A Decision Support Methodology for Strategic Planning Under Uncertainty in Maritime Transportation

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“A Decision Support Methodology for Strategic Planning Under Uncertainty in Maritime Transportation”

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Background

The design of a fleet, regarding size and mix is a complex problem where market variables such as demand, transport rates and transport costs are important in the dimensioning decision.

From the point where a ship owner controls a fleet with several vessels in a given trade, the fleet design problem become slightly different. Changes in the current market situation where spot volumes are introduced in new ports can change the trade or change the fleet. The restriction in this given port can be of the sort than none of the existing vessels can enter the port and different new fleet configurations should be investigated before making the decision of acquiring additional capacity in the fleet.

The focus in the master thesis is on the connection point between optimization of the fleet on the basis of introduced changes and ship design, by evaluating different fleet and vessel configurations as input in an existing and modified optimization model.

Overall aim and focus

Thus, the overall objective of the master thesis is to evaluate different ship designs which can generate the highest profit for the fleet as a whole in the new market opening. Other investigations such as finding the break-even point of new available spot volumes which make it profitable to change the existing fleet are of interest.

Further, to use input from a real case, where an existing trade is investigated by analysing the impacts on this trade by introducing different changes. The problem at hand should be solved by modifying an existing optimization model regarding fleet size and mix problems.

Scope and main activities

The candidate should presumably cover the following main points:

1. Provide a description of the background to the problem at hand and describing the current market conditions, assumed development in the future, the trade at hand and the ports with corresponding restrictions where the volumes are introduced.
2. Modify an existing optimization model to fit the task

3. Solving the problem by use of real case data as input to the model. By analyzing different fleet and vessel configurations in the different case scenarios a recommendation is made.

4. Discuss the results with a critical point of view, and determine the break-even point for spot volumes.

Modus operandi

At NTNU, Professor Stein Ove Erikstad will be the responsible advisor, together with Odd Torstein Mørkve at Det Norske Veritas.

The MSc project is within the topic area of the project IDEAS, TRADS and MARFLIX and is thus eligible for travelling grants from this project, such as the use of the existing optimization model especially from TRADS. What real case data to use is not yet decided.

The work shall follow the guidelines given by NTNU for the MSc Thesis work. The work load shall be in accordance with 30 ECTS, corresponding to 100% of one semester.

Stein Ove Erikstad
Professor/Responsible Advisor
Preface
This report is the result of the M.Sc. Thesis – Marine Systems Design, with collaboration between The Norwegian University of Science and Technology (NTNU) and Det Norske Veritas (DNV) spring 2012. The focus in the master thesis with a work load equivalent to 30 credits has been on the fleet size and mix problem, the corresponding mathematical models and the implementation of these.

The aim of the master thesis is to develop a suitable optimization model that can be used as a strategic decision support tool for shipowners, and a methodology considering the uncertainties related to the decision. Initially the scope existed of analyzing the effects on the fleet by different changing parameters, such as market data and external conditions and a relevant case from an industry actor was meant to substantiate the result. The large uncertainties related to the changing parameters resulted in moving the focus towards methodology handling these new aspects.

The models was implemented in a optimization software and Xpress IVE was chosen since the Department of Marine Technology have license for this software and because it is used in optimization courses followed at the Department of Industrial Economics and Technology Management. Valuable lessons regarding advanced language features have helped in the process and thanks to my fellow students for interesting discussions must also be mentioned in this context.

Special thanks will be given to persons contributing to the completion of this master thesis, Kjetil Fagerholt for providing relevant sources regarding fleet optimization which covered several modes of operation in maritime transportation. Eivind Dale at DNV proNavis for his efforts in the process of finding a relevant industry case and in the end I would like to thank the advisors Stein Ove Erikstad and Odd Torstein Mørkve for good input during the work.

Trondheim, 8th June 2012

Håvard Abusdal
Abstract
Measured in volume approximately 80% of world trade is carried at sea and with just as many different actors the shipping industry acts close to a perfect market. The highly volatile nature of the industry with unexpected market fluctuations is the basis for the major decisions shipping companies are making. Especially the fleet size and mix problem in a strategic setting involving fleet changes during several planning periods as a company growth policy. This decision is therefore highly dependent on correct timing for those who want to succeed and an introduction to the shipping industry is given to state these properties.

In this thesis various optimization models solving the fleet size and mix problem are presented where the best suited model structure related to the topic is chosen. This model is of deterministic nature, meaning that all input values are known, and based upon predefined routes. The decision regarding the fleet composition during several planning periods is aiming at determining an optimal fleet for a given market. The validity of the results solely relies on input data, which is highly uncertain into an unknown future. The predictions need to coincide with the real life development in order for the results to maintain its validity.

Two different trades are used as cases, solved with the models presented. Some input parameters are changed and the differences are investigated. The main findings imply that only relative small changes of the input parameters resulted in very different decisions. The related loss of making the wrong decision is observed in the region of 100 – 200 million USD during three years. This large loss potential and the uncertainty related to the input parameters leads to a need for a method minimizing these effects.

An approach is developed to treat uncertainties minimizing the losses by finding a robust fleet capable of handling a large set of generated future scenarios, called the “Scenario Algorithm”. The approach is divided into three main steps; the scenario generating step where development are based on historical fluctuations, a deterministic solution with the given scenario as basis and finally storing of all the solutions with a statistical analysis of the output.

The algorithm is used on the two cases with two different scenario generating approaches, based on an exponential- and a continuous uniform distribution. The fleet size and mix decisions which appeared with the highest frequency were chosen, and gave a consistent estimate based on risk aversion decreasing the potential of making losses.

The approaches presented in this thesis is not meant to give a correct answer on how the future will be, but help the decisions makers reduce the uncertainty connected to the strategic decision. The deterministic model give valuable information with a given scenario as input, but the model is only capable of evaluate the scenarios individually. The result found by the scenario algorithm evaluating scenarios collectively is therefore of higher value since it provide a more robust solution.
Sammendrag

Omtrent 80 % av verdenshandelen, målt i volum foregår på havet og mange ulike aktører gjør skipsfarten til å virke tilnærmet et perfekt marked. Det er en svært volatil bransje med uvante svingninger i markedet og dette er grunntag for flere store beslutningene rederiene tar. Spesielt avgjørelsen om størrelse og sammensetning av flåten i en strategis sammenheng, som involverer endring av flåten over tid som et middel for selskapets vekst. Beslutningen som tas er derfor svært avhengig av riktig timing for de som ønsker å lykkes og en introduksjon til skipsfartsnæringen gis for å bekrefte for disse sammenhengene.

I denne avhandlingen er ulike optimaliseringsmodeller for å løse flåte- størrelse og sammensetningsproblemet presentert, og modellen som egner seg best knyttet til temaet er valgt. Denne modellen er av deterministisk art, hvilket betyr at alle inndataverdier er kjent, og basert på forhåndsekspliserte ruter. Beslutning om flåtens sammensetning over flere planleggingsperioder tar sikte på å bestemme en optimal flåte for et gitt marked. Gyldigheten av resultatene er utelukkende avhengig av inndata, som er svært usikre i en ukjent fremtid. Antagelsene som gjøres må tilsvare utfallet i virkelighetens utvikling for at resultatet skal opprettholde sin gyldighet.

To ulike frakter analyseres med modellene som presenteres. Systemene er undersøkt ved å endre noen inndataparametere og se på effekten av dette i resultatene. De viktigste funnene indikerer at kun relativt små endringer av inndataparametere resulterte i svært ulike beslutninger. Det tilhørende tapet ved å gjøre en feil beslutning er observert i området 100 til 200 millioner USD i løpet av tre år. Dette store tapspotensialet og usikkerheten knyttet til inndataparametere fører til et behov for en metode som minimerer disse effektene.

En tilnærmning er utviklet for å behandle usikkerhet ved å minimerere tapene ved å finne en robust flåte i stand til å håndtere et stort sett av genererte framtidige scenarier, kalt "Scenario Algoritme". Tilnærmningen er delt inn i tre trinn; generering av scenarier basert på historiske svingninger, en deterministisk løsning med det genererte scenariet som grunnlag og til slutt lagring av alle løsningene med en tilhørende statistisk analyse.

Algoritmen er brukt på de to fraktene med to ulike tilnærminger for å generere scenarier, basert på en eksponentiell- og en kontinuerlig uniform fordeling. Resultatet som framkom med den høyeste frekvensen ble valgt, og ga et konsistent estimat basert på risikoaversjon som reduserte tapspotensialet.

De tilnærmingsene som er presenteres i denne avhandlingen er ikke ment å gi et korrekt svar på hvordan fremtiden vil bli, men hjelpe beslutningstakerne redusere usikkerheten knyttet til den strategiske beslutningen. Den deterministiske modellen gir verdifull informasjon med et gitt scenario som input, men modellen er bare i stand til å evaluere scenariene individuelt. Resultatet funnet ved bruk av scenario algoritmen vurderer scenarier kollektivt og er derfor av høyere verdi siden det gir en mer robust løsning.
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1 Introduction and background

A major decision that shipping companies are making is the composition of the fleet. This decision has traditionally been based on the gut-feeling of experienced analysts and qualitative evaluations. By using different developed mathematical models and methods through Operational Research in the planning process as support, the analysts and decision makers has a larger foundation for making the right evaluation.

In 2007 the total amount of cargo transported by ships was 7 billion tons (UNCTAD, 2012), making the world economy highly dependent on maritime transport. These values are increasing every year, and the total amount in the end of 2010 was almost 9 billion tons (Eurostat, 2011), which gives an average annual growth rate of about 8.7%. Potential opportunities and new challenges in maritime transportation arises because of globalization of the world trade, changes in trade patterns together with the world’s price development and energy situation. New restrictions may be introduced and the optimal fleet size and mix may change significantly as a function of a shifting future market development.

History has shown that a company owning or chartering the right size and mix of vessels has a greater potential of gaining higher revenues compared to a company lacking vessels, but operate those vessels at an excellent level. The decision of what a fleet should be composed of regarding size and mix is a fundamental and very important decision for all shipping companies. The profit potential for a shipping company is decided by the fleet size, number of vessels and the fleet mix, variety of vessel types that best fits a given market. The available fleet is the total resources at hand for a company, meaning that the fleet is the limiting factor regarding operational optimization of allocating capacities.

Parameters such as volume, demand, transport rates, transport costs and potential vessels are the basic inputs in a fleet composition problem. These parameters are associated with great uncertainty. If the fleet decision is based on a wrong forecast of the parameters mentioned above it may have negative impacts on the profit with potential high losses. The decision of building an expensive vessel is therefore connected with high uncertainties and should be based on a thorough evaluation of potential gains, losses and of the risks involved.

Fleet composition can be divided into two main focus areas; operational- and strategic fleet composition, with a distinct border between the two. Operational fleet composition at a tactical level is related to the problem with capacity adjustment within an existing fleet in a short period of time. The solution is to allocate the best suited capacities to the most profitable cargoes available in the market. The decisions are in most cases made with certain information eliminating the effect of market fluctuations.

The main decisions in the strategic setting are which new vessels should be bought or chartered in, which vessels should be sold or chartered out, and how to cope with demand fluctuations which can move heavily after the decisions are made. This kind of decision making is analyzed within a long period of time making the effects of market fluctuations highly relevant. The focus in this report is strategic fleet design and especially fleet changes during several planning periods. Methods solving the fleet size and mix problems aim at determining an optimal fleet for a given market. The owner may or may not have an
Introduction and background

existing fleet as a starting point, but it is only in rare situations relevant to determine a completely new fleet size and mix. In most practical situations, the problem is how to adjust or extend an existing fleet.

Two main different mathematical approaches to the fleet size and mix problem are presented. In the first model is the routing and scheduling included in the optimization and the second is based upon a predefined set of routes as input. The method that is most reasonable within the focus area in this report is a model based upon predefined routes and existing vessel designs. This is because the number of ports visited within a given trade in the maritime setting often is limited, making the route possibilities easy to predefine and analyze. In addition is the focus towards strategic fleet renewal and not towards routing and scheduling. The model is further extended by including time periods making the model able to analyze the optimal fleet composition regarding changes during several time periods in a given market situation.

The mathematical model is implemented in Xpress and used to analyze a given case, where an existing fleet, a trade and market situation is given. Changes in these parameters are introduced and the most profitable fleet meeting these changes during time is found. The expected output is an optimal fleet renewal plan during several planning periods, meaning that decisions whether to take in or out vessels in the fleet every planning period is made according to the solution. The main element of uncertainty when this approach is used is the input data, if these are representing the expected situation the fleet will meet during the planning period without correctness the output will act accordingly. This introduces a high risk of making the wrong decision and the loss potential is large.

It is important to have a critical view when the solution is analyzed, because the models are giving one unique solution which is optimal without consideration of the vulnerability in the transport layout. This can have dramatic effects, if one vessel serving one route alone breaks down. This introduces large costs when a new vessel is introduced in the fleet in haste with large rates. How can these elements be introduced in the models, or in the post analysis to minimize the risks involved?

A model able to handle the changes during time and analyze both worst- and best case scenarios is desirable. Is it therefore possible to eliminate this effect by finding a robust fleet which is able to cope with several future market outcomes and how can this be solved without introducing difficult stochastic behavior in the mathematical models?
Introduction and background

1.1 The shipping industry

The marine industry is highly dependent of the global financial situation, and the dramatic effects of the financial crisis during 2009 led to a decrease in gross domestic product in several countries. This caused a 23% fall in world trade compared with the year before (UNCTAD, 2012). Parameters such as freight rates and demand changes because of the global financial situation, and causes the maritime economics to behave in a cyclic pattern with repetition of peaks and troughs in for example demand and freight rates. The reactions to these movements are in most cases made after the peaks or troughs are seen, making an imbalance between supply of tonnage and demand. This is also caused by large lead times related to acquisition of new vessels. According to (UNCTAD, 2012) the world fleet grew by 7% in 2009 despite the large decrease in trade. The dynamics of the maritime economic parameters is typically referred to as shipping market cycles. (Stopford, 2009) Describes it by three different overlapping cycles, namely long-term, short-term and seasonal cycles shown in Figure 1, separated by:

1. Long-term cycles, typically triggered by major changes in the industries of seaborne commodities
2. Short-term cycles, mainly driven by the world economy status
3. Seasonal cycles, due to the seasonality of many seaborne commodity trades (e.g. agriculture)

![Figure 1 – Long, short and seasonal cyclical components (Stopford, 2009)](image)

Shipping companies are operating in this fluctuating environment making the strategic decision how to manage their own fleet over time crucial, when the decision is based upon uncertain parameters as described above. Owning a fleet able to cope with the changes during time is desirable, but the fluctuations make the planning process hard and it is necessary to compromise.

Is it possible to generate future market scenarios based upon the fluctuation seen during history? The key question follows: What tends to drive these changes? World economy is the main driver, but the relationship between sea trade and world industry is not simple or direct. Two different aspects of the world economy may bring about change in the demand for sea transport, namely the business cycle and the trade development cycle. Business cycles are usually measured by considering the growth rate of real gross domestic product. Despite being termed cycles, these fluctuations in economic activity do not follow a mechanical or predictable period pattern. These cycles lays the foundation for freight cycles, creating a cyclic pattern in demand of ships.
Fluctuations in the rate of economic growth work through into seaborne trade and the recent history of these trade cycles is evident from Figure 2, which shows the close relationship between the growth rate of sea trade and GDP. The business cycle is of major importance to anyone analyzing the demand side of shipping market.

The variation from year to year is presented above and in reality is the world gross domestic product (GDP) and world sea trade increasing from year to year. The values are therefore fluctuating around a long term growing trend. The increase is presented in Figure 3, and is approximately 4% annually in the period from 1994 to 2009.
The shipping industry is in addition divided into three main modes of operation, namely: liner-, industrial- and tramp shipping. In liner shipping the routing and scheduling are in most cases predetermined and is compared to a regular bus line very alike, with a fixed itinerary and schedule. In industrial shipping the operator of the fleet is also the cargo owner trying to minimizing the costs. A tramp shipping company operates similar to a taxi service, following cargoes trying to maximize profit by taking the best available cargoes in the spot market in addition to a certain amount of contract cargoes that are fixed, as in the liner segment. The difference between the three modes above is not always distinct, where companies may operate in several modes simultaneously. The operation mode is also an important input when a strategic fleet size and mix decision model is developed.

1.2 The open hatch bulk carrier
The world fleet consisted of approximately 480 open hatch bulk carriers in 2006 ranging in size from 10,000 to 69,000 dwt. (Stopford, 2009). They are designed to offer direct access to the hold through hatches which extend the full width of the vessel, allowing large cargo units to be lowered into place. Where possible the holds and hatches are designed around standard cargo unit sizes, including containers, with special attention paid to shipboard cargo-handling systems, such as gantry cranes or slewing cranes. All this is expensive because when the hatches are widened extra steel is needed to provide strength and cargo-handling gear adds to the cost. As a result, a high-specification open hatch bulk carrier can cost up to 50% more than a conventional vessel of the same size and this aspect increase the barrier of making the decision of acquisitions when more capital is tied up.

The conventional bulk carrier fleet existed of 4,440 vessels in the size range from 10,000 to 60,000 dwt, including Handy and Handymax vessels, in 2007. (Stopford, 2009). Compared to the open hatch bulk
carrier fleet, is this a very large difference. This is naturally a consequence of commodity demand and transport need in the world, but also a consequence of high-specification and high investment costs and therefore is the available number of open hatch bulk carriers limited.

The main drivers affecting the supply side are the rate of newbuildings and scrapping of old vessels and this rate is controlling the balance of number of vessels. This makes the supply slow and ponderous in its response to changes in demand. Merchant ships generally take a year to build and delivery may take 2-3 years if the shipyards are busy. This further increases the element of uncertainty, by making the newbuilding decision 2-3 years in advance of the market the new vessels are built to service.

1.3 Environmental aspect

With the present climate change in mind, the environmental aspect is very important to address when operating in any modality in the transportation industry. According to (IMO, 2009), maritime transportation accounted for 3.3 % of global greenhouse gas emissions with a total of 1064 million tons CO$_2$ in 2007. If no action is taken the study predicts an expected increase by 150 % - 250 % within 2050.

With increasing need for transportation and further globalization the emissions from ships will increase considerably if not changes are introduced. These changes can be of technological- or of operational nature. Interesting questions are therefore; can focus towards operational excellence, solving the routing and scheduling problem at the best possible way have any impact on the emissions? Can the decision on a strategic level, owning the right fleet decrease the emissions?

The environmental focus introduces new challenges throughout the whole organization of a shipping company, from changes in technical specifications and operational changes. Also making a strategic decision, where the decisions going from being based upon economical parameters alone, to taking the environmental aspect into consideration basing the decision on parameters related to emissions.

The total emissions can be reduced by reducing vessel sailing speed, but when this parameter is changed additional vessels may be needed to fulfill the contractual obligations limiting the total obtainable gains.

Another change which has positive impacts on the emissions is the use of larger vessels transporting more cargo with larger time intervals, exploiting the effects of economies of scale. By changing the fleet size and mix the emission per freight work may decrease as much as 30%, indicated by (Asbjørslet, Linstad, & Pedersen). Where freight work is the amount of lifted cargo (payload) times nautical mile traveled. Reduction potential up to 20 % is also obtainable by owning the right fleet of vessels, meaning that the ship owner has the optimal fleet size and mix with the highest deadweight utilization as possible keeping both emissions and costs to a minimum.
Deciding the best suited model structure

2 Deciding the best suited model structure

This chapter contains a discussion of what model structure to use in the following implementation and case analysis.

In the project thesis (Abusdal, 2011) fall 2011 a literature study of existing methods regarding fleet size and mix problems and their belonging methods where made. The findings where that there exist several developed methods related to the fleet- size and mix problem, where the bulk of the methods are tailor made to fit a given situation. The situation is normally a given trade where one ship owner is operating a fleet, or a shifting market situation is introducing a new trade where a new fleet is needed. The difference between these two strategies is to develop a model that introduce new- or recommend scraping of ships in an existing fleet, or to build a new fleet without a basis. The first case is the most common where for example new volumes are introduced in the trade and the model is developed to decide how the fleet should be utilized to either minimize costs or maximize profit.

There are two main model structures represented in the literature which is investigated. The first model structure finds the best routing and scheduling in addition to the fleet size and mix, meaning that the model finds the best order in which the different customers are visited by the best suited vessels in the fleet. The second structure is based upon predefined routes, meaning that the model is deciding which of the predefined routes the different vessels should travel and how many vessels that are needed to fulfill the requirements in the most profitable way. Within these two structures are many different approaches developed in different instances, and two are presented in the following.

2.1 The fleet size and mix vehicle routing problem

The fleet size and mix vehicle routing problem (FSMVRP) is a mixed integer programming (MIP) formulation consisting of defining the type, the number of vehicles of each type, as well as the order in which to serve the customers with each vehicle when a company has to distribute goods to a set of customers geographically spread, with the objective of minimizing the total costs. The following formulation is an extension of the model presented in (Assad, Gheysen, & Golden, 1984) which also includes time as a parameter given in (Rand & Salhi, 1993).

Sets and indices:

\[ V \] – set of vessels, indexed by \( v \)

Let \( G = (N, A) \) be a graph where \( N = \{0\} \cup \{1, \ldots, n\} \cup \{n + 1\} \). \( C = \{1, \ldots, n\} \) defines the set of customers and \( \{0\} \) and \( \{n + 1\} \) is the depot. \( V = \{1, \ldots, K\} \) is the set of different vehicle types. \( A \subseteq N \times N \) represent the traveling possibilities between nodes, where \((i, i), (i, 0), (n+1, i)\); \( i \in N \) are excluded.

Parameters:

\( q_v \) – capacity of vessel type \( v \)
\( f_v \) – fixed acquisition cost for vessel type \( v \)
\( d_j \) – demand at customer \( j \)
\( c_{ij} \) – the cost a vessel of type \( v \) of traveling from node \( i \) to node \( j \)
\( T_v \) – the maximum time a vessel of type \( v \) can spend
\( t_{ij} \) – the time a vessel of type \( v \) use to travel the link \((i, j)\)
Deciding the best suited model structure

Decision variables:

\(y_{ij}\) – flow of goods i to j

\(x_{ij}^v\) – number of vessels of type v which travels directly from customer i to customer j, 0 otherwise

\(r_{ij}^v\) – the time a vessel of type v has left before reaching its maximal value \(T_v\) after covering the link \((i,j)\)

In the following are symmetric traveling costs, vehicle costs and travel time between links independent of type assumed.

Objective function:

\[
\min z = \sum_{v \in V} \sum_{i \in N} f_v x_{ij}^v + \sum_{v \in V} \sum_{(i,j) \in A} c_{ij} x_{ij}^v \\
\text{subject to } \sum_{v \in V} \sum_{i \in N} x_{ij}^v = 1, j \in C \tag{2.1-1}
\]

\[
\sum_{i \in N} x_{ij}^v - \sum_{i \in N} x_{ji}^v = 0, v \in V, j \in C \tag{2.1-2}
\]

\[
\sum_{i \in N} y_{ij} - \sum_{i \in N} y_{ji} = d_j, j \in C \tag{2.1-3}
\]

\[
y_{ij} - \sum_{v \in V} q_v x_{ij}^v \leq 0, (i,j) \in A \tag{2.1-4}
\]

\[
\sum_{i \in N} y_{io} = 0 \tag{2.1-5}
\]

\[
r_{ij}^v - T_v x_{ij}^v \leq 0, v \in V, (i,j) \in A \tag{2.1-6}
\]

\[
r_{0j}^v - T_v x_{0j}^v + t_{0j} x_{0j}^v \leq 0, v \in V, j \in C \tag{2.1-7}
\]

\[
\sum_{i \in N} r_{ij}^v - \sum_{j \in N} r_{pj}^v = \sum_{j \in N} t_{pj} x_{pj}^v, v \in V, p \in C \tag{2.1-8}
\]

\[
x_{ij}^v \in \{0,1\}, v \in V, (i,j) \in A \tag{2.1-9}
\]

\[
y_{ij} \geq 0, (i,j) \in A \tag{2.1-10}
\]

\[
r_{ij}^v \geq 0, (i,j) \in A \tag{2.1-11}
\]

(1) Is the objective function minimizing costs, where the first term gives the total fixed cost of the vehicle used and the second term gives the total variable costs

(2) States that each customer is visited exactly once

(3) Ensures that a vessel of the same type as the one arriving at a customer will also leave the customer

(4) Represent the movement of goods assuming that all customer demands must be satisfied

(5) Makes sure that goods can travel from i to j only when there is a vessel traveling from i to j, and that the total load on the link \((i,j)\) cannot exceed the capacity of the vessel assigned

(6) Ensures that nothing is returned to the depot

(7) Denotes that the spare time on link \((i,j)\) is no more than the maximum time available for the vessel
Deciding the best suited model structure

(8) Ensures that the spare time after covering links leaving the depot does not exceed the maximum time minus the time required to travel to the first customer
(9) States that every time a vessel travels between two customers, the spare time is reduced by the time used on that connection
(10) Defines that each arc in the network is 1 if used and 0 if it is not used by a vessel of type v
(11) Ensures that the flow and the spare time are non-negative

The model presented above is best applied on land based transport problems where the number of customers can be large and the routing of the different vehicles is the biggest issue. The problem of finding the best routing and scheduling is often very large and the calculating operations are many. In addition is the difference between the vehicles small; hence is the assumption of vehicle independent traveling cost sound. In the maritime transportation problem the routing often decided a priori, within a specific trade for example between continents visiting few ports.

2.2 Model based upon predefined routes

A model where the amount of feasible routes for a specific vessel is represented in a set is therefore a better starting point for the later extensions. The model in the following is based on predefined routes, where there exist a route in a given trade only fulfilling a contract of affreightment and a route that both are fulfilling the contractual obligations and can visit ports with available spot cargoes. The contract cargoes are mandatory, but the sport cargoes are optional. If the spot cargoes are chosen, capacities are removed from the fleet covering the contract cargoes. By operating an additional vessel in the fleet spot cargoes can be lifted and the mathematical model in the following decides if this is profitable or not.

Sets and indices:
V – set of vessels, indexed by v
R – set of routes, indexed by r

Parameters:
Q^{COA} – annual quantity of goods within a COA
Q^{SPOT} – a upper bound to the quantity of goods existing in the spot market
R – revenue per ton transported goods in the spot market
q_v – capacity of vessel type v
C_v^F – annual fixed costs of vessel type v
C_v^O – daily variable operating costs of vessel type v
T_{vr} – round-trip time in days of route r and vessel type v
T_v^{TOT} – time vessel type v is available for transport of goods

Decision variables:
y_v – the number of ships of type v
x_{vr} – number of round-trips sailed by vessel v on route r every year
Deciding the best suited model structure

**Objective function:**

\[
\begin{align*}
\text{max } z &= \sum_{v \in V} R q_v x_{vt} - \sum_{v \in V} C^F_v y_v - \sum_{v \in V} \sum_{r \in R} C^0_{vr} T_{vr} x_{vr} \\
\text{subject to } \sum_{v \in V} \sum_{r \in R} q_v x_{vr} &\geq Q^{COA} \\
\sum_{v \in V} \sum_{r \in R} q_v x_{vr} &\leq Q^{SPOT} \\
\sum_{r \in R} T_{vr} x_{vr} &\leq T_v^{TOT} y_v, v \in V \\
y_v &\geq 0 \text{ and integer } v \in V \\
x_{vr} &\geq 0, v \in V, r \in R
\end{align*}
\]

1. Is the objective function maximizing profit, where the first term gives the total revenue from potential spot cargoes and the second term is fixed cost of a vessel and the last term is total variable costs
2. Ensures that the contractual obligations is fulfilled
3. Ensures that the potential spot cargos are carried
4. Ensures that sailing time is not larger than the total time available
5. Integrality and non-negativity constraint
6. Non-negativity constraint

In this model is port and vessel compatibility built into the generation of feasible routes, meaning that if for example a vessel is too long to call a port within a given route, this route is infeasible. The same is the case if the ports have time windows when the service is given and if the port lacks cranes for unloading and loading of cargo.

Other comments to the model above is that the decision variable that counts the number of roundtrips is continuous, which gives meaning because the planning period extends beyond one year. This transition effect emphasizes introducing of time in the model, by dividing the planning horizon into several periods. The revenue term in the objective function is added up only if the route has available spot cargoes, which may be done by fixing the variables that are connected to that specific route.

### 2.3 Multiple planning periods in the model

The next extension is introducing of several planning periods in the model, which consist of one year each and is a simplification of the stochastic model presented in (NTNU, 2011.11.14). Including time periods in the model is highly relevant when the planners are making a strategic decision regarding a fleet size and mix which generate the highest profit into the future. The strategic planning level has typically a time horizon larger than one year and includes market and trade information as a basis, a model which is able to adjust the fleet within the market situation is therefore desirable.
Deciding the best suited model structure

Adjustment of the fleet is done by acquiring new vessels, selling vessels owned by the company or chartering vessels if this is needed. Charters are modeled by lasting exactly one period, meaning that the vessels are returned to the charterer at the end of the planning period. The model is presented below.

Sets and indices:
- \( V \) – set of vessels, indexed by \( v \)
- \( R \) – set of routes, where \( R^S \) is a subset of \( R \) including the routes with available spot cargoes, indexed by \( r \)
- \( P \) – set of time periods, indexed by \( p \)

Parameters:
- \( Q_{p,COA} \) – annual quantity of goods within a COA in period \( p \)
- \( Q_{p,SPOT} \) – a upper bound to the quantity of goods existing in the spot market in period \( p \)
- \( R_{p,SPOT} \) – revenue per ton transported goods in the spot market
- \( q_v \) – capacity of vessel type \( v \)
- \( C_{vp,F} \) – annual fixed costs of vessel type \( v \) at the beginning of period \( p \)
- \( C_{vp,B} \) – buying costs of vessel type \( v \) at the beginning of period \( p \)
- \( C_{vp,O} \) – daily variable operating costs of vessel type \( v \) at the beginning of period \( p \)
- \( C_{vp,H} \) – cost of chartering a vessel \( v \) at the beginning of period \( p \)
- \( R_{vp,S} \) – revenue of selling vessel \( v \) at the beginning of period \( p \)
- \( E_v \) – the existing fleet at the beginning of the first planning period
- \( T_{vrp} \) – round-trip time in days of route \( r \) and vessel type \( v \) in period \( p \)
- \( T_v^{TOT} \) – time vessel type \( v \) is available for transport of goods annually

Decision variables:
- \( y_{vp,O} \) – number of vessels of type \( v \) operated in period \( p \)
- \( y_{vp,B} \) – number of vessels of type \( v \) bought at period \( p \)
- \( y_{vp,H} \) – number of vessels of type \( v \) hired in at period \( p \)
- \( y_{vp,S} \) – number of vessels of type \( v \) sold at period \( p \)
- \( x_{vrp} \) – number of round-trips sailed by vessel \( v \) on route \( r \) in period \( p \)
Deciding the best suited model structure

Objective function:

\[
\max z = \sum_{v \in V} \sum_{p \in P} R^{SPOT}_p q_v x_{vpr} - \sum_{v \in V} \sum_{p \in P} C^Y_{vpr} y_{vpr} - \sum_{v \in V} \sum_{p \in P} C^B_{vpr} y^B_{vpr} - \sum_{v \in V} \sum_{p \in P} C^H_{vpr} y^H_{vpr} - \sum_{v \in V} \sum_{p \in P} C^S_{vpr} y^S_{vpr} - \sum_{v \in V} \sum_{p \in P} C^T_{vpr} T_{vpr} x_{vpr} \]

subject to

\[
\sum_{v \in V} \sum_{p \in P} q_v x_{vpr} \geq Q^O_{p, vpr}, p \in P \]

\[
\sum_{v \in V} \sum_{p \in P} q_v x_{vpr} \leq Q^{SPOT}_p, p \in P \]

\[
\sum_{v \in V} \sum_{p \in P} T_{vpr} x_{vpr} \leq T^{TOT}_{vpr}, v \in V, p \in P \]

\[
y^{O}_{p} = E_{v} + y^{B}_{v1} + y^{H}_{v1} - y^{S}_{v1}, v \in V \]

\[
y^{O}_{v} = y^{O}_{v, p-1} + y^{B}_{v} + y^{H}_{v} - y^{S}_{v, p-1} - y^{S}_{v}, v \in V, p \in P \backslash \{1\} \]

\[
y^{S}_{v1} \leq E_{v}, v \in V \]

\[
y^{S}_{v} \leq y^{O}_{v} - y^{B}_{v}, v \in V, p \in P \backslash \{1\} \]

\[y^{O}_{v} \geq 0 \text{ and integer } v \in V, p \in P \]

\[y^{B}_{v} \geq 0 \text{ and integer } v \in V, p \in P \]

\[y^{H}_{v} \geq 0 \text{ and integer } v \in V, p \in P \]

\[y^{S}_{v} \geq 0 \text{ and integer } v \in V, p \in P \]

\[x_{vpr} \geq 0, v \in V, r \in R, p \in P \]

(1) Is the objective function maximizing profit, where the first term gives the total revenue from potential spot cargoes, the second term is fixed cost of an operating vessel, the third term is costs of hiring additional vessels, the fourth is revenue from selling existing vessels and the last term is total variable costs

(2) Ensures that the contractual obligations is fulfilled

(3) Ensures that the potential spot cargos are carried if profitable

(4) Ensures that sailing time is not larger than the total time available

(5) Initial balance constraint of the vessels in the fleet

(6) Balance constraint of the vessels in the fleet during the whole planning horizon

(7) Ensuring that the vessels being sold is not exceeding the existing vessels in the fleet

(8) Ensuring that the vessels being sold is not exceeding the available vessels in the fleet in planning period t

(9) Integrality and non-negativity constraints

(10) Non-negativity constraint

The decision is made several years in advance and based upon uncertain parameters when the model is extended into the future, especially related to market values such as rates and income. This effect is not included into the model above, and represents a weakness which can lead to large errors if the decision is solely based upon this analysis. Another effect which represents a weakness in the model is the
Deciding the best suited model structure

availability of vessels in the market and the demand for ships among other ship owner companies. This effect is not taken into account in this model.

The model above is solved with a total planning period smaller than the lifetime of the vessels and the operation by the company. The model above will therefore sell all the vessels in the final time period, because this is the most profitable decision with lack of forced cargo delivery in the next period. The company owning the vessels is continuing their operation beyond the solved planning periods, and a constraint limiting the sold vessels in the final planning period is therefore desirable.

Another extension in the model is to include scraping of vessels, the possibility of chartering out vessels and when this is profitable. This is applied to the vessels owned by the company and not the vessels chartered into the fleet.

Large vessels in the bulk- and especially in the open hatch segment have capacity to transport different cargoes in different holds on board and the optimal cargo mix, if such information exists, is desirable to include in the model based upon the given market situation.

2.4 Model used in the Pacific trade case

This section is containing the model used in the main case and the Pacific trade case, explained in chapter 4.2.1. The model presented in the following is an expansion of the model presented in chapter 2.3, by including a spot cargo amount transported on a route as decision variable, a term considering scrapping of vessels and division of owned and time chartered vessels in the fleet. The vessels time chartered have time charter hire as costs, owned vessels have operational expenditures and both have voyage related costs. The total available sailing time in the fleet is therefore a sum of the available sailing time of the owned vessels and the vessels time chartered.

The problem with sale of vessels in the final planning period is handled by introduction of a set of time periods at which vessels cannot be bought, sold chartered out or scrapped. The vessels are instead given a value in the final planning period lower than the selling price in the previous period. This value is stating what a vessel is worth at the end of the planning period and is taking the effect of aging into account. The number of available vessels in the open hatch bulk carrier fleet is dependent on transport demand and vessel supply, as described in chapter 1.2. This is represented by an upper bound on the decision variable stating the number of vessels time chartered into the fleet, together with the assumption that vessels are available first in the second planning period.

In addition is the ability to choose several spot contracts represented by routes with corresponding available spot cargo during the different time periods included in the model. New parameters and decision variables are introduced and the model is presented as a whole in the following.
Deciding the best suited model structure

Sets and indices:
- \( V \) – set of vessels, indexed by \( v \)
- \( R_i \) – set of routes in trade \( i \), indexed by \( r \)
- \( P = \{1, \ldots, p, \ldots, \bar{P}\} \) is the set of time periods where vessels are operated, and \( \bar{P} \) is the end of the planning horizon
- \( P^A = \{1, \ldots, p, \ldots, \bar{P} - 1\} \) is the set of time periods where the fleet can be adjusted

Parameters:
- \( Q^{\text{COA}}_i \) – annual demand of contract cargo in trade \( i \)
- \( Q^\text{SPOT}_r \) – an upper bound to the quantity of cargo existing in the spot market on route \( r \) in trade \( i \)
- \( R^p \) – revenue per ton transported quantity of cargo in the spot market in period \( p \)
- \( q_v \) – total capacity on board vessel type \( v \)
- \( C_{vp}^{\text{TC}} \) – daily time charter hire of vessel \( v \) at the beginning of period \( p \)
- \( C_{vp}^{\text{Voyage}} \) – daily voyage related costs of vessel type \( v \) at the beginning of period \( p \)
- \( C_{vp}^{\text{Opex}} \) – daily variable operating costs of vessel type \( v \) at the beginning of period \( p \)
- \( C_{vp}^{\text{Capex}} \) – capital expenditures of vessel \( v \) at the beginning of period \( p \)
- \( R^p_{vp} \) – revenue of selling vessel \( v \) in period \( p \)
- \( R_{vp}^\text{SC} \) – revenue of scrapping vessel \( v \) in period \( p \)
- \( R_{vp}^\text{TCO} \) – revenue of time chartering out a vessel \( v \) from the existing fleet at the beginning of period \( p \)
- \( W_v \) – states what vessel \( v \) is worth at period \( \bar{P} \)
- \( E^\text{Own}_v \) – the fleet existing of owned vessels at the beginning of the first planning period
- \( E^\text{TC}_v \) – the fleet existing of time chartered vessels at the beginning of the first planning period
- \( T^\text{Tot}_{vr} \) – total round-trip time in days of route \( r \) and vessel type \( v \)
- \( T^\text{Sea}_{r} \) – total sailing time in days on route \( r \)
- \( T^R_v \) – the total time vessel \( v \) is generating revenue annually
- \( T^\text{Cost}_v \) – the total time vessel \( v \) is generating operational costs annually
- \( T^\text{TC}_v \) – the total time vessel \( v \) is time chartered annually
- \( A_v \) – An upper bound on available time charter vessels of type \( v \) in every time period
- \( B_{vr} \) – Binary parameter equal to 1 if vessel \( v \) can travel on route \( r \), 0 otherwise

Decision variables:
- \( y^\text{Own}_{vp} \) – number of vessels of type \( v \) owned by the company in period \( p \)
- \( y^\text{TC}_{vp} \) – number of vessels of type \( v \) time charted in the fleet in period \( p \)
- \( y^\text{TCO}_{vp} \) – number of vessels of type \( v \) time charted out the fleet in period \( p \)
- \( y^S_{vp} \) – number of vessels \( v \) is sold in period \( p \)
- \( y^\text{SC}_{vp} \) – number of vessels \( v \) is scrapped in period \( p \)
- \( y^B_{vp} \) – number of vessels \( v \) bought in period \( p \)
- \( x_{vrp} \) – number of round-trips sailed by vessel \( v \) on route \( r \) in period \( p \)
- \( w_{rp} \) – quantity of spot cargo transported on route \( r \) in period \( p \)
Deciding the best suited model structure

**Objective function:**

\[
\begin{align*}
\max z &= \sum_{r \in R_i} \sum_{p \in P} R_{rp}^{SOT} w_{rp} + \sum_{v \in V} \sum_{p \in P^A} R_{vp}^S y_{vp}^S \\
&+ \sum_{v \in V} \sum_{p \in P^A} R_{vp}^{TCO} T_v^{TC} y_{vp}^{TCO} + \sum_{v \in V} \sum_{p \in P^A} R_{vp}^{SC} T_v^{SC} y_{vp}^{SC} - \sum_{v \in V} \sum_{p \in P} C_{vp}^{Capex} y_{vp}^B - \sum_{v \in V} \sum_{p \in P} C_{vp}^{Opex} T_v^{Cost} y_{vp}^B + \sum_{v \in V} W_v y_{vp}^{C_{V_{vp}}} \\
&- \sum_{v \in V} \sum_{r \in R_i} \sum_{p \in P} C_{vp}^{V_{C_{V_{vp}}}} x_{vp}^{V_{C_{V_{vp}}}} \\
\text{subject to} \quad &\sum_{v \in V} \sum_{r \in R_i} B_{vr} q_v x_{vp} \geq Q_{COA}^C, p \in P \\
&\sum_{v \in V} w_{rp} \leq \sum_{v \in V} B_{vr} q_v x_{vp}, r \in R_i, p \in P \\
&w_{rp} \leq Q_{SPOT}^C, r \in R_i, p \in P \\
&\sum_{r \in R_i} T_v^{Tot} x_{vp} \leq T_v^{R} (y_{vp}^{C_{V_{vp}}} + y_{vp}^{T_{C}}), v \in V, p \in P \\
y_{vp1}^{C_{V_{vp}}} = E_v^{C_{V_{vp}}} + y_{vp1}^B - y_{vp1}^S - y_{vp1}^{SC} - y_{vp1}^{T_{CO}}, v \in V \\
y_{vp}^{C_{V_{vp}}} = y_{vp, p-1}^{C_{V_{vp}}} + y_{vp}^B - y_{vp}^S - y_{vp}^{SC} - y_{vp}^{T_{CO}}, v \in V, p \in P \setminus \{1\} \\
y_{vp1}^{T_{C}} = E_v^{T_{C}}, v \in V \\
y_{vp1}^{cn} + y_{vp1}^{S} \leq E_v^{C_{V_{vp}}}, v \in V \\
\sum_{p \in P} y_{vp}^{T_{C}} \leq A_v, v \in V \\
y_{vp}^{C_{V_{vp}}} \geq 0 \text{ and integer, } v \in V, p \in P \\
y_{vp}^{T_{C}} \geq 0 \text{ and integer, } v \in V, p \in P \\
y_{vp}^{T_{CO}} \geq 0 \text{ and integer, } v \in V, p \in P^A \\
y_{vp}^{S} \geq 0 \text{ and integer, } v \in V, p \in P^A \\
y_{vp}^{SC} \geq 0 \text{ and integer, } v \in V, p \in P^A \\
y_{vp}^{B} \geq 0 \text{ and integer, } v \in V, p \in P^A \\
x_{vp} \geq 0, v \in V, r \in R_i, p \in P \\
w_{rp} \geq 0, r \in R_i, p \in P
\end{align*}
\]

2.4-1

2.4-2

2.4-3

2.4-4

2.4-5

2.4-6

2.4-7

2.4-8

2.4-9

2.4-10

2.4-11

2.4-12
Deciding the best suited model structure

(1) Is the objective function maximizing profit, where the first terms give the total revenue from potential spot contracts, selling and chartering out or scrapping vessels minus the last terms which describes the costs in the fleet
(2) Ensures that the contractual obligations is fulfilled
(3) Ensures that the carried amount of potential spot cargo is smaller than the capacity on board the vessels
(4) Ensures that the carried amount of potential spot cargo do not exceed the demand
(5) Ensures that total roundtrip time is not larger than the total time available in the fleet
(6) Initial balance constraint of the vessels owned in the fleet
(7) Balance constraint of the vessels owned in the fleet during the whole planning period
(8) Initial balance constraint of the time chartered vessels in the fleet
(9) Ensuring that the vessels being chartered out, scrapped or sold is not exceeding the existing vessels in the fleet at the beginning of the planning period
(10) Constraint that limits available vessels for time chartering
(11) Integrality and non-negativity constraints
(12) Non-negativity constraints
2.5 Model used in the Atlantic trade case

This section is containing the model used in the Atlantic trade case in the main case described in chapter 4.2.1. The model in the following is an expansion of the model in chapter 2.4, by including the possibility of having a cargo mix.

Sets and indices:
- $V$ – set of vessels, indexed by $v$
- $R_i$ – set of routes in trade $i$, indexed by $r$
- $C$ – set of cargoes, indexed by $c$
- $P = \{1, \ldots, \bar{P}\}$ is the set of time periods where vessels are operated, and $\bar{P}$ is the end of the planning horizon.
- $P^A = \{1, \ldots, \bar{P} - 1\}$ is the set of time periods where the fleet can be adjusted.

Parameters:
- $Q_{COA, Tot}^c$ – total annual demand of contract cargo of type $c$ in trade $i$
- $Q_{RC}^{COA}$ – annual demand of contract cargo of type $c$ in trade $i$ on route $r$
- $Q_{rc}^{SPOT}$ – an upper bound to the quantity of cargo type $c$ existing in the spot market on route $r$
- $R_{cp}^{SPOT}$ – revenue per ton transported quantity of cargo type $c$ in the spot market in period $p$
- $q_v$ – total capacity on board vessel type $v$
- $C_{vp}^{TC}$ – daily time charter hire of vessel $v$ at the beginning of period $p$
- $C_{vp}^{Voyage}$ – daily voyage related costs of vessel type $v$ at the beginning of period $p$
- $C_{vp}^{Opex}$ – daily variable operating costs of vessel type $v$ at the beginning of period $p$
- $C_{vp}^{Capex}$ – capital expenditures of vessel $v$ at the beginning of period $p$
- $R_{vp}^S$ – revenue of selling vessel $v$ in period $p$
- $R_{vp}^{SC}$ – revenue of scrapping vessel $v$ in period $p$
- $R_{vp}^{TCO}$ – revenue of time chartering out a vessel $v$ from the existing fleet at the beginning of period $p$
- $W_v$ – states what vessel $v$ is worth at period $\bar{P}$
- $E_v^{Own}$ – the fleet existing of owned vessels at the beginning of the first planning period
- $E_v^{TC}$ – the fleet existing of time chartered vessels at the beginning of the first planning period
- $T_{vr}^{Tot}$ – round-trip time in days of route $r$ and vessel type $v$
- $T_{r}^{Sea}$ – total sailing time in days of route $r$
- $T_v^R$ – the total time vessel $v$ is generating revenue annually
- $T_v^{Cost}$ – the total time vessel $v$ is generating operational costs annually
- $T_v^{TC}$ – the total time vessel $v$ is time chartered annually
- $A_v$ – An upper bound on available time charter vessels of type $v$ in every time period
- $B_v$ – Binary parameter equal to 1 if vessel $v$ can travel on route $r$, 0 otherwise

Decision variables:
- $y_{vp}^{Own}$ – number of vessels of type $v$ owned by the company in period $p$
- $y_{vp}^{TC}$ – number of vessels of type $v$ time charted in the fleet in period $p$
- $y_{vp}^{TCO}$ – number of vessels of type $v$ time charted out the fleet in period $p$
- $y_{vp}^S$ – number of vessels $v$ is sold in period $p$
- $y_{vp}^{SC}$ – number of vessels $v$ is scrapped in period $p$
- $y_{vp}^{B}$ – number of vessels $v$ bought in period $p$
- $x_{vrt}$ – number of round-trips sailed by vessel $v$ on route $r$ in period $p$
- $w_{rcp}$ – quantity of spot cargo of type $c$ transported on route $r$ in period $p$
Objective function:

$$\text{max } z = \sum_{r \in R_i} \sum_{c \in C} \sum_{p \in P} R_{cp}^{SPOT} w_{rcp} + \sum_{v \in V} \sum_{p \in P^A} R_{vp}^{S} y_{vp}^{S} + \sum_{v \in V} \sum_{p \in P^A} R_{vp}^{TC} y_{vp}^{TC} + \sum_{v \in V} \sum_{p \in P^A} R_{vp}^{SC} y_{vp}^{SC} - \sum_{v \in V} \sum_{p \in P} C_{vp}^{Capex} y_{vp}^{B} - \sum_{v \in V} \sum_{p \in P} C_{vp}^{Ope} y_{vp}^{RCost} y_{vp}^{own} + \sum_{v \in V} W_{v} y_{vp}^{own}$$

subject to

$$\sum_{v \in V} \sum_{r \in R_i} B_{vr} q_{v} x_{vpr} \geq \sum_{c \in C} Q_{c}^{COA Tot}, c \in C, p \in P$$

$$\sum_{v \in V} B_{vr} q_{v} x_{vpr} \leq \sum_{c \in C} Q_{r c}^{COA}, r \in R_i, c \in C, p \in P$$

$$\sum_{v \in V} w_{rcp} \leq \sum_{v \in V} B_{vr} q_{v} x_{vpr}, r \in R_i, c \in C, p \in P$$

$$\sum_{r \in R_i} T_{vr}^{Tot} x_{vpr} \leq T_{v}^{R}(y_{vpr}^{own} + y_{vpr}^{TC}), v \in V, p \in P$$

$$y_{vpr}^{own} = E_{v}^{own} + y_{vpr}^{B} - y_{vpr}^{S} - y_{vpr}^{SC} - y_{vpr}^{TC}, v \in V$$

$$y_{vp}^{own} = y_{v}^{own} + y_{v}^{B} - y_{v}^{S} - y_{v}^{SC} - y_{v}^{TC}, v \in V, p \in P \setminus \{1\}$$

$$y_{v}^{TC} = E_{v}^{TC}, v \in V$$

$$y_{v}^{TC}, v \in V$$

$$y_{vp}^{own} \geq 0 \text{ and integer, } v \in V, p \in P$$

$$y_{vp}^{TC} \geq 0 \text{ and integer, } v \in V, p \in P$$

$$y_{vp}^{TC} \geq 0 \text{ and integer, } v \in V, p \in P^A$$

$$y_{vp}^{B} \geq 0 \text{ and integer, } v \in V, p \in P^A$$

$$y_{vp}^{S} \geq 0 \text{ and integer, } v \in V, p \in P^A$$

$$y_{vp}^{SC} \geq 0 \text{ and integer, } v \in V, p \in P^A$$

$$x_{vpr} \geq 0, v \in V, r \in R_i, p \in P$$

$$w_{rcp} \geq 0, r \in R_i, c \in C, p \in P$$
Deciding the best suited model structure

(1) Is the objective function maximizing profit, where the first terms give the total revenue from potential spot contracts, selling and chartering out or scrapping vessels minus the last terms which describes the costs in the fleet
(2) Ensures that the contractual obligations is fulfilled
(3) Ensures that the quantity transported on one route is not larger than the available amount
(4) Ensures that the carried amount of potential spot cargo is smaller than the capacity on board the vessels
(5) Ensures that the carried amount of potential spot cargo do not exceed the demand
(6) Ensures that total roundtrip time is not larger than the total time available in the fleet
(7) Initial balance constraint of the vessels owned in the fleet
(8) Balance constraint of the vessels owned in the fleet during the whole planning period
(9) Initial balance constraint of the time chartered vessels in the fleet
(10) Ensuring that the vessels being chartered out, scrapped or sold is not exceeding the existing vessels in the fleet at the beginning of the planning period
(11) Constraint that limits available vessels for time chartering
(12) Integrality and non-negativity constraints
(13) Non-negativity constraints
Deciding the best suited model structure

3 Implementation
The mathematical models described in the previous sections have been implemented in Xpress IVE using Xpress version 1.22.04. This chapter is a short description of the implementation of the mathematical models in this software.

Optimization models can be implemented in several different software tools, where Microsoft Excel is a good tool for small instances. Xpress IVE was chosen since the Department of Marine Technology have license for this software and because it is used in optimization courses followed at the Department of Industrial Economics and Technology Management.

After creating the needed mathematical models on paper, the models are implemented in Xpress with the belonging syntax. When cases are analyzed the input values need to be constructed to fit the implementation and this was done by first organize the data in Excel and convert to readable text files afterwards. The large amount of output was first presented directly in Xpress and further presented with wanted characteristics in Excel. The structure of the optimization process is presented in Figure 4, given that the model is correctly coded and running.

![Figure 4 – The optimization process in Xpress](image)

Additional scripts are made to conduct post analysis, by writing the solution related to the investment decisions to a unique text-file and use this as input to a new script analyzing the decision at different conditions. The only varying parameters in the new script are the number of roundtrips and the amount of carried spot cargo, representing the utilization of the fleet at hand.

Development and implementation of the scenario algorithm presented in Chapter 6 proved to be challenging, with several unsuccessful attempts. The solution in the end became use of a main “forall-loop” for the iterations, where the scenario is generated, the deterministic problem is solved and the result is stored for every loop.

The advantage with this software is that the code and the mathematical model are built with the same structure without creative and artful code structure. This is making the code easy to follow and read.
4 Case study
This chapter is describing the different cases which are analyzed by the mathematical models above.

4.1 Case to test the first implementation of the model
The mathematical model described in chapter 2.2 is the first model that is implemented in Xpress, this section contains a developed test case to verify the results calculated by the implementation. The case is to invest in a fleet of vessels to ensure that contractual obligations are fulfilled. The market conditions are delivery of an annual amount of quantities from Snøhvit in Norway to Cove Point in USA, presented in Figure 5. In addition to that, there exists a spot market for transport of quantities from Trinidad to Bilbao, defined as route 2, presented in Figure 6.
Case study

With an annually contract quantity of 6000 [1000 ton/year], a quantity of 2000 [1000 ton/year] is available in the spot market and the revenue for lifting this cargo is constant during the planning period of 100 [USD/ton]. The vessel related input is presented in Error! Reference source not found..

Table 1 – Vessel input data, test case

<table>
<thead>
<tr>
<th></th>
<th>Design 1</th>
<th>Design 2</th>
<th>Design 3</th>
<th>Design 4</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacity</td>
<td>80</td>
<td>80</td>
<td>120</td>
<td>120</td>
<td>1000 ton</td>
</tr>
<tr>
<td>Annual fixed costs</td>
<td>10</td>
<td>11</td>
<td>13</td>
<td>14</td>
<td>mill USD</td>
</tr>
<tr>
<td>Daily operational costs</td>
<td>32</td>
<td>37.5</td>
<td>44</td>
<td>52</td>
<td>1000 USD</td>
</tr>
<tr>
<td>Round-trip time COA (1)</td>
<td>30</td>
<td>26</td>
<td>28</td>
<td>24</td>
<td>days</td>
</tr>
<tr>
<td>Round-trip time COA + spot (2)</td>
<td>40</td>
<td>35</td>
<td>nan</td>
<td>nan</td>
<td>days</td>
</tr>
<tr>
<td>Available sailing time per year</td>
<td>350</td>
<td>350</td>
<td>350</td>
<td>350</td>
<td>days</td>
</tr>
</tbody>
</table>

It is a draft limitation in the port of Bilbao, resulting in that vessels of design types 3 and 4 cannot enter the port and hence cannot service the spot contract. The abbreviation “nan” – not a number, is therefore used.
4.2 Main case

The shipping company at hand has a strong market position with an existing fleet of 26 ships and 10 newbuildings that will be introduced to the market and integrated in the fleet in the years to come. This is an opportunity, but also a challenge. The company is operating their fleet in six main trades with regular transport of contract cargo almost as a liner operation, but with a complicating element that some of the vessels are switching trades. The total amount of contractual cargo is approximately 7.2 million tons annually divided among several commodities. In addition is spot cargo lifted when available on the market and the planners find it profitable. The spot cargo can be situated in ports beside the liner route, called deviations and is hard to predict and model into the transport system. The number of different cargo types that is possible to lift is many and also complicating the transport model.

The objective in the case at hand is to analyze two of the trades, with respect to finding the fleet size and mix among a set of different vessel designs that can service the trades with the highest profit. Which design is best suited in the given transport setting, and what is the best fleet balance regarding owned or chartered vessels in the fleet during the planning periods? What effects have changes in speed, cargo amount, revenues or costs on this decision?

The following sections is first containing a problem description, the calculations regarding input data is presented in the case analysis part, further is the model presented in chapter 2.3 adjusted to fit the two trade cases individually in two separate parts as described in Chapter 2.4 and 2.5 respectively.

4.2.1 Problem description

The company is operating their entire fleet in six trades globally and the problem in this thesis is limited to focus on two trades, called the Pacific- and the Atlantic trade. Total roundtrip on a given trade consists of time at sea, loading at the loading ports and discharging at the discharging ports. The company deploys its fleet of ships to perform as many roundtrips as required to meet the demand.

4.2.1.1 Pacific trade

The Pacific trade is including the ports Vancouver in North America and Ulsan, Qingdao and Shanghai in the Far East with contractual transport of cargo. The problem is further limited by assuming that the supply of cargo is unlimited and only focusing on a total cargo amount without segregation into different cargo types. In addition is the routing simplified to follow a given trade pattern related to the contract cargo, presented as route 1 in Figure 7.
Case study

Figure 7 – The Pacific trade, route 1

Route 1 is starting by loading in Vancouver on the North America's west coast, discharging in Ulsan, Qingdao and Shanghai in the Far East before returning to Vancouver. The annual amount of contractual cargo is assumed as an equal distributed amount among the six trades, which gives an average of 1.2 million tons.

In both trades is there a possibility to make deviations to other ports if it exist spot cargo in these ports. This introduce one or more additional loading ports with available spot cargoes, and one or more additional discharging ports with demand for spot cargo. If the model finds it profitable these potential cargoes are chosen.

The Pacific trade includes the ports Manila and Tokyo, which are introduced as ports with demand of cargo in the spot market. The routes involving the possibility to earn money in the spot market are including these ports in different combinations. In addition to make deviations to the intended route are all the routes with spot demand also satisfying the contractual obligations. All the routes with roundtrip distance, calculated by use of an established distance matrix, the belonging number of port calls and the annual amount of requested spot cargo are presented in Appendix A.

4.2.1.2 Atlantic trade

The Atlantic trade, existing of the ports Altamira, Houston, Mobile, Panama City, Brunswick, Port Arthur and Wilmington in North America and Antwerp, Bremen, Ijmuiden and Rotterdam in Europe. This problem is expanded to include cargo mix, which make the routing more complicated since the routing is matched with respect to loading and discharging ports. The routes are found by using the method presented in chapter 4.2.2. These routes are only representing the contractual delivery of cargo, presented as route 1 and route 2 in Figure 8 and Figure 9 respectively.
**Figure 8 – The Atlantic trade, route 1**

Route 1 is starting on North America’s east coast by loading one cargo type in Port Arthur, continuing to Mobile loading three cargo types, loading one cargo type in Panama City, loading three cargo types in Brunswick and loading one cargo type in Wilmington. The route is continuing to Europe discharging two cargo types in Antwerp, four cargo types in Rotterdam, two cargo types in Bremen and loading two cargo types in Ijmuiden before sailing back Port Arthur.

**Figure 9 – The Atlantic trade, route 2**

Route 2 is starting in Europe by loading five cargo types in Bremen, two cargo types in Rotterdam, two cargo types in Ijmuiden and four cargo types in Antwerp. The route is continuing to North America’s east coast by discharging one cargo type in Panama City, discharging three in Mobile, five in Houston, one in Altamira, loading three cargo types in Brunswick and loading one cargo type in Port Arthur, before sailing back to Bremen.
In the Atlantic trade are the ports in the trade as a whole used as spot ports, with deviation from the contract route in different combinations. This is done by introducing one or more additional loading ports with available spot cargoes, and one or more additional discharging ports with demand for spot cargo. These routes also have the capability to fulfill the contractual obligations along with visiting the ports with spot cargo demand. All the routes with roundtrip distance, belonging number of port calls and the annual amount of requested contract and spot cargo mix are presented in Appendix B.

### 4.2.1.3 Vessels

The input vessel alternatives are of four different design types, two different dimensions with two different types of gearing. The different gearings are affecting roundtrip time because of loading and discharging rates, called production. The vessel data is summarized in Table 2, where cargo capacity is assumed to be 90% of deadweight. This assumption is made after investigating similar vessels through (Sea-web, 2012) and by exploiting established coefficients on use of consumables. The fleet in the beginning of the first planning period exists of zero vessels in the fleet that is owned, but two vessels of design type one are time chartered in both the cases.

<table>
<thead>
<tr>
<th>Table 2 – Vessel input data, main case</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Abbreviation</strong></td>
</tr>
<tr>
<td>Abbreviation</td>
</tr>
<tr>
<td>DWT</td>
</tr>
<tr>
<td>Capacity</td>
</tr>
<tr>
<td>OPEX</td>
</tr>
<tr>
<td>CAPEX</td>
</tr>
<tr>
<td>Gearing</td>
</tr>
<tr>
<td>Production</td>
</tr>
<tr>
<td>Cost time</td>
</tr>
<tr>
<td>Revenue time</td>
</tr>
</tbody>
</table>

### 4.2.2 Case analysis

The objective in the case is to make a strategic decision regarding fleet size and mix during a planning period. It is therefore important to address how the different values evolve during time and how other values used as input in the model is calculated.

The two routes related to contract cargo in the Atlantic trade has a unlike combination of cargo types because of different cargo supply and demand in the ports. The routes are found by first defining all possible combinations of routes, 177 in total, and further use the following method:

**Sets and indices:**
- R – set of routes, indexed by r
- P – set of ports, indexed by p
- C – set of cargo types, indexed by c
Case study

Parameters:
\( D_{rp} \) – parameter that is one if route \( r \) contains discharging port \( p \) and zero otherwise
\( L_{rp} \) – parameter that is one if route \( r \) contains loading port \( p \) and zero otherwise
\( Q_{cp}^{\text{Demand}} \) – the demand of cargo type \( c \) in port \( p \)
\( Q_{cp}^{\text{Supply}} \) – supply of cargo type \( c \) in port \( p \)
\( Q_c \) – total contract cargo of type \( c \)

Decision variable:
\( \delta_r \) – one if route \( r \) is used and zero otherwise

Objective function:
\[
\min z = \sum_{r \in R} \delta_r \quad 4.2.2-1
\]
subject to
\[
\sum_{r \in R} \sum_{p \in P} Q_{cp}^{\text{Supply}} L_{rp} \delta_r \geq Q_c, c \in C \quad 4.2.2-2
\]
\[
\sum_{r \in R} \sum_{p \in P} Q_{cp}^{\text{Demand}} D_{rp} \delta_r = Q_c, c \in C \quad 4.2.2-3
\]
\( \delta_r \in \{0,1\}, r \in R \quad 4.2.2-4 \)

1. Objective function minimizing the total amount of needed routes
2. Stating that the cargo available for loading is larger than the contract cargo
3. Ensuring that the discharged cargo is equal to the contract cargo
4. Binary constraint

By solving this system the two routes presented in Figure 8 and Figure 9, with the belonging data found in Appendix B are used as input to the fleet size and mix model.

Values are changing during time, and because of inflation and interest the capital expenditures related to acquisition of new vessels in the fleet today is larger in the next planning period, represented by the following equation:

\[
C_t = C_0 (1 + i)^t
\]

Where \( C_0 \) is the present value of the future cash flow, \( C_i \) is the nominal value of a cash flow amount in a future period \( t \) and \( i \) is the interest rate, which reflects the cost of tying up capital. This is also related to operational expenditures and time charter hire.

Some of the vessels in the fleet are operated on time charter hire, or can be time hired into the fleet. A time charter is the hiring of a vessel for a specific period of time. The charter period can vary from a single voyage to several months or years, assumed here as one planning period of one year. The owner still manages the vessel, but the charterer selects the ports and directs the vessel where to go. The charterer pays for the voyage related costs such as fuel costs, port charges, and a daily hire to the owner.
of the vessel. The shipowner pays the operational expenditures (OPEX) and takes the operational risks, i.e. the owner pays if the ship breaks down. However, the market risk is now covered by the charterer who has committed to pay the fixed amount regardless of the market.

The revenues have a different evolution pattern than the costs, where the value regarding an acquired vessel is decreasing because of aging and aggravation. This affects the selling price and time charter hire during time. The scrapping value is dependent on steel price and is fluctuating every year because of this, to simplify this value is therefore kept fixed during time.

Voyage costs are directly proportional to the fuel consumption and are therefore varying in accordance with the fuel price. The daily voyage costs on a given route are calculated by the following equation:

\[
C_{V}^{\text{Voyage}}(V) = P^{\text{Fuel}} M^{\text{Fuel}}(V), \frac{\text{USD}}{\text{day}}
\]

Where \(P^{\text{Fuel}}\) is the fuel price at a given moment and \(M^{\text{Fuel}}\) is the daily fuel consumption which is following a polynomial pattern with respect to speed, and can be simplified by the following equation:

\[
M^{\text{Fuel}}(V) = \alpha_v V^2 + \beta_v V + \gamma_v, \frac{\text{tons}}{\text{day}}
\]

Where the coefficients \(\alpha_v\), \(\beta_v\), and \(\gamma_v\) are vessel specific and \(V\) is the speed. The time a vessel is sailing is used when total voyage costs are found. Where the sailing time on route \(r\) is calculated with the following equation:

\[
T_r^{\text{Sea}}(V) = \frac{D_r}{V}, \text{[days]}
\]

Where \(D_r\) is the distance on route \(r\), the total voyage cost is therefore calculated in the model as:

\[
C_{v}^{\text{Voyage,Tot}}(V) = C_v^{\text{Voyage}}(V) T_r^{\text{Sea}}(V) x_{vr}, \text{[USD]}
\]

Where \(x_{vr}\) is the decision variable containing the number of roundtrips made annually by vessel \(v\) on route \(r\), the development during time for these values are kept fixed, but the effect on the fleet design by changing the speed and the fuel price are evaluated in the case study.

Total roundtrip time, including deadtime and time in port is needed when the number of vessels needed is estimated. This parameter is calculated by use of two new terms; deadtime and loading/discharging time for every port call stated by the following equation:

\[
P_{vr}^{\text{Tot}}(V) = T_r^{\text{Sea}}(V) + P_{r}^{\text{Call}}(T^{\text{Dead}} + C_v^{\text{Prod}}), \text{[days]}
\]

Where \(P_{r}^{\text{Call}}\) is the number of port calls for the given route, \(T^{\text{Dead}}\) is the deadtime and finally \(C_v^{\text{Prod}}\) is the loading/discharing capacity on board vessel \(v\), also called production. The weakness with this calculation is that for every port call the loading and discharging time is estimated as loading or discharging of the entire cargo capacity, unable to model partial loading or discharging considered as a conservative assumption.
Case study

The two different cases are evaluated as strategic fleet size and mix problems during several time periods with uncertain parameters kept fixed with every analysis. These results are eventually compared with the corresponding analysis by use of the scenario algorithm presented in Chapter 6.
5 Results

This chapter contains a presentation of the results after the case data are transferred into readable input files and the model is analyzed in Xpress with the given input data.

5.1 Results of the test case

The results to the developed case to verify the implementation of the initial model is presented in the table.

Table 3 – Test case results

<table>
<thead>
<tr>
<th></th>
<th>Design 1</th>
<th>Design 2</th>
<th>Design 3</th>
<th>Design 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of vessels chosen</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Number of round-trips on route 1 annually</td>
<td>0.00</td>
<td>6.25</td>
<td>0.00</td>
<td>29.17</td>
</tr>
<tr>
<td>Number of round-trips on route 2 annually</td>
<td>0.00</td>
<td>25.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

The optimal fleet configuration is containing 3 vessels of design input 2, sailing 6.25 and 25 roundtrips on route 1 and 2 every year respectively. 2 vessels of design input 4 sailing 29.17 roundtrips on route 1.

These results are verified by implementing the model in Excel and using the add-in called solver; this is possible since the problem instance is small in order of magnitude. The verification model is found in Appendix C and gives the same results as the model in Xpress.

5.2 Results of the Pacific trade case

In the following are results of the Pacific trade case presented with respect to an upper- and a lower bound on spot revenues and design speed of 14.5 knots. The solution reflects an optimal solution with perfect information given by the input conditions in each case. The decisions with respect to an upper bound on spot revenues during the planning horizon are presented in the following flow charts. The corresponding tables are found in Appendix D, where the cases with speed 12 knots and 16 knots also are presented.
Results

Figure 10 – Decisions own vessels and upper bound on revenues, Pacific

Figure 11 – Decisions time chartered vessels and upper bound on revenues, Pacific

The decisions made with a lower bound on spot revenues are presented in the following two flow charts.

Figure 12 – Decisions own vessels and lower bound on revenues, Pacific
Results

These results are showing that the vessel of design type 3 is the dominant choice, and it is the decision in each case. Showing that large and fewer vessels are preferable compared to many small ones. Design type 3 have gantry cranes making the vessel more expensive, but the operational benefits with this solution is of greater importance. The number of acquired vessels is different depending on the spot revenue. The total capacity in the fleet is adjusted with vessels on time charter when needed. Another observation is that the model seeks profit gain by selling vessels when new vessels are available on time charter after the first planning period in the case of low spot earnings.

These decisions are optimal under the given input and are only possible in an ideal world. Effects such as; delays in acquiring time charter able vessels, sailing distance from where these vessels are located, financial aspects when acquiring new vessels and if it exists potential buyers in the market are neglected in the model. These aspects would eventually cause the profit to drop or increase.

The total profit – objective value of the solutions presented above is plotted as a function of speed for both an upper and lower bound with respect to spot revenues in Figure 14. This interval states where the value of the profit lies within and the large interval describe and emphasizes the influence of uncertainties in the shipping industry. The presented solutions below are the optimal decisions made under perfect information.

Figure 13 – Decisions time chartered vessels and lower bound on revenues, Pacific
If the optimal fleet sailing at the design speed of 14.5 knots and with respect to an upper bound on spot revenues presented in Figure 10 and Figure 11 with a total profit of 86 million USD do not meet the input conditions, but instead is meeting the input conditions with respect to a lower bound the total profit becomes -136 million USD, meaning a loss of 220 million USD. If the problem is solved with perfect information with respect to the lower bound a profit of -60 million USD is obtained. If this fleet is meeting the market conditions with respect to the upper bound the new total profit is 68 million USD. The company has therefore missed 18 million USD in possible profit gain by not operating the best suited fleet.

According to the discussion above the lower bound, do not represent the actual lower bound, which represent the absolute pessimistic solution. To calculate this value a worst case with the highest profit loss is analyzed. The fleet found in the top left case, upper bound and speed 12 knots is sat under the input from the lower right case, lower bound and speed 16 knots. This is only a theoretical outcome shown in Figure 15 below.
Figure 15 - Worst case scenario in the Pacific trade case

The results presented above have only two varying input parameters, spot revenue and speed. In reality, the input parameters are more uncertain and can vary heavily from year to year. The fuel price is one of these with historical large variations. If the case with an upper bound with respect to the spot revenue and speed 14.5 knots is experience a 5% increase in the fuel price the total profit will decrease by 12% from 86 to 76 million USD, but at the same time there exists a corresponding upside if the fuel price falls. The same input parameters and speed 12 knots will experience a decrease in total profit by approximately 7% from 125 to 116 million USD under the same fuel price increase, while the system with a speed at 16 knots will experience a decrease by 18% from 48 to 39 million USD. The results are summarized in Table 4 below.

Table 4 – Influence on profit from 5% increase in fuel price, Pacific

<table>
<thead>
<tr>
<th>Speed [knots]</th>
<th>Profit [mill USD]</th>
<th>Change [%]</th>
<th>New profit [mill USD]</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>48</td>
<td>-18</td>
<td>39</td>
</tr>
<tr>
<td>14.5</td>
<td>86</td>
<td>-12</td>
<td>76</td>
</tr>
<tr>
<td>12</td>
<td>125</td>
<td>-7</td>
<td>116</td>
</tr>
</tbody>
</table>

The large error margins discussed above emphasizes the importance of doing a thorough analysis before a decision is made. In addition is the ability to develop an understanding of what the outcome in the future may be, by establishing bounds important.
Results

This can be done with a sensitivity analysis changing the profit by introducing three scenarios, one representing stagnation, one represent an optimistic- and the final represent a pessimistic situation. For simplicity is all with equal probability and a design speed of 14.5 knots is used in the analysis.

Table 5 – Three defined scenario analysis in the Pacific trade case

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Spot revenue [USD/ton]</th>
<th>Profit [mill USD]</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimistic</td>
<td>80</td>
<td>86</td>
<td>0.33</td>
</tr>
<tr>
<td>Stagnation</td>
<td>65</td>
<td>4</td>
<td>0.33</td>
</tr>
<tr>
<td>Pessimistic</td>
<td>50</td>
<td>-60</td>
<td>0.33</td>
</tr>
</tbody>
</table>

The expected profit during the planning horizon can now be found with the three scenarios defined above with the following equation:

\[
E(\text{Profit}) = \sum_{i=1}^{3} \text{Prob}_i \times \text{Profit}_i
\]

\[
E(\text{Profit}) = 0.33(86 + 4 - 60) = 10 \text{ mill USD}
\]

The analysis above is only considering one varying parameter and the possibility to consider several scenarios with several varying parameters are needed. The effect on the decision by changing more parameters is made in Chapter 6.3.1, by use of the scenario algorithm.

As seen, the presented results are highly dependent of the input situation and the prediction of the future. Relative small changes in the input situation resulted in very different decisions. If the scenario used as basis for the decision made is turning out not to develop the result can be disastrous, with losses of approximately 200 million USD and more.

The scenario algorithm presented in Chapter 6 is an approach minimizing the losses by finding a fleet capable of handling a large set of generated future scenarios. The average profit with 5000 iterations is found with two different probability distributions. The average found with the uniform distribution is used in the comparison, because of the skewed properties of the exponential distribution. The average profit became 15 million USD, only 5 million larger than the expected profit found above. This indicates that the three scenario analysis above is a reasonable approximation, with respect to expected profit. The fleet composition is harder to establish with background of this analysis and the scenario algorithm therefore provides valuable information in this respect.
5.3 Results of the Atlantic trade case

Speed 14.5 knots and an upper- and lower bound with respect to spot revenues are used in the following analysis. The related tables with additional analysis of different speeds are presented in Appendix E.
Results

These results are showing that the vessel of design type 3 is the dominant choice in this trade also. The decisions made above are consisting of acquire the same number of vessels, both in the case with low and high spot revenues. Compared to the Pacific trade where the difference is very large the loss potential is smaller and this is also affecting the interval of profits, especially the actual lower bound in Figure 21.

![Diagram]

**Figure 19 – Decisions time chartered vessels and lower bound on revenues, Atlantic**

As done in the Atlantic trade case is the loss potential investigating by analyzing the effects of operating a fleet found optimal with a different scenario as input. The fleet sailing at the design speed of 14.5 knots with respect to an upper bound on spot revenues presented in Figure 16 and Figure 17 has a total profit
of 71 million USD. This fleet is instead meeting the input conditions with respect to a lower bound, which gives a total profit -76 million USD, a loss of 150 million USD. If the problem is solved with perfect information with respect to the lower bound a profit of -75 million USD is obtained and if this fleet is meeting the market conditions with respect to the upper bound the new total profit is 68 million USD. The company has therefore missed 3 million USD in possible profit gain by not operating the best suited fleet.

Compared to the Pacific trade the potential of making losses is smaller, but the potential gains are equally smaller. The actual lower bound is found with the same procedure as in the Pacific trade case, representing the worst case.

![Figure 21 – Worst case in the Atlantic trade case](image)

Analyze the effect of a 5 % increase in fuel price for the case of an upper bound on spot revenues and the different speed situations, summarized in the table below:

**Table 6 – Influence on profit from 5 % increase in fuel price, Atlantic**

<table>
<thead>
<tr>
<th>Speed [knots]</th>
<th>Profit [mill USD]</th>
<th>Change [%]</th>
<th>New profit [mill USD]</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>53</td>
<td>-11</td>
<td>47</td>
</tr>
<tr>
<td>14.5</td>
<td>71</td>
<td>-6</td>
<td>67</td>
</tr>
<tr>
<td>12</td>
<td>86</td>
<td>-5</td>
<td>82</td>
</tr>
</tbody>
</table>
Results

Table 7 – Three defined scenario analysis in the Atlantic trade case

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Spot revenue [USD/ton]</th>
<th>Profit [mill USD]</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimistic</td>
<td>80</td>
<td>71</td>
<td>0.33</td>
</tr>
<tr>
<td>Stagnation</td>
<td>65</td>
<td>-3</td>
<td>0.33</td>
</tr>
<tr>
<td>Pessimistic</td>
<td>50</td>
<td>-75</td>
<td>0.33</td>
</tr>
</tbody>
</table>

The expected profit is:

\[ E(Profit) = 0.33(71 - 3 - 75) = -2 \text{ mill USD} \]

If this result is compared to the Pacific trade case is this result lower and the profit potential in the Atlantic trade is smaller than the Pacific trade. As seen, the loss potential is reduced by 44% in the Atlantic trade case which means that the financial risks involved are lower, but the earnings drop accordingly.

By looking at the result when several parameters are varying, done in Chapter 6.3.2, the average profit is found with the uniform distribution and became -17 million USD. 15 million USD less than the expected profit found above. This is a larger difference indicating that the three scenario analysis is doubtful in the Atlantic trade case or that the assumptions made in the scenario algorithm is not reasonable.

As stated in Chapter 1.3 the environmental aspect is important to address and three methods for reducing the emissions were presented. In the calculations above the profit is highest when the fleet is optimized with a low speed as input in both cases, and this leads to a direct reduction in emissions. Reduction potential also lies in utilizing the effect of economies of scale by using larger vessels, the solution above mainly chose design type 3, which is a vessel with large characteristics. By owning the right fleet size and mix reduction is obtainable, and this is exactly the topic of this master thesis. The seen benefits are incidental obtained, but they are still valuable observations.
6 Scenario algorithm

In the case analysis in the previous chapters are the input parameters kept fixed during time and into the future. As seen in the introduction in chapter 1.1 is the reality not behaving in this pattern, but is instead fluctuating under the influence of many uncertain parameters, such as availability of capacity in the world fleet and the demand for transport of cargo dependent on the world economic situation. These fluctuations can potential alter the result with large values, and should be taken into consideration when solving a fleet size and mix problem and eventually making a decision regarding the fleet composition.

Three strategies when analyzing situations characterized by uncertainties are; simulation of the transport model with varying market conditions, stochastic modeling including probabilities of different outcomes and finally a generation of different scenarios and analysis of each scenario with the same deterministic mathematical model.

Simulation models provide flexibility to deal with uncertainty, compared to a deterministic model only able to assess one scenario at a time, a simulation model will cover the entire specter of possible outcomes. This approach can investigate variance between model runs, and hence get an idea on how the real world outcome might become. Stochastic programming is a framework for modeling optimization problems that involve uncertainty. Whereas deterministic optimization problems are formulated with known parameters, real world problems as described above include some unknown parameters. When the parameters are known only within certain bounds, one approach to tackling such problems is called robust optimization where the goal is to find a solution which is feasible for all such data.

If simulation is used, a new model fitting the situation needs to be developed using appropriate software, this is time consuming and complicated in many instances. If an optimization model is available, another approach is to change the model at hand into handling stochastic behavior, by introducing new constraints and probabilities. This is also complicated because of the modeling aspects and because the probability belonging to each scenario is difficult to assess when the uncertain parameters are many. A third approach is therefore to use the model at hand, solving the deterministic problem with different generated scenarios as input.

This chapter contains an algorithm creating multiple scenarios as input to the deterministic model made and a final instance analyzing the output, making a statistical analysis of the results when a given scenario is solved by the model. The whole structure contains three steps and can be illustrated as follows:
The optimal algorithm has a convergence test in the output handling step, which stops the iterations when an introduced criterion is fulfilled. The solution found after a given number of iterations should be able to cope with a heavy change in the surrounding parameters, making it a robust solution.

The robust solution is therefore a fleet capable of handling most of the possible future outcomes. This is used as economical risk management, minimizing the risk of owning a fleet which will perform poorly in a given future, hence minimizing losses. In addition, an interval that contains the expected profits with a certain probability is found on basis of many different futures. This makes it possible to observe the effect of several varying parameters, when comparing with the result obtained in the deterministic analysis alone. It also gives the planner a valuable estimate of the most likely region for where the profit acts.
Scenario algorithm

6.1 Step 1 – Scenario generation
The problem with this step is how the different parameters should develop during time to represent a given future. Should the scenarios be generated randomly or follow an increasing trend with short term fluctuations? Each parameter should be considered and historical data should be investigated to find a development pattern that can be reproduced in the scenario generation function. In chapter 1.1 a discussion about maritime economics is made and the generation of the scenario should imitate the seen development.

An example is investment costs which will increase as a function of interest as described in chapter 4.2.2. The change can therefore be generated by the equation presented to follow an increasing pattern. This pattern can also be seen during time for other expenditures inside the organization.

The development of vessel value, meaning selling price and time charter rates has an opposite development, with a diminishing pattern. The loss rate is stating the pace of the decrease during the planning periods.

Some of the parameters are also dependent of each other, meaning that a correlation pattern is needed to represent the development of the parameters. Example of this is speed, which affects roundtrip time and daily fuel consumption, hence voyage costs for each vessel. These parameters are also dependent of aging and aggravation of the vessels, meaning that the vessels preform slightly worse from year to year. This can be caused by fouling, wear and tear.

6.1.1 Roundtrip times with the exponential distribution
Roundtrip time is a parameter dependent on speed and can easily be calculated when the route distances are obtained. Weather and other unforeseen events are not taken into account when a constant speed is used, and by including these factors the time varies. This can be represented by a random generation of delays and potential time savings as a deviation from the average time. An exponential distribution can be used when roundtrip times are generated, in the following matter:

The cumulative distribution function for the exponential distribution is:

\[ F(x) = 1 - e^{-\alpha x}, \quad \text{for } x \geq 0 \]

Where \( 1/\alpha \) is the mean of the distribution. By setting \( F(x) = r \) thereby yields

\[ r = 1 - e^{-\alpha x}, \quad \text{for } 0 \leq r \leq 1 \]

\[ \Rightarrow x = \frac{\ln(1 - r)}{-\alpha} \]

Where \( r \) is a random generated number, since \( 1 - r \) itself is a uniform random number; the random observation can therefore be represented by the following

\[ \text{Random observation} = \frac{\ln r}{-\alpha} = -\text{mean} \times \ln r \]
This argumentation can be exploited when roundtrip times are generated for each vessel. The variations in roundtrip times are large with use of the exponential distribution, and can be as large as 100%. This is modeled to represent outer points or worst-/best case scenarios, but with less deviation than that observed.

Another approach which better suites the transport system at hand is to use the equations in chapter 4.2.2 calculating the roundtrip times and total voyage costs, and let small deviations in the sailing speed represent the delays and/or time savings. The speed then fluctuates around the mean service speed and within a desirable interval. The equation used is presented below:

\[
C_v^{Voyage,Tot}([min, max]) = P_{Fuel} \left( \alpha_p [min, max]^2 + \beta_p [min, max] \right) + \gamma_p \left( \frac{D_r}{min, max} + \frac{T_{Dead}}{C_{prod}} \right), [USD]
\]

Where the fluctuations in speed are generated randomly, random numbers in an interval stated by \( V_{min} \) and \( V_{max} \) are used. The interval representing the random numbers can be found by using the following equation.

\[
[r_{min}, r_{max}] = e^{-\left(\frac{V_{max}V_{min}}{V_{mean}}\right)}
\]

The equation is developed from the cumulative exponential distribution function, where \( r \) is the random numbers, and \( V \) is representing the speed. A new speed is generated in each scenario, which is represented by the corresponding iteration in the algorithm and is following the pattern in Figure 23 below.
The generated speed is different in each scenario and the fluctuations are controlled with the equations above, but as seen are the generated speed skewed and give a larger downside compared to the upside.

### 6.1.2 Market data with the exponential distribution

The parameters representing the market situation are cargo quantities in the existing spot trades along with the revenue when lifting these available spot cargoes. The contract quantity is decided and is a fixed parameter, but the spot trade cargo is dependent upon many macroeconomic parameters and is highly uncertain from year to year. The cyclical behavior of the maritime economical parameters is described in chapter 1.1 stating that these parameters are fluctuating about an up going trend in the long run. This up going trend is driven by economic growth represented by gross domestic product, and is simplified to follow a fixed annual growth rate. These values can also be represented by a distribution as described above and to simplify is the exponential distribution used in this matter also. The distribution is based on a given mean, and if this mean is changing the cyclical behavior can be imitated in the following matter.

\[
Q_t^{SPOT} = \frac{\ln r}{-\alpha(t)}, \text{ where } \alpha(t) = \frac{1}{Q_t^{SPOT}} = \frac{1}{Q_t^{SPOT}(t = 0)(1 + \text{growth rate})^t}
\]

\[
\Rightarrow Q_t^{SPOT} = -\ln(r)\left(Q_t^{SPOT}(t = 0)(1 + \text{growth rate})^t\right)
\]

Where \(Q^{Trend}(t) = Q_t^{SPOT}(t = 0)(1 + \text{growth rate})^t\) represents the trend.

---

**Figure 23 – Speed generation with the exponential distribution**

The cyclical behavior can be observed in the graph, where the generated speed is shown in blue and the design speed in red. The fluctuations are controlled with the equations above, and as seen, the generated speed is skewed with a larger downside compared to the upside.
Scenario algorithm

The result by using the equation above when generating a random future spot market with a trend following the growth in world domestic gross product as a function of time is shown in Figure 24 with the corresponding annual deviations from the trend presented in Figure 25, which is generated for every scenario.

![Figure 24](image-url) - Generated development of spot cargo quantity, exponential distribution

The deviation is therefore in accordance with the fluctuations around the increasing trend and can be controlled by using random numbers within a given interval. In Figure 25 is random numbers between 0.3 and 0.5 used, giving deviations as large as 20 – 30 %.
Generation of revenues on the lifted units of spot cargo can be done in the same matter as described above; by using random numbers within a specified interval the deviations from the observed revenues the year before can be controlled to fit a historical deviation pattern into the future.

By using the exponential distribution the frequency and values on the generated scenarios are observed as unevenly distributed and do not represent the reality as desired. One approach to control the large variations is to generate random numbers within an interval, as described above.

Another approach is to use the normal distribution when generating the needed values. This distribution is better representing the events by an evenly weighted distribution around a given mean with deflections controlled by the standard deviation. The approach is presented in the following:

The normal distribution is given by the equation:

$$ f(x; \mu, \sigma^2) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{x-\mu}{\sigma} \right)^2} $$

Where $\mu$ is the mean of the distribution and $\sigma^2$ is the standard deviation, with the corresponding cumulative distribution function:
Scenario algorithm

\[ F(x; \mu, \sigma^2) = \Phi \left( \frac{x - \mu}{\sigma} \right), x \in \mathbb{R} \]

Where the function \( \Phi(x) \) is given by:

\[ \Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{t^2}{2}} dt, x \in \mathbb{R} \]

The inverse of the cumulative distribution function is needed when a random observation is generated as done in the instance regarding the exponential distribution above. The inverse function is given by:

\[ F^{-1}(r; \mu, \sigma^2) = \text{random observation} = \mu + \sigma \Phi^{-1}(r), r \text{ is a random number } \in (0,1) \]

The integral above complicates the generation, because it cannot be expressed in terms of elementary functions, but numerical methods for calculation of the equation above is well known. Both Excel and Matlab provide the needed numerical formula when a distribution mean, an assumed standard deviation and a random number between zero and one are used as input. The spread around the mean is controlled with the standard deviation, and worst-/best case scenarios are obtainable.

The software used when solving the problems in this thesis does not have this formula included; the scenarios can as an alternative be generated outside with the use of Matlab as input data. The amount of generated input is thus extremely large, since most of the data are dependent on vessels, time periods, iterations and/or routes.

### 6.1.3 Continuous uniform distribution as an alternative

Another approximation method is therefore developed on the basis of a continuous uniform distribution as described in the following. The cumulative distribution function is engineered within a wanted interval around the mean of the distribution as a function of a probability \( r \).

\[ F(r) = \text{random observation} = 2\sigma r + \mu - \sigma, \text{where } r \text{ is a random number } \in (0,1) \]

The distribution mean is represented by \( \mu \) and the deviation from the mean by \( \sigma \). A scatter plot of the three distributions is presented in Figure 26 and show the skewed property of the exponential distribution and that the uniform distribution imitates the normal distribution as wanted.
The generated market data can be developed in the same matter as deviations around the trend with the following equation:

\[ Q_t^{SPOT} = 2\sigma r + Q^{Trend}(t) - \sigma = 2\sigma r + Q^{SPOT}(t = 0)(1 + \text{growth rate})^t - \sigma \]

The deviations from the trend with the uniform distribution represent the new scenarios and are presented in Figure 27. The new scenarios are generated evenly on the downside and the upside, eliminating the skewed properties of the exponential distribution. The deviations are controlled with the parameter \( \sigma \), and is here representing a 20 % change. Deviations as large as 20 % is used to represent special outcomes, such as seen in Chapter 1.1 where the financial crisis in 2009 led to a fall in trade of approximately 23 % compared to the year before. The same effect is observed when speed was generated in the calculation of voyage costs.
**Scenario algorithm**

6.2 Step 2 – Scenario analysis

In this step is the evaluation of the scenario generated in step 1 made. This is done by using the deterministic models presented in Chapter 2.4 and 2.5, solving the fleet size and mix problem with regards to maximizing profits.

6.3 Step 3 – Output handling and statistical analysis

The final step in the algorithm is output handling and a statistical analysis of the results obtained in step 2. The output is controlled and analyzed with statistical methods, such as finding the standard deviation and a confidence interval for the given output. Each scenario gives different outcomes with respect to profit, fleet size and mix. The distribution of number of vessels chosen is presented in histograms, where the frequency of the decisions made are presented on the y-axis and the number of vessels are presented on the x-axis.

All the computational analysis is performed on a Dell Latitude E6220 computer with an Intel Core i5-2520 2.5GHz processor and 4GB of RAM running on a Windows 7 operating system.

6.3.1 Analysis of the Pacific trade case

In this analysis the algorithm is stopped after both 500- and 5000 iterations of 3 years each, meaning that the problem is solved 1500 and 15000 times respectively.
Scenario algorithm

The result, called “SA solution” is compared with the deterministic solution to investigate the performance of the robust fleet. The potential benefit by generating more scenarios, hence run more iterations and consequently use longer time in the solution process is also investigated.

Table 8 – Results of the scenario algorithm with exponential distribution, Pacific

<table>
<thead>
<tr>
<th></th>
<th>500 iterations</th>
<th>5000 iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average profit</td>
<td>-24 million USD</td>
<td>-32 million USD</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>233 million USD</td>
<td>240 million USD</td>
</tr>
<tr>
<td>95 % confidence interval lower bound</td>
<td>-47 million USD</td>
<td>-40 million USD</td>
</tr>
<tr>
<td>95 % confidence interval upper bound</td>
<td>-1 million USD</td>
<td>-24 million USD</td>
</tr>
<tr>
<td>Solution time</td>
<td>1 minute</td>
<td>15 minutes</td>
</tr>
</tbody>
</table>

As the results in Table 8 for 500 iterations shows, the standard deviation is very large approximately 1000 % of the average making the result uncertain. The profit is still within the interval [-47, -1] million USD with 95 % probability and is a conservative interval compared to the interval obtained in Figure 14. The large standard deviation makes it necessary to investigate the problem with additional iterations and 5000 iterations are used in comparison. The percentage standard deviation of the average became less, approximately 25 % and the confidence interval was also reduced with roughly 30 % on both sides. These relative small changes were obtained by increasing the solution time by a factor of 15.

Large standard deviations are obtained in both cases when an exponential distribution is used, as discussed in Chapter 6.1 this is a questionable approach with skewed properties. The continuous uniform distribution developed in the same chapter is therefore used to further verify the results.

Table 9 – Result of the scenario algorithm with uniform distribution, Pacific

<table>
<thead>
<tr>
<th></th>
<th>500 iterations</th>
<th>5000 iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average profit</td>
<td>16 million USD</td>
<td>15 million USD</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>41 million USD</td>
<td>41 million USD</td>
</tr>
<tr>
<td>95 % confidence interval lower bound</td>
<td>12 million USD</td>
<td>13 million USD</td>
</tr>
<tr>
<td>95 % confidence interval upper bound</td>
<td>20 million USD</td>
<td>16 million USD</td>
</tr>
<tr>
<td>Solution time</td>
<td>55 seconds</td>
<td>11 minutes</td>
</tr>
</tbody>
</table>

The results in Table 9 shows that this approach give much smaller standard deviations, around 250 % and much higher profits compared to the results obtained with the exponential distribution. This may be caused by the fact that the exponential distribution generates more scenarios on the down side compared to the uniform distribution, as seen in Chapter 6.1.3. The large difference between the approaches makes the analysis more doubtful. The difference between the two runs is very small, stating that the gains by running the model with more than 500 iterations can be neglected.

The chosen fleet with information from the scenario algorithm is therefore in need of some sort of post analysis. This is done by investigate the SA solution obtained in a given situation and compare with the optimal result in particular for that input situation. The interesting situation is the poorest one, stating
Scenario algorithm

the largest possible loss with the new decided fleet. This is typically a worst case scenario outside the confidence interval, which is important to assess.

The fleet size and mix decision is in the following presented in histograms showing the distribution of the decisions made, where the result obtained with 5000 iterations are used and zero vessels chosen is marked as trivial and neglected in the figures. The distribution regarding time chartered vessels are found the same in every case, both in the Pacific trade and the Atlantic trade, where 2 vessels of design 1 are kept from the existing fleet in the first planning period and taken out in the beginning of the second period basing the operation on own vessels exclusively. The distribution is shown in Figure 28.

\[ \text{Figure 28 - Distribution of time chartered vessels found with the scenario algorithm} \]

The distribution regarding owned vessels with an exponential- and a continuous uniform distribution are presented in Figure 29 and Figure 30 respectively.
The low average profit obtained with the exponential distribution is caused by a general low earnings potential in the generation of the scenarios choosing a low number of vessels on average.

Figure 29 – Distribution owned vessels found with exponential distribution, Pacific

Figure 30 – Distribution owned vessels found with uniform distribution, Pacific
With the use of the uniform distribution the weight is around 4 vessels of design 3. The fleet is now consisting of 2 vessels of design 1 on time charter and 4 vessels of design 3 which is acquired in the first planning period and kept through all planning periods. This fleet is further investigated with the same analysis as was made in the deterministic case, where only speed and spot revenues was changing parameters.

The SA fleet is decided upon more changing parameters, and in the following is a post analysis of the SA solution made. The effect on the SA solution is analyzed by the same changes in the input situations as was made in the deterministic case. This is done to compare the decision made with perfect information and the SA decision. The worst case result is plotted in the same figure, shown in Figure 31.

![Figure 31 – Plot of worst case with SA solution, Pacific](image_url)

The plot with name “Actual lower bound” represents the largest possible loss when an optimized fleet in one situation is suddenly experience changes in the surrounding parameters. The same assessment is made with the SA solution, and is represented by the line “SA lower bound”. The plot shows that the potential loss has decreased by 114 million USD.
6.3.2 Analysis of the Atlantic trade case

The same analysis as made in Chapter 6.3.1 is made in this chapter with respect to the Atlantic trade case.

Table 10 – Results of the scenario algorithm with exponential distribution, Atlantic

<table>
<thead>
<tr>
<th></th>
<th>500 iterations</th>
<th>5000 iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average profit</td>
<td>-48 million USD</td>
<td>-45 million USD</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>238 million USD</td>
<td>226 million USD</td>
</tr>
<tr>
<td>95% confidence interval lower bound</td>
<td>-72 million USD</td>
<td>-53 million USD</td>
</tr>
<tr>
<td>95% confidence interval upper bound</td>
<td>-24 million USD</td>
<td>-38 million USD</td>
</tr>
<tr>
<td>Solution time</td>
<td>2 minutes</td>
<td>18 minutes</td>
</tr>
</tbody>
</table>

The standard deviation in these particular cases is also very large and in magnitude approximately 500% of the average in both instances. This is a result of large variations in the profit from iteration to iteration due to a large interval for the given random observations and the skewed properties related to the exponential distribution as discussed in Chapter 6.1. The difference in result is very small when the number of iterations is increased extensively, the standard deviation is approximately within the same region of percentage change from the average, but as seen in the Pacific trade case is the confidence interval reduced due to more observations.

The uniform distribution is used in the following analysis and gives a higher average profit which is closer to the solution when solving the deterministic problem with fixed input values. The varying parameters have an individual deviation controlled as input with a maximum of 22% each. The accumulated standard deviation becomes larger, approximately 160% of the average and is smaller compared with the exponential distribution.

Table 11 – Results of the scenario algorithm with uniform distribution, Atlantic

<table>
<thead>
<tr>
<th></th>
<th>500 iterations</th>
<th>5000 iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average profit</td>
<td>-17 million USD</td>
<td>-17 million USD</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>28 million USD</td>
<td>28 million USD</td>
</tr>
<tr>
<td>95% confidence interval lower bound</td>
<td>-19 million USD</td>
<td>-18 million USD</td>
</tr>
<tr>
<td>95% confidence interval upper bound</td>
<td>-14 million USD</td>
<td>-16 million USD</td>
</tr>
<tr>
<td>Solution time</td>
<td>1 minute</td>
<td>22 minutes</td>
</tr>
</tbody>
</table>

As Table 11 shows is the difference between the two solutions small expect for the solution time. It can therefore be concluded that the algorithm solved with a small number of iterations give a good approximation.

The distribution of the fleet size and mix decision is presented in Figure 32 and Figure 33, related to own vessels found with the exponential- and the uniform distribution respectively.
Scenario algorithm

Figure 32 – Distribution owned vessels found with exponential distribution, Atlantic

Figure 33 - Distribution owned vessels found with uniform distribution, Atlantic
Scenario algorithm

The number of vessels which appear with the highest frequency is chosen into the robust fleet. By basing the decision on the result presented in the histogram, the SA fleet is consisting of 2 vessels of design 1 on time charter and 6 acquired vessels into the owned fleet.

The same analysis is made with the new SA fleet and presented in Figure 34, and the new solution give approximately the same result as in the deterministic case. This may be caused by the fact that the risk is already assumed as lower in Atlantic trade, making it harder to make any improvements.

---

*Figure 34 – Plot of worst case with SA solution, Atlantic*
6.4 Discussion
Most financial institutions approach risk by concentrating on the relationship between risk and profit requiring more volatile investments to pay higher profits. This is also observed in this thesis where lower profit gives lower risk. Risk, measured by the volatility of the investment where the volatility is defined as the loss potential proves that the statement above is sound and the risk in the Atlantic trade is smaller than the risk in the Pacific trade case. The algorithm decreases the loss potential, hence decreases the risks involved. But is the scenario algorithm necessary, or could the average deterministic case give an approximate solution?

An interesting aspect is therefore comparing the SA solutions with the fleet decisions found in the deterministic situation related to the scenario called stagnation, presented in Table 5 and Table 7, which is based upon average input values. Since the scenarios are randomly generated the hypothesis is that the SA solution is converging to the average solution solved with the deterministic approach. The average solutions gave approximately the same fleet, but with several adjustments during the planning periods. These adjustments are made according to the specific input situation and the SA solution rejects this advantage, hence making the solution consistent and able to cope with several scenarios.

It is of interest to use other approaches to the problems related to uncertainty in maritime transportation mentioned in the beginning of this chapter in comparison. This should be done to investigate the strengths and weaknesses with the algorithm and the validity of the solutions.
7 Conclusion
The shipping industry has as all markets two main components, supply and demand, where demand is strongly connected to the world economy. Trends in the world economy are an indication of how the shipping markets fluctuate and how the demand will develop. On the supply side are the main drivers the number of newbuildings and scrapping. The shipping industry is therefore characterized by uncertainty at all planning levels. For fleet size and mix decisions at a strategic planning level, the uncertainty is even stronger due to the long time horizons.

The optimization models presented in the beginning of this master thesis is of deterministic nature, meaning that all input parameters are known. Focus is placed upon a model solving the fleet size and mix problem during several stated planning periods with a known input situation and based on these assumptions the optimal value is found. The validity of the result relies solely of the validity of the input data and these predictions needs to coincide with the real life development in order for the result to maintain its validity. As discussed are many of the parameter used as input in the model varying differently both at long- and short terms. This makes the decision made with the given input sensitive for changes, and since it is a high probability for these predictions turning out being inaccurate the potential losses are large.

Two different trades are used as cases and in Chapter 5.2 and 5.3 is these solved with the models presented. Some input parameters are changed and the differences are investigated. The main findings imply that only relative small changes of the input parameters resulted in very different decisions. If one scenario is used as basis for the decisions made, one can only hope that the scenario will take place. Since most planners are not capable of seeing into the future and hence know the market development, another scenario may develop e.g. spot revenues will decrease instead of a planned increase, the result can be disastrous with losses in the region of 100 – 200 million USD during three years. Meaning that the losses can be even larger in the years to come, and end in bankruptcy. The forecast is highly dependent of time and gets more uncertain if the planning horizon is extended. These aspects should therefore be assessed when a deterministic optimization is used in the planning process.

The scenario algorithm presented in Chapter 6 is an approach developed to treat uncertainties by minimizing the losses by finding a fleet capable of handling a large set of generated future scenarios. The approach is divided into three main steps; the scenario generating step where development are based on historical fluctuations, a deterministic solution with the given scenario as basis and finally storing of all the solutions with a statistical analysis of the output.

The algorithm was used on the two cases with two different scenario generating approaches, based on an exponential- and a continuous uniform distribution respectively. The fleet size and mix decisions which appeared with the highest frequency were chosen, and gave a consistent estimate based on risk aversion decreasing the potential of making losses.

The approaches presented in this thesis is not meant to give a correct answer on how the future will be, but help the decisions makers reduce the uncertainty connected to the strategic decision. The
Conclusion

deterministic model give valuable information with a given scenario as input, but it is dependent on whether the scenario used actually turns out in real life. The model is thus only capable of evaluate the scenarios individually, and the result found by the scenario algorithm evaluating scenarios collectively is of higher value since it provide a more robust solution.

The optimization process is solely based upon profit maximization and is not assessing the reliability of the fleet and the routing of the vessels. This can cause even larger losses if the solution fails at an operational level. In a possible extension of this work, it might be worthwhile to apply simulation techniques for evaluating the robustness of the decision made. Questionable assessments that have been made in this thesis could then be avoided.
8 Bibliography


Bibliography


Bibliography


Appendix A – Pacific trade data

Pacific trade, defined routes

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Pacific trade, distance matrix in nautical miles

| Pacific trade, North America West Coast - Far East |
|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
|                      | Vancouver | Squamish | Qingdao | Shanghai | Ulsan | Tokyo | Manila |
| Vancouver            | 0         | 39       | 5115    | 5110     | 4644  | 4283  | 5954  |
| Squamish             | 39        | 0        | 5154    | 5149     | 4683  | 0     | 0     |
| Qingdao              | 5115      | 5154     | 0       | 312      | 510   | 0     | 0     |
| Shanghai             | 4644      | 4683     | 510     | 487      | 0     | 0     | 1028  |
| Ulsan                | 4283      | 0        | 0       | 1028     | 0     | 0     | 1779  |
| Tokyo                | 5954      | 0        | 0       | 1106     | 0     | 1779  | 0     |

Source:
http://sea-distances.com/
http://www.searates.com/reference/portdistance/

Pacific trade, route characteristics

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### Verification of results

#### Verification of the initial model

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### Appendix D – Pacific trade results

Result of the case with speed 16 knots in the Pacific trade case

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Appendix E – Atlantic trade results

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