Cooling of Transducer array used for Ultrasound radiation Force

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Preface

This thesis is submitted to the Norwegian University of Science and Technology (NTNU) for the degree of Master of Science during 2015.

The work has been carried performed at the Department of Physics, NTNU, Trondheim, with supervision from Catharina de Lange Davies, Professor in the Department of Physics at NTNU, and co supervisors Bjørn Angelsen Department of Circulation and Medical Imaging, at NTNU, and Tonni Franke Johansen SINTEF.
Summary

This thesis reviewed the power requirements for high Ultrasound Radiation Force apparatus, and the heat generation caused by this.

This thesis proposed a way to cool an ultrasound transducer. The cooling method is to have a an area on the casing of the transducer that has a high thermal conductivity, and a high emissivity. This layer is meant to passively transfer heat from the casing to the surroundings. This area on the casing is connected to the transducer stack through a copper layer that is separated for the stack by thermal paste. This cooling method was simulated using a two step method. The first step was to determine the amount of heat transferred from the highly conductive area on the casing to the surrounding, assuming constant surface temperature. The second step was to simulate the heat transfer from the transducer stack to the casing assuming evenly distributed constant heat generation in the transducer stack.

The conductive casing that was simulated was a prism surrounding the sides of the transducer stack, but not the top and bottom. This prism was off height 4cm. It was found that the casing material should have as high conductivity and emissivity as possible in order to increase heat transfer. The thickness of thermal paste was found to have a large impact on the heat transfer from the transducer stack to the conductive casing. This necessitates care when applying thermal paste to make sure it has the desired thickness.

The warmest area of the transducer would based on the simulations be close to the backing. In order to increase heat transfer out of the transducer, it was therefore proposed to add a thermal conductive layer just above the backing off the transducer. This method was shown in simulations to be effective in increasing heat transfer. With this added thermal conductive layer the transducer would stabilize around $56^\circ C$ for a heat generation of 8.3W evenly distributed throughout the transducer.
Sammendrag

Denne masteroppgaven gjennomgått kravene til høy effekt for transducere som produserer høy strålingskraft. Den har også gått igjennom hvordan dette påvirker varmegenerering.

Denne masteroppgaven har foreslått en måte å kjøle en ultralydtransducer. Kjølmetoden er å ha ett område på skallet rundt transduceren som har høy varmelednings evne og høy emmisivitet. Dette laget er koblet til transduceren gjennom ett kobberlag og ett lag med kjølepasta. Denne kjølemetoden ble simulert med en to trinns metode. Det første steget var å simulere varmeoverføring mellom skallet og omgivelsene, med antagelsen om konstant skalltemperatur. Steg to var å simulere varmeoverføringen fra ultralydtransduceren til skallet, med antagelsen om konstant varmegenererering gjevnt fordelt i transduceren.

Den høyt varmeledende delen av skallet var formet som ett prisme som omgir transduceren på alle kanter bortsett fra over og under. Dette prismet var 4 cm høyt. Det ble funnet at dette varmeledende materialet burde ha så høy varmeledning og emmisivitet som mulig for å øke varmeledning. Tykkelsen på kjølepastaen viste seg å ha stor innvirkning på varmeoverføringen mellom transduceren og skallet.

Det varmeste området i transduceren ble i simuleringene funnet å være på backingen. For å øke varmeoverføringen til skallet ble det derfor forsøkt å legge til ett ekstra varmeledningslag mellom backingen og resten av transduceren. Dette viste seg å ha god innvirkning på varmeoverføringen til skallet. Med dette ekstra varmeoverføringslaget stabiliserte transduceren seg rundt 56°C ved en varmegenerering på 8.3W.
Acknowledgement

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First, my supervisors, Professor Bjørn Angelsen Department of Circulation and Medical Imaging, at NTNU. Professor Catharina de Lange Davies, Department of Physics at NTNU, Department of Circulation and Medical Imaging. Tonni Franke Johansen at SINTEF. This thesis would not be possible without their help.

I would like to than my partner Nathalie Marquesin de Oliveira, for her invaluable assistance and support. I would like to thank my friends Hakon Seljåsen and Leif Kristian Tungodden, for their help during my studies.
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1. INTRODUCTION

This master thesis is a part of a project to create ultrasound mediated drug delivery at the Norwegian University of Science and Technology (NTNU). This project intends to use microbubbles consisting of a shell containing drugs, to deliver chemotherapy. [20] states that the typical microbubbles for ultrasound contrast have a diameter round 2µm. [21] discusses contrast microbubbles in the 1-2µm, 4-5µm and 6-8µm ranges. Ultrasound will then be used to direct these microbubbles to their destination, burst them as well as continually image the body. This sets stringent requirements on the ultrasound transducer.

This thesis focuses on evaluating the heat transfer in the ultrasound transducer. There are components in the transducer that can take damage from as low as 60 degrees. Making sure that the heat produced in the transducer is removed is important.

Aggressively growing cancer tumors often have a chaotic organization of capillaries with imperfect, highly permeable capillary walls. This wall imperfection allows the passage of nano-particles containing chemotherapy, while not crossing normal tissue. Hence protecting normal tissue against the drug. However, the increased wall permeability also gives a high leakage of fluid from the blood into the interstitium (space between cells), which combined with a weak lymphatic drainage produces an increased interstitial pressure with low pressure gradient convection of nano-particles and also free drug molecules from the capillaries and deep into the interstitium.

Ultrasound radiation force (URF) provides a similar force on fluids and particles as a pressure gradient, and can be used to increase the transport of drugs and particles from the capillaries and deep into the interstitium. The radiation force is proportional to the extinction of the acoustic radiation intensity through scattering and thermal absorption due to particles and large molecules in the fluid. To obtain a large radiation force it is necessary to transmit a high acoustic intensity, which presents challenges to reduce absorption in the transducers. It is also necessary to provide heat conduction from thermal sources and cooling of the transducer. The scattering and absorption is
also frequency dependent, so that an optimal frequency for maximal radiation force depends both on particle type and depth of the tumor.

The work contains the following parts:

1. Review of the high power requirements for URF, and the heat generation caused by this.

2. Establish model for heat generation and thermal conduction of the complete array, where structures for cooling of the array is included. Develop a computer simulation model for this structure and simulate temperature developments in the array when heat is generated.

3. Design cooling structures from the simulations for increased power operation of the arrays.
2. THEORY

2.1 Cancer

About 8.5 million deaths were caused by cancer in 2012 [28]. This accounts for 14.6% of all human deaths in that year.

Six hallmarks of cancer has been proposed by [22], sustaining proliferative signaling, evading growth suppressors, resisting cell death, enabling replicative immortality, inducing angiogenesis and activating invasion and metastasis.

1. Sustaining proliferative signaling: In healthy tissue there is a well regulated process of activating proliferative signals when needed and deactivating them when not needed. The result is that healthy tissue has a homeostasis of cell numbers, and that normal tissue architecture and function is maintained. In cancer cells this regulation does not work, and there are much higher levels of proliferate signals.

2. Evading growth suppressors: There are many mechanisms to suppress cell growth that gets activated as a result of the sustained proliferative signaling. In order for the tumor to grow these mechanisms have to be evaded.

3. Resisting cell death, resisting apoptosis. Cells in tissue are highly organized, and their numbers are tightly regulated. This regulation happens both due to control off the amount of new cells created, and by planned cell death or apoptosis. This cell suicide can is quite common, in healthy humans billions of cells die every hour in the bone marrow ad the intestine [16]. Apoptosis is an important natural barrier to cancer development. In order for a tumor to become high grade malignant apoptosis has to be avoided.

4. Enabling replicative immortality: Most healthy cells are only able to pass through a limited number of successive cell growth-and-division cycles. However cancer cells require unlimited replicative potential in order to create macroscopic tumors. The
end of chromosomes are protected by telomeres, to hinder degradation and illegitimate recombination \cite{15}. states that For every cell growth-and-division cycle these telomeres are shorten until they become so short that copying the chromosome can not happen without mistakes. In most cancer cells however there are functionally significant levels of telomerase, a DNA polymerase that add telomere repeat segments to the ends of telomeric DNA \cite{22}. The erosion of telomeres as a result of the growth-division cycle are therefore countered in these cells, leading to replicative immortality.

5. Inducing angiogenesis: The formation of new blood vessels (Angiogenesis), is a necessity in cancer tissue. As other cells, cancer cells require a supply of oxygen and nutrients, and evacuation of metabolic wastes and carbon dioxide. In the absence of angiogenesis tumors are, due to their proliferative capabilities, supply restricted, and therefore their growth is also limited. Cancer tissue overcome this obstacle by reducing the production of angiogenic inhibitors (anti-angiogenic), and increasing production of inducers (pro-angiogenic). An important part of this process is the increased production of Vascular Endothelial Growth Factor (VEGF).

6. Activating invasion and metastasis. Tumors are able to invade surrounding tissue, and to spread to far away tissues.

2.2 Chemotherapy

"This section discusses how chemotherapy works and the negative consequences of the treatment. This section also discusses how the low pressure gradient in cancer cells affect drug delivery.

Chemotherapy is a commonly used treatment for cancer. Chemotherapeutic agents are normally cytotoxic. This means that they target and kill cells that divide rapidly. Cancer cells fit this description, but they are not the only ones. Cells in the bone marrow, digestive tract and hair follicles also divide quickly and are therefore targeted. Myelosuppression is the decreased production of cells in the bone marrow. These cells include leukocytes, erythrocytes and thrombocytes. Leukocytes are responsible for providing immunity, erythrocytes are responsible for oxygen transportation in the blood and thrombocytes are responsible for normal blood clotting. The targeting of these cells by chemotherapeutic agents therefore have severe effects on the bodies
ability to handle infections, stop bleedings and supplying the cells in order to keep a normal cell activity level. As a result patients often have to be hospitalized under strict infection control, and treated with aggressive doses of antibiotics if there are signs of infection. Anemia resulting from the reduced production of erythrocytes, can greatly reduce the energy levels of the patients. Reducing the adverse effects of cancer treatment is of great importance.

In normal tissue the vasculature is orderly and efficient. There is a clear separation of arteries, arterioles and capillaries. These are evenly spread out. This is not true for cancer tissue, due to the high levels of VEGF. The vasculature in cancer tissue is disorganized. There is no clear distinction between arterioles capillaries and venules. Blood vessels in cancer tissue is also more permeable than in normal tissue. Lymphatic vessels show similar changes. Blood vessels have changing diameters, uneven shapes and can have dead ends. The result of this is that the interstitial pressure in cancer tissue is higher than normal and that there is an increased geometric resistance.

The increased interstitial pressure caused by angiogenesis, leads to problems with delivery of chemotherapeutic agents to cancer tissue. The reason for this is that the differential pressure is small compared to normal tissue. This causes osmosis to be the major delivery agent of the drug. In order to increase the differential pressure, professor Bjørn Angelsen at NTNU has proposed to use high radiation force ultrasound.

2.3 General Ultrasound theory

This section will briefly discuss acoustic wave propagation in tissue. The main source of this information is [11]. To get more in depth knowledge of this topic the reader is encouraged to read this book.

Ultrasound waves are pressure waves. These waves are primarily longitudinal in biological tissue, as transverse waves are heavily attenuated. In ultrasound waves differences in pressure cause movement of particles, which again causes differences in pressure. This is analogous to a spring mass system. The deviation of pressure from the average is analogous to the deviation of the mass from the resting position. The average speed of particles in an area is analogous to the speed of the mass in the
A particle has the equilibrium position \( \vec{r} \). During vibration of the tissue this particle is moved to the position \( \vec{r}_L \)

\[
\vec{r}_L = \vec{r} + \psi(\vec{r}, t)
\]

(2.1)

Here \( \psi(\vec{r}, t) \) is the displacement of the particle from its equilibrium position. The vibration velocity and acceleration of this particle is

\[
\vec{u}(\vec{r}, t) = \frac{\partial \psi(\vec{r}, t)}{\partial t}
\]

(2.2)

\[
\vec{a}(\vec{r}, t) = \frac{\partial \vec{u}(\vec{r}, t)}{\partial t}
\]

(2.3)

This is the Lagrange description of vibrations.

In a homogeneous isotropic material the wave equation for pressure \( p \) and vibration speed \( \vec{u} \) are respectively:

\[
\nabla^2 p(\vec{r}, t) - \frac{1}{c^2} \frac{\partial^2 p(\vec{r}, t)}{\partial t^2} = 0
\]

(2.4)

\[
\nabla^2 \vec{u}(\vec{r}, t) = -\frac{1}{c^2} \frac{\partial^2 \vec{u}(\vec{r}, t)}{\partial t^2} = 0
\]

(2.5)

here \( c = \frac{1}{\sqrt{\rho \kappa}} \) is the speed of sound. \( \rho \) is the density of the medium and \( \kappa \) is the bulk compressibility.

### 2.4 Ultrasound radiation force

Acoustic waves produce a radiation force. [10] gives the ultrasound radiation force on a small volume \( \Delta V \) as

\[
\Delta F = \frac{\sigma_e I}{c} \Delta V
\]

(2.6)

where \( \sigma_e \) is the extinction cross section, \( c \) is the ultrasound propagation velocity, and \( I \) is the ultrasound intensity. \( \sigma_e = \sigma_a + \sigma_s \), \( \sigma_a \) is the absorption cross section, \( \sigma_s \)
2. Theory

is the scattering cross section. The average ultrasound intensity is given as

\[ I = \frac{P^2}{2Z_o} \]  

where \( P \) is the pressure amplitude, and \( Z_o \) is the characteristic impedance of the medium. In the vicinity around the tumor it is desired to have \( \frac{\Delta F}{\Delta V} \) as large as possible in order to increase the flow through the tumor. \( \sigma_e \) is frequency dependent and one can optimize the frequency in order to increase \( \frac{\Delta F}{\Delta V} \). For this thesis the most relevant part is that \( \frac{\Delta F}{\Delta V} \propto P^2 \).

\( V_t \) is the voltage amplitude applied to the transducer, \( \omega_o \) is its frequency, \( W_{it} \) is the effect applied to the transducer, \( W_{ot} \).

[10] gives:

\[ P_t = Z_0 H_{tt}(\omega_0)V_t \]  

(2.8)

\[ W_{it} = N_{el} \frac{|V_t|^2}{2|Z_{el}(\omega_o)|} \cos(\Theta_{el}(\omega_o)) \]  

(2.9)

\[ W_{ot} = N_{el} A_{el} \frac{|P_t(\omega_o)|^2}{2Z_0} = N_{el} A_{el} Z_0^2 |H_{tt}(\omega_o)|^2 |V_t|^2 = N_{el} A_{el} Z_0^2 (\omega_o)|H_{tt}|^2 |V_t| \]  

(2.10)

\( W_{qr} = W_{it} - W_{ot} = \left( \frac{\cos(\Theta_{el}(\omega_o))}{|Z_{el}(\omega_o)|} - A_{el} Z_0 |H_{tt}(\omega_o)|^2 \right) \frac{1}{2} N_{el} |V_t|^2 \]  

(2.11)

\[ = \left( \frac{\cos(\Theta_{el}(\omega_o))}{|Z_{el}(\omega_o)| Z_0^2 |H_{tt}|^2} - A_{el} \frac{Z_0}{Z_0} \right) \frac{1}{2} N_{el} |P_t(\omega_o)|^2 \]  

(2.12)

A transducer will not run continuously, it will run in pulses. \( T_p \) is the length of a pulse and \( T_r \) is the time between the start of one pulse to the start off the next pulse.
The average heat power to the assembly will then be:

\[ W_{qAvg} = W_q \frac{T_p}{T_r} \quad (2.13) \]

When using URF for drug delivery it is desired to have \( \frac{\Delta F}{\Delta V} \) as large as possible. From equations 2.6, 2.7, 2.12 and 2.13 it is shown that

\[ \frac{\Delta F}{\Delta V} \propto W_{qAvg} \quad (2.14) \]

Therefore, heating of the transducer will create a constraint on the amount of URF that can be used for treatment.

### 2.5 Heat and Temperature

Temperature is a measure of the average internal kinetic energy of particles or molecules in a substance. This is related to the human sense of hot and cold. The main source for this section is [17]

Heat is the transfer of internal kinetic energy, or thermal energy. How much heat is required to change the temperature of a substance by a certain amount is material specific. The relationship is

\[ \frac{Q}{mc} = \Delta T \quad (2.15) \]

where \( Q \) is the heat, \( m \) is the mass of the substance that receives the heat, \( \Delta T \) is the change in temperature and \( c \) is a material dependent and empirical value, which for small changes in \( T \) can be considered constant. The value \( c \) is called the specific heat of a substance and has the unit \( \frac{J}{kg \cdot K} \).

Heat can also be expended or derived from a phase change in a substance, such as a solid liquefying, a liquid evaporating or the reverse processes. In this case the relationship is

\[ Q = \pm mL \quad (2.16) \]

\( m \) is again the mass of the substance changing state, \( L \), \( \frac{J}{kg} \) is an empirical value for the phase change, the sign depends on the direction of the phase change. In a transducer phase changes would cause grave problems and likely ruin the device. It is therefore important to avoid overheating.
2.6 Heat Transfer

Heat transfer refers to the transfer of thermal energy. There are three modes of heat transfer:

a) Conduction, relating to the transfer of heat due to random molecular motion.

b) Convection, relating to the transfer of heats due to a combination of diffusion and bulk movement of substances, this is common in liquids and gases.

c) Radiation, relating to the transfer of heat as photons.

2.6.1 Conduction

Conduction is due to random molecular movement. In solids this means lattice vibrations and movement of electrons. In a cube with surfaces of 1 $m^2$. (PICTURE) Two surfaces of this cube opposite to each other are held at constant temperatures $T_1$ and $T_2$, where $T_1 > T_2$. The other surfaces are completely insulated so that no heat can pass through them. The cube is made of a solid material. In this situation we will have a one dimensional heat transfer between surface 1 and 2. Since the cube is made of a solid material this heat transfer will be through conduction. The rate of heat transfer will be

$$q = \frac{dQ}{dt} = -kA \frac{T_2 - T_1}{L} = -kA \frac{\Delta T}{L}$$

(2.17)

where A is the surface area, L is the length between the surfaces, and k is the heat conductivity of the material. k has the unit $W/m\cdot K$ and is approximately constant for small changes in temperature.

For 3D situations the heat transfer is defined as the following relation [17]:

$$\vec{q}' = \frac{d\vec{Q}}{dt}dA = -k \nabla T$$

(2.18)

2.6.2 Convection

Convection is term for heat transfer taking place when there is both diffusion of heat and bulk movement of substances. This is the situation for heat transfer in gases and liquids. Even if there is no forced movement the density changes that result from temperature changes will cause movement in the gas or liquid. For a surface, at
temperature $T_s$ in contact with a fluid or gas whose temperature far away from the surface is $T_\infty$, the following relation for the heat transfer. [17]

\[ q'' = h(T_s - T_\infty) \tag{2.19} \]

This relation is known as Newton’s law of cooling. $h \left[ \frac{W}{m \cdot K} \right]$ is the convection heat transfer coefficient, which depends on the fluid properties, the surface geometry and the fluid motion.

**Free convection**

For the specialized case of a vertical plate with no forced convection, the convection heat transfer coefficient is given as

\[ \bar{h} = \frac{N\bar{u}_L \cdot k}{L} \tag{2.20} \]

where $k$ is the heat conductivity of the fluid surrounding the surface, $L$ is the height of the plate and $N\bar{u}_L$ is the Nusselt number given as [14]

\[ N\bar{u}_L = 0.68 + \frac{0.670Ra_L^{1/4}}{1 + (0.4925/Pr)^{9/16}} \tag{2.21} \]

This equation is accurate for laminar flow, which is true for $Ra_L < 10^9$. Otherwise the following equation is used.

\[ N\bar{u}_L = \left\{ 0.825 + \frac{0.387Ra_L^{1/2}}{1 + (0.4925/Pr)^{9/16} \cdot 27} \right\}^2 \tag{2.22} \]

$Ra_L$ is the Rayleigh number a dimensionless number measuring the buoyancy driven flow.

\[ Ra_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu \alpha} \tag{2.23} \]

$\nu [m^2/s]$ is the kinematic viscosity. $\alpha [m^2/s]$ i the thermal diffusivity. $\beta [k^{-1}]$ is the volumetric thermal expansion coefficient defined as

\[ \beta = -\frac{1}{\rho} \left( \frac{\partial \rho}{\partial T} \right)_P \tag{2.24} \]
for ideal gases this is simply:

$$\beta = \frac{1}{T} \tag{2.25}$$

Pr is the prandtl number, the ratio between momentum and thermal diffusivity.

$$\beta = \frac{\nu}{\alpha} \tag{2.26}$$

2.6.3 Radiation

All mater at nonzero temperatures emits photons. The energy emitted by a surface is described by the surfaces emissive power $E [W/m^2]$, this is given by the Stefan-Boltzmann law as

$$E = \epsilon \sigma T_s^4 \tag{2.27}$$

$\sigma = 5.67 \cdot 10^{-8} \frac{W}{m^2 \cdot K^4}$ is the Stefan-Boltzmann constant. $T_s$ is the surface temperature in Kelvin. $\epsilon$ is a material specific constant between 0 and 1 called emissivity. As well as emitting radiation, a surface will absorb radiation from its surrounding. All of the incoming radiation will not be absorbed, but only a fraction denoted by $\alpha$. A gray surface is an idealized surface whose emissivity does not depend on the angle of radiation and independent of wavelength [24]. For a grey surface, $\epsilon = \alpha$ [17]. At temperature $T_s$ with a surrounding at $T_{sur}$ the net rate of radiation heat transfer $q''$ is given as

$$q'' = \epsilon \sigma (T_s^4 - T_{sur}^4) \tag{2.28}$$

If there is a light source shining on the surface with irradiance I, the following equation applies:

$$q'' = \epsilon \sigma ((T_s^4 - T_{sur}^4) - I) \tag{2.29}$$

A demonstration of the importance of the emissivity is given by the Leslie cube. The cube is filled with hot water. All the surfaces have the same temperature. However the different surfaces of the cube have different emissivities and therefore the radiation from these surfaces are different. In 2.1 one side is painted black, one is painted white and the other sides are of polished aluminum. The figure 2.1 shows the surface material has a large impact on the thermal radiation.
2.7 The Finite Element Method (FEM)

"The finite element method (FEM) has become one of the most frequently used methods for solving equations from mechanics of solids and fluids and thermodynamics that describe the behavior of physical systems in the form of partial differential equations" \[18\]. An essential characteristic of FEM is mesh discretization of a continuous domain into smaller discrete sub-domains called elements \[12\]. The answers given by FEM simulations will get more accurate the finer the element mesh is made \[13\].

2.7.1 The Heat Equation

The heat equation or the heat diffusion equation is the equation that forms the basis for FEM in heat transfer. It is a combination of the conduction formula \[2.18\] and conservation of energy applied to a small control volume.

A medium is divided into a set of small cubes. The heat transfer in each of these cubes will be described by the conservation of energy in these cubes, and by equation
2.18 Conservation of energy says for one cube is given by the following expression:

\[ \dot{E}_{in} + \dot{E}_g - \dot{E}_{out} = \dot{E} \]  

In words: the change of energy in an element \( \dot{E} \) is equal to energy that flows in to the element \( \dot{E}_{in} \) plus the energy that is generated in the element \( \dot{E}_g \) minus the energy that flows out of the element \( \dot{E}_{out} \). The internal energy of the element due to its temperature is proportional to the mass of the element \( dm = \rho dV \), and to the specific heat of the substance \( c_p \)

\[ \dot{E} = \rho c_p \frac{\delta T}{\delta t} dV = \rho c_p \frac{\delta T}{\delta t} dx dy dz \]  

The energy that is created in the element can come from a variety of sources, electrical, chemical, acoustical. These will all be summed and described as \( \dot{q} \). The generated energy is then given as

\[ \dot{E}_g = \dot{q} dV = \dot{q} dx dy dz \]  

Fourier’s partial differential equation

\[ \rho c_p \frac{\delta T}{\delta t} = \text{div}(k \nabla T) \]  

must be satisfied at every point and at every time. And forms the basis of the FEM method for heat transfer for a time varying solution. [20]

2.8 Cooling methods

Cooling methods can be divided into two categories: passive cooling and active cooling. Active cooling methods refer to methods where one uses energy in order to lead to heat transfer. Passive cooling refers to methods where energy is not used to increase heat transfer. During the process of leading heat out of the system there is first an internal phase where the energy is transferred to the boundaries of the system and then an external phase where the energy is transferred from the boundary of the system to the environment. The first stage is achieved through conduction, the second through convection and radiation. The difference between the passive cooling method and the active cooling method happens in the last stage. The active cooling method uses machines to increase the convective transfer of heat. This could be done by using fans or by using methods to circulate liquids around the system. However as noted in 2.6.2 convection will happen if there is a temperature gradient.
3. SIMULATION METHOD

3.1 Transducer Design

The transducer being simulated in this thesis is shown in picture 3.1. On the top of the transducer there are two matching layers to limit reflection of acoustic waves from skin. After this there is a set of electrodes in contact with a which layer. The top electrode is ground and the bottom sends the signal to the which layer. This which layer is tuned to send out high frequency ultrasound. The next part is the heatsink of copper surrounded by two isolation layers. This copper layer is the main way for the cooling of the transducer. The next part of the transducer are a set of which layers with accompanying electrodes. The last part of the transducer is the backing.

The transducer is 150mm long and 42mm wide. The height of every layer in the transducer is given in figure 3.1.
Fig. 3.1: Design of the transducer stack. The values on the right show the thickness of every layer.
3. Simulation Method

3.2 COMSOL

The simulation program COMSOL Multiphysics is used in this thesis. The version used is 4.0a. COMSOL is a FEM program developed by COMSOL Inc. In order to use COMSOL one has to define the geometry, and material properties of the system to be simulated. Specify the initial conditions, the physical laws to be used.

3.2.1 Geometry

Fig. 3.2: Image showing the 2D geometry created in COMSOL. A mesh has been created to prepare for the simulation.
Fig. 3.3: Image showing the setup to create simulation geometry in COMSOL
3.2.2 Material

It is necessary in COMSOL to define the material parameters for the simulation. Figure 3.4 shows how this looks in COMSOL. The materials used for the simulations carried out in this thesis are further discussed in section 3.3.

![Image showing the setup to define the material parameters in COMSOL](image)

*Fig. 3.4: Image showing the setup to define the material parameters in COMSOL*
3. Simulation Method

3.2.3 Initial Condition and physical laws

For the simulation to be successful it is necessary to define the physical laws to be used. These laws are the laws from section 2.6.1. It is also necessary to state the initial conditions. The initial conditions used in this thesis are starting temperature, border temperature and heat generation. The starting temperature was always set equal to the border temperature. The exact initial values vary depending on the simulations. They will be given along with the results.

3.3 Simulation Setup

The most accurate method of simulating heat transfer from the transducer, is to create a whole 3D model of the transducer. The downside of running a full 3D model of the transducer is that it increases the computational requirements. Running a simulation with the normal transducer layers, but with 4mm width and 4mm length, found that the simulation required 15mb of ram. This scale is small compared to the actual size of the transducer. 42mm width and 150mm length. A 3D model of the transducer, created in COMSOL, stack can be seen in figure 3.5.

In the transducer from section 3.1 in a Cartesian coordinate system where the width is denoted as the x-dimension, the length is denoted as the y-dimension, and the height is denoted as the z-dimension. It is assumed that the heat generation inside the transducer is uniform in the different layers, the hottest point will be in the geometric center of the transducer as seen from above. Since finding the maximum temperature for a certain heat generation and geometry is the object of the simulation, this makes possible some simplifications of the simulation design. Instead of using a 3D model a 2D model can be used. The length of the transducer is 3.75 times its width. From 2.17 with the assumption of a uniform surface temperature $q_x = 3.75 q_y$ adding to this $A_x = 3.75 A_y$. Therefore $q_x \approx 14q_y$. The 2D simulation therefore as a first approximation only needs to take into account the x,z plane.

In order to simulate heat transfer from the transducer it was necessary to create the design to be simulated. Inside the transducer there are one or more copper plates whose task it is to transfer heat from the transducer to the outer casing. On the outer casing of the transducer there is an area of metal that exists in order to increase heat transfer between the casing and the surrounding air. The metal on the outer casing and the metal sheets in the transducer are attached. If these metal sheets were
welded together this might change the stiffness of the apparatus, and lead to possible currents due to static electricity from the outside. Therefore these metal pieces are separated by a small layer of non-conductive thermal paste. This system can be seen in figure 3.6.

The simulation has been divided into two parts. Part 1 treats heat transfer from the casing to the surroundings. Part 2 treats heat transfer from the transducer to the casing. this is shown in figure 3.7.

**Part 1: Heat transfer from casing to the surroundings.**

For the heat transfer from the casing to the surroundings. There are two mechanisms, free convection and radiation. Free convection is describe in section 2.6.2. Radiation is described in section 2.6.3. The simulation is conducted for several surface materials. These surface materials where: aluminum, anodized aluminum, polished copper, oxidized copper, copper painted black. The material parameters used are given in table 3.1.
Fig. 3.6: System for cooling the transducer.
Overview of method

- Calculation of heat transfer from casing to surroundings using functions in theory section and C#.
- Simulation of heat transfer from transducer to casing using FEM in COMSOL.

Fig. 3.7: The 2 part setup of the simulation.

<table>
<thead>
<tr>
<th>Material</th>
<th>Heat Conductivity $[\frac{W}{m \cdot K}]$</th>
<th>Emissivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>aluminum</td>
<td>237</td>
<td>0.3</td>
</tr>
<tr>
<td>Anodized aluminum</td>
<td>237</td>
<td>0.84</td>
</tr>
<tr>
<td>Polished copper</td>
<td>401</td>
<td>0.07</td>
</tr>
<tr>
<td>Oxidized Copper</td>
<td>401</td>
<td>0.87</td>
</tr>
<tr>
<td>Copper Painted black</td>
<td>401</td>
<td>0.98</td>
</tr>
</tbody>
</table>

Tab. 3.1: Material properties used to simulate heat transfer from the casing to the surroundings. [17]

<table>
<thead>
<tr>
<th>Material</th>
<th>$\rho [\frac{kg}{m^3}]$</th>
<th>$c_p [\frac{kJ}{kg \cdot K}]$</th>
<th>$\nu [\frac{m^2}{s}]$</th>
<th>$k [\frac{W}{m \cdot K}]$</th>
<th>$\alpha [\frac{m^2}{s}]$</th>
<th>Pr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>1.1614</td>
<td>1.007</td>
<td>$15.89 \cdot 10^{-6}$</td>
<td>$26.3 \cdot 10^{-3}$</td>
<td>$22.5 \cdot 10^{-6}$</td>
<td>0.707</td>
</tr>
</tbody>
</table>

Tab. 3.2: Air properties used to simulate heat transfer from the casing to the surroundings. [17]

Since radiation is an important part of heat transfer from the casing it is necessary to take into account the irradiation due to lighting on the casing. To investigate this
three cases will be used.

1. No lighting in the room.
   This is equivalent $I = 0$ in equation 2.29. This does not mean that the room has to be completely dark, it just means that the irradiation on the casing can be neglected.

2. Lighting equivalent to what is normal in an office environment. Offices have illuminance between 300 lux and 500 lux [2]. The amount of lumens/Watt ratio (also known as the Luminous efficacy of radiation) varies depending on the frequencies of the light. The maximum value is given at 555 nm (green light) at $683 \text{ lm/W}$ [29]. Light from a typical light bulb has a luminous efficacy of radiation of $15 \text{ lm/W}$, this is to a large part due to the radiation being in the infrared spectrum [25]. Sunlight has luminous efficacy of radiation of $93 \text{ lm/W}$ [29]. Light from LEDs have luminous efficacy of radiation around $60 \text{ lm/W}$ [25]. Some LEDs can however have higher efficiency, [7] shows a 5.4W LED bulb, whose output is 410-550lm. This gives a wall-plug in luminous efficacy of radiation of $76 - 102 \text{ lm/W}$, since this efficency includes electrical loss the luminous efficacy of radiation will be even a bit higher. A conservative estimate of the Light in the room will then be $500 \text{ lux} / 60 \text{ lm/W} \approx 8 \text{ W/m}^2$.

3. Direct sunlight hitting the transducer. At the surface of the earth, near the equator, the combined effect of direct sunlight and sunlight scattered from the atmosphere is $1120 \text{ W/m}^2$ [9]. This radiation changes quite a bit as you move away from the equator. It is however used as it gives an upper bound to the radiation incident in the transducer.

The heating caused by the incident Light varies a lot with the angle it hits the surface. For a grey surface the absorptivity does not change with the angle off the incident Light, the Light is however spread across a larger area if it hits at a low angle. Figure 3.8 shows Light incident on a surface with angle $\theta$. The Irradiance of the Light is $I_{in}$. The Irradiance of the light as measured by the surface is $I_{eff}$. It is assumed that the surface is a grey surface. The only difference between $I_{in}$ and $I_{eff}$ is therefore the area the Light is spread over. From Figure 3.8 it is clear that

$$I_{eff} = I_{in} \cdot \cos(\theta) \quad (3.1)$$

From (3.1) it is clear that as the angle of incidence $\theta$ goes to zero the effective irradiance $I_{eff}$ becomes negligent.
During this simulation surface temperature was assumed constant. This assumption will be discussed further in section 5.1.2 based on the acquired data. The tables in chapter 4 showing the heat transferred from the casing to the surroundings also have a column labeled $\text{Internal conduction } q_{\text{tot}}$. This is a measure of how strong the internal conduction of the surface is compared to the heat transfer to the surroundings. It is calculated the following way: There is assumed to be a 1K difference between the top of the casing and the bottom of the casing, the heat flow resulting from this is then calculated using the methods from section 2.6.1.

Part 2: Heat transfer, transducer to casing.

For the case where the generated heat in the transducer is equal to the heat transferred from the casing to the air, the temperature in the system will be constant. This static system is what is simulated in COMSOL. I.e., if the transducer generates a constant value $\dot{q}$ of heat, what will the temperature of the system be if left alone.

The simulation of the casing to air system finds the total amount of heat transferred from the casing to the air at a certain surface temperature. These result gives the initial condition for the internal simulation of the transducer. As the initial condition the whole transducer is set to the surface temperature used in the casing to air temperature. At the surface of the casing this temperature is set to be unchanged during the whole simulation. The amount of heat transferred by the casing to the air at this temperature is set to be equal to the heat generated in the transducer. The heat generation is assumed to be evenly distributed throughout the transducer. This value is divided by the total heat transfer to the surroundings. This fraction is therefore a measure of how much bigger the internal conduction would be, with a 1K temperature difference, to the heat transfer to the surroundings.

The generated heat is assumed to be evenly distributed throughout the transducer. Section 2.4 discusses how heat is generated in a transducer. The energy of a transducer is brought in by the electrodes. This electrical energy is then converted to acoustic energy in the which layers. It makes sense that a lot of the heat generated, will happen in this conversion. The which layers take up most of the space in the transducer. It is also known that absorption of acoustic energy in the other layers
Fig. 3.8: Light incident on a surface at an angle. $I_{in}$ is the Irradiance of the incoming light. $I_{eff}$ is the effective irradiance after it has been spread out across the surface.
causes heat to be generated. The duty cycle of the which layers have not been determined. In equations [2.12] and [2.13] this is shown to be an important part of calculating the heat generation, and the distribution of the heat generation between the different which layers. Due to the unknown distribution of heat generation between the LF layer, the HF layers and the other layers, it was considered best to use assume the heat to be generated evenly throughout the transducer stack.

Materials

The simulation uses four groups of material parameters for the transducer stack. These materials are copper, epoxy, ceramic and thermal paste. Figure [3.5] shows the transducer stack.

Copper is used for the electrodes, the core and the casing. COMSOL has already specified material values for copper. These values are:

- Density: $8700 \frac{kg}{m^3}$
- Specific heat capacity: $2100 \frac{J}{kg \cdot K}$
- Thermal conductivity: $0.18 - 3 \frac{W}{m \cdot K}$

Epoxy is used for the isolation layers and the matching layers. Epoxy is a relatively broad term, and refers to several materials. As the exact materials to be used for the isolation, matching layers where not clear, this general term was however used. [4] gives values in the following range for epoxies used in medical devices.

- Density: $1.02 - 1.25 \frac{kg}{m^3}$
- Specific heat capacity: $2100 \frac{J}{kg \cdot K}$
- Thermal conductivity: $0.18 - 3 \frac{W}{m \cdot K}$

The values that were used in the simulation are $\rho = 1.1 \frac{kg}{m^3}$, $c_P = 2100 \frac{J}{kg \cdot K}$, $k = 1 \frac{W}{m \cdot K}$.

Ceramic is used for the which layers. It was unknown exactly which materials were to be used in the transducer. $BaTiO_3$ is a ceramic material that has been used for transducers [5]. It is assumed that the piezoelectric layers are made of a material with
not very different properties from this. \[23\] gives the following values for $BaTiO_3$ based ceramics.

- Density: $(5.84 \pm 0.01) \cdot 10^3 \frac{kg}{m^3}$
- Specific heat capacity: $385 \pm 4 \frac{J}{kg \cdot K}$
- Thermal conductivity: $400 \pm 0.02 \frac{W}{m \cdot K}$

Thermal paste is used for the thermal paste section between the core and the casing. There are many different thermal pastes available. These are usually silicon based, with added materials to increase the thermal conductivity. Examples of added materials are ceramics, metals such as aluminum and silver, and diamonds and other carbon based materials. The thermal paste T636 from Parker Chomerics has the following specifications, $\rho = 1.2 \frac{kg}{m^3}, c_P = 1200 \frac{J}{kg \cdot K}, k = 2.4 \frac{W}{m \cdot K}$ \[6\]. There are many thermal pastes with higher thermal conductivity as an example \[8\] has thermal conductivity of $k = 16 \frac{W}{m \cdot K}$. Thermal conductivity around $k = 10 \frac{W}{m \cdot K}$ is quite normal. The values chosen for the thermal paste material in the simulation are $\rho = 1.2 \frac{kg}{m^3}, c_P = 1200 \frac{J}{kg \cdot K}, k = 10 \frac{W}{m \cdot K}$. 


4. RESULTS

4.0.1 Heat transfer from casing to surroundings.

The results in tables 4.1, 4.2, 4.3, 4.4, 4.5, 4.6, 4.7, 4.8, 4.9, 4.10, 4.11, 4.12, 4.13, 4.14, 4.15 were acquired for heat transfer between the cover and the surrounding air. Tables 4.1, 4.2, 4.3, 4.4, 4.5, 4.6, 4.7, 4.8, 4.9 are for casings with length 16cm, width 5.2cm and height 1cm.

<table>
<thead>
<tr>
<th>$T_s$ [$^\circ$C]</th>
<th>$Ra_L$</th>
<th>$q_{conv}$ [W]</th>
<th>$q_{rad}$ [W]</th>
<th>$q_{tot}$ [W]</th>
<th>Internal conduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>$4.65 \cdot 10^2$</td>
<td>0.171</td>
<td>0.0372</td>
<td>0.208</td>
<td>482</td>
</tr>
<tr>
<td>30</td>
<td>$9.21 \cdot 10^2$</td>
<td>0.392</td>
<td>0.0763</td>
<td>0.468</td>
<td>214</td>
</tr>
<tr>
<td>35</td>
<td>$1.37 \cdot 10^3$</td>
<td>0.637</td>
<td>0.117</td>
<td>0.755</td>
<td>133</td>
</tr>
<tr>
<td>40</td>
<td>$1.81 \cdot 10^3$</td>
<td>0.900</td>
<td>0.161</td>
<td>1.06</td>
<td>95</td>
</tr>
<tr>
<td>45</td>
<td>$2.25 \cdot 10^3$</td>
<td>1.18</td>
<td>0.206</td>
<td>1.38</td>
<td>73</td>
</tr>
<tr>
<td>50</td>
<td>$2.67 \cdot 10^3$</td>
<td>1.46</td>
<td>0.253</td>
<td>1.72</td>
<td>59</td>
</tr>
<tr>
<td>55</td>
<td>$3.10 \cdot 10^3$</td>
<td>1.76</td>
<td>0.303</td>
<td>2.07</td>
<td>49</td>
</tr>
</tbody>
</table>

Tab. 4.1: Heat transfer from casing to air for oxidized aluminium. No strong direct Light-source. Casing with length 16cm, width 5.2cm and height 1cm.

<table>
<thead>
<tr>
<th>$T_s$ [$^\circ$C]</th>
<th>$Ra_L$</th>
<th>$q_{conv}$ [W]</th>
<th>$q_{rad}$ [W]</th>
<th>$q_{tot}$ [W]</th>
<th>Internal conduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>$4.65 \cdot 10^2$</td>
<td>0.171</td>
<td>0.104</td>
<td>0.275</td>
<td>365</td>
</tr>
<tr>
<td>30</td>
<td>$9.21 \cdot 10^2$</td>
<td>0.392</td>
<td>0.214</td>
<td>0.605</td>
<td>166</td>
</tr>
<tr>
<td>35</td>
<td>$1.37 \cdot 10^3$</td>
<td>0.637</td>
<td>0.329</td>
<td>0.966</td>
<td>104</td>
</tr>
<tr>
<td>40</td>
<td>$1.81 \cdot 10^3$</td>
<td>0.900</td>
<td>0.577</td>
<td>1.75</td>
<td>74</td>
</tr>
<tr>
<td>45</td>
<td>$2.25 \cdot 10^3$</td>
<td>1.18</td>
<td>0.710</td>
<td>2.17</td>
<td>46</td>
</tr>
<tr>
<td>50</td>
<td>$2.67 \cdot 10^3$</td>
<td>1.46</td>
<td>0.849</td>
<td>2.61</td>
<td>38</td>
</tr>
<tr>
<td>55</td>
<td>$3.10 \cdot 10^3$</td>
<td>1.76</td>
<td>1.04</td>
<td>3.80</td>
<td>38</td>
</tr>
</tbody>
</table>

Tab. 4.2: Heat transfer from casing to air for anodized aluminium. No strong direct Light-source. Casing with length 16cm, width 5.2cm and height 1cm.

Tables 4.6, 4.7 are for ordinary office Light $8 \frac{W}{m^2}$ on the cover.
Tables 4.8, 4.9 are for direct sun Light at strength $1100 \frac{W}{m^2}$ on half of the cover.
4. Results

Tab. 4.3: Heat transfer from casing to air for polished copper. No strong direct Lightsource. Casing with length 16cm, width 5.2cm and height 1cm.

<table>
<thead>
<tr>
<th>$T_S$ [$^\circ$C]</th>
<th>$Ra_L$</th>
<th>$q_{conv}$ [W]</th>
<th>$q_{rad}$ [W]</th>
<th>$q_{tot}$ [W]</th>
<th>Internal conduction $q_{tot}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>$4.65 \cdot 10^2$</td>
<td>0.171</td>
<td>0.00869</td>
<td>0.180</td>
<td>946</td>
</tr>
<tr>
<td>30</td>
<td>$99.21 \cdot 10^2$</td>
<td>0.392</td>
<td>0.0178</td>
<td>0.409</td>
<td>415</td>
</tr>
<tr>
<td>35</td>
<td>$1.37 \cdot 10^3$</td>
<td>0.637</td>
<td>0.0274</td>
<td>0.664</td>
<td>256</td>
</tr>
<tr>
<td>40</td>
<td>$1.81 \cdot 10^3$</td>
<td>0.900</td>
<td>0.0375</td>
<td>0.937</td>
<td>181</td>
</tr>
<tr>
<td>45</td>
<td>$2.25 \cdot 10^3$</td>
<td>1.18</td>
<td>0.0481</td>
<td>1.22</td>
<td>139</td>
</tr>
<tr>
<td>50</td>
<td>$2.67 \cdot 10^3$</td>
<td>1.46</td>
<td>0.0591</td>
<td>1.52</td>
<td>111</td>
</tr>
<tr>
<td>55</td>
<td>$3.10 \cdot 10^3$</td>
<td>1.76</td>
<td>0.0708</td>
<td>1.83</td>
<td>93</td>
</tr>
</tbody>
</table>

Tab. 4.4: Heat transfer from casing to air for oxidized copper. No strong direct Lightsource. Casing with length 16cm, width 5.2cm and height 1cm.

<table>
<thead>
<tr>
<th>$T_S$ [$^\circ$C]</th>
<th>$Ra_L$</th>
<th>$q_{conv}$ [W]</th>
<th>$q_{rad}$ [W]</th>
<th>$q_{tot}$ [W]</th>
<th>Internal conduction $q_{tot}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>$4.65 \cdot 10^2$</td>
<td>0.171</td>
<td>0.108</td>
<td>0.279</td>
<td>610</td>
</tr>
<tr>
<td>30</td>
<td>$9.21 \cdot 10^2$</td>
<td>0.392</td>
<td>0.221</td>
<td>0.613</td>
<td>277</td>
</tr>
<tr>
<td>35</td>
<td>$1.37 \cdot 10^3$</td>
<td>0.637</td>
<td>0.341</td>
<td>0.978</td>
<td>174</td>
</tr>
<tr>
<td>40</td>
<td>$1.81 \cdot 10^3$</td>
<td>0.900</td>
<td>0.466</td>
<td>1.365</td>
<td>125</td>
</tr>
<tr>
<td>45</td>
<td>$2.25 \cdot 10^3$</td>
<td>1.18</td>
<td>0.597</td>
<td>1.77</td>
<td>96</td>
</tr>
<tr>
<td>50</td>
<td>$2.67 \cdot 10^3$</td>
<td>1.46</td>
<td>0.735</td>
<td>2.20</td>
<td>77</td>
</tr>
<tr>
<td>55</td>
<td>$3.10 \cdot 10^3$</td>
<td>1.76</td>
<td>0.879</td>
<td>2.64</td>
<td>64</td>
</tr>
</tbody>
</table>

Tab. 4.5: Heat transfer from casing to air for copper painted black. No strong direct Lightsource. Casing with length 16cm, width 5.2cm and height 1cm.

<table>
<thead>
<tr>
<th>$T_S$ [$^\circ$C]</th>
<th>$Ra_L$</th>
<th>$q_{conv}$ [W]</th>
<th>$q_{rad}$ [W]</th>
<th>$q_{tot}$ [W]</th>
<th>Internal conduction $q_{tot}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>$4.65 \cdot 10^2$</td>
<td>0.171</td>
<td>0.122</td>
<td>0.293</td>
<td>581</td>
</tr>
<tr>
<td>30</td>
<td>$9.21 \cdot 10^2$</td>
<td>0.392</td>
<td>0.249</td>
<td>0.641</td>
<td>265</td>
</tr>
<tr>
<td>35</td>
<td>$1.37 \cdot 10^3$</td>
<td>0.637</td>
<td>0.384</td>
<td>1.02</td>
<td>167</td>
</tr>
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<td>40</td>
<td>$1.81 \cdot 10^3$</td>
<td>0.900</td>
<td>0.525</td>
<td>1.42</td>
<td>119</td>
</tr>
<tr>
<td>45</td>
<td>$2.25 \cdot 10^3$</td>
<td>1.18</td>
<td>0.673</td>
<td>1.85</td>
<td>92</td>
</tr>
<tr>
<td>50</td>
<td>$2.67 \cdot 10^3$</td>
<td>1.46</td>
<td>0.828</td>
<td>2.29</td>
<td>74</td>
</tr>
<tr>
<td>55</td>
<td>$3.10 \cdot 10^3$</td>
<td>1.76</td>
<td>0.990</td>
<td>2.75</td>
<td>62</td>
</tr>
</tbody>
</table>
4. Results

<table>
<thead>
<tr>
<th>$T_S , [^\circ C]$</th>
<th>$Ra_L$</th>
<th>$q_{conv} , [W]$</th>
<th>$q_{rad} , [W]$</th>
<th>$q_{tot} , [W]$</th>
<th>$\text{Internal conduction}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
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<td>0.171</td>
<td>0.0784</td>
<td>0.249</td>
<td>681</td>
</tr>
<tr>
<td>30</td>
<td>$9.21 \cdot 10^2$</td>
<td>0.392</td>
<td>0.192</td>
<td>0.584</td>
<td>291</td>
</tr>
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<td>35</td>
<td>$1.37 \cdot 10^3$</td>
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<td>0.311</td>
<td>0.948</td>
<td>179</td>
</tr>
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<td>0.900</td>
<td>0.436</td>
<td>1.34</td>
<td>127</td>
</tr>
<tr>
<td>45</td>
<td>$2.25 \cdot 10^3$</td>
<td>1.18</td>
<td>0.568</td>
<td>1.74</td>
<td>97</td>
</tr>
<tr>
<td>50</td>
<td>$2.67 \cdot 10^3$</td>
<td>1.46</td>
<td>0.706</td>
<td>2.17</td>
<td>78</td>
</tr>
<tr>
<td>55</td>
<td>$3.10 \cdot 10^3$</td>
<td>1.76</td>
<td>0.850</td>
<td>2.61</td>
<td>65</td>
</tr>
</tbody>
</table>

Tab. 4.6: Heat transfer from casing to air for oxidized copper, normal office Lighting. Casing with length 16cm, width 5.2cm and height 1cm.

<table>
<thead>
<tr>
<th>$T_S , [^\circ C]$</th>
<th>$Ra_L$</th>
<th>$q_{conv} , [W]$</th>
<th>$q_{rad} , [W]$</th>
<th>$q_{tot} , [W]$</th>
<th>$\text{Internal conduction}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>$4.65 \cdot 10^2$</td>
<td>0.171</td>
<td>0.0883</td>
<td>0.259</td>
<td>656</td>
</tr>
<tr>
<td>30</td>
<td>$9.21 \cdot 10^2$</td>
<td>0.392</td>
<td>0.216</td>
<td>0.608</td>
<td>280</td>
</tr>
<tr>
<td>35</td>
<td>$1.37 \cdot 10^3$</td>
<td>0.637</td>
<td>0.351</td>
<td>0.986</td>
<td>172</td>
</tr>
<tr>
<td>40</td>
<td>$1.81 \cdot 10^3$</td>
<td>0.900</td>
<td>0.492</td>
<td>1.39</td>
<td>122</td>
</tr>
<tr>
<td>45</td>
<td>$2.25 \cdot 10^3$</td>
<td>1.18</td>
<td>0.640</td>
<td>1.82</td>
<td>94</td>
</tr>
<tr>
<td>50</td>
<td>$2.67 \cdot 10^3$</td>
<td>1.46</td>
<td>0.795</td>
<td>2.26</td>
<td>75</td>
</tr>
<tr>
<td>55</td>
<td>$3.10 \cdot 10^3$</td>
<td>1.76</td>
<td>0.957</td>
<td>2.72</td>
<td>63</td>
</tr>
</tbody>
</table>

Tab. 4.7: Heat transfer from casing to air for copper painted black, normal office Lighting. Casing with length 16cm, width 5.2cm and height 1cm.

<table>
<thead>
<tr>
<th>$T_S , [^\circ C]$</th>
<th>$Ra_L$</th>
<th>$q_{conv} , [W]$</th>
<th>$q_{rad} , [W]$</th>
<th>$q_{tot} , [W]$</th>
<th>$\text{Internal conduction}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>$4.65 \cdot 10^2$</td>
<td>0.171</td>
<td>-1.96</td>
<td>-1.79</td>
<td>-95</td>
</tr>
<tr>
<td>30</td>
<td>$9.21 \cdot 10^2$</td>
<td>0.392</td>
<td>-1.84</td>
<td>-1.45</td>
<td>-117</td>
</tr>
<tr>
<td>35</td>
<td>$1.37 \cdot 10^3$</td>
<td>0.637</td>
<td>-1.72</td>
<td>-1.09</td>
<td>-156</td>
</tr>
<tr>
<td>40</td>
<td>$1.81 \cdot 10^3$</td>
<td>0.900</td>
<td>-1.60</td>
<td>-0.70</td>
<td>-243</td>
</tr>
<tr>
<td>45</td>
<td>$2.25 \cdot 10^3$</td>
<td>1.18</td>
<td>-1.47</td>
<td>-0.292</td>
<td>-582</td>
</tr>
<tr>
<td>50</td>
<td>$2.67 \cdot 10^3$</td>
<td>1.46</td>
<td>-1.33</td>
<td>0.133</td>
<td>1276</td>
</tr>
<tr>
<td>55</td>
<td>$3.10 \cdot 10^3$</td>
<td>1.76</td>
<td>-1.19</td>
<td>0.576</td>
<td>295</td>
</tr>
</tbody>
</table>

Tab. 4.8: Heat transfer from casing to air for oxidized copper, under direct sunLight. Casing with length 16cm, width 5.2cm and height 1cm.
4. Results

Tab. 4.9: Heat transfer from casing to air for copper painted black, under direct sunlight. Casing with length 16 cm, width 5.2 cm and height 1 cm.

<table>
<thead>
<tr>
<th>$T_S$ [$^\circ$C]</th>
<th>$Ra_L$</th>
<th>$q_{conv}$ [W]</th>
<th>$q_{rad}$ [W]</th>
<th>$q_{tot}$ [W]</th>
<th>Internal conduction $q_{tot}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>$4.65 \cdot 10^2$</td>
<td>0.171</td>
<td>-2.21</td>
<td>-2.03</td>
<td>-84</td>
</tr>
<tr>
<td>30</td>
<td>$9.21 \cdot 10^2$</td>
<td>0.392</td>
<td>-2.08</td>
<td>-1.69</td>
<td>-101</td>
</tr>
<tr>
<td>35</td>
<td>$1.37 \cdot 10^3$</td>
<td>0.637</td>
<td>-1.94</td>
<td>-1.31</td>
<td>-130</td>
</tr>
<tr>
<td>40</td>
<td>$1.81 \cdot 10^3$</td>
<td>0.900</td>
<td>-1.80</td>
<td>-0.903</td>
<td>-188</td>
</tr>
<tr>
<td>45</td>
<td>$2.25 \cdot 10^3$</td>
<td>1.18</td>
<td>-1.65</td>
<td>-0.478</td>
<td>-356</td>
</tr>
<tr>
<td>50</td>
<td>$2.67 \cdot 10^3$</td>
<td>1.46</td>
<td>-1.50</td>
<td>-0.035</td>
<td>-4852</td>
</tr>
<tr>
<td>55</td>
<td>$3.10 \cdot 10^3$</td>
<td>1.76</td>
<td>-1.34</td>
<td>0.426</td>
<td>399</td>
</tr>
</tbody>
</table>
From now on all the tables will be for copper oxide or copper painted black under office Lighting. Tables 4.10, 4.11 are for casings with length 16cm, width 5.2cm and height 2cm.

<table>
<thead>
<tr>
<th>$T_S$ [°C]</th>
<th>$Ra_L$</th>
<th>$q_{conv}$ [W]</th>
<th>$q_{rad}$ [W]</th>
<th>$q_{tot}$ [W]</th>
<th>Internal conduction $q_{tot}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>3.72 · 10³</td>
<td>0.262</td>
<td>0.157</td>
<td>0.417</td>
<td>203</td>
</tr>
<tr>
<td>30</td>
<td>7.37 · 10³</td>
<td>0.607</td>
<td>0.384</td>
<td>0.991</td>
<td>86</td>
</tr>
<tr>
<td>35</td>
<td>1.10 · 10⁴</td>
<td>0.994</td>
<td>0.622</td>
<td>1.62</td>
<td>53</td>
</tr>
<tr>
<td>40</td>
<td>1.45 · 10⁴</td>
<td>1.41</td>
<td>0.873</td>
<td>2.28</td>
<td>37</td>
</tr>
<tr>
<td>45</td>
<td>1.80 · 10⁴</td>
<td>1.85</td>
<td>1.14</td>
<td>2.98</td>
<td>28</td>
</tr>
<tr>
<td>50</td>
<td>2.14 · 10⁴</td>
<td>2.31</td>
<td>1.41</td>
<td>3.72</td>
<td>23</td>
</tr>
<tr>
<td>55</td>
<td>2.48 · 10⁴</td>
<td>2.78</td>
<td>1.70</td>
<td>4.48</td>
<td>19</td>
</tr>
</tbody>
</table>

Tab. 4.10: Heat transfer from casing to air for oxidized copper, normal office Lighting. Casing with length 16cm, width 5.2cm and height 2cm.

<table>
<thead>
<tr>
<th>$T_S$ [°C]</th>
<th>$Ra_L$</th>
<th>$q_{conv}$ [W]</th>
<th>$q_{rad}$ [W]</th>
<th>$q_{tot}$ [W]</th>
<th>Internal conduction $q_{tot}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>3.72 · 10³</td>
<td>0.262</td>
<td>0.177</td>
<td>0.438</td>
<td>194</td>
</tr>
<tr>
<td>30</td>
<td>7.37 · 10³</td>
<td>0.607</td>
<td>0.432</td>
<td>1.04</td>
<td>82</td>
</tr>
<tr>
<td>35</td>
<td>1.10 · 10⁴</td>
<td>0.994</td>
<td>0.701</td>
<td>1.69</td>
<td>50</td>
</tr>
<tr>
<td>40</td>
<td>1.45 · 10⁴</td>
<td>1.41</td>
<td>0.983</td>
<td>2.39</td>
<td>36</td>
</tr>
<tr>
<td>45</td>
<td>1.80 · 10⁴</td>
<td>1.85</td>
<td>1.28</td>
<td>3.13</td>
<td>27</td>
</tr>
<tr>
<td>50</td>
<td>2.14 · 10⁴</td>
<td>2.31</td>
<td>1.59</td>
<td>3.90</td>
<td>22</td>
</tr>
<tr>
<td>55</td>
<td>2.48 · 10⁴</td>
<td>2.78</td>
<td>1.91</td>
<td>4.70</td>
<td>18</td>
</tr>
</tbody>
</table>

Tab. 4.11: Heat transfer from casing to air for copper painted black, normal office Lighting. Casing with length 16cm, width 5.2cm and height 2cm.
Tables 4.12, 4.13 are for casings with length 16cm, width 5.2cm and height 4cm.

<table>
<thead>
<tr>
<th>$T_S$ [$^\circ$C]</th>
<th>$Ra_L$</th>
<th>$q_{conv}$ [W]</th>
<th>$q_{rad}$ [W]</th>
<th>$q_{tot}$ [W]</th>
<th>Internal conduction $\frac{q_{tot}}{q_{tot}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>2.97 $\cdot 10^4$</td>
<td>0.414</td>
<td>0.314</td>
<td>0.728</td>
<td>58</td>
</tr>
<tr>
<td>30</td>
<td>5.89 $\cdot 10^4$</td>
<td>0.967</td>
<td>0.768</td>
<td>1.74</td>
<td>24</td>
</tr>
<tr>
<td>35</td>
<td>8.78 $\cdot 10^4$</td>
<td>1.59</td>
<td>1.24</td>
<td>2.84</td>
<td>15</td>
</tr>
<tr>
<td>40</td>
<td>1.16 $\cdot 10^5$</td>
<td>2.27</td>
<td>1.75</td>
<td>4.01</td>
<td>11</td>
</tr>
<tr>
<td>45</td>
<td>1.44 $\cdot 10^5$</td>
<td>2.98</td>
<td>2.27</td>
<td>5.25</td>
<td>8.1</td>
</tr>
<tr>
<td>50</td>
<td>1.71 $\cdot 10^5$</td>
<td>3.72</td>
<td>2.82</td>
<td>6.55</td>
<td>6.4</td>
</tr>
<tr>
<td>55</td>
<td>1.98 $\cdot 10^5$</td>
<td>4.50</td>
<td>3.40</td>
<td>7.90</td>
<td>5.4</td>
</tr>
</tbody>
</table>

Tab. 4.12: Heat transfer from casing to air for oxidized copper, normal office Lighting. Casing with length 16cm, width 5.2cm and height 4cm.

<table>
<thead>
<tr>
<th>$T_S$ [$^\circ$C]</th>
<th>$Ra_L$</th>
<th>$q_{conv}$ [W]</th>
<th>$q_{rad}$ [W]</th>
<th>$q_{tot}$ [W]</th>
<th>Internal conduction $\frac{q_{tot}}{q_{tot}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>2.97 $\cdot 10^4$</td>
<td>0.414</td>
<td>0.353</td>
<td>0.768</td>
<td>55</td>
</tr>
<tr>
<td>30</td>
<td>5.89 $\cdot 10^4$</td>
<td>0.967</td>
<td>0.865</td>
<td>1.83</td>
<td>23</td>
</tr>
<tr>
<td>35</td>
<td>8.78 $\cdot 10^4$</td>
<td>1.59</td>
<td>1.40</td>
<td>3.00</td>
<td>14</td>
</tr>
<tr>
<td>40</td>
<td>1.16 $\cdot 10^5$</td>
<td>2.27</td>
<td>1.97</td>
<td>4.23</td>
<td>10</td>
</tr>
<tr>
<td>45</td>
<td>1.44 $\cdot 10^5$</td>
<td>2.98</td>
<td>2.56</td>
<td>5.54</td>
<td>7.7</td>
</tr>
<tr>
<td>50</td>
<td>1.71 $\cdot 10^5$</td>
<td>3.72</td>
<td>3.18</td>
<td>6.90</td>
<td>6.2</td>
</tr>
<tr>
<td>55</td>
<td>1.98 $\cdot 10^5$</td>
<td>4.50</td>
<td>3.83</td>
<td>8.33</td>
<td>5.1</td>
</tr>
</tbody>
</table>

Tab. 4.13: Heat transfer from casing to air for copper painted black, normal office Lighting. Casing with length 16cm, width 5.2cm and height 4cm.
Tables 4.14, 4.15 are for casings with length 16cm, width 5.2cm and height 8cm.

<table>
<thead>
<tr>
<th>$T_S \ [\text{C}^\circ]$</th>
<th>$Ra_L$</th>
<th>$q_{conv} \ [\text{W}]$</th>
<th>$q_{rad} \ [\text{W}]$</th>
<th>$q_{tot} \ [\text{W}]$</th>
<th>Internal conduction $\frac{q_{tot}}{q_{rad}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>$2.81 \times 10^5$</td>
<td>0.671</td>
<td>0.627</td>
<td>1.30</td>
<td>16</td>
</tr>
<tr>
<td>30</td>
<td>$4.71 \times 10^5$</td>
<td>1.58</td>
<td>1.54</td>
<td>3.11</td>
<td>6.8</td>
</tr>
<tr>
<td>35</td>
<td>$7.02 \times 10^5$</td>
<td>2.60</td>
<td>2.49</td>
<td>5.09</td>
<td>4.2</td>
</tr>
<tr>
<td>40</td>
<td>$9.27 \times 10^6$</td>
<td>3.71</td>
<td>3.49</td>
<td>7.20</td>
<td>2.9</td>
</tr>
<tr>
<td>45</td>
<td>$1.15 \times 10^7$</td>
<td>4.88</td>
<td>4.54</td>
<td>9.42</td>
<td>2.3</td>
</tr>
<tr>
<td>50</td>
<td>$1.37 \times 10^7$</td>
<td>6.11</td>
<td>5.64</td>
<td>11.8</td>
<td>1.8</td>
</tr>
<tr>
<td>55</td>
<td>$1.59 \times 10^7$</td>
<td>7.38</td>
<td>6.80</td>
<td>14.2</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Tab. 4.14: Heat transfer from casing to air for oxidized copper, normal office Lighting. Casing with length 16cm, width 5.2cm and height 8cm.

<table>
<thead>
<tr>
<th>$T_S \ [\text{C}^\circ]$</th>
<th>$Ra_L$</th>
<th>$q_{conv} \ [\text{W}]$</th>
<th>$q_{rad} \ [\text{W}]$</th>
<th>$q_{tot} \ [\text{W}]$</th>
<th>Internal conduction $\frac{q_{tot}}{q_{rad}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>$2.81 \times 10^5$</td>
<td>0.671</td>
<td>0.701</td>
<td>1.38</td>
<td>15</td>
</tr>
<tr>
<td>30</td>
<td>$4.71 \times 10^5$</td>
<td>1.58</td>
<td>1.73</td>
<td>3.31</td>
<td>6.4</td>
</tr>
<tr>
<td>35</td>
<td>$7.02 \times 10^5$</td>
<td>2.60</td>
<td>2.80</td>
<td>5.41</td>
<td>3.9</td>
</tr>
<tr>
<td>40</td>
<td>$9.27 \times 10^6$</td>
<td>3.71</td>
<td>3.93</td>
<td>7.64</td>
<td>2.8</td>
</tr>
<tr>
<td>45</td>
<td>$1.15 \times 10^7$</td>
<td>4.88</td>
<td>5.12</td>
<td>10.0</td>
<td>2.1</td>
</tr>
<tr>
<td>50</td>
<td>$1.37 \times 10^7$</td>
<td>6.11</td>
<td>6.36</td>
<td>12.5</td>
<td>1.7</td>
</tr>
<tr>
<td>55</td>
<td>$1.59 \times 10^7$</td>
<td>7.38</td>
<td>7.66</td>
<td>15.0</td>
<td>1.4</td>
</tr>
</tbody>
</table>

Tab. 4.15: Heat transfer from casing to air for copper painted black, normal office Lighting. Casing with length 16cm, width 5.2cm and height 8cm.
The highest amount of heat transferred to the surroundings was 15.0W for a casing of height 8cm made out of copper painted black at surface temperature of $55^\circ C$. This result however has a \( \text{Internal conduction} \), \( q_{\text{tot}} \) of only 1.4, it is therefore unlikely that the surface temperature is constant. It is clear that the bigger the cover is the more heat is transferred. It is also clear that the material has a big impact on the heat transfer, especially for radiation.

4.0.2 Internal heat transfer.

This section presents results for simulations of heat transfer internally in the transducer. In all simulations the surface temperature of the transducer is held at constant and there is an evenly distributed heat generation in the transducer stack. The parameters for heat generation and surface temperature are taken from table 4.13. All simulations are therefore of a system with a copper casing if 4cm height painted black. The casing temperatures range from $40^\circ C$ to $55^\circ C$. The simulations show the impact of different thicknesses of thermal paste. They show the distribution of heat in the transducer stack. Figure 4.9 show the added heat transfer due to an extra copper layer at the bottom of the transducer stack.

Figures 4.1 and 4.2 show the static temperature distribution for a casing boundary of \( T_S = 40^\circ C \), and an evenly distributed heat generation of 4.23W. Both show the transducer in the height-width plane. The difference between the two simulations is that 4.1 has a thermal paste layer if 0.1 mm, and 4.2 has a thermal paste layer if 0.1 mm. For both simulations the warmest point is found at the center of the bottom of the transducer stack. The highest temperature for 4.1 is $43.25^\circ C$, 3.25 $^\circ C$ above \( T_S \), and for 4.2 it is $50.25^\circ C$, 10.25 $^\circ C$ above \( T_S \).
Fig. 4.1: The static temperature distribution for a casing boundary of 40°C, and an evenly distributed heat generation of 4.23W. The thermal paste is 0.1mm thick. The maximum temperature is 43.25°C.
Fig. 4.2: The static temperature distribution for a casing boundary of 40°C, and an evenly distributed heat generation of 4.23W. The thermal paste is 1mm thick. The maximum temperature is 50.25°C.
Figures 4.3 and 4.4 show the static temperature distribution for a casing boundary of $T_S = 45^\circ C$, and an evenly distributed heat generation of 5.54W. Both show the transducer in the height-width plane. The difference between the two simulations is that 4.3 has a thermal paste layer of 0.1 mm, and 4.4 has a thermal paste layer of 0.1 mm. For both simulations the warmest point is found at the center of the bottom of the transducer stack. The highest temperature for 4.3 is 49.35$^\circ C$, 4.45 $^\circ C$ above $T_S$, and for 4.4 it is 58.45$^\circ C$, 13.45 $^\circ C$ above $T_S$.

![Surface Temperature (K)](image)

Fig. 4.3: The static temperature distribution for a casing boundary of 45$^\circ C$, and an evenly distributed heat generation of 5.54W. The thermal paste is 0.1mm thick. The maximum temperature is 49.35$^\circ C$. 

Fig. 4.4: The static temperature distribution for a casing boundary of $45^\circ C$, and an evenly distributed heat generation of 5.54W. The thermal paste is 1mm thick. The maximum temperature is 58.45$^\circ C$. 
Figures 4.5 and 4.6 show the static temperature distribution for a casing boundary of \( T_S = 50^\circ C \), and an evenly distributed heat generation of 6.90W. Both show the transducer in the hight-width plane. The difference between the two simulations is that 4.5 has a thermal paste layer if 0.1 mm, and 4.6 has a thermal paste layer if 0.1 mm. For both simulations the warmest point is found at the center of the bottom of the transducer stack. The highest temperature for 4.5 is 55.45\( ^\circ C \), 5.45 \( ^\circ C \) above \( T_S \), and for 4.6 it is 66.75\( ^\circ C \), 16.75 \( ^\circ C \) above \( T_S \).

Fig. 4.5: The static temperature distribution for a casing boundary of 50\( ^\circ C \), and an evenly distributed heat generation of 6.90W. The thermal paste is 0.1mm thick. The maximum temperature is 55.45\( ^\circ C \).
Fig. 4.6: The static temperature distribution for a casing boundary of 50°C, and an evenly distributed heat generation of 6.90W. The thermal paste is 1mm thick. The maximum temperature is 66.75°C.
Figures 4.7 and 4.8 show the static temperature distribution for a casing boundary of $T_S = 55^\circ C$, and an evenly distributed heat generation of 8.33W. Both show the transducer in the height-width plane. The difference between the two simulations is that 4.7 has a thermal paste layer of 0.1 mm, and 4.8 has a thermal paste layer of 0.1 mm. For both simulations the warmest point is found at the center of the bottom of the transducer stack. The highest temperature for 4.7 is $61.55^\circ C$, 6.55$^\circ C$ above $T_S$, and for 4.8 it is $75.25^\circ C$, 20.25$^\circ C$ above $T_S$.

**Fig. 4.7:** The static temperature distribution for a casing boundary of $55^\circ C$, and an evenly distributed heat generation of 8.33W. The thermal paste is 0.1mm thick. The maximum temperature is $61.55^\circ C$.
Fig. 4.8: The static temperature distribution for a casing boundary of 55°C, and an evenly distributed heat generation of 8.33W. The thermal paste is 1mm thick. The maximum temperature is 75.25°C.
In figures 4.1, 4.2, 4.3, 4.4, 4.5, 4.6, 4.7, 4.8 the hottest point is at the bottom of the transducer stack. For this reason a simulation was run to determine the effect of an added heat conducting layer at the bottom of the transducer. This simulation is shown in figure 4.9. The material of this new layer is copper, the thickness of this layer is 2mm. The casing temperature is $T_S = 55^\circ C$, the evenly distributed heat generation is 8.33W, the thickness of the thermal paste is 1mm. Except for the new conductive layer, this is the same simulation as shown in figure 4.8. Figure TempDist9 show that with the added conductive layer the hottest point is at the top of the transducer and has a temperature of 56.15$^\circ C$, only 1.15 $^\circ C$ above the temperature of the casing.

Fig. 4.9: The static temperature distribution for a casing boundary of 55$^\circ C$, and an evenly distributed heat generation of 8.33W. The thermal paste is 1mm thick. In addition to the cooling from the core, there is also cooling from the added bar under the transducer stack. This bar is assumed to be of copper. The maximum temperature is 56.15$^\circ C$. 
Figure 4.16 shows the maximum temperatures from figures 4.1, 4.2, 4.3, 4.4, 4.5, 4.6, 4.7, 4.8. It displays the simulation using the same casing temperature and heat generation next to each other. The difference in the maximal temperatures is also shown.

<table>
<thead>
<tr>
<th>$T_S$</th>
<th>Maximal temperature, thermal paste: 0.1mm</th>
<th>Maximal temperature, thermal paste: 1mm</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>40 $^\circ C$</td>
<td>43.25$^\circ C$</td>
<td>50.25$^\circ C$</td>
<td></td>
</tr>
<tr>
<td>45 $^\circ C$</td>
<td>49.35$^\circ C$</td>
<td>58.45$^\circ C$</td>
<td></td>
</tr>
<tr>
<td>50 $^\circ C$</td>
<td>55.45$^\circ C$</td>
<td>66.75$^\circ C$</td>
<td></td>
</tr>
<tr>
<td>55 $^\circ C$</td>
<td>61.55$^\circ C$</td>
<td>75.25$^\circ C$</td>
<td></td>
</tr>
</tbody>
</table>

Tab. 4.16: Maximum temperatures for simulations made with varying thickness of thermal paste.
5. DISCUSSION

5.1 Casing to surroundings

5.1.1 Lighting

As the values presented in tables 4.4, 4.5, 4.6, 4.7, 4.8 and 4.9 it is clear that the Light on the transducer can have a large effect. Tables 4.8 and 4.9 show that under strong direct sunlight the heat transfer might actually be negative. This means that the casing leads heat into the transducer stack instead of out of the transducer stack. Comparing tables 4.7 and 4.5 show that the difference from no incident Light on the transducer, and normal office Lighting is not big. For a surface temperature of 55°C the heat flux is 2.72W under normal office Lighting and 2.75W without lighting.

If a cooling mechanism as proposed in this thesis is to be used it is necessary to avoid strong Light on the transducer. It is especially important to block strong light that comes normal to the casing. As shown in equation 3.1 Light hitting the surface at a steep angle results in much less heating than Light hitting the surface at close to a normal angle. In strong Lighting conditions it is therefore be possible to greatly reduce the heating effects caused by the strong light, by placing a shield blocking the strongest most direct Light on the casing.

5.1.2 Material

Tables 4.1, 4.2, 4.3, 4.4 and 4.5 show the cooling effect of different casing materials with different surface coatings. The results show that the material properties of the casing does not influence the convective heat transfer. Section 2.6.2 also show this as the material properties of the surface are not part of the equations. Instead the convection only depends on the temperature of the surface, the area of the surface, the material properties of the fluid surrounding the surface and the movement of this
fluid. This is however a simplification. In the calculation for convection it is assumed that the temperature of the surface is constant. If the conductivity off the surface material is low compared to the heat flow from the surface to the environment this assumption will be false. Parts of the surface in this case do not participate much in the heat transfer from surface to surroundings since these parts will be cooled down quite a bit. The end result of a thermal conductive material is to limit the amount of heat that can be transferred to the surroundings. This is measured by the last column in the table.

Comparing the tables 4.1, 4.2, 4.3, 4.4 and 4.5 with the material values given in table 3.1 show the importance of a high emissivity in order to increase radiative heat transfer. For a surface of 55°C polished copper has a radiative heat transfer of 0.708W, whereas copper painted black has a radiative heat transfer of 0.990. This is as expected from equation 2.29.

There is a very clear increase of heat transfer with surface temperature. Equation 2.29 gives $q_{rad} \propto T_S^4$. Equation 2.19 makes it seem that $q_{conv} \propto T_S$, however it also gives $q_{conv} \propto h$. Equation 2.20 gives that $h \propto N u_L$, equation 2.21 gives $N u_L \propto Ra_L^{1/2}$, and equation 2.23 gives $Ra_L \propto T_S$. The end result is that $q_{conv} \propto T_S \cdot T_S^{1/2} = T_S^3$.

It is clear from sections 5.1.1 and 5.1.2 that the ideal casing material has high conductivity and high emissivity. From the materials in table 3.1 copper painted black is the best material. This is the reason why this material is the only one used in subsequent tables. The second best material is oxidized copper, due to its high conductivity. That copper was the best material used here does not mean that there aren’t better material available.

5.1.3 Size of conductive casing

The heat transfer varies a lot with the size of the conductive casing area. Tables 4.13 and 4.14 show that with an increase in the height of the conductive area from 4cm to 8 cm the heat transferred, for $T_S = 55C\circ$ goes from 8.33W to 15.0W. In general when the height of this area is doubled the heat transferred is close to doubled. For the case of a height of 8cm the ratio of internal conduction to heat transferred to the surroundings is only 1.4. This shows that the assumption of constant surface tem-
temperature is unlikely to be true. The heat transferred to the surroundings is therefore likely to be less than 15W. 8cm height is quite a large area for a medical transducer. Especially since this area should ideally not be touched. The conductive area of 4cm is what has been used for the internal simulations using COMSOL.

5.2 Conduction inside the transducer

The simulations 4.1, 4.2, 4.3, 4.4, 4.5, 4.6, 4.7, 4.8 show that there is a large increase in the maximum temperature of the transducer for the simulation with thermal paste of 1mm compared to the ones with thermal paste 0.1mm. This is an important result to be aware of, since it means one have to be very precise when applying the cooling paste. Variations in the thickness of the cooling paste will mean that one can not be certain of the internal temperature of the transducer just based on the surface temperature of the casing and the heat generation. Due to this uncertainty it would be useful to have a temperature sensor in the transducer to avoid overheating. The bottom of the transducer stack seems a good place to place such a sensor.

In the simulations 4.1, 4.2, 4.3, 4.4, 4.5, 4.6, 4.7, 4.8 the warmest point is always the bottom of the first piezoelectric layer. This is due to this point being furthest away from the copper core. A simulation was done to test the result of adding a copper layer beneath the transducer stack, this simulation is shown in 4.8. This simulation was done with $T_S = 55^\circ C$, the heat generation was 8.33W and the thermal paste layer was 1mm. The maximum temperature in the transducer stack was 56.15$^\circ C$ only 1.15$^\circ C$ higher than $T_S$. Except for the new copper layer the simulation parameters are exactly the same as in 4.8 where the maximum temperature was 75.25$^\circ C$, 20.25 $^\circ C$ above $T_S$. The temperature gradient with the new conductive layer is only 1.15/20.25 = 5.7

Improvements to the heat transfer between the casing and surroundings, for example by using external fans, or increasing the surface area, will be more effective at lowering the maximum temperature in the transducer stack with the new conductive layer than without. Without the new conductive layer the transfer of heat from the transducer stack to the casing is a clear bottleneck. With the new conductive layer this bottleneck is removed.
The new conductive layer could lead to some problems. The new layer is next to the electrode controlling the LF2 layer. It is therefore important that there either is an electrical insulator between the electrical insulator and the new layer. Another possible solution is to use a material that is thermally conductive but not electrically conductive. An example of such a material is Aluminum Nitride with a thermal conductivity of 140-177 \( \frac{W}{m\cdot K} \). This layer also takes part of the place normally reserved for the transducer backing. If not properly designed it could therefore impact the ultrasound capabilities of the transducer stack. The new layer have to be able to work as a backing.
This thesis reviewed the power requirements for high Ultrasound Radiation Force apparatus, and the heat generation caused by this.

This thesis proposed a way to cool an ultrasound transducer. The cooling method is to have an area on the casing of the transducer that has a high thermal conductivity, and a high emissivity. This layer is meant to passively transfer heat from the casing to the surroundings. This area on the casing is connected to the transducer stack through a copper layer that is separated for the stack by thermal paste. This cooling method was simulated using a two-step method. The first step was to determine the amount of heat transferred from the highly conductive area on the casing to the surrounding, assuming constant surface temperature. The second step was to simulate the heat transfer from the transducer stack to the casing assuming evenly distributed constant heat generation in the transducer stack.

The conductive casing that was simulated was a prism surrounding the sides of the transducer stack, but not the top and bottom. This prism was off height 4cm. It was found that the casing material should have as high conductivity and emissivity as possible in order to increase heat transfer. The thickness of thermal paste was found to have a large impact on the heat transfer from the transducer stack to the conductive casing. This necessitates care when applying thermal paste to make sure it has the desired thickness.

The warmest area of the transducer would, based on the simulations, be close to the backing. In order to increase heat transfer out of the transducer, it was therefore proposed to add a thermal conductive layer just above the backing off the transducer. This method was shown in simulations to be effective in increasing heat transfer. With this added thermal conductive layer the transducer would stabilize around $56°C$ for a heat generation of 8.3W evenly distributed throughout the transducer.
6.1 Possibilities for further studies

Investigate the thickness of thermal paste required. The casing is not in direct contact with the transducer stack, but separated by a layer of thermal paste. This separation is to limit the casings effect on ultrasound generation, by increasing the rigidity of the transducer stack. It could be smart to check how thick the thermal paste would have to be in order for the transducer stack to be sufficiently isolated.

Investigate the possibility of using the new conductive layer as backing. The new conductive layer is in the position where the backing would usually be. It significantly increases heat transfer. However if it where to reduce the transducers applicability for ultrasound this method could not be used. It would therefore be useful to investigate if such a layer could fulfill the roles of a backing material as well as transfer heat.

Investigate the amount of heat generated. This thesis calculates the stable temperature that would be reached for a certain boundary temperature and heat generation. It does however not calculate how much heat would actually be generated in the transducer. It would be useful to calculate the heat generated to see if the methods proposed in this thesis are adequate for the transducer.
BIBLIOGRAPHY


