Low Power Continuous-Time Delta-Sigma ADC
The robustness of finite amplifier GBW compensation

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Problem Description

The object of this thesis is modeling and simulation of a 2-order single bit continuous-time delta sigma ADC with 10 ENOB accuracy and 50 kHz signal bandwidth using VHDL-AMS. The work will focus on implementation and testing of a compensation technique for finite amplifier GBW in the integrators, where the main goal is to see how an ADC employing the compensation technique is affected by nonidealities compared to a similar ADC without compensation.

Supervisor: Trond Ytterdal, IET
Abstract

This paper reports on the modeling and simulation of a continuous-time \( \Delta \Sigma \) analog to digital converter (ADC) in VHDL AMS. The ADC is intended for use in a microcontroller and is therefore underlying restrictions on power consumption. Continuous-time \( \Delta \Sigma \) architectures are well known for their good low-power capabilities compared to discrete-time realizations. This is due to their reduced demands to the gain bandwidth product (GBW) of the internal amplifiers in the ADCs. Continuous-time ADCs often operate with GBWs in the range of the sampling frequency \( f_s \). The ADC presented in this work is also employing a previously reported compensation technique which ideally allows the GBW to be reduced further > 20 times of this. Considering that the current drain in the amplifiers usually is proportional with GBW, this could be a promising power saving technique.

The work focuses on the development of two similar models of a 2-order continuous-time \( \Delta \Sigma \) ADC in VHDL-AMS, where one of the ADCs is using the compensation technique. The main purpose is to see how the compensated ADC is affected by nonidealities such as GBW-variation, finite amplifier gain, \( RC \)-product variation, excess loop delay and finite DAC slew rate compared to the performance of the noncompensated ADC. The required accuracy for the modeled ADCs is \( 62 \) dB Signal to Noise and Distortion Ratio (SNDR), thus an appropriate oversampling ratio (OSR) also must be found.

The simulations show that the compensated ADC has similar performance as the noncompensated ADC operating with \( GBW = 10 f_s \) when subject to the different nonidealities. With an \( OSR = 64 \) it stays within the accuracy specification for \( GBWs \geq 0.05 f_s \). This is however only valid if actual GBW stays within \( \pm 40\% \) of the GBW compensated for. For larger deviations, especially lower GBW values, the SNDR drops rapidly. It is also shown that the internal signal swing in the ADC is reduced for low GBW values. This may limit the practical achievable SNDR when subject to circuit noise. If these potential drawbacks are circumvented, the compensation technique could lead to a further decrease of the power consumption in continuous-time \( \Delta \Sigma \) ADCs.
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Contents

1 Introduction .......................... 1

2 Theory ................................ 3
  2.1 Analog to Digital Conversion ................. 3
      2.1.1 Basics of A/D conversion ................. 3
      2.1.2 White noise assumption .................. 4
      2.1.3 The Oversampling benefit ................. 5
  2.2 ΔΣ-modulators ........................ 5
      2.2.1 Noise Shaping .......................... 5
  2.3 Continuous-Time ΔΣ-Modulators ............... 8
      2.3.1 Sampling Operation ...................... 9
      2.3.2 Loop Filter Realization .................. 9
      2.3.3 Quantizer realization .................... 10
      2.3.4 Feedback realization ...................... 10
  2.4 DT to CT Conversion ..................... 10
      2.4.1 The impulse-invariant transform .......... 11
  2.5 Gain Bandwidth Product ................... 13

3 Circuit Design Considerations .............. 15
  3.1 Feedback DAC nonidealities ................ 15
      3.1.1 Excess Loop Delay ...................... 15
      3.1.2 Compensation for Excess Loop Delay ....... 17
      3.1.3 Clock Jitter .......................... 19
      3.1.4 Unequal DAC Pulse Rise/Fall Time ........ 20
  3.2 Filter Nonidealities ...................... 21
      3.2.1 Finite OTA gain ........................ 22
      3.2.2 Integrator Gain errors .................. 23
      3.2.3 Finite Amplifier Gain-Bandwidth Product .. 23
  3.3 Motivation for using amplifiers with low GBW .. 26
  3.4 Compensation for finite amplifier GBW errors ... 26
  3.5 Quantizer Nonidealities ................... 28
CONTENTS

4 System Requirements 29
  4.1 System Specification ................................. 29
  4.2 Modulator Architecture .............................. 29
  4.3 Simulation Objectives ................................. 30
    4.3.1 Feedback coefficients and OSR ................. 30
    4.3.2 Robustness of the compensated CT \( \Delta \Sigma \) Modulator . 30

5 Model Descriptions 31
  5.1 Building Blocks ..................................... 31
    5.1.1 Resistor Model .................................. 31
    5.1.2 Capacitor Model ................................ 31
    5.1.3 OTA model ...................................... 32
    5.1.4 Integrator Model ................................ 32
    5.1.5 NRZ DAC Model .................................. 32
    5.1.6 HRZ DAC Model .................................. 33
    5.1.7 Quantizer Model ................................ 33
  5.2 Modulator Models .......................... 33
    5.2.1 2-order CT \( \Delta \Sigma \) modulator ............ 33
    5.2.2 2-order GBW compensated CT \( \Delta \Sigma \) modulator .... 34
  5.3 SNDR calculation ................................. 34

6 Simulation Results and Discussion 35
  6.1 Scaling coefficients and OSR ....................... 35
  6.2 Ideal 2-order CT modulator performance .......... 37
  6.3 Finite amplifier GBW performance .................. 37
  6.4 Robustness of GBW compensation .................... 39
    6.4.1 Variations in amplifier GBW ..................... 39
    6.4.2 Finite OTA gain ................................ 40
    6.4.3 RC product variation ............................ 41
    6.4.4 Excess Loop Delay ............................... 42
    6.4.5 DAC slew rate ................................ 43
  6.5 Internal Signal Swing .............................. 44

7 Comments 45

8 Conclusions 47
  8.1 Conclusions ...................................... 47
  8.2 Future Work .................................... 48

A Scaling coefficients compensated modulator 51

B VHDL-AMS Models 53
  B.1 Resistor Model .................................. 53
  B.2 Capacitor Model .................................. 54
  B.3 OTA model ...................................... 55
List of Figures

2.1 Block diagram general ADC .............................. 4
2.2 Spectral sampling operating ............................. 4
2.3 A general ΔΣ-modulator ................................ 6
2.4 2-order ΔΣ-modulator ................................... 7
2.5 Theoretical in-band noise power ......................... 8
2.6 Block diagram of general CT ADC ....................... 8
2.7 NRZ and RZ DAC feedback pulses ....................... 10
2.8 CT-DT equivalence ...................................... 11
2.9 Block Diagram 2-order DT modulator .................. 12
2.10 CT ΔΣ modulator ...................................... 13
3.1 Excess loop delay in RZ DAC ......................... 16
3.2 Excess loop delay in NRZ DAC ......................... 16
3.3 Pole plot of 2-order CT modulator ..................... 17
3.4 Excess loop delay compensation ....................... 18
3.5 Unequal DAC pulse rise and fall times ................ 20
3.6 Asymmetric DAC pulse ................................. 21
3.7 Active RC integrator .................................. 21
3.8 Finite GBW model ..................................... 24
3.9 Integrator step response ................................ 25
3.10 GBW feedback delay compensation .............. 26
5.1 Differential Active RC integrator .................. 32
5.2 2-order CT ΔΣ model .................................. 33
6.1 SNR vs Input amplitude for different OSR .......... 36
6.2 SNDR vs Input amplitude for 2-order modulator .... 37
6.3 SNDR vs τ .............................................. 38
6.4 SNDR vs τ frequency spectrum ....................... 38
6.5 Robustness to GBW variation ......................... 39
6.6 Influence of finite OTA gain ......................... 40
6.7 Influence of RC product variation ................... 41
6.8 Influence of Excess Loop Delay ..................... 42
6.9 Influence of DAC slew rate ......................... 43
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.10</td>
<td>Influence of DACs with asymmetric slew rate</td>
<td>43</td>
</tr>
<tr>
<td>6.11</td>
<td>Signal swing at integrator outputs</td>
<td>44</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

An important part of modern microcontrollers is the analog to digital converter (ADC). If the microcontroller is supposed to do calculations based on analog signals from e.g. analog sensors, the need for an internal ADC is apparent. In a microcontroller environment there are tight restrictions on the power consumption. The growing market of battery powered hand-held applications is especially demanding low power devices.

Oversampling data converters such as ΔΣ ADCs are often used when high resolution and reasonably low signal bandwidths like 50–100 kHz is needed. This is typical single loop, first or second order discrete time ΔΣs with high oversampling ratios (OSR) and 1-bit quantizers. In these converters the power consumption is by far limited by the high sampling rates. This is because the amplifier bandwidth needed in discrete time modulators is in the range of 5-10 times the sampling frequency ($f_s$). An approach to reduce the OSR is to use higher order modulators and multibit quantizers, but this tends to heavily increase the complexity of the design.

In stead of discrete-time filters, modulators with continuous-time filters has gained popularity the past few years. These modulators only need amplifiers with bandwidths in the range of $f_s$ and also has implicit anti-alias filtering which significantly relaxes the demands on preceding filter stages. Thus the overall ADC power consumption is significantly reduced compared to discrete-time realizations.

The work presented here is using continuous-time circuitry, and is focusing on a compensation technique presented in [Ortmanns, 2004] which could decrease the required amplifier bandwidth additional 10-20 times if it is feasible to use in a circuit realization. A continuous-time ΔΣ ADC employing the compensation technique will be modelled in [VHDL-AMS], and compared to a similar ADC without the compensation during simulations. The main purpose of this work is to see how circuit imperfections and non-idealities affect the performance of the compensated ADC compared to the noncompensated one.
Chapter 2

Theory

This chapter covers the fundamentals of A/D conversion, an introduction to ΔΣ-modulation and theory regarding development of continuous-time ΔΣ-modulators.

2.1 Analog to Digital Conversion

2.1.1 Basics of A/D conversion

The conversion of a continuous-time analog signal into a digital one is done in two operations. First there is a sampling of the of the analog signal (usually with a constant sample period \( T_s \)), then a quantization of the signal amplitude is done. If the signal band of a sampled signal is less than half the sampling frequency, the sampling in time is a completely invertible process. Looking at the frequency spectrum of a sampled signal in figure 2.2 this could be understood. When a signal is sampled at uniform time intervals, this results in a periodicity of the signal spectrum at multiples of the sampling frequency \( f_s \) in the frequency domain as seen in the figure. With simple low-pass filtering it is clear that the original baseband spectrum can be reconstructed as long as the spectrums doesn’t overlap. This is achieved when

\[
f_s \geq 2f_0 = f_N
\]

(2.1)

where \( f_0 \) is the bandwidth of the input signal. This equation is known as the Nyquist theorem, and \( f_N \) is called the Nyquist frequency. An analog filter preceding the sampling operation is required to assure that the input signal bandwidth is limited to \( f_0 \). This filter is known as the antialiasing filter (AAF). A basic ADC structure is shown in figure 2.1. An ADC working at a sampling frequency that equals to \( f_N \) is called a Nyquist Rate converter, these converters are hard to design in practice because of the zero transition band required for the AAF (p.8 [Ortmanns, 2006]). To overcome this problem, this type of converters often use a slight amount of oversampling. The
oversampling ratio (OSR) is defined as

\[
OSR = \frac{f_s}{2f_0}
\]  

(2.2)

Nyquist rate converters operates in most cases with an \( OSR = 1.5 \rightarrow 10 \) [Johns, 1997]. Increasing the OSR greatly relaxes the demands to the AAF, thus simplifies the design and reduces the power and chip area of the filter (p.8 [Ortmanns, 2006]).

The quantizer encodes a continuous range of analog values into a set of predefined discrete levels. Quantization is usually uniform and the space between two adjacent output levels of the quantizer is defined as the quantizer step width:

\[
\Delta = \frac{FS}{2^{B_{int}} - 1}
\]

(2.3)

where FS is the full-scale input range and \( 2^{B_{int}} \) is the number of different output levels. Since an infinite number of input values of the sampled input signal is mapped to an finite number of values in the quantizer, the quantization is an noninvertible process.

2.1.2 White noise assumption

If the input signal \( x(n) \) has a rapidly and random varying behavior, the quantization noise \( e(n) \) can be approximated as a random number uniformly
distributed between $\pm \frac{\Delta}{2}$ and uncorrelated with its previous values, where $\Delta$ is the step size of the quantizer. It is also assumed that $e(n)$ have statistical properties independent of $x(n)$. By these properties, $e(n)$ is classified as white noise with a mean square value of $e_{\text{rms}}^2 = \frac{\Delta^2}{12}$ (p.8 [Schreier, 2005]).

2.1.3 The Oversampling benefit

When using a one-sided representation of the frequency domain, the power spectral density (PSD) of the quantization noise is:

$$S_e(f) = e_{\text{rms}}^2 \left( \frac{2}{f_s} \right)$$

Equation 2.4 implies that the quantization noise is uniformly distributed in the frequency range $0 < f < \frac{f_s}{2}$. The signal band however, might have a range from $0 < f < f_0$. The total in-band noise power is then calculated by using equation 2.2 and 2.4:

$$q_{\text{rms}}^2 = \int_0^{f_0} S_e(f) df = \frac{2f_0e_{\text{rms}}^2}{f_s} = \frac{e_{\text{rms}}^2}{OSR}$$

Equation 2.5 shows for each doubling of OSR, the in-band noise power decreases by $3dB$ or 0.5 bits. Data converters employing oversampling to benefit from this property is called oversampled converters. By increasing the OSR they can achieve higher accuracy than a Nyquist converter using the same quantizer.

2.2 $\Delta\Sigma$-modulators

Oversampled converters are also usually using $\Delta\Sigma$-modulation to decrease the in-band noise power even further. The basics of the $\Delta\Sigma$-modulation technique is presented in the following sections.

2.2.1 Noise Shaping

A general noise-shaped $\Delta\Sigma$ modulator and its linear model is shown in figure 2.3. It consists of a loop filter $H(z)$ and a quantizer. 1-bit quantizers are often used due to its inherent linearity (p.7 [Schreier, 2005]). Using the linear model, we can derive a signal transfer function (STF), $S_{TF}(z)$, and a noise transfer function (NTF), $N_{TF}(z)$. It is assumed that $u(n)$ and $e(n)$ are two independent inputs.

$$S_{TF}(z) = \frac{Y(z)}{U(z)} = \frac{H(z)}{1 + H(z)}$$

(2.6)
Figure 2.3: A general $\Delta\Sigma$-modulator and its linear model (fig. 14.6 [Johns, 1997])

\[
N_{TF}(z) = \frac{Y(z)}{E(z)} = \frac{1}{1 + H(z)} \tag{2.7}
\]

Combining these two equations gives an output function:

\[
Y(z) = S_{TF}(z)U(z) + N_{TF}(z)E(z) \tag{2.8}
\]

The zeros of $N_{TF}(z)$ will be equal to the poles of $H(z)$. This means that if $|H(z)| \gg 1$ then $|N_{TF}(z)| \ll 1$. To benefit from these noise shaping qualities, $H(z)$ is chosen such that its magnitude is large in the signal band (from $0$ to $f_0$). This will give $|S_{TF}(z)| \approx 1$, and a $|N_{TF}(z)| \approx 0$ in the signal band. Thus, the quantization noise is reduced in the signal band while the signal itself is mostly unaffected.

**First-order noise shaping**

When realizing a first order noise shaping, $N_{TF}(z)$ should have its zero at DC, resulting in a high-pass filtering of the quantization noise. Since the zeros of $N_{TF}(z)$ are equal to the poles of $H(z)$, we choose $H(z)$ to be a discrete-time integrator which have this transfer function:

\[
H(z) = \frac{1}{z - 1} \tag{2.9}
\]

Combining equation 2.6, 2.7 and 2.9 gives a STF given by

\[
S_{TF}(z) = \frac{Y(z)}{U(z)} = \frac{H(z)}{1 + H(z)} = \frac{1}{1 + \frac{1}{z-1}} = z^{-1} \tag{2.10}
\]
and a NTF given by

\[
N_{TF}(z) = \frac{Y(z)}{E(z)} = \frac{1}{1 + H(z)} = \frac{1}{1 + \frac{1}{z^{-1}}} = 1 - z^{-1}
\] (2.11)

The equations show that the STF is a simple delay, while the NTF is a first order high-pass filter. In (p.542 [Johns, 1997]) it is shown that with such a first order modulator, each doubling of the OSR gives 9dB or 1.5 bits increase in the SNR. Compared to the 3dB increase with only oversampling, this shows the effect of noise shaping.

**Higher-order noise shaping**

![Figure 2.4: 2-order ΔΣ-modulator (fig. 1.6 [Schreier, 2005])](image)

One way to further increase the resolution of the modulator, would be to use a higher-order loop filter. If another integrator and feedback path is added such as in figure 2.4, linearized analysis gives (eq. 1.8 [Schreier, 2005]):

\[
V(z) = z^{-1}U(z) + (1 - z^{-1})^2 E(z)
\] (2.12)

The NTF is now a second-order high-pass filter. It is shown in (p.9 [Schreier, 2005]) that this gives 15db or 2.5 bits increase in resolution when doubling the OSR.

By adding more integrators and feedback branches to the loop, higher order NTFs can be obtained. For \(L^{th}\)-order loops resulting in \(N_{TF}(z) = (1 - z^{-1})^L\), a function for approximation of the theoretical in-band noise power is (eq. 1.10 [Schreier, 2005]):

\[
q_{\text{rms}}^2 = \frac{\pi^{2L} e_{\text{rms}}^2}{(2L + 1)(OSR)^{2L+1}}
\] (2.13)

This function is plotted in figure 2.5

Stability considerations will reduce the practical achievable resolution of higher-order modulators. For higher-order single-bit modulators the difference is substantial. More than 60dB for a \(5^{th}\)-order modulator (p.10 [Schreier, 2005]). These stability issues arise when the modulator order is higher than second-order.
CHAPTER 2. THEORY

2.3 Continuous-Time $\Delta\Sigma$-Modulators

The $\Delta\Sigma$-theory presented so far is based on purely discrete-time (DT) circuitry. The analog input $u(n)$ to the modulator is a sampled signal, and the loop filter $H(z)$ is made of DT, $Z$-domain filters. The majority of published $\Delta\Sigma$-modulators over the last decades have also been DT-realizations using switched capacitor (SC) circuitry. This is mostly because it is easy to map the mathematics of the modulators onto the implementation (p. xxv [Cherry, 2000]), and the well known implementations and high linearity of SC-filters (p. 39 [Ortmanns, 2006]). Due to the popularity of this approach, there is a great amount of literature and tools dealing with synthesis and implementation of DT $\Delta\Sigma$-modulators, e.g. [Schreier, 2005] and [Norsworthy, 1997].

The first implementations of $\Delta\Sigma$-ADCs used continuous-time (CT) loop filters, but when the SC technique became popular in the 1980s, it was chosen in favour of its CT counterpart because of the properties already men-
2.3. CONTINUOUS-TIME ΔΣ-MODULATORS

mentioned. A typical block diagram of a CT ΔΣ ADC is shown in figure 2.6. The input signal is fed through an optional AAF and into the CT-modulator. The loop filter is now made of CT-filters such as active RC-filters and gmC-filters. The sampling operation is finally done in the internal quantizer which is clocked at the sampling frequency $f_s$. In the following sections the key advantages an disadvantages between CT contra DT realizations is presented.

2.3.1 Sampling Operation

A key advantage of the CT modulator is that the sampling operation is done inside the ΔΣ loop (figure 2.6) which makes the nonidealities in the sampling process subject to noise-shaping. In the a DT modulator the sample operation is done with a sample and hold (S/H) circuit at the input of the converter, thus every error in this operation is directly added to the input signal (p.40 [Ortmanns, 2006]). In addition to the noise shaped sampling errors, moving in the sampling operation inside the loop also results in some degree of implicit antialiasing filtering. This heavily reduces the specification of the front-end AAF and sometimes makes it unnecessary. In high-speed circuits or applications with very low OSR, the implicit antialiasing filtering can be a key argument in choosing a CT implementation (p.40 [Ortmanns, 2006]). For an in-depth understanding of this property of CT modulators, chapter 3.5 in [Ortmanns, 2006] is recommended reading.

2.3.2 Loop Filter Realization

The loop filters in both DT and CT architectures are integrators or resonators depending on the preferred transfer function. In DT modulators the signals are quickly changing pulses, and the maximum clock rate of DT modulators with SC circuits is limited by the operational transconductance amplifier (OTA) bandwidth and the settling time for the charge transfers. In CT modulators all signals are represented by analog, continuous-time waveforms, resulting in a theoretical clock rate that is in the order of a magnitude higher than its DT counterpart in the same technology. In real-life applications, this usually factor lies between three and five (p.41 [Ortmanns, 2006]). Another difference is the absolute accuracy of the filter transfer functions. In SC integrators, the integrator gain is determined by a capacitor ratio, which can have an accuracy higher than 0.1%. In CT integrators, the integrator gain is set by a $RC$ or $gmC$ product which can vary with an amount of ±50%. This results in performance degradation and in worst case unstable operation in CT modulators.
2.3.3 Quantizer realization

In both CT and DT implementations of ΔΣ-modulators, all the nonidealities of the quantization are subject to noise shaping. This is due to the placement of the quantizer inside the modulator loop. The decision time of the quantizer is on the other hand more critical in CT modulators. In a DT modulator, the quantizer has half a clock cycle to determine its value. In a CT modulator the quantizer should ideally be infinitely fast since the result is needed instantly to generate the feedback signal. This could be a source to severe performance limitations if not taken care of.

2.3.4 Feedback realization

The feedback signal of a DT modulator using SC circuitry is made by charging a capacitor to a reference voltage and then discharging it onto the the integrating capacitance of the integrator. The feedback signal of a CT modulator is a continuous waveform integrated over time, which makes it sensitive to every deviation from an ideal one. This results in reduced performance and even instability in some situations. Most CT modulators use rectangular feedback pulses in a non return to zero (NRZ) or return to zero (RZ) configuration as seen in figure 2.7.

![Figure 2.7: (a) NRZ DAC feedback pulse (b)RZ DAC feedback pulse (fig. 3.2 (a)-(b) [Ortmanns, 2006])](image)

2.4 DT to CT Conversion

Most published work on ΔΣ modulators has been focused on DT implementations. A common way to design a CT loop filter is to start with a DT
loop filter $H(z)$ with the desired specifications. Simulating the ideal modulator behaviour in the discrete-time domain is simple and very fast using tools like the Delta Sigma Toolbox for Matlab [ΔΣ Toolbox].

2.4.1 The impulse-invariant transform

![Diagram](image)

Figure 2.8: CT-DT equivalence(fig. 2.1 [Cherry, 2000])

For a successful DT to CT modulator transformation, the equivalence between these two structures must be investigated. The clocked internal quantizer of the CT modulator makes the modulator to a kind of DT system at this point (p.48 [Ortmanns, 2006]). From figure 2.8 a DT to CT equivalence is achieved if the input to the quantizers $\hat{x}(t)$ and $x(n)$ are equal at the sampling instants. This means

$$x(n) = \hat{x}(t)|_{t=nT_s}$$

(2.14)

If this is fulfilled, the bitstreams out of both modulators is the same, and thus also the noise performance (p.48 [Ortmanns, 2006]). The condition in 2.14 is satisfied if the impulse responses of the open-loop diagrams in 2.8 is are equal at sampling times. This leads to the condition in the $z$-domain (p.31 [Cherry, 2000])

$$\mathcal{Z}^{-1}\{H(z)\} = \mathcal{L}^{-1}\{\tilde{R}_D(s)\tilde{H}(s)\}|_{t=nT_s}$$

(2.15)

or in the time domain

$$h(n) = [\tilde{r}_D(t) * \hat{h}(t)]|_{t=nT_s} = \int_{-\infty}^{\infty} \tilde{r}_D(\tau)\hat{h}(t-\tau)d\tau|_{t=nT_s}$$

(2.16)

where $\tilde{r}_D(t)$ is the impulse response of the feedback DAC. This transformation is called the impulse-invariant transform because we require the
open-loop impulse responses to be the same at the sampling instants. This transformation makes it possible to design a CT loop filter \( H(s) \) that together with feedback DAC transfer function \( \hat{r}_D(t) \) exactly matches the noise shaping behaviour of a DT loop filter \( H(z) \) (p.31 [Cherry, 2000]).

To simplify the use of the transform, conversion tables are made for basic loop filter poles such as \( 1/(z - z_k)^i, i = 1, 2, 3... \) Table 2.1 covers first and second order filter poles.

Table 2.1: CT equivalents for rectangular feedback DAC pulses for first- and second-order DT low-pass loop filter poles. \( \alpha \) and \( \beta \) are the timings for the rising and falling edge of the DAC pulse relative to the sampling period. (Tab. 3.2 [Ortmanns, 2006])

<table>
<thead>
<tr>
<th>( Z )-domain</th>
<th>( S )-domain equivalents with ( f_s (\text{Hz}) = 1/T_s )</th>
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<tr>
<td>( \frac{1}{z-1} )</td>
<td>( \omega_0 = \frac{f_s}{\beta - \alpha} )</td>
</tr>
<tr>
<td>( \frac{1}{(z-1)^2} )</td>
<td>( \omega_0 = \frac{f_s^2}{\beta - \alpha}, \omega_1 = \frac{1}{2} \frac{f_s (\alpha + \beta - 2)}{\beta - \alpha} )</td>
</tr>
</tbody>
</table>

Figure 2.9: Block Diagram 2-order DT modulator(fig. 3.6 a) [Ortmanns, 2006])

Consider the second order DT modulator in figure 2.9 with the integrators \( I(z) = z^{-1}/(1 - z^{-1}) \). The loop filter would then have the transfer function

\[
LF(z) = -a_2 I(z) - a_1 a_2 I^2(z) = -\frac{a_2}{z - 1} - \frac{a_1 a_2}{(z - 1)^2}
\]

(2.17)

By using rows 1 and 2 from table 2.1 we get

\[
LF(s)_{\text{DT-CT}} = -a_2 \frac{f_s}{\beta - \alpha} - a_1 a_2 \frac{1}{2} \frac{f_s (\alpha + \beta - 2)}{\beta - \alpha} s + \frac{f_s^2}{\beta - \alpha}
\]

(2.18)

The second order CT modulator in figure 2.10 with the integrators \( Int_i = \)
2.5 Gain Bandwidth Product

The gain bandwidth product (GBW) of an amplifier is defined as:

$$GBW = A \cdot \omega_{3dB} \ [\text{rad/s}] ,$$

(2.23)

where $A$ is the dc-gain of the amplifier and $\omega_{3dB}$ is the $-3dB$ frequency. GBW is used to determine the maximum gain that can be extracted from an amplifier for a given frequency and vice versa. In common CMOS amplifiers...
the correspondence between GBW and the transconductance parameter $g_m$ is:

$$GBW = \frac{g_m}{C},$$

(2.24)

where $C$ is the load capacitance. Depending on the operating point and scaling of the transistors, the variation of $g_m$ is usually proportional with the drain current $I_d$ in practice. Hence by equation 2.24 it can be said that the amplifier GBW also is proportional to the current drain of the amplifier. In many cases lowering the GBW actually could give a larger current reduction than this due to the possibility to use more optimal transistor geometries since the transistor sizes are scaled down with the current.
Chapter 3

Circuit Design Considerations

The CT ΔΣ modulator is built of three major building blocks as in figure 2.6: a CT loop filter \( H(s) \), a clocked internal quantizer and a CT feedback DAC. Due to variations in the manufacturing process and circuit imperfections, which comes from design (e.g. OTA gain) or are intrinsic (e.g. component mismatch), each of these blocks would deviate from ideal behavior.

3.1 Feedback DAC nonidealities

A major error contributor to CT ΔΣ modulators is the feedback DAC. Errors in the outermost feedback loop adds directly to the input signal, which means that these errors are not subject to noise shaping (p.84 [Ortmanns, 2006]). If a typical rectangular feedback form is assumed (e.g NRZ and RZ), most common feedback DAC errors can be derived. The feedback pulse can be affected by timing errors which varies the position and length of the pulse. This can be a constant delay \( \tau_d \) of the pulse often refered to as excess loop delay, or a statistical variation of the position or length of the DAC pulse caused by clock jitter. The pulse edges would also inevitably have nonideally rising and falling times due to finite slew rate in the DAC.

3.1.1 Excess Loop Delay

Excess loop delay is an unwanted delay \( t_d \) between the ideal and the implemented feedback DAC pulse:

\[
t_d = \tau_d T_s,
\]

where \( \tau_d \) is the excess loop delay relative to the sampling period \( T_s \). This delay can arise from finite respond time in the DAC and delays in the path between the quantizer and the DAC. The excess loop delay causes two different nonideal effects in CT ΔΣ modulators. When the loop delay shifts the DAC pulse, but the pulse retains within its sampling period as seen
in figure 3.1 or when the loop delay shifts parts of the DAC pulse in to the next sampling period as in figure 3.2. The latter happens when using a NRZ-DAC or even sometimes with RZ-DACs in high speed modulators (p.86 [Ortmanns, 2006]).

Figure 3.1: Excess loop delay in RZ DAC

Figure 3.2: Excess loop delay in NRZ DAC (fig. 4.2 [Ortmanns, 2006])

Consider a second-order modulator as in figure 2.10 with an ideal NRZ DAC with pulse position \( \alpha = 0, \beta = 1 \), and an excess loop delay \( \tau_d \). The pulse position will then be \( \alpha = \tau_d, \beta = 1 + \tau_d \). For mathematical purposes the it is useful to divide the pulse into two separate pulses. One placed in the original sampling period with position \( \{ \alpha_1 = \tau_d, \beta_1 = 1 \} \) and another placed in the following sample period with position \( \{ \alpha_2 = 0, \beta_2 = \tau_d \} \) as shown in figure 3.2. Now it is possible to calculate the effects of this delay using table 3.1 to find the equivalent DT loop filter. \( (f_s = 1) \)

\[
\begin{align*}
-\frac{k_1}{s^2} \rightarrow LF_{CT-DT}|_1 &= -\frac{k_1(\beta_1(2 - \beta_1) - \alpha_1(2 - \alpha_1))z - k_1(\beta_1^2 - \alpha_1^2)}{2(z - 1)^2} + \frac{-k_1(\beta_2(2 - \beta_2) - \alpha_2(2 - \alpha_2))z - k_1(\beta_2^2 - \alpha_2^2)}{2(z - 1)^2} \cdot z^{-1} \\
(3.2)
\end{align*}
\]

\[
\begin{align*}
-\frac{k_2}{s} \rightarrow LF_{CT-DT}|_2 &= \frac{-k_2(\beta_1 - \alpha_1)}{z - 1} + \frac{-k_2(\beta_2 - \alpha_2)}{z - 1} \cdot z^{-1} \\
(3.3)
\end{align*}
\]

By combining these two equations and the values for \( \alpha_i \) and \( \beta_i \), the DT representation of the loop filter including the excess loop delay \( \tau_d \) is given
3.1. FEEDBACK DAC NONIDEALITIES

as

\[
LF(z, \tau_d) = \frac{(-k_1 \tau_d^2 + (2k_2 + 2k_1) \tau_d - 2k_2 - k_1)z^2}{2z(z - 1)^2} + \frac{(2k_1 \tau_d^2 + (-k_1 - 4k_2) \tau_d + 2k_2 - k_1)z}{2z(z - 1)^2} \]

\[
+ \frac{2k_2 \tau_d - k_1 \tau_d^2}{2z(z - 1)^2} \tag{3.4}
\]

This transfer function contains terms to the power \(z^{-3}\) indicating that the second-order CT modulator has turned into a third-order one due to the excess loop delay. For \(\tau_d = 0\) the expression in 3.4 is equal to an ideal second-order system. In figure 3.3 the effect of excess loop delay in a second-order modulator is illustrated. The poles is moving towards the unit circle with a growing \(\tau_d\), and finally exceeds the stability boundary. Thus excess loop delay may cause an unstable modulator.

![Figure 3.3: Pole plot of 2-order CT modulator (fig. 4.3 a) [Ortmanns, 2006)](image)

3.1.2 Compensation for Excess Loop Delay

To compensate for the increased modulator order due to excess loop delay, a technique described in [Ortmanns, 2006] and [Cherry, 2000] can be used. As seen in figure 3.4, an auxiliary feedback DAC (\(DAC_{HRZ}\)) is added to the system. A half delay RZ DAC with \(\{\alpha_h = 0.5, \beta_h = 1\}\) is a feasible implementation (p.89 [Ortmanns, 2006]). The concept for this compensation is to find the equivalent DT loop filter for the nonideal modulator in figure 3.4 and tune the coefficients to match the original ideal second-order loop filter. By using table 3.1 the loop filter of the first branch in figure 3.4 is found
Figure 3.4: Compensation for excess loop delay in second-order modulator (fig. 4.4 [Ortmanns, 2006])

Table 3.1: DT equivalents for rectangular feedback DAC pulses for first- and second-order CT low-pass loop filter poles. (Tab. 3.6 [Ortmanns, 2006])

<table>
<thead>
<tr>
<th>S-domain</th>
<th>$Z$-domain equivalents with $f_s$ (Hz) = $1/T_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{f_0}{s}$</td>
<td>$\frac{\omega_0}{z-1}$, $\omega_0 = \beta - \alpha$</td>
</tr>
<tr>
<td>$\frac{f_0^2}{s^2}$</td>
<td>$\frac{\omega_0^2 + \omega_0^2}{(z-1)^2}$, $\omega_0 = \frac{\beta^2 - \alpha^2}{2}$, $\omega_1 = \frac{[\beta(1-\beta) - \alpha(1-\alpha)]}{2}$</td>
</tr>
</tbody>
</table>
(with NRZ DAC and \( f_s = 1 \)):
\[
LF_{CT}|_1 = \frac{-k_1}{s^2} \rightarrow LF_{CT-DT}|_1 = \frac{-k_1(\beta_{11}(2 - \beta_{11}) - \alpha_{11}(2 - \alpha_{11})z - k_1(\beta_{11}^2 - \alpha_{11}^2)}{2(z - 1)^2} \]
\[
+ \frac{-k_1(\beta_{21}(2 - \beta_{21}) - \alpha_{21}(2 - \alpha_{21})z - k_1(\beta_{21}^2 - \alpha_{21}^2)}{2(z - 1)^2} \cdot z^{-1},
\]
(3.5)

where \( \{\alpha_{11} = \tau_d, \beta_{11} = 1\} \) and \( \{\alpha_{21} = 0, \beta_{21} = \tau_d\} \). For the second branch a similar loop filter is found:
\[
LF_{CT}|_2 = \frac{-k_2}{s} \rightarrow LF_{CT-DT}|_2 = \frac{-k_2(\beta_{12} - \alpha_{12})}{z - 1} + \frac{-k_2(\beta_{22} - \alpha_{22})}{z - 1} \cdot z^{-1},
\]
(3.6)
where \( \{\alpha_{12} = \tau_d, \beta_{12} = 1\} \) and \( \{\alpha_{22} = 0, \beta_{22} = \tau_d\} \). For the new branch with the HRZ feedback DAC one obtains:
\[
LF_{CT}|_h = \frac{k_h}{s} \rightarrow LF_{CT-DT}|_h = \frac{k_h(\beta_{1h} - \alpha_{1h})}{z - 1} + \frac{-k_h(\beta_{2h} - \alpha_{2h})}{z - 1} \cdot z^{-1},
\]
(3.7)
where \( \{\alpha_{1h} = 0.5 + \tau_d, \beta_{1h} = 1\} \) and \( \{\alpha_{2h} = 0, \beta_{2h} = \tau_d\} \). These three loop filters form together the DT representation of the CT modulator in 3.4 with excess loop delay \( \tau_d \) in the feedback path.

By setting \( \tau_d = 0 \) in the expression in 3.4, the DT equivalent transfer function for a ideal second order CT modulator is found:
\[
LF_{2nd|ideal}(z) = -\frac{1}{2} \left( \frac{k_1 + 2k_2}{z - 1} - \frac{k_1 - 2k_2}{2(z - 1)^2} \right)
\]
(3.8)

The new feedback coefficients is found by solving the combined equation from 3.8 and 3.5-3.7:
\[
\{LF_{CT}|_1 + LF_{CT}|_2 + LF_{CT}|_h\} = LF_{2nd|ideal}(z)
\]
(3.9)
which gives the following coefficients
\[
k_1^* = k_1, \quad k_2^* = \frac{3k_1 + 3k_2}{2} + 2k_2, \quad k_h^* = k_1 \tau_d + k_2
\]
(3.10)
where \( k_i^* \) are the coefficients of the compensated modulator in 3.4 and \( k_i \) are the coefficients of the ideal modulator. Thus with only one extra feedback DAC, the errors due to excess loop delay is ideally cancelled.

### 3.1.3 Clock Jitter

Clock jitter is statistical variations of the sampling frequency. When moving from the analog to the discrete-time domain in the quantizer, the continuous-time signal is ideally sampled with a constant time interval \( T_s = \frac{1}{f_s} \). Circuit
CHAPTER 3. CIRCUIT DESIGN CONSIDERATIONS

non-idealities and the quality of the clock source will result in a deviation of the sampling interval. This will introduce errors in the system because the samples are taken at the wrong time. A CT modulator has two error sources due to clock jitter. First the sampled internal quantizer is susceptible to jitter affected sampling errors. The placement of the quantizer inside the modulator loop make these errors heavily suppressed by noise shaping, and hence they may be neglected in practice (p.95 [Ortmanns, 2006]). The errors generated in the feedback DACs due to clock jitter is much more of concern in CT modulators. The feedback waveform is integrated over time, and a statistical variation of of this waveform results in a statistical integration error, which again gives a raised noise floor (p.95 [Ortmanns, 2006]).

For a NRZ feedback DAC pulse, equation 5.14 in [Cherry, 2000] calculates the SNR in the baseband for a modulator where the jitter noise is the dominating noise source:

$$\text{SNR}_{NRZ} = 10 \log_{10} \frac{OSR \cdot \frac{V_{in}^2}{\sigma_j^2 (\frac{n}{T_s})}}{\sigma_y^2},$$  \hspace{1cm} (3.11)

where $V_{in}$ is the input signal amplitude, $\sigma_j/T_s$ is the jitter variance relative to the clock period and $\sigma_y^2$ is the variance of $\delta_y = y(n) - y(n-1)$ where $y(n)$ is the modulator output stream.

In a NRZ modulator, the jitter only matter when the output changes. I.e. the DAC pulse doesn’t change if the output is stable. In a RZ modulator, both the rising and falling edge of the DAC pulse occur every clock cycle. This results in an increased number of transitions affected by the jitter, thus increased noise. Therefore a general rule of thumb is that a modulator with RZ DAC pulses has 6dB higher inband jitter noise than a modulator with NRZ DAC pulses (p. 116 [Cherry, 2000]).

3.1.4 Unequal DAC Pulse Rise/Fall Time

![Unequal DAC Pulse Rise/Fall Time](image)

Figure 3.5: Effect of unequal DAC pulse rise and fall times (fig. 2.10 [Cherry, 2000])

CT ΔΣ-modulators are in general sensitive to the exact shape of the DAC pulse. Any nonuniformities of the pulse tends to degrade the modulator performance. Consider a DAC output as in figure 3.5 where the rise
3.2. Filter Nonidealities

The transfer function of the loop filter defines the noise-transfer function, and thus the noise-shaping behaviour of the ΔΣ-modulator. The loop filter usually consists of several first order filters, commonly arranged in a feedback or feedforward architecture (p. 117 [Ortmanns, 2006]). In CT modulators these single filters are realized using RC-integrators, gmC-integrators or LC-resonators. In the following active RC-integrators will be discussed.

Figure 3.7 shows a typical RC-integrator with \( n \) inputs and an amplifier with transfer function \( A(s) \). The scaled integrator corner frequency \( \omega_I \)
is controlled by the $RC$-product, $R_i C$. The following ideal relation can be found for the integrator corner frequency in CT modulators (eq. 5.1 [Ortmanns, 2006]):

$$\omega_{I_i} = \frac{1}{R_i C} = |k_i f_s|$$  \hspace{1cm} (3.13)

Where $k_i$ is the CT scaling coefficients and $f_s$ is the sampling frequency. The scaling coefficients can be found e.g. using a DT to CT conversion as described in section 2.4, and then mapped to different resistor values $R_i$.

The integrator transfer function (ITF) of the $i^{th}$ input path to the integrator output with $N$ different inputs is (eq. 5.2 [Ortmanns, 2006]):

$$ITF_i(s) = \frac{k_i f_s}{s(1 + \frac{1}{A(s)}) + \frac{1}{A(s)} \sum_{l=1}^{N} k_l f_s} \bigg|_{A(s) \to \infty} \approx \frac{k_i f_s}{s},$$

which equals an ideal integrator transfer function as long as the amplifier gain $A(s)$ is infinite.

### 3.2.1 Finite OTA gain

A well studied nonideal effect in $\Delta\Sigma$ modulators is the behaviour when the integrator amplifiers has finite dc-gain. Applying finite dc-gain $A(s) = A_{dc}$ to equation 3.14 yields

$$ITF_{i,A_{dc}}(s) = \frac{k_i f_s}{s(1 + \frac{1}{A_{dc}}) + \frac{1}{A_{dc}} \sum_{l=1}^{N} k_l f_s} \approx \frac{k_i f_s}{s + \frac{1}{A_{dc}} \sum_{l=1}^{N} k_l f_s}$$  \hspace{1cm} (3.15)

The ITF is now a first-order pole transfer function with a dc gain of $A_{dc} k_i / \sum_l k_l$ and a pole at $f_s \sum_l k_l / A_{dc}$. The pole is no longer placed at dc, which again effects the placement of the zeros in the NTF of the modulator (see equation 2.11). When all the zeros in the NTF moves away from dc, this reduces the amount of attenuation of the quantization noise in the baseband of the modulator. This nonideality is usually called leaky integration and affects both CT and DT modulators in the same way (p. 120 [Ortmanns, 2006]).

Equation 5.8 in [Ortmanns, 2006] gives an expression for the integrated in-band noise (IBN) of a general second order modulator with finite amplifier dc gain $A_{dc}$:

$$IBN_2(A_{dc}) \approx \Delta^2 \frac{1}{12 k_1^2 [NRZ]^2} \left( \frac{\pi^4}{5} \frac{1}{OSR^5} + \frac{2\pi^2}{3} \frac{1}{OSR^3 A_{dc}^2} + \frac{1}{OSR \cdot A_{dc}^4} \right)$$  \hspace{1cm} (3.16)

If the dc gain is in in the range of the OSR, e.g. $A_{dc} \approx OSR$, every part of equation 3.16 is proportional to $1/OSR^5$, the ideal noise shaping supression for a second order modulator (p. 120 [Ortmanns, 2006]).
3.2. FILTER NONIDEALITIES

3.2.2 Integrator Gain errors

Variations in the integrator gain due to nonideal scaling coefficients is also a nonideal effect that has to be considered when designing CT modulators. Integrator gain is mapped to resistor-capacitor products, e.g. $1/RC$, which can vary largely over process and temperature. The absolute component values can typically variate with 20% for capacitors and 30% for resistors, which gives a worst case $RC$ variation of 56% [Project Meeting]. This surely makes this an error worth to investigate. Combining equation 3.13 and 3.14 and applying a tolerance $\delta_{RC}$ to the integrator scaling coefficient $k_i$ gives an ITF:

$$ITF_{RC}(s)|_i = \frac{1}{sR_iC(1+\delta_{RC})} = \frac{f_s k_i}{s(1+\delta_{RC})} = GE_{RC} \frac{k_if_s}{s}, \quad (3.17)$$

where $GE_{RC}$ is the resulting equivalent gain error of the integrator. Equation 5.13 in [Ortmanns, 2006] gives an expression for IBN of single-loop $N^{th}$-order modulator with gain error $GE$:

$$IBN_N(GE) \approx \frac{\pi^2 N}{2N+1} \frac{\Delta^2}{12 k_1[NRZ]^2 k_q^2 OSR^{2N+1}} \prod_{i=1}^{N} \frac{1}{GE_i^2} \quad (3.18)$$

Equation 3.18 show that the IBN is still proportional to $1/OSR^{2N+1}$, thus the modulator still shows $N^{th}$-order noise shaping.

3.2.3 Finite Amplifier Gain-Bandwidth Product

Finite amplifier gain-bandwidth product (GBW) introduces non-dominant poles in integrator transfer functions. Consider the amplifier in figure 3.7 modelled as a single pole amplifier, with a transfer function given by (eq. 1 [Ortmanns, 2004]):

$$A(s) = \frac{A_{dc}}{\omega_A + 1}, \quad GBW = A_{dc}\omega_A \quad [rad/s], \quad (3.19)$$

where $\omega_A$ is the dominant pole of the amplifier. Combining equation 3.14 and 3.19 gives an ITF for the $i^{th}$ input path due to finite GBW in the amplifier considering a large dc gain in the amplifier (eq. 3 [Ortmanns, 2004]):

$$ITF_{GBW}(s)|_i \approx \frac{k_if_s}{s} \frac{GBW}{GBW + \sum_i |k_if_s|} \frac{GBW}{s(GBW + \sum_i |k_if_s|) + 1} \quad (3.20)$$

This ITF is equivalent to an ideal scaled integrator as in equation 3.14 in series with a gain error and a additional second integrator pole. The gain error and second pole can be described as:

$$GE_{GBW} = \frac{GBW \omega_p}{GBW + \sum_i \omega_i}, \quad \omega_p = GBW + \sum_i \omega_i, \quad (3.21)$$
where \( \omega_{\text{it}} \) is the corner frequency of the \( l \text{th} \) integrator. This results in a new expression for the ITF:

\[
\text{ITF}_{\text{GBW}}(s) \approx k_i f_s \frac{GE_{\text{GBW}}}{s \frac{1}{\omega_p} + 1}
\]  

(3.22)

When neglecting the influence from the second integrator pole, the errors are similar to the gain-errors from RC-variations (sect. 3.2.2). For very low GBWs this simplification fails, thus the influence from the second integrator pole has to be investigated. To simplify the following, a variable \( c \) is defined as the ratio between \( f_s \) and GBW:

\[
c = \frac{\text{GBW}}{2\pi f_s}
\]

(3.23)

Figure 3.8 a) show a second order CT modulator with ITFs as in equation 3.22. The nonideal poles in the ITFs can be moved behind the summing nodes to achieve the modified system in figure 3.8 b). Now the feedback transfer functions are scaled single or double pole systems. Because
of the rectangular feedback pulse, the combination of the feedback transfer functions with the integrators in the forward path can be modeled as non-ideal integrators with delayed output slopes (p. 1091 [Ortmanns, 2004]). An example of this is seen in figure 3.9, where the ideal and nonideal step response of an integrator is plotted. The final model for a modulator with finite GBW is shown in figure 3.8 c). The extra integrator poles are now reduced to the feedback delays $\tau_{D2nd}$ and $\tau_{D1st}$. It should also be mentioned that the input transfer function is not affected by the poles $\omega_{pi}$ as long as $\omega_{pi} \gg f_{sig}$, where $f_{sig}$ is the input signal frequency (p. 1089 [Ortmanns, 2004]).

In (p. 1090 [Ortmanns, 2004]) expressions for the feedback delays $\tau_{Di}$ have been calculated:

$$\tau_{D1st} = \frac{1 - e^{-\omega_{p2}/f_s}}{\omega_{p2}},$$

$$\tau_{D2nd} = \frac{w_{p1}^2(1 - e^{-\omega_{p2}/f_s}) - w_{p2}^2(1 - e^{-\omega_{1}/f_s})}{\omega_{p1}^2(\omega_{p1} - \omega_{p2})},$$

where $\omega_{pi}$ are the poles calculated from eq. 3.21 for the different integrators.
3.3 Motivation for using amplifiers with low GBW

By using amplifiers with $c \geq 1$, the errors from finite GBW in CT modulators usually is negligible. As indicated in section 2.5, the current draw from an amplifier is approximately proportional with its GBW. In $\Delta\Sigma$ modulators, the design of the first integrator is crucial to the overall performance due to its placement in the noise shaping loop. In a reported 3-order CT modulator (sec. 7.3 [Ortmanns, 2006]), 55% of the overall power budget is consumed in the first integrator, 27% in the second and third modulator, 10% for the bias and 8% for the digital part. This means that 82% of the total power is consumed in the integrators, where the amplifiers usually is the most power hungry part. Thus reducing GBW with a factor of 10-20 would have a significant impact on the overall power consumption.

3.4 Compensation for finite amplifier GBW errors

It has been shown that finite amplifier GBW in the integrators can be modeled as a gain-error and an extra loop delay in the feedback branches. In section 3.1.2 a compensation technique for excess loop delay was presented, and this can also be used to compensate for the extra loop delay induced by finite amplifier GBW. Figure 3.10 shows a second-order modulator with GBW induced feedback delays and an extra HRZ-feedback DAC for compensation.

As for the excess feedback delay compensation in section 3.1.2, the non-ideal CT modulator can be transformed into a DT equivalent and matched with the ideal DT modulator to find the new feedback coefficients. The expressions for the DT equivalents for the CT loopfilter branches are equal to the ones found in equations 3.5-3.7, but the $\alpha$ and $\beta$ coefficients are changed due to having different delays in the branches. Table 3.2 shows the new $\alpha$ and $\beta$ values.
### 3.4. Compensation for Finite Amplifier GBW Errors

Table 3.2: $\alpha$ and $\beta$ coefficient mapping with finite amplifier GBW compensation

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Excess loop delay</th>
<th>GBW induced delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{11}$</td>
<td>$\tau_d$</td>
<td>$\tau_{D2nd}$</td>
</tr>
<tr>
<td>$\alpha_{21}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\beta_{11}$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\beta_{21}$</td>
<td>$\tau_d$</td>
<td>$\tau_{D2nd}$</td>
</tr>
<tr>
<td>$\alpha_{12}$</td>
<td>$\tau_d$</td>
<td>$\tau_{D1st}$</td>
</tr>
<tr>
<td>$\alpha_{22}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\beta_{12}$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\beta_{22}$</td>
<td>$\tau_d$</td>
<td>$\tau_{D1st}$</td>
</tr>
<tr>
<td>$\alpha_{1h}$</td>
<td>$0.5 + \tau_d$</td>
<td>$0.5 + \tau_{D1st}$</td>
</tr>
<tr>
<td>$\alpha_{2h}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\beta_{1h}$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\beta_{2h}$</td>
<td>$\tau_d$</td>
<td>$\tau_{D1st}$</td>
</tr>
</tbody>
</table>

In (eq. 19 [Ortmanns, 2004]) the following expressions for the scaling coefficients $k_i$ were found considering DT scaling $a_1 = a_2 = 0.5$ (the optimal scaling values from [Marques, 1998]):

$$k_{\text{sig}} = k_1 = \frac{1}{4}$$

$$k_v = 1$$

$$k_2 = \frac{6\tau_{D1st} + 4\tau_{D1st}\tau_{D2nd} - \tau_{D2nd}^2}{8\tau_{D1st}}$$

$$k_h = \frac{3\tau_{D1st} + 2\tau_{D1st}\tau_{D2nd} - \tau_{D2nd}^2}{4\tau_{D1st}}$$

(3.26)

Since the scaling coefficients in equation 3.26 are dependent of the calculated feedback-delays, and the delays again are dependent on the scaling coefficients, an iterative calculation procedure must be done. In (p. 137 [Ortmanns, 2006]) such a calculation procedure also involving compensation for the GBW induced gain-error is presented:

1. Start the calculation process with the scaling coefficients in eq. 2.20 and $k_h = 0$. Calculate the gain-error $GE_1$ with eq. 3.21 and correct for it through $k_v = 1/GE_1$

2. Calculate the resulting GBW-induced delays with eq. 3.21 and 3.24.

3. Use eq. 3.26 to assign a set of compensation coefficients.

4. Repeat steps 2 and 3 up to a certain accuracy is achieved.
5. The gain error $GE_2$ is (ideally) compensated for by the use of single-bit quantizer.

In [Ortmanns, 2004] it is shown with simulations that this compensation ideally works for very low $GBW$-values. Even with $c < 0.02$ the modulator works as intended. It is important to remember that this compensation technique require amplifiers with 1-pole transfer characteristics similar to what presented in equation 3.19.

### 3.5 Quantizer Nonidealities

Since the quantizer is located in the modulator loop at the place with the highest noise suppression, most nonidealities in quantizer has little influence on the modulator performance. E.g. offset, hysteresis and nonlinearities (p.155 [Ortmanns, 2006]). Timing induced errors like propagation and signal dependent delay however, has a more severe impact on the overall performance. Propagation delay will have the same influence as excess loop delays in the DACs, which was discussed in section 3.1.1. Even worse is signal dependent delay due to quantizer metastability. When the delay varies with the input signal amplitude, also the length of the feedback DAC pulses will be signal dependent. This is known as signal dependent timing jitter (p. 156 [Ortmanns, 2006]). However, this signal dependency can be circumvented by inserting a latch between the quantizer and the feedback DACs, giving the quantizer time to settle before its output is latched (p. 1348 [Gerfers, 2003]. If these timing issues are accounted for, the quantizer performance is of minor importance in CT $\Delta\Sigma$ modulators, compared to the nonidealities of the feedback DAC or the loop filter (p. 156 [Ortmanns, 2006]).
Chapter 4

System Requirements

4.1 System Specification

The ADC specification is as follows.

- 50 kHz signal bandwidth
- 10 effective number of bits accuracy (SNDR\(^1\) ≥ 62dB)
- Using continuous-time ΔΣ architecture

The ADC is also meant to operate in a microcontroller with a very tight power budget. Thus there are also a few other requirements that must be accounted for:

- Low voltage operation, preferably ≤ 1.5V
- Low power

4.2 Modulator Architecture

When looking for different CT modulator architectures suitable for very low power consumption and the required signal bandwidth, an article [Ortmanns, 2004] describing the finite GBW compensation technique in section 3.2.3 and 3.4 was found. Simulations performed in this article showed that for GBWs as low as \(c = 0.15\) (see eq. 3.23 for definition of \(c\)), the modulator would ideally work as intended. The article included simulations applying nonidealities such as GBW variation and RC-product variation which also showed the robustness of this compensation approach. The simulations included both second and third order modulators, but the latter had stability problems when subject to the nonidealities. A second order modulator architecture as in figure 2.10 was therefore chosen as the basis for this work, because of

---

\(^1\)signal to noise plus distortion ratio
its simplicity and the promising results applying GBW compensation with such modulators in [Ortmanns, 2004].

4.3 Simulation Objectives

4.3.1 Feedback coefficients and OSR

The first objective is to find a set of feedback coefficients and OSR to meet the accuracy specification. This should be done in the DT domain using the Delta sigma toolbox for Matlab $[\Delta \Sigma$ Toolbox].

4.3.2 Robustness of the compensated CT $\Delta \Sigma$ Modulator

The second order modulator should be simulated with and without the GBW compensation. The influence from nonidealities like finite OTA gain, finite OTA GBW, RC-product variation, excess loop delay and finite DAC slew rate should be investigated for both models to see how the compensation affects the stability of the modulator. An important aspect is to see how the compensated modulator handles these nonidealities when the amplifier GBW deviates from the value compensated for. This is not covered in [Ortmanns, 2004].
Chapter 5

Model Descriptions

The model used to simulate the system is presented in this chapter. It is written in VHDL-AMS [VHDL-AMS], an extension of the discrete event based VHDL language which also supports the description and the simulation of analog, and mixed-signal circuits and systems. The simulations are done with the Advance MS suite from Mentor Graphics [Advance MS].

5.1 Building Blocks

5.1.1 Resistor Model

The resistors are modelled as a two-terminal device with the following equation:

\[ U = I \cdot R, \]  \hspace{1cm} (5.1)

where \( U \) is the voltage between the terminals, \( I \) is the current flowing through the resistor and \( R \) is the resistance. The source code for this model is found in appendix B.1.

5.1.2 Capacitor Model

The capacitors are modelled as a two-terminal device with the following equation:

\[ I = C \cdot \frac{dV}{dt}, \quad V(0) = V_0, \]  \hspace{1cm} (5.2)

where \( I \) is the current through the capacitor, \( C \) is the capacitance, \( V \) is the voltage between the terminals and \( V_0 \) is the initial value of \( V \). The source code for this model is found in appendix B.2.
5.1.3 OTA model

The OTAs are modeled as fully differential single pole amplifiers based on the transfer function from equation 3.19

\[ A(s) = \frac{A_{dc}}{s + \frac{1}{\omega_A}}, \quad GBW = A_{dc} \omega_A \text{ [rad/s]}, \quad (5.3) \]

\[ \Delta V_{in} = V_{in+} - V_{in-}, \quad (5.4) \]

\[ V_{out+} = \frac{\Delta V_{in}}{2} \cdot A(s), \quad (5.5) \]

\[ V_{out-} = -\frac{\Delta V_{in}}{2} \cdot A(s), \quad (5.6) \]

where \( GBW \) is the gain bandwidth product of the OTA and \( A_{dc} \) is the dc-gain. The source code for this model is found in appendix B.3.

5.1.4 Integrator Model

The integrators are modeled as fully differential active RC integrators similar to the single ended version discussed in section 3.2. It is a structural model put together from the resistor, capacitor and OTA models described in the previous subsections. Figure 5.1 shows the structure of the integrators. The integrator structures are embedded into the complete modulator models in section 5.2 and the source code for these can be found in appendix B.7 and B.8.

5.1.5 NRZ DAC Model

The NRZ DAC is modeled as a clocked voltage switch which will switch between its two reference voltages if the DAC input has changed before a rising clock edge. It also incorporates delayed switching and different slew rates for rising and falling edges. The source code for this model is found in appendix B.4.
5.1.6 HRZ DAC Model

The HRZ DAC is modeled similar to the NRZ DAC model, but differs somewhat since it is zeroed at every rising clock edge and switched on at every falling edge. The OFF functionality is done by forcing the current through the switch to zero. Also this DAC model incorporates delayed switching and different slew rates for rising and falling edges. The source code for this model is found in appendix B.5.

5.1.7 Quantizer Model

The quantizer model is made as a clocked comparator with digital output. The output changes if the differential input has crossed a voltage threshold level before a rising clock edge. The source code for this model is found in appendix B.6.

5.2 Modulator Models

5.2.1 2-order CT ΔΣ modulator with active-RC integrators

![2-order CT ΔΣ modulator with active-RC integrators](image)

Figure 5.2: 2-order CT ΔΣ modulator with active-RC integrators

The model for the 2-order CT-modulator is a structural model made of the components described in the previous section. Figure 5.2 is a schematic of how the modulator is put together. It consists of two fully differential
integrators, a quantizer and four NRZ DAC elements with corresponding resistors $R_{DAC_i}$.

To avoid timing issues, the DAC clock (not shown in the figure) is a delayed version of the quantizer clock. Thus the model also includes a simple delayed clock generator. The source code for this model can be found in appendix B.7.

### 5.2.2 2-order GBW compensated CT $\Delta\Sigma$ modulator with active-RC integrators

The model for the 2-order GBW compensated CT-modulator is almost identical to the modulator without compensation. In addition to the components in figure 5.2, two HRZ DAC elements with corresponding resistors $R_{HRZ}$ has been added to the system. This is done according to the block diagram in figure 3.10, i.e. they are connected to the two inputs of OTA2 in figure 5.2. The source code for this model can be found in appendix B.8.

### 5.3 SNDR calculation

The output from the CT modulators is a serial stream of bits. To plot the output frequency spectrum and calculate the SNDR, this bitstream needs postprocessing. The [Octave] script in appendix D.1 reads the output bitstream from the VHDL testbench in appendix B.9, calculates the SNDR according to the method presented in (Appendix A.4 [Schreier, 2005]), and finally writes the calculated output noise spectrum and integrated noise spectrum to a .csv-file which can be read by a waveform viewer.
Chapter 6

Simulation Results and Discussion

6.1 Scaling coefficients and OSR

In [Marques, 1998] they found the optimal\(^1\) scaling values for a second-order DT modulator to be \(a_1 = a_2 = 0.5\). A second-order DT loop filter with these scaling coefficients is used as the basis for the CT modulators in this text. By using the impulse invariant transform from section 2.4.1 the following CT scaling coefficients were found (NRZ feedback pulse):

\[
k_{\text{sig}} = k_1 = 0.25, \quad k_v = 1, \quad k_2 = 0.375
\]

By using the [Matlab] script in appendix C.2 simulations were done to find an appropriate OSR and to verify the DT-CT transform (do a CT-DT transform). Figure 6.1 shows the simulated SNR vs. input amplitude for an OSR of 32, 48 and 64 with the [Matlab] model. The SNR peaks at 55\(\text{db}\) for OSR=32, 65\(\text{db}\) for OSR=48 and 71.5\(\text{db}\) for OSR=64. The DT CT-CT transformed loop filter turned out to be identical with the original DT loop filter, they had identical transfer functions. The specification (sec. 4.1) claims 10 \(\text{ENOB}\) or 62\(\text{dB}\) SNDR, which means only OSR=64 will give enough margins considering other noise sources and nonidealities than quantization noise. In the following simulations an OSR=64 will be used.

\(^1\)Noise shaping ability versus stability
Figure 6.1: SNR vs Input amplitude for different OSR
6.2 Ideal 2-order CT modulator performance

When using OSR=64 and the specification claims 50kHz signal bandwidth, the needed sampling rate is given by equation 2.2, \( f_s = 64 \cdot 50kHz \cdot 2 = 6.4MHz \). This is the sampling rate used in the further simulations. The resistor and capacitor values\(^2\) used are:

\[
C = 15.625pF \\
R = \frac{10k\Omega}{k_i}
\]

\[
\frac{1}{RC}|_{k_i=1} = 6.4 \cdot 10^6 = f_s
\]

where \( k_i \) is the scaling coefficient. E.g. when the scaling coefficient \( k_1 \) is 0.25, the corresponding resistor \( R_1 \) will be 40kΩ.

![Figure 6.2: SNDR vs Input amplitude for 2-order CT modulator. \( f_s = 6.4MHz, f_{in} = 9kHz \)](image)

Figure 6.2 shows the SNDR vs. input amplitude for the 2-order CT modulator [VHDL-AMS] model. The SNDR peaks at 69.3dB for an input amplitude of \(-4dBFS\). All following simulations are done in [Advance MS] with the [VHDL-AMS] models.

6.3 Finite amplifer GBW performance

Figure 6.3 shows how finite amplifier GBW in the integrators affect the performance of the CT modulator. Without compensation, the SNDR suffer already when \( c \leq 2 \). The compensated modulator works all the way

\(^2\)The capacitor and resistor values are selected to match \( f_s \), while still having resonably geometries
down to $c = 0.02$ with no visible performance loss. Figure 6.4 shows the frequency spectrums for the compensated modulator with different compensation values. There is no sign of degradation of the signal in the frequency domain due to the compensation.

Figure 6.3: SNDR vs GBW for 2-order CT modulator and ideally compensated modulator. $f_s = 6.4 MHz$, $f_{in} = 9 kHz$, $|V_{in}| = -9dBFS$

Figure 6.4: Frequency spectrum for compensated CT modulators with $c = 10$ (red), $c = 0.1$ (green), $c = 0.05$ (orange). $f_s = 6.4 MHz$, $f_{in} = 9 kHz$, $|V_{in}| = -9dBFS$
6.4 Robustness of GBW compensation

In the following sections when referred to simulations with the noncompensated model, these simulations are done with a GBW ten times the sampling frequency. (c=10)

6.4.1 Variations in amplifier GBW

Figure 6.5: SNDR when GBW deviates from compensated value. $f_s = 6.4MHz$, $f_{in} = 9kHz$, $|V_{in}| = -9dBFS$

The compensated modulator performed very well considering ideal compensation, i.e. the GBW compensated for matches the actual GBW in the amplifiers. Figure 6.5 shows how the compensated modulators are affected by variation in the actual amplifier GBW. For positive variations the performance is mostly unaffected, but for negative variations the SNDR suffers. For lower $c$-values the SNDR rolls of earlier. For $c = 0.1$ and $c = 0.05$, the degradation is acceptable down to approximately $-40\%$ deviation from compensated amplifier GBW. In the further simulations a GBW variation of $\pm 40\%$ is used.
6.4.2 Finite OTA gain

![Graph showing SNDR for different values of OTA gain and ±40% variation of GBW.](image)

In Figure 6.6 the influence from finite OTA gain is presented. The compensated modulator has been simulated with ±40% variation of the amplifier GBW. In section 3.2.1 it was stated that the OTA dc-gain should be at least ≥OSR to avoid performance degradation. This can also be seen in the figure where the SNDR falls when the gain is too low. The compensated modulator does not seem to be more affected by finite OTA gain than the noncompensated modulator. In fact, in Figure 6.6 it keeps the SNDR better for low gain values than the noncompensated modulator.
6.4. ROBUSTNESS OF GBW COMPENSATION

6.4.3 RC product variation

Figure 6.7: SNDR vs RC product deviation with ±40% variation of GBW. \( f_s = 6.4 \text{MHz}, f_{in} = 9 \text{kHz}, |V_{in}| = -9dBFS \)

In figure 6.7 the influence from RC product variation in the integrators is presented. In section 3.2.2 it was stated that the resistor value can vary with ±30% and the capacitors with ±20%. This gives a RC-product variation from −44% to +56% which also is used in the simulations. The resistor and capacitor values are adjusted with an equal amount relative to their maximum deviation. I.e. if the capacitor changes +10%, the resistor changes +15%. The figure shows that the susceptibility to RC-variations for the compensated modulator is comparable to the modulator without compensation. For the most extreme negative RC-product variation, it even seems to perform better.
6.4.4 Excess Loop Delay

![Figure 6.8: SNDR vs Excess Loop Delay with ±40% variation of GBW.](image)

In figure 6.8 the influence from excess loop delay in the feedback loops is presented. The modulator with compensation for \( c = 0.05 \) has a significant performance drop for a loop delay \( \tau_d = 7.5\% \) of \( T_s \), and becomes unstable for the highest delay values. With \( c = 0.1 \) there is no significant difference in performance compared to the modulator without compensation. A \( \tau_d \) of 7.5% gives a maximum tolerable delay \( t_d = 0.075 \cdot 156.25 \text{ ns} = 11.7 \text{ ns} \) when \( c = 0.05 \).
6.4.5 DAC slew rate

**Finite Slew Rate**

Figure 6.9: SNDR vs DAC Slew Rate with $\pm 40\%$ variation of GBW. $f_s = 6.4 MHz$, $f_{in} = 9 kHz$, $|V_{in}| = -9 dBFS$

In figure the 6.9 influence from finite DAC slew rate is presented. The SNDR starts to decrease for the compensated modulators when the slew rate is lower than $10^8 V/s$, but the noncompensated modulator is mostly unaffected down to $10^7 V/s$.

**Asymmetric Slew Rate**

Figure 6.10: SNDR vs DAC Slew “Up” Rate with $\pm 40\%$ variation of GBW and Slew “Down” Rate = $10^{12} V/s$. $f_s = 6.4 MHz$, $f_{in} = 9 kHz$, $|V_{in}| = -9 dBFS$

In figure the 6.10 the influence from asymmetric DAC pulses due to different rise and fall times in the DACs are presented. The DAC slew “down” rate was set to $10^{12} V/s$ and the slew “up” rate varies. The SNDR in the
compensated modulator falls off at $10^8 \text{ V/s}$ a decade before the noncompensated one. This might be a result of the low slew rate and not the asymmetry tough, since the results are almost equivalent to what seen for the finite slew rate simulations in figure 6.9.

6.5 Internal Signal Swing

Figure 6.11: Signal swing at integrator outputs for the second and first integrator in a compensated modulator when $GBW_{red} = 10c$ and $GBW_{green} = 0.05c$. $f_s = 6.4MHz$, $f_{in} = 9kHz$, $|V_{in}| = -9\text{dBFS}$

Figure 6.11 shows the output of the second and first integrator in an ideally compensated modulator. The red plot is with $c = 10$ and the green with $c = 0.05$. It is clear from the figure that the modulation of the signal has a reduced signal swing when $c = 0.05$. In a real system with noise this could lead to a limitation of the achievable SNDR. A transient noise analysis of the modulators would probably accommodate this, but the simulation tool [Advance MS] turned out to have a very limited functionality for this matter with mixed signal systems.
Chapter 7

Comments

The initial DT simulations showed that an OSR=64 is needed to meet the given accuracy specifications. The calculated CT scaling coefficients were also verified by doing a CT→DT conversion in [Matlab] and then check the correspondence with the original DT transfer function.

When using ideal compensation, the compensated modulator works as intended down to $c = 0.02$ (The lowest $c$-value tested in simulations). The noncompensated modulator performance is already degraded when $c \leq 2$. The simulations indicates that the modulator incorporating the compensation technique has similar robustness to nonidealities even for GBW values as low as $c \geq 0.05$ compared to the noncompensated modulator which was simulated with $c = 10$. It is important to understand that this is only valid when the deviation of the amplifier GBW is within $\pm 40\%$ of the GBW compensated for. Especially when the actual amplifier GBW is less than this, the SNDR will suffer.

It was also discovered that the internal signal swings in the modulator is lowered when $c$ is reduced. This might be a limitation for the practical achievable SNDR in a real circuit.

The simulation of the CT modulators in [Advance MS] is a rather time consuming operation. Because of the long simulation time it was not feasible to do all the simulations on different frequencies and with different input amplitudes. The SNDR plots are an indication for how the modulators perform, and the results must be treated thereafter.
Chapter 8

Conclusions

The main goal of the work presented in this text was to investigate how circuit imperfections affect the performance of a continuous-time ΔΣ ADC employing finite amplifier GBW compensation compared to a similar ADC without compensation, and to find a setup which gives a resolution of more than 10 ENOB or 62 dB SNDR. Initial discrete-time simulations were done in [Matlab] and the ADC models are made in [VHDL-AMS] and simulated with [Advance MS]. Here follows the conclusions drawn from this work, and finally some thoughts about future work on this subject.

8.1 Conclusions

The discrete-time simulations showed that an $OSR = 64$ was needed to meet the accuracy specification. Two structural models of a 2-order continuous-time modulator were made, one of them incorporating the compensation technique for finite amplifier GBW. It was verified that the GBW compensation technique works very well under ideal conditions. I.e. if the actual GBW is equal to the value compensated for it performs like an ideal non-compensated CT modulator with $c > 2$ down to $c = 0.02$. It is also shown that the performance is quite dependent on deviation of the actual amplifier GBW. The dependency also increases when lowering the $c$-value compensated for. As long as the actual GBW stays within $\pm 40\%$ of the compensation value, simulations showed that the compensated modulators with $c \geq 0.05$ has similar performance as a noncompensated modulator with $c = 10$ when subject to circuit imperfections like finite OTA gain, integrator gain variations, excess loop delay and slew rate limitations. In previous work with this compensation technique [Ortmanns, 2004], circuit imperfections were only simulated on modulators with ideal compensation. Thus the discovered relatively high sensitivity to nonidealities under GBW variations is new information.

The simulations also revealed a lower internal signal swing in the modula-
tors when the amplifier GBW was reduced. This would probably make the SNDR in the compensated modulators more susceptible to circuit noise in a circuit implementation.

Even though some potential drawbacks with the compensation technique has been found in this work, it might still be a promising track to follow. If amplifiers with stable GBW and the right 1-pole transfer characteristics is found, and the signal swing issue can be neglected or somehow counteracted, the current consumption in CT modulators could be significantly reduced.

8.2 Future Work

A natural step further would be to extend the simulations with noise analysis to see how the reduced internal signal swing affects the performance when noise is included. It could also be useful to investigate the possibility of changing the scaling coefficients to compensate for the reduced signal swing.

The sensitivity to GBW variations will also require amplifier topologies which can provide the required stability of the GBW. Finding such an amplifier architecture with a 1-pole transfer function similar to the one used in the model would also be necessary before doing a circuit implementation of the system.
Bibliography


[Schreier, 2005] Richard Schreier and Gabor C. Temes, 2005, Understanding Delta-Sigma Data Converters


[ΔΣ Toolbox] Richard Schreier, ΔΣ Toolbox
http://www.mathworks.com/matlabcentral/fileexchange

[Project Meeting] Project meeting with Trond Ytterdal and Are Hel-landsvik, 3.March 2006
Appendix A

Scaling coefficients
compensated modulator

Table A.1: Scaling coefficients for GBW compensated 2-order modulator

<table>
<thead>
<tr>
<th>c-value</th>
<th>$k_2$</th>
<th>$k_h$</th>
<th>$GE_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.8302</td>
<td>0.6964</td>
<td>0.8627</td>
</tr>
<tr>
<td>0.3</td>
<td>0.8461</td>
<td>0.6637</td>
<td>0.7904</td>
</tr>
<tr>
<td>0.2</td>
<td>0.8539</td>
<td>0.6364</td>
<td>0.7154</td>
</tr>
<tr>
<td>0.1</td>
<td>0.8500</td>
<td>0.5787</td>
<td>0.5569</td>
</tr>
<tr>
<td>0.09</td>
<td>0.8488</td>
<td>0.5672</td>
<td>0.5307</td>
</tr>
<tr>
<td>0.08</td>
<td>0.8423</td>
<td>0.5530</td>
<td>0.5013</td>
</tr>
<tr>
<td>0.07</td>
<td>0.8358</td>
<td>0.5349</td>
<td>0.4680</td>
</tr>
<tr>
<td>0.06</td>
<td>0.8263</td>
<td>0.5110</td>
<td>0.4299</td>
</tr>
<tr>
<td>0.05</td>
<td>0.8119</td>
<td>0.4777</td>
<td>0.3859</td>
</tr>
<tr>
<td>0.04</td>
<td>0.7890</td>
<td>0.4276</td>
<td>0.3345</td>
</tr>
<tr>
<td>0.03</td>
<td>0.7487</td>
<td>0.3428</td>
<td>0.2738</td>
</tr>
<tr>
<td>0.02</td>
<td>0.6654</td>
<td>0.1752</td>
<td>0.2008</td>
</tr>
</tbody>
</table>

Table A.1 shows the scaling coefficients and the first integrator gain error in a compensated modulator for different values of $c$. The values have been calculated using the Matlab script in appendix C.1.
APPENDIX A. SCALING COEFFICIENTS COMPENSATED MODULATOR
Appendix B

VHDL-AMS Models

B.1 Resistor Model

```vhdl
library ieee_proposed;
use ieee_proposed.electrical_systems.all;

entity res is
  generic ( r : resistance );
  port ( terminal n1, n2 : electrical );
end;
```

```vhdl
library ieee_proposed;
use ieee_proposed.electrical_systems.all;

entity res is
  generic ( r : resistance );
  port ( terminal n1, n2 : electrical );
end;
```
APPENDIX B. VHDL-AMS MODELS

architecture ideal of res is
quantity v across i through n1 to n2;
begin
v := i*r;
end architecture;

B.2 Capacitor Model

architecture ideal of cap is
quantity v across i through pos to neg;
begin
if domain = quiescent domain use
v := init.voltage;
else
i := cap.value*v’.dot;
end use;
end architecture ideal;
B.3 OTA MODEL

B.3 OTA model

---
Filename : OTA_ent.vhd
Std : emacs, v 1.4.17 2002/12/10 19:08:14 rs Exp $  
Description :  
Author : Jon Helge Nistad<>  
Created at : Tue Feb 7 15:17:30 2006  
Modified by : Jon Helge Nistad<>  
---

library IEEE;  
use IEEE.MATH_REAL.all;  
library ieee_proposed;  
use ieee_proposed.electrical_systems.all;

entity diffOTA is  
generic (  
gain : real := 1.0;  
GBW : real := 1.0;  
vcmn : real := 0.0);  
port (  
terminal inpp : electrical;  
terminal inpn : electrical;  
terminal outpp : electrical;  
terminal outpn : electrical);  
end entity diffOTA;

---
Filename : diffOTA_gbw.vhd
Std : emacs, v 1.4.17 2002/12/10 19:08:14 rs Exp $  
Description :  
Author : Jon Helge Nistad<>  
Created at : Tue Feb 7 15:35:55 2006  
Modified by : Jon Helge Nistad<>  
---

architecture gbw of diffOTA is  
quantity vinp across inpp to electrical_ref;  
quantity vinn across inpn to electrical_ref;  
quantity voutp across ioutp through outpp to electrical_ref;  
quantity voutn across ioutn through outpn to electrical_ref;  

--- The dominant pole  
constant wA : real := (math_2*pi*GBW/gain);  

--- Parts of the transfer function  
constant num : real_vector := (0 => gain);  
constant den : real_vector := (1.0, 1.0/wA);  
quantity vdiff_half : voltage := 0.0;

begin  
architecture gbw  
vdiff_half := (vinp–vinn)/2.0;  
voutp = vcmn+vdiff_half’ltf(num, den);  
voutn = vcmn–vdiff_half’ltf(num, den);  
end architecture gbw;

B.4 NRZ DAC Model

---
Filename : DAC_ent.vhd
Project :  
Std : emacs, v 1.4.17 2002/12/10 19:08:14 rs Exp $  
Description :  
Author : Jon Helge Nistad  
Company :  
Created : 2006–06–10  
Last update: 2006–06–10  
Platform :  
Standard : VHDL’93, VHDL-AMS, Math Packages  
---
library IEEE;
use IEEE.MATH_REAL.all;
use IEEE.std_logic_1164.all;
library ieee_proposed;
use ieee_proposed.electrical_systems.all;

entity dac is

  generic
  (vrefP : real := 5.0; -- positive reference voltage
   vrefN : real := 0.0; -- negative reference voltage
   slewUp : real := 1.0e12; -- slew "up" rate
   slewDown : real := 1.0e12; -- slew "down" rate
   delay : time := 0.0 ns); -- excess loop delay

  port
    (signal input : in std_logic;
     signal clk : in std_logic;
     terminal output : electrical);

end entity dac;

architecture ideal of dac is

  quantity vout across iout through output to electrical_ref;
  signal vinternal real := 0.0;

begin
  -- architecture

  dac : process(clk) is
  begin
    process clocked
      if clk'event and clk = '1' then
        if input = '1' then
          vinternal <= vrefN after delay; -- add the excess delay
          else
            vinternal <= vrefP after delay;
        end if;
      end if;
    end process;

    vout <= vinternal 'slew(slewUp, slewDown); -- set the output and
      apply the slewing

  end architecture ideal;

B.5 HRZ DAC Model
B.5. HRZ DAC MODEL

---

library IEEE;
use IEEE.STD_LOGIC_1164.all;
library ieee_proposed;
use ieee_proposed.electrical_systems.all;

entity hrz_dac is
    generic (vrefP : real := 1.0; — positive reference voltage
              vrefN : real := 0.0; — negative reference voltage
              slew_up : real := 1.0e15; — slew "up" rate
              slew_down : real := -1.0e15; — slew "down" rate
              delay : time := 0 ns); — excess loop delay
    port (signal input : in std_logic;
          signal clk : in std_logic;
          terminal output : electrical);
end entity hrz_dac;

---

architecture ideal of hrz_dac is

quantity vout across iout through output to electrical_ref;
signal vinternal : real := 0.0;
begin — architecture ideal

dac : process(clk) is
begin — process
    if clk'event and clk = '1' then — at rising edge
        vinternal <= 0.0 after delay; — zero the internal signal
    elsif clk'event and clk = '0' then — at falling edge
        if input = '1' then
            vinternal <= vrefP after (delay); — check input
        else
            vinternal <= vrefN after (delay);
        end if;
    end if;
end process dac;
B.6 Quantizer Model

```
library IEEE;
use IEEE.MATH_REAL.all;
use IEEE.std_logic_1164.all;
library ieee_proposed;
use ieee_proposed.electrical_systems.ALL;

entity diffquantizer is
  generic (treshold : real := 0.0);
  port (terminal inpp : electrical;
        terminal inpn : electrical;
        signal output : std_logic;
        signal clk : in std_logic);
end entity diffquantizer;
```

```
architecture ideal of diffquantizer is
begin
  vdiff := inpp - inpn;
  process (clk) is
    begin — process clocked
      if clk'event and clk = '1' then — rising clock edge
        if vdiff 'above'(treshold) then
          output <= '1';
        else
          output <= '0';
        end if;
      end if;
    end process clocked;
  end if;
  end architecture ideal;
```

B.7 2-order CT modulator Model

```
architecture ideal of Fctsdm2 is
begin
  vdiff := vpp - vnn;
  process (clk) is
    begin — process clocked
      if clk'event and clk = '1' then — rising clock edge
        if vdiff 'above'(treshold) then
          output <= '1';
        else
          output <= '0';
        end if;
      end if;
    end process clocked;
  end if;
  end architecture ideal;
```
library IEEE;
use IEEE.math_real.all;
library ieee_proposed;
use ieee_proposed.electrical_systems.all;

entity ctsdm2 is
  generic
    GBW : real := 1.0;  -- OTA GBW in Hz
    otagain : real := 1.0;  -- OTA gain
    vcmn : real := 0.0;  -- common mode voltage
    R1val : real := 1.0;  -- Value of R1
    R2val : real := 1.0;  -- Value of R2
    Rdac1val : real := 1.0;  -- Value of RDAC1
    Rdac2val : real := 1.0;  -- Value of RDAC2
    C1val : real := 1.0;  -- Value of C1
    C2val : real := 1.0;  -- Value of C2
    vrefP : real := 1.0;  -- Positive Reference Voltage
    vrefN : real := 1.0;  -- Negative Reference Voltage
    dac1delay : time := 1.0 ns;  -- Excess loop delay DAC1
    dac2delay : time := 1.0 ns;  -- Excess loop delay DAC2
    delayedclock_delay : time := 1.0 ns;  -- Delay in clock generator
    hrzdacdelay : time := 1.0 ns;  -- Excess loop delay in HRZ DAC
    dacrise : real := 1.0;  -- Slew "up" rate in the DACs
    dacfall : real := -1.0;  -- Slew "down" rate in the DACs
  port
    inpp : electrical;
    inpn : electrical;
    output : std_logic;
    clk : in std_logic;
end entity ctsdm2;
APPENDIX B. VHDL-AMS MODELS

44 gain => otagain,
45 GBW => GBW,
46 vcmn => vcmn)
47 port map (inpp => mOTA1p,
inpn => mOTA1n,
outpp => mOTA1p,
outpn => mOTA1n);
48 OTA2 : entity workDIFFOTA(gbw)
49 generic map (gain => otagain,
50 GBW => GBW,
51 vcmn => vcmn)
52 port map (inpp => mOTA2p,
inpn => mOTA2n,
outpp => mOTA2p,
outpn => mOTA2n);
53
54 R1a : entity work.ideal
55 generic map (r => R1val)
56 port map (n1 => inpp,
57 n2 => mOTA1p);
58 R1b : entity work.ideal
59 generic map (r => R1val)
60 port map (n1 => inpp,
61 n2 => mOTA1n);
62
63 R2a : entity work.ideal
64 generic map (r => R2val)
65 port map (n1 => mOTA1p,
66 n2 => mOTA2p);
67 R2b : entity work.ideal
68 generic map (r => R2val)
69 port map (n1 => mOTA1n,
70 n2 => mOTA2n);
71
72 Rdac1a : entity work.ideal
73 generic map (r => Rdac1val)
74 port map (n1 => mRdac1a,
75 n2 => mOTA1n);
76 Rdac1b : entity work.ideal
77 generic map (r => Rdac1val)
78 port map (n1 => mRdac1b,
79 n2 => mOTA1p);
80
81 Rdac2a : entity work.ideal
82 generic map (r => Rdac2val)
83 port map (n1 => mRdac2a,
84 n2 => mOTA2n);
85 Rdac2b : entity work.ideal
86 generic map (r => Rdac2val)
87 port map (n1 => mRdac2b,
88 n2 => mOTA2p);
89 C1a : entity work.ideal
90 generic map (}
B.7. 2-ORDER CT MODULATOR MODEL

cap = C1val,
inital voltage = 0.0

port map

pos = inOTA1n,
neg = outOTA1p;

C1b : entity work cap(ideal)
generic map (cap = C1val,
inital voltage = 0.0)
port map

pos = inOTA1p,
neg = outOTA1n);

C2a : entity work cap(ideal)
generic map (cap = C2val,
inital voltage = 0.0)
port map

pos = inOTA2n,
neg = outOTA2p);

C2b : entity work cap(ideal)
generic map (cap = C2val,
inital voltage = 0.0)
port map

pos = inOTA2p,
neg = outOTA2n);

quantizer : entity work diffquantizer(ideal)
generic map (threshold = 0.0)
port map

inpp => outOTA2n,
inpn => outOTA2p,
output => s1,
clk => clk);

the 4 dac blocks
dac1a : entity work dac(ideal)
generic map (vrefP = vrefP, vrefN = vrefN,
slew_up = dac_rise,
slew_down = dac_fall,
delay => dac1delay)
port map

input => s1,
clk => delayedclock,
output => inRdac1a);

dac1b : entity work dac(ideal)
generic map (vrefP = vrefN, vrefN = vrefP,
slew_up = dac_rise,
slew_down = dac_fall,
delay => dac1delay)
port map

input => s1,
clk => delayedclock,
output => inRdac1b);

dac2a : entity work dac(ideal)
generic map (vrefP = vrefP, vrefN = vrefN,
slew_up = dac_rise,
slew_down = dac_fall,
delay => dac2delay)
port map

input => s1,
clk => delayedclock,
output => inRdac2a);

dac2b : entity work dac(ideal)
generic map (vrefP = vrefN, vrefN = vrefP,
slew_up = dac_rise,
slew_down = dac_fall,
delay => dac2delay)
port map

B.8 2-order CT modulator Model with GBW compensation

Here are the delta sigma modulator put together of differen sub components

```vhdl
library MGC_AMS;
use MGC_AMS.ELDO.all;
architecture structural of ctsdm2 is
begin
  OTA1 : entity work.diffOTA(gbw)
generic map (gain => otagain, GBW => GBW, vcmn => vcmn)
port map (inpp => mOTA1p, inpn => mOTA1n, outpp => outOTA1p, outpn => outOTA1n);
OTA2 : entity work.diffOTA(gbw)
```

B.8. 2-ORDER CT MODULATOR MODEL WITH GBW COMPENSATION

```
generic map (  
gain => otagain,  
GBW => GBW,  
vmm => vmm)  
port map (  
inp => inOTA2p,  
impn => inOTA2n,  
outpp => outOTA2p,  
outpn => outOTA2n);  

R1a : entity work res(ideal)  
generic map (  
r => R1val)  
port map (  
n1 => inpn,  
n2 => inOTA1p);  

R1b : entity work res(ideal)  
generic map (  
r => R1val)  
port map (  
n1 => inpp,  
n2 => inOTA1n);  

R2a : entity work res(ideal)  
generic map (  
r => R2val)  
port map (  
n1 => outOTA1p,  
n2 => inOTA2p);  

R2b : entity work res(ideal)  
generic map (  
r => R2val)  
port map (  
n1 => outOTA1n,  
n2 => inOTA2n);  

Rdac1a : entity work res(ideal)  
generic map (  
r => Rdac1val)  
port map (  
n1 => inRdac1a,  
n2 => inOTA1n);  

Rdac1b : entity work res(ideal)  
generic map (  
r => Rdac1val)  
port map (  
n1 => inRdac1b,  
n2 => inOTA1p);  

Rdac2a : entity work res(ideal)  
generic map (  
r => Rdac2val)  
port map (  
n1 => inRdac2a,  
n2 => inOTA2n);  

Rdac2b : entity work res(ideal)  
generic map (  
r => Rdac2val)  
port map (  
n1 => inRdac2b,  
n2 => inOTA2p);  

C1a : entity work cap(ideal)  
generic map (  
cap_value => C1val,  
init_voltage => 0.0)  
port map (  
pos => inOTA1n,  
neg => outOTA1p);  

C1b : entity work cap(ideal)  
generic map (  
cap_value => C1val,  
init_voltage => 0.0)  
port map (  
```
APPENDIX B. VHDL-AMS MODELS

C2a : entity work.cap(ideal)
  generic map (  
    cap_value => C2val,  
    init_voltage => 0.0)  
  port map (  
    pos => inOTA1p,  
    neg => outOTA1n);  
C2b : entity work.cap(ideal)
  generic map (  
    cap_value => C2val,  
    init_voltage => 0.0)  
  port map (  
    pos => inOTA2n,  
    neg => outOTA2p);  
quantizer : entity work.diffquantizer(ideal)
  generic map (  
    treshold => 0.0)  
  port map (  
    inpp => outOTA2n,  
    inpn => outOTA2p,  
    output => s1,  
    clk => clk);  

the 4 dac blocks

dac1a : entity work.dac(ideal)
  generic map (  
    vrefP => vrefP,  
    vrefN => vrefN,  
    slew_up => dac_rise,  
    slew_down => dac_fall,  
    delay => dac1delay)  
  port map (  
    input => s1,  
    clk => delayedclock,  
    output => inRdac1a);  
dac1b : entity work.dac(ideal)
  generic map (  
    vrefP => vrefN,  
    vrefN => vrefP,  
    slew_up => dac_rise,  
    slew_down => dac_fall,  
    delay => dac1delay)  
  port map (  
    input => s1,  
    clk => delayedclock,  
    output => inRdac1b);  
dac2a : entity work.dac(ideal)
  generic map (  
    vrefP => vrefP,  
    vrefN => vrefP,  
    slew_up => dac_rise,  
    slew_down => dac_fall,  
    delay => dac2delay)  
  port map (  
    input => s1,  
    clk => delayedclock,  
    output => inRdac2a);  
dac2b : entity work.dac(ideal)
  generic map (  
    vrefP => vrefN,  
    vrefN => vrefP,  
    slew_up => dac_rise,  
    slew_down => dac_fall,  
    delay => dac2delay)  
  port map (  
    input => s1,  
    clk => delayedclock,  
    output => inRdac2b);  

HdacA : entity work.res(ideal)
  generic map (  
    r => Hdacval)  
  port map (  
    n1 => inHdacA,  
    n2 => outOTA2n);
HdacB : entity work.res(ideal)
generic map (r => Hdacval)
port map (n1 => inHdacB, n2 => mOTA2p);

HrdacA : entity work.hrzdac(ideal)
generic map (vrefP => vrefP, vrefN => vrefN, slewup => dac_rise,
        slew_down => dac_fall, delay => hrzdacdelay)
port map (input => s1, clk => delayedclock, output => inHdacA);

HrdacB : entity work.hrzdac(ideal)
generic map (vrefP => vrefN, vrefN => vrefP, slewup => dac_rise,
        slew_down => dac_fall, delay => hrzdacdelay)
port map (input => s1, clk => delayedclock, output => inHdacB);

END OF EXTRA HRZ ELEMENTs——

delayed clock generator
clkdelaygen : process (clk) is
begin
    if clk'event and clk = '1' then
        delayedclock <= '1' after delayedclock_delay;
    else
        delayedclock <= '0' after delayedclock_delay;
    end if;
end process clkdelaygen;

output <= s1;
end architecture structural;

B.9 VHDL testbench

---

-- File : tbsdtsm2.vhd
-- Project : 

-- Author : Jon Helge Nistad
-- Company : 
-- Created : 2006-01-17
-- Last update : 2006-06-10
-- Platform : VHDL'93, VHDL-AMS, Math Packages
-- Standard : 
-- Description : Deltasigma testbench
-- Copyright (c) 2006
-- Revisions :
-- Date Version Author Description
-- 2006-01-17 1.0 jonhelge Created

library ieee;
use ieee.math_real.all;
use ieee.std_logic_1164.all;
use ieee.std_logic_arith.all;
use ieee.std_logic_unsigned.all;
use ieee.numeric_std.all;
use STD.TEXTIO.all;
library IEEE proposed;
use IEEE proposed.electrical systems.all;
library MGC AMS;
use MGC AMS.eldo parameters all;

entity tb_deltasigma_ct2 is
end entity tb_deltasigma_ct2;

architecture bhv of tb_deltasigma_ct2 is

begin

file rapport: text open append mode is "rapport.txt";

---eldp parameters set in the .cmd top level file ---
constant amp : real := param("amp");
constant fbin : real := param("fbin");
constant OSR : real := param("OSR");
constant fin : real := param("fin");
constant GBW : real := param("GBW");
constant otagain : real := param("otagain");
constant vcmn : real := param("vcmn");
constant R1val : real := param("R1val");
constant R2val : real := param("R2val");
constant Rdac1val : real := param("Rdac1val");
constant Rdac2val : real := param("Rdac2val");
constant Hdacval : real := param("Hdacval");
constant Clval : real := param("Clval");
constant C2val : real := param("C2val");
constant vrefN : real := param("vrefN");
constant dac1delay : real := param("dac1delay");
constant dac2delay : real := param("dac2delay");
constant delayedclock_delay : real := param("delayedclock_delay");
constant hrzdacdelay : real := param("hrzdacdelay");
constant GBWcomp : real := param("GBWcomp");
constant dacrise : real := param("dacrise");
constant dacfall : real := param("dacfall");

---initiate the delta sigma modulator

dut : entity work chtdm2(structural)

generic map (
    GBW => GBW,
    otagain => otagain,
    vcmn => vcmn,
    R1val => R1val,
    R2val => R2val,
    Rdac1val => Rdac1val,
    Rdac2val => Rdac2val,
    Hdacval => Hdacval,
    Clval => Clval,
    C2val => C2val,
    vrefP => vrefP,
    vrefN => vrefN,
    dac1delay => (dac1delay+1.0 ns),
    dac2delay => (dac2delay+1.0 ns),
    delayedclock_delay => (delayedclock_delay+1.0 ns),
    hrzdacdelay => (hrzdacdelay+1.0 ns),
    dacrise => dacrise,
    dacfall => dacfall)

port map (
    inpp => A,
    inpn => B,
    output => op,
    clk => clk);
B.9. VHDL TESTBENCH

— write the header of the output file
— this is used to parse the results in the octave script

wr_header : process is
variable L : line;
variable temp : integer := 0;
beginn
write(L, OSR);
writeLine(rapport, L);
write(L, bin);
writeLine(rapport, L);
write(L, GBW);
writeLine(rapport, L);
write(L, GBWcomp);
writeLine(rapport, L);
write(L, otagain);
writeLine(rapport, L);
write(L, fin);
writeLine(rapport, L);
wait;
end process wr_header;

— Generate the input signal to the modulator
Ain == vcmn + amp * sin (math::pi * fin * NOW);
Bin == vcmn - amp * sin (math::pi * fin * NOW);

— Process for input clock generation
f clk = 6.4 MHz

stimulus : process is
begin — process stimulus
clk <= '1';
wait for 78.125 ns;
clk <= '0';
wait for 78.125 ns;
end process stimulus;

— The process that writes the output value to a file.
writefile : process(clk) is
variable L : line;
variable temp : integer := 0;
beginn
if falling_edge(clk) then — do a sample at a stable moment
if op = '1' then temp := 1;
else temp := 0;
end if;
write(L, temp); — write it to the file
writeLine(rapport, L);
end if;
end process writefile;

— A process for converting the digital output to an analog signal.
— Nice for doing measurements on the output in Ezwave
writeanaop : process(op) is
begin
if op = '1' then anaoptemp <= 1.0;
else anaoptemp <= -1.0;
end if;
end process writeanaop;

anaop == anaoptemp * ramp(1.0e-15); — ramp the analog version of output.
end architecture bhv;
Appendix C
Matlab Code

C.1 Code for calculation of compensated feedback coefficients

```matlab
%% Author Jon Helge Nistad 2006
%% GE1, k2 and kh must be set to
%% "something" be fore running script
fs = 6.4e6;
c = 0.015;
GBW = fs; % GBW = fs

%% Poles
w2 = (GBW + fs/(GE1 + k2 + kh + fs));
w1 = (GBW + 0.25 + fs + 0.25 + fs);

%% Delays
tau1 = (1 - exp(-w2/fs))/w2 + fs;
tau2 = (w1^2 - exp(-w2/fs))/w1 - exp(-w1/fs) - 1;
tau3 = w1 + tau2 = tau1 - (w1 - w2)/w1;

%% Scaling coefficients
k2 = (6 + 4 + tau1 + tau2 - tau2^2)/(8 + tau1);
k0 = (3 + tau1 + tau2 - tau2^2)/(4 + tau1);

%% Gain errors
GE1 = GBW/(w1);
GE2 = GBW/(w2);
```

C.2 Code for simulation of DT filter and DT simulation of CT loop filter

```matlab
%% Author: Jon Helge Nistad, 2005
%% Simulation and comparison of 2. order DT and CT
%% Delta sigma modulator. CT simulation is done by
%% first doing CT-DT transform of the CT loop filter.
OSR = 64;
fs = 100e3;

z = zpk('z', 1);

%% The optimal DT NTF a1=a2=0.5
NTFclfb = 1/(1 - (0.5)*(z - 1) - (0.5)*(z - 1)^2);

%% The results from the imp. inv. transform
z = [1 - 0.25; 0 - 0.375];
```

69
APPENDIX C. MATLAB CODE

```matlab
Cc = [0 1];
Dc = [0 0];

%% Making a state space description
LFc = ss(Ac, Bc,Cc, Dc);
L0c = ss(Ac, Bc(:,1),Cc, Dc(1));
L0c = zpk(L0c);

%% Do a CT-DT conversion
LF = mapCtoD(LFc, [0 1])
ABCD = [LF.a LF.b; LF.c LF.d];

%% Find the new NTF
H = calculateTF(ABCD)

%% Plot a pole-zero map of the two NTF’s
figure(1);
pzmap(H,NTFcfib);

%% Simulate the two NTFs with different input amplitudes.
amp = [-140:5:-15, -12:10:0];

snr1 = simulateSNR(H,OSR,amp,0,2,1/(4+48),16);

snr2 = simulateSNR(NTFcfib,OSR,amp,0,2,1/(4+48),16);
figure(2);
hold on;
plot(amp, snr1,'g', amp, snr2,'b');
```
Appendix D

Octave code

D.1 Octave script for SNDR calculation

```octave
## A script for reading the output file from
## the vhdl testbench.
## Author: Jon Helge Nistad

f=fopen("result.txt","r","ieee-lr");
invec=fscanf(f,"%d");
fclose(f);

## Find the length of the bitstreams
## The first value in the header is OSR
cnt=2;
while(invec(cnt) != invec(1))
cnt++;
end
bLength=cnt-1; ## The length of the bitstreams

## Calculates how many sim. runs done
nRuns=(length(invec)/bLength)

## create the two .csv files for the plots
spectra=fopen("spectra.csv","w");
integ=fopen("spectraInteg.csv","w");

## SNDR calculation as done at page 380 in schreier05 ##
w = hann(N); # Hann window
w = fliplr(w); # Rotate and flip the vector so it matches V
```
APPENDIX D. OCTAVE CODE

54 w1=norm(w,1);  
55 w2=norm(w,2);  
56 NWW=(w2/w1)^2;  
57  
58 V=fft(w.*result)/(w1/2);  
59 signal_bins=fbin-[-(nb-1)/2:(nb-1)/2];  
60 noise_bins=1-(N/2+OSR)+1;  
61 noise_bins(signal_bins)=[];  
62 snr=10*log10(sum(abs(V(signal_bins)).^2)/sum(abs(V(noise_bins)).^2));  
63  
64 ## Write the calculated SNDR to the octres file  
65 fprintf(octres,'\"Number of samples\", \"\", N);  
66 fprintf(octres,'\"SNDR\", \"\", snr);  
67 fprintf(octres,'\"GBW\", \"\", GBWact);  
68 fprintf(octres,'\"Number of\", \"\", nb);  
69 fprintf(octres,'\"SNDR\", \"\", snr);  
70 fprintf(octres,'\"GBW\", \"\", GBWact);  
71  
72  
73 ## WRITE the spectra.csv file  
74 ## First the header needed by ezwave  
75 fprintf(spectra,'\"FREQ\", \"\", \"\", GBWact);  
76 fprintf(spectra,'\"Number of\", \"\", nb);  
77 fprintf(spectra,'\"Number of\", \"\", nb);  
78 fprintf(spectra,'\"Number of\", \"\", nb);  
79 fprintf(spectra,'\"SNDR\", \"\", snr);  
80 fprintf(spectra,'\"GBW\", \"\", GBWact);  
81 fprintf(spectra,'\"Number of\", \"\", nb);  
82 fprintf(spectra,'\"Number of\", \"\", nb);  
83 fprintf(spectra,'\"Number of\", \"\", nb);  
84 fprintf(spectra,'\"SNDR\", \"\", snr);  
85 fprintf(spectra,'\"GBW\", \"\", GBWact);  
86 fprintf(spectra,'\"Number of\", \"\", nb);  
87 fprintf(spectra,'\"SNDR\", \"\", snr);  
88 fprintf(spectra,'\"GBW\", \"\", GBWact);  
89 fprintf(spectra,'\"Number of\", \"\", nb);  
90 fprintf(spectra,'\"SNDR\", \"\", snr);  
91 fprintf(spectra,'\"GBW\", \"\", GBWact);  
92  
93 ## Then write the spectrum to the file  
94 -floor(length(V)/2);  
95 for i=1:j  
96 fprintf(spectra,'%E,%E
',i,20+log10(abs(V(i))));  
97 end  
98  
99  
100 ## WRITE the spectraintegrated.csv file  
101 ## First the header needed by ezwave  
102 fprintf(spectraintegrated,'\"FREQ\", \"\", \"\", GBWact);  
103 fprintf(spectraintegrated,'\"Number of\", \"\", nb);  
104 fprintf(spectraintegrated,'\"Number of\", \"\", nb);  
105 fprintf(spectraintegrated,'\"Number of\", \"\", nb);  
106 fprintf(spectraintegrated,'\"SNDR\", \"\", snr);  
107 fprintf(spectraintegrated,'\"GBW\", \"\", GBWact);  
108 fprintf(spectraintegrated,'\"Number of\", \"\", nb);  
109 fprintf(spectraintegrated,'\"Number of\", \"\", nb);  
110 fprintf(spectraintegrated,'\"Number of\", \"\", nb);  
111 fprintf(spectraintegrated,'\"SNDR\", \"\", snr);  
112 fprintf(spectraintegrated,'\"GBW\", \"\", GBWact);  
113 fprintf(spectraintegrated,'\"Number of\", \"\", nb);  
114 fprintf(spectraintegrated,'\"SNDR\", \"\", snr);  
115 fprintf(spectraintegrated,'\"GBW\", \"\", GBWact);  
116 fprintf(spectraintegrated,'\"Number of\", \"\", nb);  
117 fprintf(spectraintegrated,'\"SNDR\", \"\", snr);  
118 fprintf(spectraintegrated,'\"GBW\", \"\", GBWact);  
119  
120 ## Then write the integrated spectrum  
121 -floor(length(V)/2);  
122 temp=0;  
123 temp2=0;  
124 for i=1:j  
125 if (bin-2<i && i<bin+2) ## separate signal bins  
126 temp2=temp2+(abs(V(i))^2);  
127 else  
128 temp=temp+(abs(V(i))^2);  
129 end  
130 end  
131  
132  
133  
134  
135  
136 ## finally close the files  
137
D.1. OCTAVE SCRIPT FOR SNDR CALCULATION

137  fprintf ( octres , "###\n") ;
138  fclose ( octres ) ;
139  fclose ( spectra ) ;
140  fclose ( integ ) ;
141  fclose ( integ ) ;