r_{Mt} \) is the return of the value-weighted market portfolio, \( SMB_t \) is the return of a diversified portfolio of small stocks minus the return on a diversified portfolio of big stocks, \( HML_t \) is the difference between the return on diversified portfolios of high and low value stocks, \( RMW_t \) is a portfolio constructed by robust minus weak operating profitability stocks and \( CMA_t \) is a portfolio of conservative minus aggressive investment firm stocks.

In order to decrease return variation that originates from firm-specific effects and thus reduce our pricing problem to analysing the effect of systematic risk factors on asset returns, we use portfolios instead of single stocks as test assets.

In the following we first explain how we construct the model components, i.e. the test portfolios and the factors. After that, in section 4.2, we describe the procedures applied to test the suitability of equation (15) for German and Norwegian stock returns.

4.1 Portfolio construction

4.1.1 Double sorting

Both the test portfolios and the factor mimicking portfolios are constructed using a double sorting technique. For that, all sample stocks are first sorted by one firm characteristic and then, in an independent second sort by another characteristic. Based on each sort, the stocks are divided into groups ranging from low to high values of the respective sorting variable. By that, every stock is marked as a group-x stock on the characteristic-1 scale and as a group-y stock on the characteristic-2 scale. Portfolios are then formed by grouping all stocks that have the same x-y combination. Double sorting hence gives us a set of portfolios consisting of stocks with similar characteristics.

Double sorting aims at isolating the effect of one factor from the effect of the other factors. As the five-factor model assumes the presence of four firm-specific effects, ideal effect isolation would be achieved by fourfold sorts that control simultaneously for the effect of all three other factors. In practice such a four dimensional sort would however lead to 256 portfolios to be filled with stocks. This is way beyond possible within our work, considering the restricted number of sample stocks. Therefore we stick to the double sorting procedure but are aware that results might be biased in the presence of factor correlation.
4.1.2 Sorting variables

(i) **Size** is defined as market capitalization at the end of each June of year $\tau$. It is calculated as

$$
\text{market cap}^i_\tau = \text{share price}^i_\tau \times \text{shares outstanding}^i_\tau
$$

For firms that have several concurrent stock issues either at the same exchange (see section 5.2.5) or at different exchanges (see section 5.2.6), share price is the weighted mean of the different share issue prices and shares outstanding is the total number of shares in all share issues. So total market capitalization is the sum of market capitalizations of each stock issue.

(ii) **Value** is defined as the ratio of a firm’s book equity at the end of fiscal year $\tau - 1$ and its market equity at the end of December of year $\tau - 1$:

$$
B/M_\tau = \frac{\text{book equity}_{\tau - 1}}{\text{market equity}_{\text{Dec} \tau - 1}}
$$

where market equity (ME) is market capitalisation calculated in Equation (16). Book equity is calculated as

$$
\text{book equity}_{\tau} = \text{stockholder equity}_{\tau} + \text{deferred taxes}_{\tau} + \text{investment tax credit}_{\tau}
$$

If the value of the stockholder equity variable provided by Compustat is missing, we use the difference of total assets and total liabilities as a proxy for book equity. Since preferred stocks form part of the aggregate market capitalization in our approach (see section 5.2.6), we do not subtract their book value from total book equity as Fama and French (1993, 2015a) do.

For companies whose fiscal year does not end in December, this approach leads to a time gap between the measurement point of book equity and that of market equity. Intuitively the numerator and denominator of the book-to-market (B/M)-ratio should be time-consistent. If however B/M-ratios at fiscal year ends are used, ratios will differ across firms not only due to differences in firm characteristics, but also because of market changes throughout the year. Fama and French (1992) find that the use of fiscal year end market equity does not affect their results significantly. We adress this issue in Section 6.6.1.

(iii) **Operating profitability** is defined as operating profit less interest expenses relative to book equity, all measured at the end of fiscal year $\tau - 1$:

$$
\text{OP}_\tau = \frac{\text{total revenues}_{\tau - 1} - \text{total operating expenses}_{\tau - 1} - \text{interest expenses}_{\tau - 1}}{\text{book equity}_{\tau - 1}}
$$

Book equity is calculated as in Equation (17). Fama and French (2014) use the sum of costs of goods sold (cogs) and selling, general and administrative expenses (xsga) instead
of total operating expenses. These two variables are however aggregated by the total operating expenses variable in Compustat. Given that we have missing values for cogs and/or xsga for many German and Norwegian firms, we use the aggregate total operating expenses variable instead.

(iv) **Investment** behavior is defined as book asset growth from year $\tau - 2$ to year $\tau - 1$:

$$\text{Inv}_\tau = \frac{\text{total assets}_{\tau - 1} - \text{total assets}_{\tau - 2}}{\text{total assets}_{\tau - 2}}$$

all measured at fiscal year-ends.

### 4.1.3 Portfolio dynamics

The double sorting procedure is conducted at the end of each June and is based on a company’s accounting measures from the previous fiscal year. The time lag aims to secure that the companies’ annual reporting process is completed, and the necessary accounting measures thus are ready and publicly available. The portfolio composition does not change until the annual portfolio updating at the end of June. After having constructed the portfolios, we trace their monthly excess return from July to the following June. This approach ensures that we use known information to explain future returns and avoid a “look ahead bias”. The portfolio excess return at month $t$ is calculated as

$$R^P_{t} = \sum_{i=1}^{n} \frac{(r^i_t - r^f_t) \times ME^i_t}{ME^P_{t}}$$

(17)

where $n$ is the number of stocks in the respective portfolio, $r^i_t$ is the individual stock return in month $t$, $r^f_t$ is the risk-free return in month $t$, and $ME^i_t$ and $ME^P_{t}$ are the individual stock’s market capitalization and the aggregate market capitalization of all stocks in the portfolio respectively, both measured at the end of month $t$.

Figure 3 sums up this section by illustrating portfolio construction graphically.

Applying the double sorting procedure to our sample data makes the number of observations differ between portfolios throughout a year, as we for many companies do not have return data for every month of the year. This is partly due to Compustat not providing the data, partly due to our sample adjustments described in Section 5, and partly because firms become listed or delisted during a year. The most reasonable solution to this is to exclude firms for which we have incomplete return data. This is however unfavorable considering our already quite small sample sizes. We thus keep the concerned observations and argue that accumulated portfolio returns get approximately right by weighting them with the observations that actually are available at each point in time.

### 4.1.4 Test assets

In their studies, Fama and French construct test assets by splitting sample stocks into five equal groups for each sorting characteristic. The interface of the double sorts leaves them with $5 \times 5$,
Figure 3
Illustration of portfolio construction. Portfolios are constructed in the end of June based on double sorts of size, book-to-market (B/M), operating profitability (OP) and investment behavior (Inv). Size is defined as market equity at the point of portfolio construction. B/M is the ratio of book-equity at the end of fiscal year $\tau - 1$ and market equity at the end of December of year $\tau - 1$. OP is total revenues less operating and interest expenses per end of fiscal year $\tau - 1$ all divided by book equity per end of fiscal year $\tau - 1$. Inv is the growth of total assets from the end of fiscal year $\tau - 2$ to the end of fiscal year $\tau - 1$. Once constructed, portfolios are held for one year before they are updated again at the end of June $\tau + 1$.

i.e. 25, test portfolios. In the case of Germany we adhere to this procedure and divide the data into quintile groups of each sorting variable. For the Norwegian data however, we deviate from this approach because quintile sorts result in too few stocks per test portfolio due to the limited number of companies in the Norwegian data set. Instead, we assign our sample stocks to only $3 \times 3$ portfolios, using 30% and 70% sample quantiles as breakpoints.

As evidence found by others indicates that size is the most prevailing effect in both German and Norwegian data (see section 2, we construct our test portfolios by first sorting sample stocks by size and then by value, profitability or investment. This gives us three different sets of test portfolios: size-B/M portfolios, size-profitability portfolios and size-investment portfolios. In table 17 in the appendix we show the mean number of stocks in each of the test portfolios constructed by this procedure. For the German data the mean number of portfolio stocks ranges mostly between 8 and 21. We find that profitability tends to increase with company size since few stocks are allocated to the portfolio comprising large size and low profitability stocks while the number of stocks in the portfolio comprising small size and low profitability stocks is above average. The same applies to investment behaviour although the effect is weaker. The mean number of stocks in each Norwegian portfolio ranges from three to ten. This constrasts to the findings for the U.S. market (see Fama & French, 1993) where each test portfolio contains between 23 and 512 stocks. Our test portfolios might thus, in spite of the
reduction of variable groups, not be diversified enough to rule out all company specific effects. We address this problem in Section 6.6.4.

In table 18 in the appendix we additionally show the mean of the respective sorting characteristics in each of the constructed test portfolios.

4.1.5 Factor mimicking portfolios

The right-hand-side (RHS) variables of the Fama and French three-factor model are factor mimicking portfolios built on two size groups (small and big) and three value groups (low, neutral and high) of the sample stocks. The reason for the different number of groups is that Fama and French (1992) find that book-to-market ratios have higher explanatory power for average stock returns than firm size. Fama and French (2015a) scrutinize the impact of factor construction by comparing three possible versions of RHS portfolio sorts: 2 × 3 sorts based on two size groups and 3 groups of the second sorting variable (i.e. B/M, OP or Inv), 2 × 2 sorts based on two size groups and two groups of the second sorting variable, and 2 × 2 × 2 × 2 sorts where each sorting variable is split into two levels, and all possible permutations of these are built. The authors find that the 2 × 2 and the 2 × 2 × 2 × 2 sorts are not significantly better than the original 2 × 3 sorts, which they used in their three-factor model. Hence we conduct our base analysis with factor portfolios constructed from 2 × 3 sorts. In Section 6.6.3 we reflect on whether the type of sort affects our results.

We split companies into small and big groups, using the domestic sample median as breakpoint. Next we divide stocks into high, neutral and low value-stocks using the 30% and 70% sample quantiles of B/M for each country. With the same procedure and breakpoints we form the three investment groups (conservative, neutral and aggressive) and the three profitability groups (weak, neutral and robust). Intersections from each of these sorts result in six portfolios, which serve as the basis for the construction of factor mimicking portfolios.

The monthly values of the SMB factor are then calculated as the difference between the simple average of the returns from the three small-stock portfolios and the simple returns from the three corresponding big-stock portfolios. The HML factor is defined in a similar way, except that the B/M dimension only produces two high-stock portfolios and two low-stock portfolios. The monthly values of the HML factor are the difference between the simple average of the returns from the two high and the two low book-to-market portfolios. Following the same procedure, the RMW and CMA factors are defined as the monthly difference between the average returns of the two robust and weak profitability portfolios, and the conservative and aggressive investment portfolios, respectively. Table 2 shows the formal factor definitions.

By constructing the factors in this way, the SMB factor is supposed to be adjusted for influences of firm value, profitability and investment. This is a result of small and big stock portfolios comprising approximately the same weighted average B/M, OP and Inv measures. The same applies to the HML, RMW and CMA factors, which are supposed to be largely free of
Table 1
Composition of the factor building blocks. Based on sorts by size, value, profitability and investment, the sample stocks are assigned to specific groups. Firm size is divided into two groups, small and big, using the sample median. The other three sorting variables are divided into three groups (low/neutral/ high for book-to-market, robust/neutral/weak for operating profitability and conservative/neutral/aggressive for investment) using 30% and 70% quantile breakpoints. The interface of the size groups and the second variable groups gives 6 factor building blocks per double sort.

Panel A: Size-B/M sorts

<table>
<thead>
<tr>
<th></th>
<th>Low</th>
<th>Neutral</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>SL</td>
<td>SN</td>
<td>SH</td>
</tr>
<tr>
<td>Big</td>
<td>BL</td>
<td>BN</td>
<td>BH</td>
</tr>
</tbody>
</table>

Panel B: Size-OP portfolios

<table>
<thead>
<tr>
<th></th>
<th>Weak</th>
<th>Neutral</th>
<th>Robust</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>SW</td>
<td>SN</td>
<td>SR</td>
</tr>
<tr>
<td>Big</td>
<td>BW</td>
<td>BN</td>
<td>BR</td>
</tr>
</tbody>
</table>

Panel C: Size-Inv portfolios

<table>
<thead>
<tr>
<th></th>
<th>Conservative</th>
<th>Neutral</th>
<th>Aggressive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>SC</td>
<td>SN</td>
<td>SA</td>
</tr>
<tr>
<td>Big</td>
<td>BC</td>
<td>BN</td>
<td>BA</td>
</tr>
</tbody>
</table>

Table 2
Composition of the four factor mimicking portfolios SMB, HML, RMW and CMA. Factor building blocks as described in table 1

<table>
<thead>
<tr>
<th>Breakpoints</th>
<th>Factor Construction</th>
</tr>
</thead>
<tbody>
<tr>
<td>size sample median</td>
<td>$SMB = (SMB_{B/M} + SMB_{OP} + SMB_{Inv})/3$</td>
</tr>
<tr>
<td></td>
<td>$SMB_{B/M} = (SH + SN + SL)/3 - (BH + BN + BL)/3$</td>
</tr>
<tr>
<td></td>
<td>$SMB_{OP} = (SR + SN + SW)/3 - (BR + BN + BW)/3$</td>
</tr>
<tr>
<td></td>
<td>$SMB_{Inv} = (SC + SN + SA)/3 - (BC + BN + BA)/3$</td>
</tr>
<tr>
<td>30th and 70th B/M sample percentiles</td>
<td>$HML = (SH + BH)/2 - (SL + BL)/2$</td>
</tr>
<tr>
<td>30th and 70th OP sample percentiles</td>
<td>$RMW = (SR + BR)/2 - (SW + BW)/2$</td>
</tr>
<tr>
<td>30th and 70th Inv sample percentiles</td>
<td>$CMA = (SC + BC)/2 - (SA + BA)/2$</td>
</tr>
</tbody>
</table>
Finally, we construct the market factor as the value-weighted return on all sample stocks, including the negative book-equity stocks, in excess of the four-week U.S. Treasury rate. Weighting is done based on a company’s market capitalization at the end of each month, relative to the sum of all sample companies’ market capitalization.

4.2 Testing procedures

In this study we use the Fama-MacBeth two-step procedure to estimate first factor exposures and then risk premiums. In the estimation of risk premiums we use a GMM approach in order to cope with possible serial correlation and the generated regressor problem. We evaluate model performance based on the GRS-test. In the following we explain the rationale behind each technique and how to apply it.

4.2.1 Fama-MacBeth regressions

The fact that we analyse multiple firm’s development over time, introduces cross-sectional correlation to our model. Companies that operate on the same market are exposed to the same environment, and thus will tend to make similar decisions at the same point in time. In times of high economic growth, many companies will show higher stock returns, higher profitability, etc. than during economic downturns. If this is the case, there is cross-sectional correlation in the data and the error terms are correlated. Applying a simple ordinary least squares (OLS) approach to the pooled data regardless of its two-dimensionality then leads to incorrect standard errors. This phenomenon is called the errors-in-variables problem (see Griliches & Hausman, 1986, for a formal description of the problem).

Asset pricing literature offers two main solutions to this problem. The first is the approach by Black, Jensen, and Scholes (1972). They group securities on the basis of their ranked beta values obtained from five years of historical data and run time-series regressions for each of these portfolios on the model factors. Factor risk premiums are then given by the sample mean of each factor. Such an approach is especially powerful when analyzing bonds and stocks at the same time, because the factor loadings (i.e. the betas) have obvious meaning as factors exposures for both types of securities.

When only stock returns are to be explained, the alternative approach developed by Fama and MacBeth (1973) is more intuitive as factor loadings in this case originate from firm characteristics. We therefore choose this technique for our analysis.

The essence of the Fama-MacBeth approach is that estimation is split into two steps. In the first step, the test assets’ exposure to each of the factors is estimated. For that, the excess returns of the $N$ test assets are regressed on the model factors in $N$ time-series regressions of
the form

\[ R_{it} = a_i + b_i(r_{Mt} - r_{Ft}) + s_iSMB_t + h_iHML_t + r_iRMW_t + c_iCMA_t + \epsilon_{it} \]  

(18)

where \( a_i \) is a constant term and \( b_i, s_i, h_i, r_i \) and \( c_i \) are the exposures to the market, size, value, profitability and the investment factor, respectively. In the second step, risk premiums are estimated by running \( T \) cross-sectional regressions of the form

\[ R_{it} = a_t + \lambda_i^{Mkt}\hat{b}_i + \lambda_i^{SMB}\hat{s}_i + \lambda_i^{HML}\hat{h}_i + \lambda_i^{RMW}\hat{r}_i + \lambda_i^{CML}\hat{c}_i + \epsilon_t \] 

(19)

where \( a_t \) is a constant term, \( \hat{b}_i, \hat{s}_i, \hat{h}_i, \hat{r}_i \) and \( \hat{c}_i \) are the factor loadings found in step one and \( \lambda_t^L \) is the risk premium linked to factor \( L \). The \( T \) risk premium estimates from these cross-sectional regressions are averaged to receive one single risk premium estimate for each factor:

\[ \hat{\lambda}^L = \frac{1}{T} \sum_{t=1}^{T} \hat{\lambda}_t^L \] 

(20)

Factor loadings measure to which extent the test portfolio returns are affected by the model factors. Risk premiums are a measure of how much extra excess return, ceteris paribus, a test portfolio gives due to one unit increase in exposure to factor \( L \), i.e. how the factor exposure is priced.

Figure 4 illustrates the Fama-MacBeth procedure graphically.

4.2.2 GMM regressions

The second-stage Fama-MacBeth regression uses explanatory variables that are estimates from the first-stage regression. Estimating risk premiums by an OLS approach does not take into consideration that the factor loadings are generated regressors and will thus lead to incorrect standard errors (see Pagan, 1984, for a more detailed description of the problem and its implications).

Additionally, as risk premiums are estimated based on cross-sectional regressions, time-variation in factor loadings is not taken into account. Petersen (2009) shows that the Fama-MacBeth standard errors are biased in the presence of serial correlation. We run rolling window estimations of the first-stage Fama-MacBeth regressions to scrutinize the assumption of constant factor loadings and find that it indeed might be violated in our case. We do not show this analysis in detail here as it is not the focus of this thesis. The interested reader though finds an exemplary visualization of some of the results in Figures 8 and 9 in the appendix.

Both the issues due to generated regressors and those due to time-varying betas can be addressed by using a Generalized Method of Moments (GMM) approach to estimate risk premiums.

The GMM approach was first formulated by Hansen (1982). It is just another way of looking at estimation problems as known methods like OLS can be mapped into the GMM framework.
**Step 1:** Obtain the test assets’ exposure to each of the factors

![Diagram](Diagram1.png)

**Step 2:** Obtain the test assets’ risk premiums due to factor exposure

![Diagram](Diagram2.png)

**Figure 4**
Illustration of the Fama-MacBeth procedure.
We confine the discussion here to giving a basic intuition of the GMM approach. Apart from Hansen (1982), the interested reader is referred to Cochrane (2005, chapters 10, 11 and 13) for an extensive overview of estimation features and implications.

Economic theory provides a set of so-called moment conditions which should hold. A perfect model would make all conditions to be precisely met. In practice GMM finds the best model estimators by minimizing the overall deviation from the set of conditions. The resulting deviations from each moment condition gives information about the importance of each condition for the estimation. Therefore, in a second step, estimation errors are again minimized, this time by weighting each error with its importance found in the previous step. By using information originating from the data itself, GMM produces unbiased estimates with robust standard errors.

Hansen and Singleton (1982) derive and discuss the GMM approach for the setting of the consumption-based model that is the fundament of all asset pricing approaches (see discussion in section 2). This implies the economic theory needed for our approach, although we do not draft the implications of investors utility functions on the GMM procedures, as this is not the focus of this thesis.

Recall from section 2 that the fundamental asset pricing equation can be expressed as

\[ p_t = E_t[m_{t+1} x_{t+1}] \]  \hspace{1cm} (21)

where \( p_t \) is an asset’s price, \( m_{t+1} \) is the stochastic discount factor and \( x_{t+1} \) is the asset’s future cash flow. This equation can easily be transformed into the moment condition

\[ 0 = E_t[m_{t+1} x_{t+1} - p_t] = E_t[u_t(\theta)] \]  \hspace{1cm} (22)

which states that the pricing error \( u_t(\theta) \), which is dependent on some unspecified parameters \( \theta \), should in expectation be zero. As a correct asset pricing model should price all assets, there is one moment condition (22) for each test asset.

By means of the Law of Iterated Expectations, equation (21) can be transformed into the Euler equation

\[ 1 = E_t[m_{t+1} r_{t+1}] \]  \hspace{1cm} (23)

where \( r_{t+1} \) is an asset’s return. The Euler equation implies the following moment conditions for models that estimate returns instead of prices

\[ 0 = E_t[m_{t+1} r_{t+1} - 1] \]  \hspace{1cm} (24)

In our setting of estimating the five-factor model for a set of \( N \) test portfolios, we thus have \( N \) moment conditions of the form (24). GMM estimation then finds estimates for the model parameters by minimizing all \( N \) pricing errors simultaneously.
4.2.3 Evaluating model performance

In addition to assessing the absolute performance of the five-factor model, we also look at its performance relative to a model without the two new factors. The intuition behind this is the principle of parsimony, i.e. that a more extensive model is only appropriate when it adds considerable informational value to the sparser model. If model fit does not increase significantly, the simpler model gives about the same information and should hence be preferred.

As one removes the profitability and the investment factor, one arrives at the Fama-French three-factor model with the market, size and value factors as independent variables. As discussed in section 2, others find evidence that all three factors have explanatory power on the German stock market (see e.g. Ziegler et al., 2003). The three-factor model thus seems like a proper benchmark model for the German data. For Norway though, the latest research rejects the significance of the value factor for explaining Norwegian stock returns (Næs et al., 2009). Hence, in this case, using a two-factor model that contains the market and the size factor as benchmark appears more appropriate.

In an attempt to embed this intuition more directly in our specific data, we run stepwise regressions that backwards eliminate unnecessary factors based on the Akaike Information Criterion. To enhance clarity we do not show the results of these regressions here. The algorithm can however be found in our documentation of used codes. We can report that the stepwise regressions do not converge to one specific model that shows best performance for all test portfolios, neither for the German nor the Norwegian data. A two-factor model comprising the market and size factors generally appears to fit best to the small stock portfolios, while the value factor seemingly is more relevant in regressions on large stock portfolios. In order not to lose the main thread of this thesis we leave out operating with individually adjusted benchmark models. Instead, we build on existing evidence and argue that the five-factor model is useful for describing German stock returns when it improves model fit relatively to the three-factor model and enlarges the understanding of Norwegian stock returns when it enhances model fit compared to the mentioned two-factor model.

In the context of the first-stage Fama-MacBeth regressions, model performance can be assessed by looking at the absolute size of the estimated intercepts. If a model is correctly specified and thus captures all return variation, its pricing error $a_t$ equals zero (Merton, 1973). In our setting of multiple simultaneous regressions on the set of test portfolios, this condition is fulfilled if all $N$ regression intercepts jointly equal zero. This can be tested with the modified F-test developed by Gibbons et al. (1989), in the following only referred to as GRS-test. It has the form

$$
\frac{T}{N} \times \frac{T - N - L}{T - L - 1} \times \frac{\hat{\alpha}' \hat{\Sigma}^{-1} \hat{\alpha}}{1 + \mu' \hat{\Omega}^{-1} \mu} \sim F(N, T - N - L)
$$

where $N$ is the number of simultaneous regressions or test portfolios, $T$ is the number of time periods (in our case months), $L$ is the number of explanatory variables i.e. factors in the
model, $\hat{\alpha}$ is an $N \times 1$ vector of estimated intercepts, $\hat{\Sigma}$ is an estimate of the residual covariance matrix, $\mu$ is an $L \times 1$ vector of factor portfolios’ sample means, and $\hat{\Omega}$ is an estimate of the factor portfolios’ covariance matrix. The null of the GRS-test is $H_0 : \alpha_i = 0, \forall i = 1, \ldots, N$. If the null hypothesis holds, the GRS-statistic will be close to zero. The larger the regression intercepts are in absolute value, the greater the GRS-statistic.

When estimating risk premia in the GMM-setting, model fit can be assessed by evaluating the size of the pricing errors produced by the GMM. This is done by the test of overidentifying restrictions, also called J-test (see Cochrane, 2005, chapter 10). It has the form

$$T \times [g_T(\hat{\theta})'S^{-1}g_T(\hat{\theta})] \sim \chi^2(N - L)$$

where $T$ is the sample size, $g_T(\hat{\theta})$ is the sample mean of the pricing errors, $S$ is the variance-covariance matrix of $g_T$, $N$ is the number of test portfolios (moments) and $L$ is the number of model factors (parameters). Under the null of this test, the second-stage GMM estimate is $\chi^2$-distributed with $N - L$ degrees of freedom. Models are thus rejected when J-statistic is too high.
5 Data

5.1 Motivation for building factors from raw data

As the number of publications dealing with empirical multi-factor models is increasing, more and more researchers make their constructed factors publicly accessible. The main source of factors based on U.S. data is Kenneth French’s own database (French, 2016). Brückner, Lehmann, Schmidt, and Stehle (2015) give an overview of the available sources of German factor data. Extensive databases which cover several countries are among others provided by P. S. Schmidt, Von Arx, Schrimpf, Wagner, and Ziegler (2014) and Marmi and Poma (2012). Ødegaard (2016a) provides extensive data on factors for the Norwegian stock market. Nevertheless, none of these databases provide data for the Fama and French-style profitability and investment factors for Germany and Norway as we write this thesis. One possible approach would be to use existing data for the market, size and value factor and only construct the two missing factors from raw data. Brückner et al. (2015) argue however that the underlying assumptions for factor construction differ considerably from one database to another so that the choice of data provider in itself can lead to different model outcomes. With this in mind, we conclude that it is most consistent to construct all model factors from raw data so that all are based on the same set of assumptions and that we are certain about all assumptions made.

All data analysis is conducted in R. The interested reader can look up programming details or follow our steps of analysis by looking at the R code attached in the appendix. To enhance clarity, we divide the code into several parts. Later parts are however based on the output of earlier parts and cannot be used as individual R programs. As we show the code in the order of appearance in the program we use, our results can still be reproduced by running all codes. Further, we want to spare the reader many iterations of the same content and thus only show one example code where we actually run the same analysis several times. In this sense we show the code for the analysis of the German data, for one currency and for the base scenario only. Most of the codes are based on our own understanding. We however follow the "recipe" of Diether (2001) for calculating the GRS-statistic and use the code provided by Ødegaard (2016b) as basis for GMM estimation of the risk premiums.

5.2 Sample construction

Our main source of data is the Compustat Global database. From there we extract two data files for each country, one that contains all available security data and one that contains all firms’ accounting data. Table 3 gives an overview of the sample variables that we retrieve and use in the following analysis. The original German security data set contains stock information of in total 1210 companies, while the original German accounting data file contains data of 1065 firms. In the Norwegian sample the corresponding total numbers of firms are 433 in the security data set and 354 in the accounting data set. In order to make the data usable for our
purpose of analysis, we filter and adjust the raw data and calculate new variables on the basis of the given ones. Tables 4 and 5 give an overview of how each of these steps affects the number of observations, firms and time periods in our data set. In the following subsections we discuss each step in further detail. After all security or accounting data-specific adjustments, we merge the two revised data files into one single sample. In this step the number of observations is reduced quite a bit, since we only keep observations for which we have both accounting and stock price data for at least one time period. Our final data set then contains in total 973 German and 268 Norwegian firms. Figure 5 shows the number of firms per sample month. It is obvious that the number of sample firms differs a lot from the beginning of our sample period to its end. The German data set comprises about 100 companies in the beginning of the nineties and around 450 companies from year 2001 until the end of our sample period. The Norwegian data set contains no more than 20 companies for the early nineties and about 100 companies from year 2007 until the end of our sample period.

5.2.1 Time period and compounding

From Compustat we extract all available data for the period of June 1989 to December 2015. This data enables us to investigate returns from July 1991 until December 2015, i.e. 294 months, since the analysis of asset returns at any time $t$ requires accounting data with a lag of two years.
Table 3
Overview of the sample variables retrieved from Compustat Global and used in our study.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Content</th>
</tr>
</thead>
<tbody>
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<td><strong>Common ID variables</strong></td>
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<td>Date</td>
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</tr>
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<tr>
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<td>Costs of Goods Sold</td>
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<td>Total Liabilities</td>
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<td>Deferred Taxes and Investment Tax Credit</td>
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Table 4
Germany: Development of the number of sample observations from the original data file to our final data set. The single steps are described in detail throughout section 5.

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<table>
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<th>years</th>
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* months
Table 5
Norway: Development of the number of observations from the original data file to our final data set. The single steps are described in detail throughout section 5.

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* months
(more about this in Section 4.1.3 and Figure 3). The reason for this sample choice is twofold. Firstly, we want to align our study with the studies of international stocks markets by Fama and French (2012, 2015b) as well as Novy-Marx (2013) which all are based on this very time horizon. Secondly, Compustat Global only provides data from the mid-80s due to poor reporting in prior periods. Especially small stocks were not present in the global database before 1989. In recent periods a higher proportion of companies reports its fundamentals.

The accounting data in Compustat Global is naturally reported annually. While one can choose between daily and monthly compounding of security data for North-American countries, Compustat does only provide daily data in its global database. We refer to the general practise of testing multi-factor models on the basis of monthly stock returns (see for example Fama and French (2015a), Næs et al. (2009) and Stehle and Schmidt (2014)) and convert the daily securities data obtained from Compustat to monthly data. As monthly stock prices in Compustat are defined as close prices for each calendar month, we aim to reduce our dataset by keeping only end-of-month observations. Our data set contains several cases where a company’s reported last day of the month deviates considerably from the actual last trading day of the respective month. This occurs whenever a stock is not traded at the last days of a month, a case that mostly applies to small stocks. As we later on interpolate between the end-of-month observations to calculate monthly returns, keeping mid-month observations would lead to uneven return intervals, which could not be compared to regular risk-free rate periods. Consequently, we restrict our sample to end-of-month observations only. We however allow the reported data to deviate slightly from actual last trading days – might it be due to inaccurate reporting or international differences in local holidays – by defining reported last day of the month observations as end-of-month observations if they fall on the 26th or later. This reasoning is supported by the fact that our data shows a clear breakpoint between the number of observations with an end-of-month day lower than the 26th and the number of observations with an end-of-month day of at least 26 with the vast majority of observations belonging to the latter group.

5.2.2 Operational vs financial firms

The financial statements of most financial firms differ fundamentally from those of operational firms and leverage has a different role in each of the firm types. The following example illustrates this point. While operational firms’ capital ideally should consist of at least 50% equity, banks (as an example of a financial firm) normally hold no more than 10% equity. This is the result of a bank’s role as financial intermediary and asset transformer, rather than a sign of financial distress. As the measures of firm characteristics that we define in our further analysis do not take these fundamental differences into consideration, including financial firms in the analysis may lead to biased results. Therefore we exclude financial firms, which are identified by a standard industrial classification (SIC) code between 6000 and 7000, from our sample. This filtering is already done when we retrieve the data file from Compustat, so financial firms are
5.2.3 Exchange rates

In our study, we take the view of an U.S. investor who can choose to invest in the German or Norwegian market as an alternative to his U.S. home market. Therefore we convert all data in our sample to US Dollars (USD). The security data from Compustat does only contain local currency observations after we have removed the stocks listed at foreign exchanges (see Section 5.2.5), so in this case we only need exchange rates between USD and Deutsche Mark (DEM), Euro (EUR) and Norwegian Kroners (NOK). The accounting data comes however with a twist. As some domestic companies have their main operations in foreign countries, their accounting numbers are reported in foreign currencies. These are Australian Dollars (AUD), Great Britain Pounds (GBP), and USD within the German sample, and EUR and USD in the Norwegian sample. We keep these companies in our sample as long as they are legally registered at any German or Norwegian stock exchange, and thus have stocks that are traded in domestic currency on the local market. In the German security file we have 259 observations, or 7 companies, which do not fulfill this requirement since they are all denoted in South African Rand (ZAR). We omit these observations from our sample. We obtain most exchange rates from the U.S. Federal Reserve. The DEM/USD exchange rate however, is the MSCI rate taken from Datastream. All original exchange rate series contain daily closing spot rates. From these we calculate two sets of exchange rates for each currency which contain rates for each fiscal year end month of each accounting year. The first set consists of balance sheet rates, defined as the exchange rate of the last day of a company’s fiscal year, i.e. the end-of-month rate depending on the fiscal year cycle. The second set comprises profit and loss exchange rates which we define as the mean of all daily exchange rates contained in a company’s fiscal year.

5.2.4 The risk-free rate

In line with the assumption of an U.S. investor, we use the 4-week U.S. Treasury bill, provided by the Center for Research of Security Prices (CRSP), as a proxy for the risk-free rate. Since this time series comprises daily returns, we calculate monthly returns by multiplying with 30. This is a simplification given that the return periods we are looking at not always have the span of exactly 30 days. An alternative would be to align the time span of the risk-free rates with the specific time span of each stock return period. As these rates are quite small in size compared to the stock returns, we assume though that the simpler alternative is sufficient.

5.2.5 Choice of stock exchanges

Since our Compustat query asks for all listed German and Norwegian stocks, the original data set contains not only stocks that are listed at a local stock exchange, but also those that are traded at foreign stock exchanges. Most of the companies that list at foreign exchanges also
have stocks at least at one local exchange. We observe that the foreign listings of this kind are duplicates of the local listings, as they show the exact same number of shares outstanding and their stock price is simply converted to the foreign exchange’s local currency. Consequently, we remove the observations listed at the foreign exchange to avoid double counting. This logic is in line with Fama and French (1993), where the authors exclude American Depository Receipts (ADRs), i.e. cross listings of foreign stocks on the U.S. market (see A. Karolyi, 2003), from their sample. In addition to these double-listings, there are a few companies – four German firms and one Norwegian firm – in our sample, which are listed on foreign exchanges only. We exclude these observations from our sample since they might be subject to foreign rules and regulations e.g. with regard to taxation or accounting and thus not be fully comparable to locally listed stocks. Besides, these observations might lead to exchange rate uncertainties that do not apply to the locally listed stocks. By excluding them from the sample, we focus on country specific effects only. In the case of Germany we keep stocks listed at all eight domestic stock exchanges, including those traded at Xetra. For the Norwegian data we keep stocks listed at Oslo Stock Exchange and those traded over the counter (OTC).

5.2.6 Common vs preferred stocks

Analyses of American stock data are normally restricted to common stocks as the nature of U.S. preferred stocks is closer to bonds than to stocks (Brückner et al., 2015) and therefore have quite different dynamics. In Germany it is very common that enterprises issue both ordinary and preferred stocks. As argued by Stehle and Schmidt (2014), German preferred stocks have more in common with ordinary stocks than with bonds and should hence be included when studying stock returns. Therefore we keep preferred stocks in our sample when analyzing the German stock data. In Norway preferred stocks are less common although some companies do issue so called A- and B-stocks. Reading several annual reports of concerned companies gave us the impression that the definition of these stock types is quite vague and differs from company to company. In most cases which we examined, A-stocks were classified as common shares while B-stocks were defined as either another series of common shares or as shares with restricted voting rights, restricted dividend payments etc. In some other cases B-stocks are classified as common shares, while A-stocks are subject to restrictions. The ambiguity in the definition of B-stocks makes it difficult for us to evaluate whether Norwegian B-stocks should be included or excluded from our sample. As Compustat defines all Norwegian shares to be common, irrespective of the existence of different share series, we assume resemblance to the German model and include all Norwegian shares in our analysis.

5.2.7 Penny stocks

Shares of very low value, commonly called penny stocks, can distort our analysis of returns since even slight stock price increases are noted as very high returns. This is misleading when
it does not reflect the rise of a growth company but simply shows minimal price fluctuations of inconsiderable stocks. The U.S. Securities and Exchange Commission (2013) defines a penny stock as a security that is “issued by a very small company that trade(s) at less than $5 per share.” According to the NASDAQ stock market rules (Nasdaq, Inc., 2016), stocks with this property do not qualify for being listed yet or get delisted. This implies that studies of the U.S. stock market automatically do not include penny stocks. In Germany however, there was no delisting rule until recently (Stehle & Schmidt, 2014). Consequently, our German data set contains a lot of penny stocks of companies that went bankrupt but where not delisted. To identify these disadvantageous observations, we need a proper definition of German penny stocks. One of the current Frankfurt Stock Exchange general standard listing requirements is that the “company must provide evidence of a minimum nominal capital of EUR 750,000,- […] paid-in-capital and the nominal value of shares must not be less than one Euro.” (FrankfurtStockExchange.de, 2016). We use this as the definition for German penny stocks and remove all observations from the sample that do not fulfil these requirements. For the time period before 1999, requirements are applied to prices denoted in DEM by using the irrevocably fixed conversion rate of 1,95583 DEM/EUR set by The Council of the European Union (1998). In the absence of a formal rule for first sample years, we apply the market capitalization requirement to all sample years, although the threshold value of EUR 750,000,- due to inflation might be too high for earlier years. To check if this simplification has an impact on our results, we have looked closer at the companies that become excluded by this procedure and found that most penny stocks occur after year 2000. This lets us assume that an inflation adjustment of the market capitalization requirement would not change our results significantly. The delisting rules for the Oslo Stock Exchange say that a company must not have a share price of less than one NOK during a period spanning more than six months (Oslo Børs, 2016b). Because of the six-month rule, some companies do have an unaccepted stock price for shorter periods. These observations are excluded from our analysis.

5.2.8 Calculation of returns

As Compustat does not provide a return variable, we compute monthly individual stock returns as

\[ r^i_t = \frac{p^i_t \cdot adj^i_t}{p^i_{t-1} \cdot adj^i_{t-1}} - 1 \]

(27)

where \( p^i_t \) is the monthly close price of stock \( i \) and \( adj^i_t \) is the Compustat adjustment factor that adjusts prices for stock splits and stock dividends. Stock splits and dividends lead to changes of the actual stock price although the total company value remains unchanged. A simple example of this is a 2-for-1 stock split where 100 company stocks at the price of 50 are diluted to 200 stocks at the price of 25. Due to this, actual prices before and after such an occurrence are
not directly comparable and return computation across these prices would give misleading
er results. The adjustment factor incorporates these cases. It takes the value 1 when a stock series
are not subject to any stock splits or stock dividends. A 2-for-1 stock split in year \(t\) leads to
adjustment factors of 2 for all periods prior to year \(t\). Adjustment factors are cumulative, so an
additional stock split in year \(t - 2\) leads to adjustment factors of 4 for all years prior to year
\(t - 2\). Adjustments for stock dividends are indicated in a similar manner.

In Figure 6 we show the return distributions that we get by analyzing the German and
Norwegian data for the sample period spanning July 1990 to December 2015. What sticks out
is that our sample contains a disproportionately high number of stocks with a return equal
to zero. A return of exactly zero can only occur when neither the number of shares, nor the
stock price has changed from one period to another. Particularly the latter is quite improbable
given steady stock market movements. A closer look into the data gives us the impression that
returns exactly equal to zero indicate insufficient reporting, since the respective observations
also lack other information. Another explanation for zero or close-to-zero returns can be low
trading volumes and hence poor liquidity of the stock. A closer look at our data shows though
that using a trading volume threshold does leave large parts of the zero-return observations
unexplained. We therefore choose to toss out all observations with zero return, although this
is a simplified technical step and not directly based on a natural selection criterion.

Table 6 shows key measures of the return distribution at the two stock markets. It raises
the question of how to treat observations with extreme returns. On the one hand, investors are
particularly concerned about just these extreme realizations, on the other hand it is ordinary
practise to exclude extreme values in empirical work to avoid that results are distorted by
these outliers (see for example Aharoni et al., 2013; Novy-Marx, 2013). In principle, a correctly
specified asset pricing model should price all assets, i.e. also extreme cases. Looking closer
at our data, we observe that the extreme returns are mostly caused by single firms that have
quite low stock prices but are still not covered by the penny stock definition. In line with the
discussion in section 5.2.7 we consider it misleading to keep these observations in the sample
and thus choose to follow ordinary practice and trim our sample returns by 0.5 percent on
each side of the distribution. The right hand side of Table 6 shows that these adjustments lead
to a much more normalized return distribution. The right hand side of Figure 6 confirms this
graphically. It shows the return distribution after all adjustments, and compares it to a normal
distribution that has the same mean and standard deviation as the sample data. We observe
that returns are bell-curve shaped, but have a higher kurtosis than the corresponding normal
distribution. The application of a Jarque Bera test to the trimmed data sample rejects the null
hypothesis of normally distributed returns. This is in line with others’ findings concerning
return distributions. Eberlein and Keller (1995) find that daily return data of DAX companies
has a hyperbolic distribution.
Table 6
Descriptive statistics of the end-of-month sample stock returns for the period of July 1990-December 2015, before and after adjustments. Both samples include only stocks listed at local stock exchanges and penny stocks are removed. The adjusted sample equals the original sample less the 0.5% highest and lowest observations and does not contain returns that equal exactly zero.

<table>
<thead>
<tr>
<th></th>
<th>Original sample</th>
<th>Adjusted sample</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Germany</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Min</td>
<td>-95.39 %</td>
<td>-52.00 %</td>
</tr>
<tr>
<td>Mean</td>
<td>0.65 %</td>
<td>0.00 %</td>
</tr>
<tr>
<td>Max</td>
<td>13900.00 %</td>
<td>83.10 %</td>
</tr>
<tr>
<td>Std</td>
<td>44.72 %</td>
<td>14.02 %</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>63270.95</td>
<td>4.46</td>
</tr>
<tr>
<td>Skewness</td>
<td>212.74</td>
<td>0.70</td>
</tr>
<tr>
<td><strong>Panel B: Norway</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Min</td>
<td>-90.43 %</td>
<td>-47.06 %</td>
</tr>
<tr>
<td>Mean</td>
<td>0.70 %</td>
<td>0.55 %</td>
</tr>
<tr>
<td>Max</td>
<td>471.43 %</td>
<td>66.67 %</td>
</tr>
<tr>
<td>Std</td>
<td>16.11 %</td>
<td>13.92 %</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>48.53</td>
<td>2.43</td>
</tr>
<tr>
<td>Skewness</td>
<td>2.83</td>
<td>0.57</td>
</tr>
</tbody>
</table>
Figure 6
Distribution of individual monthly stock returns before and after adjustments, July 1990-December 2015. Returns are computed as the change in stock price from month $t - 1$ to month $t$, divided by the stock price in $t - 1$. All stock prices are adjusted for stock splits and dividends. The adjusted sample excludes the 0.5% lowest and 0.5% highest observations as well as returns that equal exactly zero.

5.2.9 Negative book equity and book assets

When a firm shows both negative operating profit and negative book equity (BE), the operating profitability (OP) measure will turn out to be positive. This is against the idea that a high level of OP should be assigned to firms that are more profitable than their competitors. Mixing highly profitable firms with those firms that do very badly, will prone to distort our analysis. Therefore we do not use firms with negative book equity when constructing the profitability measure. For consistency negative BE firms are omitted as well when defining the other sorting variables. They are however included in the market portfolio as this is supposed to reflect all stock variety. An equivalent reasoning lets us omit firms that record a negative book value of assets (at) as keeping them would lead to misleading measures of investment. Tables 4 and 5 show however that this is only a minor problem in our data set.
6 Results

6.1 Overview of the test portfolio returns

6.1.1 Simple sorts

To get a first idea of which effects might be present in our German and Norwegian stock data, we look at the return spreads between the two extreme portfolios that result from sorting and splitting sample stocks by the single sorting characteristics. Here we only look at one anomaly at the time while we in Section 6.1.2 adress returns of portfolios based on double sorting. Figure 7 shows the cumulative return of the two extreme portfolios throughout the 294 months of our sample period for each of the two countries. Panel (a) shows the size spread, Panel (b) the value spread, Panel (c) the profitability spread and Panel (d) the investment spread.

In the sort on size, German megacap stocks outperform microcap stocks until the year 2004. Then the trend reverses and microcap stocks give a higher return than megacap stocks until the end of our sample period. For the Norwegian data we find a large and unambiguous spread with microcap stocks clearly outperforming megacap stocks throughout the entire sample period.

When sorting by firm value we cannot find clear return differences between German high and low book-to-market stocks until the year 2003. Henceforth, high B/M stocks beat low B/M stocks, consistent with Fama and French’s findings. For the Norwegian data we cannot identify the same pattern. From year 2000 on, high value stocks show a lower accumulated return than low value stocks, but since the spread then is not increasing in the course of time, we cannot report a persistent value effect for the Norwegian data.

The sort on operating profitability does not reveal any particular return pattern of the German data. For the Norwegian data we find that weak profitability stocks generate higher returns than robust profitability stocks. This is the opposite of the findings for U.S. companies.

Splitting stocks by investment behavior does not seem to generate any return spread, neither for German nor for Norwegian stocks.

This first analysis leaves us with the conjecture that neither the profitability nor the investment factor explain return differences of the German stock data. For the Norwegian data a profitability factor might have explanatory power while the investment factor does not seem to play a role. These results are nevertheless based on appearance alone so we will support them with actual data analysis in the following sections. Note as well that the returns that are shown here are not adjusted for market risk. It thus remains to prove that premiums found exceed the market premium. We adress this topic in Section 6.5.

6.1.2 Double sorts

Table 7, shows the average monthly percentage excess returns on the test portfolios formed by double sorts on size and B/M (Panel A), size and OP (Panel B), as well as size and Inv (Panel C).
Figure 7
Spread between the two extreme portfolios of simple sorts by (a) company size, (b) value (B/M), (c) profitability (OP) and (d) investment (Inv), July 1991 to December 2015, 294 months. Size is a firm’s market capitalization at the end of June of year \( \tau \). B/M is book equity at the end of fiscal year \( \tau - 1 \) divided by market equity measured at the end of December of year \( \tau - 1 \). OP is defined as total revenues less total operating expenses and total interest expenses all divided by book equity, all measured at the end of fiscal year \( \tau - 1 \). Inv is the growth of total assets from the end of fiscal year \( \tau - 2 \) to the end of fiscal year \( \tau - 1 \). Portfolios are updated in the end of each June, with weights hold constant throughout the following 12 months. The graph shows monthly value-weighted accumulated portfolio return in percent, not adjusted for market risk. The green lines show German portfolios, the black lines the Norwegian ones.
We cannot detect any clear size-pattern in the German data. In the size-B/M sorts, average returns seem to evolve randomly in the vertical dimension when controlling for firm value. The only exception is an extraordinary high average return of the firms that both are in the group of the smallest firms and in the group of lowest value. This risk premium might reflect the investors’ uncertainty about the development of this obviously most risky group of stocks. The size-OP sorts resemble more the classical size effect patterns, as average returns of small firms are always higher than those of big firms, ceteris paribus. In between those two extreme points of the size scale we can however not detect a clear downwards trend. The returns of the mid-level size groups show in almost all cases a peak before falling again. For the size-Inv sorts we find falling average returns for the firms with the most conservative investment behavior, while returns increase in size for the firms with the most aggressive investment behavior. For mid-level investment behavior returns are oscillating without a clear trend. This ambiguity is consistent with our findings in Section 6.1.1 that none of the German extreme size portfolios dominates its counterpart over the whole sample period. In line with the large spread that we observe in Panel (a) of Figure 7, we find the existence of a classical size effect in the Norwegian data as the microcap portfolios’ average return falls consistently with increasing firm size. An example is the size-BM sort where monthly average excess returns for microcaps are 1.85%, 1.91% and 2.02%, with respective megacap-returns equal to 0.78%, 0.92% and 0.42%. This trend is present in all portfolio sorts except for the companies with the most aggressive investment behavior. In this case a premium applies to large companies investing heavily.

The tendency of German high value stocks to outperform low value stocks that we found in Section 6.1.1, can also be seen in Panel A from Table 7. When controlling for size, the lowest value portfolios have average returns much below the high value portfolios. An exception is the high premium applying to the group of smallest and lowest value firms. While only looking at the endpoints of the value scale gives a clear picture, returns are not falling steadily with increasing value, but oscillate for the mid-level value groups. This might be due to the unclear value effect before the year 2003. For Norwegian data we do not find any significant spread between the two extreme value portfolios. This absence of the value effect is also reflected in the double sort return data as we find increasing returns for microcap stocks but an oscillating and rather decreasing return trend for medium-sized and megacap firms.

The spread analysis above suggests that there is no profitability effect in the German data. Table 7 shows however that when controlling for firm size, stocks of the most profitable firms outperform those of weak profitability firms. This is valid for all size groups except the microcap portfolios. For Norwegian stocks we assumed an opposite profitability effect. As the size effect is ruled out, we now find robust stocks outperforming weak stocks for both micro- and megacap firms. While we find a monotonous rise in average returns within the megacap group, which differs from 0.57% in the weak profitability group, via 0.71% in the mid-level OP group to 0.73% in the robust OP group, return development of the microcap stocks is not that steady. Additionally, in the group of medium-sized companies weak profitability stocks have higher
returns than robust profitability stocks.

The graphical analysis in Figure 7 lets us assume that we do not have any investment effect in neither the German nor the Norwegian data. Table 7 shows however that at the German stock market conservative investment strategies lead mainly to higher stock returns than aggressive investing. For megacap firms the opposite applies. The same pattern is observable in the Norwegian data. We find Norwegian average returns to be falling steadily as investment evolves from conservative to aggressive except for microcap stock returns which are higher for aggressive than for conservative investment stocks. To give an example, the average excess return of microcap-portfolios falls from 3.12%, to 1.96%, ending at 0.88%.

The analysis in this section leaves us still uncertain about whether or not operating profitability and investment behaviour are useful to explain asset returns in Germany and Norway. To find more evidence, we conduct regression analyses in Section 6.3. Before that we give an overview of the returns of the factor mimicking portfolios.

### 6.2 Overview of the factor mimicking portfolio returns

The average return of the five factor mimicking portfolios, their standard deviations and t-statistics are shown in Panel A of Table 8. We find that for Germany only the market factor is significantly different from zero with an average return of 0.84%. For Norway, both the market and the size factor have effects that are significantly different from zero at the 5% level. The average return of the market portfolio is 1.01%, while the average return of the SMB factor portfolio is 0.73%.

Table 19 in the appendix gives more insight into why most of the factors do not have a significant effect in explaining asset returns in our data. We list the average percent returns, standard deviations and t-statistics of the RHS portfolios that form the basis for factor portfolio construction. Almost all of these portfolios have significantly positive average returns at least at the 5%-level.

These do not differ that much given relatively high standard deviations. When these building blocks however are combined to the hedging portfolios in the way described in Table 2, large parts of the long- and short position returns cancel out and hence give average factor portfolio returns that are insignificantly different from zero, given the relatively high return variation.

In Panel B of Table 8 we further show the correlations between the factor portfolio returns. In an ideal model, explanatory factors should be independent from each other, i.e. the closer the correlation coefficients are to zero, the better. We do not detect this in our data set. For Germany we get market portfolio returns that are negatively correlated with all other factor portfolios. Particularly the quite high negative correlation of -0.44 between the market and the SMB portfolio stands out. Further the returns of SMB and HML have a positive correlation of 0.27, while RMW and CMA have a positive correlation of 0.31. For Norway we find overall
Table 7
Average monthly excess return per test portfolio in the period from July 1991 to December 2015 by three different double sorts, all numbers in %. Portfolios are constructed as described in Table 17. Each portfolio’s return is the sum of the returns of the stocks it contains, weighted with the respective firm’s market capitalization. Excess returns are portfolio returns minus the 4-week U.S. Treasury bill rate.

<table>
<thead>
<tr>
<th></th>
<th>Germany</th>
<th>Norway</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low</td>
<td>2</td>
</tr>
<tr>
<td><strong>Panel A: Size-B/M portfolios</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Small</td>
<td>2.08</td>
<td>0.26</td>
</tr>
<tr>
<td>2</td>
<td>0.42</td>
<td>1.22</td>
</tr>
<tr>
<td>3</td>
<td>0.35</td>
<td>0.76</td>
</tr>
<tr>
<td>4</td>
<td>0.60</td>
<td>0.96</td>
</tr>
<tr>
<td>Big</td>
<td>0.85</td>
<td>0.54</td>
</tr>
<tr>
<td><strong>Panel B: Size-OP portfolios</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Small</td>
<td>1.23</td>
<td>1.32</td>
</tr>
<tr>
<td>2</td>
<td>0.67</td>
<td>0.72</td>
</tr>
<tr>
<td>3</td>
<td>0.57</td>
<td>0.81</td>
</tr>
<tr>
<td>4</td>
<td>0.85</td>
<td>0.91</td>
</tr>
<tr>
<td>Big</td>
<td>-0.07</td>
<td>0.64</td>
</tr>
<tr>
<td><strong>Panel C: Size-Inv portfolios</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Small</td>
<td>1.62</td>
<td>1.34</td>
</tr>
<tr>
<td>2</td>
<td>0.82</td>
<td>0.63</td>
</tr>
<tr>
<td>3</td>
<td>1.00</td>
<td>0.55</td>
</tr>
<tr>
<td>4</td>
<td>0.89</td>
<td>1.18</td>
</tr>
<tr>
<td>Big</td>
<td>0.67</td>
<td>0.90</td>
</tr>
</tbody>
</table>
Summary statistics of monthly factor returns, July 1991-December 2015. Averages and standard deviations are shown in percent. Mkt is the value-weighted return on a portfolio of all sample stocks in excess of the four-week US Treasury bill rate. SMB is the size factor, HML is the value factor, RMW is the profitability factor and CMA is the investment factor. Factors are constructed as described in Table 2.

<table>
<thead>
<tr>
<th></th>
<th>Mkt</th>
<th>SMB</th>
<th>HML</th>
<th>RMW</th>
<th>CMA</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>0.84</td>
<td>0.16</td>
<td>0.48</td>
<td>-0.06</td>
<td>-0.10</td>
</tr>
<tr>
<td><strong>Std</strong></td>
<td>5.77</td>
<td>3.95</td>
<td>5.17</td>
<td>5.77</td>
<td>5.44</td>
</tr>
<tr>
<td><strong>t-stat</strong></td>
<td>2.51</td>
<td>0.70</td>
<td>1.59</td>
<td>-0.17</td>
<td>-0.30</td>
</tr>
</tbody>
</table>

**Panel B: Correlations**

<table>
<thead>
<tr>
<th></th>
<th>Mkt</th>
<th>SMB</th>
<th>HML</th>
<th>RMW</th>
<th>CMA</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mkt</strong></td>
<td>1.00</td>
<td>-0.44</td>
<td>-0.11</td>
<td>-0.16</td>
<td>-0.26</td>
</tr>
<tr>
<td><strong>SMB</strong></td>
<td>1.00</td>
<td>0.27</td>
<td>0.06</td>
<td>0.12</td>
<td></td>
</tr>
<tr>
<td><strong>HML</strong></td>
<td>0.11</td>
<td>1.00</td>
<td>0.06</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td><strong>RMW</strong></td>
<td>-0.16</td>
<td>0.06</td>
<td>1.00</td>
<td>0.31</td>
<td></td>
</tr>
<tr>
<td><strong>CMA</strong></td>
<td>-0.26</td>
<td>0.15</td>
<td>0.31</td>
<td>1.00</td>
<td></td>
</tr>
</tbody>
</table>

less correlation between the factor returns. What sticks out is the negative correlation of -0.37 between the SMB and the RMW factor returns.

High factor correlations seem to be an issue in the U.S. as well. Fama and French (2015a) find a positive correlation coefficient equal to 0.70 between HML and CMA. Additionally they find negative correlations between CMA and the market portfolio, and between HML and RMW equal to -0.39 and -0.36 respectively.

Since we find relatively much correlation between the model factors, we conduct auxiliary regressions where four factors are used to describe average returns on the fifth. Auxiliary regressions that have an intercept that is statistically insignificant from zero, indicate factor redundancy. In that case the explanatory power of the fifth factor is fully absorbed by the other four factors. Regression results are shown in Table 20 and Table 21 in the appendix. In Germany (Panel A) the auxiliary regression on the market factor is the only one that produces an intercept significantly different from zero (at the 1% level). For Norway both the regression on the market factor and the one on the size factor have intercepts indistinguishable from zero, with significance levels of 1% and 5% respectively. According to this simple method HML, CMA and RMW (and for Germany SMB additionally) are all redundant in describing average
monthly excess returns. To investigate this further we perform the regression approach by Fama and MacBeth (1973) in the next section.

6.3 Factor exposures

Tables 9 to 14 show the factor exposures and their significance estimated by the first-step Fama-MacBeth regressions on test portfolios sorted on size-B/M, size-OP and size-Inv. The right-hand-side of the tables shows the coefficients produced by a five-factor model while the left-hand-side of table shows the equivalent factor loadings estimated by the respective benchmark model.

We start with an analysis of the factor exposures of the German test portfolios, shown in Tables 9 to 11. We observe that the three-factor and the five-factor model produce very similar factor exposure estimates and significance levels for the common factors. In the following discussion, we therefore focus on the estimated coefficients of the five-factor model.

We find that all test portfolios have a significant exposure of about 1 to the market factor, regardless of the portfolio sort. Besides, we observe that all test portfolios but those belonging to the largest size group are significantly exposed to the size factor. Exposures are highest for small-size/low-value stocks (maximum exposure of $\hat{b}=1.24$ in the size-BM sort) and decrease both with increasing company value and with increasing firm size. This pattern is present for all three portfolio sorts.

For portfolios sorted on size-BM we find significant exposure to the value factor for most portfolios of the largest two size groups and the group with lowest firm value. According to factor construction, exposures are negative for the low-value group (more negative the larger the firm size) and increase with firm value. Large high-value stocks have the largest exposure to the value factor ($\hat{h}=0.63$). Of the test portfolios sorted on size-OP, most of the mid and largest size group show some significant exposure to the value factor. Sign and size of the exposure varies though from portfolio to portfolio without a clear pattern. Size-Inv sorted portfolios are negatively exposed to the value factor when they contain aggressive stocks and mostly positively exposed to value when they comprise large-size stocks.

We find that only some few portfolios that are part of the largest size group of the size-BM sorts have a slightly positive exposure to the profitability factor. For several size-BM sorted portfolios of the lowest value group we find a slight negative exposure to profitability. This is though only at a 10%-significance level. As expected, portfolios sorted on size-OP show positive profitability exposure when they contain robust stocks and negative profitability exposure when they are formed by weak profitability stocks. Size-Inv sorted test portfolios that are comprised of aggresive stocks have a negative exposure between -0.10 and -0.26. Additionally, portfolios of the largest size group have significant exposure to profitability.

Four size-BM sorted portfolios show a negative exposure to the investment factor. These are the group of large firm size and low value in addition to some small-size/medium-value portfo-
lios. For the size-OP sorted portfolios we find mostly negative investment exposure for medium-
high profitability stocks and positive investment exposure for most weak profitability stocks.
Sorts on size-Inv give positive investment exposure in the group of large-size/conservative
stocks, while the exposure is negative for large-size/agegressive stocks.

The estimated factor exposures of the Norwegian test portfolios are shown in Tables 12 to 14.

As for the German stocks, we find that all Norwegian test portfolios have a market exposure
of about 1. High Size-exposure is prevalent in all three sorts for all small- and medium-sized
portfolios (with \( s \) ranging from 0.31 to 0.84). Additionally, big-size stocks of the size-BM sort
show significant slight size exposure.

As expected due to the construction of the value factor, we find that high-value size-BM
sorted portfolios have highly positive value exposure and that low-value portfolios have highly
negative value exposure. Small-size size-BM portfolios do however not show significant value
exposure. Further we find that most portfolios of the largest size group of the size-OP sorts
show a slight negative exposure to the value factor. For the size-Inv sorts we observe significant
value exposure for four of the nine portfolios. The size is however low and the sign is varying.

We find that both large-size and high-value portfolios sorted by size-BM have negative
profitability exposure. Little surprising, low-profitability size-OP sorted stocks show negative
exposure to the profitability factor while the opposite is true for high-profitability stocks of this
sort. This does however not apply to small firm stocks. There are four size-Inv sorted portfolios
that have significant exposure to the profitability factor, all of them negative. Small-size stocks
do in this case not show any significant profitability exposure.

Investment exposure can almost not be found in the size-BM and size-OP sorts. Naturally
size-Inv sorted portfolios show negative exposure to the investment factor when they contain
aggressive stocks and positive investment exposure when they contain conservative stocks.
Again, small-size stocks do not show significant exposure.

6.4 GRS-test

After having analysed the single factor exposures, we now look at the overall model performance
both in absolute and in relative terms.

As described in section 4.2.3, a good model is characterised by a zero pricing error. We thus
first look at the single estimated intercepts shown in Tables 9 to 14 and then at the GRS-test
which tests for all intercepts being jointly zero. The results of the GRS-test are shown in Table
15.

Among the 25 five-factor regressions on the German test portfolios sorted by size and
book-to-market, six result in intercepts that are significantly different from zero at a 10% level.
Positive pricing errors are found in the regressions of small size stocks with very low and very
high company value (\( \alpha=0.91\% \) and \( \alpha=0.44\% \)) as well as rather large-sized firms with high value
Table 9
Factor exposures of German size-BM portfolios. Results of the N first stage Fama-MacBeth time-series regressions on test portfolios formed on double sorts on size and investment behavior. FF5F is the five-factor regression

\[ R_{it} = \alpha_i + b_1(r_{Mt} - r_{Ft}) + s_i \text{SMB}_t + b_2 \text{HML}_t + r_i \text{RMW}_t + c_i \text{CMA}_t + \epsilon_{it} \]

where \( R_{it} \) is the excess return on portfolio \( i \) above the risk free rate of return \( r_{Ft} \), \( \alpha_i \) is the pricing error, \( r_{Mt} \) is the return of the value-weighted market portfolio, \( \text{SMB}_t, \text{HML}_t, \text{RMW}_t \) and \( \text{CMA}_t \) are the returns of the factor mimicking portfolios formed as described in Table 2. FF3F is the regression given above without \( \text{RMW} \) and \( \text{CMA} \) as explanatory variables. July 1991-December 2015, 294 months.

<table>
<thead>
<tr>
<th>BM →</th>
<th>( \hat{\alpha} )</th>
<th>( \hat{b} )</th>
<th>( \hat{s} )</th>
<th>( \hat{h} )</th>
<th>( \hat{\epsilon} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>0.009*** -0.008 -0.001 0.000 0.004**</td>
<td>0.009* -0.008 -0.001 0.000 0.004**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-0.004 0.001 -0.001 0.001 -0.001</td>
<td>-0.004 0.001 -0.001 0.001 -0.001</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-0.006** -0.002 0.001 -0.002 0.002</td>
<td>-0.006** -0.002 0.001 -0.002 0.002</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-0.002 0.001 -0.002 0.002 0.005**</td>
<td>-0.002 0.001 -0.003 0.002 0.005*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Big</td>
<td>0.001 -0.003** 0.002 -0.004** 0.001</td>
<td>0.001 -0.003** 0.001 -0.004* 0.001</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Significance levels: "p<0.1; **p<0.05; ***p<0.01"
Table 10
Factor exposures of German size-OP portfolios. Results of the \( N \) first stage Fama-MacBeth time-series regressions on test portfolios formed on double sorts on size and investment behavior. \( FF5F \) is the five-factor regression

\[
R_{it} = a_i + b_i(r_{Mt} - r_{Ft}) + s_iSMB_t + h_iHML_t + r_iRMW_t + c_iCMA_t + \epsilon_{it}
\]

where \( R_{it} \) is the excess return on portfolio \( i \) above the risk free rate of return \( r_{Ft} \), \( \alpha_i \) is the pricing error, \( r_{Mt} \) is the return of the value-weighted market portfolio, \( SMB_t, HML_t, RMW_t \) and \( CMA_t \) are the returns of the factor mimicking portfolios formed as described in Table 2. \( FF3F \) is the regression given above without \( RMW \) and \( CMA \) as explanatory variables. July 1991-December 2015, 294 months.

<table>
<thead>
<tr>
<th>OP</th>
<th>( FF5F ) Low</th>
<th>( FF5F ) 2</th>
<th>( FF5F ) 3</th>
<th>( FF5F ) 4</th>
<th>( FF5F ) High</th>
<th>( FF3F ) Low</th>
<th>( FF3F ) 2</th>
<th>( FF3F ) 3</th>
<th>( FF3F ) 4</th>
<th>( FF3F ) High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>1.24***</td>
<td>1.01***</td>
<td>0.99***</td>
<td>0.73***</td>
<td>0.91***</td>
<td>1.13***</td>
<td>0.97***</td>
<td>0.93***</td>
<td>0.79***</td>
<td>0.84***</td>
</tr>
<tr>
<td>2</td>
<td>1.07***</td>
<td>1.00***</td>
<td>0.92***</td>
<td>0.89***</td>
<td>0.86***</td>
<td>1.06***</td>
<td>0.97***</td>
<td>0.93***</td>
<td>0.91***</td>
<td>0.88***</td>
</tr>
<tr>
<td>3</td>
<td>1.05***</td>
<td>1.01***</td>
<td>0.82***</td>
<td>0.75***</td>
<td>0.74***</td>
<td>1.02***</td>
<td>0.99***</td>
<td>0.83***</td>
<td>0.78***</td>
<td>0.77***</td>
</tr>
<tr>
<td>4</td>
<td>0.77***</td>
<td>0.76***</td>
<td>0.62***</td>
<td>0.53***</td>
<td>0.53***</td>
<td>0.66***</td>
<td>0.71***</td>
<td>0.62***</td>
<td>0.55***</td>
<td>0.54***</td>
</tr>
<tr>
<td>Big</td>
<td>0.17</td>
<td>0.08</td>
<td>0.03</td>
<td>-0.20***</td>
<td>0.16***</td>
<td>0.13</td>
<td>0.07</td>
<td>0.05</td>
<td>-0.15***</td>
<td>0.21***</td>
</tr>
</tbody>
</table>

\( \hat{a} \) and \( \hat{b} \) are the returns of the pricing error, \( \hat{c} \) are the returns of the factor mimicking portfolios. Significance levels: *p<0.1; **p<0.05; ***p<0.01
Table 11
Factor exposures of German size-Inv portfolios. Results of the \( N \) first stage Fama-MacBeth time-series regressions on test portfolios formed on double sorts on size and investment behavior. \( FF5F \) is the five-factor regression

\[
R_{it} = \alpha_i + b_1(r_{Mt} - r_{Ft}) + s_iSMB_t + h_iHML_t + r_iRMW_t + c_iCMA_t + \epsilon_{it}
\]

where \( R_{it} \) is the excess return on portfolio \( i \) above the risk free rate of return \( r_{Ft} \), \( \alpha_i \) is the pricing error, \( r_{Mt} \) is the return of the value-weighted market portfolio, \( SMB_t, HML_t, RMW_t \) and \( CMA_t \) are the returns of the factor mimicking portfolios formed as described in Table 2. \( FF3F \) is the regression given above without \( RMW \) and \( CMA \) as explanatory variables. July 1991-December 2015, 294 months.

<table>
<thead>
<tr>
<th>Inv</th>
<th>Low</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>High</th>
<th>Low</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{a} )</td>
<td>0.005**</td>
<td>0.003</td>
<td>-0.004</td>
<td>-0.004</td>
<td>-0.004</td>
<td>0.005**</td>
<td>0.003</td>
<td>-0.004</td>
<td>-0.004</td>
<td>-0.004</td>
</tr>
<tr>
<td>( \hat{b} )</td>
<td>1.08***</td>
<td>1.00***</td>
<td>0.96***</td>
<td>0.89***</td>
<td>0.91***</td>
<td>1.07***</td>
<td>1.01***</td>
<td>0.95***</td>
<td>0.86***</td>
<td>0.88***</td>
</tr>
<tr>
<td>( \hat{c} )</td>
<td>1.11***</td>
<td>1.10***</td>
<td>1.02***</td>
<td>1.01***</td>
<td>0.71***</td>
<td>1.09***</td>
<td>1.11***</td>
<td>1.04***</td>
<td>1.01***</td>
<td>0.62***</td>
</tr>
<tr>
<td>( \hat{d} )</td>
<td>0.001</td>
<td>-0.03</td>
<td>-0.01</td>
<td>0.06</td>
<td>-0.09</td>
<td>0.01</td>
<td>-0.04</td>
<td>-0.00</td>
<td>0.07</td>
<td>-0.08</td>
</tr>
<tr>
<td>( \hat{e} )</td>
<td>0.01</td>
<td>0.05</td>
<td>0.02</td>
<td>-0.26***</td>
<td>0.03</td>
<td>0.05</td>
<td>0.02</td>
<td>-0.26***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \hat{f} )</td>
<td>-0.11***</td>
<td>0.01</td>
<td>0.05</td>
<td>0.04</td>
<td>-0.10**</td>
<td>0.02</td>
<td>0.01</td>
<td>0.04</td>
<td>-0.00</td>
<td>-0.04</td>
</tr>
<tr>
<td>( \hat{g} )</td>
<td>-0.05</td>
<td>-0.10***</td>
<td>0.05</td>
<td>-0.04</td>
<td>-0.13***</td>
<td>-0.05</td>
<td>-0.02</td>
<td>0.13***</td>
<td>0.07</td>
<td>-0.21***</td>
</tr>
<tr>
<td>( \hat{h} )</td>
<td>0.14***</td>
<td>-0.02</td>
<td>0.13***</td>
<td>0.12***</td>
<td>-0.10***</td>
<td>0.10**</td>
<td>-0.01</td>
<td>0.17***</td>
<td>0.12***</td>
<td>-0.10***</td>
</tr>
<tr>
<td>( \hat{i} )</td>
<td>0.05</td>
<td>0.02</td>
<td>0.06</td>
<td>-0.16***</td>
<td>0.07</td>
<td>0.03</td>
<td>0.02</td>
<td>-0.26***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \hat{j} )</td>
<td>0.02</td>
<td>-0.05</td>
<td>-0.02</td>
<td>-0.00</td>
<td>-0.04</td>
<td>0.08**</td>
<td>0.02</td>
<td>-0.08**</td>
<td>-0.05</td>
<td>-0.03</td>
</tr>
<tr>
<td>( \hat{k} )</td>
<td>0.18***</td>
<td>0.12***</td>
<td>-0.04</td>
<td>0.02</td>
<td>0.07</td>
<td>0.45***</td>
<td>0.30***</td>
<td>0.02</td>
<td>-0.12***</td>
<td>-0.48***</td>
</tr>
</tbody>
</table>

Note: Significance levels: "p<0.1; "p<0.05; ""p<0.01
Table 12
Factor exposures of Norwegian size-BM portfolios. Results of the first stage Fama-MacBeth time-series regressions on test portfolios formed on double sorts on size and investment behavior. FF5F is the five-factor regression

\[ R_{it} = \alpha_i + b_i(r_{Mt} - r_{Ft}) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + \epsilon_{it} \]

where \( R_{it} \) is the excess return on portfolio \( i \) above the risk free rate of return \( r_{Ft} \), \( \alpha_i \) is the pricing error, \( r_{Mt} \) is the return of the value-weighted market portfolio, \( SMB_t, HML_t, RMW_t \) and \( CMA_t \) are the returns of the factor mimicking portfolios formed as described in Table 2. Mkt + SMB is the regression given above without \( HML, RMW \) and \( CMA \) as explanatory variables. July 1991-December 2015, 294 months.

<table>
<thead>
<tr>
<th>BM →</th>
<th>Mkt + SMB</th>
<th>FF5F</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low 2 High</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \hat{\alpha} )</td>
<td>( \hat{\alpha} )</td>
</tr>
<tr>
<td></td>
<td>( \hat{\beta} )</td>
<td>( \hat{\beta} )</td>
</tr>
<tr>
<td></td>
<td>( \hat{s} )</td>
<td>( \hat{s} )</td>
</tr>
<tr>
<td></td>
<td>( \hat{h} )</td>
<td>( \hat{h} )</td>
</tr>
<tr>
<td></td>
<td>( \hat{c} )</td>
<td>( \hat{c} )</td>
</tr>
</tbody>
</table>

|      | Low 2 High |      |      |
| Small | 0.001 0.004 0.005 | 0.001 0.004 0.005 |
| 2     | 0.000 0.003 -0.002 | -0.000 0.003 -0.000 |
| Big   | -0.006 0.000 -0.006 | -0.007*** 0.000 -0.003 |
|       | 1.09*** 0.92*** 1.01*** | 1.09*** 0.90*** 1.01*** |
| 2     | 0.89*** 0.82*** 1.00*** | 0.90*** 0.82*** 0.91*** |
| Big   | 1.03*** 0.94*** 1.09*** | 1.07*** 0.93*** 1.05*** |
|       | 0.64*** 0.67*** 0.75*** | 0.58*** 0.62*** 0.81*** |
| 2     | 0.65*** 0.43*** 0.54*** | 0.59*** 0.43*** 0.44*** |
| Big   | 0.30*** -0.17*** 0.13* | 0.13** -0.14*** 0.15** |
|       | -0.10 -0.00 0.07    | -0.16*** 0.02 0.35*** |
| 2     | -0.16*** 0.02 0.35*** | -0.55*** 0.06*** 0.44*** |
| Big   | -0.11*** 0.03 -0.11*** | -0.14*** 0.03 -0.11*** |
|       | -0.04 0.13* 0.11    | -0.04 0.13* 0.11    |
| 2     | -0.03 0.02 -0.06    | -0.04 0.13* 0.11    |
| Big   | 0.05 0.07*** -0.07  | -0.04 0.13* 0.11    |

Note: Significance levels: *p<0.1; **p<0.05; ***p<0.01
Table 13
Factor exposures of Norwegian size-OP portfolios. Results of the first stage Fama-MacBeth time-series regressions on test portfolios formed on double sorts on size and investment behavior. FF5F is the five-factor regression

\[ R_{it} = \alpha_i + \beta_i (r_{Mt} - r_{Ft}) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + \epsilon_{it} \]

where \( R_{it} \) is the excess return on portfolio \( i \) above the risk free rate of return \( r_{Ft} \), \( \alpha_i \) is the pricing error, \( r_{Mt} \) is the return of the value-weighted market portfolio, \( SMB_t, HML_t, RMW_t \) and \( CMA_t \) are the returns of the factor mimicking portfolios formed as described in Table 2. Mkt + SMB is the regression given above without \( HML, RMW \) and \( CMA \) as explanatory variables. July 1991-December 2015, 294 months.

<table>
<thead>
<tr>
<th>OP →</th>
<th>Low 2 High</th>
<th>Low 2 High</th>
<th>FF5F</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \hat{a} )</td>
<td>( \hat{a} )</td>
<td></td>
</tr>
<tr>
<td>Small</td>
<td>0.005 0.005 0.006</td>
<td>0.005 0.005 0.006</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-0.003 0.000 0.002</td>
<td>-0.002 0.000 0.002</td>
<td></td>
</tr>
<tr>
<td>Big</td>
<td>-0.005 -0.004 -0.002</td>
<td>-0.003 -0.004 -0.003</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \hat{b} )</td>
<td>( \hat{b} )</td>
<td></td>
</tr>
<tr>
<td>Small</td>
<td>1.05*** 0.80*** 0.95***</td>
<td>1.03*** 0.83*** 0.96***</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1.20*** 0.81*** 0.80***</td>
<td>1.09*** 0.80*** 0.82***</td>
<td></td>
</tr>
<tr>
<td>Big</td>
<td>1.16*** 1.03*** 1.00***</td>
<td>1.05*** 1.01*** 1.05***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \hat{s} )</td>
<td>( \hat{s} )</td>
<td></td>
</tr>
<tr>
<td>Small</td>
<td>0.79*** 0.63*** 0.74***</td>
<td>0.72*** 0.72*** 0.79***</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.84*** 0.49*** 0.38***</td>
<td>0.59*** 0.50*** 0.47***</td>
<td></td>
</tr>
<tr>
<td>Big</td>
<td>0.07 0.13** -0.11**</td>
<td>-0.32*** 0.03 0.02</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \hat{h} )</td>
<td>( \hat{h} )</td>
<td></td>
</tr>
<tr>
<td>Small</td>
<td></td>
<td>-0.02 0.06 -0.03</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.13*** 0.07* 0.02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Big</td>
<td>-0.06 -0.09** -0.06**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \hat{c} )</td>
<td>( \hat{c} )</td>
<td></td>
</tr>
<tr>
<td>Small</td>
<td></td>
<td>-0.10 0.12* 0.11</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-0.48*** -0.01 0.14***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Big</td>
<td>-0.70*** -0.12*** 0.24***</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Significance levels: *p<0.1; **p<0.05; ***p<0.01
Table 14
Factor exposures of Norwegian size-Inv portfolios. Results of the $N$ first stage Fama-MacBeth time-series regressions on test portfolios formed on double sorts on size and investment behavior. FF5F is the five-factor regression

$$R_{it} = \alpha_i + b_i (r_{Mt} - r_{Ft}) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + \epsilon_{it}$$

where $R_{it}$ is the excess return on portfolio $i$ above the risk free rate of return $r_{Ft}$, $\alpha_i$ is the pricing error, $r_{Mt}$ is the return of the value-weighted market portfolio, $SMB_t$, $HML_t$, $RMW_t$ and $CMA_t$ are the returns of the factor mimicking portfolios formed as described in Table 2. $Mkt + SMB$ is the regression given above without $HML$, $RMW$ and $CMA$ as explanatory variables. July 1991-December 2015, 294 months.

<table>
<thead>
<tr>
<th>Inv $\rightarrow$</th>
<th>Low</th>
<th>2</th>
<th>High</th>
<th>Low</th>
<th>2</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>$\hat{a}$</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Small</td>
<td>0.016***</td>
<td>0.004</td>
<td>-0.008</td>
<td>0.015***</td>
<td>0.004</td>
<td>-0.008*</td>
</tr>
<tr>
<td>2</td>
<td>0.000</td>
<td>-0.000</td>
<td>-0.007</td>
<td>0.001</td>
<td>0.001</td>
<td>-0.007</td>
</tr>
<tr>
<td>Big</td>
<td>-0.004</td>
<td>-0.006**</td>
<td>-0.001</td>
<td>-0.005*</td>
<td>-0.006**</td>
<td>0.001</td>
</tr>
<tr>
<td><strong>$\hat{b}$</strong></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Small</td>
<td>1.05***</td>
<td>0.95***</td>
<td>0.95***</td>
<td>1.05***</td>
<td>0.94***</td>
<td>0.97***</td>
</tr>
<tr>
<td>2</td>
<td>1.03***</td>
<td>0.82***</td>
<td>0.93***</td>
<td>0.98***</td>
<td>0.78***</td>
<td>0.96***</td>
</tr>
<tr>
<td>Big</td>
<td>1.04***</td>
<td>1.00***</td>
<td>1.04***</td>
<td>1.00***</td>
<td>0.98***</td>
<td>1.07***</td>
</tr>
<tr>
<td><strong>$\hat{s}$</strong></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>Small</td>
<td>0.75***</td>
<td>0.72***</td>
<td>0.77***</td>
<td>0.79***</td>
<td>0.64***</td>
<td>0.81***</td>
</tr>
<tr>
<td>2</td>
<td>0.91***</td>
<td>0.35***</td>
<td>0.49***</td>
<td>0.84***</td>
<td>0.31***</td>
<td>0.47***</td>
</tr>
<tr>
<td>Big</td>
<td>0.07</td>
<td>0.07</td>
<td>0.08</td>
<td>-0.02</td>
<td>0.04</td>
<td>-0.01</td>
</tr>
<tr>
<td><strong>$\hat{h}$</strong></td>
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<td></td>
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</tr>
<tr>
<td>Small</td>
<td>0.04</td>
<td>-0.05</td>
<td>0.03</td>
<td>0.11**</td>
<td>0.15***</td>
<td>-0.10*</td>
</tr>
<tr>
<td>2</td>
<td>-0.12***</td>
<td>0.00</td>
<td>-0.04</td>
<td>-0.12***</td>
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<td>-0.04</td>
</tr>
<tr>
<td>Big</td>
<td>-0.16***</td>
<td>-0.12***</td>
<td>0.01</td>
<td>-0.10***</td>
<td>-0.05</td>
<td>-0.13***</td>
</tr>
<tr>
<td><strong>$\hat{c}$</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Small</td>
<td>0.10</td>
<td>0.01</td>
<td>-0.12*</td>
<td>0.16***</td>
<td>-0.05</td>
<td>-0.16**</td>
</tr>
<tr>
<td>2</td>
<td>0.45***</td>
<td>0.07*</td>
<td>-0.57***</td>
<td>0.45***</td>
<td>0.07*</td>
<td>-0.57***</td>
</tr>
</tbody>
</table>

**Note:** Significance levels: *p<0.1; **p<0.05; ***p<0.01
(a=0.53%) while negative pricing errors are produced by regressions on large size stocks with medium value (a= −0.31% and a= −0.36%) as well as growth stocks with medium firm size (a= −0.62%). Here it seems that it is the extremes of both portfolio sorts that lead to bad overall model fit. Fama and French (1993, 2015a) find that for the U.S. stock market low-value microcap stocks are troublesome. We detect this effect also in the German data although the problem is not limited to this kind of stocks.

Table 10 shows that four out of the 25 regressions on the portfolios formed on size and profitability lead to pricing errors that are significantly different from zero at a 10% level. Negative intercepts are produced by regressions on medium-sized growth stocks (a= −0.52%) and medium value stocks (a= −0.42%) as well as large size growth stocks (a= −0.86%). A significantly positive error can be observed in the regression of high-value medium-sized stocks (a=0.54%). Here the bad model fit seems mainly anchored in the stocks of medium size firms.

In the 25 regressions on portfolios sorted by size and investment behavior we find four intercepts to deviate from zero at a 10% significance level. A positive pricing error exists for the regression on microcap growth stocks (a=0.53%) whereas we detect negative errors for medium-sized/medium-and high-value stocks (a= −0.44% and a= −0.55%) as well as large medium value stocks (a= −0.31%). These findings do not seem to follow any particular pattern.

A three-factor model gives pricing errors for the same test portfolio returns. In addition these pricing errors are at about the same size and are similarly significant. This suggests that the five-factor model does not explain German stock returns better than the three-factor model.

The regressions on the Norwegian size-BM portfolios give a negative pricing error for the low-value megacap portfolio. Using portfolios sorted on size and operating profitability as test portfolios does not lead to significant pricing errors. Size-Inv sorts result in significant intercepts for low-value micro- and megacap stocks as well as for medium-value megacaps and high-value microcaps. We find that the five-factor model produces more pricing errors than the two-factor benchmark model and thus assume also in this case that the five-factor model does not add value to the analysis of Norwegian stock returns.

In Table 15 we present the results of the GRS-test applied to the first-step Fama-MacBeth regressions. Panel A shows the results for the double sorts on size and value, Panel B those for the sorts on size and operating profitability, and Panel C those for the sorts on size and investment behavior. We compare the models by their GRS-statistic and its p-value, the average value of the 25 (9) regression intercepts, as well as the adjusted R². Intercepts significantly different from zero are contrary to the null-hypothesis of jointly insignificant pricing errors and thus cause the GRS-value to increase. The GRS-test will therefore indicate bad model performance in those cases where a regression model leads to too many high intercepts in absolute terms. A low GRS-value means that the model fits well in describing average excess returns of the test portfolios. The p-value gives the probability of obtaining the observed or a higher GRS-value if the null hypothesis that all intercepts jointly equal zero holds. For high
p-values, good model fit can hence not be rejected.

In the case of Germany we find that all p-values are lower than 0.1, meaning that at a 10% level, the null hypothesis that all pricing errors jointly equal zero is rejected for all tested models. We therefore conclude that none of the tested models is sufficient in explaining total variation in monthly average excess returns of German stocks. Fama and French (2015a) find the same result for all models they test but argue that one model still is to be preferred when it shows better performance in relative terms. As all metrics remain almost unaffected by the transition from the three-factor to the five-factor model, we hence reason that extending the three-factor model with a profitability and an investment factor does not add value to an analysis of German stock returns.

Looking at the Norwegian data, we see relatively high p-values for both the five-factor model and the two-factor benchmark for portfolios formed on size-BM and size-OP sorts. This indicates good absolute model fit although the adjusted $R^2$ measures at slightly above 0.5 are considerably lower than in Fama and French (2015a) where the authors find the five-factor model to absorb more than 90% of the test portfolios’ return variation. Comparing the performance of the five-factor model with that of the two-factor benchmark, we see that the latter gives lower GRS-values both for the size-OP and the size-Inv sorts, whereas GRS-values are equal for the size-BM sorts. The conclusion we draw from this is that the five-factor model does not outperform a two-factor model comprising the market and size factors in describing Norwegian stock returns. This conclusion is in line with the results from Table 21 that the explanatory power of the HML, RMW and CMA factors is absorbed by the other model factors.

### 6.5 Risk premiums

The risk premiums associated with one unit of additional exposure to the RHS factors are presented in Table 16. We present in Panel A the size-B/M sorts, in Panel B the size-OP sorts, and in Panel C the size-Inv sorts. Again, we compare the premiums estimated by the five-factor model with those of the respective benchmark model.

As in the previous analysis, we find that none of the German models shows good model fit. Only the market factor is priced in regressions on size-BM sorted portfolios, no factors are priced in when size-OP sorted portfolios are used as test assets and only the value factor is priced by size-Inv sorts. In particular we find neither the profitability nor the investment factor to be priced risk factors.

The good model fit of the two Norwegian models on size-BM and size-OP sorted portfolios that we observed in the first stage Fama-MacBeth regressions can also be observed here. Irrespective of the test portfolio sort, the five-factor model does neither give significant profitability nor investment premiums. As in the two-factor model, only the market and the size factor are priced. Both models with size-Inv sorted test portfolios are rejected by the J-test and the five-factor model in this case does not result in significant profitability or investment premiums.
Table 15

GRS-test based on data from July 1991-December 2015, 294 months. The GRS statistic tests if the intercepts of all $N$ time-series regressions given by Equation (18) jointly equal zero. $A|a_i|$ is the average absolute intercept value, adj. $R^2$ is the adjusted coefficient of determination. We show the test results for the five-factor model (FF5F) and compare them to the results of two benchmark models. For the German portfolios, we use the three-factor model (FF3F) as benchmark. The benchmark for the Norwegian portfolios is a two-factor model with the market portfolio (Mkt) and the size factor (SMB) as explanatory variables.

<table>
<thead>
<tr>
<th>Panel A: Size-B/M portfolios</th>
<th>Germany</th>
<th></th>
<th>Norway</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GRS</td>
<td>p-value</td>
<td>$A</td>
<td>a_i</td>
</tr>
<tr>
<td>FF5F</td>
<td>1.48</td>
<td>0.07</td>
<td>0.0028</td>
<td>0.66</td>
</tr>
<tr>
<td>benchmark</td>
<td>1.47</td>
<td>0.07</td>
<td>0.0028</td>
<td>0.64</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Size-OP portfolios</th>
<th>Germany</th>
<th></th>
<th>Norway</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GRS</td>
<td>p-value</td>
<td>$A</td>
<td>a_i</td>
</tr>
<tr>
<td>FF5F</td>
<td>2.10</td>
<td>0.00</td>
<td>0.0024</td>
<td>0.63</td>
</tr>
<tr>
<td>benchmark</td>
<td>2.12</td>
<td>0.00</td>
<td>0.0025</td>
<td>0.62</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel C: Size-Inv portfolios</th>
<th>Germany</th>
<th></th>
<th>Norway</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GRS</td>
<td>p-value</td>
<td>$A</td>
<td>a_i</td>
</tr>
<tr>
<td>FF5F</td>
<td>1.70</td>
<td>0.02</td>
<td>0.0023</td>
<td>0.66</td>
</tr>
<tr>
<td>benchmark</td>
<td>1.70</td>
<td>0.02</td>
<td>0.0023</td>
<td>0.64</td>
</tr>
</tbody>
</table>

6.6 Robustness checks

To increase the reliability of our findings discussed above, we test to which extent they are sensitive to various assumptions we made throughout our analysis. We do this by changing one assumption at a time, holding everything else constant, and checking if the five-factor model then outperforms the benchmark model in relative terms. For that we again compare the GRS-statistics of the two models. In this section we summarize and evaluate these robustness tests. Results are shown in Tables 22 and 23 in the appendix.

6.6.1 Avoiding time gaps due to different fiscal year ends

In section 4.1.2 we expounded that the value measure we build is in many cases based on book equity and market equity measured at two different points in time. There are two possible fixes to this issue. Firstly, one can decide to omit all firms whose fiscal year does not end in December. Secondly, one can adjust the measurement time of market equity to that of book-equity. Both alternatives and their effects are discussed in the following.

Keeping only companies in the sample whose fiscal year ends in December ensures a constant time gap between the measurement time of book-to-market and the period in which
Table 16
Risk premiums estimated by the T cross-sectional second stage Fama-MacBeth regressions

\[ R_{it} = \lambda_t^{Mkt} \hat{b}_i + \lambda_t^{SMB} \hat{s}_i + \lambda_t^{HML} \hat{h}_i + \lambda_t^{RMW} \hat{r}_i + \lambda_t^{CM} \hat{c}_i + \epsilon_t \]

where \( R_{it} \) is the excess return of the test portfolios formed on the double sorts indicated in Panel A-C, \( \hat{b}_i, \hat{s}_i, \hat{h}_i, \hat{r}_i \) and \( \hat{c}_i \) are the factor loadings found in the first stage Fama-MacBeth regression and \( \lambda_t^L \) is the risk premium for risk factor \( L \). Estimation is done by GMM as described in section 4.2.2. Coef is the estimated risk premium awarded for factor \( L \) in %, Std is its standard deviation in % and t-stat is its t-statistic. Low p-values of the J-statistic indicate bad model fit. We compare the results of the five-model model with those of the respective benchmark model. The benchmark for German portfolios is the Fama-French three-factor model. The benchmark for the Norwegian portfolios is a two-factor model with the market and the size factor as explanatory variables.

<table>
<thead>
<tr>
<th></th>
<th>Germany</th>
<th></th>
<th>Norway</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \lambda_t^{Mkt} )</td>
<td>( \lambda_t^{SMB} )</td>
<td>( \lambda_t^{HML} )</td>
<td>( \lambda_t^{CM} )</td>
</tr>
<tr>
<td><strong>Panel A: size-BM</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a) FF5F</td>
<td>Coef 0.68</td>
<td>-0.05 0.31</td>
<td>-0.46 0.31</td>
<td>0.77 1.46</td>
</tr>
<tr>
<td></td>
<td>Std 0.32</td>
<td>0.24 0.40</td>
<td>0.72 0.61</td>
<td>0.39 0.36</td>
</tr>
<tr>
<td></td>
<td>t-stat 2.12</td>
<td>-0.21 0.77</td>
<td>-0.65 0.50</td>
<td>1.98 4.03</td>
</tr>
<tr>
<td></td>
<td>J-test ( \chi^2(20) = 43.92, p = 0.002 ) ( \chi^2(4) = 2.83, p = 0.59 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b) benchmark model</td>
<td>Coef 0.61</td>
<td>-0.05 0.34</td>
<td></td>
<td>0.84 1.06</td>
</tr>
<tr>
<td></td>
<td>Std 0.31</td>
<td>0.23 0.40</td>
<td></td>
<td>0.36 0.33</td>
</tr>
<tr>
<td></td>
<td>t-stat 1.98</td>
<td>-0.20 0.84</td>
<td></td>
<td>2.32 3.21</td>
</tr>
<tr>
<td></td>
<td>J-test ( \chi^2(22) = 44.64, p = 0.003 ) ( \chi^2(7) = 11.50, p = 0.12 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Panel B: size-OP</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a) FF5F</td>
<td>Coef 0.39 0.25</td>
<td>0.42 0.68</td>
<td>0.95</td>
<td>0.74 1.74</td>
</tr>
<tr>
<td></td>
<td>Std 0.32</td>
<td>0.25 0.64</td>
<td>0.45 0.64</td>
<td>0.46 0.57</td>
</tr>
<tr>
<td></td>
<td>t-stat 1.20</td>
<td>0.98 0.66</td>
<td>1.52 1.49</td>
<td>1.62 3.06</td>
</tr>
<tr>
<td></td>
<td>J-test ( \chi^2(20) = 45.86, p = 0.001 ) ( \chi^2(4) = 1.00, p = 0.91 )</td>
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</tr>
<tr>
<td>b) benchmark model</td>
<td>Coef 0.42 0.25</td>
<td>0.47</td>
<td></td>
<td>0.72 1.49</td>
</tr>
<tr>
<td></td>
<td>Std 0.31</td>
<td>0.25 0.59</td>
<td></td>
<td>0.38 0.36</td>
</tr>
<tr>
<td></td>
<td>t-stat 1.36</td>
<td>1.00 0.79</td>
<td></td>
<td>1.89 4.17</td>
</tr>
<tr>
<td></td>
<td>J-test ( \chi^2(22) = 51.15, p = 0.000 ) ( \chi^2(7) = 4.79, p = 0.69 )</td>
<td></td>
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<tr>
<td><strong>Panel C: size-Inv</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a) FF5F</td>
<td>Coef 0.45 0.11</td>
<td>1.53 -0.62</td>
<td>0.22</td>
<td>0.60 1.38</td>
</tr>
<tr>
<td></td>
<td>Std 0.33</td>
<td>0.26 0.76</td>
<td>0.67 0.36</td>
<td>0.47 0.45</td>
</tr>
<tr>
<td></td>
<td>t-stat 1.36</td>
<td>0.41 2.01</td>
<td>-0.92 0.62</td>
<td>1.27 3.06</td>
</tr>
<tr>
<td></td>
<td>J-test ( \chi^2(20) = 42.65, p = 0.002 ) ( \chi^2(4) = 20.21, p = 0.000 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b) benchmark model</td>
<td>Coef 0.38 0.22</td>
<td>1.52</td>
<td></td>
<td>0.58 1.36</td>
</tr>
<tr>
<td></td>
<td>Std 0.32</td>
<td>0.26 0.62</td>
<td></td>
<td>0.40 0.38</td>
</tr>
<tr>
<td></td>
<td>t-stat 1.18</td>
<td>0.86 2.45</td>
<td></td>
<td>1.46 3.62</td>
</tr>
<tr>
<td></td>
<td>J-test ( \chi^2(22) = 42.80, p = 0.005 ) ( \chi^2(7) = 20.31, p = 0.005 )</td>
<td></td>
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</tr>
</tbody>
</table>
we track returns. Panel A of Table 22 indicates that this adjustment does not improve the relative performance of the five-factor model on Norwegian portfolios as GRS-values are lower for the two-factor benchmark regardless of the portfolio sort. For the German data we find the five-factor model to produce a lower GRS-statistic in regressions on size-OP sorted test portfolios. The difference is however no more than 0.12 and the average absolute values of the intercepts is almost unchanged. We hence question that we see a real improvement here.

Aligning the measurement point of market equity with a firm’s fiscal year-end removes the time inconsistency between the numerator and the denominator of the book-to-market ratio. Market equity throughout the year is however dependent on the general market development. A drawback of this approach is thus that differences in the B/M-ratio can be caused by a change of external conditions throughout the year rather than due to cross-sectional differences. In Panel B of Table 22 we show the effects of this alignment on the GRS-test. We find that the GRS-values of the five-factor regressions on German size-OP and size-Inv sorted portfolios are now smaller than those of the corresponding three-factor regressions. The difference is however so slight that its significance is questionable. Looking at the Norwegian stock market, the adjustment leads to a relatively better performance of the five-factor model when size-OP sorted portfolios are used as test assets. We conduct the Fama-MacBeth second-stage regressions in this case, but find neither good model fit nor that any factors are priced. Therefore we do not show the results here.

6.6.2 Redefining penny stocks

In 5.2.7 we discussed uncertainty about the German penny stock definition. Due to the lack of a formal definition of German penny stocks we referred to the current listing requirements of the Frankfurt Stock Exchange and calculated penny stocks as those stocks that not only have a stock price of less than one Euro but also have a market capitalization lower than 750.000,- Euro. The latter restriction leaves us though with many very low-valued stocks in the sample, so if it is wrong to extrapolate from this listing requirement to which stocks should be kept in the sample, this might change results noticeably. To test if this occurs, we drop the market cap requirement and define penny stocks simply as those stocks that have a value of less than one Euro. The results of this change are shown at the left side of Panel C of Table 22. Since the GRS-statistics of the two models still are almost equal for all portfolio sorts, we reject that our penny stock definition affects model choice.

6.6.3 Better differentiation between LHS and RHS sorts

Our test portfolios and RHS portfolios are based on the same sorting variables. As in Fama and French (2015a) the difference between them is that the test portfolios are the results of “finer sorts” than the RHS portfolios and that the factors do not build on observations with medium firm characteristics. In our base scenario for Norway the limited sample size leads us
to reduce the number of LHS portfolios from 25 to 9 (see Section 4.1.4). These $3 \times 3$ LHS sorts might be too similar to the $2 \times 3$ RHS sorts that form the basis of factor portfolio construction. We test the importance of this assumption by building the factor portfolios on $2 \times 2$ RHS portfolios instead. In this case sample medians are used as breakpoints. The reduction from $2 \times 3$ portfolios to $2 \times 2$ portfolios is only slight, but a further reduction is not possible when double sorting still should be used. Besides, the $2 \times 2$ sort increases the number of securities in each RHS portfolio considerably, as we no longer exclude the mid 40% of the sample. The right-hand side of Panel C of Table 22 shows the effects of this change on the GRS metrics. We get an almost identical size-BM model quality, a small improvement for the size-OP sorts, and a slightly less accurate model for portfolios sorted on size-Inv compared to the base-scenario. The changes applied do however not increase the relative performance of the five-factor model.

### 6.6.4 Shorter time horizon

Finally, we test to which extent the GRS approach is sensitive to the choice of the sample time period. Effects are shown in Table 23.

(i) **Germany**

Since it is possible that stock return characteristics changed when Germany became part of the European Monetary Union and replaced DEM with EUR, we divide our sample period into pre and post Euro introduction time and see if the conclusions about the usefulness of the five-factor model varies between the two subsamples. The resulting effects are shown in the left part of Panel A and Panel B.

We observe that both models that use size-OP sorted portfolios as test assets seem to fit the data of the DEM-period. In this case the GRS-statistic of the five-factor model is lower than that of the three-factor model. For all other models, both those on the DEM-period subsample and those on the EUR-period subsample, good model fit is rejected by the GRS-test. The five-factor model produces lower GRS-values than the three-factor model for the DEM-period when the dependent variables are portfolios formed on size-BM sorts. The same applies to size-OP and size-Inv sorted test portfolio regressions based on the EUR-period. Differences are however marginal again and can thus be questioned.

To investigate the case of size-OP portfolios on the DEM-period data more in detail, we proceed to conduct the Fama-MacBeth second step regressions for this special case. Results are shown in the left part of Panel B of Table 24, in the appendix. We find that in these regressions size is a priced factor while the profitability and investment, as before, are not priced.

In sum, our conclusions about the usefulness of the five-factor model for describing German stock returns do not change when we split the sample into a DEM and a EUR period.
This gives us the impression that the currency transition is not a crucial explanatory variable for German stock returns.

(ii) **Norway**

The reason for using portfolios instead of single stocks is that one eliminates firm-specific risk by diversification. Ødegaard (2016c) finds that one needs at least ten Norwegian stocks to form a diversified portfolio. In Figure 5 we see that the number of companies in our Norwegian sample is quite low before year 2000. Table 17 confirms that the number of stocks in some of the Norwegian test portfolios is considerably less than ten. Therefore we examine if our conclusions change when we postpone the sample period beginning from 1991 to the year 2000. Results are shown in the right part of Panel A.

We find that the reduction of the sample period has a major impact on the GRS-test statistic compared to our base scenario results. The GRS-value is clearly reduced, and the increased adjusted $R^2$ measure for most models and sorting procedures indicateds better model fit. This strengthens our presumption that our base-scenario is applied to portfolios that do not completely rule out company-specific effects. In addition to general enhanced model fit, we now observe that the five-factor model outperforms the two-factor model regardless of the portfolio sort, both with respect to the GRS-statistic and to the adjusted $R^2$. We therefore show the updated results of the Fama-MacBeth second stage regressions in the right part of Table 24 in the appendix. We find that even the better-fitted models do not price the profitability and the investment factors.

To scrutinize the Norwegian data even more intensively, in another test we reduce the sample period to the years 2007-2015. Thereby we secure that the number of sample companies in each month is mostly three-digit. However, this also leads to the financial crisis playing a more crucial role on returns, as it now amounts for a large part of the sample period. Besides the overall sample size is then unfavourably small. The effect of this sample period cut is shown in the right part of Panel B in Table 23. We see that this adjustment leads to increased GRS-values when size-BM or size-OP sorted test portfolios are used. The model fit of models based on size-Inv sorts is now improved. The relative performance of the five-factor model is though worse than that of the two-factor model. This adjustment thereby does not change conclusions drawn above.
7 Conclusion and outlook

We evaluate the Fama-French five-factor model’s applicability to the German and Norwegian stock markets in the period from July 1991 to December 2015. Our findings suggest that the market portfolio and the size factor have a significantly effect in describing cross-sectional excess returns. These results support among others the analyses by Heston et al. (1995), Næs et al. (2009) and Amel-Zadeh (2011). At the same time we conclude that the five-factor model does not add any value relative to our country-specific benchmark models when trying to describe return variation across companies in the two markets. In several robustness tests, we scrutinize the sensitivity of this result to the assumptions made throughout the analysis. These tests show that our restrictions are adequately reasonable since they do not affect our final conclusions.

The pricing errors produced by the first-stage Fama-MacBeth regressions on German stock data are almost identical for the three-factor and the five-factor model. Small companies with low value appear to be troublesome, a result that confirms earlier documentations of Fama and French (1993, 2015a). The problem is however not limited to this stock type, as we find that both models misspecify the returns of several other portfolios, regardless of test portfolio sort. To evaluate the absolute and relative performance of each model we use the GRS-test. Results show that none of the two model specifications gives good model fit. Additionally there is no indication that the five-factor model performs better than the three-factor model. The GMM estimation of risk premiums in the German market shows that only the sort on size and value achieves to price the market factor, while other factors largely remain unpriced. As the GRS-test, the J-test though suggests that both models, regardless of the test portfolio choice, are inadequate asset-pricing models.

In the Norwegian sample, we find almost all pricing errors from sorts on size-BM and size-OP to be indistinguishable from zero for both models tested. The GRS-test does in these cases indicate good overall model fit. In relative terms, the five-factor model does not achieve a lower GRS-statistic than the two-factor benchmark model. This is among others due to the five-factor model giving a significantly negative pricing error for the portfolio containing large firms with high company value. In the factor premium analysis, the J-test indicates good model fit of both Norwegian models that have size-BM and size-OP sorted test portfolios. We then find that the market and the size factors are priced, while there is no evidence for neither a profitability nor an investment premium. We infer that the five-factor model does not give more information about the variation in Norwegian stock returns than the sparser two-factor model.

We test if these results are sensitive to several assumptions that we made when constructing our data sample. Additionally, we examine whether conclusions change if the models are applied to shorter period subsamples. Our findings reject both the former and the latter.

The meaningfulness of our results can still be questioned in several ways. Criticism can on
the one hand be directed at the fundamentals of our approach and on the other hand at our
implementation.

There are many researchers who question the validity of an empirical search for systematic
risk factors due to its lack of theoretical foundation. They regard the process as pure data mining
(Lo & MacKinlay, 1990) and show that even factors that obviously are unrelated to risk exposure
can appear to have explanatory power (Ferson, Sarkissian, & Simin, 1999). Others argue that
empirically found factor premiums are a result of irrational investor behavior rather than a
compensation for systematic risk (Lakonishok, Shleifer, & Vishny, 1994, 5). As we scrutinize
the applicability of the Fama-French five-factor model to other markets, we presuppose that
this kind of models is in general useful for describing asset returns and do not dig deeper into
the basic principle.

Regarding our implementation, we see three aspects that are subject to potential improve-
ment.

1. We restrict our analysis to factors that form part of the five-factor model, and do not
take into account that there actually are other factors like liquidity, momentum or
macroeconomic variables that have been found to be significant in multifactor models
(see e.g. Pástor & Stambaugh, 2003; Jegadeesh & Titman, 1993; Næs et al., 2009). As
omitting relevant risk factors may lead to biased results, we could have tested if an
inclusion of these factors leads to significant profitability and investment factors.

2. We use test portfolios that are formed on the same characteristics as the factors themselves.
This potentially creates artificially high correlation between regressor and regressand.
Lewellen, Nagel, and Shanken (2010) therefore advocate that results found for this setting
should be verified by using other test portfolios that correlate less with the factors. A
widespread approach to do so is to use portfolios comprising stocks sorted by industry
(see e.g. Næs et al., 2009; Fama & French, 1997).

3. Due to their deviating balance sheet structure we exclude financial firms from our analysis
although a good asset pricing model is supposed to price all assets. By constructing
measures for financial firms that align their firm characteristics to those of industrial
firms we could have included them in the analysis so that conclusions apply even more
generally.

All of these aspects give rise to future research possibilities and can be used to verify or revise
the findings of our thesis.
Table 17
Average number of stocks per test portfolio in the period July 1991-December 2015 by three different double sorts.

At the end of each June sample stocks are independently sorted by the firm characteristics size, book-to-market (B/M), operating profitability (OP) and Investment (Inv) and assigned to different groups. German stocks are divided into five groups using quintile breakpoints, Norwegian stocks are divided into three groups using 30% and 70% quantile breakpoints. The interface of the size groups (small to big) and the second variable groups (low to high) gives 25 test portfolios for Germany and 9 portfolios for Norway. Size is a firm’s market capitalization at the end of June of year $\tau$. B/M is book equity at the end of fiscal year $\tau - 1$ divided by market equity measured at the end of December of year $\tau - 1$. OP is defined as total revenues less total operating expenses and total interest expenses all divided by book equity, all measured at the end of fiscal year $\tau - 1$. Inv is the growth of total assets from the end of fiscal year $\tau - 2$ to the end of fiscal year $\tau - 1$. The sample is restricted to firms with positive book-equity.

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Table 18
Average firm size, book-to-market (B/M), operating profitability (OP) and investment (Inv) of the stocks in the 25 (9) test portfolios, July 1991-December 2015. Portfolios are constructed as described in Table 17.

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(a) size-INV PF11, Exposure to all five factors

(b) size-INV PF11, Exposure to the profitability factor

c) size-INV PF11, Exposure to the investment factor

Figure 8
Germany: Variation of the factor loadings over time, July 1991 to December 2015. We run rolling window regressions with 50 observations at a time to see if the factor loadings are constant over the course of our sample period. We only show results for the size-INV sorted test portfolio containing conservative microcap firm stocks. In Panel (a) we show the variation of all factor loading estimates, while Panels (b) and (c) show the estimates of the portfolios exposure to the profitability (RMW) and the investment factor (CMA) with confidence bands.
(a) size-Inv PF11, Exposure to all five factors

(b) size-Inv PF11, Exposure to the profitability factor

(c) size-Inv PF11, Exposure to the investment factor

Figure 9
Norway: Variation of the factor loadings over time, July 1991 to December 2015. We run rolling window regressions with 50 observations at a time to see if the factor loadings are constant over the course of our sample period. We only show results for the size-Inv sorted test portfolio containing conservative microcap firm stocks. In Panel (a) we show the variation of all factor loading estimates, while Panels (b) and (c) show the estimates of the portfolios exposure to the profitability (RMW) and the investment factor (CMA) with confidence bands.
Table 19
Average monthly percent returns, standard deviations and t-statistic of the factor building blocks, July 1991-December 2015. All sample stocks are independently sorted and divided into two size groups (small and big), three value groups (low, neutral and high), three profitability groups (weak, neutral and robust) and three investment groups (conservative, neutral and aggressive). The breakpoint for the size groups is the sample mean. The three other variables are cut on the basis of the 30th and 70th percentile. The interface of the size groups and the respective other variable groups gives six RHS portfolios which are the building blocks for the construction of the factor mimicking portfolios.

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Germany: Auxiliary regressions to identify collinearity, July 1991-December 2015 (294 months). Four factors are regressed on the fifth to see to which extent one factor’s average return is explained by the others’. Mkt is the value-weighted return on a portfolio of all sample stocks in excess of the four-week U.S. Treasury bill rate. SMB is the size factor, HML is the value factor, RMW is the profitability factor and CMA is the investment factor. Factors are constructed as shown in Table 2.

<table>
<thead>
<tr>
<th>Dependent variable:</th>
<th>Mkt</th>
<th>SMB</th>
<th>HML</th>
<th>RMW</th>
<th>CMA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.009***</td>
<td>0.003</td>
<td>0.004</td>
<td>0.002</td>
<td>0.0001</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.002)</td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Mkt</td>
<td></td>
<td>−0.300***</td>
<td></td>
<td>−0.186***</td>
<td>−0.176***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.035)</td>
<td></td>
<td>(0.058)</td>
<td>(0.058)</td>
</tr>
<tr>
<td>SMB</td>
<td>−0.662***</td>
<td></td>
<td>0.383***</td>
<td>−0.374***</td>
<td>0.061</td>
</tr>
<tr>
<td></td>
<td>(0.078)</td>
<td></td>
<td>(0.084)</td>
<td>(0.091)</td>
<td>(0.088)</td>
</tr>
<tr>
<td>HML</td>
<td>0.051</td>
<td>0.176***</td>
<td></td>
<td>0.074</td>
<td>0.102*</td>
</tr>
<tr>
<td></td>
<td>(0.059)</td>
<td>(0.038)</td>
<td></td>
<td>(0.063)</td>
<td>(0.059)</td>
</tr>
<tr>
<td>RMW</td>
<td>−0.163***</td>
<td>−0.148***</td>
<td></td>
<td>0.063</td>
<td>0.267***</td>
</tr>
<tr>
<td></td>
<td>(0.054)</td>
<td>(0.036)</td>
<td>(0.054)</td>
<td></td>
<td>(0.053)</td>
</tr>
<tr>
<td>CMA</td>
<td>−0.173***</td>
<td>0.027</td>
<td>0.099*</td>
<td>−0.302***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.054)</td>
<td>(0.039)</td>
<td>(0.058)</td>
<td>(0.060)</td>
<td></td>
</tr>
</tbody>
</table>

Observations: 294

R²: 0.263
Adjusted R²: 0.252
Residual Std. Error: 0.050 (df = 289)
F Statistic: 25.720*** (df = 4; 289)

Note: Standard errors in parentheses. Significance levels: *p<0.1; **p<0.05; ***p<0.01
Table 21
Norway: Auxiliary regressions to identify collinearity, July 1991-December 2015 (294 months). Four factors are regressed on the fifth to see to which extent one factor’s average return is explained by the others’. Mkt is the value-weighted return on a portfolio of all sample stocks in excess of the four-week U.S. Treasury bill rate. SMB is the size factor, HML is the value factor, RMW is the profitability factor and CMA is the investment factor. Factors are constructed as shown in Table 2.

<table>
<thead>
<tr>
<th>Dependent variable:</th>
<th>Mkt</th>
<th>SMB</th>
<th>HML</th>
<th>RMW</th>
<th>CMA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.011*** (0.003)</td>
<td>0.007** (0.003)</td>
<td>-0.002 (0.004)</td>
<td>0.001 (0.004)</td>
<td>0.002 (0.004)</td>
</tr>
<tr>
<td>Mkt</td>
<td>-0.129*** (0.045)</td>
<td>0.064 (0.074)</td>
<td>-0.163** (0.067)</td>
<td>0.076 (0.063)</td>
<td></td>
</tr>
<tr>
<td>SMB</td>
<td>-0.212*** (0.074)</td>
<td>-0.347*** (0.093)</td>
<td>-0.630*** (0.079)</td>
<td>0.017 (0.081)</td>
<td></td>
</tr>
<tr>
<td>HML</td>
<td>0.041 (0.047)</td>
<td>-1.133*** (0.036)</td>
<td>0.224*** (0.052)</td>
<td>0.062 (0.050)</td>
<td></td>
</tr>
<tr>
<td>RMW</td>
<td>-0.122** (0.050)</td>
<td>-0.286*** (0.036)</td>
<td>0.266*** (0.062)</td>
<td>-0.017 (0.055)</td>
<td></td>
</tr>
<tr>
<td>CMA</td>
<td>0.065 (0.054)</td>
<td>0.009 (0.042)</td>
<td>0.084 (0.069)</td>
<td>-0.019 (0.063)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Observations</th>
<th>294</th>
<th>294</th>
<th>294</th>
<th>294</th>
<th>294</th>
</tr>
</thead>
<tbody>
<tr>
<td>R²</td>
<td>0.050</td>
<td>0.207</td>
<td>0.091</td>
<td>0.214</td>
<td>0.013</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>0.037</td>
<td>0.196</td>
<td>0.079</td>
<td>0.203</td>
<td>-0.001</td>
</tr>
<tr>
<td>Residual Std. Error</td>
<td>0.059 (df = 289)</td>
<td>0.046 (df = 289)</td>
<td>0.074 (df = 289)</td>
<td>0.068 (df = 289)</td>
<td>0.063 (df = 289)</td>
</tr>
<tr>
<td>F Statistic</td>
<td>3.821*** (df = 4; 289)</td>
<td>18.852*** (df = 4; 289)</td>
<td>7.274*** (df = 4; 289)</td>
<td>19.629*** (df = 4; 289)</td>
<td>0.961 (df = 4; 289)</td>
</tr>
</tbody>
</table>

Note: Standard errors in parentheses. Significance levels: *p<0.1; **p<0.05; ***p<0.01
Table 22
GRS-test based on alternative assumptions, part 1. July 1991-December 2015, 294 months. The GRS statistic tests if the intercepts of all $N$ time-series regressions given by Equation (18) jointly equal zero. $A|a_i|$ is the average absolute intercept value, adj. $R^2$ is the adjusted coefficient of determination. We test if the model performance of the five-factor model (FF5F) dominates that of the respective benchmark model (FF3F in the case of Germany, Mkt+SMB in the case of Norway).

<table>
<thead>
<tr>
<th></th>
<th>Germany</th>
<th>Norway</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GRS p-value</td>
<td>$A</td>
</tr>
<tr>
<td><strong>Panel A:</strong> only firms with fiscal year-end = December</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a) Size-B/M:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FF5F</td>
<td>1.75</td>
<td>0.02</td>
</tr>
<tr>
<td>benchmark</td>
<td>1.57</td>
<td>0.05</td>
</tr>
<tr>
<td>b) Size-OP:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FF5F</td>
<td>1.64</td>
<td>0.03</td>
</tr>
<tr>
<td>benchmark</td>
<td>1.76</td>
<td>0.02</td>
</tr>
<tr>
<td>c) Size-Inv:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FF5F</td>
<td>1.63</td>
<td>0.03</td>
</tr>
<tr>
<td>benchmark</td>
<td>1.63</td>
<td>0.03</td>
</tr>
<tr>
<td><strong>Panel B:</strong> Market equity aligned with fiscal year-end</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a) Size-B/M:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FF5F</td>
<td>1.55</td>
<td>0.05</td>
</tr>
<tr>
<td>benchmark</td>
<td>1.49</td>
<td>0.07</td>
</tr>
<tr>
<td>b) Size-OP:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FF5F</td>
<td>2.39</td>
<td>0.00</td>
</tr>
<tr>
<td>benchmark</td>
<td>2.43</td>
<td>0.00</td>
</tr>
<tr>
<td>c) Size-Inv:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FF5F</td>
<td>1.79</td>
<td>0.01</td>
</tr>
<tr>
<td>benchmark</td>
<td>1.82</td>
<td>0.01</td>
</tr>
<tr>
<td><strong>Panel C:</strong> penny stock adjustment</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a) Size-B/M:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FF5F</td>
<td>1.31</td>
<td>0.15</td>
</tr>
<tr>
<td>benchmark</td>
<td>1.32</td>
<td>0.15</td>
</tr>
<tr>
<td>b) Size-OP:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FF5F</td>
<td>2.14</td>
<td>0.00</td>
</tr>
<tr>
<td>benchmark</td>
<td>2.16</td>
<td>0.00</td>
</tr>
<tr>
<td>c) Size-Inv:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FF5F</td>
<td>1.94</td>
<td>0.01</td>
</tr>
<tr>
<td>benchmark</td>
<td>1.93</td>
<td>0.01</td>
</tr>
</tbody>
</table>
### Table 23

GRS-test based on alternative assumptions, part 2. The GRS-statistic tests if the intercepts of all $N$ time-series regressions given by Equation (18) jointly equal zero. $A|a_i|$ is the average absolute intercept value, adj. $R^2$ is the adjusted coefficient of determination. We test if the model performance of the five-factor model (FF5F) dominates that of the respective benchmark model (FF3F in the case of Germany, Mkt+SMB in the case of Norway). For the German data set we test if only looking at the DEM period, i.e. 1990-1998 (Panel A) or only looking at the EUR period, i.e. 1999-2015 (Panel B) leads to different conclusions. For the Norwegian data set we test if only analysing the sample period 2000-2015 (Panel A) or 2007-2015 (Panel B) affects model choice.

<table>
<thead>
<tr>
<th></th>
<th>Germany</th>
<th>Norway</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GRS p-value A</td>
<td>a_i</td>
</tr>
<tr>
<td><strong>Panel A:</strong></td>
<td>DEM period</td>
<td>2000-2015</td>
</tr>
<tr>
<td></td>
<td>FF5F 1.95 0.02 0.0036 0.60</td>
<td>benchmark 2.05 0.01 0.0036 0.60</td>
</tr>
<tr>
<td></td>
<td>FF5F 0.89 0.60 0.0030 0.57</td>
<td>benchmark 0.96 0.53 0.0029 0.56</td>
</tr>
<tr>
<td></td>
<td>FF5F 2.13 0.01 0.0031 0.57</td>
<td>benchmark 1.92 0.02 0.0031 0.55</td>
</tr>
<tr>
<td><strong>Panel B:</strong></td>
<td>EUR period</td>
<td>2007-2015</td>
</tr>
<tr>
<td></td>
<td>FF5F 1.68 0.03 0.0031 0.67</td>
<td>benchmark 1.67 0.03 0.0031 0.67</td>
</tr>
<tr>
<td></td>
<td>FF5F 3.09 0.00 0.0034 0.66</td>
<td>benchmark 3.16 0.00 0.0034 0.64</td>
</tr>
<tr>
<td></td>
<td>FF5F 1.94 0.01 0.0029 0.69</td>
<td>benchmark 2.09 0.00 0.0029 0.67</td>
</tr>
</tbody>
</table>

Note: $A|a_i|$ denotes the average absolute intercept value.
Table 24

Results of the T cross-sectional second stage Fama-MacBeth regressions

\[ R_{it} = a_t + \lambda_{Mt}^{12}b_t + \lambda_{SM}^{12}S_t + \lambda_{HML}^{12}h_t + \lambda_{RMW}^{12}r_t + \lambda_{CML}^{12}c_i + \epsilon_t \]

where \( R_{it} \) is the excess return of the LHS portfolios formed on the double sorts indicated in Panel A-C, \( a_t \) is the pricing error, \( b_t, S_t, h_t, r_t \) and \( c_i \) are the factor loadings found in the first stage Fama-MacBeth regression and \( \lambda_L \) is the risk premium for risk factor \( L \). Estimation is done by GMM as described in section 4.2.2. Coef is the estimated risk premium awarded for factor \( L \) in %, Std is its standard deviation in % and t-stat is its t-statistic. Low p-values of the J-statistic indicate bad model fit. We compare the results of the five-model model with those of the respective benchmark model. The benchmark for German portfolios is the Fama-French three-factor model. The benchmark for the Norwegian portfolios is a two-factor model with the market and the size factor as explanatory variables. The results for Germany are based on the subperiod 1991-1998, i.e. the DEM period. The results for Norway are based on the subperiod 2000-2015.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \lambda_{Mt} )</td>
<td>( \lambda_{SM} )</td>
</tr>
<tr>
<td><strong>Panel A: size-BM</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a) FF5F</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coef</td>
<td>0.69</td>
<td>1.15</td>
</tr>
<tr>
<td>Std</td>
<td>0.48</td>
<td>0.44</td>
</tr>
<tr>
<td>t-stat</td>
<td>1.43</td>
<td>2.60</td>
</tr>
<tr>
<td>J-test</td>
<td>( J(\chi^2(4)) = 4.68, p = 0.32 )</td>
<td>( J(\chi^2(7)) = 6.51, p = 0.48 )</td>
</tr>
<tr>
<td>b) benchmark model</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coef</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Std</td>
<td></td>
<td></td>
</tr>
<tr>
<td>t-stat</td>
<td></td>
<td></td>
</tr>
<tr>
<td>J-test</td>
<td>( J(\chi^2(7)) = 6.51, p = 0.48 )</td>
<td>( J(\chi^2(7)) = 6.51, p = 0.48 )</td>
</tr>
<tr>
<td><strong>Panel B: size-OP</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a) FF5F</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coef</td>
<td>0.55</td>
<td>-1.32</td>
</tr>
<tr>
<td>Std</td>
<td>0.52</td>
<td>0.29</td>
</tr>
<tr>
<td>t-stat</td>
<td>1.05</td>
<td>-4.59</td>
</tr>
<tr>
<td>J-test</td>
<td>( J(\chi^2(20)) = 37.17, p = 0.011 )</td>
<td>( J(\chi^2(4)) = 3.27, p = 0.51 )</td>
</tr>
<tr>
<td>b) benchmark model</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coef</td>
<td>0.40</td>
<td>-1.11</td>
</tr>
<tr>
<td>Std</td>
<td>0.50</td>
<td>0.26</td>
</tr>
<tr>
<td>t-stat</td>
<td>0.80</td>
<td>-4.31</td>
</tr>
<tr>
<td>J-test</td>
<td>( J(\chi^2(22)) = 40.77, p = 0.009 )</td>
<td>( J(\chi^2(7)) = 7.26, p = 0.40 )</td>
</tr>
<tr>
<td><strong>Panel C: size-Inv</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a) FF5F</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coef</td>
<td>0.64</td>
<td>1.26</td>
</tr>
<tr>
<td>Std</td>
<td>0.47</td>
<td>0.51</td>
</tr>
<tr>
<td>t-stat</td>
<td>1.36</td>
<td>2.49</td>
</tr>
<tr>
<td>J-test</td>
<td>( J(\chi^2(4)) = 23.05, p = 0.000 )</td>
<td>( J(\chi^2(4)) = 23.05, p = 0.000 )</td>
</tr>
<tr>
<td>b) benchmark model</td>
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<td></td>
</tr>
<tr>
<td>Coef</td>
<td>0.65</td>
<td>0.90</td>
</tr>
<tr>
<td>Std</td>
<td>0.45</td>
<td>0.38</td>
</tr>
<tr>
<td>t-stat</td>
<td>1.45</td>
<td>2.36</td>
</tr>
<tr>
<td>J-test</td>
<td>( J(\chi^2(7)) = 24.79, p = 0.000 )</td>
<td>( J(\chi^2(7)) = 24.79, p = 0.000 )</td>
</tr>
</tbody>
</table>
### Balance sheet exchange rates ###

# Download daily exchange rates
USDEUR = Quandl("FED/RXI_US_N_B_EU", 
  start_date = "1988-01-01", 
  end_date = "2015-12-31")

# Change order such that dates are descending
attach(USDEUR)
USDEUR <- USDEUR[order(Date),]

# Extract end-of-month dates
USDEUR <- USDEUR[endpoints(USDEUR$Date, on = "months"),]
colnames(USDEUR)[2] <- "USDEUR"

# Split the date into separate year and month columns
USDEUR$Year <- as.numeric(format(USDEUR$Date, format = "%Y"))
USDEUR$Month <- as.numeric(format(USDEUR$Date, format = "%m"))
USDEUR <- subset(USDEUR, select=c("Year","Month","USDEUR"))

# Make complete time frame
USDEUR <- data.frame(  rep(1988:2015, each=12),
  rep(1:12, times=28),
  c(rep(0, times=132), USDEUR$USDEUR))
colnames(USDEUR)[1:3] <- c("Year","Month","USDEUR")
is.na(USDEUR$USDEUR) <- USDEUR$USDEUR == 0

# Form a matrix of balance sheet exchange rates
USDEUR_Bal <- data.frame(subset(USDEUR, Month == 1, 
  select = c("Year","USDEUR")),
  subset(USDEUR, Month == 2, select = "USDEUR"),
  subset(USDEUR, Month == 3, select = "USDEUR"),
  subset(USDEUR, Month == 4, select = "USDEUR"),
  subset(USDEUR, Month == 5, select = "USDEUR"),
  subset(USDEUR, Month == 6, select = "USDEUR"),
  subset(USDEUR, Month == 7, select = "USDEUR"),
  subset(USDEUR, Month == 8, select = "USDEUR"),
  subset(USDEUR, Month == 9, select = "USDEUR"),
  subset(USDEUR, Month == 10, select = "USDEUR"),
  subset(USDEUR, Month == 11, select = "USDEUR"),
  subset(USDEUR, Month == 12, select = "USDEUR"))
colnames(USDEUR_Bal)[2:13] <- c("USDEUR_1","USDEUR_2","USDEUR_3","USDEUR_4","USDEUR_5","USDEUR_6","USDEUR_7","USDEUR_8","USDEUR_9","USDEUR_10","USDEUR_11","USDEUR_12")
### Profit and loss statement exchange rates ###

# Download daily exchange rates USDEUR

```r
USDEUR_avg = Quandl("FED/RX1_US_N_B_EU",
    start_date = "1988-01-01",
    end_date = "2015-12-31")
```

```r
colnames(USDEUR_avg)[2] <- 'USDEUR'
```

# Split the date into separate year and month columns

```r
USDEUR_avg$Year <- as.numeric(format(USDEUR_avg$Date, format = "%Y"))
USDEUR_avg$Month <- as.numeric(format(USDEUR_avg$Date, format = "%m"))
```

```r
USDEUR_avg <- data.table(USDEUR_avg)
```

# Calculate mean rate per sample month and make descending order

```r
USDEUR_avg <- USDEUR_avg[, list(USDEUR_avg = mean(USDEUR)),
    by = c("Year", "Month")]
USDEUR_avg <- USDEUR_avg[order(Year, Month),]
```

# Make complete time frame

```r
USDEUR_avg <- data.frame(rep(1988:2015, each=12),
    rep(1:12, times=28),
    c(rep(0, times=132), USDEUR_avg$USDEUR(avg))
```

```r
colnames(USDEUR_avg)[1:3] <- c("Year", "Month", "USDEUR_avg")
```

# Calculate rolling mean for each fiscal year

```r
USDEUR_avg <- data.frame(USDEUR_avg$Year, USDEUR_avg$Month, 
    SMA(USDEUR_avg$USDEUR_avg, n=12, na.rm=T))
```

```r
colnames(USDEUR_avg)[1:3] <- c("Year", "Month", "USDEUR")
```

```r
is.na(USDEUR_avg$USDEUR) <- USDEUR_avg$USDEUR == 0 |
    (USDEUR_avg$Year == 2016 & USDEUR_avg$Month > 3)
```

# Form a matrix of P/L statement exchange rates

```r
USDEUR_avg <- data.frame(subset(USDEUR_avg, Month == 1, 
    select = c("Year", "USDEUR")),
    subset(USDEUR_avg, Month == 2, select = "USDEUR"),
    subset(USDEUR_avg, Month == 3, select = "USDEUR"),
    subset(USDEUR_avg, Month == 4, select = "USDEUR"),
    subset(USDEUR_avg, Month == 5, select = "USDEUR"),
    subset(USDEUR_avg, Month == 6, select = "USDEUR"),
    subset(USDEUR_avg, Month == 7, select = "USDEUR"),
    subset(USDEUR_avg, Month == 8, select = "USDEUR"),
    subset(USDEUR_avg, Month == 9, select = "USDEUR"),
    subset(USDEUR_avg, Month == 10, select = "USDEUR"),
    subset(USDEUR_avg, Month == 11, select = "USDEUR"),
    subset(USDEUR_avg, Month == 12, select = "USDEUR"))
```

```r
colnames(USDEUR_avg)[2:13] <- c("USDEUR_1", "USDEUR_2",
    "USDEUR_3", "USDEUR_4",
    "USDEUR_5", "USDEUR_6",
    "USDEUR_7", "USDEUR_8",
    "USDEUR_9", "USDEUR_10",
    "USDEUR_11", "USDEUR_12")
```
R code 2

Risk-free rate

# CRSP Treasuries, riskfree series 4-week monthly
Risk_free <- read.csv("Risk_free_rate_4_week_m.csv", sep=".",
stringsAsFactors=F)

colnames(Risk_free)[2] <- "Date"
colnames(Risk_free)[3] <- "Rf_d"
colnames(Risk_free)[4] <- "Duration"

# Reduce date frame
Risk_free <- subset(Risk_free, select=c("Date","Rf_d","Duration"))

# Change to numeric and make a date column
Risk_free <- transform(Risk_free,
                    Date=as.Date(as.character(Date), format="%m/%d/%Y"))

# Split the date into separate year and month columns
Risk_free$Year <- as.numeric(format(Risk_free$Date, format = "%Y"))
Risk_free$Month <- as.numeric(format(Risk_free$Date, format = "%m"))

# Calculate monthly yield
Risk_free$Rf_m <- Risk_free$Rf_d * Risk_free$Duration
Risk_free$Rf_30d <- Risk_free$Rf_d * 30

Risk_free <- subset(Risk_free, select=c("Year","Month","Rf_m","Rf_30d"))

R code 3

Formatting security data

GERMANY_SD <- read.csv("GERMANY_SD_90-15.csv", sep=".", stringsAsFactors=F)

# generate easier column names:
colnames(GERMANY_SD)[3:4] <- c("Date","company")

# Change to NUMERIC and make a DATE column
GERMANY_SD <- transform(GERMANY_SD,
                  Date=as.Date(as.character(Date), format="%Y%m%d"),
                  cshoc=as.numeric(gsub("","",cshoc)),
                  prccd=as.numeric(gsub("","",.prccd)))

# Extract last company observation per month
GERMANY_SM <- GERMANY_SD[diff(as.numeric(substr(GERMANY_SD$Date, 9, 10))) < 0, TRUE], ]

# Split the date into separate year, month and day columns
GERMANY_SM$Year <- as.numeric(format(GERMANY_SM$Date, format = "%Y"))
GERMANY_SM$Month <- as.numeric(format(GERMANY_SM$Date, format = "%m"))
GERMANY_SM$day <- as.numeric(format(GERMANY_SM$Date, format = "%d"))

# Keep stocks traded at German Stock Exchanges only
# 115 - Berlin
# 149 - Dusseldorf
# 154 - Frankfurt
# 163 - Hamburg
# 165 - Hannover
# 171 - IBIS Germany, today called "Xetra"
# 212 - Munich
# 257 — Stuttgart

GERMANY_SM <- subset(GERMANY_SM, exchg == 115 | exchg == 149 |
                      exchg == 154 | exchg == 163 | exchg == 165 |
                      exchg == 171 | exchg == 212 | exchg == 257)

# Keep common and preferred stocks only
# 0 — Common, ordinary
# 1 — Preferred, preference, etc.
GERMANY_SM <- subset(GERMANY_SM, tpci == 0 | tpci == 1)

# Keep only end-of-month observations
GERMANY_SM <- subset(GERMANY_SM, day > 25)

# Create a data file with desired variables
GERMANY_SM <- subset(GERMANY_SM, select =
                      c("Year", "Month", "iid", "gvkey", "company",
                       "currccd", "prccd", "cshoc", "tpci", "ajexdi"))

# Calculate adjusted prices as basis for correct return computation
GERMANY_SM$adjpr <- GERMANY_SM$prccd / GERMANY_SM$ajexdi

# Make an ID-column that combines gvkey and issue number
# (there are some companies that have the same name but different gvkey/ISIN,
# therefore company name is not enough)
GERMANY_SM <- within(GERMANY_SM, id <- paste(gvkey, iid, sep="/"))

# Make complete time frame to ensure that returns are calculated for ONE month
# 1. Find all unique combinations of Year and Month
# 2. Find all companies in the sample
# 3. Combine dates and companies (cartesian product) -> gives desired grid
# 4. Merge the desired grid with the information we have.
# all=T gives "NA" in rows without information
GERMANY_SM <- as.data.frame(GERMANY_SM)
df2 <- unique(GERMANY_SM[, c("Year", "Month")])
df3 <- unique(GERMANY_SM[, c("company", "id")])
df2 <- merge(df2, df3)
GERMANY_SM <- merge(GERMANY_SM, df2, all=T)

# For the return calculation we define a growth function
Growth <- function(x) {c(NA, diff(x) / x[-length(x)])}

GERMANY_SM <- data.table(GERMANY_SM)
GERMANY_SM[, RETURN := Growth(adjpr), by = id]

# Calculate shares total and price, over all exchanges
# and all tpci's for each company at every time
GERMANY_SM <- data.table(GERMANY_SM)
GERMANY_SM[order(company, Year, Month),]
GERMANY_SM <- GERMANY_SM[, list(Shares_tot = sum(cshoc),
                       Price_tot = weighted.mean(prccd, cshoc),
                       RETURN = weighted.mean(RETURN, cshoc),
                       by = c("Year", "Month", "gvkey", "company", "currccd"))]

GERMANY_SM <- na.omit(GERMANY_SM)

# Add a column with market cap. denoted in different currencies
GERMANY_SM$ME_mixed_curr <- (GERMANY_SM$Shares_tot * GERMANY_SM$Price_tot)

# Merge GERMANY_SM and currency data by "Year", and "Month."
GERMANY_SM <- merge(GERMANY_SM, USDEDM, by = c("Year", "Month"))
GERMANY_SM <- merge(GERMANY_SM, USDEUR, by = c("Year", "Month"))

# Add a column containing market cap denoted in USD
GERMANY_SM <- data.frame(GERMANY_SM)
h <- paste0("USD", GERMANY_SM$currdd)
GERMANY_SM$ME_usd <- GERMANY_SM$ME_mixed_curr * as.numeric(bound(1:rown(GERMANY_SM), match(h, names(GERMANY_SM)))))

### Remove penny stocks ###

# Add a column containing ME denoted in EUR
GERMANY_SM$EURDEM <- ifelse(GERMANY_SM$Year < 1999, (1/1.95583), 1)

GERMANY_SM$ME_eur <- ifelse(GERMANY_SM$currdd == "DEM", GERMANY_SM$ME_mixed_curr * GERMANY_SM$EURDEM, GERMANY_SM$ME_mixed_curr)

# Add a column containing share price denoted in EUR:
GERMANY_SM$Price_tot_EUR <- ifelse(GERMANY_SM$currdd == "DEM", GERMANY_SM$Price_tot * GERMANY_SM$EURDEM, GERMANY_SM$Price_tot)

GERMANY_SM$pennystock <- ifelse(GERMANY_SM$Price_tot_EUR < 1 & GERMANY_SM$ME_eur < 750000, 1, 0)

GERMANY_SM <- subset(GERMANY_SM, pennystock == 0)

### RETURN CALCULATION CONTINUES ###

# Sort by company, then by time
GERMANY_SM <- data.table(GERMANY_SM)
GERMANY_SM <- GERMANY_SM[order(company, Year, Month).]

# Extract desired variables
GERMANY_SM <- subset(GERMANY_SM, select = c("Year", "Month", "gvkey", "company", "RETURN", "ME_usd"))

# Denote ME in Millions
GERMANY_SM$ME_usd <- (GERMANY_SM$ME_usd / 1000000)

# Add column with risk free rate of return to existing data frame, 
# and sort by company and date
GERMANY_SM <- merge(GERMANY_SM, Risk_free, by = c("Year", "Month"), all=T)
GERMANY_SM <- GERMANY_SM[order(Year, Month, company).]

# Calculate excess return, and extract desired variables
GERMANY_SM$Excess_ret <- (GERMANY_SM$RETURN - GERMANY_SM$RF_30d)
GERMANY_SM <- subset(GERMANY_SM, select = c("Year", "Month", "gvkey", "company", "ME_usd", "RETURN", "Excess_ret"))

R code 4
Return descriptive statistics and histograms

# Returns original sample
r_gb <- GERMANY_SM$RETURN
# Descriptive statistics

Before_trim <- data.frame(
  min(r_gb, na.rm=T),
  mean(r_gb, na.rm=T),
  max(r_gb, na.rm=T),
  sd(r_gb, na.rm=T),
  kurtosi(r_gb, na.rm=T),
  skew(r_gb, na.rm=T))


Before_trim <- data.frame(t(Before_trim))

### Return adjustments ###

# Remove returns that are exactly zero
GERMANY_SM <- subset(GERMANY_SM, RETURN != 0)

# Trim for outliers (the 0.5% highest and lowest observations)
qnt_up <- quantile(GERMANY_SM$RETURN, .995, na.rm=T)
qnt_low <- quantile(GERMANY_SM$RETURN, .005, na.rm=T)

GERMANY_SM <- subset(GERMANY_SM, RETURN >= qnt_low & RETURN <= qnt_up)

# Returns adjusted sample
r_ga <- GERMANY_SM$RETURN

# test returns on normal distribution
jarque.bera.test(GERMANY_SM$RETURN)

# Descriptive statistics

After_trim <- data.frame(
  min(r_ga, na.rm=T),
  mean(r_ga, na.rm=T),
  max(r_ga, na.rm=T),
  sd(r_ga, na.rm=T),
  kurtosi(r_ga, na.rm=T),
  skew(r_ga, na.rm=T))


After_trim <- data.frame(t(After_trim))

# Histograms

par(mfrow=c(2,2))

h_gb <- hist(r_gb, xlim=c(-0.75,0.75), breaks=10000, col="red",
             xlab="stock_return", main="Germany, original sample")

h_ga <- hist(r_ga, xlim=c(-0.75,0.75), breaks=150, col="red",
             xlab="stock_return", main="Germany, adjusted sample")

xfit <- seq(min(r_ga), max(r_ga), length=100)
yfit <- dnorm(xfit, mean=mean(r_ga), sd=sd(r_ga))
yfit <- yfit * diff(h_ga$sids[1:2]) * length(r_ga)
lines(xfit, yfit, col="black", lwd=2)

h_nb <- hist(r_nb, xlim=c(-0.75,0.75), breaks=500, col="red",
             xlab="stock_return", main="Norway, original sample")

h_na <- hist(r_na, xlim=c(-0.75,0.75), breaks=150, col="red".),
Formatting accounting data

R code 5

# Change to numeric and make a date column
GERMANY_FY <- transform(GERMANY_FY,
  Date = as.Date(as.character(Date), format="%Y-%m-%d"),
  at = as.numeric(gsub("-*","." , at)),
  cogs = as.numeric(gsub("-*","." , cogs)),
  lt = as.numeric(gsub("-*","." , lt)),
  revt = as.numeric(gsub("-*","." , revt)),
  seq = as.numeric(gsub("-*","." , seq)),
  txdictc = as.numeric(gsub("-*","." , txdictc)),
  sic = as.numeric(gsub("-*","." , sic)),
  xsga = as.numeric(gsub("-*","." , xsga)),
  xint = as.numeric(gsub("-*","." , xint)),
  xopr = as.numeric(gsub("-*","." , xopr)),
  xopro = as.numeric(gsub("-*","." , xopro)),
  xintd = as.numeric(gsub("-*","." , xintd)),
  exchg = as.numeric(gsub("-*","." , exchg)),
  Year = as.numeric(gsub("-*","." , Year)),
  fy = as.numeric(gsub("-*","." , fy)),
  gvkey = as.numeric(gsub("-*","." , gvkey))))

# Divide balance sheet and result variables in separate data frames
GERMANY_FY_bal <- subset(GERMANY_FY,
  select=c("Year", "at", "lt", "seq", "txdictc"))
GERMANY_FY_res <- subset(GERMANY_FY,
  select=c("Year", "at", "lt", "seq", "txdictc", "gvkey", "company",
  "cursd", "at", "lt", "seq", "txdictc"))

# Merge GERMANY_FY_bal with balance sheet exchange rates
GERMANY_FY_bal <- merge(GERMANY_FY_bal, USD_AUD_Bal, by = "Year")
GERMANY_FY_bal <- merge(GERMANY_FY_bal, USDDEM_Bal, by = "Year")
GERMANY_FY_bal <- merge(GERMANY_FY_bal, USDEUR_Bal, by = "Year")

# Merge GERMANY_FY_res with P/L exchange rates
GERMANY_FY_res <- merge(GERMANY_FY_res, USD_AUD_avg, by = "Year")
GERMANY_FY_res <- merge(GERMANY_FY_res, USDDEM_avg, by = "Year")
Sorting variable construction

# Denote balance sheet variables in USD
i <- paste0("USD", GERMANY_FY_bal$curcd, ",", GERMANY_FY_bal$fyr)
tmp <- as.numeric(GERMANY_FY_bal[cbind(1:nrow(GERMANY_FY_bal), match(i, names(GERMANY_FY_bal)))]))
GERMANY_FY_bal[,c("at_usd","lt_usd","seq_usd","txdite_usd")]
<- GERMANY_FY_bal[,6:9] * ifelse(is.na(tmp), 1, tmp)

# Denote P/L variables in USD
l <- paste0("USD", GERMANY_FY_res$curcd, ",", GERMANY_FY_res$fyr)
tmp <- as.numeric(GERMANY_FY_res[cbind(1:nrow(GERMANY_FY_res),
match(1, names(GERMANY_FY_res)))]))
GERMANY_FY_res[,c("revt_usd","cogs_usd","xsga_usd",
"xint_usd","xopr_usd","xopro_usd")]
<- GERMANY_FY_res[,6:11] * ifelse(is.na(tmp), 1, tmp)

R code 6
Sorting variable construction

# Combine all desired balance sheet and P/L variables into one frame
GERMANY_FY <- subset(GERMANY_FY_bal, select=c("Year","fyr","gvkey","company",
"at_usd","lt_usd","seq_usd","txdite_usd"))
GERMANY_FY <- cbind(GERMANY_FY, GERMANY_FY_res[,c("revt_usd","cogs_usd",
"xsga_usd","xint_usd",
"xopr_usd","xopro_usd")])

# Omit companies with zero assets (sign of insufficient reporting)
GERMANY_FY <- data.table(GERMANY_FY)
GERMANY_FY <- subset(GERMANY_FY, at_usd!=0)

# Define stockholder equity
GERMANY_FY[,stockholder_equity := ifelse(!is.na(seq_usd), seq_usd,
(at_usd - lt_usd))]

# Calculate book-equity
GERMANY_FY$BE_usd <- (GERMANY_FY$stockholder_equity + GERMANY_FY$txdite_usd)

# Define market frame as base for market factor construction
GERMANY_mkt <- data.frame(GERMANY_FY)
GERMANY_mkt <- data.frame(GERMANY_mkt$Year, GERMANY_mkt$company,GERMANY_mkt$BE_usd)
colnames(GERMANY_mkt)[1:3] <- c("Year","company","BE_usd")

# Omit negative– and zero–BE firms
GERMANY_FY <- subset(GERMANY_FY, !BE_usd<=0)

# Define profitability
GERMANY_FY[,profitability := (revt_usd - xopr_usd - xint_usd) / BE_usd]

# make complete time frame to secure monthly return periods
GERMANY_FY <- as.data.frame(GERMANY_FY)
df2 <- unique(GERMANY_FY[, "Year", drop=F])
df3 <- unique(GERMANY_FY[,c("company","gvkey")])
df2 <- merge(df2, df3)

76
GERMANY_FY <- merge(GERMANY_FY , df2 , all=T)

# Define investment
GERMANY_FY <- data.table(GERMANY_FY)
GERMANY_FY <- GERMANY_FY[order(company , Year) ,]
GERMANY_FY[, investment := Growth(at_usd), by = company]

GERMANY_FY <- na.omit(GERMANY_FY)

# Extract ME and add a year column
df2 <- subset(GERMANY_SM , select=c("Year","Month","company","ME_usd"))
df2 <- subset(df2 , Month == 12 , select=c("Year","company","ME_usd"))

# Extract BE, Profitability and Investment
df3 <- subset(GERMANY_FY , select=c("Year","company","BE_usd","profitability","investment"))

# Merge to one file
GERMANY <- merge(df2 , df3 , by = c("Year","company"))
GERMANY <- data.table(GERMANY)
GERMANY <- GERMANY[order(Year , company),]

# Calculate Book-to-Market
GERMANY$BM <- (GERMANY$BE_usd / GERMANY$ME_usd)
GERMANY <- subset(GERMANY , select=c("Year","company","BM","profitability","investment"))

GERMANY$Year <- (GERMANY$Year + 1)

# Make one file containing BE, OP and Inv at time t and ME at June of t+1
# Extract ME at June of t+1
df4 <- subset(GERMANY_SM , Month == 6 , select=c("Year","company","ME_usd"))
colnames(df4)[3] <- "size"
GERMANY <- merge(GERMANY , df4 , by = c("Year","company"), all=T)

GERMANY <- na.omit(GERMANY)

# Construct the market factor
GERMANY_mkt <- merge(df2 , GERMANY_mkt , by = c("Year","company"), drop=F)
GERMANY_mkt <- na.omit(GERMANY_mkt)

GERMANY_mkt$Year <- (GERMANY_mkt$Year + 1)
GERMANY_mkt <- merge(GERMANY_mkt , df4 , by = c("Year","company"), all=T)

GERMANY_mkt <- na.omit(GERMANY_mkt)

R code 7
Form test portfolios

# Add columns that show which quintile the observation belongs to
GERMANY <- ddply(GERMANY , .(Year) , transform ,
size_gr = cut(size , breaks = c(quantile(size , seq(0,1,by=0.2)) ) ,
labels=c("1","2","3","4","5"),include.lowest = T))

GERMANY <- ddply(GERMANY , .(Year) , transform ,
BM_gr = cut(BM , breaks = c(quantile(BM , seq(0,1,by=0.2)) ) ,
labels=c("1","2","3","4","5"),include.lowest = T))

GERMANY <- ddply(GERMANY , .(Year) , transform ,

77
```r
Inv_gr = cut(investment, breaks = c(quantile(investment, seq(0.1, by=0.2)), labels=c("1", "2", "3", "4", "5")), include.lowest = T)

GERMANY <- ddply(GERMANY, .(Year), transform,
                 OP_gr = cut(profitability, breaks = c(quantile(profitability, seq(0.1, by=0.2)), labels=c("1", "2", "3", "4", "5")), include.lowest = T))

# Add columns containing information about which size_BM, size_Inv, and
# size_OP portfolio the observation belongs to
GERMANY <- within(GERMANY, size_BM <- paste0(size_gr, BM_gr))
GERMANY <- within(GERMANY, size_OP <- paste0(size_gr, OP_gr))
GERMANY <- within(GERMANY, size_Inv <- paste0(size_gr, Inv_gr))

# To compare yearly portfolio updates to monthly company returns,
# we need to have each observation 12 times
GERMANY_LHS <- data.frame(GERMANY[rep(seq_len(nrow(GERMANY)), each=12),])
GERMANY_mkt <- data.frame(GERMANY_mkt[rep(seq_len(nrow(GERMANY_mkt)), each=12),])

# Add a month column that runs from 1−12 for each company
GERMANY_LHS$Month <- rep(c(7:12), 1:6) times=nrow(GERMANY_LHS)/12
GERMANY_mkt$Month <- rep(c(7:12), 1:6) times=nrow(GERMANY_mkt)/12

GERMANY_LHS <- data.table(GERMANY_LHS)
GERMANY_LHS[, Year_ret := ifelse(Month > 6, Year, Year+1)]

GERMANY_mkt <- data.table(GERMANY_mkt)
GERMANY_mkt[, Year_ret := ifelse(Month > 6, Year, Year+1)]

# Merge data frames to include return data, and subset desired variables
colnames(GERMANy_SM)[1] <- "Year_ret"

GERMANY_LHS <- merge(GERMANy_LHS, GERMANY_SM, by = c("Year_ret", "Month", "company"))
GERMANY_mkt <- merge(GERMANy_mkt, GERMANY_SM, by = c("Year_ret", "Month", "company"))

GERMANY_LHS <- data.frame(GERMANy_LHS)
GERMANY_LHS <- data.frame(GERMANy_LHS[,1:3], GERMANY_LHS[,17:19], GERMANY_LHS[,4:15])

GERMANY_mkt <- data.frame(GERMANy_mkt)
GERMANY_mkt <- data.frame(GERMANy_mkt[,1:3], GERMANY_mkt[,7], GERMANY_mkt[,9:11])
colnames(GERMANy_mkt)[4] <- "size"
colnames(GERMANy_mkt)[5] <- "ME_usd"

# Change back column name to avoid mistakes later on
colnames(GERMANy_SM)[1] <- "Year"

GERMANY_LHS <- data.frame(GERMANy_LHS)
comp_per_month <- aggregate(GERMANy_LHS$Month, by=list(GERMANy_LHS$Year_ret, GERMANy_LHS$Month), FUN="length")
colnames(comp_per_month)[1:3] <- c("Year_ret", "Month", "observations")
```

R code 8

Test portfolio characteristics

# Calculate mean excess return of each LHS portfolio at each time period
GERMANY_LHS <- data.table(GERMANy_LHS)
Ret_size_BM <- GERMANy_LHS[, list(R_eweight = mean(Excess_ret),
                                    R_vweight = weighted.mean(Excess_ret, ME_usd)),
```
### Table 1 from FF2014

# Calculate time-series mean per portfolio
T1 <- Ret_size_BM[ , list(Ret_size_BM = mean(R_vweight)) , by = c("size_BM")]
colnames(T1)[1] <- "Portfolio"

# Denote numbers in %
T1$Ret_size_BM <- T1$Ret_size_BM * 100

T1_size_BM <- t(matrix(T1$Ret_size_BM,nrow=5,ncol=5,byrow=F))

# Number of observations in each LHS portfolio
PF_obs <- aggregate(GERMANY_LHS$size_BM, by=list(GERMANY_LHS$Year, GERMANY_LHS$Month, GERMANY_LHS$size_BM), FUN="length")
colnames(PF_obs)[1:4] <- c("Year","Month","Portfolio","size_BM_obs")

PF_obs <- data.table(PF_obs)
PF_obs <- PF_obs[,list(size_BM_obs = mean(size_BM_obs)),by = Portfolio]

---

**R code 9**

Double sorting as basis of factor construction

# Create columns containing the stock observations' unique group
GERMANY_LHS <- data.frame(GERMANY_LHS)
GERMANY_RHS <- data.frame(GERMANY_LHS[,1:11])

GERMANY_RHS <- ddply(GERMANY_RHS, .(Year), transform, 
  size_gr = ifelse(size > median(size, na.rm=T), "B","S")
)

GERMANY_RHS <- ddply(GERMANY_RHS, .(Year), transform, 
  BM_gr = ifelse(BM > quantile(BM,0.7,na.rm=T), "H", 
    ifelse(BM < quantile(BM,0.3,na.rm=T), "L","N")))

GERMANY_RHS <- ddply(GERMANY_RHS, .(Year), transform, 
  Inv_gr = ifelse(investment > quantile(investment,0.7,na.rm=T), 
    "A",ifelse(investment < quantile(investment,0.3,na.rm=T), 
      "C","N")))

GERMANY_RHS <- ddply(GERMANY_RHS, .(Year), transform, 
  OP_gr = ifelse(profitability > quantile(profitability,0.7,na.rm=T), 
    "R",ifelse(profitability < quantile(profitability,0.3,na.rm=T), 
      "W","N")))

# Combine groups
GERMANY_RHS <- within(GERMANY_RHS, size_BM <- paste0(size_gr,BM_gr))
GERMANY_RHS <- within(GERMANY_RHS, size_OP <- paste0(size_gr,OP_gr))
GERMANY_RHS <- within(GERMANY_RHS, size_inv <- paste0(size_gr,Inv_gr))

GERMANY_RHS <- na.omit(GERMANY_RHS)

# Number of observations in each RHS portfolio

# size_BM
RHS_obs_size_BM <- aggregate(GERMANY_RHS$size_BM, 
  by=list(GERMANY_RHS$Year,GERMANY_RHS$Month, GERMANY_RHS$size_BM), FUN="length")
colnames(RHS_obs_size_BM)[1:4] <- c("Year","Month","Portfolio","size_BM_obs")
RHS_obs_size_BM <- data.table(RHS_obs_size_BM)
RHS_obs_size_BM <- RHS_obs_size_BM[, .list(size_BM_obs = mean(size_BM_obs), by = Portfolio)]

R code 10
Construction of the SMB factor

# Calculate monthly mean return first within each of the 6 size BM groups
GERMANY_SMB_BM <- data.table(GERMANY_RHS)
GERMANY_SMB_BM[ , .list(r_eweight = mean(RETURN), r_vweight = weighted.mean(RETURN, ME_usd), ME_usd = sum(ME_usd)), by = c('Year_ret', 'Month', 'size_BM', 'size_gr')] = P(ortfolio)

# In this subset calculate the mean return by size group
GERMANY_SMB_BM[ , .list(r_eweight = mean(r_eweight), r_vweight = weighted.mean(r_vweight, ME_usd)), by = c('Year_ret', 'Month', 'size_gr')] = P(ortfolio)

# Sort such that small stocks come before big stocks
GERMANY_SMB_BM[order(Year_ret, Month, -size_gr),]

# Calculate monthly mean return for small minus big stocks
GERMANY_SMB_BM[ , .list(SMB = diff(-r_vweight)), by = c('Year_ret', 'Month')] = P(ortfolio)

# Repeat the same procedure for size_OP and size_INV

# Calculate mean total SMB based on the three SMB sub-factors
GERMANY_5F <- data.frame(GERMANy_SMB_BM$Year_ret, GERMANY_SMB_BM$Month, (GERMANy_SMB_BM$SMB_BM + GERMANY_SMB_INV$SMB_INV + GERMANY_SMB_OP$SMB_OP) / 3)
colnames(GERMANy_5F)[1:3] = c('Year_ret', 'Month', 'SMB')

R code 11
Construction of the HML, RMW and CMA factors

# Calculate monthly mean return first within each of the 6 size BM groups
GERMANY_SMB_BM <- data.table(GERMANY_RHS)
GERMANY_SMB_BM[ , .list(r_eweight = mean(RETURN), r_vweight = weighted.mean(RETURN, ME_usd), ME_usd = sum(ME_usd)), by = c('Year_ret', 'Month', 'size_BM', 'size_gr')] = P(ortfolio)

# In this subset calculate the mean return by size group
GERMANY_SMB_BM[ , .list(r_eweight = mean(r_eweight), r_vweight = weighted.mean(r_vweight, ME_usd)), by = c('Year_ret', 'Month', 'size_gr')] = P(ortfolio)

# Extract High and Low BM stocks only, i.e. omit Neutral BM stocks
GERMANY_SMB_BM[ !GERMANy_SMB_BM$gr == 'N',]

# Sort
GERMANY_SMB_BM[order(Year_ret, Month, BM_gr),]

# Calculate monthly mean return for High minus Low stocks
GERMANY_SMB_BM <- GERMANY_SMB_BM[, list(HML = diff(-r_vweight)), by = c("Year_ret", "Month")]

# Add the HML column to GERMANY_5F
GERMANY_5F$HML <- GERMANY_SMB_BM$HML

# Repeat the same for size_OP and size_Inv

---

R code 12

Construction of the market factor

GERMANY_mkt <- data.table(GERMANY_mkt)

GERMANY_mkt <- GERMANY_mkt[, list(RM_vweight = mean(Excess_ret, na.rm=T),
                                 RM_vweight = weighted.mean(Excess_ret, ME_usd, na.rm=T)),
                                 by = c("Year_ret", "Month")]

GERMANY_5F <- merge(GERMANY_5F, GERMANY_mkt, by = c("Year_ret", "Month"))

---

R code 13

Figure extreme portfolio spread

### size sorts Germany and Norway ###

# Define x, here equal to monthly cumulative returns for Megacaps.

x_N <- data.table(subset(NORWAY_LHS, size_gr == 3))
x_N <- x_N[, list(RETURN = weighted.mean(RETURN, ME_usd)),
           by = c("Year_ret", "Month")]
x_N <- x_N[order(Year_ret, Month),]
x_N$NSCum_ret <- cumsum(x_N$RETURN)
x_N <- ts(x_N$NSCum_ret, start=c(1991,7), freq=12)

x_G <- data.table(subset(GERMANY_LHS, size_gr == 5))
x_G <- x_G[, list(RETURN = weighted.mean(RETURN, ME_usd)),
           by = c("Year_ret", "Month")]
x_G <- x_G[order(Year_ret, Month),]
x_G$GCum_ret <- cumsum(x_G$RETURN)
x_G <- ts(x_G$GCum_ret, start=c(1991,7), freq=12)

# Define y, here equal to monthly cumulative returns for Microcaps.

y_N <- data.table(subset(NORWAY_LHS, size_gr == 1))
y_N <- y_N[, list(RETURN = weighted.mean(RETURN, ME_usd)),
           by = c("Year_ret", "Month")]
y_N <- y_N[order(Year_ret, Month),]
y_N$NSCum_ret <- cumsum(y_N$RETURN)
y_N <- ts(y_N$NSCum_ret, start=c(1991,7), freq=12)

y_G <- data.table(subset(GERMANY_LHS, size_gr == 1))
y_G <- y_G[, list(RETURN = weighted.mean(RETURN, ME_usd)),
           by = c("Year_ret", "Month")]
y_G <- y_G[order(Year_ret, Month),]
y_G$GCum_ret <- cumsum(y_G$RETURN)
y_G <- ts(y_G$GCum_ret, start=c(1991,7), freq=12)

setwd(".../Output/Plots")

# Make a plot, where 'jpeg' saves the plot.
jpeg("Size_VW", width=1000, height=400)
plot(x_N, type="l", lwd=2, xlim=c(1990,2015), ylim=c(-0.5,8.1), axes=F,
main="Size_sor ts", xlab="Year_ret", ylab="Cumulative_Return")
axis(1, at=1990:2015, labels=1990:2015);
axis(2); box()

# add several time-series
lines(y_N, col="Black", lty=2, lwd=2)
lines(x_G, col="Green", lty=1, lwd=2)
lines(y_G, col="Green", lty=2, lwd=2)

# add background lines
grid(nx=T, ny=NULL, lty=2)

# format the legend
par(mar=c(5.1, 4.1, 0, 2.1), xpd=T)
legend("topleft", inset=c(0, 0), legend=c("Megacaps", "Microcaps"), lwd=c(2,2),
  lty=c(1,2), col=c("black", horiz=TRUE))
legend("topleft", inset=c(0, 0.1), legend=c("Germany", "Norway"), lwd=c(2,2),
  lty=c(1,1), col=c("green", "black"), horiz=TRUE)

# turn off the plot display and save instead
dev.off()

R code 14
Summary statistics for factor returns

factor_ret <- data.frame(Mean = sapply(GERMANY_5F[ , 3:8], mean, na.rm=T),
                       Std = sapply(GERMANY_5F[ , 3:8], sd, na.rm=T))

# Denote numbers in %
factor_ret$Mean <- factor_ret$Mean * 100
factor_ret$Std  <- factor_ret$Std  * 100

ttest = sapply(GERMANY_5F[ , 3:8], t.test, na.rm=T)
ttest <- t(t(ttest))
factor_ret$Mean <- as.numeric(ttest[ , 1])
factor_ret <- t(factor_ret)

# Correlation between factors
GERMANY_5F <- transform(GERMANY_5F, SMB = as.numeric(SMB), HML = as.numeric(HML),
CMA = as.numeric(CMA), RMW = as.numeric(RMW),
RM_eweight = as.numeric(RM_eweight),
RM_vweight = as.numeric(RM_vweight))
factor_corr <- data.frame(cor(GERMANY_5F[ , c(3:6, 8)]))

R code 15
Auxiliary regressions

# Regression function
SMB <- lm(SMB ~ RM_vweight + HML + RMW + CMA, data = GERMANY_5F)

# Make table with regression coefficients, t-stats and R-squared
T2 <- data.frame(1:5, summary(SMB)$coefficients[ , c(1, 3)], summary(SMB)$r.squared)
colnames(T2)[1:4] <- c("Row", "SMB_intercept", "SMB_t_value", "SMB_r_squared")
R code 16

Stepwise regressions

```r
# size-BM sorts
# FF3F as starting point
lapply(split(Ret_size_BM, Ret_size_BM$size_BM), function(d) stepAIC(lm(R_vweight ~ RM_vweight + SMB + HML, na.action=na.exclude, d=Ret_size_BM)))

# FF5F as starting point
lapply(split(Ret_size_BM, Ret_size_BM$size_BM), function(d) stepAIC(lm(R_vweight ~ RM_vweight + SMB + HML + RMW + CMA, na.action=na.exclude, d=Ret_size_BM)))
```

R code 17

Estimate factor loadings by Fama-MacBeth first-step regressions

```r
# Make complete return tables for all 3 sorts
# size_BM sorts
Ret_size_BM <- merge(Ret_size_BM, GERMANY_5F, by=c("Year_ret","Month"))

Ret_size_BM <- as.data.frame(Ret_size_BM)
df2 <- unique(Ret_size_BM[,"Year_ret",drop=F])
df3 <- unique(Ret_size_BM[,"Month",drop=F])
df4 <- unique(Ret_size_BM[,"size_BM",drop=F])
df2 <- merge(df2, df3)
df2 <- merge(df2, df4)
Ret_size_BM <- merge(Ret_size_BM, df2, all=T)

Ret_size_BM <- subset(Ret_size_BM, !(Year_ret == 1991 & Month < 7))
Ret_size_BM[is.na(Ret_size_BM)] <- 0

# Repeat for size_OP and size.Inv sorts

### GRS–test ###

# Parameters:
# L = number of RHS factors in the model
# t = number of months in the sample
# N = number of LHS portfolios
L <- 5
t <- nrow(GERMANY_5F)
N <- length(unique(Ret_size_BM$size_BM))

# Time series regression:
# Break up Ret_size_BM by LHS–PFs, fit the specified model to each PF

# FF5F
models <- dply(Ret_size_BM, "size_BM", function(Ret_size_BM)
  lm(R_vweight ~ RM_vweight + SMB + HML + RMW + CMA, na.action=na.exclude, data = Ret_size_BM))

# Calculate intercept vector
betas <- ldply(models, coef)
alpha_hat <- as.vector(betas[,"(Intercept)"])

# Calculate residual matrix
residuals <- ldply(models, residuals)
epsilon_hat <- as.matrix(residuals[,2:ncol(residuals)])
epsilon_hat_t <- as.matrix(t(residuals[,2:ncol(residuals)]))
```
# Calculate estimate of the covariance matrix of residuals:
sigma_hat <- (epsilon_hat * epsilon_hat_t) / (t-L-1)

# Calculate vector of factor means
Ret_size_BM <- data.frame(Ret_size_BM)
mju_bar <- as.vector(apply(Ret_size_BM[,c("R_vweight","SMB","HML","RMW","CMA")],
2, mean, na.rm=T))

# Calculate factor matrix
factormatrix <- as.matrix(germany_5F[,c("RM_vweight","SMB","HML","RMW","CMA")])

# Calculate estimate of the covariance matrix of the factors
F_bar <- matrix(rep(t(mju_bar),each=t),nrow=t)
omega_hat <- (t(factormatrix-F_bar) %*% (factormatrix-F_bar)) / (t-1)

# Compute the GRS statistic
W_u <- (t(alpha_hat) %*% solve(sigma_hat, alpha_hat)) / (1+(t(mju_bar) %*% solve(omega_hat, mju_bar)))
GRSstat <- (t/N)*(t-N-L)/(t-N-1) * W_u

### Content of the GRS table ###
# GRS statistic
GRSstat

# p-value of the GRS statistic
pf(GRSstat, N, t-N-L, lower.tail=F)

# average absolute value of the PF intercepts
mean(abs(alpha_hat))

# average adj R-squared
T5_adjR2 <- laply(models, function(mod) summary(mod)$adj.r.squared)
mean(T5_adjR2)

### Table with factor exposures ###
# intercepts:
alpha_hat <- laply(models, function(mod) summary(mod)$coefficients[1,1])
T5_a <- t(matrix(alpha_hat, nrow=5, ncol=5, byrow=F))
xtable(T5_a, digits=3)

# p-value:
T5_p_a <- laply(models, function(mod) summary(mod)$coefficients[,4])
T5_p_a <- T5_p[,1]
T5_p_a <- t(matrix(T5_p_a, nrow=5, ncol=5, byrow=F))

# betas:
T5_b <- laply(models, function(mod) summary(mod)$coefficients[,1])
T5_b <- T5_b[,2]
T5_b <- t(matrix(T5_b, nrow=5, ncol=5, byrow=F))
xtable(T5_b, digits=2)

# p-value:
T5_p_b <- laply(models, function(mod) summary(mod)$coefficients[,4])
T5_p_b <- T5_p[,2]
T5_p_b <- t(matrix(T5_p_b, nrow=5, ncol=5, byrow=F))
Rolling window regressions to assess the factor loadings’ time variation

```r
### Portfolio 11, Inv-sort

test <- subset(Ret_size_inv, size_inv == 11)

# Rolling window estimation
Betas <- matrix(NA, nrow=245, ncol=12)
for(i in 50:nrow(test)){
  fit1 <- lm(RM_yc ~ RM_yc + SMB + HML + RMW + CMA, data = test[(i-49):i,])
  Betas[i-49,1:6] <- coef(fit1)
  Betas[i-49,7:12] <- summary(fit1)$coefficients[,2]
}

colnames(Betas)[1:12] <- c("Intercept", "Mkt", "SMB", "HML", "RMW", "CMA",
  "Intercept_SE", "Mkt_SE", "SMB_SE", "HML_SE", "RMW_SE", "CMA_SE")
Betas <- data.frame(Betas)
Betas[, c("Mkt_SE+2", "SMB_SE+2")] <- c(Betas$Mkt+Betas$Mkt_SE+2, Betas$SMB+Betas$SMB_SE+2)
Betas[, c("Mkt_SE-2", "SMB_SE-2")] <- c(Betas$Mkt-Betas$Mkt_SE+2, Betas$SMB-Betas$SMB_SE+2)

# store the estimated factor loadings
mkt <- data.table(Betas[,2])
smb <- data.table(Betas[,3])
hml <- data.table(Betas[,4])
rmw <- data.table(Betas[,5])
cma <- data.table(Betas[,6])

# store the estimated confidence bands
smb_SE_pluss <- data.table(Betas[,14])
smb_SE_minus <- data.table(Betas[,16])

# figure that shows all estimated factor loadings
jpeg(‘PF11_3’, width=1000, height=400)
plot(smb, lty=1, lwd=2, xlim=c(1,245), ylim=c(-0.7,1.8), axes=F, main="PF_11_insorts_Germany",
  xlab="Time(Months)", ylab="Factor Loadings")
axis(1);
axis(2); box()
lines(mkt, col="Black", lty=1, lwd=2)
lines(smb, col="Green", lty=1, lwd=2)
lines(hml, col="blue", lty=1, lwd=2)
lines(rmw, col="red", lty=1, lwd=2)
lines(cma, col="yellow", lty=1, lwd=2)
grid(nx=T, ny=NULL, lty=2)
par(mar=c(5.1, 4.1, 0, 2.1), xpd=T)
legend("topleft", inset=c(0, 0), legend=c("Mkt", "SMB", "HML", "RMW", "CMA"),
  lwd=c(2,2,2,2,2),
  lty=c(1,1,1,1), col=c("black", "green", "blue", "red", "yellow"), horiz=TRUE)
dev.off()

# figure that shows only one factor and its standard deviation
jpeg(‘PF11_2’, width=1000, height=400)
plot(smb, lty=2, lwd=2, xlim=c(1,245), ylim=c(-0.2,1.6), axes=F, main="PF_11_insorts_Germany",
  xlab="Time(Months)", ylab="Factor Loadings")
axis(1);
axis(2); box()
lines(smb, col="Green", lty=1, lwd=2)
lines(smb_SE_pluss, col="Green", lty=2, lwd=2)
lines(smb_SE_minus, col="Green", lty=2, lwd=2)
grid(nx=T, ny=NULL, lty=2)
par(mar=c(5.1, 4.1, 0, 2.1), xpd=T)
```

85
R code 19
Estimating risk premiums with GMM

X ← data.table(Ret_size_Inv[,1:5])
X ← X[order(Year_ret, Month),]
X$Date ← paste(X$Year_ret, X$Month, sep = ".")
X ← as.data.frame(X)
X ← X[,c("Date", "size_Inv", "R_vweight")]
R ← matrix(X[,"R_vweight"], nrow=294, ncol=25, byrow=T)
PF ← unique(X$size_Inv)
colnames(R) ← PF
GERMANY_5F ← data.table(GERMANY_5F)
fact ← GERMANY_5F[order(Year_ret, Month),]
X ← as.matrix(cbind(R,fact))
g1 ← function(parms,X) {
  a ← parms[1:25]
  b ← parms[26:50]
  s ← parms[51:75]
  h ← parms[76:100]
  r ← parms[101:125]
  c ← parms[126:150]
  mcond ← c()
  for (i in 1:25) {
    e ← X[,i]-a[i]-b[i]*X[,"RM_vweight"]-s[i]*X[,"SMB"]-h[i]*X[,"HML"]-r[i]*X[,"R_MW"]
    e[i]*X[,"CMA"]
    mcond ← cbind(mcond, e)
    mcond ← cbind(mcond, e*X[,"RM_vweight"])
    mcond ← cbind(mcond, e*X[,"SMB"])
    mcond ← cbind(mcond, e*X[,"HML"])
    mcond ← cbind(mcond, e*X[,"R_MW"])
    mcond ← cbind(mcond, e*X[,"CMA"])
  }
  return (mcond)
}
t1 ← as.matrix(betas[2:7])
res1 ← gmm(g1,X,t1)

gm_a ← res1$coefficients[1:25]
gm_b ← res1$coefficients[26:50]
gm_s ← res1$coefficients[51:75]
gm_h ← res1$coefficients[76:100]
gm_r ← res1$coefficients[101:125]
gm_c ← res1$coefficients[126:150]
gm_coef ← matrix(res1$coefficients, nrow = 25, ncol = 6, byrow = F)

g2 ← function(parms,X) {
  lambdaB ← parms[1]
lambdaS ← parms[2]
```r
lambdaH <- parems[3]
lambdaR <- parems[4]
lambdaC <- parems[5]
mcond <- e()

for (i in 1:25) {
  e <- X[,i]-lambdaB*gmm_b[i]-lambdaS*gmm_s[i]-lambdaH*gmm_h[i]-lambdaR*gmm_r[i]-lambdaC*gmm_c[i]
mcond <- cbind(mcond, e)
}
return(mcond)
}

t2 <- c(0.01, 0.01, 0.01, 0.01, 0.01)
res2 <- gmm(g2, X, t2)
summary(res2)
stargazer(res2)

gmm2_coef <- 100*coef(summary(res2))[, "Estimate"]
gmm2_std <- 100*coef(summary(res2))[, "Std. Error"]
gmm2_t <- coef(summary(res2))[, "t_value"]

gmm2 <- rbind(gmm2_coef, gmm2_std, gmm2_t)

xtable(gmm2, digits = 2)
```
References


