Increased information flow between hydropower scheduling models through extended cut sharing

Knut Skogstrand Gjerden*, Hans Ivar Skjelbred, Birger Mo

*SINTEF Energy Research, Sem Sælands vei 11, 7050 Trondheim, Norway

Abstract

We present initial results and description of a method for coupling long term hydro scheduling models to short term hydro scheduling models. The method is based on an established approach but extends on the principle to increase the available information of the future estimates provided by the long term model.

1. Introduction

In hydropower scheduling, numerical models are used for various tasks depending on by planning horizon and user needs. In a research setting, use cases could be long-term analyses investigating grid structure/expansions or future scenario analyses, while in an operational scheduling setting the use case could be an immediate scheduling problem for bidding in power markets.

This span has led to a hierarchy of models starting with very long term models at one end where the scheduling horizon can be several years, the geographical span can be extensive and the level of detail usually coarse. The scope of the models then decrease in scheduling horizon and geographical span and increase in level of detail to models covering a year or several months or weeks. At the other end of the hierarchy the models tend to cover only a small geographical region with a scheduling horizon of only days or a few weeks. However, the level of detail is much greater than for the long term models.

This hierarchy is coupled through information shared between the different models so that results from one model is input to the next model. One example is that a long-term market model produce power price forecasts that one inputs to long-term or seasonal models. These models again provide estimates on, e.g., the value of stored water at a certain time, which again can be used as input to more detailed short-term models.

In this paper, we present details of a method which uses the hydropower scheduling optimisation problem structure to provide more information in the link between long-term and short-term models. The method has been developed...
using and applied to two models developed by SINTEF Energy Research, both of which are in operational use by several of the market actors in the Nordic power markets. The method should be readily applicable to any set of models using a similar solution approach; this will become apparent in the following section.

### Nomenclature

All sizes are vectors unless explicitly noted otherwise.

- $J_t$: object function of the optimisation problem at time $t$, scalar
- $\alpha_t$: future cost estimate at time $t$, scalar
- $x_t$: vector containing all variables at time $t$ except the reservoir volumes
- $v_t$: reservoir volumes at time $t$
- $c_t$: vector containing all direct costs associated with $x_t$
- $q_t$: physical inflow
- $z_t$: normalised inflow
- $m_t$: mean of $q_t$
- $Q_t$: estimated standard deviation of $q_t$, diagonal matrix representation
- $\phi_t$: transition matrix in the inflow model
- $\xi_t$: noise term in the inflow model
- $A_V$: matrix containing the hydrosystem topology
- $d_t$: firm power demand at time $t$
- $S_t$: power balances at time $t$ required to meet the firm power demand, matrix
- $\lambda^r_t$: hydro storage cut coefficient for cut $r$ at time $t$
- $\nu^r_t$: inflow cut coefficient for cut $r$ at time $t$
- $b^r_t$: right-hand side of cut description $r$ at time $t$, scalar

### 2. Problem definition and information sharing

The two models used in this study are the long-term optimisation tool ProdRisk [1–5] and the short-term optimisation tool SHOP [6–8]. Both solve in essence the same optimisation problem, consisting of the optimisation problem for $t \in T$ periods defined by equations (1)-(6). We focus here on the equalities of the models, leaving most details to their respective cited papers. The long-term model has a typical scheduling horizon of one to five years with $t = 1$ week and a geographical extension of one to a few river systems. The short-term model has a typical scheduling horizon of one to two weeks and is typically applied to one production area with a common price. The short-term model has a more detailed and physically accurate description of the river system and the hydropower generation.

The objective of both models is to optimise the utilisation of the hydro resources through minimisation of the future cost of operation,

$$J_t = \min (\alpha_t + c^T_t x_t) ,$$

subject to global and local constraints

$$
\begin{align*}
  v_t &= v_{t-1} + q_t + A_V x_t, \\
  S_t x_t &= d_t, \\
  \alpha_t + (\lambda^r_t)^T v_t + (\nu^r_t)^T z_t &\geq b^r_t, \quad r = 1, \ldots, R, \\
  x^\min_t &\leq x_t \leq x^\max_t, \\
  v^\min_t &\leq v_t \leq v^\max_t.
\end{align*}
$$
These constraints can be either physical limitations or man-imposed. Examples of physical limitations are available inflow or storage capacity and examples of imposed constraints are regulations on reservoir levels or (upper or lower) restrictions on water flow in the river system due to environmental or aesthetic concerns.

Information on the estimated future is transferred between models as cuts of the form (4) which relates the expected future cost to the inflow and volume of water in all reservoirs at a given power price. Mathematically speaking, the set of $R$ cut equations are collectively describing a future-cost hyperplane in the state space of reservoir levels. This is illustrated for a two-reservoir system in Fig. 2. The key point is that the value of water in any reservoir at any time is dependent on the water level and inflow in all other reservoirs. The short-term model uses this description (or an interpolation between two such cuts) as the starting state for further optimization.

![Expected income for a fictitious two-reservoir system](image)

**Fig. 1.** Expected income for a fictitious two-reservoir system. The expected income is a function of the state of all reservoirs making it a hyperplane of the $n+1$ state space comprising of $n$ reservoirs and the future cost.

Inflow is input to both models. The short-term model uses deterministic inflow while the long-term model contains an inflow model using first-order autoregressive approach on normalised (to eliminate seasonal variations) input data:

$$z_t = \phi z_{t-1} + \xi_t$$  
$$q_t = Q_t z_t + m_t.$$  (7) (8)

Price is input to the short-term model and assumed deterministic over the modelling period, which is usually one or two weeks. There is a strong auto-correlation in power prices which influences the water values. The long-term model has to take this into account and contains the price model described in [9]. The price model is a discrete Markov chain representation based on a price forecast. The forecast is typically obtained from a larger scale market model. From the forecast, a model consisting of a grid of $N$ price nodes $p_{ij}$ by $T$ time steps $t$ is constructed. The value (power price) a price node represents is calculated based on price scenarios from the forecast. Then node-transition probabilities, $P(p_{ij+1}|p_{ij}), (i,j) \in N$, are calculated from the price scenarios through an optimization process. The result is a Markov chain traversing the price nodes $p_{ij}$ from $t = 0$ to $t = T$. For further details, we refer the reader to [9].

Separate cuts are calculated for each node and time step in the Markov chain. To reduce calculation times the number of price nodes in one time step is typically limited to less than 10 (default 7). The short-term model takes the current power price (along with a deterministic prediction for the modelling period) as input, which usually ends up being somewhere in between two price levels.

In order to calculate the future cost information, the long-term model uses a combination of stochastic dynamic programming (SDP) and stochastic dual dynamic programming (SDDP) in an iterative approach to reach an optimal strategy for resource management. The iteration consists of a calculation backwards in time and a simulation forwards in time. When the future cost as calculated both backward and forward converge, the optimal solution is found. In each backwards phase, cuts are generated. These state descriptions limit (cut) the total state space for the optimization problem, thus building an increasingly detailed description of the future. A typical value for $R$ is 500, and ProdRisk
optimises for the node prices in the price model, so the total number of cuts available in the long-term model is in the thousands, of which only a handful were selected to be used as input to the short-term model. A cut in itself can consist of any valid values bounded by \( \pm \infty \) and any constraints in the optimization problem and only describes a possible state; only when it is binding in the optimization problem can it become a realization and represent water values.

The new coupling method proposed consists in essence of sharing the entire future-cost hyperplane description\(^1\) between the models. The information is readily available as it is essential/central to the SDDP solution algorithm, so the only cost of the method is the increased amount of data used in the coupling. This data overhead is very small relative to the gain which has several aspects.

**More cuts** are shared. This means a better description of the future cost state space and should result in a better optimisation in the short-term model. Instead of selecting in the long-term model which power price you expect to run the short-term model for, **cuts for all prices** in the long-term model are available to the short-term model thus providing a tighter price-coupling. This also allows for the short-term model to be run for more sets of input parameters without re-running the long-term model. This is important in an operational setting in the power markets.

Finally, there is more information per cut through inflow correlation. In the old method, the shared cuts were “corrected” so that the inflow information in them were referenced to zero auto-correlation because the long-term model has no information on how the short-term model was to be run. When the full cut description is shared, the short-term model can couple inflow series (which is input to both models) to specific reservoirs in the cut description and correct for the specific inflow at the time of optimisation (the short-term model has an updated inflow description as input).

To sum up, extended cut sharing between the long-term and short-term models includes more information with finer detail and correlations in both inflow and price.

### 3. Results

The models are run on a medium-sized Norwegian watercourse comprising of 16 reservoirs and 8 power plants.

Results are obtained by running the long-term model once to generate the coupling information in the old scheme and once to generate the new coupling information. Then, the short-term model is run several times either using the full description from the long-term model or using the old single-price description. Using the old coupling, the short-term model is first run using the price data from the long-term model and then re-run with this data scaled some percentage up and down to achieve results for a spread around the mean value. This one way of creating bid curves for the power markets. The new coupling provides this spread by default with the correlation between price and water values inherent in the cuts and the short-term model is run for the same prices as for the old coupling, interpolating between prices when needed. The difference for the short-term model is then not in the input price or inflow, but the relevance of the information coupling between value of water and reservoir levels. The new method uses values from the long-term optimization where the old method used extrapolation from a single optimization.

Fig. 2 shows the new and old cut information shared between the models. As previously stated, more information is shared, resulting in a broader coverage of the optimisation state space available to the short-term model which should result in a more accurate overall description as the price varies. The end result should be a more accurate production-price curve which could be used for marked bids. Note that the coverage of possible water values is asymmetric, with a much denser coverage for lower values. This asymmetry should lead to improved results in/from the short-term model for lower prices, because the upper region already had some coverage with the old cut information.

Extracting the sum production for the hydropower system at hand, we find the aforementioned asymmetry in the sum production curve, Fig. 3. The figure shows the sum of production in the system for a spread in the estimated market price. The sum production changes less with the new cut coupling due to the correlation between expected price and expected water value. In essence, low correlation in price and water value leads to higher correlation in price and production. A higher correlation in price and water value indicates a system which responds better to market signals and yields a flatter production curve, which is desirable with power producers. The new coupling

\(^1\)Technically plural as the power price is not part of (4) so there is one description per price level.
results in a production curve more in tune with variations in the price because of the price information being part of the complete cut description. The effect is greatest for lower prices, as anticipated. Remember that Fig. 2 indicated the coverage of available cut information, it does not show which cuts are binding and thus water values can not be inferred from this figure. Fig. 3 shows the result of the realized water values.

4. Conclusions

We have presented a new method for coupling long-term and short-term hydropower-system optimisation models. The method utilises more information which includes correlations in price and inflow in the transferred water values. This first study demonstrates changes of about 10% in suggested production from the short-term model when compared to using the old approach. The change is expected direction, but we have not studied the optimality of the effect. The analysis performed has so far only examined the effect of the additional price coupling. We expect also an improvement when using the additional inflow coupling available through the new approach, but this has not been studied as of yet.
Acknowledgements and authors affiliations

The authors are currently employed at SINTEF Energy Research and work on the scheduling models used as background for this paper. Those models are owned by SINTEF Energy Research. SINTEF Energy Research is a private non-profit research organisation. The development work leading to this paper was funded by the hydropower producers Hydro, Agder Energi and E-CO Energi.

References


