ABSTRACT: The existence of a no-flow boundary is important for reservoir management. This is because fluid cannot flow across a sealing boundary. In many cases no-flow boundaries do not show up on seismic. Sub-seismic faults may be detected by well testing.

This study deals with mathematical modeling of interference tests in fractured reservoirs. The objective is to help locate sub-seismic faults to improve reservoir management. We concentrate on fractured networks of fractal type. An interference test involves at least two wells, an active well and an observation well. The latter is idle. The resulting mathematical model includes the single well responses as a special case.

An equation to predict the pressure behavior of interference and single well tests in the presence of a no-flow boundary has been derived. Two extreme cases were identified. These depend on the distance between the active well and the fault which could be infinitely large (case 1) or zero (case 2). All other cases fall in between these.

It was found that the influence of a now-flow boundary shows up as a transition between the two limiting curves. Initially the pressure response will follow the curve of case 1. After the transition the pressure signature will merge with curve two. A larger distance leads to later transition.

KEYWORDS: Well testing, fractured reservoir, fractal reservoir.

1. Introduction

We concentrate on fracture networks of fractal behavior. The fractured reservoir is without secondary porosity. Such reservoirs are characterized by fracture networks of different architecture. Well test interpretation may provide important input to the geological model of the reservoir. This is because pressure transient data reflect reservoir dynamics. In case of an interference test, the entire volume in between wells is sampled. The traditional interpretation methodology is type curve matching. A type curve is a graph of the solution to the dimensionless flow equation in a log-log coordinate system. If the right model has been obtained, then the well test response and the type curve are characterized by the same shape. In fact, if one is put on top the other, the two curves will overlap. This property is the basis of classical type curve analysis, Earlougher (1977).

Chang and Yortsos (1990) proposed a one-phase model of fluid flow in fractal networks. Their solution is for single well tests. Beier (1990) applied the Chang and Yortsos fractal model to a non-fractured but disordered inhomogeneous reservoir. Aprillan et al. (1993)
studied the pressure behavior of interference tests. Their methodology was based on a similarity solution and did not accommodate wellbore storage, skin or possible faults.

2. Methodology

We assume an infinite reservoir and constant production in the active well. The Laplace space solution of the governing equation, see Chang and Yortsos (1990), was obtained by classical theory of differential equations. This in turn was transformed back to time domain by a numerical technique, Stehfest (1970).

Much has been written about the behavior of fractals. The Society of Petroleum Engineers (SPE) data base has been searched for a solution to the problem of an interference test with wellbore storage and skin at the active well. We were unable to locate one. Hence we decided to derive a solution rather than search additional databases. The effect of a no-flow boundary was generated by the method of images.

Sandal et al. (1978) found the pressure in an observation well due to a production well with wellbore storage and skin for a homogeneous reservoir. The same methodology is applicable to a fractal reservoir. The resulting equation for the Laplace transform of pressure behavior in the observation well is:

\[
\bar{p}_{D}(r_{D}) = \frac{r_{D}^{\nu(1-\delta)}K_{v}(\xi r_{D}^{\nu(1-\delta)})}{s\left(C_{D}s\left[K_{v}(\xi) + S\sqrt{s}K_{v-1}(\xi)\right] + \sqrt{s}K_{v-1}(\xi)\right)} \tag{1}
\]

and in the production well:

\[
-\bar{p}_{wD} = \frac{K_{v}(\xi) + S\sqrt{s}K_{v-1}(\xi)}{s\left(C_{D}s\left[K_{v}(\xi) + S\sqrt{s}K_{v-1}(\xi)\right] + \sqrt{s}K_{v-1}(\xi)\right)} \tag{2}
\]

\[
\nu = 1 - \delta \tag{3}
\]

\[
\xi = \left[2/(\theta + 2)\right]\sqrt{s} \tag{4}
\]

\[
\Phi = (2 + \theta)/2 \tag{5}
\]

\[
\delta = \frac{D}{(\theta + 2)} \tag{6}
\]
where: \( C_D \) is wellbore storage constant  
D is fractal dimension  
d is Eucledian dimension  
S is skin  
K is modified Bessel function of second kind, of any real order \( \nu \)  
\( \theta \) is connectivity index  
r is the distance between wells  
s is Laplace variable

The dimensionless variables are

\[
\frac{t_D}{r_D^{2+2d}} = \frac{k(r)r_D^{2+2d}}{\varphi(r)\mu r_w r_{obs}^2} \gamma(t) \tag{7}
\]

\[
r_D = \frac{r}{r_w} \tag{8}
\]

\[
p_{D_{obs}} = \frac{k_w G_r^{d-2}}{q\mu} (p_i - p_{qf}) \tag{9}
\]

\[
p_D(r_D) = \frac{k_w G_r^{d-2}}{q\mu} (p_i - p(r_D)) = \frac{k(r) G_r^{d-2} r_D^{d+\theta-D}}{q\mu} (p_i - p(r_D)) \tag{10}
\]

Following Aprillan (1990), we define

\[
TDF = \frac{t_D}{r_D^{2+2d}} = \frac{k(r_{obs})}{\varphi(r_{obs})\mu c r_{obs}^2} \gamma(t) \tag{11}
\]

\[
PDF = \frac{p_{D_{obs}}(r_D)}{r_D^{d+\theta-D}} = \frac{k(r_{obs}) G_r^{d-2}}{q\mu} (p_i - p_{obs}) \tag{12}
\]

\( k_w \) is the permeability around the active well and \( k(r) \) is the permeability at a distance \( r \) from the active well. Well test interpretation may be carried out by type curve matching as explained by Earlougher (1977).

d=1, 2 and 3 corresponds to flow in a Cartesian, radial and spherical coordinate system respectively. The constant \( G \) assumes different values depending on the Eucledian dimension:  
\( d = 1 \rightarrow G = A, \quad d = 2 \rightarrow G = 2\pi h \quad \text{and} \quad d = 3 \rightarrow G = 4\pi \).

The distance between the active well and the observation well is:

\[
r_{obs} = \sqrt{(\Delta x_{obs})^2 + (\Delta y_{obs})^2} \tag{13}
\]
No-flow boundaries were simulated by the method of images. Suppose a no-flow fault is located at a distance $\Delta y_F$ away from the active well. The fault is of infinite extension and runs parallel to the x-axis. The physical boundary may be replaced by an image well at the same distance, $\Delta y_F$, behind the fault. Interference between the real and fictitious (image) well will create a no-flow boundary at the position of the fault. The total response of the two wells may be calculated by superposition. This is because of the linearity of the governing equations. The distance between the observation well and image well is:

$$r_{img} = \sqrt{\Delta x^2_{obs} + (\Delta y_{img} \pm \Delta y_{obs})^2}$$

(14)

where $\Delta y_{img} = 2\Delta y_F$. The sign depends on which well, active or observation well, has the shortest distance to the fault. It is positive when the active well is more close to the fault than the observation well and vice versa.

The pressure drop in the production well in the presence of a fault is:

$$p_{Dobs}(t_D) = p_D(r_{Dobs}, t_D) + p_D(r_{Dimg}, t_D)$$

(15)

3. Results

The effect of the distance to the boundary on the pressure behavior is shown in Fig. 1 and on the logarithmic pressure derivative in Fig. 2. A dimensionless distance of 2000 corresponds to 200 m for a petroleum reservoir. This is because the radius of the wellbore is around 10 cm. For a shallower reservoir the wellbore radius could be greater. We find that the effect of the wellbore storage constant and skin is negligible when the dimensionless distance between the active and observation well, $r_{Dobs}$, is 2000. As expected the effect of the fault shows up earlier with decreasing distance, $r_{Dimg}$.

<table>
<thead>
<tr>
<th>Table 1: Input data: Effect of the distance to a fault</th>
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<tbody>
<tr>
<td><strong>Run</strong></td>
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<tr>
<td>$r_{Dimg}$</td>
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<td>$d$</td>
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<td>$D$</td>
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<td>$S$</td>
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<td>$C_0$</td>
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4. Discussion

The fractal model implies the assumption of an inhomogeneous reservoir. The rock properties depend on the spatial coordinate in the form of power law expressions. In most cases the rock properties will be decreasing functions of the distance away from the active well. This means that the permeability and porosity at the observation well will be different as seen from the active well and the image well. This is because the two wells are located at different distances away from the observation well. Then one could question the validity of the superposition by simple addition of pd-functions. Apparently pd- and td- functions (for the two wells) are scaled different ways since they are based on rock properties evaluated at different distances away from the real and the imaginary image well. The primary dimensionless variables,
however, are scaled according to the rock properties at the active well, \( r_w \), see equations (7) and (10), left hand side. Hence direct addition of \( \Phi \)-functions is possible.

A negative skin factor may be accounted for by use the equivalent wellbore radius, \( r_{we} \). For cylindrical geometry, \( d=2 \), the relationship between equivalent and actual wellbore radius is given by:

\[
r_{we} = r_w e^{-s}
\]

Suppose the skin factor of a stimulated well is -3. Then the effective radius is 20 times larger than the radius drilled. Then the dimensionless distances to the observation well and the image well are reduced by the same factor. Under such conditions we find that that the wellbore storage factor and skin may have an influence on the pressure signature. The wellbore storage constant is more important than the skin for an interference test.

5. Conclusions and references

A numerical model to simulate the effect of faults in a fractal reservoir has been implemented. The numerical model depends on the Laplace space solution to the governing equation. The solution was converted back to time domain by a numerical technique.

There are two limiting curves for a reservoir with a fault. The lower one is characterized by no fault at all or equivalently a fault at an infinite distance. The upper one is characterized by a fault in the immediate neighborhood of the active well, \( \Delta y_F = 0 \). When a fault is placed anywhere in between the limiting positions, the response is characterized by a transition from the lower to the upper curve. The start of the transition period is delayed with increasing distance, \( \Delta y_F \). Hence there is a possibility to estimate the distance to a sealing fault.

The wellbore storage factor and skin of the active well may have an effect on the pressure signature in the observation well in case the dimensionless distance between the two wells is in the order of 10. This situation is likely to occur if the active well has been stimulated.

REFERENCES


