Sequential Investment in Emerging Technologies under Policy Uncertainty

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Abstract

Investment in emerging technologies is particularly challenging, since, apart from uncertainty in revenue streams, firms must also take into account both policy uncertainty and the random arrival of innovations. We assume that the former is reflected in the sudden provision and retraction of a support scheme, which takes the form of a fixed premium on top of the output price. Thus, we develop an analytical framework for sequential investment in order to determine how price, technological, and policy uncertainty interact to affect the decision to invest sequentially in successively improved versions of an emerging technology. We show that greater likelihood of subsidy retraction lowers the incentive to invest, whereas greater likelihood of subsidy provision facilitates investment. However, embedded options to invest in improved technology versions raise the value of the investment opportunity, thereby mitigating the impact of subsidy retraction and making the impact of subsidy provision more pronounced. Additionally, by allowing for sequential policy interventions, we find that the impact of policy uncertainty becomes less pronounced as the number of policy interventions increases.

Keywords: investment analysis, real options, policy uncertainty, technological uncertainty
1. Introduction

Investment in emerging technologies is typically made in the light of technological uncertainty, which is often reflected in the random arrival of innovations. Consequently, within an environment of increasing economic uncertainty, the viability of private firms depends crucially on which technology they adopt and when. For example, subsidies for renewable energy (RE) technologies fuelled a boom in solar panel manufacturing in China and allowed solar production capacity to increase significantly. Combined with the decrease in the price of silicon, the main component of traditional solar panels, this reduced the competitive advantage of US companies, many of which either went bankrupt or were purchased by Chinese companies (The New York Times, 2013). While various papers analyse how investment in technological innovations is affected by price and technological uncertainty (Grenadier & Weiss, 1997; Chronopoulos & Siddiqui, 2015), insights on the interaction of these features with policy uncertainty are not equally developed. In fact, in most cases, insights are based on numerical or simulation methods, which are crucial for studying more complex settings, but do not allow for analytical tractability. However, the latter is necessary for understanding the implications of policy uncertainty for investment, for example, why the incentive to either accelerate or postpone investment increases as the likelihood of subsidy retraction increases depending on the specifications of a model (Adkins & Paxson, 2015; Boomsma & Linnerud, 2015). In turn, this will also enable a better understanding of any implications resulting from the potential to invest sequentially in more efficient technologies that become available at random points in time. Hence, incorporating technological and policy uncertainty in an analytical framework for sequential investment is crucial in understanding the optimal investment policy in sectors characterized by high R&D activity.

Indeed, although emerging technologies often enjoy government support, the absence of a clear policy framework, which is frequently reflected in the sudden provision or retraction of a support scheme, discourages investment decisions. For example, although promises of 10% annual returns boosted the Spanish solar industry in 2008, the subsequent reduction of subsidies at different points in time increased producers’ reluctance to commit to future investments (The Economist, 2013). Similarly, although Siemens had decided to invest 160 million in offshore wind turbines, it subsequently required that policy uncertainty is resolved and that the UK Government maintained its commitments to RE subsidies (Financial Times, 2015). Furthermore, empirical research based on small hydropower projects has indicated that uncertainty regarding future subsidy provision increases the incentive to postpone investment. In fact, even promises to include existing projects in a prospective support scheme may not be as successful in promoting investment decisions as
policymakers may expect (Linnerud et al., 2014).

Despite recent attempts to incorporate policy uncertainty within real options models, insights involving the combined impact of price, technological, and policy uncertainty are limited, as these features are frequently analysed in isolation. We address this disconnect by incorporating these features in a real options framework for sequential investment in technological innovations. Thus, we provide insights not only on how price, policy, and technological uncertainty interact to affect the optimal investment policy, but also on how policymakers can devise more efficient policy mechanisms in order to incentivise investment in emerging technologies. The scope of our model does not include the option to choose between alternative projects (Grenadier & Weiss, 1997; Chronopoulos & Siddiqui, 2015), but emphasises on how price, policy, and technological uncertainty interact to affect sequential investment decisions. Our results indicate that greater likelihood of subsidy retraction (provision) postpones (accelerates) investment, while increasing number of policy interventions lower the impact of policy uncertainty on the propensity to invest. Additionally, the option to invest sequentially in improved versions of a technology raises the value of the investment opportunity, and, thus, may either mitigate the impact of policy uncertainty or make it more pronounced. These results have important implications for the current policymaking process in many countries that seek to stimulate investment in RE power plants. Indeed, many countries implement a variety of policy interventions and selective support schemes, without taking into account particular features of investment projects or considering that private companies may act more cautiously in the light of the uncertainties emerging from frequent switches between policy regimes.

We proceed by discussing some related work in Section 2 and introduce assumptions and notation in Section 3. In Section 4.1, we address the problem of optimal investment timing taking into account only price and technological uncertainty. We introduce policy uncertainty in Section 4.2 and 4.3 in the form of sudden retraction and provision of a subsidy, respectively. In Section 4.4, we allow for the sudden provision of a retractable subsidy, and, in Section 4.5 we allow for infinite provisions and retractions. Section 5 presents numerical results for each case, while Section 6 concludes the paper and offers directions for further research.

2. Related Work

The seminal work of McDonald & Siegel (1985) and Dixit & Pindyck (1994) has spawned a substantial literature in the area of investment under uncertainty. A strand of this literature illustrates the amenability of real options theory to emerging technologies and the energy sector (Schwartz and Zozaya-Gorostiza 2003; Rothwell, 2006; Siddiqui & Fleten, 2010; Lemoine, 2010). Nevertheless, analytical formulations of problems that address investment in RE projects typically
do not combine crucial features such as price, policy, and technological uncertainty. Indeed, most of this literature either addresses the impact of technological uncertainty on investment decisions ignoring the implications of policy uncertainty (Majd & Pindyck, 1987; Schwartz and Zozaya-Gorostiza, 2003) or allows for policy uncertainty without taking into account the sequential nature of investment in emerging technologies (Boomsma et al., 2012; Adkins & Paxson, 2015). Consequently, models that incorporate price, technological, and policy uncertainty in analytical frameworks for sequential investment in technological innovations remain somewhat underdeveloped.

In the area of investment under policy uncertainty, Boomsma et al. (2012) develop a real options model in order to investigate how investment behavior is affected by regulatory uncertainty as well as changes of support scheme. They show that the value of an investment opportunity under policy uncertainty is greater than under RE certificate trading, which is higher than under a premium feed-in-tariffs. In the same line of work, Boomsma & Linnerud (2015) find that the prospect of subsidy retraction increases the rate of investment if it is applied to new projects, while it slows down investment if it has a retroactive effect. Adkins & Paxson (2015) develop an analytical model for investment under price, quantity, and policy uncertainty. The latter is reflected in the random provision and retraction of a subsidy, which takes the form of a fixed premium on quantity. Their results indicate that the prospect of a permanent subsidy retraction (provision) facilitates (postpones) investment. Additionally, they find that the value of the option to invest increases as the correlation between the price of electricity and quantity of electricity produced increases, since this raises the aggregate volatility. Chronopoulos et al. (2016) ignore quantity uncertainty, yet allow for discretion over capacity and sequential policy interventions. They find that the greater likelihood of a subsidy retraction may facilitate investment, yet results in smaller projects. Although these papers address the impact of policy uncertainty on investment timing and capacity sizing decisions, they ignore the implications of technological uncertainty and how sequential investment opportunities may impact the optimal investment policy.

Examples of analytical frameworks for sequential investment under uncertainty include Majd & Pindyck (1987), who show how traditional valuation methods understate the value of a project by ignoring the flexibility embedded in the time to build. Dixit & Pindyck (1994) develop a model for sequential investment assuming that the value of the project depreciates exponentially and that the investor has an infinite number of investment option. In the same line of work, Gollier et al. (2005) compare a sequence of small nuclear power plants with a single nuclear power plant of large capacity. Their results indicate that the value of modularity may even trigger investment in the initial module at an electricity price level below the now-or-never net present value (NPV) threshold. By comparing a lumpy to a stepwise investment strategy, Kort et al. (2010) show that
higher price uncertainty raises the attractiveness of the former by increasing the reluctance to make costly switches between different stages.

Allowing for technological uncertainty, Balcer & Lippman (1984) find that the optimal timing of technology adoption under infinite switching options is influenced by expectations about future technological changes and that increasing technological uncertainty tends to delay adoption. Grenadier & Weiss (1997) develop a model for sequential investment in order to study how the innovation rate and technological growth impact the optimal technology adoption strategy, and find that a firm may adopt an available technology even though more valuable innovations may occur in the future. Farzin et al. (1998) assume that technological innovations follow a Poisson process and find that the NPV rule can be used as an investment criterion in most cases. By contrast, Doraszelski (2001) identifies an error in Farzin et al. (1998) and shows that a firm will always defer investment when it takes the value of waiting into account. Huisman & Kort (2004) analyze how technological uncertainty impacts the competitive equilibrium and find that when technological uncertainty becomes sufficiently large, the competition changes from a preemption game into a war of attrition. Chronopoulos & Siddiqui (2015) develop an analytical framework for sequential investment and analyze how the endogenous relationship between price and technological uncertainty impacts the optimal technology adoption strategy and the associated investment rule. While these papers present a comprehensive modeling of investment in technological innovations, they ignore the implications of policy and technological uncertainty for sequential investment.

In this paper, we develop a real options framework for sequential investment under price, policy, and technological uncertainty. In line with Adkins & Paxson (2015), we assume that the output price follows a geometric Brownian motion, while technological innovations and policy uncertainty follow independent Poisson processes that are not affected by price uncertainty. Nevertheless, our results deviate from those of Adkins & Paxson (2015), since we show that greater likelihood of subsidy retraction lowers the incentive to invest (Boomsma et al., 2015), whereas greater likelihood of subsidy provision facilitates investment. This happens because, like Boomsma et al. (2015), we assume that the impact of policy uncertainty on the value of the project is governed by an exponential distribution without being subject to a linear approximation, and, therefore, is more pronounced. Additionally, we find that, although an embedded option to invest in a more efficient technology may mitigate the impact of policy uncertainty in the case of sudden subsidy retraction, in the case of subsidy provision, the opportunity for sequential investment makes the impact of policy uncertainty more pronounced. Finally we find that, under infinite provisions and retractions, the impact of policy uncertainty is less pronounced and diminishes when the rate of policy interventions increases.
3. Assumptions and Notation

We consider a price-taking firm with a perpetual option to invest in \( n = 1, 2 \) successively improved versions of a technology, each with infinite lifetime, facing price, technological, and policy uncertainty. Given a probability space \((\Omega, \mathcal{F}, \mathbb{P})\), we assume that technological and policy uncertainty follow independent Poisson processes, \( \{M_i(t), t \geq 0\} \), where \( \lambda_i \geq 0 \) denotes the intensity of the Poisson process, \( t \) is continuous and denotes time, and \( i = \{\tau, p\} \) (denoting technological and policy uncertainty, respectively). Intuitively, \( M_i(t) \) counts the number of random times \( y_m, m \in \mathbb{N} \) that occur between 0 and \( t \), and \( T_m = y_m - y_{m-1} \) is the time interval between subsequent Poisson events. Furthermore, we assume that there is no operating cost associated with each technology and that the output price at time \( t \), \( E_t \), is independent of \( M_i(t) \) and follows a GBM, which is described in (1). We denote by \( \mu \) the annual growth rate, by \( \sigma \) the annual volatility, by \( dZ_t \) the increment of the standard Brownian motion, and by \( \rho \geq \mu \) the subjective discount rate.

\[
dE_t = \mu E_t dt + \sigma E_t dZ_t, \quad E_0 \equiv E > 0 \tag{1}
\]

We also denote the output of technology version \( n \) by \( D_n \) \((D_2 \geq D_1)\) and the corresponding investment cost by \( I_n \). We let \( a = 0, 1 \) denote the presence \( (a = 1) \) or absence \( (a = 0) \) of a subsidy that can be provided and retracted \( b \) and \( c \) times, respectively. Thus, the time of investment in technology version \( n \) is denoted by \( \tau_{b,c}^{n,a} \), while \( \varepsilon_{b,c}^{n,a} \) is the corresponding optimal investment threshold. For example, under sudden provision of a permanent subsidy, the optimal time to invest in the second technology is \( \tau_{1,0}^{2,0} \), while the corresponding optimal investment threshold is \( \varepsilon_{1,0}^{2,0} \). Finally, \( F_{b,c}^{n,a}(\cdot) \) is the maximised expected NPV from investing in technology \( n \), while \( \Phi_{b,c}^{n,a}(\cdot) \) is the expected value (NPV) of the active project inclusive of embedded options.

The firm’s value function at different states of operation is indicated in Figure 1 and is determined via backward induction. Therefore, we assume initially that the firm is operating the second technology, and, thus, holds the value function \( \Phi_{b,c}^{2,0}(E) \). Prior to the adoption of the second technology, the firm is operating the first one holding a single embedded investment option, \( F_{2,0}^{b,c}(E) \), which the firm will exercise at time \( \tau_{2,0}^{b,c} \) in order to obtain the value function \( \Phi_{2,0}^{b,c}(E) \). Before the arrival of the second technology, the firm holds the value function \( \Phi_{b,c}^{1,0}(E) \), which consists of the expected value from operating the first technology and the embedded option to invest in the second one, that has yet to become available. Finally, before time \( \tau_{b,c}^{1,0} \) the firm holds an option to invest in the first technology, \( F_{1,0}^{b,c}(E) \), with a single embedded option to invest in the second, that has yet to become available.
4. Analytical Results

4.1. Benchmark Case: Investment without Policy Uncertainty

We assume that a firm has the option to invest in each technology facing only price and technological uncertainty. First, we assume that the firm is already operating the first technology and holds a single embedded option to invest in the second one. The expected value of the revenues from operating the second technology net of investment expenses is indicated in (2), where the first term on the right-hand side is the expected revenues while the second term is the total investment cost, which includes the cost of investment in the first technology.

\[ \Phi_{b,c}^{1,a}(E) = D_2 E (1 + ay) \rho - \mu - I_1 - I_2 \]  

Next, the value of the option to invest in the second technology is indicated in (3). The first two terms on the top part of (3) reflect the expected value of the profits from operating the first technology, while the third term represents the option to invest in the second one. The bottom part of (3) is the expected profits from operating the second technology and \( \beta_1 > 1 \) is the positive root of the quadratic \( \frac{1}{2} \sigma^2 \beta (\beta - 1) + \mu \beta - \rho = 0 \) (all proofs can be found the appendix).

\[ F_{b,c}^{0,0}(E) = \begin{cases} \frac{D_1 E (1 + ay)}{\rho - \mu} - I_1 + A_{2,a}^{0,0} E^{\beta_1}, & E < \varepsilon_{2,a}^{0,0} \\ \Phi_{2,a}^{0,0}(E), & E \geq \varepsilon_{2,a}^{0,0} \end{cases} \]  

The optimal investment threshold, \( \varepsilon_{2,a}^{0,0} \), and the endogenous constant, \( A_{2,a}^{0,0} \), are obtained analytically by applying value-matching and smooth-pasting conditions to the two branches of (3) and are indicated in (4).

\[ \varepsilon_{2,a}^{0,0} = \frac{(\rho - \mu) \beta_1 I_2}{(\beta_1 - 1)(D_2 - D_1)(1 + ay)} \quad \text{and} \quad A_{2,a}^{0,0} = \left( \frac{1}{\varepsilon_{2,a}^{0,0}} \right)^{\beta_1} \left( \frac{(D_2 - D_1)(1 + ay) \varepsilon_{2,a}^{0,0}}{\rho - \mu} - I_2 \right) \]  

Next, we assume that the firm is operating the first technology holding an embedded option to adopt the second, which has yet to become available. The dynamics of the value function \( \Phi_{1,a}^{0,0}(E) \) are described in (5), where \( E_E \) denotes the expectation operator that is conditional on the initial output price \( E \). The first term on the right-hand side of (5) represents the immediate
profit from operating the first technology. As the second term indicates, with probability $\lambda_r dt$ the second technology will arrive and the firm will receive the value function $F_{2,a}^{0,0}(E)$, whereas, with probability $1 - \lambda_r dt$, no innovation will occur and the firm will continue to hold the value function $\Phi_{1,a}^{0,0}(E)$.

$$\Phi_{1,a}^{0,0}(E) = [D_1 E (1 + ay) - \rho I_1] dt + (1 - \rho dt) \left\{ \lambda_r dt \mathbb{E}_E \left[ F_{2,a}^{0,0}(E + dE) \right] + (1 - \lambda_r dt) \mathbb{E}_E \left[ \Phi_{1,a}^{0,0}(E + dE) \right] \right\}$$  \hspace{1cm} (5)

By expanding the right-hand side of (5) using Itô’s lemma, we can rewrite (5) as in (6), where $A_{1,a}^{0,0} \leq 0$ and $B_{1,a}^{0,0} \geq 0$ are determined analytically via value-matching and smooth-pasting conditions between the two branches and $\delta_1 > 1, \delta_2 < 0$ are the roots of the quadratic $\frac{1}{2} \sigma^2 \delta (\delta - 1) + \mu \delta - (\rho + \lambda_r) = 0$. The first two terms on the top part of (6) represent the expected profit from operating the first technology, while the third term is the option to invest in the second technology, adjusted via the fourth term because the second technology has yet to become available. The first three terms on the bottom part of (6) represent the expected profit from operating the second technology and the fourth term the likelihood of the price dropping in the waiting region prior to the arrival of an innovation.

$$\Phi_{1,a}^{0,0}(E) = \begin{cases} \frac{D_1 E (1+ay)}{\rho - \mu} - I_1 + A_{2,a}^{0,0} E^{\beta_1} + A_{1,a}^{0,0} E^{\delta_1}, & E < \varepsilon_{1,a}^{0,0} \\ \frac{\lambda_r D_2 (\rho - \mu) D_1 (1+ay)}{(\rho - \mu)(\rho - \mu + \lambda_r)} - \frac{\lambda_r I_2}{\lambda_r + \rho} - I_1 + B_{1,a}^{0,0} E^{\delta_2}, & E \geq \varepsilon_{1,a}^{0,0} \end{cases}$$  \hspace{1cm} (6)

Finally, the value function $F_{1,a}^{0,0}(E)$ is indicated in (7), where the optimal investment threshold, $\varepsilon_{1,a}^{0,0}$, and the endogenous constant, $C_{1,a}^{0,0} \geq 0$, are determined numerically via value-matching and smooth-pasting conditions between the two branches. The top part on the right-hand side of (7) is the value of the option to invest, while the bottom part is the expected value from operating the first technology inclusive of the embedded option to invest in the second.

$$F_{1,a}^{0,0}(E) = \begin{cases} C_{1,a}^{0,0} E^{\beta_1}, & E < \varepsilon_{1,a}^{0,0} \\ \Phi_{1,a}^{0,0}(E), & E \geq \varepsilon_{1,a}^{0,0} \end{cases}$$  \hspace{1cm} (7)

4.2. Permanent Subsidy Retraction

We extend the previous framework by assuming that a subsidy is available and that it may be retracted permanently at a random point in time. If the subsidy lasts exactly $T_1$ years, then the expected value of the revenues of the project is $\mathbb{E}_E \left[ \int_0^\infty e^{-\rho t} D_2 E_t dt + \int_0^{T_1} e^{-\rho t} D_2 E_t y dt \right] = \frac{D_2 E}{\rho - \mu} + \frac{D_2 E_y [1 - e^{-\rho (\mu + T_1)}]}{\rho - \mu}$. Since $T_1 \sim \exp(\lambda_p)$, evaluating the expectation of this expression with respect to $T_1$ and subtracting the investment cost we obtain (8). Notice that the subsidy will never
be retracted if $\lambda_p = 0$, while greater $\lambda_p$ raises the likelihood of subsidy retraction and lowers the expected NPV of the project.

\[
\Phi_{2,1}^{0,1}(E) = \frac{D_2E}{\rho - \mu} + \int_0^\infty \lambda_pe^{-\lambda_pT_1} \frac{D_2Ey}{\rho - \mu} \left[1 - e^{-(\rho - \mu)T_1}\right] dT_1 - (I_1 + I_2)
\]
\[
= \frac{D_2E}{\rho - \mu} + \frac{D_2Ey}{\rho + \lambda_p - \mu} - (I_1 + I_2)
\]
(8)

Next, we assume that the firm is operating the first technology and holds a single embedded option to invest in the second. The dynamics of the firm’s value function are described in (9), where the first term on the right-hand side reflects the immediate profit from operating the first technology. As the second term indicates, the option to invest in the second technology will be exercised in the permanent absence of a subsidy with probability $\lambda_p dt$, whereas, with probability $1 - \lambda_p dt$, no policy intervention will take place and the firm will continue to hold the option to invest in the second technology in the presence of a retractable subsidy.

\[
F_{2,1}^{0,1}(E) = [D_1E(1 + y) - \rho I_1] dt + (1 - \rho dt) \left\{ \lambda_p dt \mathbb{E}_E \left[F_{2,0}^{0,0}(E + dE)\right] + (1 - \lambda_p dt) \mathbb{E}_E \left[F_{2,1}^{0,1}(E + dE)\right]\right\}
\]
(9)

By expanding the right-hand side of (9) using Itô’s lemma and solving the resulting ordinary differential equation, we obtain (10), where $\varepsilon_{2,1}^{0,1}$ and $A_{2,1}^{0,1} \geq 0$ are determined via value-matching and smooth-pasting conditions, while $\eta_1 > 1$, $\eta_2 < 0$ are the roots of the quadratic $\frac{1}{2} \sigma^2 \eta(\eta - 1) + \mu \eta - (\rho + \lambda_p) = 0$. The first three terms in the top part of (10) represent the expected profit from operating the first technology. The fourth term is the option to upgrade to the second one in the absence of a subsidy, adjusted via the fifth term since the subsidy is currently available.

\[
F_{2,1}^{0,1}(E) = \begin{cases} 
\frac{D_2E}{\rho - \mu} + \frac{D_1Ey}{\rho - \mu + \lambda_p} - I_1 + A_{2,0}^{0,0}E^\beta_1 + A_{2,1}^{0,1}E^{\eta_1}, & E < \varepsilon_{2,1}^{0,1} \\
\Phi_{2,1}^{0,0}(E), & E \geq \varepsilon_{2,1}^{0,1}
\end{cases}
\]
(10)

Next, we step back and assume that an innovation has not taken place yet, but may occur over an infinitesimal time interval $dt$ with probability $\lambda_r dt$. The dynamics of the value function $\Phi_{1,1}^{0,1}(E)$ are described in (11), where the first term on the right-hand side represents the immediate profit from operating the first technology version, while the second term reflects the expected value in the continuation region. Notice that if the subsidy is retracted with probability $\lambda_p dt$, then either an innovation will take place with probability $\lambda_r dt$ and the firm will receive the value function $F_{2,0}^{0,0}(E)$, or no innovation will take place with probability $1 - \lambda_r dt$ and the firm will continue to hold the value function $\Phi_{1,0}^{0,0}(E)$. Similarly, if no policy intervention occurs with probability $1 - \lambda_p dt$, then the firm will either receive the value function $F_{2,1}^{0,1}(E)$ with probability $\lambda_r dt$, or it will hold the value function $\Phi_{1,1}^{0,1}(E)$ with probability $1 - \lambda_r dt$.

\[
\Phi_{1,1}^{0,1}(E) = [D_1E(1 + y) - \rho I_1] dt + (1 - \rho dt) \left\{ \lambda_p dt \mathbb{E}_E \left[F_{2,0}^{0,0}(E + dE)\right] + (1 - \lambda_r dt) \times \mathbb{E}_E \left[F_{1,0}^{0,0}(E + dE)\right]\right\} + (1 - \lambda_p dt) \left( \lambda_r dt \mathbb{E}_E \left[F_{2,1}^{0,1}(E + dE)\right] + (1 - \lambda_r dt) \mathbb{E}_E \left[\Phi_{1,1}^{0,1}(E + dE)\right]\right\}
\]
(11)
The expression of $\Phi_{1,1}^{0,1}(E)$ is indicated in (12), where $A_{1,1}^{0,1} \leq 0$ and $B_{1,1}^{0,1} \leq 0$ are determined numerically via value-matching and smooth-pasting conditions, while $\theta_1 > 1$, $\theta_2 < 0$ are the roots of the quadratic $\frac{1}{2} \sigma^2 \theta (\theta - 1) + \mu \theta - (\rho + \lambda_p + \lambda_r) = 0$. The first three terms in the top part of (12) represent the expected profit from operating the first technology, while the fourth term is the option to invest in the second one without policy uncertainty, adjusted by the fifth term since the second technology has yet to become available. The two remaining option terms reflect the necessary adjustment due to policy uncertainty. Also, the first four terms in the bottom part of (12) represent the expected profit from operating the second technology, while the fifth term represents the likelihood of the price dropping in the waiting region before the arrival of the second technology, adjusted by the final term due to policy uncertainty.

$$
\Phi_{1,1}^{0,1}(E) = \begin{cases} 
\frac{\theta_1}{\rho - \mu} + \frac{D_1 E}{\rho - \mu + \lambda_p} - I_1 + A_{2,0}^{0,0} E^{\beta_1} + A_{1,0}^{0,0} E^{\beta_1} + A_{2,1}^{0,1} E^{\beta_1} + A_{1,1}^{0,1} E^{\beta_1} + \lambda_r I_2 E^{\beta_1} \left( \frac{\lambda_p + \lambda_r - \lambda_r}{\lambda_r + \rho} \right) - \frac{\lambda_r I_2}{\lambda_r + \rho} - I_1 + B_{1,0}^{0,0} E^{\beta_1} + B_{1,1}^{0,1} E^{\beta_1}, & E < \epsilon_{1,1}^{0,1} \\
\Phi_{1,1}^{0,1}(E), & E \geq \epsilon_{1,1}^{0,1} 
\end{cases}
$$

The dynamics of the option to invest in the first technology are described in (13). Notice that, over an infinitesimal time interval $dt$, either the subsidy will be retracted with probability $\lambda_p dt$ and the firm will receive the option to invest in the absence of a subsidy, or no policy intervention will take place with probability $1 - \lambda_p dt$ and the firm will continue to hold the value function $F_{1,1}^{0,1}(E)$.

$$
F_{1,1}^{0,1}(E) = (1 - \rho dt) \left\{ \lambda_p dt E^\mu \left[ F_{1,0}^{0,0}(E + dE) \right] + (1 - \lambda_p dt) E^\mu E \left[ F_{1,1}^{0,1}(E + dE) \right] \right\} 
$$

The expression of $F_{1,1}^{0,1}(E)$ is indicated in (14), where $\epsilon_{1,1}^{0,1}$ and $C_{1,1}^{0,1}$ can be obtained numerically via value-matching and smooth-pasting conditions. The first term in the top part of (14) is the option to invest in the absence of a subsidy, adjusted by the second term since the subsidy is currently available. The bottom part represents the expected value from operating the first technology inclusive of the embedded option to invest in the second one.

$$
F_{1,1}^{0,1}(E) = \begin{cases} 
C_{1,0}^{0,0} E^{\beta_1} + C_{1,1}^{0,1} E^{\eta_1}, & E < \epsilon_{1,1}^{0,1} \\
\Phi_{1,1}^{0,1}(E), & E \geq \epsilon_{1,1}^{0,1} 
\end{cases}
$$

Although $\epsilon_{1,1}^{0,1}$ and $C_{1,1}^{0,1}$ are obtained numerically, we can investigate the impact of $\lambda_p$ and $\lambda_r$ on the optimal investment rule by expressing $F_{1,1}^{0,1}(E)$ as in (15).

$$
F_{1,1}^{0,1}(E) = \left( \frac{E}{\epsilon_{1,1}^{0,1}} \right)^{\beta_1} \left[ \Phi_{1,1}^{0,1}(\epsilon_{1,1}^{0,1}) - C_{1,1}^{0,1} \epsilon_{1,1}^{0,1} \right], \quad E < \epsilon_{2,1}^{0,1} 
$$

The optimal investment rule is obtained by applying the first-order necessary condition (FONC) to (15) and is indicated in (16), where we equate the marginal benefit (MB) of delaying investment to the marginal cost (MC). The first two terms on the left-hand side consist of the stochastic discount.
factor multiplied by the incremental project value created by waiting until the price is higher. These terms are positive, decreasing functions of the output price, as waiting longer allows the project to start at a higher initial price, yet the rate at which this benefit accrues diminishes due to the effect of discounting. The third term represents the reduction in the MC of waiting due to saved investment cost. Similarly, the first two terms on the right-hand side reflect the opportunity cost of forgone cash flows discounted appropriately. The fourth and third term on the left- and right-hand side, respectively, reflect the loss in option value from not having the second version yet. Specifically, the fourth term on the left-hand side is the MB from postponing the loss in value, whereas the third term on the right-hand side is the MC from a potentially greater impact of the loss from waiting for a higher threshold price. The final three option terms on both sides are all corrections for policy risk in each state.

\[
\left( \frac{E}{\epsilon_{1,1}} \right)^{\beta_1} \left[ \frac{D_1}{\rho - \mu} + \frac{D_1 y}{\rho - \mu + \lambda_p} + \frac{\beta_1 I_1}{\epsilon_{1,1}} - \beta_1 A_{1,0}^{0.0,0,1,1,1} \delta_{1,1} - \beta_1 A_{1,1}^{0.1,0,1,1,1} \theta_{1,1} - \beta_1 C_{1,1}^{1,0,1,1,1} + \eta_1 A_{2,1}^{0.1,0,1,1,1} \right] \\
= \left( \frac{E}{\epsilon_{1,1}} \right)^{\beta_1} \left[ \frac{\beta_1 D_1}{\rho - \mu} + \frac{\beta_1 D_1 y}{\rho - \mu + \lambda_p} - \delta_{1,1} A_{1,0}^{0.0,0,1,1,1} - \theta_{1,1} A_{1,1}^{0.1,0,1,1,1} + \eta_1 c_{1,1}^{1,0,1,1,1} + \beta_1 A_{2,1}^{0.1,0,1,1,1} \right] (16)
\]

As shown in Proposition 1, greater likelihood of subsidy retraction lowers the MB by more than the MC, thereby raising the incentive to postpone investment. Intuitively, the incentive to invest early in order to take advantage of the subsidy for a longer period is mitigated by the rapid reduction in the value of the active project.

**Proposition 1.** Greater likelihood of subsidy retraction raises the optimal investment threshold.

The relative loss in option value due to subsidy retraction is \( \frac{F_{1,1}^{0.0,0.0}(E) - F_{1,1}^{0.1,1}(E)}{F_{1,1}^{0.0,0.0}(E)} \). If \( \lambda_p = 0 \), then the subsidy will never be retracted, and the relative loss in option value is zero. By contrast, as \( \lambda_p \) increases, the relative loss increases, since \( C_{1,1}^{1,0} E^{\nu_1} \to 0 \Rightarrow F_{1,1}^{0.1}(E) \to F_{1,1}^{0.0}(E) \), as shown in Proposition 2. Also, \( \frac{C_{1,1}^{0.0} - C_{1,1}^{1,0}}{C_{1,1}^{0.0}} < 1 \), which implies that the relative loss in option value will always be below one, since the firm can invest even in the absence of a subsidy.

**Proposition 2.** \( \frac{F_{1,1}^{0.0,0.0}(E) - F_{1,1}^{0.1,1}(E)}{F_{1,1}^{0.0,0.0}(E)} \in \left[ 0, 1 - \frac{1}{(1+y)^{\nu_1}} \right] \)

4.3. Provision of a Permanent Subsidy

As the increasing replacement of fossil-fuel with RE facilities may deteriorate the financial risk-return performance of incremental investments (Muñoz and Bunn, 2013), subsidies may be required to support green investments. Like in Section 4.2, we assume that there is a single policy intervention, and, therefore, we denote the random time at which it takes place by \( T_1 \). The expected NPV of the project if the subsidy is provided at time \( T_1 \) years is \( \mathbb{E}_E \left[ \int_0^\infty e^{-\rho t} D_2 E dt + \int_{T_1}^\infty e^{-\rho t} D_2 E dt dt \right] = \frac{D_2 E}{\rho - \mu} + \frac{D_2 E \left[ e^{-(\rho - \mu) T_1} \right]}{\rho - \mu} \), and since \( T_1 \sim \exp(\lambda_p) \), taking the expectation of this expression with respect
to $T_1$ we obtain (17).

$$\Phi_{2,0}^{1,0}(E) = \frac{D_2E}{\rho - \mu} + \frac{\lambda_y D_2Ey}{(\rho + \lambda_y - \rho - \mu)} - (I_1 + I_2)$$

The dynamics of the option to invest in the second technology are described in (18), where the first term on the right-hand side represents the instantaneous profit from operating the first technology. The second term indicates that, depending on the provision of a subsidy, the firm will receive either the value function $F_{2,0}^{0,0}(E)$ with probability $\lambda_y dt$, or $F_{2,0}^{1,0}(E)$ with probability $1 - \lambda_y dt$.

$$F_{2,0}^{1,0}(E) = \left[D_1E - \rho I_1\right] dt + (1 - \rho dt) \left\{ \lambda_y dt E \left[ F_{2,1}^{0,0}(E + dE) \right] + (1 - \lambda_y dt) E \left[ F_{2,0}^{1,0}(E + dE) \right] \right\}$$

The expression of $F_{2,0}^{1,0}(E)$ is indicated in (19), where $\varepsilon_{2,0}^{1,0}$, $A_{2,0}^{1,0}$, $B_{2,0}^{2,0}$, $C_{2,0}^{1,0}$, are determined numerically via value-matching and smooth-pasting conditions between the three branches. Note that, unlike the case of sudden subsidy retraction, $F_{2,0}^{1,0}(E)$ is now defined over three different regions of $E$: (i) if $E < \varepsilon_{2,1}^{0,0}$ then the firm would not invest even in the presence of a subsidy, (ii) if $\varepsilon_{2,1}^{0,0} \leq E < \varepsilon_{2,0}^{1,0}$ then the firm would invest immediately if the subsidy is provided, and (iii) if $E \geq \varepsilon_{2,0}^{1,0}$, then investment will take place immediately even in the absence of the subsidy.

$$F_{2,0}^{1,0}(E) = \begin{cases} \frac{D_1E}{\rho - \mu} + \frac{\lambda_yD_1E}{(\rho - \mu)(\rho + \lambda_y - \mu)} - I_1 + A_{2,0}^{1,0}E^{\beta_1} + A_{2,0}^{1,0}E^{\eta_1}, & E < \varepsilon_{2,1}^{0,0} \\ \frac{\lambda_yD_1E(1+y) + (\rho - \mu)D_1E}{(\rho - \mu)(\rho - \mu + \lambda_y)} - I_1 + B_{2,0}^{2,0}E^{\eta_2} + C_{2,0}^{1,0}E^{\eta_2}, & \varepsilon_{2,1}^{0,0} \leq E < \varepsilon_{2,0}^{1,0} \\ \Phi_{2,0}^{1,0}(E), & E \geq \varepsilon_{2,0}^{1,0} \end{cases}$$

Next, the dynamics of the value function $\Phi_{1,0}^{1,0}(E)$ are described in (20), where the first term on the right-hand side reflects the instantaneous profit from operating the first technology. As the second term indicates, within an infinitesimal time interval dt a subsidy will be provided with probability $\lambda_y dt$ and then the firm will receive either the value function $F_{2,1}^{0,0}(E)$ or $\Phi_{1,1}^{0,0}(E)$ depending on the arrival of an innovation. By contrast, a subsidy will not be provided with probability $1 - \lambda_y dt$, and, depending on the arrival of an innovation, the firm will receive either the value function $F_{2,0}^{1,0}(E)$ or $\Phi_{1,0}^{1,0}(E)$.

$$\Phi_{1,0}^{1,0}(E) = \left[D_1E - \rho I_1\right] dt + (1 - \rho dt) \left\{ \lambda_y dt E \left[ F_{2,1}^{0,0}(E + dE) \right] + (1 - \lambda_y dt) E \left[ \Phi_{1,1}^{0,0}(E + dE) \right] \right\} + (1 - \lambda_y dt) \left\{ \lambda_y dt E \left[ F_{2,0}^{1,0}(E + dE) \right] + (1 - \lambda_y dt) E \left[ \Phi_{1,0}^{1,0}(E + dE) \right] \right\}$$

Notice that (20) must be solved separately for each of the expressions of $F_{2,1}^{0,0}(E)$, $\Phi_{1,1}^{0,0}(E)$, and $F_{2,0}^{1,0}(E)$ that are indicated in (3), (5), and (19), respectively. Like $F_{2,0}^{1,0}(E)$, $\Phi_{1,0}^{1,0}(E)$ is defined over three different regions of $E$. Hence, following the same approach as in Section 4.2, we obtain the
the impact of and represents the MC associated with the second technology not being available. The third term to not invest should the output price fall below the investment threshold, two branches of (23), and, thus, obtain (24). The first term on the left-hand side represents the expression for $\Phi_{1,0}(E)$ that is described in (21), where $A_{1,0}^{1.0}$, $B_{1,0}^{1.0}$, $C_{1,0}^{1.0}$ and $D_{1,0}^{1.0}$ are determined via value-matching and smooth-pasting conditions between the three branches.

\[
\Phi_{1,0}(E) = \begin{cases} \\
\frac{D_1 E}{\rho - \mu} + \frac{\lambda_2 D_1 E}{(\rho - \mu + \lambda_p)} - I_1 + A_{2,1}^{0.0} \epsilon_1 + A_{2,0}^{1.0} \epsilon_1 + A_{1,1}^{0.0} \epsilon_1 + A_{1,0}^{1.0} \epsilon_1, & E < \epsilon_{2,0}^0 \\
\left[\frac{\lambda_2 D_1 (\rho - \mu) D_1}{\rho - \mu + \lambda_p} + \frac{\lambda_2 D_1}{\rho - \mu + \lambda_p}\left(\frac{\lambda_2 D_1}{\rho - \mu + \lambda_p}\right)^2 \frac{1}{1 + \frac{\lambda_2 D_1}{\rho - \mu + \lambda_p} + \frac{\lambda_2 D_1}{\rho - \mu + \lambda_p}\left(\frac{\lambda_2 D_1}{\rho - \mu + \lambda_p}\right)^2 \frac{1}{1 + \frac{\lambda_2 D_1}{\rho - \mu + \lambda_p}} + \frac{D_1 E}{\rho - \mu + \lambda_p}\right] \\
- \left[\frac{\lambda_2 (1 + y)}{\rho - \mu + \lambda_p} + \frac{\lambda_2 y}{\rho - \mu + \lambda_p} + 1\right] \frac{\lambda_2 D_1 E}{(\rho - \mu)^2 \left(1 + \frac{\lambda_2 D_1}{\rho - \mu + \lambda_p} + \frac{\lambda_2 D_1}{\rho - \mu + \lambda_p}\left(\frac{\lambda_2 D_1}{\rho - \mu + \lambda_p}\right)^2 \frac{1}{1 + \frac{\lambda_2 D_1}{\rho - \mu + \lambda_p}} + \frac{D_1 E}{\rho - \mu + \lambda_p}\right] \\
\times \frac{D_1 E}{\rho - \mu + \lambda_p} + \frac{\lambda_2 D_1}{\rho - \mu + \lambda_p} - I_1 + B_{1,1}^{0.0} \epsilon_2 + D_{1,0}^{1.0} \epsilon_2, & E \geq \epsilon_{2,0}^0 
\end{cases}
\]

Finally, the dynamics of the option to invest in the first technology with a single embedded option to upgrade to the second one are described in (22).

\[
F_{1,0}^{1.0}(E) = (1 - p dt) \left\{ \lambda_p dt \mathbb{E} \left[ F_{1,1}^{0.0}(E + dE) \right] + (1 - \lambda_p dt) \mathbb{E} \left[ F_{1,0}^{1.0}(E + dE) \right] \right\}
\]

The expression for $F_{1,0}^{1.0}(E)$ is indicated in (23), where $\epsilon_{1,0}^{0.0}$, $C_{1,0}^{1.0}$, $H_{1,0}^{1.0}$, and $J_{1,0}^{1.0}$ are determined numerically via value-matching and smooth-pasting conditions. The first term in the top branch of (23) reflects the value of the option to invest in the presence of a subsidy, adjusted via the second term due to policy uncertainty. The first two terms in the second branch represent the expected value of the project if the subsidy is provided, while the third term is the option to invest in the second technology, adjusted for technological uncertainty via the fourth term. The last two terms reflect the likelihood of the price either dropping below $\epsilon_{1,1}^{0.0}$ or increasing beyond $\epsilon_{1,0}^{1.0}$.

\[
F_{1,0}^{1.0}(E) = \begin{cases} \\
C_{1,1}^{0.0} \epsilon_1 + G_{1,0}^{1.0} \epsilon_1 + \frac{\lambda_2 D_1 (1 + y)}{(\rho - \mu) (\rho - \mu + \lambda_p)} - \frac{\lambda_2 D_1}{\rho - \mu + \lambda_p} + A_{2,1}^{0.0} \epsilon_1 + \frac{\lambda_2 D_1}{\rho - \mu + \lambda_p} + A_{1,1}^{0.0} \epsilon_1, & E < \epsilon_{1,1}^{0.0} \\
+ H_{1,0}^{1.0} \epsilon_1 + J_{1,0}^{1.0} \epsilon_1, & \epsilon_{1,1}^{0.0} \leq E < \epsilon_{1,0}^{1.0} \\
\Phi_{1,0}^{1.0}(E), & E \geq \epsilon_{1,0}^{1.0} 
\end{cases}
\]

Although it is not possible to express the value of the option to invest as in (15), we can analyse the impact of $\lambda_p$ on $\epsilon_{1,0}^{1.0}$ by applying the FONC to the value matching condition between the bottom two branches of (23), and, thus, obtain (24). The first term on the left-hand side represents the extra benefit from allowing the project to start at a higher output price, the second term reflects the reduction in the MC due to saved investment cost, and the third term the MB of being able to not invest should the output price fall below the investment threshold, $\epsilon_{1,1}^{0.0}$. The first term on the right-hand side is the MC of the forgone cash flows, while the second term is always positive and represents the MC associated with the second technology not being available. The third term
on the left-hand side reflects the increase in the MB of waiting due to the likelihood of a subsidy, whereas the third term on the right-hand is the corresponding MC of waiting because the subsidy is not available yet. The fourth term on the right-hand side is the MC of waiting, since the output price might drop below the investment threshold prior to the arrival of an innovation.

\[
\left( \frac{E}{\epsilon_{1,0}} \right)^{\eta} \left[ \frac{D_1}{\rho - \mu + \lambda_p} + \frac{\eta_1 I_1}{(\rho + \lambda_p)\epsilon_{1,0}} + \theta_1 A_{1,0,1,0}^{1,0} t_{1,0} + (\eta_1 - \eta_2) H_{1,0,1,0}^{1,0,1,0} \right] \\
= \left( \frac{E}{\epsilon_{1,0}} \right)^{\eta} \left[ \frac{\eta_1 D_1}{\rho - \mu + \lambda_p} - \frac{(\delta_1 - \eta_1) \lambda_T}{\lambda_T - \lambda_p} A_{1,1,0,0}^{1,0,1,0} t_{1,0} + \lambda_1 A_{1,0,1,0}^{1,0,1,0} \right]
\]

(24)

As shown in Proposition 3, greater likelihood of subsidy provision lowers the MB by more than the MC, thereby decreasing the optimal investment threshold.

**Proposition 3.** Greater likelihood of subsidy provision lowers the optimal investment threshold.

The relative loss in option value due to policy uncertainty is \( \frac{F_{1,1,1,1}^{0,0}(E) - F_{1,1,0,1}^{0,0}(E)}{F_{1,1,1,1}^{0,0}(E)} \), and, unlike the case of sudden subsidy retraction, decreases with greater \( \lambda_p \). Indeed, for \( \lambda_p = 0 \) the subsidy will never be provided and the relative loss in option value is maximised, whereas, it decreases with greater \( \lambda_p \), since the expected value of the project increases.

**Proposition 4.** \( \frac{F_{1,1,1,1}^{0,0}(E) - F_{1,1,0,1}^{0,0}(E)}{F_{1,1,1,1}^{0,0}(E)} \in \left[ 1 - \frac{1}{(1+y)^2}, 0 \right] \)

#### 4.4. Provision of a Retractable Subsidy

Here, we assume that a subsidy that was provided at time \( T_1 \) may be retracted at time \( T_2 \). The expected present value of the subsidy is \( \mathbb{E}_E \left[ \int_{T_1}^{T_2} e^{-\rho t} D_2 E_t dt \right] = \frac{D_2 E \left[ e^{-\rho T_1} - e^{-\rho T_2} \right]}{\rho - \mu} \) and since \( T_m \sim \exp(\lambda_p), m = 1, 2 \), the expected value from operating the second technology is indicated in (25). Unlike (17), the subsidy will be available for a smaller time period, and, therefore, its expected value is reduced, i.e., \( \frac{\lambda_p}{(\rho - \mu + \lambda_p)^2} \leq \frac{\lambda_p}{(\rho - \mu)(\rho - \mu + \lambda_p)} \).

\[
\Phi_{2,0}^{1,1}(E) = \frac{D_2 E}{\rho - \mu} + \int_0^\infty \lambda_p e^{-\lambda_p T_1} \int_{T_1}^{\infty} \lambda_p e^{-\lambda_p (T_2 - T_1)} D_2 E y \left[ e^{-\rho y T_1} - e^{-\rho y T_2} \right] dy dT_2 dT_1 - (I_1 + I_2)
\]

(25)

Next, we assume that the firm operates the first technology version and holds a single embedded replacement option. The latter, will either be exercised in the presence of a retractable subsidy with probability \( \lambda_p dt \), or in the absence of a subsidy that has yet to be provided with probability \( 1 - \lambda_p dt \). Thus, the dynamics of the value function \( F_{2,0}^{1,1}(E) \) are described in (26). Notice that the ordinary differential equation that is obtained by expanding the right-hand side of (26) using
Itô’s lemma must be be solved for each expression of $F_{2,1}^{0,1}(E)$, that is indicated in (10). Thus, the expression for $F_{2,0}^{1,1}(E)$ is indicated in (D-1).

$$F_{2,0}^{1,1}(E) = [D_1 E - \rho I_1] \, dt + (1 - \rho dt) \left\{ \lambda_p dt E_T \left[ F_{2,1}^{0,1}(E + dE) \right] + (1 - \lambda_p dt) E_T \left[ F_{2,0}^{1,1}(E + dE) \right] \right\} \quad (26)$$

The dynamics of the value function $\Phi_{1,0}^{1,1}(E)$ are indicated in (27), where the first term on the right-hand side reflects the immediate profit from operating the first technology. The second term indicates that either a subsidy will be provided with probability $\lambda_p dt$ and then the firm will either receive the value function $F_{2,1}^{0,1}(E)$ or $\Phi_{1,1}^{0,1}(E)$ conditional on the arrival of an innovation. Alternatively, a subsidy will not be provided with probability $1 - \lambda_p dt$ and contingent on the arrival of the second technology version the firm will receive either the value function $F_{2,0}^{1,1}(E)$ or $\Phi_{1,0}^{1,1}(E)$. Solving the differential equation that is obtained by expanding the right-hand side of (27) using Itô’s lemma, we obtain the expression that is indicated in (D-2).

$$\Phi_{1,0}^{1,1}(E) = [D_1 E - \rho I_1] \, dt + (1 - \rho dt) \left\{ \lambda_p dt E_T \left[ F_{2,1}^{0,1}(E + dE) \right] + (1 - \lambda_p dt) E_T \left[ \Phi_{1,1}^{0,1}(E + dE) \right] \right\}$$

$$+ (1 - \lambda_p dt) \left( \lambda_p dt E_T \left[ F_{2,0}^{1,1}(E + dE) \right] + (1 - \lambda_p dt) E_T \left[ \Phi_{1,0}^{1,1}(E + dE) \right] \right) \quad (27)$$

Similarly, the dynamics of the value of the option to invest in the first technology version are described in (28). Notice that, over an infinitesimal time interval $dt$, either the subsidy will become available and the option will be exercised in the presence of a retractable subsidy, or no policy intervention will take place and the firm will continue to hold the value function $F_{1,1}^{1,1}(E)$. Solving (28) for each expression of $F_{1,1}^{1,1}(E)$ that is indicated in (14), we obtain (D-3).

$$F_{1,0}^{1,1}(E) = (1 - \rho dt) \left\{ \lambda_p dt E_T \left[ F_{1,1}^{0,1}(E + dE) \right] + (1 - \lambda_p dt) E_T \left[ F_{1,0}^{1,1}(E + dE) \right] \right\} \quad (28)$$

As it will be shown numerically, the likely retraction of the subsidy after its initial provision decreases the incentive to invest compared to a permanent subsidy provision. Intuitively, the subsidy is less valuable, and, therefore, it is optimal to postpone investment.

4.5. Infinite Provisions and Retractions

Here, we assume that a subsidy is subject to infinite provisions and retractions. Taking into account that $\frac{\lambda_p}{(\rho - \mu + \lambda_p)^2} + \frac{\lambda_p^2}{(\rho - \mu + \lambda_p)^3} + \frac{\lambda_p^2}{(\rho - \mu + \lambda_p)^4} \ldots = \frac{\lambda_p}{(\rho - \mu + \lambda_p)^2} \left( 1 - \frac{\lambda_p^2}{(\rho - \mu + \lambda_p)^2} \right) = \frac{\lambda_p}{(\rho - \mu)(\rho - \mu + 2\lambda_p)}$, the expected value of the profits from operating the second technology version is indicated in (29).

$$\Phi_{2,0}^{\infty,\infty}(E) = \frac{D_2 E}{\rho - \mu} + \frac{\lambda_p D_2 E y}{(\rho - \mu)(\rho - \mu + 2\lambda_p)} - (I_2 + I_1) \quad (29)$$
Next, we assume that the subsidy is initially available. In the first period we get the value of a retractable subsidy, \( \frac{D_2Ey}{(\rho-\mu+\lambda_p)} \), and subsequently the subsidy has to be provided and retracted similar to the previous case. The expected value becomes \( \frac{(\rho-\mu+\lambda_p)D_2Ey}{(\rho-\mu)(\rho-\mu+2\lambda_p)} \), and the expected revenue from operating the second technology is indicated in (30).

\[
\Phi_{2,1}^{\infty}(E) = \frac{D_2E}{\rho-\mu} + \frac{(\rho-\mu+\lambda_p)D_2Ey}{(\rho-\mu)(\rho-\mu+2\lambda_p)} - (I_2 + I_1)
\]

The dynamics of the option to invest in the second technology version are described in (31) for \( a = 0, 1 \). The first term on the right-hand side indicates the instantaneous profit from operating the first technology version, while the subsequent term represents the expected continuation value if the subsidy is either provided or retracted.

\[
F_{2,a}^{\infty}(E) = [D_1E(1+ya) - \rho I_1] dt + (1 - \rho dt) \left\{ \lambda_p dt E_E [F_{2,1-a}^{\infty}(E + dE)] + (1 - \lambda_p dt) E_E [F_{2,a}^{\infty}(E + dE)] \right\}
\]

Similarly, the dynamics of the firm’s value function when it operates the first technology version holding a single embedded option to invest in the second one, are described in (32). The first term on the right-hand side indicates the immediate profit from operating the first technology, while the remaining represents the expected continuation value that depends on the arrival of the second technology.

\[
\Phi_{1,a}^{\infty}(E) = [D_1E(1+ya) - \rho I_1] dt + (1 - \rho dt) \left\{ \lambda_p dt E_E [F_{2,1-a}^{\infty}(E + dE)] + (1 - \lambda_p dt) E_E [\Phi_{1,a}^{\infty}(E + dE)] \right\}
\]

Finally, the dynamics of the value functions in the initial state are indicated in (33).

\[
F_{1,a}^{\infty}(E) = (1 - \rho dt) \left\{ \lambda_p dt E_E [F_{1,1-a}^{\infty}(E + dE)] + (1 - \lambda_p dt) E_E [F_{1,a}^{\infty}(E + dE)] \right\}
\]

5. Numerical Results

For the numerical results we assume that \( \rho = 0.1, \mu = 0.01, \sigma = 0.24, y = 0.1, I_1 = 500, I_2 = 1500, D_1 = 8, D_2 = 16, \) and \( \lambda_p, \lambda_r \in [0,1] \). Consequently, the second technology covers greater demand than the first one, yet is three time more expensive. Figure 2 illustrates the impact of technological and policy uncertainty on the optimal investment threshold in the second (left panel) and the first technology (right panel) under sudden subsidy retraction. Notice that the threat of permanent subsidy retraction decreases the firm’s incentive to invest and raises the optimal investment threshold, as shown in Proposition 1. Intuitively, this is a consequence of the functional
form of the value of the active project. For example, in Adkins & Paxson (2015) the active project is a linear function of $\lambda_p$, yet, in our paper, the impact of policy uncertainty on the value of the active project is exponential. Consequently, the incentive to invest early in order to take advantage of the subsidy for a longer period does not compensate the loss in project value due to subsidy retraction. Nevertheless, the possibility to upgrade to a more efficient technology mitigates the impact of policy uncertainty. This happens because the prospect of sequential investment increases the value of the initial investment opportunity and mitigates the loss in option value due to subsidy retraction. Additionally, greater price uncertainty raises the opportunity cost of investment, and, in turn, the value of waiting, thereby increasing the incentive to postpone investment. These results have important implications for both private firms and policymakers. Indeed, the former can take into account the impact of policy uncertainty on the value of the project and the option to invest, and, thus, make more informed investment and operational decisions, while, the latter, can devise more effective policy mechanisms by taking into account how firms respond to policy uncertainty in the light sequential investment decisions.

![Figure 2: Impact of $\lambda_p$ and $\lambda_e$ on the optimal investment threshold in the second (left) and the first technology (right) under sudden subsidy retraction.](image)

Unlike the case of sudden subsidy retraction, the left panel in Figure 3 illustrates that if a firm holds a single investment option, then greater likelihood of subsidy provision accelerates investment, as shown in Proposition 3. This result becomes more pronounced in the presence of an embedded option to invest in a more efficient technology (right panel). Intuitively, a greater likelihood of subsidy provision raises the value of the investment opportunity, and, in turn, the firm’s incentive to invest. This implies that the rapid increase in project value due to subsidy provision mitigates the firm’s incentive to wait for the subsidy to become available.
Figure 3: Impact of $\lambda_p$ and $\lambda_r$ on the optimal investment threshold in the second (left) and the first technology (right) under sudden subsidy provision.

Figure 4 illustrates how the impact of policy uncertainty on the optimal investment threshold can be decomposed with respect to the MB and MC of delaying investment. Notice that greater likelihood of subsidy retraction lowers both the MB and the MC, yet the MC curve shifts down by more than the MB curve, and, as a result, the two curves intersect at a higher threshold (left panel). Intuitively, the extra cost from delaying investment is reflected partly in the loss due to the absence of the second technology, which becomes more pronounced at a higher output price and as the likelihood of subsidy retraction increases. By contrast, as the right panel illustrates, greater likelihood of subsidy provision decreases both the MB and MC of delaying investment, yet the MB decreases by more, thereby decreasing the marginal value of delaying investment, and, in turn, the optimal investment threshold.

Figure 4: Impact of $\lambda_p$ on the MB and MC of delaying investment for a permanent sudden retraction (left) and a permanent provision (right), $\lambda_r = 0.02$. 
The relative loss in option value due to sudden subsidy retraction and provision is illustrated in the left and right panel of Figure 5, respectively. According to the left panel, greater likelihood of subsidy retraction raises the relative loss in option value, as shown in Proposition 2, and this result becomes more pronounced as the rate of innovation increases. By contrast, the right panel illustrates that in the case of sudden subsidy provision the relative loss in option value decreases with greater \( \lambda_p \), as shown in Proposition 4. Again, this result becomes more pronounced as the rate of innovation increases. This is in line with Propositions 1 and 3, as it implies that the incentive to postpone (accelerate) investment increases with greater likelihood of subsidy retraction (provision), and this becomes more pronounced in the presence of embedded options. Notice also that the relative loss in option value is never zero because the firm can always exercise an investment option whether a subsidy is present or not, albeit at a higher price threshold.

![Figure 5: Impact of \( \lambda_p \) and \( \lambda_r \) on the relative loss in option value under sudden subsidy retraction (left) and provision (right)](image)

Figure 6 illustrates the impact of \( \lambda_p \) and \( \lambda_r \) on the optimal investment threshold in the case of provision of a permanent and a retractable subsidy. Notice that the likelihood of subsidy retraction after its initial provision mitigates the impact of policy uncertainty on the optimal investment threshold. More specifically, relative to the case of permanent subsidy provision, the likelihood that the subsidy will be available temporarily decreases the investment incentive and raises the optimal investment threshold. Nevertheless, as the right panel illustrates, the possibility to upgrade an existing technology by adopting a more efficient version makes the impact of subsequent policy interventions less pronounced.

The impact of \( \lambda_p \) and \( \lambda_r \) on the optimal investment thresholds under infinite provisions and retractions is illustrated in Figure 7. Even though the optimal investment thresholds present a similar
behavior as in the case of permanent subsidy provision and retraction, increasing number of policy interventions make the impact of policy and technological uncertainty less pronounced. Notice also that the investment incentive is greater if the subsidy is currently available, i.e., $a = 1$. This happens because, due to discounting and policy uncertainty, the first policy intervention has a greater impact on the expected value of the project and the investment decision. Additionally, the investment thresholds for $a = 0$ (the subsidy is initially absent) and $a = 1$ (the subsidy is initially present) converge when the rate of policy interventions increases. In fact, as the right panel illustrates, this convergence becomes more pronounced in the presence of embedded investment options.

Figure 7: Impact of $\lambda_p$ and $\lambda_r$ on the optimal investment threshold in the second (left) and the first technology (right) under infinite provisions and retractions.
6. Conclusions

In an era of increasing uncertainty, firms in sectors such as energy, manufacturing, and telecommunications require managerial strategies that are responsive to market conditions. For example, the implications of the structural transformation of the power sector for both market participants and policymakers are considered to be crucial as they are expected to change substantially the wholesale market dynamics (Sensfuß et al., 2008). Within this environment, private firms are required to make accurate investment decisions, while policymakers must take into account how private firms respond to different sources of uncertainty in order to incentivize investment. In this paper, we develop a real options framework in order to investigate how price, policy, and technological uncertainty interact to affect the decision to invest sequentially in an emerging technology. We consider the case of sudden subsidy retraction, sudden provision of a permanent and retractable subsidy, as well as infinite provisions and retractions. The results are pertinent to sectors such as RE and R&D, that are subject to frequent policy interventions and where the rate of innovation is high.

We show that greater likelihood of subsidy retraction (provision) decreases (increases) the investment incentive and postpones (facilitates) investment. Allowing for sequential investment opportunities mitigates the impact of policy uncertainty in the case of subsidy retraction but can make the impact of policy uncertainty more pronounced in the case of sudden subsidy provision. Additionally, increasing number of policy interventions mitigate the impact of policy uncertainty on investment by reducing the expected value of the subsidy. Interestingly, the results indicate that the impact of policy uncertainty on investment decisions depends crucially on how policy uncertainty is reflected in the functional form of the value of the project. Indeed, a linear approximation of the impact of policy uncertainty on the value of the active project implies that greater likelihood of subsidy retraction (provision) increases (decreases) the investment incentive (Adkins & Paxson, 2015; Chronopoulos et al., 2016). By contrast, if the impact of policy uncertainty on the value of the project is exponential, and, therefore, more pronounced, then the opposite result is observed, i.e., greater likelihood of subsidy retraction (provision) postpones (accelerates) investment. Consequently, understanding how the rate of policy interventions may influence the propensity to invest is particularly crucial for the design of policies that aim to promote long-term investment decisions.

In order to relax the assumption of a GBM, other stochastic processes, e.g., mean-reverting process or arithmetic Brownian motion, may be implemented within the same framework. Additionally, the current framework can be extended by investigating the interaction between price and policy uncertainty via a two-factor model, whereby price and policy uncertainty are modelled via
correlated geometric Brownian motions. Also, it would be interesting to include strategic interactions via duopolistic competition and analyse how the presence of a rival may impact not only investment but also capacity sizing decisions (Dangl, 1999). This will provide further insights on how policy measures may enhance or reduce the competitive advantage of power plants depending on their asymmetries, related to cost and operational flexibility. For example, a carbon-price floor can influence the value of operational flexibility, thereby inducing investment in a RE facility by decreasing the value of operational flexibility embedded in a commodity-based facility (Chronopoulos et al., 2014). Moreover, allowing for the choice between two technologies would enable further insights regarding the optimal technology adoption strategy under policy uncertainty.

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APPENDIX

A. Benchmark Case

The value function $F_{2,a}^{0,0}(E)$ is described in (A-1).

$$F_{2,a}^{0,0}(E) = D_1 E (1 + ya) dt - \rho I_1 dt + (1 - \rho dt) \mathbb{E}_E \left[ F_{2,a}^{0,0}(E + dE) \right]$$  \hspace{1cm} (A-1)

Using Itô’s lemma, we expand the right-hand side of (A-1), and, thus, we obtain (A-2).

$$\frac{1}{2} \sigma^2 E^2 F_{2,a}^{0,0'}(E) + \mu E F_{2,a}^{0,0'}(E) - \rho F_{2,a}^{0,0}(E) + D_1 E (1 + ya) - \rho I_1 = 0$$  \hspace{1cm} (A-2)

Notice that the solution of the homogeneous part of (A-2) is $F_{2,a}^{0,0}(E) = A_{2,a} E^{\beta_1} + B_{2,a} E^{\beta_2}$. However, $E \to 0 \Rightarrow B_{2,a} E^{\beta_2} \to \infty$, and, therefore, $B_{2,a}^{0,0} = 0$. The expression for $F_{2,a}^{0,0}(E)$ is indicated in (3), where $A_{2,a}$ and $B_{2,a}$ are determined analytically via value-matching and smooth-pasting conditions and are indicated in (4).

Next, the firm is operating the first technology version and holds an option to invest in the second one. The dynamics of the value function are described in (5), and by expanding the right-hand side of (5) using Ito’s lemma, we obtain (A-3).

$$\frac{1}{2} \sigma^2 E^2 \Phi_{1,a}^{0,0'}(E) + \mu E \Phi_{1,a}^{0,0'}(E) - (\rho + \lambda_\tau)\Phi_{1,a}^{0,0}(E) + \lambda_\tau F_{1,a}^{0,0}(E) + D_1 E (1 + ya) - \rho I_1 = 0$$  \hspace{1cm} (A-3)

The endogenous constants $A_{1,a}$ and $B_{1,a}$ are determined via value-matching and smooth-pasting conditions.
between the two branches of (6) and are indicated in (A-4) and (A-5), respectively.

\[ A_{1,0}^{0,0} = \frac{\varepsilon_{2,0}^{0,0} - \delta_{1}}{\delta_{2} - \delta_{1}} \left[ \lambda_{r} (\delta_{2} - 1) (D_{2} - D_{1}) (1 + \rho \lambda_{p}) - \frac{\delta_{2} \lambda_{r} I_{2}}{\lambda_{r} + \rho} + (\beta_{1} - \delta_{2}) A_{2,0}^{0,0} \varepsilon_{2,0}^{0,0} \right] \]  
\[ B_{1,0}^{0,0} = \frac{\varepsilon_{2,0}^{0,0} - \delta_{2}}{\delta_{1} - \delta_{2}} \left[ \lambda_{r} (1 - \delta_{1}) (D_{2} - D_{1}) (1 + \rho \lambda_{p}) - \frac{\delta_{1} \lambda_{r} I_{2}}{\lambda_{r} + \rho} - (\beta_{1} - \delta_{1}) A_{2,0}^{0,0} \varepsilon_{2,0}^{0,0} \right] \]  

(A-4)

(A-5)

B. Permanent Subsidy Retraction

**Proposition 1.** Greater likelihood of subsidy retraction raises the optimal investment threshold.

**Proof:** Notice that greater \( \lambda_{p} \) lowers the expected value of the project, thereby reducing both the MB and the MC of delaying investment. Also, notice that the first five terms on the left-hand side of (16) are less sensitive to changes in \( \lambda_{p} \) than the first four terms on the right-hand side, since \( \theta_{1} \geq \eta_{1} \geq \beta_{1} \geq 1 \) and \( \theta_{1} \geq \delta_{1} \geq \beta_{1} \geq 1 \). Next, we denote the last terms on the left- and right-hand side by \( G = \beta_{1} C_{1,1}^{0,1} + \eta_{1} A_{2,1}^{0,1} \) and \( H = \eta_{1} C_{1,1}^{0,1} + \beta_{1} A_{2,1}^{0,1} \), respectively. If \( \lambda_{r} = 0 \), then the ratio between \( C_{1,1}^{0,1} \) and \( A_{2,1}^{0,1} \) is equal to \( \left( \frac{(D_{2} - D_{1}) I_{1}}{D_{1} I_{2}} \right)^{\eta_{1}} \frac{I_{2}}{I_{1}} \).

\[ G = \beta_{1} C_{1,1}^{0,1} + \eta_{1} A_{2,1}^{0,1} = \left[ \beta_{1} + \eta_{1} \left( \frac{(D_{2} - D_{1}) I_{1}}{D_{1} I_{2}} \right)^{\eta_{1}} \frac{I_{2}}{I_{1}} \right] C_{1,1}^{0,1} \]  
\[ H = \eta_{1} C_{1,1}^{0,1} + \beta_{1} A_{2,1}^{0,1} = \left[ \eta_{1} + \beta_{1} \left( \frac{(D_{2} - D_{1}) I_{1}}{D_{1} I_{2}} \right)^{\eta_{1}} \frac{I_{2}}{I_{1}} \right] C_{1,1}^{0,1} \]  

(B-1)

(B-2)

Note that \( \left( \frac{(D_{2} - D_{1}) I_{1}}{D_{1} I_{2}} \right)^{\eta_{1}} \frac{I_{2}}{I_{1}} < 1 \), which implies that \( \frac{\partial G}{\partial \lambda_{p}} < \frac{\partial H}{\partial \lambda_{p}} \), and, in turn, that the MC decreases by more than the MB. If \( \lambda_{r} > 0 \), then both the MB and the MC of delaying investment increase due to the embedded investment option, yet the impact of \( \lambda_{r} \) only impacts \( C_{1,1}^{0,1} \), and, therefore, the overall effect remains unchanged. \( \square \)

**Proposition 2.** \( \frac{F_{1,1}^{0,0}(E) - F_{1,1}^{0,1}(E)}{F_{1,1}^{0,0}(E)} \in \left[ 0, 1 - \frac{1}{(1+y)^{\beta_{1}}} \right] \)

**Proof:** The relative loss in option value is outlined in (B-3).

\[ \frac{F_{1,1}^{0,0}(E) - F_{1,1}^{0,1}(E)}{F_{1,1}^{0,0}(E)} = \left( C_{1,1}^{0,0} - C_{1,1}^{0,1} \right) E^{\beta_{1}} - C_{1,1}^{0,1} E^{\eta_{1}} \]  
\[ \frac{C_{1,1}^{0,0} E^{\beta_{1}}}{C_{1,1}^{0,1}} \]  

(B-3)

Notice that \( \lambda_{p} = 0 \Rightarrow F_{1,1}^{0,0}(E) = F_{1,1}^{0,1}(E) \Rightarrow \frac{F_{1,1}^{0,0}(E) - F_{1,1}^{0,1}(E)}{F_{1,1}^{0,0}(E)} = 0 \). By contrast, as \( \lambda_{p} \) increases, the relative loss increases since \( C_{1,1}^{0,1} \rightarrow 0 \). Also, notice that \( \varepsilon_{2,1}^{0,0} = \varepsilon_{1,0}^{2,0} + \frac{A_{2,1}^{0,0} (1 + y)}{\lambda_{r}} \), and, \( \varepsilon_{1,1}^{0,0} = \varepsilon_{1,0}^{1,0} + \frac{A_{1,1}^{0,0} (1 + y)}{\lambda_{r}} \). Thus, \( A_{1,1}^{0,0} = (1 + y)^{\beta_{1}} A_{1,1}^{0,0} \) and by substituting \( \varepsilon_{1,1}^{0,0} = A_{1,1}^{0,0} \) and \( A_{2,1}^{0,0} \) in the expression for \( C_{1,1}^{0,0} \), we obtain (B-4).

\[ C_{1,1}^{0,0} = (1 + y)^{\beta_{1}} \left( \frac{D_{1} \varepsilon_{1,0}^{1,0}}{\rho - \mu} - I_{1} + A_{2,1}^{0,0} \varepsilon_{1,0}^{0,0} + A_{1,1}^{0,0} + A_{2,1}^{0,0} \right) = (1 + y)^{\beta_{1}} C_{1,1}^{0,0} \]  

(B-4)
Hence, $\frac{C_{1,0}^{0,0}}{C_{1,0}} = (1 + y)^{\beta_1}$, and, thus, $\frac{F_{1,1}^{0,0}(E) - F_{1,1}^{0,1}(E)}{F_{1,1}^{1,0}(E)} = 1 - \frac{1}{(1+y)^{\beta_1}}$.

**C. Provision of a Permanent Subsidy**

**Proposition 3.** Greater likelihood of subsidy provision lowers the optimal investment threshold.

**Proof:** If $\lambda_r = 0$, then (24) can be rewritten as in (C-1), since $\theta_1 = \eta_1$.

$$\left( E \frac{1}{\varepsilon_{1,0}} \right)^{\eta_1} \left[ \frac{D_1}{\rho - \mu + \lambda_p} + \frac{\eta_1 \rho I_1}{(\rho + \lambda_p) \varepsilon_{1,0}} + (\eta_1 - \eta_2) H_{1,0}^{1,0} \varepsilon_{1,0}^{1,0} \eta_2 - 1 \right] = \left( E \frac{1}{\varepsilon_{1,0}} \right)^{\eta_1} \left[ \frac{\eta_1 D_1}{\rho - \mu + \lambda_p} \right]$$  \hspace{1cm} (C-1)

By inserting the expression for $H_{1,0}^{1,0} = \frac{1}{(\eta_1 - \eta_2) \varepsilon_{1,0}^{1,0}} \left( \eta_1 - \beta_1 \right) C_{1,1}^{0,0} \varepsilon_{1,1}^{1,0} \beta_1 = (\eta_1 - 1) \lambda_p D_1 \varepsilon_{1,1}^{0,0} \varepsilon_{1,1}^{1,0} + \eta_1 \frac{\lambda_p I_1}{\rho + \lambda_p}$ in (C-1), subtracting the left from the right-hand side, and taking the derivative with respect to $\lambda_p$ we obtain (C-2).

\[
\frac{\partial}{\partial \lambda_p} \left[ \left( \eta_1 D_1 (\rho - \mu) + \lambda_p D_1 \right) \left[ \varepsilon_{1,1}^{1,0} (1 + y) - \varepsilon_{1,0}^{1,0} \varepsilon_{1,0}^{1,0} \eta_2 \right] \right] + \eta_1 \rho I_1 \left[ \varepsilon_{1,0}^{1,0} - \varepsilon_{1,1}^{1,0} \right] \rho + \lambda_p \\
\eta_1 \frac{\lambda_p I_1}{\rho + \lambda_p} < 0
\]  \hspace{1cm} (C-2)

Starting with the second term on the left-hand side of (C-2), we notice that:

\[
\frac{\partial}{\partial \lambda_p} \eta_1 = \frac{\partial \eta_1}{\partial \lambda_p} (\rho + \lambda_p) - \eta_1 = \frac{\partial \eta_1}{\partial \lambda_p} (\rho + \lambda_p) - \sqrt{\frac{\rho + \lambda_p}{\sigma^2}} < 0
\]  \hspace{1cm} (C-3)

And by inserting $\frac{\partial \eta_1}{\partial \lambda_p} = \frac{1}{\sigma^2 \sqrt{\left( \frac{\mu}{\sigma^2} - \frac{1}{2} \right)^2 + \frac{2(\rho + \lambda_p)}{\sigma^2}}}$ into the inequality $\frac{\partial \eta_1}{\partial \lambda_p} (\rho + \lambda_p) - \sqrt{\frac{\rho + \lambda_p}{\sigma^2}} < 0$ we obtain $0 < \left( \frac{\mu}{\sigma^2} - \frac{1}{2} \right)^2 + \frac{(\rho + \lambda_p)}{\sigma^2}$, which cannot be negative. Next, we take the partial derivative of the first term on the left-hand side of (C-2) with respect to $\lambda_p$ and we obtain (C-4).

\[
\frac{\partial}{\partial \lambda_p} \eta_1 D_1 (\rho - \mu) + \lambda_p D_1 \left[ \frac{\partial \eta_1}{\partial \lambda_p} (\rho + \lambda_p) - \eta_1 + 1 \right] \rho - \mu + \lambda_p \\
(\rho - \mu)(\rho - \mu + \lambda_p)^2
\]  \hspace{1cm} (C-4)

Similarly, we can show that $\frac{\partial \eta_1}{\partial \lambda_p} (\rho + \lambda_p - \mu) - \eta_1 + 1 < 0$, and, that, $\varepsilon_{1,1}^{1,0} (1 + y) - \varepsilon_{1,0}^{1,0} \varepsilon_{1,0}^{1,0} < 0$. But $\varepsilon_{1,1}^{0,0} (1 + y) = \varepsilon_{1,0}^{0,0}$. The final term in (C-2) is negative because $\frac{\partial \eta_2}{\partial \lambda_p} < 0$, while the other terms are positive. Consequently, the MB of delaying investment decreases by more than the MC. If $\lambda_r > 0$, then both the MB and the MC of delaying investment increase due to the embedded investment option, however, since policy uncertainty impacts the embedded investment option in the same way, the overall effect is maintained.

**Proposition 4.** $\frac{F_{1,1}^{0,0}(E) - F_{1,1}^{1,0}(E)}{F_{1,1}^{1,0}(E)} \in \left[ 1 - \frac{1}{(1+y)^{\beta_1}}, 0 \right]$.

**Proof:** Notice that the relative loss in option value when $\lambda_p = 0$ is $\frac{F_{1,1}^{0,0}(E) - F_{1,1}^{1,0}(E)}{F_{1,1}^{1,0}(E)} = \frac{C_{1,1}^{0,0} - C_{1,0}^{0,0}}{C_{1,1}^{0,0}}$. 

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which, from Proposition 2, is $1 - \frac{1}{(1+\varepsilon)^2}$. Furthermore, when $\lambda_p$ increases the value of the adjustment term, $G_{1,0}^{1,0}$, approaches zero, and the relative loss is zero. Since $\frac{\partial}{\partial \lambda} G_{1,0}^{1,0} > 0$ and $G_{1,0}^{1,0} \leq 0$ the relative loss is decreasing when we increase $\lambda_r$.

\[ \square \]

D. Provision of a Retractable Subsidy

By expanding the right-hand side of (26) using Itô’s lemma, we obtain an ordinary differential equation, whose solution is indicated in (D-1). The first three terms in the top part of (D-1) reflect the expected profit from operating the first technology. The third term represents the option to invest in the second technology in the permanent absence of a subsidy, adjusted via the last term, since the subsidy will be provided and subsequently retracted. The first four terms in the middle branch represent the expected profit from operating the second technology, while the fifth term represents the likelihood of the price dropping in the waiting region. The last term is the option to upgrade to the second technology.

\[
F_{1,0}^{1,1}(E) = \begin{cases} 
\frac{D_1 E}{\rho - \mu} + \frac{\lambda_p D_1 E}{\mu} - I_1 + A_{2,0}^{0,0} E^{\theta_1} + \frac{\lambda_p A_{2,0}^{0,1}}{\mu} \ln E + A_{2,0}^{1,1} E^\eta, & E < \varepsilon_{2,1} \\
\frac{\lambda_p D_2 E + (\rho - \mu) D_1 E}{\rho} - \frac{\lambda_p E}{\rho + \lambda_p} I_2 - I_1 + B_{2,0}^{1,1} E^{\theta_2} + C_{2,0}^{1,1} E^{\eta}, & \varepsilon_{2,1} \leq E < \varepsilon_{2,0} \\
\Phi_{1,0}^{1,1}(E), & E \geq \varepsilon_{2,0}
\end{cases}
\]

Similarly, by expanding the right-hand side of (27) using Itô’s lemma and solving the resulting ordinary differential equation for each expression of $F_{1,0}^{1,1}(E)$ that is indicated in (D-1), we obtain (D-2). Note that $A_{1,0}^{1,1}$, $B_{1,0}^{1,1}$, $C_{1,0}^{1,1}$ and $D_{1,0}^{1,1}$ are determined by the value-matching and smooth-pasting conditions between the three branches.

\[
\Phi_{1,0}^{1,1}(E) = \begin{cases} 
\frac{D_1 E}{\rho - \mu} + \frac{\lambda_p D_1 E}{\mu} - I_1 + A_{2,0}^{0,0} E^{\theta_1} + A_{1,0}^{0,0} E^{\theta_1} + A_{1,0}^{1,1} E^\eta, & E < \varepsilon_{2,1} \\
\frac{\lambda_p D_2 E + (\rho - \mu) D_1 E}{\rho} + \frac{\lambda_p A_{2,0}^{0,1}}{\mu} \ln E + \frac{\lambda_p A_{2,0}^{1,1}}{\mu} E^\eta + \frac{\lambda_p A_{2,0}^{1,1}}{\mu} \ln E - A_{2,0}^{1,1}, & \varepsilon_{2,1} \leq E < \varepsilon_{1,1} \\
\Phi_{1,0}^{1,1}(E) - \frac{(\rho + \lambda_p + \lambda_2) + (\rho + \lambda_p + \lambda_2)}{\rho + \lambda_p + \lambda_2} I_1 + B_{1,0}^{1,1} E^{\theta_2} + C_{1,0}^{1,0} E^{\eta}, & \varepsilon_{1,1} \leq E < \varepsilon_{2,0} \\
\frac{\lambda_p D_2 E + (\rho - \mu) D_1 E}{\rho} + \frac{\lambda_p A_{2,0}^{0,1}}{\mu} \ln E + \frac{\lambda_p A_{2,0}^{1,1}}{\mu} E^\eta + \frac{\lambda_p A_{2,0}^{1,1}}{\mu} \ln E - A_{2,0}^{1,1}, & E \geq \varepsilon_{2,0}
\end{cases}
\]

Finally, the expression for $F_{1,0}^{1,1}(E)$ is indicated in (D-3), where $\epsilon_{1,0}^{1,1}$, $G_{1,0}^{1,1}$, $H_{1,0}^{1,1}$, and $J_{1,0}^{1,1}$ are determined by value-matching and smooth-pasting conditions between the three branches. The first term on the top branch of (D-3), is the option to invest in the permanent presence of a subsidy, adjusted via the second term due to policy uncertainty. The second branch reflects the expected
project value if the subsidy becomes available, and the bottom branch is expected project value when the price is high enough so that investment is optimal even in the absence of a subsidy.

\[ F_{1,0}^{1,1}(E) = \begin{cases} \frac{\lambda_2 A_{1,1}^0}{\sigma^2 - \eta_1 \sigma^2 - \mu} F_{\infty}^{\infty} + \left( \frac{\lambda_2 A_{1,1}^0}{\sigma^2 - \eta_1 \sigma^2 - \mu} \ln E + \frac{G_{1,0}^{1,1}}{\sigma^2 - \eta_1 \sigma^2 - \mu} \right) E^{\eta_1} & , E < \varepsilon_{1,1}^{0,1} \\ \frac{\lambda_2 A_{1,1}^0}{\sigma^2 - \eta_1 \sigma^2 - \mu} F_{\infty}^{\infty} + \frac{\lambda_2 A_{1,1}^0}{\xi_0 \lambda_2 + \lambda_2 A_{1,1}^0} E^{\eta_1} & , \varepsilon_{1,1}^{0,1} \leq E < \varepsilon_{1,0}^{1,1} \\ \Phi_{1,0}^{1,1}(E) & , E \geq \varepsilon_{1,0}^{1,1} \end{cases} \] (D-3)

E. Infinite Provisions and Retractions

By expanding the right-hand side of (31) using Itô’s lemma and adding the differential equations corresponding to \( a = 0,1 \) we obtain \( \bar{f}(E) = F_{2,1}^{\infty,\infty}(E) + F_{2,0}^{\infty,\infty}(E) \), whereas by subtracting them we obtain \( f(E) = F_{2,1}^{\infty,\infty}(E) - F_{2,0}^{\infty,\infty}(E) \).

\[ \frac{1}{2} \sigma^2 E^2 \bar{f}''(E) + \mu E \bar{f}'(E) - \rho \bar{f}(E) + 2D_1 E + D_1 Ey - 2 \rho I_1 = 0 \] (E-1)

\[ \frac{1}{2} \sigma^2 E^2 f''(E) + \mu E f'(E) - \rho f(E) - 2 \lambda_p f(E) + D_1 Ey = 0 \] (E-2)

If \( a = 1 \), then the expression of \( F_{2,1}^{\infty,\infty}(E) \) is indicated in (E-3), where \( \xi \) is the solution to the quadratic \( \frac{1}{2} \sigma^2 \xi(\xi - 1) + \mu \xi - (\rho + 2 \lambda_p) = 0 \). The first two terms on the top part represent the expected profit, while the last two terms is the adjusted value of the option to invest.

\[ F_{2,1}^{\infty,\infty}(E) = \begin{cases} \frac{D_1 E}{\rho - \mu} + \frac{\lambda_p D_1 Ey}{(\rho - \mu)(\rho - \mu + 2 \lambda_p)} - I_1 + A_{2,1}^{\infty,\infty} E^{\beta_1} + B_{2,0}^{\infty,\infty} E^{\xi_1} & , E < \varepsilon_{2,1}^{\infty,\infty} \\ \Phi_{2,1}^{\infty,\infty}(E) & , E \geq \varepsilon_{2,1}^{\infty,\infty} \end{cases} \] (E-3)

Similarly, if \( a = 0 \), then the expression of \( F_{2,0}^{\infty,\infty}(E) \) is indicated in (E-4). The endogenous constants \( A_{2,1}^{\infty,\infty}, B_{2,0}^{\infty,\infty}, C_{2,0}^{\infty,\infty}, D_{2,0}^{\infty,\infty} \) and the investment thresholds \( \varepsilon_{2,1}^{\infty,\infty}, \varepsilon_{2,0}^{\infty,\infty} \) are obtained numerically by value-matching and smooth-pasting between the branches in (E-3) and (E-4). The first three terms in the top branch of (E-4) represent the expected value from operating the first technology, while the fourth term is the value of the option to invest in the second technology, adjusted via the fifth term due to policy uncertainty. The first four terms in the second branch, represent the expected value of the project, while the last two terms represent the likelihood of the price either dropping below \( \varepsilon_{2,1}^{\infty,\infty} \) or rising above \( \varepsilon_{2,0}^{\infty,\infty} \).

\[ F_{2,0}^{\infty,\infty}(E) = \begin{cases} \frac{D_1 E}{\rho - \mu} + \frac{\lambda_p D_1 Ey}{(\rho - \mu)(\rho - \mu + 2 \lambda_p)} - I_1 + A_{2,1}^{\infty,\infty} E^{\beta_1} - B_{2,0}^{\infty,\infty} E^{\xi_1} & , E < \varepsilon_{2,1}^{\infty,\infty} \\ \frac{\lambda_2 A_{2,1}^0}{\sigma^2 - \eta_2 \sigma^2 - \mu} E^{\eta_2} + \left( \frac{\lambda_2 A_{2,1}^0}{\sigma^2 - \eta_2 \sigma^2 - \mu} \ln E + \frac{G_{2,0}^{\infty,\infty}}{\sigma^2 - \eta_2 \sigma^2 - \mu} \right) E^{\eta_1} & , \varepsilon_{2,1}^{\infty,\infty} \leq E < \varepsilon_{2,0}^{\infty,\infty} \\ \Phi_{2,0}^{\infty,\infty}(E) & , E \geq \varepsilon_{2,0}^{\infty,\infty} \end{cases} \] (E-4)
Next, the dynamics of the firm’s value function before the arrival of the second technology are described in (32) for \( a = 0, 1 \). By expanding the right-hand side of (32) using Itô’s lemma we obtain (E-5).

\[
\frac{1}{2} \sigma^2 E^2 \phi_{1,a}^{\infty,\infty'} (E) + \mu E \phi_{1,a}^{\infty,\infty'} (E) - (\rho + \lambda_p + \lambda_r) \phi_{1,a}^{\infty,\infty} (E) + \lambda_p \phi_{1,1-a}^{\infty,\infty} (E) + \lambda_r F_{2,a}^{\infty,\infty} (E) + D_1 E (1 + y a) - \rho I_1 = 0 \tag{E-5}
\]

Following a similar approach, we let \( \bar{\phi}(E) = \Phi_{1,1}^{\infty,\infty} (E) + \Phi_{1,0}^{\infty,\infty} (E) \). Notice that (E-5) has to be solved for each expression of \( F_{2,a}^{\infty,\infty} (E) \) that is indicated in (E-3), and, thus, the expression of \( \bar{\phi}(E) \) is indicated in (E-6).

\[
\bar{\phi}(E) = \begin{cases} 
\frac{(2+y)D_1 E}{\rho - \mu} - 2I_1 + 2A_{2,1}^{\infty,\infty} E^{\beta_1} + A_{1,1}^{\infty,\infty} E^{\delta_1}, & E < \varepsilon_{2,1} \\
\frac{\lambda_r D_2 E}{\rho - \mu + 2\lambda_p} + \frac{\lambda_r D_2 E}{\rho - \mu + \lambda_r} + \frac{\lambda_r D_2 E}{\rho - \mu + \lambda_r} + \frac{\lambda_r D_2 E}{\rho - \mu + \lambda_r} E^{\mu}, & E < \varepsilon_{2,0} \\
+ B_{2,1}^{\infty,\infty} E^{\beta_2} + C_{1,1}^{\infty,\infty} E^{\delta_1}, & E < \varepsilon_{2,1} \\
\frac{(2+y)D_1 E}{\rho - \mu + \lambda_r} + \frac{\lambda_r (2+y)D_2 E}{\rho - \mu + \lambda_r} - 2I_1 - \frac{2\lambda_r I_2}{\rho - \mu + \lambda_r} + D_{1,1}^{\infty,\infty} E^{\delta_2}, & E \geq \varepsilon_{2,0}
\end{cases} \tag{E-6}
\]

Similarly, we set \( \underline{\phi}(E) = \Phi_{1,1}^{\infty,\infty} (E) - \Phi_{1,0}^{\infty,\infty} (E) \), and the expression of \( \underline{\phi}(E) \) is indicated in (E-7). Note that \( \kappa \) is the solution to the quadratic \( \frac{1}{2} \sigma^2 \kappa (\kappa - 1) + \mu \kappa - (\rho + \lambda_r + 2\lambda_p) = 0 \). Consequently, \( \Phi_{1,1}^{\infty,\infty} (E) \) and \( \Phi_{1,0}^{\infty,\infty} (E) \) can be expressed as a linear combination of (E-6) and (E-7).

\[
\underline{\phi}(E) = \begin{cases} 
\frac{D_1 E}{\rho - \mu + 2\lambda_p} + 2B_{2,0}^{\infty,\infty} E^{\xi_1} + G_{1,1}^{\infty,\infty} E^{\kappa_1}, & E < \varepsilon_{2,1} \\
\frac{D_1 y + \lambda_r [D_2 - D_{1,1}^{\infty,\infty} (E)]}{\rho - \mu + \lambda_r} + \frac{\lambda_r D_2 y}{\rho - \mu + \lambda_r + 2\lambda_p}, & E < \varepsilon_{2,0} \\
- \frac{\lambda_r [C_{2,0}^{\infty,\infty} E^{\xi_2} + D_{2,0}^{\infty,\infty} E^{\xi_2}]}{\lambda_r + \lambda_p} + H_{1,1}^{\infty,\infty} E^{\kappa_2} + J_{1,1}^{\infty,\infty} E^{\kappa_1}, & E < \varepsilon_{2,1} \\
\frac{D_1 E}{\rho - \mu + \lambda_r + 2\lambda_p} + \frac{\lambda_r D_2 y}{\rho - \mu + \lambda_r + 2\lambda_p} + K_{1,1}^{\infty,\infty} E^{\kappa_2}, & E \geq \varepsilon_{2,0}
\end{cases} \tag{E-7}
\]

Finally, the option to invest in the first technology is indicated in (E-8) for \( a = 1 \). The first term in the top branch of (E-8) reflects the value of the investment opportunity, adjusted via the second term since subsidy is currently available, while the bottom branch is the value of the active project.

\[
F_{1,1}^{\infty,\infty} (E) = \begin{cases} 
I_{1,1}^{\infty,\infty} E^{\beta_1} + M_{1,0}^{\infty,\infty} E^{\xi_1}, & E < \varepsilon_{1,1} \\
\Phi_{1,1}^{\infty,\infty} (E), & E \geq \varepsilon_{1,1}
\end{cases} \tag{E-8}
\]

If \( a = 0 \), then the option to invest in the first technology is described in (E-9), and, like in Section 4.3 and 4.4, it is defined over three different regions of \( E \). The first term on the top branch of (E-9) is the option to invest, adjusted via the second term due to policy uncertainty. The second branch reflects the value of the project provided that the subsidy becomes available, and the bottom branch
is the expected value of the project if the price is sufficiently high so that investment would take place even in the absence of a subsidy.

$$F_{1,0}^{\infty, \infty}(E) = \begin{cases} L_{1,1}^{\infty, \infty} E_{1}^{\beta_1} - M_{1,0}^{\infty, \infty} E_{1}^{\xi_1} & \text{if } E < \varepsilon_{1,1}^{\infty, \infty} \\
\frac{1 + \frac{y}{p - \mu}}{2(p - \mu + 2\lambda r)} \left( \frac{\lambda_p D_1 E}{(p - \mu + \lambda r)} - \frac{\lambda_p I_1}{p + \lambda r} + \frac{1}{2} A_{1,1}^{\infty, \infty} E_{1}^{\beta_1} + \frac{\lambda_p A_{1,1}^{\infty, \infty} E_{1}^{\xi_1}}{2(p - \mu + \lambda r)} \right) - \frac{B_{2,0}^{\infty, \infty} E_{1}^{\xi_1}}{2} + N_{1,0}^{\infty, \infty} E_{1}^{\eta_1} + O_{1,0}^{\infty, \infty} E_{1}^{\eta_1} & \text{if } \varepsilon_{1,1}^{\infty, \infty} \leq E < \varepsilon_{1,0}^{\infty, \infty} \\
\Phi_{1,0}^{\infty, \infty}(E) & \text{if } E \geq \varepsilon_{1,0}^{\infty, \infty} \end{cases}$$

References


