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Precautionary Storage in Electricity Markets

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Abstract

As renewable energy depends on meteorological shocks and is non-controllable, the overall energy production becomes riskier with the rising renewable share. Although this has led to a renewed interest in storage technologies, not much consideration has been given to energy storage due to precautionary motives. In our study, we look at to what extent a convex marginal utility (prudence) and a convex marginal cost (frugality) can spur precautionary energy storage. We set up a simple theoretical model of energy consumption and production with intermittent renewable sources, dispatchable systems, and energy storage. First, we characterize the optimum and demonstrate how prudence and frugality can lead to higher levels of energy storage. By applying our findings to perfectly competitive markets, we further show that prudence and frugality increase the market energy price through higher demand for energy storage and decrease price volatility. Our analysis can have implications for inventory decisions in various other industries where firms face capacity constraints and are exposed to production risks.
Keywords: Precautionary energy storage; Intermittency; Renewable energy; Fossil fuel energy; Prudence; Frugality; Rational Expectations Equilibrium
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1 Introduction

In light of the agreement reached at 2015 Paris Climate Conference (COP21), which requires countries to limit their emissions to keep the global temperature rise well below 2°C, the renewable share of energy generation is expected rise considerably in the years to come. As renewable energy (RE) is inherently variable and uncertain, however, the overall

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energy production becomes riskier with the rising renewable share. A number of strategies exist to deal with the challenges posed by intermittent RE generation.[1] The use of dispatchable generation is one example in this regard. It refers to the production of electricity using steam and internal combustion turbines (e.g., natural gas power plants) to avoid mismatches between supply and demand. A demand response is another way to enhance the electrical grid's resilience and enable a greater use of RE. It relates to the presence of end-use consumers in electricity markets who can monitor and change their electricity consumption in response to changes in the electricity price (DOE, 2006).[2]

One other way of enhancing the reliability of the grid is energy storage. Energy storage systems absorb energy during periods of excess capacity and release it when the output from RE is low and dispatchable generation is expensive to balance the power system. Key benefits include providing balancing services, such as load following, supplying power during brief disturbances, and serving as substitutes for network transmission and distribution upgrades (Wang et al., 2012).[3] Currently, the cost of electricity storage is high. However, with the development of better storage systems with larger storage capacities, they may become game changing technologies.[4]

When consumers are responsive, and energy generators –in particular, dispatchable generators– are responsible to match electricity supply with demand, two precautionary motives can lead to a higher demand for energy storage. One is prudence with respect to electricity consumption, which is formally equivalent to a positive third derivative of the utility function. The other is frugality, which is formally equivalent to a convex marginal cost of dispatchable generation. We refer to the property of a convex marginal cost function as frugality, since, in the presence of uncertainty, it endows a cost minimizing producer with the same motivations as that of a prudent consumer. In Section 2, we will motivate the properties of prudence and frugality and give a first intuition as to why they encourage energy storage.

In this study, we show how prudence and frugality drive precautionary energy storage. We first look at a social planner's problem and examine how storage decisions are influenced in the presence of a convex marginal utility (prudence) and a convex marginal cost (frugality). We then turn to a decentralized setting and discuss how current and future energy prices, the price volatility, and the use of energy systems are influenced by prudence, frugality, the degree of intermittency and price elasticities. Our results indicate that

[1] Intermittency means that RE generation depends on meteorological shocks and is non-controllable (Steffen and Weber, 2013).

[2] Smart meters and applications that record and display data on energy consumption allow consumers to access real-time knowledge about prices, be more responsive and thus control their power usage. When active engagement is not practical, consumers can also have access to smart appliances that can react to prices based on criteria set by the consumer (Hamilton et al., 2012). With sustained investments, it is projected that the smart grid will provide a communications network for the energy industry by 2020; that is, a system of interconnected energy networks similar to the Internet in terms of its provisions for business and personal communications (RMI, 2014).

[3] Load following refers to changes in power generation to meet the energy demand; that is, the load.

[4] The energy storage industry is experiencing strong growth and it is expected that the industry will have a global net worth of $10.8 billion in 2018 (RMI, 2014).

prudence and frugality can cause precautionary energy storage. Even in the absence of prudence, we demonstrate that frugality can still allow for precautionary storage and vice versa. Furthermore, a higher degree of intermittency can boost energy storage when prudence, frugality, or both, is present. Higher demand and supply elasticities diminish the effect of prudence and frugality, respectively, on precautionary energy storage. For a highly elastic demand, demand response becomes a good substitute for energy storage and in turn lower the need for precautionary energy storage. When energy supply is more price elastic, dispatchable generation becomes a better substitute for storing energy.

To the best of our knowledge, our findings with regard to the impact of the precautionary motives on electricity storage and prices are novel within the energy economics literature. This is also the first study to look at the effects of the producers' risk attitudes, that is, frugality, on precautionary storage. Frugality can have implications for inventory decisions in various other industries (petroleum, food, transportation, lumber, primary and fabricated metals and industrial machinery industries to name a few) where firms face capacity constraints and are exposed to production risks. Therefore, its scope of application is not limited to the energy market.[5]

The remainder of the paper is organized as follows. Section 2 motivates the properties of prudence and frugality and gives a first intuition as to why they encourage energy storage. Section 3 reviews the related literature. Section 4 presents the model, states the social planner's problem and discusses its solution. Section 5 turns to a decentralized setting and looks at the role of prices in coordinating the energy market in the presence of the precautionary motives. Section 6 concludes.

2 Motivations for prudence and frugality

Prudence

Let us explain what it means to be prudent in our framework. Consider a consumer with a (gross) surplus function, $U(e)$, which is increasing, $U' > 0$, and concave, $U'' < 0$, in electricity consumption, e.[6] Suppose that the consumer is exposed to a zero-mean consumption risk, $\tilde{x}$. The difference between certain and expected surplus is given by

$$k(e) \equiv U(e) - \mathbb{E}[U(e + \tilde{x})].$$

[5]Our model shares similar features with the standard competitive commodity storage models. This is mainly related to the fact that in this literature the production, that is, the harvest, also depends on weather conditions and can be stored. Yet, we are not aware of any study that formally demonstrates the implications of precautionary motives for commodity storage. In this regard, our study has the potential to fill a gap within this literature.

[6]We consider a consumer with a quasi-linear utility function over electricity consumption and a numéraire commodity. Thus, $U(e)$ is the monetary value of utility derived from consuming e kilowatt-hour of electricity.

Due to the Jensen's inequality, $k(e)$ is positive if $U(e)$ is concave. In other words, uncertainty is costly for the consumer when he/she is risk averse.

A consumer is said to be prudent with respect to electricity consumption if the cost of uncertainty, $k(e)$, decreases as consumption, e, increases. In differential terms, this is equivalent to $k'(e)$, given by

$$k'(e) = U'(e) - \mathbb{E}[U'(e + \tilde{x})],$$

being negative, which is ensured by the convexity of the marginal surplus; that is, $U''' > 0$. Again, this results from the Jensen's inequality. As consuming stored energy is one way to increase e, and thus, to decrease the cost of uncertainty, $U''' > 0$ –that is, prudence– gives a *prima facie* argument for energy storage.

Now let us explicate how the consumption risk can emerge for an electricity consumer. Experience shows that intermittent RE increases the volatility in the price of electricity (Jonsson et al., 2010; IPCC, 2012; Ketterer, 2014). Thus, when a consumer is endowed with an equipment that can inform her about the market price, she will adjust her consumption in response to changing prices (e.g., use less electricity at times when electricity prices are high). Nevertheless, the price-taking behavior will also expose her to consumption risk.

Focusing on income lotteries, the evidence for prudence can be found in the experimental research literature. In line with the prediction of precautionary saving theory, Noussair et al. (2014) indicate that the majority of individual decisions is consistent with prudence.[7] Crainich et al. (2013) provide theoretical arguments to show that prudence is more prevalent than risk aversion, as risk lovers can also demonstrate it. This prediction is shown to hold in Ebert and Wiesen (2014) and Deck and Schlesinger (2014). Accordingly, prudence may be a more universal trait, which suggests that narrowing down risk preferences to the second-order may obscure valuable information. There are also empirical studies such as Chavas and Holt (1996) and Guiso et al. (1996) that support prudence. Carroll and Samwick (1998) indicate that wealth holdings are positively and significantly related to income uncertainty.[8]

Frugality

In this subsection, we shall expound frugality. Consider a producer with an increasing cost function $C(q)$, where q is the level of production. Suppose that the firm faces a zero-mean production risk, $\tilde{x}$. Here, $\tilde{x}$ represents the variation in the residual demand that the firm has to match with its supply. The difference between the expected and the certain cost of production

[7] Noussair et al. (2014) also argue that the degree of prudence has implications in a wide range of economic applications such as bargaining, bidding in auctions, rent seeking, discounting, sustainable development and climate change, and tax compliance.

[8] Carroll and Kimball (2008) argue that, although there is evidence for prudence, it is measured differently with different data; that is, the degree of the same motive changes among different data sets.

is as follows:

$$\rho(q) \equiv \mathbb{E}[C(q+\tilde{x})] - C(q).$$

Due to the Jensen's inequality, the firm is exposed to a penalty of uncertainty when $C'' > 0$ (i.e., the cost function is convex). In other words, increasing marginal cost implies that uncertainty is costly for the firm: $\rho(q) > 0$.

A producer is said to be frugal with respect to energy generation if the cost of uncertainty, $\rho(q)$, increases as production, q, increases. This is equivalent to

$$\rho'(q) \equiv \mathbb{E}[C'(q+\tilde{x})] - C'(q)$$

being positive, for which the convexity of the marginal cost (i.e., $C''' > 0$) is sufficient. Once again, this results from the Jensen's inequality. As using stored energy is one way to decrease q, and thus, to decrease the cost of uncertainty, $C''' > 0$ –i.e., frugality– provides a second *prima facie* reason for energy storage.

By analyzing production and inventory data, Cecchetti et al. (1997) find evidence supporting a positive third derivative of the cost function, and note that, from an operational perspective, a firm is capacity constrained when faced with a convex marginal cost curve. Indeed, a convex marginal (production) cost curve has a transparent economic interpretation, which indicates that it becomes increasingly expensive to make large and positive changes to meet the residual demand.

Now let us explain how the production risk emerges for a fossil fuel power generator. Variations in energy demand are typically limited and more predictable compared with the variations in supply (Nyamdash et al., 2010; Hart et al., 2012; Ummels et al., 2007). However, due to the low operating cost of intermittent RE that leads to its earlier dispatch (Denholm et al., 2010), the residual load is intermittent. Therefore, after accounting for RE, a capacity constrained dispatchable generator that has to supply the residual load can incur high operating costs especially during periods of peak demand and low renewable energy generation. As a result, a frugal firm will intend to balance its capacity-constrained supply and the residual load in such a way that it minimizes its expected cost.

3 Related literature

The optimal dispatch of energy and energy storage was addressed earlier in the operations research literature. In a model of hydroelectric and dispatchable systems, Little (1955) studies hydroelectric generation under uncertainty. Disregarding fluctuations in energy demand, the study determines optimal energy dispatch and water storage policies. Borrowing most of his assumptions from Little (1955), Koopmans (1957) calculates the optimal energy generation and storage policies in the presence of complete certainty.[9] He shows how dispatchable

[9]Koopmans (1957) argues that the purpose is to develop concepts and tools that will be useful in a systematic analysis of cases involving uncertainty.

generation and storage decisions are related to the energy prices and storage rents.

With a few exceptions, however, the economics of pumped-storage hydroelectricity (PSH) has not attracted many researchers so far. An early work on the economics of PSH is Jackson (1973) where the motivation to use PSH is due to its ability to meet the varying load as nuclear power cannot be ramped up and down rapidly. In his analysis, Jackson assumes that storage is always optimal, and hence, the technology always pumps water to an upper reservoir. In contrast, Gravelle (1976) shows the conditions under which storage is efficient. Assuming that demand deterministically varies between off-peak and peak periods, he shows that storage allows the substitution of less costly off-peak production for highly valued peak production. In return, peak consumption is substituted for off-peak consumption. Horsley and Wrobel (2002) build on the framework provided by Koopmans (1957) and study the optimal operation of existing PSHs and the valuation of energy and storage rents in the presence of uncertain inflows.

Crampes and Moreaux (2010) build their work on Jackson (1973) and Gravelle (1976). Unlike Horsley and Wrobel (2002), who assume an exogenously given demand and perfectly efficient conversion, they investigate the optimal dispatchable generation and PSH when energy demand varies deterministically between peak and off-peak periods and there are losses in converting energy. Assuming a merit order in using dispatchable generators, the study first calculates a frontier that separates storage and no-storage solutions given technical conditions such as operation cost characteristics and energy losses. The authors then calculate the socially optimal allocation given consumer preferences. When dispatchable generation is used to pump water to an upper level reservoir, the welfare losses corresponding to this off-peak period is compensated by welfare gains in the peak period when stored water is used. In line with Jackson (1973) and Gravelle (1976), the study then discusses the implementation of an optimal energy dispatch in competitive markets where agents are price takers. The calculations show that the peak and off-peak price differential is reduced when storage is feasible.

The literature on commodity storage has relevant implications for the economics of energy storage. Wright and Williams (1982, 1984) examine the welfare effects of storage in a market with stochastic supply and indicate that the welfare effects of storage depend on the specification of the inverse demand function (that is, the slope and curvature of the demand curve). The authors introduce a parameter that is analogous to the coefficient of relative prudence (cf. Kimball, 1990) and argue that agents will pay for a mean-preserving decrease in the variability of the commodity when relative prudence is bigger than one (Wright and Williams, 1984; Williams and Wright, 1991). Given the storage and current production (that is, the amount on hand), the authors derive a storage rule numerically. Accordingly, when the stored amount is less than a particular threshold, all of the stored commodity will be consumed, and vice versa. Numerical simulations indicate that storage is more likely and the marginal propensity to store at the threshold increases when there is a higher degree of variability in supply (Wright and Williams, 1982).

Regarding the relationship between the degrees of variability in RE and energy storage, one finds similar results in the operations research and economics literature. Tuohy and

O'Malley (2011) argue that intermittency increases the benefit driven from the flexibility offered by PSH and makes energy storage more attractive. Evans et al. (2013) demonstrate that water storage becomes more welfare-enhancing with higher uncertainty.

It is surely possible to find more studies that associate higher levels of storage to higher degrees of variation in the RE supply. However, the role that precautionary motives play is not elaborated upon adequately. Evans et al. (2013) assume a linear demand schedule (i.e., $U'' > 0$ and $U''' = 0$) and a convex supply schedule (i.e., $C'' > 0$ and $C''' > 0$) for dispatchable generation. As we will show, frugality will lead to precautionary energy storage, unless capacity constraints are explicitly considered for each dispatchable unit. Evans et al. (2013) do not address such a relationship. In Bobtcheff (2011), the cost of dispatchable generation is constant and not subject to any capacity constraints; that is, her model disregards frugality. She numerically shows that a social planner keeps more water in a reservoir when faced with higher uncertainty and explains that this action is due to prudence. However, she does not present a formal analysis.

In our work, we are interested in storage technologies that are more suitable for energy management applications. These applications have the ability to shift the bulk of energy for a duration of several hours or more (Denholm et al., 2010), and hence, can insulate the rest of the power grid from substantial changes in the power supply and demand. One example of energy management applications is electric energy time shift, which means charging a storage device when electricity prices are low (e.g., storing excess wind power during periods of low energy demand) and then discharging the device when electricity prices are high (Lichtner et al., 2010; Kim et al., 2012). High energy batteries, pumped hydro (the most widely used form of electrical energy storage), and compressed air energy storage are the technologies for this type of applications (Denholm et al., 2010).

Although we focus on uncertainty in RE only, we do not neglect variations in demand and employ a deterministic demand that varies between off-peak (or night) and peak (day) periods.[10] Even though we work with a deterministically varying demand function, it can be noted that the residual load is intermittent. This is due to the low operating cost of intermittent RE that leads to its earlier dispatch. After accounting for RE, the net but intermittent load is met by the peaking power plants or "peakers".

Our work discusses storage and no-storage solutions and indicates that intermittency can lead to a higher level of storage when agents are prudent and frugal. The latter is due to the structure of the energy markets where, after accounting for the RE, the dispatchable generators supply the residual load. Thus, different than the literature on commodity storage, the demand for storage not only depends on the curvature of the demand curve but also on the supply curve.[11]

[10] Compared with the variations in supply, the variations in demand tend to be limited and more predictable (Nyamdash et al., 2010; Hart et al., 2012; Ummels et al., 2007).

[11] In our study, we could investigate the conditions under which energy storage would increase investments in RE and the other way around, and analyze how the results would depend on prudence and frugality. Yet, such an inquiry would require the use of derivatives with orders higher than three. We plan to pursue such issues in future work.

By applying our findings to perfectly competitive markets, we show that precautionary motives can lead to a higher spot market electricity price through higher a demand for energy storage, and in turn, decrease future price as well as its variability. Furthermore, while a higher price elasticity of demand decreases the effect of prudence (that is, consumption adjustment becomes a stronger substitute for stored energy), a higher supply elasticity diminishes the precautionary storage motive from frugality. This is because the intermittent residual load can be more easily met by dispatchable systems. We further demonstrate that a higher degree of intermittency leads to higher level of energy storage. Lastly, precautionary storage depends positively on the coefficients of relative prudence and frugality. Thus, in response to the overall energy production risk, energy storage, and therefore, the spot market electricity price, will increase with higher levels of relative prudence and frugality.

4 The model

We consider a two-period model. In the initial period the demand for energy is low. Let us call this the off-peak period. In the final period, we call it the peak period, the demand is high. Thus, the marginal gross surplus derived from the same level of electricity consumption is higher in the peak period. Algebraically, this can be shown as $U'(e-\epsilon) \geq U'(e)$ where e is energy consumption and ϵ is a positive constant. Let $U_0(e) \equiv U(e)$ and $U_1(e) \equiv U(e-\epsilon)$ denote the gross surplus function in the off-peak and on-peak periods, respectively.

Energy can be supplied from dispatchable generation, renewable sources, and energy storage systems:

$$q_t = y_t + z_t + s_t - \alpha s_{t+1}, \tag{1}$$

where q_t is total energy supply ($t = 0, 1$), y_t is dispatchable generation, z_t is RE, s_t is the level of stored energy. For $\alpha > 1$, $1/\alpha$ is the round-trip efficiency parameter. It is the ratio of energy recovered to the initially stored energy.[12] Hence, a certain percentage of stored energy is lost with time.[13] We assume that the power grids are smart, that is, the transmission and distribution systems of electricity are added with digital sensors and remote controls (Ambec and Crampes, 2012; van de Ven et al., 2013; Evans et al., 2013). This assumption instantaneously lets the prices adjust, such that the energy supply meets the demand at all times: $e_t = q_t$. Thus, there is no overloading of the power grids.

While z_0 is observed prior to making decisions in the initial period, z_1 is uncertain and therefore is denoted by $\tilde{z}_1$. In the rest of the analysis, we indicate that a variable is random by placing a tilde over it. Once the RE system is installed, the unit cost of generating RE

[12]It is possible to assess different types of storage technologies by using different round-trip efficiency parameters.

[13]Given various storage technologies with differing round-trip efficiencies, we could consider a unique merit order of using storage systems. Although, such an assumption would diminish the level of energy storage and take our model one step closer to reality, it would not affect our key results.

becomes so low that we consider it as zero (Ambec and Crampes, 2012; Evans et al., 2013; Førsund and Hjalmarsson, 2011).[14] Thus, the renewable system operates at its capacity, $\bar{z}$. Yet, as the weather conditions are uncertain, so is the RE generation. Let $\tilde{z}_1$ be independently and identically distributed (i.i.d.) with a commonly known cumulative distribution function, $F(z)$ and a compact support $[0, \bar{z}]$, and have mean, μ, and variance, σ^2.

The cost function for dispatchable generation is denoted by $C(y)$. It is increasing in dispatchable generation, $C'(y) > 0$, with $C''(y) > 0$ and $C'''(y) \geq 0$, where C', C'' and C''' are the first-, second- and third-order derivatives of the cost function, respectively. When the marginal cost is increasing, one can think of a unique merit order of using individual generators: initially the power plants with the lower marginal costs of energy generation will be brought online (such as a coal-fired power plant), followed by costlier ones (such as a natural gas power plant with carbon capture and storage). We assume that given the market price for energy, there is no constraint on the availability of y, that is, there is a large existing generating capacity portfolio that can meet the demand when RE is not adequate to supply the total load (Joskow, 2011; Bobtcheff, 2011; Tsitsiklis and Xu, 2015). Yet, when $C'''(y) > 0$, one can think of an implicitly assigned capacity constraint such that the effect of convexity dominates for high levels of dispatchable generation.

$U(q)$ is the gross surplus function over kilowatt-hour consumption of energy. It is assumed that $U' > 0$, $U'' < 0$ and $U''' \geq 0$, where U', U'' and U''' are the first-, second- and third- order derivatives of the surplus function, respectively. Thus, under perfect competition, the inverse demand schedule is downward sloping and convex.

We study the model as a social planner's problem, in which the planner makes energy generation as well as storage and consumption decisions. The planner's problem is the following:

$$\max_{\{q_0, q_1, y_0, y_1, s_1\}} \quad U_0(q_0) - C(y_0) + \mathbb{E}\left[U_1(\tilde{q}_1) - C(\tilde{y}_1)\right] \tag{2a}$$

$$\text{subject to} \quad q_0 \geq 0, \tilde{q}_1 - \epsilon \geq 0, \; y_0 \geq 0, \; \tilde{y}_1 \geq 0, \tag{2b}$$

$$\bar{s} \geq s_1, \; s_1 \geq 0 \; \text{ and } \; s_0 \geq 0 \; \text{ given.} \tag{2c}$$

As the weather in the next period is uncertain, we use $\mathbb{E}[\cdot]$ to denote the expected net surplus in period 1. Energy consumption (net of ϵ) is positive and dispatchable generation can equal zero (that is, become idle) when the RE generation is sufficiently high (cf. Eq.(2b)). As we focus on a relatively short time horizon, such as one day, we take the RE generating and storage capacities, $\bar{z}$ and $\bar{s}$, respectively, as fixed. When there is sufficiently high RE generation such that the storage capacity is reached, we assume that the remaining energy will be consumed. Furthermore, stored energy cannot be negative; that is, we cannot borrow energy from the future to consume today. Throughout the study, we assume $s_0 = 0$. This assumption does not change the main results of the study, which identify prudence and frugality as the main drivers of precautionary storage. However, we shall comment on the possible effects of $s_0 > 0$ later

[14] The only cost of RE generation is the opportunity cost of not generating more than the capacity of the system.

in the study. For simplicity, we neglect discounting between the first and final periods.[15] Lastly, we assume that the energy demand is independent of the weather conditions.

Solving the model

We solve the problem recursively. Given RE generation in the last period, z_1, and the available amount of stored energy, s_1, the problem in period 1 is as follows:

$$\max_{\{q_1, y_1\}} U_1(q_1) - C(y_1)$$
$$\text{subject to} \quad q_1 - \epsilon \geq 0, y_1 \geq 0.$$

The first-order necessary condition for a maximum yields:[16]

$$U_1'(y_1 + z_1 + s_1) \leq C'(y_1), \text{ with equality if } y_1 > 0. \tag{3}$$

If the level of energy supplied by the renewable systems and energy storage is sufficiently high such that the marginal surplus will become less than the marginal cost of fossil fuel energy, then no dispatchable generation will take place: $U_1'(z_1 + s_1) < C'(0)$. Otherwise, $U_1'(y_1 + z_1 + s_1) = C'(y_1)$ and the dispatchable systems will be active. As a result, one can calculate a threshold level, τ, such that when $z_1 > \tau$, the dispatchable systems will become idle, and vice versa:[17]

$$y_1^* \geq 0 \quad \text{if} \quad z_1 \leq \tau, \tag{4a}$$
$$y_1^* = 0 \quad \text{otherwise} \quad (\text{i.e., } z_1 > \tau), \tag{4b}$$

where we denote the optimal dispatchable generation decision by $y_1^* \equiv y(z_1 + s_1)$. When the weather conditions are such that the level of RE is lower than τ, Eq. (4a) demonstrates that the dispatchable systems will be used to meet the residual demand. In contrast, when RE generation is sufficiently high, the dispatchable systems will be shutdown.[18] Given y_1^*, the maximum value function for period 1 is

$$W_1(z_1, s_1) = U_1(y_1^* + z_1 + s_1) - C(y_1^*). \tag{5}$$

[15]This is a reasonable assumption given our focus on a relatively short time horizon. When considering the allocation of production between seasons (summer and winter) in a hydropower system with reservoir constraints, it will be beneficial to introduce discounting.

[16]The second-order condition for a maximum is satisfied by $U_1''(q_1) - C''(y_1) < 0$.

[17]Using $U_1'(z_1 + s_1) < C'(0)$, one can calculate τ as $z_1 > \tau \equiv U_1'^{-1}(C'(0)) - s_1$.

[18]When there is an interior solution for dispatchable generation, the comparative statics provide $\frac{\partial y_1^*}{\partial z_1} = \frac{U_1''}{C_1'' - U_1''} < 0$ and $\frac{\partial y_1^*}{\partial s_1} = \frac{U_1''}{C_1'' - U_1''} < 0$ where $C_1'' \equiv C''(y_1)$. The analysis indicates that a higher (lower) RE decreases (increases) dispatchable generation. In a similar way, a higher (lower) level of stored enegy decreases (increases) y_1^*. In contrast, when $z_1 > \tau$, the dispatchable systems are kept idle. Thus, $\partial y_1^*/\partial z_1 = \partial y_1^*/\partial s_1 = 0$.

The problem in period 0 is then the following:

$$\max_{\{q_0,y_0,s_1\}} U_0(q_0) - C(y_0) + \mathbb{E}\left[W_1(\tilde{z}_1, s_1)\right]$$
$$\text{subject to } \; q_0 \geq 0, y_0 \geq 0,$$
$$\bar{s} \geq s_1, s_1 \geq 0.$$

The first-order necessary condition for dispatchable generation at a maximum is:[19]

$$U_0'(y_0^* + z_0 - \alpha s_1) \leq C'(y_0^*), \text{ with an equality if } y_0^* > 0. \tag{6}$$

Using the maximum value function in Eq. (5) and the Envelope Theorem, the first-order condition with respect to s_1 is

$$U_0'(y_0^* + z_0) \geq \frac{1}{\alpha}\mathbb{E}\left[U_1'(\tilde{y}_1^* + \tilde{z}_1)\right] \qquad \text{if } s_1^* = 0, \tag{7a}$$

$$U_0'(y_0^* + z_0 - \alpha s_1^*) = \frac{1}{\alpha}\mathbb{E}\left[U_1'(\tilde{y}_1^* + \tilde{z}_1 + s_1^*)\right] \qquad \text{if } \bar{s} > s_1^* > 0, \tag{7b}$$

$$U_0'(y_0^* + z_0 - \alpha \bar{s}) \leq \frac{1}{\alpha}\mathbb{E}\left[U_1'(\tilde{y}_1^* + \tilde{z}_1 + \bar{s})\right] \quad \text{otherwise } \Big(\text{i.e., if } s_1^* = \bar{s}\Big), \tag{7c}$$

where $y_0^* \equiv y(z_0 - \alpha s_1^*)$.[20] From the social planner's perspective, the willingness to store energy is determined by the expected marginal surplus from energy consumption in the next period. For

$$q_0^* \equiv y_0^* + z_0 - \alpha s_1^* \text{ and } \tilde{q}_1^* \equiv \tilde{y}_1^* + \tilde{z}_1 + s_1^*,$$

if it is not optimal to store energy, that is, $s_1^* = 0$, there is an expected loss from energy storage: $U_0'(q_0^*) \geq \frac{1}{\alpha}\mathbb{E}\left[U_1'(\tilde{q}_1^*)\right]$. Otherwise, energy is stored until its current and expected social values are equalized. If, however, $s_1^* = \bar{s}$, the marginal expected benefit from storing energy is at least as high as the marginal cost of energy storage; that is, $U_0'(q_0^*) \leq \frac{1}{\alpha}\mathbb{E}\left[U_1'(\tilde{q}_1^*)\right]$.[21]

In studying the effect of energy storage on welfare, we start from a situation of certainty. Suppose that the energy system is composed of baseload power plants as well as dispatchable and energy storage systems. Power plants such as nuclear and coal-fired plants that produce at low marginal costs and are devoted to the production of baseload supply have slow ramp rates, and therefore, are not flexible to switch on and off. As they are inflexible in practicing "load following," electric power companies try to operate them at full output as much as possible (Denholm et al., 2010). Let $\mu > 0$, which is a constant, denote this capacity.

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[19] Similar to the problem in the final period, the second-order condition for a maximum is satisfied: $U_0''(q_0) - C''(y_0) < 0$.

[20] The second-order condition for a maximum gives $\alpha^2 U_0''(q_0) + \mathbb{E}[U_1''(q_1)] < 0$.

[21] Notice that when $s_0 > 0$, the marginal cost of energy storage becomes lower. This will make it more likely that energy will be stored and transferred to the next period. For the limiting case of no RE generation (i.e., $\bar{z} = 0$), the reader is referred to Appendix A.

Suppose now that we introduce some noise $\tilde{x}$ around μ in period 1 such that $\tilde{z}_1 = \mu + \tilde{x}$, $\mathbb{E}[\tilde{x}] = 0$ and $\mathbb{E}[\tilde{x}^2] = \sigma^2$. Accordingly, $\tilde{z}_1$ represents the intermittent RE with mean μ and variance σ^2. Our purpose here is to determine whether the optimal level of energy storage under intermittent and variable generation is greater than the corresponding level without uncertainty.

Let s_1^+ be the optimal level of energy storage when $\tilde{z}_1 = \mu$ in the future with certainty:

$$s_1^+ = \arg\max\ U_0(y(z_0 - \alpha s_1) + z_0 - \alpha s_1) - C(y(z_0 - \alpha s_1)) + W_1(\mu + s_1).$$

Without any uncertainty, the only factor that leads to energy storage is the higher valuation of energy in the peak period.

Furthermore, suppose that s_1^* is the optimal level of energy storage when there is uncertain RE generation:

$$s_1^* = \arg\max\ U_0(y(z_0 - \alpha s_1) + z_0 - \alpha s_1) - C(y(z_0 - \alpha s_1)) + \mathbb{E}[W_1(\tilde{z}_1 + s_1)].$$

Following these definitions, we present our major result by Theorem 1:

Theorem 1. *For every μ and $\tilde{x}$ with $\mathbb{E}[\tilde{x}] = 0$, $s_1^* \geq s_1^+$ if and only if:*

$$F(\tau)\big(\psi_U U_1'''(\check{q}_1^*) + \psi_C C'''(\check{y}_1^*)\big) + \big(1 - F(\tau)\big)U_1'''(\hat{q}_1^*) \geq 0. \tag{8}$$

where $\psi_U \equiv (C_1''^3)/(C_1'' - U_1'')^3$, $\psi_C \equiv (-U_1''^3)/(C_1'' - U_1'')^3$, $\check{q}_1^* \equiv q(y_1^* + z_1 + s_1^* | z_1 \leq \tau)$, $\hat{q}_1^* \equiv q(z_1 + s_1^* | z_1 > \tau)$, $\check{y}_1^* \equiv y(z_1 + s_1^* | z_1 \leq \tau)$ and $F(\tau)$ is the probability of $z_1 \leq \tau$.

Proof. The proof is provided in Appendix B. □

When there is intermittent and variable energy generation, Theorem 1 shows that a higher level of energy storage will be welfare improving if and only if Eq. (8) holds. Conditional on $z_1 \leq \tau$, that is, there is dispatchable generation, ψ_U and ψ_C are weights attached to U_1''' and C_1''', respectively. When $z_1 > \tau$, there will be no dispatchable generation, and in turn, no risk that will emerge for the production side of the economy. In this case, all weight will be attached to U_1'''. Notice that there can be precautionary storage even if $U_1''' < 0$ and $C_1''' > 0$ or $C_1''' < 0$ and $U_1''' > 0$. Therefore, it is the probability weighted sum of $\psi_U U_1'''(\check{q}_1^*) + \psi_C C'''(\check{y}_1^*)$ and $U_1'''(\hat{q}_1^*)$, which matters for precautionary energy storage.

Let y_0^+ and q_0^+ represent the fossil fuel energy generation and energy consumption, respectively, in period 0 without uncertainty in period 1. We then obtain the following result:

Corollary 1. *$s_1^* \geq s_1^+$ implies $y_0^* \geq y_0^+$ and $q_0^* \leq q_0^+$.*

Proof. Theorem 1 shows that for every μ and $\tilde{x}$ with $\mathbb{E}[\tilde{x}] = 0$, $s_1^* \geq s_1^+$ if and only if Eq. (8) is positive. Given $y_0 \equiv y(z_0 - \alpha s_1)$ and $q_0 \equiv q(y_0 + z_0 - \alpha s_1)$, $\partial y_0/\partial s_1 \geq 0$ and $\partial q_0^*/\partial s_1 < 0$. □

Corollary 1 indicates that a higher level of energy storage will cause a lower off-peak energy consumption, and in turn, a lower welfare in the initial period. Nevertheless, by transferring the social surplus to the peak period using energy storage systems, a higher welfare in the future is expected to more than compensate for this loss.

When dispatchable generation is the marginal resource in the initial period, that is, $y_0^* > 0$, the dispatchable systems will supply the extra amount of energy for storage. On the other hand, if it is optimal to keep the dispatchable systems idle in the initial period, that is, $y_0^* = 0$, the economy can increase energy storage by only consuming less electricity in the initial period. Yet, if precautionary storage due to uncertain renewable energy generation is sufficiently high, dispatchable systems will need to be brought online. This is shown in the following corollary.

Corollary 2. *Suppose $y_0^+ = 0$ and Eq. (8) holds. Then, there is a τ^s such that*

$$\begin{aligned} &y_0^* > y_0^+ \quad \textit{if} \quad s_1^* > \tau^s, \\ &y_1^* = y_0^+ \quad \textit{otherwise} \ \ (\textit{i.e.}, \tau^s \geq s_1^*). \end{aligned}$$

Proof. From Eq. (6), $U_0'(z_0 - \alpha s^+) \leq C'(0)$ if $y_0^+ = 0$. Thus, there is a threshold level of storage $\tau^s (\geq s^+)$ that satisfies

$$U_0'(y_0^+ + z_0 - \alpha \tau^s) = C'(y_0^+), \tag{10}$$

If Eq. (8) holds and $s^* > \tau^s$, then Eq. (10) will be violated as $U'' < 0$. Accordingly, $y^* > y^+ = 0$. On the other hand, if $\tau^s \geq s^*$, $U_0'(z_0 - \alpha s^*) \leq C'(0)$ and $y_0^* = y_0^+ = 0$. □

In our study, our main focus is on prudence and frugality. Theorem 1 has a stronger corollary in this regard (the proof is trivial and omitted):

Corollary 3. $U_1''' \geq 0$ *and* $C_1''' \geq 0$ *are sufficient for* $s_1^* \geq s_1^+$.

Hence, if $U_1''' \geq 0$ and $C_1''' \geq 0$, Eq. (8) holds and it is optimal to store a higher level of energy under uncertainty. When there is no prudence, $U_1''' = 0$, frugality alone will lead to precautionary energy storage. The same is true when $C_1''' = 0$ and $U_1''' \geq 0$.

Although it was Kimball (1990) who coined the term prudence, the analysis of precautionary demand for savings was done earlier by Leland (1968) and Sandmo (1970). Within an expected utility framework, they indicate that a risky future income increases savings if and only if the third-order derivative of the utility function is positive (that is, the agents are prudent).

Frugality, however, is not fully investigated in the literature. Yet, by analyzing production and inventory data, Cecchetti et al. (1997) find evidence that supports a positive third derivative of the cost function and note that, from an operational perspective, a firm is capacity constrained when faced with a convex marginal cost curve. Considering the fact that the capacity constrained dispatchable systems follow the load when RE and energy storage are not adequate to cover the optimal level of energy demand, it can become increasingly costly to make large and positive changes to meet the residual demand. In this regard, frugality can lead to precautionary energy storage.

5 Competitive market equilibrium with energy storage

In this section, we look at the role of prices in coordinating the energy market by taking into consideration the precautionary motives that we have been discussing thus far. This task stipulates a well-defined market equilibrium concept. In Appendix C, by assuming price-taking behavior in the electricity markets, we characterize the optimal behavior of consumers, producers and energy storage firms, depict the formation of expectations, and define the competitive rational expectations equilibrium.

As there are no externalities or other distortions in the model, the competitive rational expectations equilibrium quantities correspond to the allocation dictated by the social planner. This allows us to carry forward the results from our analysis of the social planner's problem. We assume that the consumers have identical preferences and model their behavior by a representative consumer.[22] In this regard, the marginal surplus function can be denoted by $P_0^* \equiv P(q_0^*) = U_0'(q_0^*)$ and $\tilde{P}_1^* \equiv P(q_1^*) = U_1'(\tilde{q}_1^*)$, where $q_0^* \equiv q(P_0^*)$ and $\tilde{q}_1^* \equiv q(\tilde{P}_1^*)$ are the aggregate demand functions given the retail prices P_0^* and $\tilde{P}_1^*$. From Theorem 1 and Corollary 1, we shall establish the following proposition:

Proposition 1. *Precautionary energy storage leads to an increase in P_0^*, which is followed by a reduction in $\tilde{P}_1{}^*$ and its variance.*

Proof. From Theorem 1, $s_1^* \geq s_1^+$ if and only if Eq. (8) holds. Precautionary energy storage (i.e., $s_1^* \geq s_1^+$) implies $q_0^* \leq q_0^+$ (Corollary 1). As $U_0'(q_0^*) = P_0^*$ and $U'' < 0$, $q_0^* \leq q_0^+$ leads to $P_0^* \geq P_0^+$, where P_0^+ is the retail price in the absence of precautionary storage. On the other hand, $\partial q_1^*/\partial s_1 > 0$. As $U_1'(\tilde{q}_1^*) = \tilde{P}_1^*$ and $U_1'' < 0$, $q_1^* \geq q_1^+$ implies that $\tilde{P}_1^*$ will decrease. Lastly, $\text{Var}(\tilde{P}_1^*) = \mathbb{E}\big[\tilde{P}_1^{*2}\big] - \mathbb{E}\big[\tilde{P}_1^*\big]^2$. Taking the partial derivative of $\text{Var}(\tilde{P}_1^*)$ with respect to q_1 gives

$$\frac{\partial}{\partial \tilde{q}_1}\text{Var}(P(\tilde{q}_1^*)) = 2\Big[\mathbb{E}\big[P(\tilde{q}_1^*)P'(\tilde{q}_1^*)\big] - \mathbb{E}\big[P'(\tilde{q}_1^*)\big]\mathbb{E}\big[P'(\tilde{q}_1^*)\big]\Big].$$

For $P(q_1)$ decreasing and $P'(q_1)$ increasing, $\mathbb{E}[P(\tilde{q}_1^*)P'(\tilde{q}_1^*)] \leq \mathbb{E}[P(\tilde{q}_1^*)]\mathbb{E}[P'(\tilde{q}_1^*)]$ (see Lemma 1 in Gurland (1967)). Accordingly, $\partial\text{Var}(P(\tilde{q}_1^*))/\partial\tilde{q}_1 \leq 0$. □

[22] We consider a quasi-linear utility function over electricity consumption and a numéraire commodity. Accordingly, $U(q)$, which is the (gross) surplus function, is the monetary value of utility derived from consuming q kilowatt-hour of electricity. In economic theory, using such preferences is a standard assumption when discussing issues related to a single market in a general equilibrium framework. This approach can be justified in the absence of income effects (see Mas-Colell et al., 1995, chap. 10), which we do not consider in our study.

Proposition 1 indicates that the precautionary demand for energy storage will increase the retail price of electricity and cause a lower level of off-peak energy consumption. The higher amount of energy that is carried to the next period will lead to a decline in the future electricity price, $\tilde{P}_1$, for every realization of $\tilde{z}_1$, that is, the meteorological shock and in turn RE. Furthermore, precautionary energy storage will also allow for a decrease in price uncertainty; that is, $\partial \mathrm{Var}(P(\tilde{q}_1^*))/\partial \tilde{q}_1 \leq 0$.

As the existing energy systems worldwide can generally be characterized by small shares of RE (Lund et al., 2012), let us focus on the case where the dispatchable generators always supply the residual load. In our model, this translates into $F(\tau) = 1$. Thus, even with favorable weather conditions, the RE generation cannot meet the energy demand. In this case, the necessary and sufficient condition for precautionary storage is $\psi_U U_1''' + \psi_C C_1''' \geq 0$ (see Theorem 1). We can call this the "prudence-frugality index" (PF-index), which is a weighted sum of the degree of convexity (that is, the curvature) in the demand curve and the dispatchable energy supply curve. In this regard, it is an indicator of the degree of precaution in the market.

Assuming that the dispatchable systems always meet the residual load, the second-order Taylor approximation of the expected retail price of electricity (cf. the right-hand side of Eq. (7b)) around μ, which is the mean-level RE generation, will give

$$P_0^* \simeq \frac{1}{\alpha}\left[P_1 + \frac{1}{2}\sigma^2\left(\psi_{_U} U_1''' + \psi_{_C} C_1'''\right)\right], \tag{11}$$

where $P_1 = U_1'(\bar{q}_1^*)$ is the electricity price that corresponds to $\bar{q}_1 \equiv q\big(y(\mu+s_1^*)+\mu+s_1^*\big)$, the peak period level of electricity consumption evaluated at the mean RE generation.[23] Thus, the spot market price (approximately) equals the product of $\frac{1}{\alpha}$ multiplied by the sum of the market price evaluated at μ and the product of the PF-index and the degree of intermittency, σ^2.

One can rearrange Eq. (11) to obtain

$$P_0^* \simeq \frac{1}{\alpha}\left[1 + \frac{1}{2}\left(\psi_{_U}\left(\frac{\sigma}{\bar{q}_1}\right)^2 \frac{\xi_r^p}{\eta_d} + \psi_{_C}\left(\frac{\sigma}{\bar{y}_1}\right)^2 \frac{\xi_r^f}{\eta_s}\right)\right] P_1. \tag{12}$$

where $\xi_r^p \equiv -\bar{q}_1 \frac{P_1''}{P_1'}$ and $\xi_r^f \equiv \bar{y}_1 \frac{C_1'''}{C_1''}$ are the coefficients of relative prudence and frugality, respectively; $\eta_d \equiv \left|\frac{d\bar{q}_1/\bar{q}_1}{dP_1/P_1}\right|$ and $\eta_s \equiv \frac{d\bar{y}_1/\bar{y}_1}{dP_1/P_1}$ are price elasticities of demand and dispatchable energy supply, respectively.[24] This leads us to the following remark:

Remark For an electricity market where the dispatchable generation always supplies the residual load, P_0^* ($\tilde{P}_1^*$ and $\mathrm{Var}(\tilde{P}_1^*)$) is augmented (reduced) by a lower α, η_d and η_s, and a higher σ , ξ_r^p, ξ_r^f, $\psi_{_U}$ and $\psi_{_C}$.

[23] Refer to Appendix D for the calculations. Note that $F(\tau) = 1$.

[24] Equivalently, the demand and dispatchable energy supply elasticities can be written as $\eta_d \equiv -\frac{P_1}{P_1'\bar{q}_1}$ and $\eta_s \equiv \frac{C_1'}{C_1''\bar{y}_1}$, respectively.

This remark indicates that a more efficient storage technology, that is, a lower α, will create arbitrage opportunities and lead to a higher demand for energy storage. A higher level of stored energy will in turn lead to a higher current price of energy, and a lower future price and price volatility. An increase in price elasticity of demand makes demand response a better substitute for energy storage and diminishes the impact that prudence can have on precautionary energy storage.[25] A higher elasticity of supply, that is, a more responsive dispatchable energy generation, also causes a lower level of energy storage. Hence, both the supply- and demand-side elasticities have similar effects. If, however, the supply elasticity is low (e.g., think of a baseload power plant, which has low supply elasticity due to its poor flexibility in adjusting its output), there will be a higher level energy storage on precautionary grounds.

An increase in the variations of RE generation, and thus, an increase in σ, implies a higher level of precautionary storage and price of electricity in the initial period. Yet, if the degree of deviations in RE with respect to the level of consumption, $\sigma/\bar{q}_1$, is small, intermittency is less of a problem for the market. Thus, an increase in σ may have a limited impact on precautionary storage. If, however, $\sigma/\bar{q}_1$ is big, electricity consumption can be exposed to significant deviations. Therefore, a higher amount of energy will be stored. Additionally, when $\sigma/\bar{y}_1$ is big, there can be costly attempts in the dispatchable generation industry to supply the residual demand when the level of RE gets low. This will lead to a rise in the level of precautionary storage and hence the spot market electricity price in equilibrium. Lastly, precautionary storage depends positively on the coefficients of prudence and frugality. Thus, in response to the overall energy production risk, energy storage, and therefore, the spot market electricity price, will increase with relative prudence and frugality. The peak period electricity price and its variance will decrease.

When consumers are prudent and the supply schedule is linear, that is, $P_1'' > 0$ and $C_1''' = \xi_r^f = 0$, the risk attitudes on the consumers' side will drive the demand for precautionary storage. On the other hand, when the price schedule is linear, that is, $P_1'' = \xi_r^p = 0$, and the fossil fuel power industry is characterized by a convex supply schedule, $C_1''' > 0$, it will be the producers' side that will derive the demand for precautionary storage. Moreover, considering constant retail pricing for electricity, e.g., fixed prices in peak and off-peak hours, consumers will not be subject to consumption risk and will have no incentives to change their demand with respect to variations in RE, and thus, wholesale prices. Nevertheless, due to the intermittent RE generation, the dispatchable energy suppliers will be subject to the changes in the residual load. Therefore, it will still be the producers' side that will derive the demand for precautionary energy storage.

An interesting feature of our results is the weights assigned to prudence and frugality.

[25] A similar result can be found in the commodity storage literature, where Wright and Williams (1982, 1984) show that higher demand elasticity decreases the scope for commodity storage. The electricity data, nevertheless, indicates that the relative price response is rather low. Accordingly, the short-run (1–5 years) residential own-price elasticity of electricity demand in absolute value is estimated at 0.3 (EPRI, 2008). The same number averaged for potential system peak hours for the summer months is estimated to be 0.15 (Taylor et al., 2005). Surveying the evidence from the recent experiments with dynamic pricing of electricity, Faruqui and Sergici (2010) report that the own price elasticities in peak usage range from 0.02 to 0.10. A low price elasticity of demand will emphasize the role of prudence in precautionary energy storage.

Miranda and Helmberger (1988) show that price variability is more sensitive to the demand elasticity than the supply elasticity. When translated to our case, this can imply a greater weight on the effect of demand elasticity on the current energy price, and thus, ψ_U being greater than ψ_C. For energy markets, measuring this effect can be an interesting problem, and an empirical investigation may supply crucial information for electricity pricing.

Let us now consider an electricity market with 100% RE, that is, $F(\tau) = 0$. Norway, where nearly the whole electricity is generated by the hydropower systems (Førsund, 2007, p. 95) is one example in this regard.[26] In this case, the arbitrage equation is given by

$$P_0^* \simeq \phi \left[1 + \frac{1}{2} \Big(\frac{\sigma}{\bar{q}_1} \Big)^2 \frac{\xi_r^p}{\eta_d} \right] P_1. \tag{13}$$

This leads us to the following remark:

Remark For an electricity market with 100% RE, P_0 ($\tilde{P}_1^*$ and $\text{Var}(\tilde{P}_1^*)$) is augmented (reduced) by a lower η_d and α, and a higher σ and ξ_r^p.

The interpretation as to how the equilibrium level of energy storage and current and future electricity prices are affected by the degree of intermittency, price elasticity of demand and coefficient of relative prudence remains the same. Note, however, that instead of purchasing energy from the dispatchable energy industry, the energy storage firms will obtain the desired level of energy from the RE generators.

For the general case in which dispatchable systems are occasionally shut down, we have

$$P_0^* \simeq F(\tau)\check{P}_0 + (1 - F(\tau))\hat{P}_0, \tag{14}$$

where $\check{P}_0$ and $\hat{P}_0$ are given by Eqs. (12) and (13), respectively. Note that while $\check{P}_0$ corresponds to the cases where dispatchable systems are active, $\hat{P}_0$ corresponds to the cases when they are kept idle. Eq.(14) shows that the more often the dispatchable systems are online, that is, the higher $F(\tau)$ is, frugality will have a higher impact on precautionary energy storage and in turn the price of electricity, and the other way around.

[26] Iceland's electricity sector with minor contributions from dispatchable systems is another example. With hydropower accounting for 74% and geothermal for 26%, electricity generation was produced solely from renewables in 2010 (IEA, 2013). Although our focus is on variable and intermittent RE, in the absence of dispatchable systems our model can easily be converted to a model with a reservoir hydroelectric system. For example, suppose z_0 is the current flow of water to a reservoir and $\tilde{z}_1$ is the uncertain flow in the next period. Let s_1 be the water that is stored in the reservoir for use in the peak period. Disregarding losses due to surface evaporation and leakages (i.e., $\alpha = 1$), $q_0 = z_0 - s_1$ is the consumption of energy that is generated by letting the water flow through water turbines. Considering such a system and the allocation of production between seasons (summer and winter) in this regard, it will be convenient to relax the "no discounting" assumption.

6 Conclusion

Energy storage is addressed in many studies in the literature. Yet, the extent to which precautionary motives can spur energy storage, and in turn, electricity pricing, is not well known. The model we develop provides a simple setup to assess the impact of a convex marginal utility (prudence) and a convex marginal cost (frugality) on energy storage. We characterize the optimum and show how prudence and frugality can lead to a higher level of energy storage, that is, precautionary energy storage. Even in the absence of prudence, frugality can still contribute toward precautionary storage, and the other way around. Decentralizing the optimal allocation allows us to see the role that prices can play in coordinating the energy market in the presence of the precautionary motives. Our analysis indicates that prudence and frugality increase spot market energy prices through higher demand for energy storage. Moreover, they lead to a decline in the future electricity price and its volatility. Further results present important lessons about the direct and indirect impacts precautionary motives can have on electricity prices and energy generation decisions.

In our study, we took an idealized approach to effectively highlight the impact of precautionary motives on energy storage and electricity pricing. This naturally enables one to consider deviations from this idealized setting. Market power problem is hard to overlook in the electricity industries. Accordingly, it will be interesting to investigate the role that energy storage firms can play in decreasing the ability of power generators to exercise market power. Furthermore, one can study design distortions such as price-cap regulation. It will also be worthwhile to investigate the price dynamics in a fully dynamic model.

One other way to extend our study is to incorporate capacity decisions into the model. While the capacity decisions would be associated with long-term commitments in the economy, the latter, which corresponds to the present study, would be related to short-term decisions. When consumers respond to price fluctuations and thus can shift their peak demand, the potential to invest in peaking power plants can decrease in the long run (Jessoe and Rapson, 2014; Tsitsiklis and Xu, 2015). Whether precautionary motives and therefore precautionary energy storage can contribute to this potential and lead to further investments in RE capacity will be interesting to look at.

Appendices

A Energy storage in the absence of RE generation

As a limiting case, suppose that energy can only be produced using dispatchable systems and there is energy storage. Therefore, $\bar{z} = 0$. For an interior solution for dispatchable generation in both periods; that is, $U_0'(q_0^*) = C'(y_0^*)$ and $U_1'(q_1^*) = C'(y_1^*)$, one gets

$$\frac{C'\big(y_0^*\big)}{C'\big(y_1^*\big)} = \frac{1}{\alpha}. \tag{15}$$

Eq. (15) satisfies intertemporal efficiency. There is an equality between the marginal rate of transformation of off-peak energy into peak energy and cost of energy transformation. A similar result can be seen in Crampes and Moreaux (2010), where α is the level of energy required to add one unit to the stock of energy in a water reservoir for use in period 1 when the demand is high. If the absolute value of the slope of the isocost curve, $C'(y_0^*)/C'(y_1^*)$, is greater than $\frac{1}{\alpha}$, no energy is stored in period 0. This is because the cost of storage on the margin is bigger than its value in the peak period. In contrast, if $C'(y_0^*)/C'(y_1^*) < \frac{1}{\alpha}$, the available storage capacity is completely utilized.

B Proof of Theorem 1

To prove our main result, we will need the following lemma:

Lemma 1. *If $\psi_U U_1''' + \psi_C C_1''' \geq 0$, then an increase in risk (or, degree of intermittency) in the sense of Rothschild and Stiglitz (1970) (RS) increases s_1.*

Proof. Let E and F represent the cumulative distribution functions of $\tilde{n}_1$ and $\tilde{z}_1$, respectively. Assume that F is a mean-preserving spread of E in the sense of RS. Thus, although both systems have the same average level of RE generation, the density function $f(\tilde{z}_1)$ has more weight in the tails, and is more risky. Then, for every non-decreasing concave function, υ, we have the following:

$$\int_a^b \upsilon(m)dE(m) \geq \int_a^b \upsilon(m)dF(m).$$

Taking $\upsilon \equiv -U_1'$ yields

$$\int_a^b U_1'(m)dE(m) \leq \int_a^b U_1'(m)dF(m).$$

Then, for $b = \bar{z}$, $a = 0$, and $U_1' = U_1'(y(\tilde{j} + s_1) + \tilde{j} + s_1)$ for $\tilde{j} = \tilde{n}_1, \tilde{z}_1$, Theorem 2(A) in RS states that an increase in risk leads to a higher s_1 if U' is convex in $\tilde{j}$; that is,

$$\frac{\partial^2 U_1'}{\partial \tilde{j}^2} = U_1'''\Big(\frac{\partial y_1}{\partial \tilde{j}} + 1\Big)^2 + U_1''\frac{\partial^2 y_1}{\partial \tilde{j}^2} \geq 0. \tag{16}$$

As

$$\frac{\partial y_1}{\partial j} = \frac{U_1''}{C_1'' - U_1''} < 0, \tag{17}$$

we get

$$\frac{\partial^2 y_1}{\partial \tilde{j}^2} = \frac{C_1''^2 U_1''' - U_1''^2 C_1'''}{(C_1'' - U_1'')^3}. \tag{18}$$

Substituting Eqs. (17) and (18) in Eq. (16) then gives

$$\psi_U U_1''' + \psi_C C_1''' \geq 0. \tag{19}$$

□

The expected marginal surplus can be written as

$$\mathbb{E}\big[U_1'(\tilde{q}_1^*)\big] = F(\tau)\mathbb{E}[U_1'(\tilde{q}_1^*)|\tilde{z}_1 \leq \tau] + (1 - F(\tau))\,\mathbb{E}[U_1'(\tilde{q}_1^*)|\tilde{z}_1 > \tau]. \tag{20}$$

While $\mathbb{E}[U_1'(\tilde{q}_1^*)|\tilde{z}_1 \leq \tau]$ represents the conditional expected marginal surplus from consuming energy supplied by both the dispatchable and renewable systems, $\mathbb{E}[U_1'(\tilde{q}_1^*)|\tilde{z}_1 > \tau]$ is the conditional expected marginal surplus when consuming energy only from the renewable systems. Thus, the latter corresponds to cases in which dispatchable systems are kept idle. Moreover, $F(\tau)$ is the probability of $\tilde{z}_1 < \tau$ and vice versa.

Lemma 2. *If $F(\tau) = 1$, then for every μ and $\tilde{x}$ with $\mathbb{E}[\tilde{x}] = 0$, $s_1^* \geq s_1^+$ if and only if:*

$$\psi_U U_1''' + \psi_C C_1''' \geq 0. \tag{21}$$

Let us prove sufficiency using Lemma 1. Given s_1^+, let $V(s_1^+, \mu)$ be the maximum value function for the intertemporal optimization problem under certainty. Further, let $\mathbb{E}[V(s_1^*, \tilde{z}_1)]$ be the expected value of the maximum value function for the intertemporal optimization problem when RE is uncertain. Given $\tilde{z}_1 = \mu + \tilde{x}$ and $\mathbb{E}[\tilde{x}] = 0$, the first order conditions with respect to s_1 for $V(s_1, \mu)$ and $\mathbb{E}[V(s_1, \tilde{z}_1)]$, that is, $V_s(s_1, \mu) = 0$ and $\mathbb{E}[V_s(s_1, \tilde{z}_1)] = 0$, respectively, yield:

$$-U_0'(y(z_0 - \alpha s_1^+) + z_0 - \alpha s_1^+) + \frac{1}{\alpha}[U_1'(y(\mu + s_1^+) + \mu + s_1^+)] = 0,$$
$$-U_0'(y(z_0 - \alpha s_1^*) + z_0 - \alpha s_1^*) + \frac{1}{\alpha}\mathbb{E}[U_1'(y(\tilde{z}_1 + s_1^*) + \tilde{z}_1 + s_1^*)] = 0.$$

If $V(s_1^+, \tilde{z}_1)$ is convex in $\tilde{z}_1$, then $\mathbb{E}[V_s(s_1^+, \tilde{z}_1)] \geq V_s(s_1^+, \mu) = 0$, or equivalently, $\mathbb{E}[U_1'(y(\tilde{z}_1+$

$s_1^+) + \tilde{z}_1 + s_1^+] \geq U_1'(y(\mu + s_1^+) + \mu + s_1^+)$. If we take μ and 0 as the mean and the variance of $\tilde{n}_1$, respectively, then, by Lemma 1, $\mathbb{E}[U_1'(y(\tilde{z}_1 + s_1^+) + \tilde{z}_1 + s_1^+] \geq U_1'(y(\mu + s_1^+) + \mu + s_1^+)$. Hence, the expected marginal benefit of increasing energy storage is positive when $s_1 = s_1^+$, and thus, $s_1^* \geq s_1^+$. This ends the proof for sufficiency.

If $s_1^* \geq s_1^+$ for every μ and $\tilde{x}$ with $\mathbb{E}[\tilde{x}] = 0$, then this must also be true for small zero-mean risks. The small risk allows us to focus on 2nd Taylor approximation around μ:

$$V_s(s_1^+, \mu + \tilde{x}) \simeq V_s(s_1^+, \mu) + \tilde{x} V_{sz}(s_1^+, \mu) + \frac{1}{2}\tilde{x}^2 V_{szz}(s_1^+, \mu) + \mathrm{O}\left(\tilde{x}^3\right) \tag{22}$$

where $V_{sz} = \frac{1}{\alpha}\frac{\partial U'}{\partial \tilde{z}_1}$, $V_{szz} = \frac{1}{\alpha}\frac{\partial^2 U'}{\partial \tilde{z}_1^2}$ (see Eqs. (16) and (18)), and $\mathrm{O}\left(\tilde{x}^3\right)$ is the remainder. By assuming that the risk is small, we can ignore the remainder term. From the first order condition, $V_s(s_1^+, \mu) = 0$. Taking the expectation of both sides yields:

$$\mathbb{E}[V_s(s_1^+, \mu + \tilde{x})] \simeq \frac{1}{2}\sigma^2 V_{szz}(s_1^+, \mu) \tag{23}$$

For a small risk, if $s_1^* > s_1^+$, then $\mathbb{E}[V_s(s_1^+, \mu + \tilde{x})] \geq 0$. For $\mathbb{E}[V_s(s_1^+, \mu + \tilde{x})] \geq 0$ to be positive, $V_{szz} \geq 0$ must be positive. One can calculate that $V_{szz} \geq 0$ is equivalent to Eq. (19). This completes the proof for necessity.

Lemma 3. *If $F(\tau) = 0$, then for every μ and $\tilde{x}$ with $\mathbb{E}[\tilde{x}] = 0$, $s_1^* \geq s_1^+$ if and only if*

$$U_1''' \geq 0. \tag{24}$$

Proof. The proof is similar to that of Lemma 2, except that for $t = 0, 1,\ y_t^* = 0$. □

Proof of Theorem 1. Following the proof of Lemma 2, if $V(s_1^+, \tilde{z}_1)$ is convex in $\tilde{z}_1$, $\mathbb{E}[V_s(s_1^+, \tilde{z}_1)] \geq V_s(s_1^+, \mu) = 0$ or equivalently $\mathbb{E}[U_1'(y(\tilde{z}_1 + s_1^+) + \tilde{z}_1 + s_1^+] \geq U_1'(y(\mu + s_1^+) + \mu + s_1^+)$. When $s_1 = s_1^+$ and there is uncertainty, by Lemma 2 and Lemma 3, the marginal benefit of increasing energy storage is positive if

$$F(\tau)\big(\psi_U U_1'''(\check{q}_1^*) + \psi_C C'''(\check{y}_1^*)\big) + \big(1 - F(\tau)\big)U_1'''(\hat{q}_1^*) \geq 0. \tag{25}$$

If $s_1^* \geq s_1^+$ for every μ and $\tilde{x}$ with $\mathbb{E}[\tilde{x}] = 0$, then this must also be true for small zero-mean risks. Given that there is a small zero-mean risk allows us to focus only on the second-order Taylor approximation. This yields the following:

$$\mathbb{E}[V_s(s_1^+, \mu + \tilde{x})] \simeq F(\tau)\frac{1}{2}\check{\sigma}^2 V_{szz}(s_1^+, \check{\mu}) + (1 - F(\tau))\frac{1}{2}\hat{\sigma}^2 V_{szz}(s_1^+, \hat{\mu}). \tag{26}$$

For a small risk, if $s_1^* > s_1^+$, then $\mathbb{E}[V_s(s_1^+, \mu + \tilde{x})] \geq 0$. For $\mathbb{E}[V_s(s_1^+, \mu + \tilde{x})] \geq 0$ to be positive, $F(\tau)\frac{1}{2}\check{\sigma}^2 V_{szz}(s_1^+, \check{\mu}) + (1 - F(\tau))\frac{1}{2}\hat{\sigma}^2 V_{szz}(s_1^+, \hat{\mu})$ must be positive. By Lemma 2 and Lemma 3, this probability weighted sum is positive. This completes the proof for necessity. □

C Competitive rational expectations equilibrium

On the demand side, we assume that all consumers have identical preferences. This allows us to model their behavior by a representative consumer. The first-order necessary conditions for the consumer problem yield

$$\begin{aligned} U_0'(q_0^*) &= P_0^*, \\ U_1'(\tilde{q}_1^*) &= \tilde{P}_1^*, \end{aligned} \tag{27}$$

where $q_0^* \equiv q(P_0^*)$ and $\tilde{q}_1^* \equiv q(\tilde{P}_1^*)$ are the aggregate demand functions given the market prices P_0^* and $\tilde{P}_1^*$.

Regarding the production side of the economy, fossil-fuel and RE generators and energy storage firms are price-taking competitors. There is a continuum of RE generators with measure normalized to one. Given that the unit cost of generating energy is so low –which we consider as zero– the RE generation is price inelastic. Thus, each RE generator operates at its capacity. However, as the weather conditions are uncertain, so is the energy produced by each generator. Therefore, given $P_t > 0$, the profit of each RE generator in both periods is $\pi_{it} = P_t^* z_{i_t}$, where $\bar{z}_i \geq z_{i_t} \geq 0$ and $\bar{z}_i$ is the installed capacity of RE generator i. The total RE generation then satisfies

$$z_t^* \equiv z(P_t^*) = z_t \equiv \int_0^1 z_{i_t} di, \tag{28}$$

where $\bar{z} \geq z_t \geq 0$ and $\bar{z} \equiv \int_0^1 \bar{z}_i di$.

There is a unique merit order of fossil fuel power plants with measure normalized to one. Given P_t, dispatchable generation in the industry extends up to the dispatchable unit for which the marginal cost of dispatchable generation equals the market price.[27] The profit maximization problem of each dispatchable generator is as follows:

$$\max_{y_{j_t}} \ \pi_{jt} = P_t^* y_{jt} - c_j y_{jt}, \ \text{subject to} \ \bar{y}_{jt} \geq y_{jt} \geq 0,$$

where y_{j_t} is the energy generation from dispatchable unit j at time t and $c_j > 0$ is a constant. The first order necessary condition of profit maximization for a generator is

$$\begin{aligned} P_t^* &\leq c_j \quad \text{if} \quad y_{j_t}^* = 0, \\ P_t^* &= c_j \quad \text{if} \quad \bar{y}_j > y_{j_t}^* > 0, \\ P_t^* &\geq c_j \quad \text{if} \quad y_{j_t}^* = \bar{y}_j. \end{aligned} \tag{29}$$

Given that $y_{j_t}^* \equiv y_j(P_t^*)$ is the profit maximizing level of energy that dispatchable generator j is willing to supply at price P_t^*, the dispatchable energy (aggregate) supply function in both

[27]For the units that are brought online earlier, the individual marginal costs equal the market price minus the shadow prices of the individual capacity constraints.

periods is

$$y_t^* \equiv y(P_t^*) = \int_0^1 y_{jt}^* dj.$$

We characterize the energy storage sector by a continuum of energy storage firms with identical technologies. When a storage firm decides to store more energy, it rationally anticipates the future energy price based on the available information (that is, the supply schedule of the dispatchable energy industry, the aggregate demand schedule, the processes that affect the weather, and –in turn– the RE generation). With the ultimate motivation to maximize profits, energy storage firms apply the principle of rational behavior to the acquisition and processing of information and the formation of anticipations. In this sense, they are rational profit maximizers. When storage firms are fully aware of the economic implications of intermittency, they will, for example, change their energy storage levels in anticipation of the effects from intermittent RE rather than wait for these effects to occur in the electricity market. By anticipating the future RE generation, and thus, the future price, the net anticipated profit of energy storage firm ℓ from storing s_{ℓ_1} is

$$\pi_{\ell_1}^a = \frac{1}{\alpha} P_1^a s_{\ell_1} - P_0^* s_{\ell_1}, \tag{30}$$

where P_1^a and P_0^* are the anticipated and current equilibrium spot prices, respectively. Each storage firm maximizes its anticipated profits subject to a non-negativity constraint, $s_{\ell_1} \geq 0$, and a capacity constraint, $\bar{s}_\ell \geq s_{\ell_1}$. As each storage firm shares the same rational expectations with every other firms, the anticipated price is not indexed by a particular storage firm. The first-order condition for the maximization problem yields

$$\begin{aligned} \frac{\partial \pi_{\ell_1}^a}{\partial s_{\ell_1}} &= \frac{1}{\alpha} P_1^a - P_0^* \leq 0, \text{ with equality if } s_{\ell_1}^* > 0, \\ &= \frac{1}{\alpha} P_1^a - P_0^* \geq 0, \quad \text{otherwise} \quad s_{\ell_1}^* = \bar{s}_\ell. \end{aligned} \tag{31}$$

The fact that these firms share the same rational expectations, and therefore, anticipate the same market clearing future price indicates that, in equilibrium, the anticipated profit from a marginal unit of energy storage cannot be positive. Otherwise, profit-seeking entrepreneurs would eliminate any type of disequlibria by adjusting the individual levels of energy storage.[28] This allows us to describe Eq. (31) as the condition for market equilibrium rather than the first-order condition for an energy storage firm's optimization problem. The relationship between the industry level of energy storage and the anticipated profit can then

[28]This will hold for interior solutions. If the industry storage capacity binds, the storage firms will make positive profits.

be summarized by

$$P_0^* \geq \frac{1}{\alpha} P_1^a, \quad s_1^* = 0,$$
$$(32) \qquad P_0^* = \frac{1}{\alpha} P_1^a, \quad \bar{s} > s_1^* > 0,$$
$$P_0^* \leq \frac{1}{\alpha} P_1^a, \quad s_1^* = \bar{s},$$

where $s_1^* = \int_0^1 s_{\ell_1}^* d\ell$, $s_1^* \equiv s_1(P_0^*, P_1^a)$ and $\bar{s} = \int_0^1 \bar{s}_\ell d\ell$.

In the sense that the energy storage firms' expectations are rational and they make informed predictions of future prices, the subsequent market prices that arise from the decisions based on these expectations will confirm their anticipations. Hence, the expected price will be consistent with the level of energy storage that is governed by the anticipated price. Then, in a competitive rational expectations equilibrium

$$(33) \qquad P_1^a = \mathbb{E}[\tilde{P}_1^*],$$

that is, the anticipated price will be confirmed in equilibrium.

Having depicted the formation of expectations and the response of competitive energy storage firms to current and anticipated prices and the qualitative relationship between price and profit maximization for each storage firm, we make the following definition:

Definition 1. *Competitive rational expectations equilibrium is a price vector,* $\mathbb{P} = \{P_0^*, \tilde{P}_1^*, P_1^a\}$, *and an allocation vector,* $\mathbb{Q} = \{q_0^*, \tilde{q}_1^*, z_0^*, \tilde{z}_1^*, y_0^*, \tilde{y}_1^*, s_1^*\}$, *that solve Eqs.* (27), (28), (29), (31), *and* (33), *such that markets clear:* $q_0^* = y(P_0^*) + z(P_0^*) - \alpha s_1(P_0^*, P_1^a)$ *and* $\tilde{q}_1^* = y(\tilde{P}_1^*) + z(\tilde{P}_1^*) + s_1(P_0^*, P_1^a)$.

In equilibrium, the prices, P_0^* and $\tilde{P}_1^*$, are implicitly defined by

$$P_0^* \equiv P\big(y^*(P_0^*) + z_0 - \alpha s_1^*(P_0^*)\big),$$
$$\tilde{P}_1^* \equiv P\big(y^*(\tilde{P}_1^*) + \tilde{z}_1 + s_1^*\big),$$

respectively. In the presence of uncertainty, the storage firms will increase the amount of stored energy until the net expected price, $\frac{1}{\alpha}\mathbb{E}[\tilde{P}_1^*]$, equals the current spot price of energy.[29]

[29]Note also that for $P_0^* > \frac{1}{\alpha}\mathbb{E}[\tilde{P}_1^*]$, $s_1^* = 0$, that is, when the net expected price is below the current price, then the energy storage is zero. If the capacity constraint in the energy storage industry is met, then the net expected future price is above the current price at storage capacity: $P_0^* \leq \frac{1}{\alpha}\mathbb{E}[\tilde{P}_1^*]$, $s_1^* = \bar{s}$.

D The second-order Taylor approximation of the intertemporal efficiency condition

Let $g(\tilde{z}_1) \stackrel{\text{def}}{=} C'(\tilde{y}_1^*)$ and $h(\tilde{z}_1) \stackrel{\text{def}}{=} U_1'(\tilde{q}_1^*)$, where and $\tilde{y}_1^* = y(\tilde{z}_1 + s_1^*)$ and $\tilde{q}_1^* = \tilde{y}_1^* + \tilde{z}_1 + s_1^*$. Given s_1, taking a second-order Taylor series expansion around the conditional means $\check{\mu} \equiv \mathbb{E}\left[z_1 | z_1 < \tau\right]$ for $g(\tilde{z}_1)$ and $\hat{\mu} \equiv \mathbb{E}\left[z_1 | z_1 > \tau\right]$ for $h(\tilde{z}_1)$ yield:

$$g(\tilde{z}_1) \simeq g(\check{\mu}) + (\tilde{z}_1 - \check{\mu})g'(\check{\mu}) + (1/2)(\tilde{z}_1 - \check{\mu})^2 g''(\check{\mu}), \tag{34a}$$

$$h(\tilde{z}_1) \simeq h(\hat{\mu}) + (\tilde{z}_1 - \hat{\mu})h'(\hat{\mu}) + (1/2)(\tilde{z}_1 - \hat{\mu})^2 h''(\hat{\mu}). \tag{34b}$$

Here, $g'(\check{\mu}) \equiv \check{C}_1'' \partial y_1^* / \partial \check{\mu}$, $h'(\hat{\mu}) \equiv \hat{U}_1''$, $g''(\check{\mu}) \equiv \check{C}_1''' \left(\partial y_1^* / \partial \check{\mu}\right)^2 + \check{C}_1'' \partial^2 y_1^* / \partial^2 \check{\mu}$ and $h''(\hat{\mu}) \equiv \hat{U}'''$, where $\check{C}_1'' \equiv C''(\check{y}_1^*)$, $\hat{U}_1'' \equiv U_1''(\hat{q}_1^*)$, $\check{C}_1''' \equiv C'''(\check{y}_1^*)$ and $\hat{U}_1''' \equiv U_1'''(\hat{q}_1^*)$.

For an interior solution for dispatchable generation, $U_1' = C_1'$, one can calculate the second order derivative for the optimal dispatchable generation decision using Eq. (17):

$$\frac{\partial^2 y_1^*}{\partial^2 \check{\mu}} = \frac{\check{C}_1''^2 \check{U}_1''' - \check{U}_1''^2 \check{C}_1'''}{(\check{C}_1'' - \check{U}_1'')^3}. \tag{35}$$

Hence,

$$g''(\check{\mu}) = \frac{\check{C}_1''^3}{(\check{C}_1'' - \check{U}_1'')^3} \check{U}_1''' + \frac{-\check{U}_1''^3}{(\check{C}_1'' - \check{U}_1'')^3} \check{C}_1'''. \tag{36}$$

Calculating the conditional expectations, that is, $\mathbb{E}[g(\tilde{z}_1) | \tilde{z}_1 \leq \tau]$ and $\mathbb{E}[h(\tilde{z}_1) | z_1 > \tau]$, gives the following:

$$\mathbb{E}[g(\tilde{z}_1) | \tilde{z}_1 \leq \tau] \simeq g(\check{\mu}) + \frac{1}{2} \check{\sigma}^2 g''(\check{\mu}), \tag{37a}$$

$$\mathbb{E}[h(\tilde{z}_1) | \tilde{z}_1 > \tau] \simeq h(\hat{\mu}) + \frac{1}{2} \hat{\sigma}^2 h''(\hat{\mu}), \tag{37b}$$

where $\check{\sigma}^2 = \mathbb{E}\left[(\tilde{z}_1 - \check{\mu})^2 | \tilde{z}_1 \leq \tau\right]$ and $\hat{\sigma}^2 = \mathbb{E}\left[(\tilde{z}_1 - \hat{\mu})^2 | \tilde{z}_1 > \tau\right]$ are conditional variances.

From

$$\mathbb{E}\left[U_1'(\tilde{q}_1^*)\right] = F(\tau)\mathbb{E}[U_1'(\tilde{q}_1^*) | \tilde{z}_1 \leq \tau] + (1 - F(\tau))\, \mathbb{E}[U_1'(\tilde{q}_1^*) | \tilde{z}_1 > \tau],$$

Eqs, (36), (37a) and (37b), and the fact that $h''(\hat{\mu}) \equiv \hat{U}'''$, one can rewrite Eq. (7b) using a second-order Taylor approximation as in the following:

$$\begin{aligned} U_0'(q_0^*) &= \frac{1}{\alpha}\left[F(\tau)\mathbb{E}[C'(\tilde{y}_1^*) | \tilde{z}_1 \leq \tau] + (1 - F(\tau))\, \mathbb{E}[U_1'(\tilde{q}_1^*) | \tilde{z}_1 > \tau]\right] \\ &= \frac{1}{\alpha}\left[F(\tau)\left(\check{C}_1' + \frac{1}{2}\check{\sigma}^2\left(\psi_U \check{U}_1''' + \psi_C \check{C}_1'''\right)\right) + (1 - F(\tau))\left(\hat{U}_1' + \frac{1}{2}\hat{\sigma}^2 \hat{U}_1'''\right)\right] \end{aligned} \tag{38}$$

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