PIKETTY’S LAWS AND LOGIC

Torgeir Høien
SKAGEN Funds

August 2014
Piketty’s laws and logic

Torgeir Høien
August 2014

ABSTRACT:

Thomas Piketty’s *Capital in the Twenty-First Century* is a valuable contribution to the history of wealth and income distribution. But Piketty’s theory about wealth and income distribution as encapsulate in his laws of capitalism leaves much to be desired. One of the laws is devoid of substance, the others are convoluted reformulations of relationships that have been known for a long time. And neither of them have any bearing on the distribution of wealth and income. Also, Piketty’s use of logical terms is at times obscure, as when he talks about “contradictions” in capitalism. Thus, Piketty’s laws are irrelevant for his predictions, and his logical vocabulary is deceptive.
## Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>The laws</td>
<td>3</td>
</tr>
<tr>
<td>The logical vocabulary</td>
<td>7</td>
</tr>
<tr>
<td>Conclusion</td>
<td>9</td>
</tr>
</tbody>
</table>
The laws

In Capital in the Twenty-First Century Thomas Piketty states three “fundamental laws of capitalism”. What do these laws say and imply?¹

The first law says that the ratio of capital income to national income equals the rate of return on the capital stock multiplied by the ratio of the capital stock to national income. (p. 52).

National income is GDP minus depreciation plus net foreign income during a period, e.g. 2014. The capital stock is the market value of all potentially marketable wealth. That is, capital goods, land, houses, and net financial claims on foreigners, at a point in time, e.g. Dec. 31 2013.²

The rate of return on the capital stock is capital income/capital stock. Hence, the first law states that:

\[
\frac{\text{capital income}}{\text{national income}} = \frac{\text{capital income}}{\text{capital stock}} \cdot \frac{\text{capital stock}}{\text{national income}}
\]

Simple fractional algebra reduces this equation to \(1 = 1\). Now, while it might be useful to be reminded of the Aristotelian law of identity, only a disciple of Ayn Rand might be tempted to assert that capitalism rests on the fact that, ontologically speaking, \(a = a\), i.e. “a thing is what it is”. And it’s not my impression that Piketty is a Randian.

The second law says that in the long run the ratio of the capital stock to national income equals the ratio of net savings rate to the growth rate of national income. (p. 166).

Net savings is the addition to the capital stock between two points in time, e.g. the capital stock on Dec. 31 2014 minus the capital stock on Dec. 31 2013. That is, it is gross savings minus depreciation.

Ignoring capital gains, as Piketty does, the net savings rate equals the capital stock growth divided by national income, e.g. savings in 2014 is:³

\[
\frac{\text{capital stock Dec. 31 2014} - \text{capital stock Dec. 31 2013}}{\text{national income 2014}}
\]

The growth rate of national income is national income one year minus national income the previous year divided by national income the previous year, e.g. the growth rate in 2014 is:

\[
\frac{\text{national income 2014} - \text{national income 2013}}{\text{national income 2013}}
\]

¹ Strictly speaking: two laws and a “fundamental contradiction of capitalism”. See below.
² While Piketty includes houses, he excludes other consumer durables, like cars and television sets – presumably because one does not pay taxes on such durables after they are purchased and such goods are not counted as wealth in national accounts.
³ I ignore the fact that the capital stock grows continuously during the year, not just on the last day of the year. This has no implication for the results.
Assume we are in the long run, i.e. when the economy has adapted to any shocks, and Piketty’s second law applies. Using the definitions above, the second law states that:

\[
\frac{\text{capital stock Dec. 31 2013}}{\text{national income 2014}} = \frac{\text{capital stock Dec. 31 2014} - \text{capital stock Dec. 31 2013}}{\text{national income 2014} - \text{national income 2013}}
\]

Now, if both sides are multiplied by \((\text{national income 2014}/\text{capital stock Dec. 31 2013})\), the second law becomes:

\[
1 = \frac{\text{capital stock Dec. 31 2014} - \text{capital stock Dec. 31 2013}}{\text{capital stock Dec. 31 2013}} \frac{\text{national income 2014} - \text{national income 2013}}{\text{national income 2013}}
\]

Multiply both sides by \((\text{national income 2014} - \text{national income 2013})/\text{national income 2013}\). Piketty’s second law can then be written as:

\[
\frac{\text{national income 2014} - \text{national income 2013}}{\text{national income 2013}} = \frac{\text{capital stock Dec. 31 2014} - \text{capital stock Dec. 31 2013}}{\text{capital stock Dec. 31 2013}}
\]

In other words: Piketty’s second law states that in the long run the growth rate of national income = the growth rate of the capital stock. And that’s it.

There is nothing new here. The second law has been a standard prediction of economic models since Solow’s seminal 1956 paper on economic growth. Note that this equation does not imply that the growth of national income is determined by the growth of the capital stock, or vice versa.

The third law, which Piketty doesn’t call a “law” but “the central contradiction of capitalism” – more on this later – says that the rate of return on capital is higher than the growth rate of national income. (p. 351).

The return on capital is capital income. The rate of return on capital is capital income/capital stock. Since, according to the second law, the growth rate of national income equals the growth rate of the capital stock, this inequality says that:

\[
\frac{\text{capital income 2014}}{\text{capital stock Dec. 31 2013}} > \frac{\text{capital stock Dec. 31 2014} - \text{capital stock Dec. 31 2013}}{\text{capital stock Dec. 31 2013}}
\]

If both sides of the inequality are multiplied by capital stock Dec. 31 2013, the inequality becomes:

\[
\text{capital income 2014} > \text{capital stock Dec. 31 2014} - \text{capital stock Dec. 31 2013}
\]
In other words, Piketty’s “central contradiction of capitalism” says that capital income during a period is higher than net savings. Since capital income is net of depreciation, this implies that a part of capital income is consumed.

That the rate of return on capital is higher than the growth rate of national income is, as noted by Ray and other commentators, a prediction of established economic theory. Hence there is nothing new in Piketty’s inequality.

Why do standard models predict that the rate of return on the capital stock is higher than the growth rate of national income? Without going into the detail: if the rate of return on capital is lower than the rate of growth of national income, the economy uses too many resources to accumulate new capital. Consumption now and in the future can then be increased by saving less. With a slower growth rate of the capital stock, the marginal productivity of capital services rises, lifting r above g.

Some commentators have hailed Piketty’s “central contradiction of capitalism” as the Holy Grail of new political economy. Professor Gudmund Hernes compares Piketty’s “central contradiction of capitalism” to Einstein’s $E = MC^2$. This, to put it mildly, is an exaggeration. Piketty is no Einstein. But, as Rays writes, Piketty’s inquiry has established that a standard prediction of economic theory has been correct over long time spans – but with the past century as an exception. Thus, Piketty and his collaborators might, if we are generous, be compared to Arthur Eddington who, during a total solar eclipse in 1919, confirmed one implication of Einstein’s general theory of relativity – that the sun deflects light.

How does all of this underpin Piketty’s main thesis, which states that we are at the dawn of a new area of high and growing inequality?

The first law is obviously irrelevant for anything. The second law implies that the capital stock in the long run will not grow faster than national income, i.e. that the ratio of the capital stock to national income remains constant. This does not have any bearing on the distribution of the capital stock or national income. The third law, known as $r > g$, with $r$ for the rate of return on capital and $g$ for the growth rate of national income, also does not have any repercussions for the distribution of the capital stock or national income. If anyone thinks otherwise it must be that they read $r > g$ as implying that the growth rate of capital income is higher than the growth rate of national income.

That’s not the case; $r$ is the rate of return on capital, not the growth rate of capital income. In fact, $r > g$, implies constant labor and capital shares of national income. How large the shares are, however, depends upon the size of the gap between $r$ and $g$. Thus, contrary to what Piketty claims, $r > g$ does not lead to growing inequality. Inequality only spirals if the difference between $r$ and $g$ continuously increases – and if the distribution of labor and capital income is uneven among households.

An example might clarify:

Suppose that national income in Petitistan was USD 97.09 in 2013. Capital income was USD 29.13 and labor income was USD 67.96. Suppose that Petitistan’s capital stock on Dec 31. 2013 was USD 600. Assume that $r$ is 5 and $g$ is 3 during 2014, i.e. the rate of return on the capital stock is 5 percent in 2014.
and the growth of national income is 3 percent from 2013 to 2014. How does capital income then evolve relative to labor income?

5 percent of USD 600 is USD 30. Thus capital income increases 3 percent from 2013 to 2014, the same as national income, which is USD 100 in 2014. Labor income in 2014 is USD 100 minus USD 30, i.e. USD 70, a growth rate of 3 percent. Despite that \( r > g \), therefore, both capital and labor income grow at the same pace as national income, i.e. 3 percent.

Whatever happens to the stock of capital depends upon savings.

According to Piketty’s second law, and standard economic theory, the growth rate of capital is equal to the growth rate of national income. 3 percent of USD 600 is USD 18. Thus the stock of capital increases to USD 618 on Dec. 31 2014. This satisfies the condition we got by combining the second and third laws, that capital income is greater than savings. If \( r \) remains at 5 and \( g \) remains at 3, the story repeats itself in 2015. The return on capital is USD 30.9, an increase of 3 percent. National income rises to USD 103. Labor income is USD 103 minus USD 30.9, i.e. USD 72.1 – an increase of 3 percent.

But what if \( r \) rises to 6 in 2015 while \( g \) remains 3? Then capital income in 2015 is USD 37.08, an increase of 23.6 percent. With national income at USD 103, labor income drops to USD 65.92, i.e. a 5.8 percent decrease. Thus the functional distribution of income is altered in favor of capital income and to the detriment of labor income. Suppose that thereafter the capital stock grows at the same rate as national income. Then the capital stock is USD 636.54 on Dec. 31 2015. What if both \( r \) and \( g \) remain at 6 and 3 in 2016? Capital income increases to USD 38.19 in 2016, an increase of 3 percent, which is equal to the growth rate of national income. Thus labor income also rises 3 percent in 2016.

Importantly, therefore, the year after the shift in the capital and labor share of national income when the gap between \( r \) and \( g \) widens, the functional income distribution is constant.

The effect is equivalent if \( g \) drops for a given \( r \). One only gets a one-time level effect on the distribution of income from a change in the \( r > g \) inequality. Contrary to what many seem to think, Piketty’s “central contradiction of capitalism” does not predict a continuously decreasing labor to capital income ratio.

In order to get a workers’ dystopia, where laborers get a continuously smaller pie of national income, the gap between the rate of return on the capital stock and the growth rate of national income must widen constantly – and only capitalists must save.

Admittedly, while Piketty does not state the third law this way, his predictions for this and the next century rest on assumptions that the rate of return on capital gradually rises and that the rate of growth of national income gradually falls. (pp. 356-357). Whether these assumptions, which are not grounded in a coherent theory, are reasonable, is beyond the remit of this paper.

I don’t quarrel with Piketty’s forecasts about income distribution if the future. My point is that his laws have no bearing on his predictions, implying that they are no more than speculations.
Piketty, who laments the use of mathematics in economics, should have dropped two equations and one inequality from his mighty tome. To support his main thesis he needs a different theory than what’s encapsulated in his laws. Piketty’s verbal reasoning might contain the gist of such a theory. But his laws and his “central contradiction in capitalism” are likely to share the fate of Keynes' “fundamental equations”. Keynes put forth these equations, which purported to explain national income and the price level in his two-volume *A Treatise of Money* in 1930. They quickly dropped in economists’ memory hole. The Keynesianism we remember is the one he formulated in his *General Theory of Employment, Interest, and Money* in 1936.

**The logical vocabulary**

As mentioned, Piketty refrains from calling his third law a “law”. He instead refers to it as the “central contradiction of capitalism”. On page 571 he writes “this inequality \[ r > g \] expresses a fundamental logical contradiction”.

This is an idiosyncratic use of the word “contradiction”, derived, probably, via Piketty’s reading of Marx from the scribbling of Hegel. In logic a contradiction is a statement, or a theory, that’s necessarily false, i.e. false no matter how the world works. Piketty clearly does not mean that \( r > g \) is necessarily false. On the contrary, he thinks that the inequality is correct and that it has dire consequences for capitalistic income distribution. It’s these consequences, which might undermine the public support for capitalism, which he misleadingly refers to as a “contradiction”.

Capitalism is not a theory, it’s an economic system. There might be a flurry of contradictions in theories about capitalism, but to claim that capitalism as an economic system is contradictory just breed confusion. The same sort of confusion Ludwig von Mises sowed when he, three years after the Bolshevik revolution, said that “socialism was impossible”.

Piketty’s first law is the identity \( 1 = 1 \). Viewed semantically, i.e. as a proposition, it is the opposite of a contradiction. It’s a statement, or a theory, which is necessarily true no matter how the world works. Logicians call such propositions tautologies.

Piketty himself describe his first law is tautological. But it “should nevertheless be regarded as the first fundamental law of capitalism, because it expresses a simple, transparent relationship among the three most important concepts for analyzing the capitalist system: the capital/income ratio, the share of capital in income, and the rate of return on capital”. (p. 52). I think it is just the opposite. Writing \( \alpha = r \times \beta \) instead of \( 1 = 1 \) alienates more than in illuminates.

Tautologies do have their place in logic and mathematics, where all valid theorems, no matter how hard they are to prove, are tautological. Bertrand Russell and Alfred North Whitehead, when they deduced

---

6 In fairness to von Mises: that «socialism is impossible» is how his phrasing is usually remembered. What he actually wrote in 1920 was that “rational economic activity is impossible in a socialist commonwealth”.
mathematics from logic in *Principia Mathematica*, arrived at the “occasionally useful” proposition that “1 + 1 = 2” on page 86 in volume two.

Outside of logic and mathematics, however, the usefulness of tautologies is limited. Tautologies are used when a new empirical or theoretical phenomenon is given a name, as when the existence of a new elementary particle, subsequently named the Higgs boson, was suggested in 1964. But such definitions are of course never elevated to the status of scientific laws.

I’m not sure if Piketty has a clear notion of what a tautology is. He criticizes the role of time preference in the standard theory of the rate of return on capital for being tautological, writing that “this ‘theory’, like many theoretical models in economics, is somewhat tautological (one can always explain any behavior by assuming that the actors involved have preferences – or ‘utility functions’ in the jargon of economics – that lead them to act the way).” (p. 359).

But this does not describe the way economists use the concept of preferences, whether it’s preferences for goods during a period, or preferences for goods in one period relative to the next, to explain behavior. On the contrary, since preferences are unobservable, economists assume that preferences are *constant*. Economists then predict altered behavior when observable parameters, like prices, change. Since Paul Samuelson and Milton Friedman wrote about economic methodology in the late 1940s and early 1950s, this conception of economics has been mainstream.

Moreover, even if a change in behavior were to be explained by a change in preferences, one would not thereby engage in tautological reasoning.

Asserting that an increase in the rate of return on the capital stock is due to an increased preference for goods now relative to later, for example, is not tautological since the two concepts don’t have the same meaning. The problem is rather that since time preferences are unobservable, such a prediction cannot be checked against empirical evidence. Economists, therefore, abstain from such explanations.

Interestingly, Piketty himself violates this methodological norm when he discusses the rise of labor income among top managers. Dismissing the idea that increased compensation is due to higher marginal productivity, he asserts that top managers lay claim on a larger share of national income due to altered social norms. (pp. 332-333).

It’s difficult to define social norms without referring to common preferences for acceptable behavior. Thus explaining increasing wage inequality by referring to altered social norms is, as I understand it, an explanation in terms of shifting preferences. Admittedly, Piketty writes that in order to explain a change in social norms one has to draw on expertise outside of economics. But he does not elaborate on how evidence from such expertise supports his thesis. Anyway, while Piketty’s explanation of higher wages to top managers doesn’t follow established economic methodology, I wouldn’t characterize his reasoning as tautological. Not all problematic aspects of economic analysis boils down to tautological thinking.

The notion of laws, let alone “fundamental laws” in economics, that Piketty exposes, is also dubious.
Logically, science attempts to produce novel and falsifiable hypotheses that correctly predict empirical phenomena. Repeatedly non-falsified, but falsifiable, hypotheses might gain the status of scientific laws in fields like physics. But since it, due to a lack of controlled experiments among other things, typically is beyond the reach to conclusively test hypothesis in the social sciences, one should refrain from calling an economic hypothesis – or any hypothesis about human behavior – a law.

Even the best candidates for economic laws fail. The existence of Giffen goods falsifies the law that demand is a decreasing function of price. At best, economics can establish statistical generalizations that normally hold, like the proposition that \( r > g \).

**Conclusion**

While there is much to admire in Piketty’s voluminous book, especially his data-driven discussion of wealth and income inequality in previous centuries, his attempt to predict the evolution of inequalities in this century and the next is unconvincing. Hence when Paul Krugman, in his review of the book said that Piketty “offers what amounts to a unified field theory of inequality”, he engaged in hyperbole.

Piketty’s theoretical framework, upon closer inspection, turns out to be a castle of sand, at least to the extent the theory is capsuled in two equations and one inequality. If there is more to Piketty’s theory than these three mathematical expressions exposes, and there might be, then he should do what Keynes did after his failed 1930 book, and write a new, more convincing one.

By claiming to have discovered the two fundamental laws and the central contradiction of capitalism in three convoluted mathematical expressions, and by his idiosyncratic use of logical vocabulary, Piketty exemplifies simultaneously what Jon Elster respectively has called hard and soft obscurantism. Is this what happens when a French intellectual has a stint as a professor at a “university near Boston” (p. 31)?
1/10  
**Isachsen, Arne Jon**  
Finanskrisen – Konsekvenser for USA

2/10  
**Isachsen, Arne Jon**  
Korrupsjon i Kina – Kommet for å bli?

3/10  
**Isachsen, Arne Jon**  
Om spekulasjon i valutamarkedet

4/10  
**Steigum, Erling**  
Norsk økonomi etter 1980 – fra krise til suksess

5/10  
**Isachsen, Arne Jon**  
Fortellinger om samfunnet

6/10  
**Isachsen, Arne Jon**  
Hva Kina vil

7/10  
**Isachsen, Arne Jon**  
Kinas makt

1/11  
**Børsum, Øystein**  
Exchange Rate Risk in the Government Pension Fund Global

2/11  
**Isachsen Arne Jon**  
Penge- og valutapolitikk i Kina

3/11  
**Alstadheim Ragna**  
Om nye virkemidler i pengepolitikken – avgrensning mellom pengepolitikken og finanspolitikken

4/11  
**Isachsen Arne Jon**  
Økonomi er risikosport

5/11  
**Steigum Erling**  
The Norwegian Banking Crisis in the 1990s: Effects and Lessons

6/11  
**Isachsen Arne Jon**  
Felles valuta krever felles vaner

1/12  
**Isachsen Arne Jon**  
Is China Falling Apart?

2/12  
**Isachsen Arne Jon**  
På vandring

3/12  
**Nerheim, Reidun Grue**  
Investeringsstrategien for statens pensjonsfond utland

4/12  
**Steigum, Erling**  
Sovereign wealth funds for macroeconomic purposes

1/13  
**Isachsen, Arne Jon**  
Bollinger champagne
Mork, Knut Anton
Etter oljen: Utfordringer for norsk økonomi

Andersen, Torben M.
Finanspolitik og den økonomiske og monetære union

Isachsen, Arne Jon
Bank og Finans – Hva har hendt? Og hva har vi lært?

Utne, Amund
EØS-avtalen

Isachsen, Arne Jon
Stabilt eller robust?

Bjørnland, Hilde C. and Leif Anders Thorsrud
Ringvirkninger
Norsk økonomi og olje

Isachsen, Arne Jon
De gamle er eldstr

Steigum, Erling and Øystein Thogersen
A crisis not wasted – Institutional and structural reforms behind Norway’s strong macroeconomic performance

Isachsen, Arne Jon
Ulikhetens pris

Høien, Torgeir
Nibor-mysteriet

Isachsen, Arne Jon
Noen betraktninger om Oljefondet

Isachsen, Arne Jon
Bank og pølsebod – Hva er forskjellen?

Aamdal, Kyrre
NIBOR – Intet mysterium

Høien, Torgeir
Piketty’s laws and logic
No 1/2000  Hamilton, Carl , Øystein Thøgersen, Marianne Andreassen og Harald Magnus Andreassen  
Norsk pengepolitikk, Norges Banks rolle og bankens gjennomføring av pengepolitikken i 1999 og første del av 2000

No 2/2001  Andreassen, Harald Magnus, Paul De Grauwe, Haakon Solheim and Øystein Thøgersen  
A review of inflation targeting, the Norwegian monetary regime and its institutional arrangements and Norges Bank’s actual monetary policy and communication

An Independent Review of Monetary Policy and Institutions in Norway

No 4/2003  Ekel, Thomas, Anne Kari Haug, Kjetil Houg and Erling Steigum  

No 5/2004  Bjørnland, Hilde C., Thomas Ekel, Petra M. Geraats and Kai Leitemo  

No 6/2005  Dørum, Øystein, Steinar Holden and Arne Jon Isachsen  

No 7/2006  Dørum, Øystein and Steinar Holden  

No 8/2007  Goodfriend Marvin, Mork Knut Anton and Söderström Ulf  

No 9/2008  Juel Steinar, Molnar Krisztina and Røed Knut  

No 10/2009  Bergman Michael, Juel Steinar and Steigum Erling  
Monetary Policy and the Financial Turmoil.

No 11/2010  Bjørnland Hilde C., Clarida Richard, Holvik Elisabeth and Steigum Erling  
An Independent Evaluation of Monetary Policy in Norway.

No 12/2011  Bjørnland Hilde C. and Wilhelmsen Bjorn Roger  
An Independent Evaluation of Monetary Policy in Norway.

No 13/2012  Torvik Ragnar, Vredin Anders and Wilhelmsen Bjorn Roger  
An Independent Evaluation of Monetary Policy in Norway.

No 14/2013  Boye Katrine Godding and Sveen Tommy  
An Independent Evaluation of Monetary Policy in Norway.

No 15/2014  Mork Knut Anton, Freixas Xavier and Aamdal Kyrre  
An Independent Evaluation of Monetary Policy in Norway.