Vehicle Weight in Gipps' Car-Following Model

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Abstract:

Car-following models are mathematical models, which describe the situation where vehicles drive behind each other on a single lane road section with no overtaking possibilities. The purpose of the models is to estimate how a vehicle reacts to the behavior of the vehicle ahead. A weakness in these models is that they do not take the weight of each vehicle into account. It can however be shown that a vehicle’s weight affects its driving behavior.

The purpose of this master’s thesis is to investigate the ability of Gipps’ car-following model to reproduce differences in driving behavior caused by differences in vehicles’ weight. A modified version of the model where weight is included as a parameter is to be estimated and compared to the original model and field data.

The method used was to make a Matlab script which simulates vehicle driving on a road section with Gipps’ model. The model results were compared to data collected from a point detector equipped with Weigh-in-motion technology. Time-gap distributions were chosen as the measure against which the models were compared.

In the estimated modified model the vehicles are assigned a gross weight, which is picked from an inputted weight distribution. The deceleration parameters of the vehicles are then varied according to vehicle weight.

The conclusion of the study is that the original Gipps’ model can be calibrated to produce accurate results in uncongested flow. However several vehicle types would need to be defined in order for this to hold for all vehicles. A modified version of the model where vehicle weight is included as a parameter rather than separating vehicles according to type, produced accurate time-gap distributions for all vehicles, with a lower number of input parameters than the original model.

However there are aspects of the driving behavior and other traffic situation for which the modified model is not compared. The areas of application of this modified model are also limited today because it requires detailed data on the vehicle weight distribution, which is not widely available and of poor quality.

Keywords:

1. Car-Following
2. Gipps
3. Microsimulation
4. Heavy vehicles
Preface

This report is written as a part of my master’s thesis in transportation at NTNU in the spring of 2013. The topic is vehicle weight in Gipps’ car-following model.

This topic was chosen because I worked with traffic simulation software in my specialization project in the fall of 2012, and I found it interesting to get a deeper understanding of the foundation of such tools. In addition to this, the topic enabled me to get insight in the field of research and development.

The topic is related to the research conducted by Odd André Hjelkrem, which concerns traffic flow of heavy vehicles. However the work presented in this thesis is performed by myself. I want to thank Odd for good guidance and valuable support throughout the work process. I also want to thank main supervisor Eirin Ryeng.

Trondheim, June 7, 2013

Sebastian Nerem
Abstract

Car-following models are mathematical models, which describe the situation where vehicles drive behind each other on a single lane road section with no overtaking possibilities. The purpose of the models is to estimate how a vehicle reacts to the behavior of the vehicle ahead. A weakness with these models is that they do not take the weight of each vehicle into account. This parameter is only represented indirectly through acceleration and deceleration parameters. It can however be shown that a vehicle's weight affects its driving behavior.

The purpose of this master’s thesis is to investigate the ability of Gipps’ car-following model to reproduce differences in driving behavior resulting from differences in vehicles’ weight. A modified version of Gipps’ model, where a weight parameter is incorporated, is estimated. Both models are compared to field data.

Gipps’ car-following model is a safety distance model. This model type estimates the behavior of a vehicle based upon an assumption that it wishes to keep a safe following distance to the vehicle ahead. That is if the vehicle ahead commences braking, the vehicle should be able to make a complete stop without collision.

The method used is to make a Matlab script, which simulates vehicles driving on a road section by using Gipps’ model. Primarily the simulation is run with two vehicle types: cars and heavy vehicles. After suggesting a modified version of the model, the script is updated to implement this. Time-gap is chosen as the variable of comparison. This variable influences how much space one vehicle occupies on the road, and is thus important when determining road capacity.

The data used for comparison is collected at a point detector equipped with Weigh-in-motion technology. Due to the low availability of such data, only data from uncongested traffic flow was available. The uncertainty of vehicle weights collected using this technology is also higher than desired.

The model simulates a given number of vehicles on a straight single lane road section of a given length. The simulation is run with different entry flows for a given number of replications. Data is retrieved from the model at a simulated point detector. Two different references were used for calibration: (1) Macroscopic calibration of flow-speed relationship; and (2) Microscopic calibration of the distribution of time-gaps shorter than 6 seconds.
The original model produced accurate results for the flow-speed relationship and the time-gap distribution for all vehicles. However the time-gap distributions for smaller portions of heavy vehicles, grouped according to weight, did not fit to the field distributions.

The proposed modified model is based upon eliminating the need to separate the vehicles into discrete vehicle types, but to introduce a weight parameter which for each vehicle is picked from a continuous weight distribution. The acceleration and deceleration parameters are made dependent on vehicle weight. The latter are multiplied with a weight dependent factor, which is determined by a double exponential function. Due to insufficient data, the relationship between the acceleration parameter and weight could not be determined in this study.

A better fit between the field and simulated time-gap distributions for heavy vehicles grouped according to vehicle weight into intervals of 10,000 kg was obtained by using the modified model.

The conclusion of the study is that the original Gipps’ model can be calibrated to reproduce the characteristics of uncongested flow accurately. However more than two vehicle types with different input parameters would need to be defined for the time-gaps to be modeled correctly for all vehicles. A modified version of the model where vehicle weight is included as a parameter rather than separating vehicles according to type, produced accurate time-gap distributions for all vehicles, with a lower number of input parameters than the original model.

However there are aspects of the driving behavior and other traffic situation for which the modified model is not compared. The areas of application of this modified model is also limited today because it requires detailed data on the vehicle weight distribution, which is not widely available and of poor quality.
Bilfølgemodeller er matematiske modeller, som beskriver situasjonen der kjøretøy kjører etter hverandre i ett kjørefelt uten mulighet for forbikjøring. Formålet med modellene er å estimere hvordan et kjøretøy reagerer på oppførselen til forankjørende. En svakhet ved disse modellene er at de ikke inkluderer vekten av hvert kjøretøy. Denne parameteren er bare representert indirekte gjennom aksellerasjons- og retardasjonsparametere. Det kan derimot vises at et kjøretøys vekt påvirker hvordan det oppfører seg.

Formålet med denne masteroppgaven er å undersøke evnen til Gipps’ bilfølgemodell til å gjengi forskjeller i kjøretøyoppførsel som er resultat av kjøretøyvekt. En modifisert versjon av Gipps’ modell, hvor en vektparameter er inkludert, skal estimeres. Begge modellene sammenliknes med feltdata.

Gipps’ bilfølgemodell er en sikkerhetsavstandsmodell. Denne modelltypen beregner kjøretøyoppførsel ut fra en antakelse om at et kjøretøy ønsker å holde en trygg avstand til forankjørende. Det vil si at hvis forankjørende bremser, så skal kjøretøyet kunne stoppe helt uten at det oppstår kollisjon.

Metoden som er brukt er å lage et Matlab-skript, som simulerer kjøretøy som kjører på en vegstrekning ved hjelp av Gipps’ modell. I første omgang kjøres simuleringen med to kjøretøytyper: biler og tunge kjøretøy. Etter at en modifisert versjon av modellen er forslått, oppdateres skriptet slik at denne blir implementert. Tidsluke er valgt som sammenlikningsvariabel. Denne variablene påvirker hvor mye plass hvert kjøretøy opptar på vegen, og er dermed viktig når vegkapasitet skal bestemmes.

Feltdataene som er brukt i sammenlikningen er samlet inn ved hjelp av en punktdetektor som er utstyrt med Weigh-in-motion-teknologi. På grunn av den dårlige tilgjengeligheten av slike punkter, var det ikke anledning til å få data fra en køsituasjon. I tillegg er det knyttet høy usikkerhet til data, som er samlet inn ved hjelp av denne teknologien.

Modellen simulerer et gitt antall kjøretøy på en rett vegstrekning av en gitt lengde. Simuleringen er kjørt med forskjellige inngangstrafikkmengder og et gitt antall replikasjoner er laget. Data er hentet ut fra modellen fra en simulert punktdetektor. To forskjellige referanser ble brukt til kalibrering: (1) Makroskopisk kalibrering
av forholdet mellom trafikkmengde og fart; og (2) Mikroskopisk kalibrering av fordelingen av tidsluker mindre enn 6 sekunder.

Den originale modellen produserer resultater som er godt tilpasset feltdata for forholdet mellom trafikkmengde og fart og tidslukefordelingene for alle kjøretøy. Derimot var det dårlig tilpassning for feltdataene for tidslukefordelingene til mindre andeler av de tunge kjøretøyene, gruppert etter vekt.

Den foreslåtte modifiserte modellen er basert på at at kjøretøyene ikke skal separeres etter type, men å introdusere en vektparameter som for hvert kjøretøy bestemmes ut fra en kontinuerlig vektfordeling. Akselerasjons- og retardasjonsparametrerene er gjort avhengige av kjøretøyvekt. Sistnevnte multipliseres med en vektavhengig faktor, som bestemmes av en dobbel eksponesialfunksjon. På grunn av manglende data kunne ikke forholdet mellom akselerasjon og vekt bestemmes i denne studien.

Bedre tilpasning for tidslukefordelingene for tunge kjøretøy gruppert etter vekt i 10 000 kg-intervaller var resultatet av å bruke den modifiserte modellen.

Konklusjonen av studien er at Gipps' modell kan kalibreres til å produsere godt tilpassede resultater i en ikke overbelastet trafikksituasjon. Men flere enn to kjøretøytyper med ulike inngangsparametere må defineres for at tidslukenes skal modelleres riktig for alle kjøretøy. En modifisert versjon av modellen der kjøretøyvekt er inkludert som en parameter, fremfor å gruppere kjøretøyene etter type, produserte nøyaktige tidslukefordelinger for alle kjøretøy. Med et mindre antall inngangsparametere enn i den originale modellen.

Men det er aspekter ved kjøretøyoppførselen og andre trafikksituasjoner, som modellen ikke er testet mot. Bruksområdene til modellen er dessuten begrenset i dag fordi den krever detaljerte data om vektfordelingen blant kjøretøyene. Dette er lite tilgjengelig og av lav kvalitet.
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Terms and Abbreviations

Subscripts and superscripts

0  Basis
a  Free driving situation
b  Car-following situation
cr Critical
detector Detector location
e  Entry
f  Free flow
g  Gap
j  Jam
max  Maximum
min  Minimum
n  Vehicle n, the observed vehicle
n − 1 Vehicle n − 1, the downstream vehicle
s  Section
t  Time

Parameters and coefficients

a  Acceleration, [m/s²]
b  Deceleration, [m/s²]
ˆb  Estimated deceleration, [m/s²]
c  Constant
E  Expected value
F  Cumulative probability function
h  Headway, [s] or [m]
¯h  Average headway, [s] or [m]
k  Density, [veh/km]
l  Length of vehicle, [m]
¯l  Average length of vehicle, [m]
L  Length of section, [m]
p  Parameter
q  Flow rate, [veh/h]
R²  Coefficient of determination
s  Effective length of vehicle, [m]
t  Time, [s]
T  Time period, [s]
u  Speed, [km/h] or [m/s]
¯u  Average speed, [km/h] or [m/s]
U  Desired speed, [km/h] or [m/s]
w  Gross vehicle weight, [kg]
$x$  Position, [m]
$\alpha$  Weight dependent factor
$\gamma$  Parameter used in displaced exponential distribution, [s$^{-1}$]
$\mu$  Mean
$\sigma$  Standard deviation
$\tau$  Reaction time / simulation step, [s]

**Abbreviations**

AIMSUN  Advanced Interactive Microscopic Simulator for Urban and Non-urban Networks
RMSP  Root Mean Squared Percent Error
veh  Vehicle
WIM  Weigh-in-motion
Chapter 1

Introduction

1.1 Background

The growth in automobile usage in the 20th century lead to increasing interest in models, which could describe vehicle behavior. However, as the driving task is dependent on human decision making, they are difficult to make. Much research has been conducted and several models describing different traffic situations have been made. Many of them form the basis of computer traffic simulation software, which has become an important tool for analyzing and planning infrastructure projects.

An important type of model in this context is the car-following model. This describes the situation where vehicles are driving behind each other in a single lane on the road, and estimates how a vehicle reacts to the behavior of the vehicle ahead. Several approaches have been made, one of which is Gipps’ car-following model.

A weakness in this model, and other commonly used car-following models, is that it does not take the weight of each vehicle into account. This variable is only represented indirectly through acceleration and deceleration parameters. It can however be shown, that a vehicle’s weight affects its driving behavior (Brackstone et al. 2009, Sayer et al. 2000, Cristoforo et al. 2004, Aghabayk et al. 2012). This might produce errors in the simulation results, especially when modeling traffic streams with a non-uniform weight distribution or a high proportion of heavy vehicles. In the later years, an increase in heavy vehicles on the road network has been observed. Thus increasing the importance of these vehicles' behavior being modeled correctly in traffic simulation software.
CHAPTER 1. INTRODUCTION

1.2 Purpose

The purpose of this master’s thesis is to investigate the ability of Gipps (1981)'s car-following model to reproduce differences in car-following behavior caused by vehicles’ weight, with focus on spacing between vehicles.

A modified version of Gipps’ model, where vehicle weight is included as a parameter, is to be estimated and compared to the original model. Whether this modified model is practical to use will be discussed.

1.3 Scope

The thesis is limited to investigating Gipps’ car-following model for a straight single lane road section with no inclination and no overtaking possibilities.

The model will be compared to and calibrated against field traffic data collected at a point detector equipped with Weigh-in-motion technology on a two-lane highway.

1.4 Outline

The thesis is divided into eight chapters.

In chapter two, there will be given a summary of the traffic flow theory, which is relevant for the analysis. The theory behind Gipps’ car-following model is presented, and relevant literature concerning vehicle weight and driving behavior is reviewed.

The method used is presented in chapter three. Whereas the data collection is described in chapter four; together with relevant statistics from the dataset concerning traffic flow and vehicles.

Chapter five covers the development of and results from simulation using Gipps’ original model. The modified model is presented in chapter six.

The results from the two models are compared in chapter seven and discussed in chapter eight.
Chapter 2

Theory and Literature

This chapter covers the theory behind traffic flow, car-following models and vehicle weight. Relevant literature is also reviewed.

The literature was found by performing searches in computer databases; most notably Google Scholar, Ovid and TRB. Some articles were also provided by supervisor Odd A. Hjelkrem.

2.1 Traffic flow theory

Car-following models describe the situation where vehicles drive behind each other on a road section. Before going into further detail on such models, the theory behind traffic flow and variables used in this thesis are presented. The summary is based on the monograph edited by Gartner et al. (1997).

Traffic flow is described on two different levels: microscopic and macroscopic. On the former characteristics of single vehicles are described, whereas the latter models traffic as a moving liquid.

2.1.1 Microscopic variables

Microscopic traffic flow theory describes individual vehicles. The two main variables for this purpose are each vehicle’s position and speed.

Speed, \( u \)

Two different measurements are used for the speed of a vehicle: section speed and point speed. The section speed is the time, \( T \), it takes a vehicle to cover a
distance, \( L \).

\[ u_s = \frac{L}{T} \]  \hspace{1cm} (2.1)

The second measurement is the point speed, \( u_t \), which is the instantaneous speed of a vehicle at a point on the road. It can be looked upon as a section speed where the distance is a limit value approaching zero.

**Vehicle spacing**

When focus is on more than one vehicle, the spacing between vehicles driving behind each other can be described using two different variables: headway and gap. A vehicle’s headway is defined as the distance between the front of the vehicle and the front of the vehicle ahead. The gap is the distance between the front of the vehicle and the rear of the vehicle ahead. The major difference is that the headway is dependent on the length of the vehicle ahead. Whereas the gap only describes the empty space between the two vehicles. The two variables are illustrated in Figure 2.1.

![Figure 2.1: Headway and gap between two vehicles.](image)

Both headway and gap can be given in two different units: space and time. The former yields only the physical distance, while the latter is the time it takes the vehicle to cover this distance. Hence the vehicle’s speed affects the time-spacing.

For further reference the following terms regarding vehicle spacing are used:

- **Time-gap**: Gap measured in seconds.
- **Space-gap**: Gap measured in meters.
- **Time-headway**: Headway measured in seconds.
- **Space-headway**: Headway measured in meters.

**Vehicle spacing distributions**

When multiple vehicles follow each other on a road section, they do not necessarily keep the same distance to the vehicle ahead. These differences in vehicle spacing
in a traffic stream can be modeled using statistical distributions. Cowan (1975) presents four headway models, each with different complexity. The first model, given in Equation (2.2), yields an exponential distribution with expected value equal to the inverse of the flow rate\(^1\), \(q\). The distribution gives the probability of a vehicle in a traffic stream keeping a headway to the vehicle ahead of \(h\) seconds. It assumes independent observations. This means that the headway chosen by one vehicle is not affected by the headways chosen by other vehicles in the traffic stream. Cowan’s notation has be altered in the equations below to be in accordance with the rest of the thesis.

\[
F(h) = \begin{cases} 
0 & h < 0 \\
1 - e^{-qh} & h \geq 0 
\end{cases}
\]

\[
E(h) = q^{-1}
\]  

The second model uses a displaced exponential distribution. This model is equal to the first model apart from the fact that it assumes that all headways in the traffic stream are longer than a given minimum headway, \(h_{\text{min}}\), see Equation (2.3).

\[
F(h) = \begin{cases} 
0 & h < h_{\text{min}} \\
1 - \gamma\left(\frac{h - h_{\text{min}}}{\gamma}\right) & h \geq h_{\text{min}} 
\end{cases}
\]

\[
E(h) = h_{\text{min}} + \frac{\gamma}{h_{\text{min}} + 1}
\]

The third and fourth model involve that one portion of the vehicles are taking their predecessor’s headway while the remaining vehicles are driving freely. However as these two models are not used in this thesis, the theory behind them is not explained further.

### 2.1.2 Macroscopic variables

On the macroscopic level focus is not on single vehicles. Traffic is rather described as a moving liquid consisting of several vehicles. The main variables used in this context are flow rate, density and average speed. These are explained below.

---

\(^1\)Macroscopic variable, which describes the number of vehicles passing a point per time unit. See Section 2.1.2 on page 5.
Flow rate, $q$

Flow rate, $q$, is the number of vehicles, $N$, counted at a fixed point on the road over a time period $T$. It is usually given in vehicles per hour.

$$q = \frac{N}{T} \quad (2.4)$$

The time period, during which the vehicles are counted, is equal to the sum of the individual time-headways of the counted vehicles. From this it can be derived that the flow rate equals is the inverse of the average time-headway, $\bar{h}_t$.

$$q = \frac{1}{\bar{h}_t} \quad (2.5)$$

Density, $k$

Density, $k$, is the number of vehicles counted at a point in time over a spatial distance, $L$. It is usually given in vehicles per kilometer.

$$k = \frac{N}{L} \quad (2.6)$$

Similar to the flow rate, the spatial distance, over which the vehicles are counted, is equal to the sum of the individual space-headways. The relationship between density and average space-headway is thus the same as between flow rate and average time-headway:
2.1. TRAFFIC FLOW THEORY

\[ k = \frac{1}{h_s} \]  \hspace{1cm} (2.7)

**Average speed, \( \bar{u} \)**

Like the individual vehicle speed, the average speed is given as either mean point speed or mean section speed. Mean point speed is calculated by taking the arithmetic mean of observed vehicle speeds at a point:

\[ \bar{u}_t = \frac{1}{N} \sum_{i=1}^{N} u_i \]  \hspace{1cm} (2.8)

Mean section speed is found by dividing a distance by the average time used by vehicles to cover this distance.

\[ \bar{u}_s = \frac{L}{N \sum_i T_i} \]  \hspace{1cm} (2.9)

It can be shown that this is equivalent to the harmonic mean of individual vehicle speeds.

\[ \bar{u}_s = \frac{1}{\frac{1}{N} \sum_i \frac{1}{u_i}} \]  \hspace{1cm} (2.10)

**Fundamental equation**

The relationship between the three main variables of macroscopic traffic flow is given in Equation (2.11), which is called the fundamental equation. This states that flow rate is the product of density and average section speed.

\[ q = \bar{u}_s k \]  \hspace{1cm} (2.11)

In addition to this, several estimates for the relationship between speed and density have been proposed. Greenshields’ linear relationship, given in Equation (2.12), is commonly used. Here \( u_f \) denotes the speed at free flow and \( k_j \) denotes the density at traffic jam. By applying this to the fundamental equation the relationships between \( q, u \) and \( k \) illustrated in Figures 2.3, 2.4 and 2.5 are obtained.

\[ u = u_f \left(1 - \frac{k}{k_j}\right) \]  \hspace{1cm} (2.12)
CHAPTER 2. THEORY AND LITERATURE

Figure 2.3: Relationship between density and speed.

Figure 2.3 shows the linear relationship which forms the basis of Greenshields’ model. In this example the free flow speed is 90 km/h. The density declines with increasing speed. Meaning that the faster a vehicle drives, the more space it occupies on the road.

Figure 2.4: Relationship between flow rate and speed.

The relationship between flow rate and speed, shown in Figure 2.4, forms a parabolic function. Maximum flow rate in this example is 1500 veh/h with a corresponding speed at capacity of 45 km/h. The figure shows that one flow rate, except for the
maximum flow rate, can occur at two different speeds. This corresponds to uncongested traffic flow for the higher speed and congested flow for the lower speed.

As for flow and speed, the relationship between density and flow rate, shown in Figure 2.5, also forms a parabolic function.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{density_flow.png}
\caption{Relationship between density and flow rate.}
\end{figure}

### 2.2 Car-following models

Traffic simulation software are tools used for simulating and describing the traffic flow in a road network. In order to do this the software uses different mathematical and logical models, which describe various traffic situations.

Car-following models are one such kind of model. They describe the interactions between vehicles driving behind each other on a single lane roadway with no overtaking possibilities on a microscopic level. (Busch 2011).

#### 2.2.1 Terminology

Terms used when describing car-following models in this thesis are explained below and in Figure 2.6.
CHAPTER 2. THEORY AND LITERATURE

Figure 2.6: Terminology used when describing car-following models.

<table>
<thead>
<tr>
<th>Term</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed vehicle</td>
<td>The vehicle, of which the following behavior is estimated. Denoted as vehicle $n$.</td>
</tr>
<tr>
<td>Downstream vehicle</td>
<td>The vehicle ahead of the observed vehicle. Denoted as vehicle $n-1$.</td>
</tr>
<tr>
<td>Upstream vehicle</td>
<td>The vehicle behind the observed vehicle. Denoted as vehicle $n+1$.</td>
</tr>
<tr>
<td>Time step</td>
<td>The simulated time interval, at which the vehicles’ speeds and positions are updated.</td>
</tr>
<tr>
<td>Reaction time</td>
<td>The time it takes for the driver of a vehicle to react to a stimulus from the downstream vehicle.</td>
</tr>
</tbody>
</table>

2.2.2 Car-following model classification

Brackstone & McDonald (1999) divide car-following models into the following five categories: Gazis-Herman-Rothery models, safety distance/collision avoidance models, linear models, psychophysical/action point models, and fuzzy-logic based models.

The Gazis-Herman-Rothery (GHR) model dates back to the 1950s and 1960s. It calculates a response, the observed vehicle’s acceleration, based on a stimulus from the downstream vehicle. The stimulus may for example be difference in speed and position between the two vehicles.

Safety distance models are not based on stimulus and response like the GHR model, but seek to find a safe following distance to the downstream vehicle. Gipps’ car-following model, which is analyzed in this thesis, falls under this classification.

Linear models are similar to the GHR model, but include additional terms. These make the acceleration of the observed vehicle dependent on whether one or more downstream vehicles are braking.

Psychophysical models simulate the behavior of the observed vehicle differently according to the distance to the downstream vehicle. This type of model defines a threshold where the vehicle starts adjusting its behavior according to the downstream vehicle rather than driving freely.

Fuzzy-logic based models use logical operations to estimate driving behavior. For example can a definition of a ‘too close’ following distance be inputed into the
model. Then the model can make the observed vehicle commence braking if it is true that the distance to the downstream vehicle is within the definition of 'too close'.

2.3 Gipps’ car-following model

2.3.1 Model description

This car-following model is a safety distance model proposed by Gipps (1981). Today the model is used in some microsimulation softwares, for example AIMSUN (Transport Simulation Systems 2010). It was designed to posses the following three properties:

1. It should mimic the behavior of real traffic.
2. The parameters in the model should correspond to obvious characteristics of drivers and vehicles.
3. It should be well behaved when the interval between successive recalculations of speed and position is the same as the reaction time.

As opposed to earlier models, the second point listed above made calibration easier as the user could select parameter values connected to actual driving behavior. For example the maximum acceleration of a vehicle.

In practice the model is used to simulate a number of vehicles on a single lane road section over a period of time. This time period is divided into many equally large time steps, at which the speed of each vehicle is estimated. From this the vehicles’ positions can be derived. The size of one time step is set equal to the reaction time, as mentioned in the third point listed above. How long the reaction time is must be determined by calibration. However, to give an impression of the order of magnitude; Gipps’ suggestion is $\frac{2}{3}$ seconds.

The following three constraints form the basis for the speed calculation:

1. A vehicle will not exceed the driver’s desired speed.
2. The acceleration of a vehicle should first increase with speed, and then decrease to zero as the desired speed is approached.
3. If the downstream vehicle commences braking, the observed vehicle should be able to make a complete stop without collision.

The first two constraints are limiting when the vehicle drives freely and is able to choose its own speed. This means when the distance to the downstream vehicle is large enough, that this vehicle does not affect the driving behavior of the observed vehicle. Whereas the third constraint limits the speed when the driver needs to take the behavior of the downstream vehicle into account. This applies to congested
traffic flow, and in uncongested flow when the distance to the downstream vehicle is lower than a certain limit, which depends on the values of the input parameters.

To be able to function in both situations, the model calculates two different speeds for each vehicle at each time step. One for the free driving situation, \( u^a_n \), given by Equation (2.13a) and \( u^b_n \) for the car-following situation, given by Equation (2.13b). The actual speed of the vehicle is then determined by Equation (2.14) to be the lowest of the two, meaning the most limiting constraint. Gipps’ original notation has been slightly altered in the equations.

\[
\begin{align*}
 u^a_n(t + \tau) &= u_n(t) + 2.5a_n\tau \left(1 - \frac{u_n(t)}{U_n}\right) \sqrt{0.025 + \frac{u_n(t)}{U_n}} \tag{2.13a} \\
 u^b_n(t + \tau) &= b_n\tau + \sqrt{b_n^2\tau^2 - b_n\left[2[x_n-1(t) - s_{n-1} - x_n(t)] - u_n(t)\tau - \frac{u_{n-1}(t)^2}{\hat{b}}\right]} \tag{2.13b} \\
 u_n(t + \tau) &= \min\{u^a_n(t + \tau), u^b_n(t + \tau)\} \tag{2.14}
\end{align*}
\]

Where

- \( a_n \) is the maximum acceleration which the driver of vehicle \( n \) is willing to undertake.
- \( b_n \) is the most severe deceleration that the driver of vehicle \( n \) wishes to undertake.
- \( \hat{b} \) is the most severe deceleration of vehicle \( n-1 \) as estimated by the driver of vehicle \( n \).
- \( s_{n-1} \) is the effective size of vehicle \( n-1 \). This includes the physical length of vehicle \( n-1 \) and a safety margin, into which the driver of vehicle \( n \) is not willing to intrude even at rest.
- \( U_n \) is the desired speed of vehicle \( n \).
- \( x_n(t) \) is the location of the front of vehicle \( n \) at time \( t \).
- \( u_n(t) \) is the speed of vehicle \( n \) at time \( t \).
- \( \tau \) is the reaction time, which is constant for all vehicles and equal to the simulation step.

Some of the parameters used in the model are illustrated in Figure 2.7.

### 2.3.2 Example

An example of how the model works is illustrated in Figure 2.8. It shows the results of a simple simulation of two vehicles, with a duration of 100 seconds. The first vehicle (leader vehicle) drives with a constant speed of 15 m/s. When the simulation starts the second vehicle (following vehicle) is located 500 meters behind...
2.3. GIPPS’ CAR-FOLLOWING MODEL

the leader, and has no speed. The speed of this vehicle is from this point forward calculated using Gipps’ model. The desired speed is 25 m/s. The remaining input parameters used in this example are given in Appendix B.

Graph (a) in Figure 2.8 shows the two speeds, $u_n^a$ and $u_n^b$, which are calculated for the following vehicle at each time step. It shows that when the distance between the two vehicles, given in Graph (c), is large then $u_n^a$ is smaller than $u_n^b$ and is thus chosen as the vehicle’s speed. The former increases up to the desired speed of 25 m/s, which the following vehicle keeps until the 60th second.

Because the following vehicles keeps a higher speed than the leader vehicle, the distance between the two decreases with time. This is illustrated by the vehicle trajectories in Graph (b) and the space-gap between the vehicles in Graph (c). Graph (a) shows that $u_n^b$ decreases in a similar manner as the gap. At the 60th second this speed is lower than $u_n^a$, and the following vehicle’s speed is now set equal to $u_n^b$. Both the vehicle spacing and $u_n^b$ keep decreasing until the latter is equal to the leader vehicle’s speed of 15 m/s. At this point the distance between the vehicles is chosen by the following vehicle based on the premise that if the leader vehicle were to commence braking, the following vehicle should be able to make a complete stop without collision.

2.3.3 Comments

In the free driving situation only the vehicle’s current speed, the maximum acceleration, the reaction time and the ratio between current speed and desired speed affect the observed vehicle’s speed. Whereas the speed calculated in congested traffic flow, and in uncongested flow when vehicle spacing is small, depends on the maximum deceleration, the reaction time, the distance to the downstream vehicle, the current speed of both the observed and the downstream vehicle, and the estimated deceleration rate of the downstream vehicle. The last mentioned parameters will also affect how a vehicle reacts to changes in the behavior of the downstream vehicle.

An advantage of Gipps’ model compared to other car-following models is the low number of input parameters which makes calibration easier. On the other hand this
means that the acceleration and deceleration parameters are very comprehensive. They need to take both the physical properties of each vehicle and the psychological aspects of each driver into account. So although these two parameters are defined as maximum values, it does not mean that they represent the maximum acceleration and deceleration capabilities of the vehicle. Two persons, who drive the same type of vehicle, might not be willing to accelerate equally fast or brake equally hard.

In order to create diversity among vehicles and drivers most of the input parameters for each vehicle are chosen from a normal population with a given mean and standard deviation in AIMSUN (Transport Simulation Systems 2010). For some
parameters also an interval is defined, within which all values must be. This makes each vehicle unique.

2.4 Course of simulation studies

When applying traffic simulation models they need to be adapted to the particular road network or situation, which is to be modeled. Because the performance of car-following models and other traffic models are dependent on numerous input parameters, this task can be complex and might yield incorrect results when not performed correctly.

To assist in this situation, the Federal Highway Administration (2004) has formulated guidelines for applying traffic microsimulation modeling software. They suggest that the overall process should proceed in the following seven steps:

1. Identification of study purpose, scope and approach
2. Data collection and preparation
3. Base model development
4. Error checking
5. Calibration
6. Alternatives analysis
7. Final report and technical documentation

The study performed in this thesis does not comprise an ordinary traffic simulation process, where an established model is used to analyze different alternatives. In this case the model itself is to be evaluated. However, some of the points presented above still apply, as a simulation model needs to be developed in order to perform the intended analysis. Therefore the course formulated in the guidelines will be followed. Further details on the performed steps are elaborated as the topics are treated in the analysis.

2.5 Vehicle weight

This study devotes attention to heavy vehicles in Gipps’ model. Therefore a definition of heavy vehicles will be given in this section. Also the current ways of incorporating weight in the model are summarized, and relevant literature concerning vehicle weight and driving behavior is reviewed.
2.5.1 Weight classification of vehicles

In Norway a heavy vehicle is defined as a vehicle with allowed gross weight above 3500 kg. This definition is for example used in conjunction with driver’s licenses (Samferdselsdepartementet 2004). The class is further subdivided into additional classes, most notably: light trucks with allowed gross weights between 3500 and 7500 kg, and heavy trucks with allowed gross weights above 7500 kg. General maximum gross weight is 50,000 kg, and maximum length is 19.5 meters. However special vehicles with gross weight 60,000 kg and length up to 25.25 meters are allowed on some road sections (Norwegian Public Roads Administration 2012a). Heavy vehicles are generally not allowed to drive faster than 80 km/h, but some buses are exempt from this restriction (Samferdselsdepartementet 1986).

Heavy vehicles may also be defined according to length. Especially when using traffic data where vehicle weight is not available. Then a heavy vehicle is defined as a vehicle longer than 5.6 meters (Norwegian Public Roads Administration 2011).

As the term heavy vehicle comprises such a large weight span, properties of vehicles differ greatly within the class. From pick-up trucks and large SUVs, which may share many properties with passenger cars, to fully loaded semi trailers.

Vehicles of equal weight might also show different properties according to their purpose. A city bus or an urban delivery truck needs to be able to accelerate and decelerate quickly due to frequent stops. However this is less important for a truck designated for long haul goods transport. Here driving at constant speed with good fuel economy might be of greater importance.

For further reference the following terminology will be used for vehicle weight classes:

- **Cars**: Vehicles lighter than 3500 kg.
- **Heavy vehicles**: All vehicles heavier than 3500 kg.
- **Light trucks**: Vehicles weighing between 3500 and 7500 kg.
- **Heavy trucks**: Vehicles heavier than 7500 kg.

2.5.2 Vehicle weight in Gipps’ car-following model

AIMSUN includes heavy vehicles by allowing the user to define different vehicle types to appear in the model (Transport Simulation Systems 2010). Each type is set to constitute a certain portion of all vehicles. This does in theory make it possible to include several types of heavy vehicles, grouped for example according to weight or purpose. But this possibility might not be used because each added vehicle type comprises greater calibration efforts, as each type has its own set of input parameters. Heavy vehicles are thus often gathered into one class, using average input parameters for all heavy vehicles, which creates a source of uncertainty.

Ravishankar & Mathew (2011) have made an improved version of Gipps’ model, where they introduced vehicle-type specific parameters. They found that data col-
lected from this model provided a better fit to field data than the original model. The difference compared to AIMSUN’s approach is that this improved model involved assigning different input parameters to different pairs of leader and follower vehicles. In AIMSUN the properties of the downstream vehicle have less influence. In a simulation model of a traffic stream consisting of cars and trucks this improved model would include four different types of vehicle pairs: car-car (leader-follower), car-truck, truck-car, and truck-truck. Hence four sets of input parameters. A negative side with this approach is that extensive calibration efforts are required with such a high number of parameters.

The study is interesting because it shows that inclusion of vehicle-type specific parameters does make an improvement to Gipps’ model. It also indicates that focus should be devoted to the vehicle type of both the observed and the downstream vehicle.

### 2.5.3 Vehicle weight and driving behavior

The amount of literature, which concerns correlations between vehicle weight and driving behavior is limited. However some studies have been conducted. The literature which is relevant for choices and assessments made in this thesis are reviewed in this section.

Sayer et al. (2000) found in an empirical study that drivers of passenger cars followed light trucks at shorter distances than they followed other passenger cars.

An investigation of factors influencing the following headway was also conducted by Brackstone et al. (2009). They also concluded that the type of leader vehicle influenced the headway, with drivers following closer to trucks/vans than cars. Results from the same study showed that level of flow and road type did not seem to have an effect on the following headways.

Cristoforo et al. (2004) found a strong relationship between the weight of combination vehicles, that is vehicles consisting of more than one unit, and acceleration and deceleration. The conclusion was drawn on the basis of a series of acceleration and braking tests for different types of vehicle combinations, such a semi-trailer and road train, the latter is a combination vehicle with more than one trailer. They found that the average acceleration from 0–80 km/h ranged from 0.28 m/s$^2$ for gross weight 40 tonnes to 0.14 m/s$^2$ for gross weight 160 tonnes. Average deceleration from 80–0 km/h varied from 3.16 m/s$^2$ for gross weight 40 tonnes to 2.40 m/s$^2$ for gross weight 160 tonnes. It should be noted that these are maximum acceleration and deceleration rates capable by the vehicles. The study was conducted in Australia, where higher gross vehicle weights are allowed than in Norway.

In the traffic simulation guidelines reviewed earlier, the Federal Highway Administration (2004) presents some vehicle parameters, which they claim can be used in absence of better data. Their parameter suggestion are given for five different vehicle types: passenger cars, single-unit truck, semi-trailer truck, double-bottom
trailer truck and bus. Parameter suggestions are given for vehicle length, maximum speed, maximum acceleration, maximum deceleration and jerk, which is the rate of change in acceleration.

Most interesting in this context is acceleration and deceleration. The former ranges from 0.61 m/s$^2$ for double-bottom trailer trucks to 3.05 m/s$^2$ for passenger cars. However the maximum deceleration is set to 4.58 m/s$^2$ for all vehicle types.

That the deceleration rate is constant for all vehicle classes does not agree with Cristoforo et al. (2004). And if these guidelines are followed, the simulation may yield incorrect results for vehicle weight affected driving behavior, even though several weight classes are included.

The weight of a vehicle can also influence the distance it keeps to the downstream vehicle. A model for headway spacing estimation based on a vehicle’s weight is presented by Nouveliere et al. (2012). The model was estimated based on field data. It involves that the headway increases with increasing vehicle weight.

Aghabayk et al. (2012) have conducted an investigation on which variables affect the following behavior of heavy vehicles. Correlations between the acceleration of heavy vehicles and different stimuli were investigated. The most significant stimuli were found to be the speed difference between the observed and following vehicle, and the acceleration of both vehicles. The authors state a need for a vehicle-following model, which incorporates heavy vehicle driver’s behavior. Such a model could be used in traffic micro-simulations to provide more accurate modeling of traffic phenomena.
Chapter 3

Method

The method used to perform the planned investigation is to simulate a real road section, from which traffic flow data and vehicle weight data are available, by using Gipps’ car-following model. Then a modified simulation model, which includes a weight parameter, will be estimated. The same road section will be simulated using this model. Finally the results from the two models will be compared and discussed.

A measure, against which the models are to be compared, needs to be defined. This could be how an observed vehicle in the model reacts to different maneuvers performed by the downstream vehicle. Alternatively could one or more single variables be compared. Based on the literature reviewed in the previous section acceleration rates, deceleration rates, and vehicle spacing seems to be affected by the weight of the vehicle. These variables are thus eligible for comparison.

Vehicle spacing is chosen as the comparison variable. Mainly because it was the only variable which could be derived from the available data, which is explained in the next chapter. This is though an important variable because it influences how much space one vehicle occupies on the road, which again is of importance when determining road capacity. This may be of interest when performing a microsimulation analysis. In addition, vehicle spacing may be particularly relevant for an analysis of Gipps’ car-following model, because this is a safety-distance model. As mentioned the foundation, upon which this type of model is built, is the following distance to the vehicle ahead.

Of the different kinds of spacing variables, time-gap is chosen for the comparison. That is the distance measured in seconds between the rear of the downstream vehicle and the front of the observed vehicle. Primarily because this variable is independent of the length of the vehicle ahead, as opposed to the headway. Secondly, time-gap is advantageous over space-gap because it includes the vehicle’s speed.

It could be argued that headways should be chosen over time-gaps, as vehicle
length also influences how much space a vehicle occupies on the road. However, when analyzing potential deviations between headways collected in the field and from the model, it may be difficult to determine whether the source of error is the vehicle length or the vehicle spacing. Of the two variables, only gap is solely a result of driving behavior.

Only short time-gaps are interesting in the context of car-following. Al-Kaisy & Karjala (2010) found that the car-following interaction on a two-lane rural highway ceased to exist when headway exceeded a value of approximately six seconds. When the distance is larger, the behavior of the observed vehicle is not affected by the downstream vehicle. Other factors, such as if the driver is satisfied with the vehicle’s current speed may be dominating. And the vehicle spacing distribution will depend on the portion of vehicles driving in a platoon. Even though it is interesting to investigate if these aspects are modeled correctly, it is the car-following behavior which is in focus in this study. Because of this it is chosen to only investigate time-gaps shorter than six seconds. This value is chosen, even though gaps are shorter than headways due to the exclusion of the downstream vehicle’s length, because it is stated in the article that six seconds is an approximate value.
Chapter 4

Data

According to the Federal Highway Administration (2004), the step in a simulation process succeeding the identification of study purpose and approach is data collection. There are three types of traffic data, which can be used for the purpose of this study:

- **Point data**
  Data recorded from all vehicles which pass a single point on the road. An advantage with point data is that large data amounts can be collected with little effort. This makes it possible to derive traffic flow and vehicle characteristics at the detector location with little uncertainty. The most important disadvantage is that it only captures one single moment in time. What happens before and after a vehicle passes the location is not known.

- **Time-series data**
  Data recorded from a single vehicle over a period of time, collected for example by using GPS. An advantage with this type of data is that more traffic flow characteristics can be derived from it. Such as acceleration, which is impossible to find from point data. It will also show the variation in variables over time. However, as time-series data take more time to collect, obtaining data from many vehicles demands great efforts. Thus, will general traffic flow and vehicle characteristics derived from time series data contain greater uncertainty than point data.

- **Video data**
  Video of a road section recorded over a certain time-period. This makes it possible to study the behavior of multiple vehicles over a longer time period. However, traffic flow data must be derived visually, and will therefore contain uncertainty. Collecting large amounts of data is also time consuming, as the videos usually need to be analyzed in real time. Another disadvantage is that vehicle weight data is unavailable, except on the level that trucks can be separated from cars according to observation.
A car-following model produces microscopic traffic flow data for single vehicles over a period of time, hence time-series data. This means that in theory, this type of data collected in the field would be ideal for the analysis. However, this study aims at investigating how well the model reproduces differences in driving behavior caused by vehicles’ weight. Thus is data from vehicles, with many different gross weights needed. In order to reduce uncertainty it is also desirable to obtain data from several vehicles with the same weight.

Collecting time-series data for such a large number of vehicles would comprise great efforts. Video data is ruled out due to the lack of vehicle weight data and the uncertainty in traffic flow data. Therefore, point data are used in this analysis. These data are collected continuously by automatic equipment, and are generally widely available.

4.1 Collection

4.1.1 Method

The data was collected using ViperWIM Weigh-in-Motion (WIM) technology produced by Applied Traffic. Unlike conventional point detectors, which only measure variables related to speed and time, this system also measures vehicle weight. A major advantage with this system is that it can measure weight while the vehicle is in motion. To do this is uses a combination of an inductive loop detector and piezoelectric cables (Applied Traffic Limited 2010).

Inductive loops are electric cables which form a coil and are buried in the road surface. A voltage is applied to the cables and the metal in passing vehicles will affect the magnetic field, which is created above the coils. By taking advantage of this effect the inductive loop can be used to detect the presence of a vehicle. (Norwegian Public Roads Administration 2011).

Piezoelectric cables are placed beneath the road surface. When pressure is applied to them they output a charge, which can be used to detect vehicle presence. However, the charge varies according to the pressure. Therefore the cables can also be used to measure weight. (Applied Traffic Limited 2010).

4.1.2 Time and location

Even though point detector data generally are widely available, the range of usable locations is limited due to the purpose of the study. The following should be satisfied on the data collection location:

- Vehicle weight data must be available.
  
  Only a small number of detectors are equipped with WIM technology.
4.1. COLLECTION

- The WIM point should be recently calibrated. This is particularly important because there have been some problems regarding the accuracy of this technology, and that it needs to be calibrated often to produce usable data.\(^1\)

- The road should only have one lane in each direction. Lane changing and overtaking is not modeled by a car-following model. Such situations should thus not be present at the data collection location.

- Interfering aspects should not be present in close vicinity to the detector. For example large intersections or other factors which are likely to affect the traffic flow. This is because the point detector only captures one point in time, and it is desirable that most vehicles are not accelerating or decelerating. Only this way the speed estimate will be correct.

- Ideally both uncongested and congested traffic flow is present. This way the model could be compared against both traffic situations.

A detector located on road E6 at Stamplusmyra in Verdal municipality in Nord-Trøndelag county in Norway was chosen. Straight road sections are found on each side of the detector location and there are no large intersection in close vicinity, which may affect the traffic flow. The road has one lane in each direction at this point. However the most important reason for this choice was that it was recently calibrated.\(^2\) Vehicles with gross weights up to 60,000 kg and lengths up to 25.25 meters are allowed here (Norwegian Public Roads Administration 2012a). The speed limit is 70 km/h (Google 2013).

Data were collected from both lanes on August 1 and August 4–19 2012, over the whole 24-hour period on each day. Only incomplete datasets were available from the detector from August 2–3. These dates were therefore excluded.

4.1.3 Uncertainty

There have been problems regarding the quality of weight data registered with Weigh-in-motion technology. An investigation was performed for the particular point, from which the data used in this study is collected, in the summer of 2012 (Norwegian Public Roads Administration 2012b). That is just a few months before the data collection took place. It showed a relative error of 10 % for the gross weights of vehicles heavier than 40,000 kg and an error of 20–25 % for the remaining heavy vehicles. The system was more likely to overestimate the vehicle weight. This source of error is important to have in mind when interpreting the results.

Charges sent by the piezoelectric cables may vary according to temperature dependent changes in the road surface. As a consequence the vehicle weights registered may vary according to weather conditions. Temperature can vary over one day

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\(^1\)Stated by Torbjørn Haugen in e-mail 16.05.2013.

\(^2\)Stated by supervisor Odd André Hjelkrem.
and mean temperature might change during the time period of twenty days, during which the data was collected. This might yield different results according to the time of a vehicle observation. But there is a mechanism in the registration equipment designated for adjusting these differences (Applied Traffic Limited 2010). However Aakre (2011) still found indications of a 1 % decrease in deviation for each increasing °C. Due to the general uncertainty connected to the results, it is chosen not to initiate any further measures regarding temperature.

In the investigation of the WIM point performed by the Norwegian Public Roads Administration (2012b), a better accuracy was found for the length registrations than the weight registrations. The uncertainty regarding vehicle speeds was by Heggland (2012) found to be low (±1 km/h). An investigation regarding the uncertainty of time-gaps does not seem to have been conducted. As this also is a measurement concerning space and time it is assumed that the uncertainty is not poorer than for speed and length.

4.2 Statistics

Data retrieved from the detector for each vehicle were:

- Point-speed
- Time-gap
- Vehicle length
- Vehicle weight
- Detection time
- Lane

The purpose of this study is to investigate the performance of the model, not to analyze a particular road network. Therefore it was desirable to make the vehicle observations independent of the collection time and lane, and rather connect them to a certain traffic condition. Such as the flow rate, the section speed or the density. In this case flow rate was chosen. This is the macroscopic variable, which can be measured with least uncertainty at a single point. Both section speed and density need to be measured over a spatial distance. Only estimates for these two variables can therefore be derived from point data.

The flow rates were found by counting the number of vehicle observations in each lane in 15-minute intervals. It is assumed that observations from the two lanes are independent of each other. These values were then multiplied by four to find the equivalent hourly flow rate. The duration of the time interval was chosen on the basis of Gartner et al. (1997), who recommend 15 minutes as a minimum for estimating hourly flow rates.
4.2. STATISTICS

Connecting the vehicle observations to these estimated flow rates may be a source of error. Data from a vehicle which passes the detector right before a 15-minute interval ends will be connected to another flow rate than data from a vehicle which passes right after the beginning of the next interval. Even though they might be only seconds apart. However, the flow rates estimated for succeeding 15-minute intervals are not expected to differ greatly from each other. This source of error is therefore considered not to be of crucial significance.

The relevant data were retrieved from the data file produced by the detector by making a Matlab script, which is included in Appendix D.1. This also calculated the flow rates and connected them to the vehicle observations.

Some vehicle observations, which were obvious results of registration errors, were removed manually. For example vehicles registered with zero speed or unnaturally high gross weights.

4.2.1 Vehicle Statistics

The statistics presented below are based on the 247,645 vehicle observations recorded in August. Due to the large amount of data, the registrations are not included in this report. They can however be found in the digital attachment.

Vehicle weight

Because vehicle weight is of particular interest, some statistics concerning this variable are presented in this section. As mentioned, the uncertainty concerning weight data is high.

Table 4.1: Vehicle weight statistics.

<table>
<thead>
<tr>
<th>Category</th>
<th>Gross weight</th>
<th>Portion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cars</td>
<td>0–3500 kg</td>
<td>86 %</td>
</tr>
<tr>
<td>Heavy vehicles</td>
<td>&gt; 3500 kg</td>
<td>14 %</td>
</tr>
<tr>
<td>Light trucks</td>
<td>3501–7500 kg</td>
<td>8 %</td>
</tr>
<tr>
<td>Heavy trucks</td>
<td>&gt; 7500 kg</td>
<td>6 %</td>
</tr>
</tbody>
</table>

Table 4.1 shows the share of vehicles in the different weight categories presented in Chapter 2. Cars (and trucks lighter than 3500 kg) constitute the majority of the vehicles passing the detector in the observed time period.

Figure 4.1 shows the distribution of gross weights among the vehicles in the dataset. The probability density function and the cumulative distribution function for all the vehicles are shown in Figures 4.1a and 4.1b. They show that the majority of the vehicles have gross weights below 5000 kg. The vehicles are counted in intervals of 100 kg.
CHAPTER 4. DATA

(a) Vehicle weight distribution. Probability density function.

(b) Vehicle weight distribution. Cumulative distribution function.

(c) Distribution of vehicle weight below 7500 kg.

(d) Distribution of vehicle weights above 7500 kg.

Figure 4.1: Vehicle weight distributions.

The weight distribution only for vehicles lighter than 7500 kg is shown in Figure 4.1c. It shows that the median is just above 2000 kg. There is a small peak on the curve between 0 and 500 kg, which is probably caused by scooters and motorcycles.

The remaining distribution, for vehicles heavier than 7500 kg, is shown in Figure 4.1d. In this distribution vehicles are counted in intervals of 1000 kg, and it should be noted that the axis are different from the remaining figures due to lower frequencies. This curve is not as smooth as the one in Figure 4.1c, which is probably due to a lower number of registered vehicles within this weight group. The highest frequencies are found between 10,000 and 30,000 kg. For higher weights the frequencies declines gradually.

Table 4.2 shows the mean weights, standard deviation and median weights for all vehicles separated by classification. Vehicles lighter than 3500 kg have a mean gross weight close to the median and the standard deviation is less than a third of this value. For all heavy vehicles the standard deviation is larger compared to
Table 4.2: Mean weight, standard deviation and median weight for all vehicles.

<table>
<thead>
<tr>
<th>Category</th>
<th>Cars</th>
<th>Heavy</th>
<th>Light tr.</th>
<th>Heavy tr.</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight</td>
<td>≤ 3500 kg</td>
<td>&gt; 3500 kg</td>
<td>3501–7500 kg</td>
<td>&gt; 7500 kg</td>
<td>&gt; 0 kg</td>
</tr>
<tr>
<td>Mean, $\mu$</td>
<td>2188 kg</td>
<td>14,498 kg</td>
<td>4428 kg</td>
<td>27,254 kg</td>
<td>3896 kg</td>
</tr>
<tr>
<td>St.dev., $\sigma$</td>
<td>602 kg</td>
<td>14,377 kg</td>
<td>843 kg</td>
<td>12,574 kg</td>
<td>6864 kg</td>
</tr>
<tr>
<td>Median</td>
<td>2193 kg</td>
<td>5510 kg</td>
<td>4178 kg</td>
<td>25,462 kg</td>
<td>2309 kg</td>
</tr>
</tbody>
</table>

The mean, meaning that the weight span of this vehicle class is greater than for cars. The median of 5510 kg is close to the lower weight boundary of this class, considering that it comprises all vehicles weighing up to 60,000 kg. This is however expected as Table 4.1 shows that more than half the vehicles in this class weigh less than 7500 kg.

Vehicle length

Vehicle length is an input parameter in Gipps’ model, and is therefore covered in this section.

Table 4.3: Mean length, standard deviation and median length for all vehicles.

<table>
<thead>
<tr>
<th>Category</th>
<th>Cars</th>
<th>Heavy</th>
<th>Light tr.</th>
<th>Heavy tr.</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight</td>
<td>≤ 3500 kg</td>
<td>&gt; 3500 kg</td>
<td>3501–7500 kg</td>
<td>&gt; 7500 kg</td>
<td>&gt; 0 kg</td>
</tr>
<tr>
<td>Mean, $\mu$</td>
<td>5.5 m</td>
<td>10.8 m</td>
<td>7.6 m</td>
<td>14.9 m</td>
<td>6.2 m</td>
</tr>
<tr>
<td>St.dev., $\sigma$</td>
<td>0.9 m</td>
<td>5.0 m</td>
<td>2.2 m</td>
<td>4.4 m</td>
<td>2.8 m</td>
</tr>
<tr>
<td>Median</td>
<td>5.4 m</td>
<td>9.3 m</td>
<td>6.8 m</td>
<td>16.9 m</td>
<td>5.5 m</td>
</tr>
</tbody>
</table>

Table 4.3 shows the mean, standard deviation and median for the lengths of the vehicles. It shows a tendency of increasing mean length and standard deviation for increasing weight. Minimum vehicle length registered is 1.1 meter, this is probably a motorcycle.

4.2.2 Traffic flow statistics

Flow rate and speed

Figure 4.2 shows all the hourly flow rates estimated from 15-minute intervals for both lanes in the data collection time period. One point represents one 15-minute interval. The number on the x-axis corresponds to noon on that particular date in August 2012. The figure shows that the flow rate mostly is below 800 veh/h. The maximum occurring flow rate during the data collection period is 1128 veh/h, the minimum is 4 veh/h.
Speed statistics are shown in Table 4.4. The maximum difference in mean point speed is 1 km/h and is found between cars and heavy trucks. The deviation from the speed limit of 70 km/h is also largest among heavy trucks; the mean speed is 2.6 km/h below this value.

Figure 4.3a shows the relationship between flow rate and speed in the field data. Each point represents one 15-minute interval. The calculation of the average section speed for each interval was based upon an assumption that a steady state traffic flow was present at the detector location, and that the speed of each vehicle was constant within a certain distance upstream and downstream from the detector. Average section speed was thus calculated by taking the harmonic mean of the point speeds of the vehicles passing the detector in a particular interval. This method was considered to be valid as there are few interfering aspects around the detector. A source of error is that if a vehicle did not keep constant speed, this method will
4.2. STATISTICS

(a) Average section speed and average flow rate for all 15-minute intervals.

(b) Average section speed for flow rate intervals of 100 veh/h.

Figure 4.3: Relationship between flow rate and speed in the field data.

over or underestimate the section speed.

The figure shows that the spread in average section speed is higher at flow rates below approximately 50 veh/h than for higher flow rates. This is partly because a lower number of vehicle observations is used to estimate the average section speeds. One vehicle with high speed will thus have greater effect on the average at low flow rates than at high. Another explanation is that when fewer vehicles are present on the road, then vehicles can more easily drive at higher speeds due to less interference from other vehicles.

In Figure 4.3b the vehicles have been grouped according their assigned flow rates, into intervals of 100 veh/h. The mean speed is calculated by taking the mean of the estimated section speeds in all 15-minute intervals with a flow rate within the given interval. For example is the average speed for all vehicles observations recorded with a flow rate in the interval 100–200 veh/h just above 70 km/h.

Both graphs in Figure 4.3 show the same pattern: The average speed declines with increasing flow rate.

If Greenshield’s linear relationship between density and speed is assumed, the flow-speed curve in Figure 4.3b is similar to the highlighted region of the curve in the macroscopic flow-speed graph shown in Figure 4.4. This means that the only traffic state found on this particular road is uncongested flow. No congestion occurs and the road has probably not met its capacity.

Vehicle spacing

Figure 4.5a shows average time-gaps for traffic flow rate intervals of 100 veh/h. For example is the average time-gap for all vehicle observations recorded with a flow rate in the interval 100–200 veh/h between 20 and 25 seconds. The graph shows
Figure 4.4: Portion of flow-speed curve derived from Greenshields’ relationship, which is present in the dataset.

Figure 4.5: Average time-gap for flow rate intervals of 100 veh/h.

that the average time-gap declines rapidly for low flow rates, but stabilizes below 5 seconds when the flow rate exceeds 700 veh/h. A reason for this can be that more vehicles drive in platoons at high flow rates than at low. Because there is less space available on the road.

However the vehicle spacings, which are interesting in this context, are the ones chosen by a vehicle, when it follows another vehicle. Consequently not vehicle spacings recorded in the free driving situation. These are excluded in Figure 4.5b by only including time-gaps smaller than 6 seconds when calculating the averages. It shows a stable time-gap for all flow rates.
4.2. STATISTICS

Figure 4.6 shows the relationship between average time-gap shorter than 6 seconds and vehicle weight. The graph shows that from 0–5000 kg to 15,000–20,000 kg the average time-gap increases by 0.5 seconds. This may sound like a small value, but at average speed 68.3 km/h (∼19.0 m/s) this equals 9.5 meters, which corresponds to almost two average car lengths.

Between 20,000 and 30,000 kg the curve makes a 0.1 second decline. A possible explanation for this is that vehicles within this weight interval are empty large trucks. Whereas vehicles in the previous weight interval, which kept a larger average time-gap, are fully loaded smaller trucks. The relationship between engine power and vehicle weight may be different in these two situations. The uncertainty in the vehicle weight data may also be a reason for the decline.

The average time-gap for vehicles with gross weight in the interval 30,000–50,000 kg remains close to 3.0 seconds. In the last weight interval included in the graph, 50,000–55,000 kg, the average time-gap rises to 3.15 seconds, which is a sudden deviation compared to the remaining weight intervals. It may be that such heavy vehicles keep longer gaps to the downstream vehicle compared to lighter vehicles. However, the weight distribution in Figure 4.1d shows that the number of vehicle observations within this group is lower than for the remaining groups. Thus will each time-gap have a greater influence on the average, and a lower number of long time-gaps will be able increase the average more than for the remaining weight groups.

Figures 4.7a and 4.7b show the time-gap distributions among the vehicles in the dataset. The figures only contain time-gaps shorter than 6 seconds, as these are of particular interest. The vehicles have been divided into groups according to their gross weights, which are separated at intervals of 10,000 kg. According to Fig-
Conditional probabiliy density function.

Figure 4.7: Time-gap distributions for different weight groups.

ure 4.1d on page 26 there does not seem to be a peak on the weight distribution at these interval boundarys which may suggest that the boundarys should be moved. An exception from this separation is made for the lowest group, which is subdivided into the intervals 0–3500 kg and 3500–10,000 kg, in order to separate cars from the heavy vehicles. The time-gaps are counted in intervals of 0.5 seconds. For example does the highest point on the curve for vehicles with gross weights between 0 and 3500 kg show the frequency of time-gaps between 1.5 and 2.0 seconds. A distribu-

Figure 4.7: Time-gap distributions for different weight groups.

It is chosen not to look at smaller weight intervals. This is because the number of vehicle observations within each interval would be smaller, and greater uncertainty will be connected to the distributions. Besides, the large uncertainty connected to vehicle weights in the dataset make it less interesting to look at very small weight intervals. For example if the uncertainty becomes as large as the range of the interval.

The graphs in Figure 4.7 show that the time-gap distributions change according to vehicle weight. The time-gaps, which appear at the highest frequency for vehicles with gross weights between 0 and 10,000 kg, are between 1.5 and 2.0 seconds. Whereas these are between 2.0 and 2.5 seconds for vehicles heavier than 10,000 kg. The maximum frequency found in the latter distributions does also seem to decline with weight. The difference between the distributions for weight groups above 10,000 kg are relatively small compared to the differences between weight groups below this value.

Because the WIM point used for data collection produced weight data with a high level of uncertainty, and had a tendency to overestimate vehicle weights, the relationship between exact values of gross weight and both average time-gaps and time-gap distribution may not be correct. The most important conclusion to draw
from these data is that time-gaps seem to change according to vehicle weight. However the changes might be both smaller or larger, due to the uncertainty.

4.2.3 Comments

The collected data is not ideal. Firstly, there is an issue of quality. Vehicle weights registered using WIM technology are through earlier tests found to have poor accuracy. Secondly, the only traffic situation present at the detector location was uncongested flow. This excludes the possibility to analyze the car-following model’s behavior in congested conditions.

However due to the necessity of vehicle weight data in this study, combined with few alternatives when it comes to data collection technologies and available collection locations, further data collection is not commenced.
Chapter 5

Original Model

This chapter covers the analysis of Gipps’ original model.

The microsimulation software, which was available when working on this thesis, did not offer the flexibility in changing the underlying mathematical models necessary for performing the analysis, in this case introduce a weight parameter. The solution to this issue was to construct a simulation model from scratch in Matlab. Matlab is a programming language, which can be used to analyze data, develop algorithms, and to create models and applications (MathWorks 2013).

Creating a new simulation model for this purpose entails both advantages and disadvantages. Primarily it offers full freedom to do any changes one might desire to carry out, and to retrieve any desirable data. Secondly it makes it possible to investigate the car-following model isolated. Commercial microsimulation softwares, like AIMSUN, use many different models to determine the behavior of a vehicle. The results might therefore be affected by other models in addition to the car-following model. Exclusion of other models is also likely to reduce the simulation time. However, creating a simulation model from scratch means that more effort and time need to be put into the model development compared to using already available software.

5.1 Model Development

The model simulates a given number of vehicles, $N$, on a straight road section of length $L$, as illustrated in Figure 5.1. The simulated section has one lane where no overtaking is possible. This provides similar conditions to those found on the location where the data was collected. Here there are only one lane in each direction and areal photographs (Finn.no 2013) show road marking of type warning line, which indicates that overtaking is not recommended (Samferdelsdepartementet 2005). And it is assumed that traffic data collected in the two lanes are independent
of each other. Vehicles are entering the modeled area with a flow rate of $q_e$. From this point forward their speeds are calculated using Gipps’ model. The script is initially designed to simulate two vehicle types: cars and heavy vehicles.

To be able to compare the output of the model to the collected field data on equal basis, a point detector is established in the model at a given distance from the entry point.

Vehicle observations collected in the field were connected to a flow rate in order to make them independent of registration time. For the model to produce results from a similar variety of flow rates, the script runs several simulations for different entry flow rates.

In order to obtain a sufficient amount of vehicle observations, the simulation can be replicated a given number of times.

### 5.1.1 Script

The entire Matlab script is attached in Appendix D.2. Its mode of operation is illustrated in Figure 5.2 and explained in the following sections.
Figure 5.2: Flow chart of the simulation process. The number in each box corresponds to the step numbers in the explanation below and in the Matlab script attached in Appendix D.2.
CHAPTER 5. ORIGINAL MODEL

Step 1: Initialization

The script starts with defining some variables that are constant for all replications. These are:

- Portion of heavy vehicles
- The entry flow rates, with which the simulation is to be run. For example, flow rates from 200 to 900 veh/h at 50 veh/h intervals. The script will then first run a simulation with entry flow 200 veh/h, then one with 250 veh/h, then with 300 veh/h, and so on up to the last simulation which is run with an entry flow of 900 veh/h.
- The number of replications.

Step 2 and 3: Replications and entry flow

This point is the beginning of the replications loop and the entry flow loop. The former repeats everything inside the loop the number of times, which was defined in step one. The code inside the latter loop is repeated for each entry flow defined in the first step.

Step 4: Input parameters

This step starts by defining the number of vehicles, which are to be simulated for each entry flow in each replication. Every vehicle is assigned an ID, starting at one for the first vehicle in the simulated traffic stream. Then all the vehicles are assigned input parameters. Further details are explained in Section 5.2.

Step 5: Simulation start

This step initiates simulation of the vehicles and is the beginning of the time step loop. The code inside this loop is repeated the number of time steps it takes for all vehicles to drive through the simulation area.

The first vehicle enters the area at time $t = 0$ with a predefined entry speed and position at the model area’s boundary. The remaining vehicles wait.

Step 6: Select vehicle

As elaborated in Section 2.3, the speed and position of the vehicles are updated at each time step. This step is the beginning of the vehicles loop, which for each time step repeats itself the same number of times as there are vehicles inside the simulation area.
5.1. MODEL DEVELOPMENT

Step 7: Apply Gipps’ model

For each repetition of the vehicles loop started in step 6, the speed and position of one vehicle, which is inside the simulation area, is updated by calculating a new speed using Gipps’ model. All previously calculated speeds and positions are stored in an array.

Step 8: Last vehicle?

If all vehicles present in the simulation area have been updated the vehicles loop is terminated and the script continues to step 9. Otherwise it returns to step 6 and selects the next vehicle.

Step 9: Vehicles left area?

The script only simulates a limited road section. Hence vehicles, which have covered this section need to be removed from the simulation. In this step the script checks if the position of the vehicle in the simulation area with the lowest ID exceeds the area’s boundary. If yes, then this vehicle is removed from the simulation. Otherwise the script continues to step 10 with no changes. As no overtaking is possible it is no need to check if any other vehicle also has left the area.

Step 10: Time for next vehicle arrival?

Vehicles, which have been waiting, must enter the area as time progresses. A method of determining the time between each vehicle entry at a given entry flow is therefore needed. Vehicle spacing distributions, which are described in section 2.1.1, can be used for this purpose.

The purpose of this model is to study the effects of Gipps’ car-following model. Hence a very detailed headway model for the vehicle arrivals is not needed, as the headways registered at a certain distance from the entry point will be affected by the car-following model. The main purpose is to regulate the flow rate. Due to this, the two most advanced headway models presented by Cowan (1975) are not used in the model.

The choice is then between an exponential and a displaced exponential distribution. The latter is chosen for the model. As described earlier, this uses a minimum headway. That prevents vehicles from entering the model area with an unnaturally small headway. In the model a vehicle’s position is given at its front bumper. A generated headway smaller than the length of the preceding vehicle would in practice mean that one vehicle enters the area inside another. Something which naturally must be avoided.
CHAPTER 5. ORIGINAL MODEL

The first vehicle enters the area immediately at simulation start. Then, and each time a new vehicle enters, the time until the next vehicle entry is picked at random from the displaced exponential distribution, and rounded to the nearest whole time step.

If the simulated time, that is the number of elapsed time steps multiplied by the size of one time step, is equal to the time for the next vehicle entry a new vehicle is added to the simulation area. As long as there still are vehicles which have not yet entered.

Step 11: All vehicles left?

If all vehicles have driven through the simulation area, that is the position of the last vehicle exceeds the area’s boundary, the time step loop is terminated. Otherwise the script adds one time step to the simulation clock and returns to step 5.

Step 12: Retrieve detector data

During simulation, the model stores all calculated positions and speeds for all vehicles. However it is desirable to compare data from the simulated point detector. Vehicle data from this location must therefore be extracted.

Due to the fact that the vehicles’ speeds and positions are only updated once during each time step, the model has a discrete time axis. As a result the exact point in time, at which a vehicle is located the detector location, $x_{\text{detector}}$, must be determined by interpolation between the closest registered positions before and after this. In the script this is done by finding the first recorded position of vehicle $n$ exceeding $x_{\text{detector}}$, and then the corresponding point in time, $t_1$. Then the time when the front of the vehicle passes the detector, $t_{\text{detector}}$, is calculated using Equation (5.1). The time when the rear of the vehicle passes the detector is calculated using the same equation and replacing $x_{\text{detector}}$ with $x_{\text{detector}} + l_n$.

$$t_{\text{detector}} = \frac{x_{\text{detector}} - x_n(t_1 - \tau)}{x_n(t_1) - x_n(t_1 - \tau)} + (t_1 - \tau) \quad (5.1)$$

Because a vehicle’s speed is constant during one time step, using interpolation for this purpose involves no error. It also means that the speed of the vehicle at the detector location can be set equal to the speed calculated at the last time step before the vehicle reaches the detector. The time-gap between vehicle $n$ and the downstream vehicle $n - 1$ is determined by subtracting the time when the rear of vehicle $n - 1$ passes the detector from the passing time of the front of vehicle $n$.

In order to compare the model results to the field data, flow rates in the model were also registered at 15-minute intervals. These are not necessarily exactly equal to the flow rates, with which the vehicles entered the model. This is because the
vehicle spacings will be affected by the car-following model. The first 15-minute interval starts when the first vehicle passes the detector.

However as the duration of each simulation run is determined by the number of vehicles simulated, \( N \), and the entry flow, \( q_e \), the simulated time period may not be dividable into whole 15-minute intervals. Thus the last time interval might have a duration shorter than 15-minutes. For example if the time period between the first and last vehicle passes the detector is \( T = 65 \) minutes. This includes four whole 15-minute intervals from 0–60 minutes. There will however be a remaining interval with duration five minutes. As all vehicle observations are connected to an estimated flow rate calculated from a 15-minute interval, there is an issue of what to do with vehicle observations recorded in shorter intervals.

There are three solutions to this issue:

1. Change the Matlab script, so that the simulated time period is a restraint rather than the number of vehicles.

2. The number of vehicles counted in the last time interval is multiplied by a factor, which is dependent on the length of the interval, in order to find the equivalent hourly flow rate. For example multiply the number of vehicles by 12 if the duration of the last interval is 5 minutes.

3. Discard the vehicle observations from the last time interval of each simulation run, if this interval is shorter than 15 minutes.

The first option would solve the problem completely, but would comprise the most effort as several changes need to be made in the script. In addition, one does no longer have direct control of the number of vehicles in the simulation. If this number is too high Matlab might run out of memory and the simulation is terminated. Option number two might produce incorrect results, as hourly flow rates estimated from very small time intervals involve a higher level of uncertainty.

The last option presented above is chosen. This will also solve the problem completely, but with the negative effect that some vehicles are simulated for no reason. Hence simulation time will be longer than necessary.

**Step 13: All entry flows?**

If all entry flows defined in step 1 have been simulated the entry flow loop is terminated, and the script continues to step 14. Otherwise it returns to step 3 and repeats the process with the next entry flow.

**Step 14: All replications?**

If the desired number of replications have been made the script terminates. Otherwise it returns to step 2 and a new replication is made.
5.2 Input parameters

This section covers the basis for and the assumptions made when determining the input parameters. However, the values of some of the parameters must be determined by calibration, which is described in Section 5.4.

There are two different types of input parameters: vehicle related and model related. The former are the parameters used in the equations which constitute Gipps’ model, and may differ from vehicle to vehicle. Whereas the latter are global parameters which affect all vehicles.

The type of each vehicle is chosen at random with a probability of being heavy of 14%, as found in Chapter 4. The vehicle is then assigned input parameters according to the vehicle type, as described in the next section. Most of these parameters are chosen by picking a random number from a normal population with a given mean and standard deviation.

5.2.1 Vehicle parameters

**Acceleration, $a_n$**

As the only data available for calibration were point data, picking a value for the acceleration parameter is difficult. Therefore the default values used by AIMSUN are chosen.

These are $\mu = 3.0 \text{ m/s}^2$ and $\sigma = 0.2 \text{ m/s}^2$ for cars; and $\mu = 1.0 \text{ m/s}^2$ and $\sigma = 0.5 \text{ m/s}^2$ for heavy vehicles.

Early simulation test runs showed that when picking random values from a normal population with a mean close to zero and a large standard deviation, the model assigned negative values for the acceleration parameter of some vehicles. This would in practice mean that a vehicle brakes when trying to accelerate. To avoid this the script checks if the acceleration value is lower than $0.5 \text{ m/s}^2$. If yes, the acceleration value will be set to $0.5 \text{ m/s}^2$.

This will though entail that vehicles with acceleration rate equal $0.5 \text{ m/s}^2$ are overrepresented compared to vehicles with other acceleration rates.

**Deceleration, $b_n$ and $\dot{b}$**

It should be noted that deceleration rates are always referred to as negative values in this report.

Similar to the acceleration, the deceleration parameter was restricted to not be higher than $-0.5 \text{ m/s}^2$. In order to avoid positive deceleration rates.
The parameter $\hat{b}$, which is the maximum deceleration the driver of vehicle $n-1$ is willing to take on as estimated by the driver of vehicle $n$, is particularly hard to determine. This parameter depends on the properties of two vehicles, as opposed to the other parameters, which only concern one.

Different ways of determining this parameter are given in the literature. Wilson (2001) found that with a deceleration rate, $b_n$, lower than the one estimated for the downstream vehicle, Gipps’ model produces unrealistic results. Practically this means that the driver of a vehicle believes that he or she can decelerate faster than the downstream vehicle. Based on this, it is chosen that $\hat{b}$ always should be lower than $b$. That is, the ratio between $\hat{b}$ and $b$ should never be lower than one. This means that the driver of a vehicle estimates that the driver of the downstream vehicle can decelerate faster than himself. This is an incentive for the driver to keep a distance to the downstream vehicle.

\[
\hat{b} \leq b_n
\]
\[
\frac{\hat{b}}{b_n} \geq 1
\]  
\hspace{1cm} (5.2)

In his validation of the original model Gipps (1981) set $\hat{b}$ according to Equation (5.3). The minimum of $-3$ m/s$^2$ and the average between $b_n$ and $-3$ m/s$^2$.

\[
\hat{b} = \min\{-3, \frac{b_n - 3}{2}\}
\]  
\hspace{1cm} (5.3)

In AIMSUN two alternatives are given for estimation of $\hat{b}$. It is either assumed that the driver can estimate the downstream vehicle’s deceleration perfectly, see equation (5.4a). Alternatively it is set to the average of the decelerations rates of vehicle $n$ and vehicle $n-1$, see Equation (5.4b). (Transport Simulation Systems 2010)

\[
\hat{b} = b_{n-1}
\]  
\hspace{1cm} (5.4a)

\[
\hat{b} = \frac{b_n + b_{n-1}}{2}
\]  
\hspace{1cm} (5.4b)

In their study of vehicle type dependent parameters in Gipps’ model Ravishankar & Mathew (2011) chose $\hat{b}$ directly from a normal population, with a mean lower than for $b_n$.

From a practical point of view, the driver of a vehicle has little knowledge of the deceleration ability of the downstream vehicle. The aggression of this vehicle’s driver is hard to determine, and the maximum braking power may differ from vehicle to vehicle. Due to this it is considered reasonable to choose $\hat{b}$ at random from a normal population. However the model verifies that $\hat{b} \leq b_n$, holds for each vehicle pair. If this is not satisfied, then $\hat{b}$ is set equal to $b_n$. 

CHAPTER 5. ORIGINAL MODEL

Effective vehicle size, \( s_n \)

The vehicle lengths for both cars and heavy vehicles were determined using the respective means and standard deviations found in the dataset. Because of a high standard deviation for the length of heavy vehicles, a minimum and maximum value was set for this vehicle type, in order to avoid unnaturally short vehicles, or vehicle lengths exceeding the legal maximum value on the road. Maximum length was thus set to 25.25 meters. Minimum length was set to 5.6 meters, which as mentioned is the limit for classifying a vehicle as heavy according to its length.

The size of the safety margin is initially set to AIMSUN’s suggestion of 1.0 meter. However this parameter may be changed in the calibration process.

Desired speed, \( U_n \)

This parameter has one mean and one standard deviation for each vehicle type. Values must be determined through calibration. Further details are explained in Section 5.4.

Reaction time (simulation step), \( \tau \)

This parameter is constant for all vehicles and both vehicle types. Must be determined through calibration. The initial reaction time when calibration is commenced is set equal to Gipps’ suggestion of \( \frac{2}{3} \) second.

5.2.2 Global parameters

Entry flows and vehicle arrivals

The entry speed is set to \( u_e = 15 \) m/s for all vehicles. This does not affect the results, as the speeds from that point forward are determined by Gipps’ model.

By using the maximum allowed vehicle length of 25.25 meters and the entry speed of \( u_e = 15 \) m/s, the minimum required headway for the vehicle arrival distribution is calculated using Equation (5.5).

\[
  h_{\text{min}} = \frac{25.25 \text{ m}}{15 \text{ m/s}} = 1.68 \text{ s}
\]

(5.5)

To add a safety margin, this value is rounded up to \( h_{\text{min}} = 2.0 \) seconds.

The choice of which flow rates the model is to be run with is based on Figure 5.3. It shows the distribution of flow rates registered for the vehicle observations in the field dataset. The flow rates are counted in intervals of 50 veh/h. The graph shows that there are few observations above 950 veh/h. To have sufficient foundation
5.2. INPUT PARAMETERS

Figure 5.3: Distribution of flow rates in the field dataset.

for the comparison, it is chosen to set the highest entry flow to this value. The
lowest entry flow is set to 200 veh/h. This value is chosen even though the dataset
contains vehicle observations at lower flow rates. However at low flow rates, the
amount of time-gaps smaller than 6 seconds might be low, and are therefore less
interesting.

To ensure observations within each 100 veh/h interval, it is chosen to use entry
flow from 200–950 veh/h at 50 veh/h intervals.

Number of vehicles and replications

The number of vehicles simulated in each simulation run was set to $N = 800$. This
value is chosen because a greater number caused Matlab to run out of memory.
The script could probably have been constructed differently to avoid this situation.
However, a sufficiently high amount of vehicle observations for comparison with
the dataset was obtained by running several replications. Thus the script was not
changed. Problems with shortage of memory did not occur with this solution as
many variables are cleared between each replication.

The desired number of vehicle observations was set equal to the number of ob-
servations found in the dataset, which was $N = 247,645$. With 800 vehicles
being simulated in each simulation run, the product of the number of replications
and the number of entry flows being simulated in each replication must be:
$247,645/800 \approx 310$. If entry flows from 200 to 950 veh/h at intervals of 50 veh/h
are to be simulated, this means that $310/16 = 19.4$ replications need to me made.
Because the number of replications must be an integer, it is rounded up to 20
replications.
The heavy vehicle share is equal for all entry flows for simplification reasons. This may however be a source of error, as this may differ in the field.

Length of simulation area and detector location

The choice of detector location will have an effect on the results. The time-gaps, which are of interest in this analysis, are those produced by Gipps' model. If the detector is placed in too close vicinity to the entry point, the registered vehicle spacing might be affected by the vehicle arrival distribution used to feed vehicles into the simulation area.

Therefore a steady state traffic flow should be present at the detector location. In this situation the majority of the vehicles in the model will be in situation one or two listed below, but vehicles in the third situation may also occur.

1. The vehicle is driving freely. Hence the desired speed of the vehicle is lower than that of the downstream vehicle. It is thus driving at a constant speed equal to its desired speed.

2. The vehicle is following one or more vehicles—driving in a platoon. That is the desired speed of the vehicle is equal to or higher than that of the downstream vehicle or the first vehicle in the platoon. It is thus following this vehicle at its desired speed.

3. The vehicle is accelerating or decelerating in order to reach one of the above situations.

Another argument against a too short simulation area is that at low flow rates, the distance between two simulated vehicles might be large. The model is not constructed for an empty simulation area. Minimum area length must therefore be longer than the longest expected headway.

The chosen length of the simulation area is based upon a hypothetical extreme situation. The total number of vehicles to be simulated should be \( N = 247,646 \approx 250,000 \). The largest headway calculated from the displaced exponential distribution which statistically will appear once every 250,000th vehicle is 157 seconds, provided the minimum entry flow rate and minimum headway given in the previous section (calculation is attached in Appendix E.2). The simulation area must be long enough to ensure that if only one vehicle is present in the simulation area, for example the first simulated vehicle, this must not leave before the next enters. If a vehicle is able to keep a speed of for example 35 m/s (126 km/h), and the time until the next vehicle arrival is 157 seconds, the area length needs to be:

\[
L = 157 \text{ s} \cdot 35 \text{ m/s} \approx 5500 \text{ m}
\]  

(5.6)

The simulated detector is placed 500 meters before the end of the simulation area, \( x_{\text{detector}} = 5000 \text{ m} \), in order to avoid effects the boundary could have on the results.
5.3 Error checking

The purpose of the step in the simulation process called error checking, as described by the Federal Highway Administration (2004), is to ensure that the vehicles in the model behave correctly before the calibration process is commenced. If not, calibrated parameters might be distorted to compensate for overlooked coding errors.

Error checking proceeds in three stages:

1. Software error checking
2. Input coding error checking
3. Animation review to spot less obvious input errors

The first two steps were executed by performing a thorough review of the Matlab script, in order to eliminate any programming or input errors.

Unlike many traffic simulation softwares, animated simulation is not included in this Matlab script. The third step of error checking is therefore performed by looking at vehicle trajectories from one single simulation run. These can be used to check two important issues:

1. A vehicle cannot overtake another vehicle.
2. When approaching a downstream vehicle, which is keeping a lower speed, the observed vehicle should reduce speed gradually until the downstream vehicle’s speed is reached at the desired vehicle spacing.

Figure 5.4: Vehicle trajectories from the model.

Figure 5.4 shows how the positions of vehicles simulated by the model change over time. Each line represents one vehicle. The speed of the vehicle is the line’s
slope. Entry flow in this example was 750 veh/h and 200 vehicles were simulated. However only the time period from 60 to 240 seconds and the distance from 0 to 1000 meters are included in the figure. It shows that no lines cross each other, which is the correct behavior according to the first point in the list above.

Lines in the figure, which meet another line with a less steep slope, assumes this line’s slope rather than crossing it. This corresponds to the situation where a vehicle approaches another vehicle which keeps a lower speed. The figure shows that the lines change slope gradually, meaning that the vehicles reduce speed gradually. This is in accordance with the second point listed above, and is the transition between the free driving and car-following situation.

The figure shows the vehicle headways chosen by the vehicle arrival distribution along the time-axis at position zero. These are not equal to the ones at positions greater than zero. This shows that the vehicles spacing changes as the vehicles are affected by Gipps’ model.

5.4 Calibration

5.4.1 Method

According to the Federal Highway Administration (2004), the objective of calibration is to improve the ability of the model to accurately reproduce local traffic conditions. Their calibration guidelines will be followed in this analysis.

Available calibration parameters should be divided into two basic categories:

- Parameters that one is certain about and do not wish to adjust.
- Parameters that one is less certain about and willing to adjust.

In this case, the parameters which are considered certain are the ones that can be derived from the field data. These are vehicle lengths and the portion of heavy vehicles. The remaining input parameters are eligible for calibration.

A three-step strategy is recommended for calibration: capacity calibration, route choice calibration, and system performance calibration.

Field data presented in Chapter 4 show that the road did not meet its capacity during the data collection period. Therefore a capacity calibration is difficult to perform. The second point is irrelevant in this case, as the model only comprises one straight section of road. Hence, the vehicles can only choose one route.

The remaining step is system performance calibration. The purpose of this step is to calibrate the overall performance of the model.

It is chosen to perform calibration on both the macroscopic and the microscopic level. On the former, the relationship between flow rate and speed will be calibrated. These variables are chosen over density because they can be estimated
5.4. CALIBRATION

from the dataset with least uncertainty. The microscopic calibration will comprise fitting the time-gap distribution, as the focus in this study is on this variable. Distributions are more accurate than average time-gaps. For example will different distributions be able to yield the same average value.

5.4.2 Macroscopic calibration

Hourdakis et al. (2003) have developed a practical procedure for calibrating microsimulation models. As a goodness of fit measure between field data and model results, they use the Root Mean Squared Percent Error (RMSP). This value is defined as:

\[
RMSP = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left( \frac{x_i - y_i}{y_i} \right)^2}
\]  

(5.7)

where \( x_i \) is the simulated measurement and \( y_i \) is the actual traffic measurement. Hourdakis et al. (2003) used this value to compare flow rates at \( n \) different points in time. However, as this model does not simulate a specific time period, this measurement is not relevant. It is rather chosen to compare the modeled and field average speeds for different flow rate intervals. Because the RMSP describes sizes of errors, it should ideally be as low as possible. For flow rate measurements Hourdakis et al. (2003) suggest that the RMSP should be lower than 15 %.

Although the average section speed could be measured accurately in the model, it is chosen to estimate this value the same way as in the field data. That is the average section speed is calculated by taking the harmonic mean of individual vehicle speeds at the detector location. This is done in order to compare the results on equal basis.

In order to find the optimal input parameters, the Golden Selection Search Algorithm has been used. This is explained in Appendix C. The algorithm was implemented in a Matlab script, which is attached in Appendix D.4.

The input parameter chosen for calibration is the desired speed. This is the parameter which is considered to have the greatest influence on the flow-speed relationship in uncongested flow, where many vehicles will be able to choose their own speed. Both mean and standard deviation for cars and heavy vehicles is to be calibrated. However values of the input parameters for cars might affect the heavy vehicles and vice versa because they are driving in the same traffic stream.

The mean desired speed in the model is expected to be higher than the average speed observed in the field. This is because many vehicles will not be able to keep their desired speeds, if they drive behind a vehicle with a lower desired speed. A vehicle will however never drive faster than its desired speed, as this is a premise in Gipps’ model.
The standard deviation will regulate how much speed varies with flow rate. A high standard deviation will increase the difference between the mean desired speed and the lowest desired speed among the vehicles. This may lead to more vehicles driving in platoons, which again will lead to fewer vehicles being able to keep their desired speeds. Thus will the model output be a lower mean speed. The higher the flow rate, the more vehicles are present on the road in a given time period. This means more vehicles driving in the platoons.

The number of times the search algorithm is to be repeated must be chosen based on a compromise between accuracy and computation time. As each simulation lasts several minutes, a high number of repetitions will make the calibration process very time consuming. It is chosen to round the value to the nearest 0.1 m/s, as this is the accuracy of vehicle speeds registered in the field.

Calibration was initiated for cars with a search area between 20 and 22 m/s. With the desired accuracy of $\pm 0.1 \text{ m/s}$, a resulting interval of 0.2 m/s is needed. Because each repetition narrows the search interval by a factor of 0.618, the required number of repetitions is found by using Equation (5.8).

\[
\text{Initial interval} \times \text{Reduction factor} = \text{Resulting interval} \\
(22 - 20) \times 0.618^n = 0.2 \\
n \ln 0.618 = \ln \frac{0.2}{2} \\
n = \frac{\ln \frac{0.2}{2}}{\ln 0.618} = 4.78
\]

As $n$ must be an integer, $n = 5$ repetitions is chosen for the search algorithm.

Figure 5.5 shows how the RMSP changes for different values of the mean and the standard deviation for the desired speed of cars. The graph shows that the model produces the best fit when $\mu_U = 20.7 \text{ m/s}$ and $\sigma_U = 1.4 \text{ m/s}$. These values correspond to a RMSP value of 0.5 %. However, the graph shows that the RMSP values found for other combinations of mean and standard deviation only differ by about 0.2 percentage points. As expected, it also appears that the higher the standard deviation is, the higher the means must be.

Figure 5.6 shows how the RMSP changes for different values of the mean and the standard deviation for the desired speed of heavy vehicles. Early calibration attempts showed that the algorithm’s search area needed to be adjusted to 19–21 m/s. The graph shows that the model produces the best fit when $\mu_U = 20.2 \text{ m/s}$ and $\sigma_U = 1.8 \text{ m/s}$. These values correspond to a RMSP value of 0.9 %. The standard deviation for heavy vehicles is higher than for cars. This corresponds to the standard deviations found in the field. A reason could be that the heavy vehicle class comprises a large span of vehicle types, and must thus comprehend a greater difference in vehicle characteristics than the car class. Something, which also could contribute to the higher RMSP value, thus a poorer fit.
5.4. CALIBRATION

5.4.3 Microscopic calibration

Numerous input parameters, which may affect the vehicle spacing, make it difficult to use the search algorithm, which was used for the macroscopic calibration, for microscopic calibration. Therefore the time-gap distribution calibration was performed by manually changing the input parameters and visually determining the fit between the field and modeled distributions. Without compromising the fit for
the relationship between flow rate and speed.

It should be noted that Rakha & Gao (2010) present a way of calibrating all the input parameters in Gipps’ model by using macroscopic variables. This particular method does however require data from other traffic states than uncongested flow, which was the only one present at the detector location, such as jam density and flow and speed at capacity. One could argue that if Greenshields’ relationship is assumed, these variables could be estimated by fitting a parabolic function to the flow-speed data, and thus predict the traffic stream characteristics of congested flow. However, this approach would involve a high level of uncertainty. In addition to this, Rakha & Gao (2010) state that their calibration method offers a sub-optimal fit if data show that the traffic stream speed is sensitive to the traffic stream flow in the uncongested regime. The data reviewed in Chapter 4 suggest that this correlation is present in the used dataset.

A negative side of not being able to apply this calibration method is that manual calibration is likely to be more time consuming.

In order to make the calibration process more effective and less time consuming, each input parameter’s effect on time-gap in the model is investigated. The situation studied is when a vehicle has a higher desired speed than the downstream vehicle, which drives at constant speed. The vehicle will after a period of time, catch up to the downstream vehicle and assume this vehicle’s speed at a certain distance from it. Because both vehicles’ speeds are equal and constant, \( u_n(t + \tau) = u_n(t) = u_{n-1}(t) = u \) applies.

The time-gap, \( t_g \), between the two vehicles in this situation can be expressed according to Equation (5.9). The entire derivation is attached in Appendix E.1.

\[
    t_g = \frac{3}{2} \tau + \frac{u}{2} \left( \frac{1}{b} + \frac{1}{b_n} \right) + \frac{1}{u} (s_{n-1} - l_{n-1}) \tag{5.9}
\]

Each input parameter is changed one by one, while the other remain constant. The values used for the input parameters, which are not under investigation, are set to the values given in Appendix B. The vehicles keep a constant speed of \( u = 20 \text{ m/s} \).

**Acceleration, \( a_n \)**

The acceleration parameter, \( a_n \), has no influence on the time-gap calculated from Equation (5.9). Because this parameter is only present in the first of the two speed equations in Gipps’ model (see Equation (2.13a) on page 12). A vehicle’s speed is only determined by this equation when it is driving freely and accelerating to or keeping its desired speed. Consequently when the distance to the downstream vehicle is large. When the gap is smaller and the vehicle is following another vehicle, the second equation determines the speed (see Equation (2.13b)). Hence this parameter has no influence on small gaps.
5.4. CALIBRATION

Deceleration, $b_n$

Figure 5.7: The relationship between deceleration and time-gap.

Figure 5.7 shows how the time-gap chosen by the following vehicle changes when the deceleration parameter varies between $-10.0$ and $-1.0 \text{ m/s}^2$. Time-gap increases as the deceleration rate increases. For high deceleration values the time-gap is more sensitive to change in this parameter compared to low decelerations values. This parameter seem to have a great influence on the vehicle spacing. The difference in time-gap between a deceleration of $-2.0$ and $-3.0 \text{ m/s}^2$ is close to 1.5 seconds. At a speed of 20 m/s this corresponds to a difference in space-gap of 30 meters.

Effective size of downstream vehicle, $s_{n-1}$

This parameter includes the length of the downstream vehicle and a safety margin, into which the following vehicle is not willing to intrude even at rest. Only the latter will have an effect on the time-gap, as the time-gap is calculated from the rear of the downstream vehicle. Figure 5.8 shows how the time-gap chosen by the follower vehicle changes as the safety margin varies from zero to about one car length (5 meters). The time-gap increases linearly as the safety margin increases. This is expected as the value only specifies the point, in relation to the position of the downstream vehicle, where the follower needs to be able to make a complete stop if the downstream vehicle should commence braking. However the effect of changes in the safety margin are small compared to changes in the deceleration rate. The difference in time-gap between a safety margin of 1 and 2 meters is 0.05 seconds, which corresponds to 1 meter when driving at 20 m/s. Hence a change in the input parameter is observed with equal magnitude in the time-gap.
Figure 5.8: The relationship between safety margin and time-gap.

**Estimated deceleration of downstream vehicle, \( \hat{b} \)**

Figure 5.9: The relationship between estimated deceleration of downstream vehicle and time-gap.

Figure 5.9 shows how the time-gap changes with variation in the deceleration rate of the downstream vehicle as estimated by the driver of the observed vehicle. Time-gap increases as this deceleration parameter decreases. The faster the driver of the following vehicle believes that the leader vehicle is able to decelerate, the longer distance he needs to keep in order to be able to make a complete stop during the
same period of time. As for the $b_n$ parameter, the increase is more severe for high deceleration rates than for low.

**Relationship between $\hat{b}$ and $b_n$**

![Figure 5.10: The effect of the ratio between estimated deceleration of the down-stream vehicle and deceleration of the observed vehicle on time-gap.](image)

Figure 5.10 shows how the time-gap is affected as the ratio between the two deceleration parameters in the model, $\frac{\hat{b}}{b_n}$, is varied between 0.5 and 2, and $b_n = -3 \text{ m/s}^2$. The figure shows that the time-gap increases as the difference between the two parameters increases.

It also shows that when $\frac{\hat{b}}{b_n} < 1$, that is $\hat{b} > b_n$, the time-gap becomes small and eventually below zero, which is not acceptable as time-gap only takes positive values. This supports the before mentioned demand that $\hat{b} \leq b_n$ must hold for all vehicle pairs.

**Reaction time, $\tau$**

The relationship between reaction time, which is equal to the simulation step, and the time-gap chosen by the following vehicle is linear, as shown in Figure 5.11. Time-gap increases as reaction time increases. The physical interpretation of this is that the longer it takes for the driver of the following vehicle to react to the driving behavior of the lead vehicle, the greater distance he needs to keep in order to be able to make a stop without collision.
CHAPTER 5. ORIGINAL MODEL

Figure 5.11: The relationship between reaction time (simulation step) and time-gap.

Vehicle speed, $u_n$

Figure 5.12: The relationship between vehicle speed and time-gap.

Other parameters included in the speed equation in Gipps’ model are the speeds of both the lead and following vehicles. In this particular situation these are equal, as the speed of the follower is limited by the speed of the leader. Figure 5.12 shows that the relationship between the speed that both vehicles keep and the time-gap is linear. Time-gap increases with increasing speed. This is expected as
the following vehicle needs a longer time to decelerate when the speed is higher. However the effect of changes in this parameter is less significant than changes in other parameters.

**Calibration in practice**

How the different input parameters affect the time-gap were investigated in the previous sections. This section covers how this information can be used when calibrating the time-gap distribution produced by the model.

A factor, which makes the calibration procedure more complex, is that most of the input parameters are picked at random from a normal population with a mean and a standard deviation. This does not only mean that the results will be affected by randomness, and that there will be diversity in the parameters among the vehicles, but also that each input parameter has two degrees of freedom.

- (a) Change in mean value of deceleration $b_n$.
- (b) Change in standard deviation of deceleration $b_n$.
- (c) Change in safety margin included in effective vehicle size $s_{n-1}$.
- (d) Change in reaction time $\tau$.

Figure 5.13: Examples of calibration of time-gap distributions.
By taking into account the effects of changes in the input parameters, which were investigated in the previous sections, the following changes in mean and standard deviation can be made to the input parameters to adjust the time-gap distribution produced by the model. Some examples are shown in Figure 5.13.

- Reduce mean of deceleration
  This means that the vehicles’ deceleration parameters generally will have lower values, and more vehicle will thus keep a shorter distance to the down-stream vehicle. The result will be that the peak on the time-gap distribution is moved closer to zero. However the shape of the distribution curve may change because the relationship between deceleration rate and time-gap is non-linear. The same effect can be achieved by increasing the mean of the estimated deceleration. This may however not be as effective, because this parameter depends on the deceleration of the observed vehicle due to the demand that $\hat{b} \leq b_n$ must hold for all vehicle pairs.

- Increase mean of deceleration
  This will have the opposite effect of reducing the mean. More vehicles will have higher deceleration rates, and will thus keep a longer distance to the vehicle ahead. The peak of the time-gap distribution will be moved further away from zero.

- Reduce standard deviation of deceleration
  By reducing the standard deviation of the deceleration parameter, more vehicles will have a deceleration rate closer to the mean. That is the deceleration capabilities of the vehicles and drivers are less diverse. The result will be a more concentrated time-gap distribution. The distribution curve will have a more evident peak, and a higher corresponding frequency. The frequency of time-gaps further away from the peak will decrease.

- Increase standard deviation of deceleration
  Increasing the standard deviation of deceleration will lead to more vehicles having deceleration parameters further away from the mean. The vehicles in the model will thus have more diverse deceleration capabilities. This will result in a less concentrated time-gap distribution. That is, the distribution curve will have a less evident peak and the corresponding frequency will deviate less from the frequencies of other time-gaps.

- Change safety margin
  Changing the size of the safety margin, into which the observed vehicle is not willing to intrude, will shift the distribution curve towards the left or the right, depending on whether the size is decreased or increased. However changing this parameter when calibrating the model may not be effective. In order to increase the time-gap of all vehicles of one vehicle type by one second, the safety margin would have to be extended by 20 meters at speed 20 m/s. The effect is more significant when calibrating the model for congested conditions, where speeds are lower.
5.4. **CALIBRATION**

- Change reaction time
  Changing the reaction time will also lead to a shift in the curve. However this change will affect all vehicle types, as this parameter is not vehicle type dependent.

The microscopic calibration process proceeds in the following steps:

1. Run simulation.

2. Look at resulting time-gap distribution and investigate where there are deviations compared to the field data. Make sure that the macroscopic flow-speed relationship has not been compromised.

3. Change input parameters according to the deviations and the guidelines mentioned in this section.

4. Run new simulation. If the results are adequate then the model is considered calibrated. If not, return to step 2.

Calibration was performed for cars and heavy vehicles separately.

### 5.4.4 Calibration results

The input parameters resulting from the calibration process are given in Tables 5.1 and 5.2, for cars and heavy vehicles.

Table 5.1: Input parameters for cars. Parameters with no standard deviation are constant, and not picked from a normal distribution

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Unit</th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum acceleration</td>
<td>$a_n$</td>
<td>m/s²</td>
<td>3.0</td>
<td>0.2</td>
</tr>
<tr>
<td>Maximum deceleration</td>
<td>$b_n$</td>
<td>m/s²</td>
<td>2.9</td>
<td>1.0</td>
</tr>
<tr>
<td>Vehicle length</td>
<td>$l_n$</td>
<td>m</td>
<td>5.5</td>
<td>0.9</td>
</tr>
<tr>
<td>Effective vehicle length</td>
<td>$s_n$</td>
<td>m</td>
<td>$l_n + 1.1$</td>
<td></td>
</tr>
<tr>
<td>Desired speed</td>
<td>$U_n$</td>
<td>m/s</td>
<td>20.7</td>
<td>1.4</td>
</tr>
<tr>
<td>Estimated deceleration</td>
<td>$b$</td>
<td>m/s²</td>
<td>6.2</td>
<td>1.0</td>
</tr>
<tr>
<td>Reaction time</td>
<td>$\tau$</td>
<td>sec</td>
<td>0.8</td>
<td></td>
</tr>
</tbody>
</table>

The values presented in the tables above were the ones which provided the best fit. The most notable difference between the input parameters for the two vehicle types is that both deceleration parameters have a lower mean for heavy vehicles than for cars.

The tables show that each vehicle type requires 11 input parameters. The reaction time is excluded in this number because it is not vehicle type differentiated.
Table 5.2: Input parameters for heavy vehicles. Parameters with no standard deviation are constant, and not picked from a normal distribution

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Unit</th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum acceleration</td>
<td>$a_n$</td>
<td>m/s$^2$</td>
<td>1.0</td>
<td>0.5</td>
</tr>
<tr>
<td>Maximum deceleration</td>
<td>$b_n$</td>
<td>m/s$^2$</td>
<td>2.5</td>
<td>1.0</td>
</tr>
<tr>
<td>Vehicle length</td>
<td>$l_n$</td>
<td>m</td>
<td>10.8</td>
<td>5.0</td>
</tr>
<tr>
<td>Effective vehicle length</td>
<td>$s_n$</td>
<td>m</td>
<td>$l_n + 1.0$</td>
<td></td>
</tr>
<tr>
<td>Desired speed</td>
<td>$U_n$</td>
<td>m/s</td>
<td>20.2</td>
<td>1.8</td>
</tr>
<tr>
<td>Estimated deceleration</td>
<td>$\hat{b}$</td>
<td>m/s$^2$</td>
<td>5.5</td>
<td>0.9</td>
</tr>
<tr>
<td>Reaction time</td>
<td>$\tau$</td>
<td>sec</td>
<td>0.8</td>
<td></td>
</tr>
</tbody>
</table>

5.4.5 Validation

It should be verified that the model generally produces adequate results with the calibrated input parameters, not just for the relationship between flow rate and speed and the time-gap distributions, which the model was calibrated against. Therefore the model results for a different variable relationship is compared to similar field data. In this case the relationship between flow rate and all time headways, that includes those greater than 6 seconds, is chosen. The result is shown in Figure 5.14. Results from the model seem to fit to the field data. The largest deviations are found below 200 veh/h and above 900 veh/h.

![Flow-timegap](image)

Figure 5.14: Average timegap for flow rate intervals of 100 veh/h for field data and original model.

The model should also be validated in order to verify that the fit between the field data and the model output generally holds for that particular road section, not just
5.4. CALIBRATION

for the dataset used for calibration. Therefore the model output was compared to a second dataset collected on September 1–11 and September 20–30 2012 from the same detector.

Validation dataset

There are some potential uncertainties in the validation dataset compared to the original dataset. Because these data were collected about one month later, the surface temperatures may have been lower. However this should not be of significance due to the built-in adjustment of this in the equipment. Another factor is that one additional month has elapsed since calibration of the WIM point compared to when the original dataset was collected. As frequent calibration is found to be important for the accuracy, this might affect the results.

An investigation on some vehicle weight statistics is initiated in order to map potential differences between the datasets. The median weight for all vehicles in the validation dataset is 2311 kg, which only differs from the median weight of the original dataset of 2309 kg by 1 %. For cars the median weights are accordingly 2189 kg and 2188 kg for the validation and the original dataset. This also constitute a 1 % difference. Correspondingly for heavy vehicles 8327 kg and 5510 kg. The difference in median weight for this vehicle class is 51 %.

That the median weight for all vehicles and for cars are almost equal for the two datasets can be used for arguing that the weight curve is not shifted. Median weights for heavy vehicles however differ by 51 %. A reason for this can be found by looking at the share of each vehicle group. In the original dataset, heavy trucks constitute 43 % of the heavy vehicles. In the validation dataset this share is 51 %. A higher heavy truck to light truck ratio may be the reason for the higher median weight found for heavy vehicles in the validation dataset. However if this difference is caused by more heavy trucks on the road in the second data collection time period, or errors in the registrations is not known.

Validation results

Using the original dataset, the RMSP when comparing the simulated and field relationship between flow rate and speed for all vehicles was $RMSP_{\text{calibration}} = 0.61\%$. When comparing the simulated results against the validation dataset, $RMSP_{\text{validation}} = 0.75\%$ was obtained. The difference of 0.14 percentage points is in this case accepted, and the model is considered valid. A reason for the deviation may be the differences in the weight distribution between the two datasets.
5.5 Results

Due to high number of vehicle observation all vehicle data derived from the model is not attached in this report. It can however be found in the digital attachment. The field dataset used when describing the results is from August.

![Flow-speed relationship](image)

Figure 5.15: Relationship between flow rate and estimated average section speed in field data and from the original model.

Figure 5.15 shows the estimated average section speeds for flow rate intervals of 100 veh/h for all vehicles in the field dataset from August and simulated by the original Gipps’ model. The curves have the same shape, and the maximum deviation between the two points is less than 1 km/h. That the model provides results that fit to the observed traffic situation is not surprising, as Gipps’ model is a recognized car-following model, and used in commercial traffic simulation softwares. Besides, the traffic conditions on the data collection location were not complex, and there were few interfering aspects.

The reason for deviation could either be that the model is not able to model the field conditions accurately, or it could be that more extensive calibration efforts would have provided an even better fit.

The resulting time-gap distributions are shown in Figures 5.16a and 5.16b. The former shows the probability density function and the latter shows the cumulative distribution function of time-gaps smaller than 6 seconds for all vehicles. The time-gaps are counted in intervals of 0.5 seconds. The graphs show that the distributions are close to each other, but that the simulation model produces to many time-gaps shorter than 1 second and too few time-gaps longer than 4 seconds.

However, also in this case the deviation could be caused by both lacking ability
5.5. RESULTS

(a) Probability density function. 

(b) Cumulative distribution function.

Figure 5.16: Distribution of time-gaps shorter than 6 seconds for all vehicles in field data and from the original model.

by the model to reproduce the correct time-gap distribution, or lacking calibration efforts.

Figure 5.17: Distribution of time-gaps shorter than 6 seconds for cars in field data and original model.

Figures 5.17a and 5.17b show the probability density function and cumulative distribution function for time-gaps for cars, that is vehicles lighter than 3500 kg. The distributions are similar to the distributions for all vehicles shown in Figure 5.16, and the deviations are also here for time-gaps shorter than 1 second and longer than 4 seconds. The similarities are not surprising as cars constitute 86 % of all vehicles, and will thus have great influence on the results for all vehicles.

The time-gap distributions for all heavy vehicles, that is vehicles heavier than 3500 kg, are shown in Figures 5.18a and 5.18b. The modeled distribution is also in this case similar to the distribution derived from the field data. The graphs show
that more time-gaps longer than 2.5 seconds are found among the heavy vehicles compared to cars, and consequently fewer that are shorter than this value.

Figures 5.19 and 5.20 show the modeled and the field probability density functions and cumulative distribution functions for the time-gap distributions of all vehicles, divided into six weight groups. The group boundaries are set at intervals of gross weight 10,000 kg. An exception is made for the lowest group, which is further divided into two groups containing vehicles with gross weight 0–3500 kg and 3500–10,000 kg in order to separate cars from the heavy vehicles.

The top left graph in each figure, which shows the time-gap distribution for vehicles with gross weight from 0–3500 kg, is the same as the one shown in Figure 5.17. As mentioned above, it shows that the model provides a good fit for this vehicles type. The top right graph shows a good fit also for the vehicles weighing between 3500 and 10,000 kg, but the probability density functions shows a slight overestimation of the lengths of the time-gaps. However for the remaining four distributions, which are for vehicles heavier than 10,000 kg, the model produces a poorer fit to the field distributions. For these four weight groups the model distribution curve is shifted to the right of the field curve. This indicates that the models produces too many short time-gaps.

It is not unexpected that the average modeled distribution for all heavy vehicles is close to the field distribution for vehicles weighing between 3500 and 10,000 kg. As mentioned in Chapter 4, light trucks constitute more than half of the heavy vehicles. Their time-gaps will thus have great effect on the average.

The consequence of underestimating the time-gaps of heavy vehicles is that the model estimates that these vehicles occupy less space on the road than they do in the field. This may again lead to an overestimation of road capacity.

The distributions in Figures 5.19 and 5.20 show the problem, which arises when
5.5. RESULTS

Figure 5.19: Probability density function for the distribution of time-gaps shorter than 6 seconds for vehicles in different weight groups.

using average input values for a vehicle class that comprises such a large span of vehicle types. Even though a model provides a good fit between the field and simulated distribution for all the heavy vehicles combined, the time-gap distributions for smaller groups of heavy vehicles deviate from this.

One might ask how the model can produce a good fit for the relationship between flow rate and speed even though it does not reproduce correct vehicle spacings for different groups of heavy vehicles. A reason for this is that cars constitute the majority of the vehicles present on the road section. And the time-gap distribution for this vehicle type was modeled with a good fit. This will have considerable
effect on the combined results. In addition to this, the fit for the average time-gap distribution for the heavy vehicles was good. This involves that the underestimation of the time-gaps for some heavy vehicles are counteracted by overestimation of the time-gaps for other heavy vehicles. An important factor is also that the road section, which was used as a reference, had not met its capacity. This means that there was still spare space on the road. Deviations in short time-gaps could be counteracted by longer time-gaps, which occur when vehicles are driving freely. This could however not happen in congested conditions, where most time-gaps are short and empty road space is scarce.

Figure 5.20: Cumulative distribution function for the distribution of time-gaps shorter than 6 seconds for vehicles in different weight groups.
Chapter 6

Modified Model

6.1 Problems with the original model

The issue with the original model, when it was applied with two vehicle types, was that the time-gap distributions for portions of the heavy vehicles, grouped according to gross weight, did not fit to the field data. Even though the fit between modeled and field average time-gap distribution for all the heavy vehicles was good.

One could argue that if the original model is applied with a higher number of vehicle types, each representing a smaller group of heavy vehicles, the time-gap distributions could be fitted better by calibration. However as a complete set of 11 vehicle related input parameters would need to be defined for each vehicle type, the calibration process would be comprehensive.

Although the number of additional vehicle types would not necessarily need to be high to make an improvement. For example does Figure 4.7 on page 32 show that the relative differences between the field time-gap distribution for vehicles heavier than 10,000 kg are small compared to the distributions for vehicles lighter than 10,000 kg. This could support an approach where the heavy vehicles are separated into two types, for example light trucks and heavy trucks, in the model.

An analysis of the original model with this vehicle configuration is however not initiated. Focus is rather devoted to an investigation on whether the model can be modified to completely eliminate the need for separation of vehicles according to type.
6.2 Weight incorporation

One possible solution to this issue is to introduce a weight parameter, $w_n$, to the model which stands for the gross weight of vehicle $n$ in kilograms. This will yield a continuous weight distribution among the vehicles in the model, rather than the current practice of assigning them into discrete weight groups. Each vehicle’s weight will then be assigned by picking a random value from an inputed weight distribution, which is preferably collected in the field. The behavior of a vehicle will then be affected by the vehicle weight. If this approach is successful only one set of input parameters will need to be defined.

6.2.1 Goals

The goal is that the improved model should possess the following properties:

1. It should be able to reproduce the differences in driving behavior caused by differences in vehicle weight. In this case the time-gap distributions for different vehicle weight groups will be used as a reference.

2. The number of new constants which are introduced in addition to the weight parameter should be as small as possible.

3. The original three properties, which Gipps’ model was built to possess, should not be compromised. See Section 2.3.

The first goal is based upon the motivation for modifying the model; that the time-gap distributions for the heavy vehicles did not fit to field data. Because the performance of the original model probably could be improved by including more vehicle types, which would comprise multiple new input parameters, the number of new parameter is preferably kept low for the modified model to offer an advantage. Hence the second goal in the list above. To make sure that the modifications do not compromise the overall performance of Gipps’ original model, which may be the reason for its success, it is desirable to keep the properties which it was originally built to possess.

6.2.2 Method

The two equations, from which a vehicle’s speed is determined in Gipps’ model, are repeated in Equation 6.1.

$$u_n^a(t + \tau) = u_n(t) + 2.5a_n\tau \left( 1 - \frac{u_n(t)}{U_n} \right) \sqrt{0.025 + \frac{u_n(t)}{U_n}}$$  \hspace{1cm} (6.1a)
There are three possible approaches when a new parameter is to be introduced to the equations:

1. Add a new element to the equations where the new parameter is included.

2. Multiply one or more of the existing elements with the new parameter, or a factor which is determined by a function of the new parameter.

3. Exchange one of the existing elements.

A new element can be added to one or both of the equations if the new parameter has direct influence on vehicle speed. The new element must be of unit meters per second, to be in accordance with the remaining elements of the equation.

The second approach presented above could be followed if the new parameter does not necessarily have a direct influence on the vehicle speed, but affects one or more of the other input parameters of the model.

To exchange one of the existing elements might be useful if it can be shown that it contributes to incorrect vehicle behavior.

The second approach is chosen because there is limited data to support a direct influence of vehicle weight on vehicle speed in all situations. There is also no evidence to support that one of the existing elements causes incorrect vehicle behavior.

In order to achieve a complete weight incorporation in Gipps’ model, the weight parameter needs to be introduced to both speed equations. However as vehicle spacing is chosen as the variable of comparison, in addition to the fact that it is impossible to derive information on acceleration rates from traffic data collected at a single point, the latter equation is devoted most effort. This is Equation (6.1b).

### 6.2.3 Analysis

The chosen method can proceed in two different ways:

1. Analyze the effect of the new parameter on each single element of the equation.

2. Analyze the effect of the new parameter on each existing parameter in the equation. That is, all elements where this parameter is present will be affected. And the parameter is indirectly made weight dependent.

The latter is chosen because this more directly can be related to driving behavior and findings in the dataset and the reviewed literature.
The chosen approach also serves another advantage. By making one or more of the existing input parameters dependent on vehicle weight, which means that the values will vary according to the weight of the vehicle, the model will still operate within its original frames. That is, the same modeled behavior of vehicles with one particular weight could be achieved by calibrating the input parameters in the original model. The validity of the model is thus not compromised. This approach does in theory involve applying the original model with a high number of vehicle types. For example if the weight distribution ranges from 0 to 60,000 kg, and has a resolution of 100 kg, this would correspond to 600 vehicle types in the original model.

As discussed in Section 5.4.3 the parameters affecting small gaps between downstream vehicle \(n-1\) and observed vehicle \(n\) are:

- \(b_n\): The most severe deceleration that the driver of the observed vehicle is willing to undertake.
- \(s_{n-1}\): The effective size (length + safety margin) of the downstream vehicle.
- \(\hat{b}\): The most severe deceleration that the driver of the downstream vehicle is willing to undertake, as estimated by the driver of the observed vehicle.
- \(\tau\): The reaction time.
- \(u_n(t)\) and \(u_{n-1}(t)\): The speeds of both the observed and downstream vehicle in the preceding time step.

Changing the speed parameters according to vehicle weight would not be practical. This is the speed which is calculated for the vehicles in the preceding time step, and the vehicles have kept these speeds for the duration of the time step. Making these parameters weight dependent would thus be to change something which has happened in the past.

The driver’s reaction time, \(\tau\), is also not eligible for being affected by the vehicle’s weight. This is because Gipps’ model is based upon a constant reaction time for all vehicles. As the model’s simulation step also equals the reaction time, making this parameter vehicle dependent would make the simulation procedure more complex.

The safety margin included in the effective size of the leader vehicle, \(s_{n-1}\), could be used for the purpose. The result of multiplying this parameter with a factor, which increases with the vehicle’s weight, would be that vehicle spacing increases with vehicle weight. However this would only affect the vehicle spacing, not other aspects of the driving behavior. For example would a heavy vehicle keep a longer gap to the downstream vehicle than a car, which is in accordance with the data collected at the detector, but the deceleration rate of the two vehicle types would be equal. This does not agree with the findings of Cristoforo et al. (2004), who found decreasing deceleration rates with increasing vehicle weight. Besides, no literature has been found to support the assumption that the safety margin behind the downstream vehicle, into which the driver will not intrude even at rest, increases with the observed vehicle’s weight.
6.2. WEIGHT INCORPORATION

Figure 6.1: Relationship between vehicle weight and average time-gap shorter than 6 seconds and fitted curve.

Only the deceleration parameters remain as eligible for being affected by the vehicle’s weight, both the one of the observed vehicle and the one estimated for the downstream vehicle. As shown in Section 5.4, these have the greatest impact on vehicle spacing. Choosing the most severe deceleration rate the driver of a vehicle wishes to undertake according to the vehicle’s weight can be justified according to the findings of Cristoforo et al. (2004). In addition, the calibration performed for the original Gipps’ model, which is covered in Section 5.4, yielded the best fit compared to the field data when mean deceleration rates were lower for heavy vehicles than for cars.

The next step is to determine how the deceleration parameters should be reduced in order to replicate the differences in vehicle spacing caused by differences in vehicle weight. Assuming a mere linear decline would probably yield incorrect results, as the average time-gap was found not to increase linearly with vehicle weight.

Another alternative is to use the deceleration rates found in the tests performed by Cristoforo et al. (2004) to derive a function describing the relationship between vehicle weight and deceleration rate. However vehicles with gross weight below 40 tonnes were not included in these tests. The values of the function below this weight would thus be difficult to determine. Another argument against this approach is that the deceleration rates found in the study were maximum deceleration rates capable by the vehicles. The deceleration rates in Gipps’ model need to take both vehicle and driver characteristics into account.

A third option is to look at the relationship between vehicle weight and time-gap. A regression performed by Matlab shows that the average time-gaps for vehicle weight grouped in 5000 kg intervals are estimated well ($R^2 = 0.92$) by a double
exponential function, see Figure 6.1 and Equation (6.2), where \( w \) is vehicle weight and \( \bar{t}_g \) is average time-gap.

\[
\bar{t}_g = 2887 \exp(1.231 \cdot 10^{-6} w) - 888.9 \exp(-0.0002977 w)
\]  

One possible approach is to assume that the relationship between the deceleration rate of a modeled vehicle and the vehicle’s weight is similar to the relationship between vehicle weight and time-gap. Meaning that it also can be described by a double exponential function, which is shown in its general form in Equation 6.3, where \( c_1 – c_4 \) are constants.

\[
c_1 \exp(c_2 w) + c_3 \exp(c_4 w)
\]

By using the double exponential function, some characteristics of the relationship between vehicle weight and average time-gap will be lost. For example the decline in average time-gap for vehicles with gross weights between 20,000 and 30,000 kg. Such characteristics could have been included with a different kind of function, for example a high degree polynomial function, but that would entail more constants. It is also not certain if this decline is caused by actual differences in driving behavior, is just coincidental, or caused by registration errors. Besides, a regression curve fitted to average values is not able to incorporate the standard deviation within each weight group.

Ideally the relationship between the estimated most severe deceleration of the downstream vehicle, \( \hat{b} \), and vehicle weight is the same as the one between \( b_n \) and vehicle weight. This would involve the lowest number of new constants, given that the constants are equal. Three possible options arise in this case: The parameter can either be (1) affected by the weight of the observed vehicle; (2) affected by the weight of the downstream vehicle; or (3) independent of vehicle weight. The former would in practice mean that drivers of light and heavy vehicles estimate another vehicle’s deceleration ability differently. An assumption which cannot be supported by literature. The practical interpretation of the second option is that the driver of a vehicle estimates another vehicle’s deceleration ability according to the weight of the other vehicle, and thus will the other vehicle’s weight indirectly affect the spacing between the vehicles. This is supported by Sayer et al. (2000) and Brackstone et al. (2009), who found that the size of the downstream vehicle influenced the following headway. It must however be assumed that this holds for other vehicle pairs as well as car following truck. In addition, Ravishankar & Mathew (2011) found that including properties of the downstream vehicle into the equations in Gipps’ model yielded more accurate results. Therefore it is chosen to let \( \hat{b} \) be adjusted according to the weight of vehicle \( n – 1 \). The third option is thus discarded.

This means that the deceleration parameters used in the modified Gipps’ model will be calculated by multiplying basis parameters \( b_0 \) and \( \hat{b}_0 \) with weight dependent factors \( \alpha_n \) and \( \alpha_{n-1} \). The basis parameters should be picked randomly from a
normal population for each vehicle. If the basis parameters were only picked once, all vehicles with the same gross weight would behave equally. However both vehicle performance and driver characteristics may differ among vehicles of equal weight.

\[ b_n = \alpha_n b_0 \]  
\[ \hat{b} = \alpha_{n-1} \hat{b}_0 \]  

(6.4a)  
(6.4b)

The weight dependent factor, \( \alpha_n \), is determined for each vehicle by the double exponential function.

\[ \alpha_n = c_1 \exp(c_2 w_n) + c_3 \exp(c_4 w_n) \]  

(6.5)

The second speed equation in Gipps’ model assumes the following form:

\[ u_n^b(t + \tau) = \alpha_n b_0 \tau + \sqrt{(\alpha_n b_0)^2 \tau^2 - \alpha_n b_0 \left[ 2[x_{n-1}(t) - s_{n-1} - x_n(t)] - u_n(t) \tau - \frac{u_{n-1}(t)^2}{\alpha_{n-1} b_0} \right]} \]  

(6.6)

The weight factor, \( \alpha_n \)

Figure 6.2 shows the form of the double exponential function, which is thought to be eligible for this purpose. The function makes a rapid decline for low gross vehicle weights, while the decline for higher weights is smaller. A similar pattern was observed for the increase in average time-gaps. The function consists of three
parts: one with a steep slope, one with a gradual slope, and a transition zone between the two.

The values of the constants used for the function in Figure 6.2 were: \( c_1 = 0.9 \), \( c_2 = -2 \cdot 10^{-7} \), \( c_3 = 0.1 \), and \( c_4 = -3 \cdot 10^{-4} \).

(a) How constant \( c_1 \) affects the weight factor.
(b) How constant \( c_2 \) affects the weight factor.
(c) How constant \( c_3 \) affects the weight factor.
(d) How constant \( c_4 \) affects the weight factor.

Figure 6.3: How the different constants in the double exponential function affect its value.

The graphs in Figure 6.3 show how the four constants, \( c_1 \), \( c_2 \), \( c_3 \) and \( c_4 \), affect the weight factor. The first constant, \( c_1 \), regulates at which value the function assumes a gradual slope. Constant \( c_2 \) affects the slope of the gradual part of the function. The third constant changes the point where the function intersects the y-axis. Whereas the fourth constant determines how abrupt the transition zone is and the slope of the steep part of the function.

Both \( c_1 \) and \( c_3 \) determine the function’s relation to the y-axis. And it seems that the intersection point is located at \( \alpha = c_1 + c_3 \). By deciding that zero weight always should correspond to a weight factor equal to one, then \( c_3 \) can be expressed by \( c_1 \):
\( c_3 = 1 - c_1 \). The number of constants is thereby reduced. The function assumes the form given in Equation (6.7). Former constant \( c_4 \) is renamed \( c_3 \).

\[
\alpha_n = c_1 \exp(c_2 w_n) + (1 - c_1) \exp(c_3 w_n) \quad (6.7)
\]

**Interpretation of constants \( c_1, c_2 \) and \( c_3 \)**

The factor \( \alpha_n \) itself is without units because the product of the weight factor and the deceleration rate needs to have the same units as the deceleration rate. The constant \( c_1 \) is also without unit as it is multiplied by an exponential expression. Constants \( c_2 \) and \( c_3 \) are multiplied by the vehicle weight \( w_n \) and must therefore have unit \( \text{kg}^{-1} \).

The interpretation of this is not intuitive. The constants can for example be looked upon as parameters describing resistance in deceleration. Another interpretation is that constants \( c_2 \) and \( c_3 \) are reference weights describing some extreme situation. For example could \( 2 \cdot 10^{-7} \text{ kg}^{-1} \cdot w \text{ kg} \) be rewritten as \( w \text{ kg}/20,000,000 \text{ kg} \). However such a high reference weight cannot be connected to any practical aspect of driving or vehicle specifications.

### 6.3 Simple test

The modified model needs to be tested to verify that the modifications have the desired effect. That is, the weight dependent deceleration factors should make the model reproduce the weight dependent time-gap differences observed in the field. Before a full-fledged simulation study of the modified model is initiated, the effect that the modifications have on time-gaps calculated using Equation (6.8), which is derived in Appendix E.1, is investigated. This is to check if the chosen approach seems reasonable, or if another method of including weight should be considered.

\[
t_g = \frac{3}{2} \tau + \frac{u}{2} \left( \frac{1}{b} + \frac{1}{b_n} \right) + \frac{1}{u} (s_{n-1} - l_{n-1}) \quad (6.8)
\]

As mentioned earlier, the time-gap between a vehicle and the downstream vehicle can be calculated using this equation if the following holds:

- The downstream vehicle drives with constant speed.
- The observed vehicle has a higher desired speed than the current speed of the downstream vehicle.
- The observed vehicle has caught up with the downstream vehicle, and keeps this vehicle’s speed.
The time-gap expression for the modified model is given in Equation (6.9).

\[
t_g = \frac{3}{2} \tau + \frac{u}{2} \left( \frac{1}{\alpha_{n-1} b_0} + \frac{1}{\alpha_n b_0} \right) + \frac{1}{u} (s_{n-1} - l_{n-1})
\]  

(6.9)

Figure 6.4 shows how the time-gap calculated using Equation (6.9) varies with gross weight of the observed vehicle. The downstream vehicle was assumed to be a car with gross weight 2500 kg. The parameter values used in this example are \( u = 20 \) m/s and as described in Appendix B. The constants used to determine the weight factor were the ones used in the example in the previous section: \( c_1 = 0.9 \), \( c_2 = -2 \cdot 10^{-7} \), and \( c_3 = -3 \cdot 10^{-4} \).

The graph’s shape is similar to the curve fitted to the average time-gaps in Figure 6.1. However at this point the results must not be over interpreted. There are several factors, which have not been taken into account in this simple test, that will affect how the model performs in the simulation model developed in the previous chapter. Such as the effect of randomness and variation on the input parameters, and that some vehicles might not keep a constant speed. However, this simple test indicates that the modified model may have the desired effect.

6.4 Simulation

Because the results from the simple test seem convincing, a full-fledged simulation is initiated. This comprises making the necessary changes to the Matlab script for it to adopt the modified model. The modified Matlab script is attached in Appendix D.3.
6.4. SIMULATION

6.4.1 Script changes

The changes which need to be made in the script, are:

- The speed equations from the original Gipps’ model must be exchanged with the ones for the modified model.
- New values for the input parameters and the three constants in the weight factor must be determined.
- Vehicle weight must be picked at random from an inputed distribution.

Vehicle weight

A crucial part of implementing the modified model is that the weight distribution among the vehicles in the model is correct. A method of ensuring this is to use the vehicle weight distribution derived from the field data. Each vehicle’s weight is picked at random from this distribution. The resolution of the distribution is in this case set to 100 kg.

The script does this by first finding a random value between zero and one. The vehicle weight is chosen as the value on the cumulative weight distribution with probability equal to the random number.

Motorcycles are not included in this study. The modified model is therefore also not fitted to this type of vehicle. Minimum vehicle weight is set to 500 kg.

The modified model entails reducing the deceleration parameters of heavy vehicles in the model according to their weight. However this reduction will also apply to vehicles lighter than 3500 kg. This is because the value of the weight factor $\alpha_n$ varies within the weight interval of cars. There is however nothing to support that there are systematical differences in time-gap or deceleration rate within this class. As opposed to heavy vehicles where the ratio between gross weight and braking power can vary according to for example whether a vehicle is loaded or unloaded.

There are three alternatives for treating this case:

1. Still keep cars and heavy vehicles separated in two vehicle classes. The modified model is only applied to the heavy vehicles.

2. Adjust the vehicle weight distribution so that all vehicles lighter than 3500 kg are assigned the same weight. The input parameters for cars will thus not be weight dependent.

3. Make no changes. There is already variation in the input parameters of the cars due to the standard deviation of the normal distribution, from which they are picked. Weight dependent deceleration parameters will provide a new degree of variation, but may not yield incorrect results as long as both the standard deviation and the constants for the weight factor are calibrated.
The first option would undermine one of the advantages with the modified model, which is a low number of input parameters and elimination of the need to separate vehicles according to type. Of the remaining options the third is chosen over the second. This is because it can be advantageous to keep the weight distribution among the cars in the model, in case the modified model is to be used in an analysis where this is needed. However if the modified proves to produce incorrect results for cars, the second option should be considered.

**Vehicle length**

Vehicle length is also an input parameter in Gipps’ car-following model. When applying the model with different vehicle types, the lengths of the vehicles of each type are determined by picking a random value from a normal population. The mean and standard deviation can vary between vehicle types.

However, this does not work when only one set of input parameters are selected for all vehicles. If the vehicle lengths were chosen at random from a normal population in this case, they would not correspond to the vehicles’ weight. The result may be too long cars or too short heavy vehicles.

A solution to the problem is to seek a correlation between vehicle weight and vehicle length. This way the length of a vehicle can be determined using the vehicles’ weight, which is picked from the weight distribution.

![Figure 6.5: Relationship between average vehicle length and vehicle weight.](image)

Figure 6.5 shows the average vehicle length for vehicle weight intervals of 500 kg. The averages are fitted with a second degree polynomial function shown with a black line the figure, and written in Equation (6.10), where vehicle length, \( l \), is
given \( i \) meters and vehicle weight, \( w \), in kg. The polynomial provides a good fit with \( R^2 = 0.98 \).

\[
\bar{l}(w) = -5.8758 \cdot 10^{-9} w^2 + 0.00057928 w + 4.5758 
\] (6.10)

A weakness in using this relationship is that it is only based on average vehicle lengths and does not take the standard deviation within each weight interval into account. For example is the longest vehicle length obtainable when using the function 19 meters. This is lower than the maximum allowed length of 25.25 meters. There is also a matter of uncertainty. As mentioned in Chapter 4, the uncertainty is smaller for vehicle lengths than for vehicles weights collected from the WIM detector. Thus by connecting an accurate vehicle length to an inaccurate vehicle weight, the accuracy of the length is reduced.

The vehicle lengths will also be less affected by randomness than before. With this solution all vehicles, which have of the same weight, will have the same length. However, as vehicle weight in this case is picked in intervals of 100 kg, there are 600 possible vehicle weights, and thus the same amount of possible vehicle lengths. But should the weight distribution have a lower resolution, the diversity in vehicle lengths might be unnaturally low. One solution could be to let the constant part of the equation be picked randomly from a normal population.

That this method of determining vehicle length yields equally or more accurate values as the original model is important for the modified model to be successful. If not, the improved performance due to more accurate modeling of the spacing between vehicles could be counteracted by more inaccurate vehicle lengths. Both variables affect how much space a vehicle occupies on the road, and are important when modeling congested traffic conditions.

**Desired speed**

The original model produced the most accurate results when the desired speed of cars and heavy vehicles were different. This is however not possible in the modified model, as this is based upon only one set of input parameters. Thus only one mean and one standard deviation for the desired speed for all vehicles. This might yield incorrect vehicle speeds.

One solution to the issue is to make the desired speed weight dependent, in addition to the deceleration parameters. However, this would entail additional constants, which need to be calibrated. This should be avoided because one of the goals when developing the modified model is too keep the number of new constants as low as possible.

Besides, using only one mean and one standard deviation for the desired speed for all vehicles may not create significantly deviating results. The data reviewed in Chapter 4 shows that the mean speed for cars and heavy trucks only differed by
1 km/h. This is however for that particular road section, where the speed limit is 70 km/h. On other roads, especially where the speed limit is higher and perhaps more difficult or even illegal for heavy vehicles to keep, this issue might be of greater significance.

One simple solution, which would prevent the heavy vehicles from being assigned too high desired speeds, is to set a maximum value for this input parameter for vehicles heavier than a certain limit. For example 3500 kg. This would not necessarily entail greater calibration efforts, as the value could be set equal to the maximum recorded speed of a heavy vehicle in the simulation area, the maximum allowed speed for heavy vehicles of 80 km/h, or the speed at which a built-in speed restricter in the heavy vehicles is enabled. This solution is not adequate if the desired speed declines gradually with increasing weight.

**Deceleration parameters**

It is expected that the base deceleration parameters, $b_0$ and $\hat{b}_0$, need to be assigned higher mean values than the ones in the original model. This is because the value of the weight dependent factor, $\alpha_n$, equals one at zero weight, after which it declines with increasing weight. For example for the mean weight of cars, the factor will have a value lower than one.

**Acceleration**

With this approach of modifying the model, a logical solution for the acceleration parameter is to let this be affected by vehicle weight as well. Cristoforo et al. (2004) showed that acceleration rates declined with increasing gross weight. However how the acceleration parameter in the model should change according to weight is not possible to determine with the available data.

Ideally the same weight dependent parameter, which is used for the deceleration parameter, can be used for the acceleration parameter. But as this cannot be confirmed it is chosen to set the acceleration parameter equal to the one used for cars in the original model. The acceleration for heavy vehicles will not be modeled correctly, but because time-gaps shorter than 6 seconds are to be compared, the results are not affected. As discussed earlier, the acceleration parameter has no effect on small gaps.

**6.5 Calibration**

The calibration of the modified model proceeded in the same steps as for the original model. First the macroscopic flow-speed relationship was calibrated. Then, the time-gap distributions were fitted visually. The new constants were also calibrating by using time-gap distributions.
The resulting input parameters for the modified model are given in Table 6.1.

Table 6.1: Input parameters for the modified model. Parameters with no standard deviation are constant, and not picked from a normal distribution.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Unit</th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum acceleration</td>
<td>$a_n$</td>
<td>m/s$^2$</td>
<td>3.0</td>
<td>0.2</td>
</tr>
<tr>
<td>Maximum deceleration</td>
<td>$b_n$</td>
<td>m/s$^2$</td>
<td>3.0</td>
<td>1.2</td>
</tr>
<tr>
<td>Vehicle length</td>
<td>$l_n$</td>
<td>m</td>
<td>acc.</td>
<td>$w_n$</td>
</tr>
<tr>
<td>Effective vehicle length</td>
<td>$s_n$</td>
<td>m</td>
<td>$l_n + 1.0$</td>
<td></td>
</tr>
<tr>
<td>Desired speed</td>
<td>$U_n$</td>
<td>m/s</td>
<td>20.5</td>
<td>1.4</td>
</tr>
<tr>
<td>Estimated deceleration</td>
<td>$\hat{b}_n$</td>
<td>m/s$^2$</td>
<td>6.7</td>
<td>1.2</td>
</tr>
<tr>
<td>Reaction time</td>
<td>$\tau$</td>
<td>sec</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>$c_1$</td>
<td></td>
<td>0.78</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>$c_2$</td>
<td>kg$^{-1}$</td>
<td>$-9 \cdot 10^{-7}$</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>$c_3$</td>
<td>kg$^{-1}$</td>
<td>$-2.5 \cdot 10^{-4}$</td>
<td></td>
</tr>
</tbody>
</table>

The table shows that the total number of vehicle related input parameters in the modified model is 14. The original model with two vehicle types required 23 input parameters. With one additional vehicle type this number would have increased to 34.

6.5.1 Validation

Validation of the model does also proceed in the same steps as for the original model.

Figure 6.6 shows the relationship between flow rate and all time-gaps produced by the modified model. The largest deviation is for low flow rates. This is not surprising because the model is only calibrated against time-gaps shorter than 6 seconds. These are more likely to occur at higher flow rates.

The RMSP value for the flow-speed relationship of the modified model using the original dataset was 0.55 %. For the validation dataset this value was 0.66 %. A deviation of only 0.11 percentage points is accepted.

Based on these two tests, the modified model is considered valid.

6.6 Results

Figure 5.15 shows the estimated average section speeds for flow rate intervals of 100 veh/h for all vehicles in the field dataset and simulated by the modified Gipps’ model. The curves have similar shape, and the maximum deviation between average
Figure 6.6: Average timegap for flow rate intervals of 100 veh/h for field data and modified model.

Figure 6.7: Relationship between flow rate and estimated average section speed in field data and from the modified model.

speeds in the same flow rate interval is less than 1 km/h. This is not more than in the original model. Thus does this relationship not seem to have been compromised.

The resulting time-gaps from the simulation with the modified model are shown in Figures 6.8 and 6.9. The former shows the probability density functions and the latter shows the cumulative distribution functions for the time-gaps for different
weight groups. The figure shows that the field graph and the simulation graph for heavy vehicles are close to each other, which indicates that the distributions are similar.

Some deviations do appear. Most notably for vehicles with gross weights in the intervals 10,000–20,000 kg and 40,000–50,000 kg. This could be caused by insufficient calibration or lacking ability of the model to reproduce the field distributions. The latter is not unlikely, because the relationship between average time-gap and vehicle weight was only estimated by the double exponential function, there was no perfect fit.

From the probability density function for cars it seems that there are deviations in time-gaps shorter than 2 seconds. There seem to be too many time-gaps shorter than one second, and too few between one and two seconds. However the effect of the deviations on the cumulative distribution function do not seem to be of greater significance than deviations on distributions for other weight groups. Therefore are no further calibration efforts initiated.

It can however be argued that the accuracy for the distribution of time-gaps for cars should be prioritized higher than other weight groups. This is because cars constitute 86 % of all vehicles, and will thus have greater effect on the overall performance of the model than any other vehicle type.
Figure 6.8: Probability density functions for the distributions of time-gaps shorter than 6 seconds for vehicles in different weight groups from the modified model.
Figure 6.9: Cumulative distribution functions for the distributions of time-gaps shorter than 6 seconds for vehicles in different weight groups from the modified model.
Chapter 7

Comparison

It is chosen to compare the two models on three different points:

- Vehicle spacing distributions
  This is the chosen comparison variable.

- Vehicle spacing grouped by different pairs of leader and follower vehicles
  As it was chosen that the weight of the downstream vehicle should affect
  the estimated deceleration of the downstream vehicle, it is interesting to
  investigate whether this has had any effect.

- Vehicle lengths
  It was stated earlier that these need to be modeled equally accurate or more
  accurate in the modified model compared to the original. It should thus be
  investigated whether this is satisfied.

7.1 Vehicle spacing

7.1.1 Time-gap distributions

Figures 7.1 and 7.2 show the probability density functions and the cumulative
distribution functions for different weight groups. Field data as well as results
from both the original and the modified model are included in the figures.

For all heavy vehicles is the black dotted line is closer or equally close to the
blue circled line than the red, asterisk marked line. This shows that the modified
model produces results that are better fitted to the time-gap distributions for heavy
vehicles observed in the field.

The top left graph in each figure shows the distribution for cars. As mentioned
under the results for the modified model, there is a deviation between the distribu-
CHAPTER 7. COMPARISON

Figure 7.1: Distribution of time-gaps shorter than 6 seconds for vehicles in different weight groups from the both the original and the model.

tion produced by the modified model and the distributions from the field and the original model. However the deviations do not seem to have a considerable effect on the cumulative distribution function, especially compared to the deviations between the cumulative distribution functions from the original model and the field data for heavy vehicles.

Distributions for vehicles weighing between 3500 and 10,000 kg are shown in the top right graph in each figure. It seems that the modified model and the original model produce almost equally accurate results for this weight group. And that both models slightly overestimate the time-gaps.
7.1. VEHICLE SPACING

Figure 7.2: Distribution of time-gaps shorter than 6 seconds for vehicles in different weight groups from the both the original and the modified model. The remaining four graphs in each figure, which show the time-gap distributions for vehicle with gross weights between 10,000 and 50,000 kg show the same pattern. While the original model underestimates the lengths of the time-gaps compared to the field data, the modified model produces more accurate distributions. Some deviations appear, however these are small compared to the deviation in the distribution from the original model.

Figure 7.3 also shows the cumulative distribution functions for different weight groups. However, here the groups intervals are shifted 5000 kg from the ones in Figure 7.2. The graphs show that the modified model produces a good fit also if
CHAPTER 7. COMPARISON

Figure 7.3: Distribution of time-gaps shorter than 6 seconds for vehicles in different weight groups from the both the original and the modified model.

Figure 7.4 shows the cumulative distribution functions for time-gaps compared to the validation dataset from September. It shows that the modified model still provides an equally good or better fit to the time-gap distribution for all weight groups compared to the original model. Though the deviations are greater than compared to the original dataset. However it was shown that the ratio between heavy trucks and light trucks was different in the two datasets. This indicates a different weight distribution among the heavy vehicles. That is, the distribution of vehicle weights within each weight group shown in the figure may differ from the
7.1. VEHICLE SPACING

Figure 7.4: Distribution of time-gaps shorter than 6 seconds for vehicles in different weight groups from the both the original and the modified model compared to the September dataset.

original dataset. This will cause differences in the time-gap distributions. Based on this uncertainty it is chosen not to interpret the deviations in the figure as signs of invalidity in the model.

**Goodness of fit test**

The goodness for fit between two distributions can be estimated by different statistical tests. One of which is the $\chi^2$-test (Blakstad 1995). This is applied on the
time-gap distributions for heavy vehicles for the original and the modified model. For each weight group in each model the following hypothesis were tested:

$H_0$: The time-gap distribution from the model follows the time-gap distribution in the field data.

$H_1$: The time-gap distribution from the model is different from the time-gap distribution in the field data.

The null-hypothesis is rejected if a calculated $\chi^2$-value is higher than a critical value $\chi^2_{cr}$. A Matlab script was made to calculate the $\chi^2$ values. This is attached in Appendix D.5.

The time-gaps smaller than 6 seconds were distributed into intervals of size 0.5 seconds as in Figure 7.1. The number of bins were 11, thus 10 degrees of freedom in the $\chi^2$-distribution. At a 0.05 level of significance, the critical value is $\chi^2_{cr} = 18.31$.

Table 7.1: $\chi^2$-values

<table>
<thead>
<tr>
<th>Weight interval</th>
<th>$\chi^2$ Original model</th>
<th>$\chi^2$ Modified model</th>
</tr>
</thead>
<tbody>
<tr>
<td>3500-10,000 kg</td>
<td>254</td>
<td>122</td>
</tr>
<tr>
<td>10,000-20,000 kg</td>
<td>826</td>
<td>32</td>
</tr>
<tr>
<td>20,000-30,000 kg</td>
<td>1310</td>
<td>31</td>
</tr>
<tr>
<td>30,000-40,000 kg</td>
<td>1610</td>
<td>24</td>
</tr>
<tr>
<td>40,000-50,000 kg</td>
<td>1598</td>
<td>26</td>
</tr>
</tbody>
</table>

Table 7.1 shows the $\chi^2$-values, which were calculated for each weight interval of heavy vehicles for the time-gap distributions produced by the original and modified model. It shows that no values are lower than the critical value $\chi^2_{cr} = 18.31$. The conclusion of all tests is therefore that the null-hypothesis is rejected at a 0.05 level of significance. That is, the modeled time-gap distributions are not the same as the ones derived from the field data. The table does however show that the $\chi^2$-values for the modified model are lower than for the original model, and thus closer to the critical value. This supports that the modified model provides a better fit for the time-gap distributions of heavy vehicles than the original model.

7.1.2 Vehicle pairs

The modified model includes the weight of the downstream vehicle by letting the parameter $\hat{b}$ be adjusted by this vehicle’s weight. This section covers an investigation on whether the modified model improves the accuracy of time-gaps between different pairs of leader and follower vehicles where one or both are heavy.

The different pairs of vehicles which are compared are: car–car (leader–follower), car–heavy vehicle, heavy vehicle–car, and heavy vehicle–heavy vehicle. It was desirable to subdivide the heavy vehicles in to further groups in this analysis, but then the number of available vehicle pairs was so low that the distributions were not smooth. Especially for the case of heavy vehicle following heavy vehicle.
7.1. VEHICLE SPACING

Figure 7.5: Cumulative distributions of time-gaps shorter than 6 seconds grouped by pair of leader and follower vehicles.

Figure 7.5 shows the cumulative distributions of time-gaps sorted by pairs of leader and follower vehicle. The top right graph shows that both the original and the modified model produces accurate results for the case where heavy vehicle follows a car. In the situations where car follows car and car follows a heavy vehicle, the modified model produces a better fit than the original model. For the case of heavy vehicle following heavy vehicle there does not seem to be a considerable difference between the two models. In all cases of deviation the distribution curve is shifted to the right, meaning that the lengths of time-gaps are underestimated.

Table 7.2 shows the deviation in average time-gap shorter than 6 seconds from the field data in percent for different pairs of leader and follower vehicles in the original and modified model. It shows the same as Figure 7.5; the lowest deviation is found for heavy vehicle following car, and the largest improvement is found for car following heavy vehicle. The improvement for heavy vehicle following heavy vehicle is 0.1 percentage point, and is not considered to be of significance.

The largest deviations for the modified model are found in the cases where a heavy vehicle is the leader. This may be an indication that the same constants in the function determining the weight dependent reduction factor cannot be used for
both $b_n$ and $\hat{b}$.

Table 7.2: Deviations in average time-gap below 6 seconds for different pairs of leader and follower vehicle in the original and the modified model.

<table>
<thead>
<tr>
<th>Vehicle pair</th>
<th>Original model</th>
<th>Modified model</th>
</tr>
</thead>
<tbody>
<tr>
<td>(leader–follower)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Car-car</td>
<td>-3.4 %</td>
<td>0.7 %</td>
</tr>
<tr>
<td>Car–heavy</td>
<td>-1.7 %</td>
<td>-0.3 %</td>
</tr>
<tr>
<td>Heavy–car</td>
<td>-10.4 %</td>
<td>-5.6 %</td>
</tr>
<tr>
<td>Heavy–heavy</td>
<td>-5.3 %</td>
<td>-5.2 %</td>
</tr>
</tbody>
</table>

### 7.2 Vehicle length

As mentioned, the modified model should not produce a poorer fit for the vehicle lengths than the original model. This is because this variable, as well as the vehicle spacing, determines how much space a vehicle occupies on the road. Less accurate results for vehicle lengths could counteract the improved accuracy gained for the vehicle spacing.

![Vehicle length distribution for cars and heavy vehicles](image)

Figure 7.6: Cumulative vehicle length distributions for cars and heavy vehicles.

The graphs in Figure 7.6 show the cumulative distribution functions for the vehicle lengths derived from the field data and from the original and modified models. They show that neither the original nor the modified model produce an accurate fit for the length distributions. For the heavy vehicles the length distribution from the modified model does not seem to have more significant deviations from the field distribution than the original model.

The deviations are greater for cars than for heavy vehicles. The distribution curve produced by the original model has a less steep slope than the field curve. This
7.2. VEHICLE LENGTH

indicates that the standard deviation is too high. The distribution curve from the modified model has a slope which is more similar to the field curve. It does however seem to be shifted to the right. This indicates that the car lengths produced by the modified model are too long. Both errors might however be solved by changing the length input parameter.

It is surprising that the modeled vehicle lengths are different from the field lengths in the original model, even though the mean and standard deviation derived from the field data was used. A reason for this is that the model assumes that the vehicle lengths follow a normal distribution, but this is not necessarily true in the field.

Figure 7.7 shows the vehicle length distributions from the field data and the original and modified models after the length input parameter was changed. In the original model the mean and standard deviation for the length of cars were changed to \( \mu = 5.4 \) meters and \( \sigma = 0.4 \) meters. In the modified model the constant part of the function describing the relationship between weight and length was reduced to 4.2, see Equation (7.1). The figures show that a better fit for the length distribution of cars was obtained for both models.

\[
\bar{l}(w) = -5.8758 \cdot 10^{-9} w^2 + 0.00057928 w + 4.2 \tag{7.1}
\]

The conclusion to be drawn from this is that neither model produces an accurate fit for the vehicle lengths compared to the field data, when applied directly with length statistics derived from these data. Deviations were greater for cars than for heavy vehicles. This may be of significance because cars constitute the majority of the vehicles. The fit can however be improved if vehicle length is a variable, against which the models are calibrated.
Chapter 8

Discussion

8.1 Sources of error

Primarily, using a model is in itself a source of error. Models will never be able to reproduce real life phenomenas a hundred percent accurately. They can only approximate them. This means that even though the models used in this analysis seem to reproduce real traffic behavior accurately, there is no guarantee that this holds when for example using them to analyze different alternatives. A model should always be applied with caution.

The field data used in this analysis were important for calibrating, validating and comparing the original and the modified model. Thus may errors connected to these data have effect on the results and the drawn conclusion. Gross vehicle weights registered with Weigh-in-motion technology have proven to be uncertain. Vehicle weights play an important role in the analysis performed in this thesis, and the development of the proposed modified model. This uncertainty may therefore have had a considerable effect on the results. In the worst case the modified model may not be valid after all.

However, the car-following model was calibrated to produce a good fit to the traffic situation observed at the detector location. So even though the weight distribution, and heavy vehicle portion may not be correct, it is possible that re-calibration could adjust the differences between falsely registered and actual vehicle weight related variable relationships, if correct data were present. For example should systematical errors, such as general overestimation of vehicle weights be possible to compensate by changing the values of the constants in the modified model. The results would be more questionable if the systematical error changed according to weight. For example underestimation for some vehicle weight intervals and overestimation for others. A such situation was however not found by the Norwegian Public Roads Administration (2012b).
The choice of using point data over time-series data does also increase the uncertainty in the results. Vehicle speeds and time-gaps were compared under the assumption that both of these variables were constant at the detector location. However, a vehicle passing the detector might have been accelerating or decelerating, or it may have been in the process of changing its gap to the downstream vehicle. A high number of vehicle observations used when deriving traffic flow data and few interfering aspects around the detector should though reduce this source of error.

Some vehicle observations, which were obvious results of false registrations, were removed from the dataset in order to reduce this source of error. However, as the number of observations was very high, validating each registration would comprise much effort. Therefore, some obvious false registrations might not have been spotted. The high number of vehicle observations used in the comparison should however reduce the significance of this source of error, as each single registration has little influence.

The analysis is performed by implementing Gipps’ model in a Matlab script. Although a thorough review of the script has been performed numerous times in order to eliminate errors, the possibility still exists that something might have been overlooked. However, as the model produced credible results, any possible script error is not considered to have a considerable influence.

Time-gap distributions which were compared in this analysis were only separated by vehicle weight, not traffic situation. This involves that possible differences in time-gaps as a result of for example differences in vehicle speeds are not taken into account in the analysis. However there were found to be no considerable differences in traffic flow conditions during the data collection time period. This source of error is therefore not considered be of significance compared to the present uncertainty in data.

8.2 Limitations of the study

A major limitation of this study is that the model is only calibrated against and compared to point data. This only captures the car-following behavior at a single point in time. One consequence of this was that the model’s acceleration parameter could not be determined. The values and the modifications, which were applied to the deceleration parameters, will also contain a source of error. These were based upon the assumption that the premise of Gipps’ model, which is that a vehicle follows the downstream vehicle at a safe distance, holds for heavy vehicles. These parameters are however not compared to real deceleration rates, and for example how a heavy vehicle reacts to different maneuvers performed by the downstream vehicle. Even though there is literature to support a declining deceleration rate for increasing vehicle weight, there is no guarantee that the decline follows the same relationship as increasing time-gaps. If not, another way of including weight should
8.3. THE MODIFIED MODEL

be considered. For example making more input parameters weight dependent or adding a new element to the equation, which adjusts the difference in relationship between vehicle weight and deceleration and vehicle weight and time-gap. This would however entail several new constants, thus making the model less advantageous over the original.

The only traffic situation, which was simulated, was uncongested flow on a straight road section. There were no queues, intersections, curves or inclination. If the modified model is well behaved in these situations is therefore uncertain.

Both models were only calibrated against and compared to field data from one single location. Even though the models were validated against a second dataset, which was collected during a different period of time, one cannot be certain that the modified model performs well on other locations.

The main reason for these limitations was the limited availability of data, which included vehicle weight.

8.3 The modified model

8.3.1 Satisfaction of goals

Three goals for the modified model were presented in the previous chapter: (1) It should be able to reproduce the differences in driving behavior caused by differences in vehicle weight; (2) The number of new constants should be as low as possible; and (3) The original three properties of Gipps’ model should not be compromised. This section covers an investigation on whether these three goals are satisfied.

The results show that the model is able to reproduce differences in vehicle spacing caused by vehicle weight. However, there are other aspects of the driving behavior of heavy vehicles, which the model was not tested against in this study. Satisfaction of the first goal can therefore not be confirmed completely.

Determination of three new constants is required when applying the modified model. A solution with even fewer constants could perhaps have been found, but this would result in less freedom to calibrate the model. Such freedom is important as the relationship between driving behavior and vehicle weight might differ from place to place and situation to situation. In addition to this, the uncertainty connected to the weight data was high, and weight related variable relationships may thus differ from the ones in the dataset. It may therefore be wise not to further reduce the degrees of freedom.

The only changes, which were made to the model, were to make the deceleration parameters weight dependent. This does in practice mean applying the original model with a high number of vehicle types, each with different input parameters. Due to this it is assumed that the premises of Gipps’ model still hold. That is the modified model is also able to mimic the behavior of real traffic and that it is
well behaved when the reaction time equals the simulation step, like the original model. However, the three new constants in the modified model do not correspond to obvious characteristics of drivers and vehicles. At this point it does not satisfy the goals.

The last point raises the question of what is more practical to use; the modified model with few, but some non-relateable input parameters, or the original model with several vehicle types and more input parameters, but all of which are vehicle and driver related? The answer to this can depend on the data, which are available for calibration. If sufficient data to calibrate the three new constants in the modified model is present, this can be practical to use. However if calibration data is scarce, then it might be more practical to use vehicle and driver related parameters, for which suggestions can be obtained from various sources.

8.3.2 Advantages and disadvantages

The modified model eliminates the need to separate vehicles according to vehicle type, and it reduces the number of input parameters compared to using the original Gipps’ model with several vehicle types. The modified model has a total of 14 vehicle related input parameters, whereas the original has 11 input parameters per vehicle type. This is advantageous because applying the model requires less calibration efforts. However, this can also be disadvantageous because it reduces the amount of freedom in the model. For example will some parameters, which can be vehicle type differentiated in the original model, be equal for all vehicles in the modified model; such as desired speed. It must though be expected that some freedom is lost, when a model is simplified.

A premise for using the modified model is that detailed vehicle weight distribution data is present. This is a disadvantage because such data is not widely available. Vehicle weight data is also not possible to collect visually. One is therefore dependent on technical registration equipment. In addition to this, the quality of the data that is available is questionable. Both the availability and the quality may however be improved in the future. One could argue that average weight distributions which are assumed to be present on different road types could be used. However, this creates a source of error. If the used average distribution differs considerably from the actual distribution, the modified model might not be advantageous to use.

In addition to the need for a detailed weight distribution, a successful application of the modified model also depends on vehicle weight related traffic flow data, for example time-gap distributions, which were used in this study. Without such data it will not be possible to calibrate the model adequately. If a WIM detector is used for collection of vehicle weight data, this will also provide data on vehicle spacing.

Another argument against using the modified model is that the assumed relationship between vehicle spacing and gross vehicle weight may not hold in all situations. The driving style of heavy vehicle drivers can vary between different places, for example cities and rural areas. Differences might be even grater between countries.
Such possible differences in driving style also support that the model should not be used unless sufficient calibration data is present.

However, the double exponential function that determines the weight dependent factor has three degrees of freedom. This means that there are several possibilities for adjusting the model by calibration. Even though the function, which in this modified model is used to determine the weight dependent factor, does not yield correct results for all situations and areas, it shows that changing the deceleration parameters of the model according to gross vehicle weight seems to improve the accuracy of the results.

Another possible disadvantage is that introducing vehicle weight into Gipps’ model may create a conflict with the already present length parameter. Both weight and length are physical properties of a vehicle, and it can be shown that there is a correlation between them. When the original model was applied with the mean length and standard deviation derived from the field data, and the modified model was applied with length as a function of weight, they did not produce an accurate fit for the vehicle length distribution. The deviation was largest for cars. This was however improved by changing the length input parameter.

One of the advantages with the modified model is more accurate vehicle spacings. But how much space one vehicle occupies on the road will not be correct if the vehicle lengths are not in accordance with the field data. It is therefore recommended that the model should be calibrated against vehicle lengths.

### 8.3.3 Areas of application

When could the modified model be used? As discussed earlier the original model produced a good fit for the relationship between flow and speed, and overall time-gap distribution, in the analyzed traffic situation. The same results were obtained with the modified model. An improvement is not clear unless focus is on groups of heavy vehicles separated according to weight, and is most evident for vehicles heavier than 10,000 kg. The improvement in time-gap accuracy is however expected to have greater effect in congested conditions, where road space is scarce.

An area of application for the modified model could be when heavy vehicles are of particular interest in a simulation study. If this is the case, correct behavior of these vehicles is important. However, one could argue that accurate behavior also can be achieved by applying the original Gipps’ model with several vehicle types. For example if six vehicle types are created, each representing with gross weight ranging in a 10,000 kg interval. Using the modified model would still be advantageous as it comprises less calibration efforts.

One such situation could be when creating simulation models of road networks with a particularly high heavy vehicle portion. For example roads road leading to and from areas with a high density of industries dependent on road based goods transport. However, if the heavy vehicles in this area easily can be placed into a
limited number of weight groups, such as empty and fully loaded trucks, then the original Gipps' model might suffice.

On the other hand, if the weight distribution of the heavy vehicles in the modeled area is non-uniform, then the modified model may be advantageous to use.

There may however be other areas of application for the model in relation to the situations which were not tested in this study. That is, where other aspects of the driving behavior than vehicle spacing on a straight road section are important. These may also benefit from a continuous weight distribution. For example, acceleration of heavy vehicles on roads with inclination.

But the data show that the average time-gap increases rapidly for low weights, and slowly for higher weight. This can support an approach with the original where all heavy vehicles of higher weight are gathered into one vehicle type, while the number of vehicle types for lower weight are higher. This does however also entail a high number of vehicle types and input parameters. The modified model offers freedom to adjust the weight factor also for vehicles of low weights. This approach can therefore also be simplified with the modified model.

A factor which may limit the area of the application of the model today is that it needs to be implemented into microsimulation software in order to be usable when analyzing more complex traffic situations, where other models in addition to the car-following model are needed. Other models, such as lane change models, may not be adjusted to incorporate a weight parameter. That is they still depend on vehicles being separated according to type. If this is the case then applying the modified model will yield less advantages.
Conclusion

The purpose of this master’s thesis was to investigate the ability of Gipps’ car-following model to reproduce differences in driving behavior caused by differences in vehicle weight. A modified version of the model where vehicle weight is included as a parameter was to be estimated and compared to the original model and data collected in the field.

Data was collected from a point detector, which was equipped with Weigh-in-motion technology. This produced both traffic flow and vehicle weight statistics, however with a significant uncertainty within the latter. Time-gap was chosen as the variable, against which the models were to be compared.

The results show that Gipps’ original model can be calibrated to reproduce time-gap distributions and the relationship between flow rate and speed accurately in uncongested flow for cars and all heavy vehicles combined. However the latter comprises a large span of vehicle types. And when focusing on smaller portions of these vehicles, grouped according to gross weight, the modeled time-gap distributions deviate from the field distributions. That is, the ability of Gipps’ car-following model to reproduce differences in driving behavior caused by differences in vehicles’ weight is inadequate when only applied with two vehicle types. However a separation of heavy vehicles into multiple vehicle types is expected to improve this. An approach that would involve a high number of input parameters, because each vehicle type requires 11 additional input parameters.

The modified model is based upon eliminating the need to separate the vehicles into discrete vehicle types, but to introduce a weight parameter which is picked from a continuous weight distribution. The deceleration parameters in the model are made dependent on vehicle weight, and are reduced with increasing weight according to a double exponential function. The results were more accurate time-gap distributions for all groups of heavy vehicles, with only 14 input parameters.

Due to limitations of the study, the success of the modified model cannot be confirmed completely. The model results were only compared to point data. It was thus not possible to determine a potential weight dependent reduction of the acceleration parameter. In addition to this only the effects of the deceleration parameter on time-gaps was compared, not actual deceleration rates.
The area of application for a car-following model, which includes a weight parameter is limited today. This is because it demands a detailed vehicle weight distribution to yield advantages over the original model. Detectors that collect vehicle weight data are not widely available. In addition to this the quality of these data is questionable.
Further Work

It is recommended that further work is aimed at the limitations of this study.

The proposed modified model should be compared against time-series data to verify that the weight dependent reduction of the deceleration parameters produce correct deceleration rates, not just correct vehicle spacing. In addition to this, such data can also be used to determine which potential changes that need to be made to the acceleration parameter.

The investigation on the modified model’s effect on the time-gaps between different vehicle pairs indicated that the weight dependent factor affecting the estimated deceleration of the downstream vehicle should be calculated with different constants than the ones affecting the deceleration of the observed vehicle. Further work with the model should involve a more detailed investigation on this topic.

This was an initial study in the field of incorporating vehicle weight into Gipps’ car-following model. And it was found to make an improvement on a straight road section with no interferences. Further the modified model should be tested on new locations and in different traffic situations, such as congestion and urban traffic, and with other road alignments. This can for example be curves and road sections with inclination. This will show if the model is generally valid, or if its application should be limited to certain conditions. It will also show how the new constants vary according to location and situation.

Including the model in microsimulation software would also be of interest, as this will show how well the model performs together with other types of models, such as lane-change models.

In the longer term, a similar analysis can be performed for other car-following models. This may show if some models are better fitted for weight incorporation than others.
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Appendices
Appendix A

Task Description
Vehicle Weight in Gipps’ Car-Following Model

Kjøretøyvekt som parameter i Gipps’ bilfølgemodell

BACKGROUND

Car-following models are mathematical models which describe the behavior of a vehicle when following a downstream vehicle. They are used in traffic engineering, and play an important role in traffic simulation software.

It is found that a vehicle’s weight affects its driving behavior. This parameter is not represented directly in today’s most commonly used car following models, only indirectly through acceleration and deceleration parameters. This may produce errors in the simulation results, especially when modeling traffic streams with a non-uniform vehicle weight distribution or a high proportion of heavy vehicles.

TASK DESCRIPTION

The purpose of this master’s thesis is to investigate the ability of Gipps’ car-following model to model the differences in car-following behavior caused by vehicles’ weight. An improved Gipps’ model where weight is incorporated is to be estimated and compared to the original model and collected data.

The thesis can be divided into the following subtasks. The candidate shall:

- Perform a literature review of how vehicle weight affects driving behavior.
- Explain how car-following models work, with focus on Gipps’ model.
- Compare Gipps’ model to vehicle data, primarily from a two lane highway.
- Discuss how new parameters can be incorporated into Gipps’ model, and estimate an improved model where vehicle weight is represented.
- Compare the improved model with the original Gipps’ model and the data set.
- Discuss the results.
General about content, work and presentation

The text for the master thesis is meant as a framework for the work of the candidate. Adjustments might be done as the work progresses. Tentative changes must be done in cooperation and agreement with the professor in charge at the Department.

In the evaluation thoroughness in the work will be emphasized, as will be documentation of independence in assessments and conclusions. Furthermore the presentation (report) should be well organized and edited; providing clear, precise and orderly descriptions without being unnecessary voluminous.

The report shall include:

- Standard report front page (from DAIM, [http://daim.idi.ntnu.no/](http://daim.idi.ntnu.no/))
- Title page with abstract and keywords. (template on: [http://www.ntnu.no/bat/skjemabank](http://www.ntnu.no/bat/skjemabank))
- Preface
- Summary and acknowledgement. The summary shall include the objectives of the work, explain how the work has been conducted, present the main results achieved and give the main conclusions of the work.
- Table of content including list of figures, tables, enclosures and appendices.
- If useful and applicable a list explaining important terms and abbreviations should be included.
- The main text.
- Clear and complete references to material used, both in text and figures/tables. This also applies for personal and/or oral communication and information.
- Text of the Thesis (these pages) signed by professor in charge as Attachment 1..
- The report must have a complete page numbering.

Advice and guidelines for writing of the report is given in: “Writing Reports” by Øivind Arntsen. Additional information on report writing is found in “Råd og retningslinjer for rapportskriving ved prosjekt og masteroppgave ved Institutt for bygg, anlegg og transport” (In Norwegian). Both are posted on [http://www.ntnu.no/bat/skjemabank](http://www.ntnu.no/bat/skjemabank)

Submission procedure

Procedures relating to the submission of the thesis are described in DAIM ([http://daim.idi.ntnu.no/](http://daim.idi.ntnu.no/)). Printing of the thesis is ordered through DAIM directly to Skipnes Printing delivering the printed paper to the department office 2-4 days later. The department will pay for 3 copies, of which the institute retains two copies. Additional copies must be paid for by the candidate/external partner.

On submission of the thesis the candidate shall submit a CD with the paper in digital form in pdf and Word version, the underlying material (such as data collection) in digital form (eg. Excel). Students must submit the submission form (from DAIM) where both the Ark-Bibl in SBI and Public Services (Building Safety) of SBII has signed the form. The submission form including the appropriate signatures must be signed by the department office before the form is delivered Faculty Office.

Documentation collected during the work, with support from the Department, shall be handed in to the Department together with the report.
According to the current laws and regulations at NTNU, the report is the property of NTNU. The report and associated results can only be used following approval from NTNU (and external cooperation partner if applicable). The Department has the right to make use of the results from the work as if conducted by a Department employee, as long as other arrangements are not agreed upon beforehand.

**Tentative agreement on external supervision, work outside NTNU, economic support etc.**
Separate description to be developed, if and when applicable. See [http://www.ntnu.no/bat/skjemabank](http://www.ntnu.no/bat/skjemabank) for agreement forms.

**Health, environment and safety (HSE)**
[http://www.ntnu.edu/hse](http://www.ntnu.edu/hse)
NTNU emphasizes the safety for the individual employee and student. The individual safety shall be in the forefront and no one shall take unnecessary chances in carrying out the work. In particular, if the student is to participate in field work, visits, field courses, excursions etc. during the Master Thesis work, he/she shall make himself/herself familiar with “Fieldwork HSE Guidelines”. The document is found on the NTNU HMS-pages at [http://www.ntnu.no/hms/retningslinjer/HMSR07E.pdf](http://www.ntnu.no/hms/retningslinjer/HMSR07E.pdf)

The students do not have a full insurance coverage as a student at NTNU. If you as a student want the same insurance coverage as the employees at the university, you must take out individual travel and personal injury insurance.

**Start and submission deadlines**
The work on the Master Thesis starts on January 14, 2012

The thesis report as described above shall be submitted digitally in DAIM at the latest at 3pm June 10, 2013

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Other supervisors: Odd André Hjelkrem

Trondheim, January 24, 2013. (revised: 14.05.2013)
Appendix B

Input Parameters Used in Example

The following input parameters are used in the simulations conducted for example purposes:

\[
\begin{align*}
    a_n &= 2.0 \text{ m/s}^2 \\
    b_n &= -3.0 \text{ m/s}^2 \\
    \dot{b} &= -6.0 \text{ m/s}^2 \\
    s_{n-1} &= 6.0 \text{ m} \\
    \tau &= 0.8 \text{ s} \\
    U_n &= 25 \text{ m/s}.
\end{align*}
\]
Appendix C

Golden Selection Search Algorithm

The Golden Selection Search Algorithm is included in the microsimulation guidelines developed by the Federal Highway Administration (2004). It is a method for finding the value of a single parameter, that minimizes the squared error between a model and observations. This algorithm is an efficient way of narrowing down the interval, in which the optimal solution is located.

The algorithm proceeds in the following six steps:

1. Identify the minimum and maximum acceptable values, $p_{\text{min}}$ and $p_{\text{max}}$, for the parameter to be optimized.

2. Compute the squared error for maximum and minimum values.

3. Identify two interior parameter values for testing.
   The interior parameter values, $p_1$ and $p_2$, are selected using the following equations:
   
   \[ p_1 = p_{\text{min}} + 0.382 \left( p_{\text{min}} - p_{\text{max}} \right) \]  
   \[ p_2 = p_{\text{min}} + 0.618 \left( p_{\text{min}} - p_{\text{max}} \right) \]  

4. Compute squared error for the two interior parameters.

5. Identify the three parameter values that appear to bracket the optimum.
   The parameter value, $p_1$ or $p_2$, which produces the lowest square error is identified. The values to the left and right of this value is now chosen as the new minimum and maximum values. For example if $p_1$ produces the smallest error, then then $p_{\text{min}} = p_{\text{min}}$ and $p_{\text{max}} = p_2$. In this way the search area is narrowed.
6. Return to step 3 and repeat until the uncertainty is satisfactory. The process is repeated with the new minimum and maximum values. When the search range is as small as desired, the process is terminated.

Figure C.1: Golden Selection Search Algorithm. Source: Federal Highway Administration (2004).
D.1 Retrieve traffic data

```matlab
%% RETRIEVE DETECTOR DATA

clear % Clear all variables

files = [1 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19]; % Dates to be included

gaps = []; % Allocate variable

for i = 1:length(files) % Loop including all dates
    fileName = sprintf('1208%02d.txt',files(i)); % Set correct filename
    [tid felt fart lengde tidsluke headway vekt] = importfile_all(fileName,4,inf); % Import data from file to
    % variables

    q = zeros(2,96); % Allocate flow rate var.

    for j = 1:length(tid) % Loop for counting flow rates
        aaa = cell2mat(tid(j)); % Convert text date to number
        bbb = str2double(aaa(12:13)); % Extract hour number
        ccc = str2double(aaa(15:16)); % Extract minute number

        timecode = floor(bbb*4 + ccc/15 + 1); % Find number of 15 min int.

        q(felt(j), timecode) = q(felt(j), timecode) + 1; % Count flow rate for correct
        % lane and 15 min. interval

    end
```

119
q = q .* 4;            % Estimate equivalent hourly
% flow rate

for j = 1:length(tid) % Loop for sorting other vars.
    aaa = cell2mat(tid(j)); % Convert text date to number
    bbb = str2double(aaa(12:13)); % Extract hour number
    ccc = str2double(aaa(15:16)); % Extract minute number

    timecode = floor(bbb*4 + ccc/15 + 1); % Find number of 15 min int.

    gaps(end + 1, :) = [tidsluke{j} felt{j} q{felt{j},timecode} ...
                        lengde{j} fart{j} vekt{j} headway{j}]; % Add variables to result
    % array, which can be used
    % for analysis.
end
end
D.2. ORIGINAL MODEL

D.2 Original model

```matlab
1 \% ORIGINAL GIPPS CAR FOLLOWING MODEL
2
3 \% STEP 1
4 qs = [200:50:950]; % Entry flows to be simulated
5 weight_bins = [2500 7500]; % Available weights in weight distribution
6 weight_cum = [0.86 1.0]; % Share of heavy vehicles
7 % 2500 = car, 7500 = truck
8 weight_cum = [0.86 1.0]; % Share of heavy vehicles
9 % 1 - 0.86 = 0.14
10
11 \% Allocate variables
12 gapsm = []; qall = []; % Allocate variables
13
14 \% STEP 2
15 for replication = 1:20 % Number of replications
16 % Replications loop starts here
17 \% STEP 3
18 for m = 1:length(qs) % Entry flow loop starts here
19 clearvars –except m replication qs weight_cum weight_bins gaps gapsm ...
20 gapsn qall gapsv % Clear all variables, except those that
21 % need to be remembered between each
22 % replication
23
24 \% STEP 4
25 n = 800; % Number of vehicles in each simulation
26 n = n + 1; % First vehicle is not simulated
27
28 weight = zeros(n,1);a = zeros(n,1);b = zeros(n,1);lv = zeros(n,1);s = ...
29 zeros(n,1);V = zeros(n,1);bs = zeros(n,1); % Allocate space for variables
30
31 for i = 1:n % Pick vehicle weight from weight
32 weight(i) = weight_bins(find(weight_cum > rand(1),1)); % Here only two options: car and heavy
33
34 if (weight(i)<=3500) % If car
35 a(i) = normrnd(3, 0.2); % Acceleration
36 b(i) = -normrnd(2.9, 1); % Deceleration
37 lv(i) = normrnd(5.5, 0.9); % Length of vehicle
38 s(i) = lv(i) + 1.1; % Length + safety distance
39 V(i) = normrnd(20.7,1.4); % Desired speed
40 bs(i) = -normrnd(6.2,1); % Estimated deceleration
41
42 elseif (weight(i)>3500) % If heavy
43 a(i) = normrnd(1, 0.5); % Acceleration
44 if (a(i) < 0.5)
45 a(i) = 0.5;
46 end
47 b(i) = -normrnd(2.5, 1); % Deceleration
48 lv(i) = normrnd(10.8, 5); % Length of vehicle
49 if (lv(i) > 25.25) % Maximum and minimum value
50 lv(i) = 25.25; % for vehicle length
51
```
```matlab
elseif (lv(i) < 2.7)
    lv(i) = 5.6;
end

s(i) = lv(i) + 1;  % Length + safety distance
V(i) = normrnd(20.2,1.8);  % Desired speed
if (V(i) > 25)  % Maximum value for desired speed
    V(i) = 25;
end

bs(i) = -normrnd(5.5,0.9);  % Estimated deceleration

ers = find(b>0-0.5);  % Deceleration parameter
b(ers) = -0.5;  % restraint
ers = find(bs(1:n-1)>b(2:n));  % Estimated deceleration
bs(ers) = b(ers+1);  % restraint

T = 0.8;  % Reaction time / sim.step
l = 5600;  % Length of study area [m]
detector = 5100;  % including start position
% Detector location

% Vehicle arrivals
tm = 2;  % Minimum headway [s]
lambda = (qs(m)/3600)/(1-tm*qs(m)/3600);  % Lambda in exponential distribution
next = 0;  % Time for next vehicle entry

v = zeros(n,n*15);  % Allocate variable, save all recorded speed
x = zeros(n,n*15);  % Allocate variable, save all recorded positions
v(1:n,1) = 15;  % Initial speed
x(1:n,1) = 100;  % Entering position

vv = v(:,1);  % Last recorded speed
xx = x(:,1);  % Last recorded position

inside = [2 2];  % ID of first and last vehicle in study area
xaxis = zeros(1,n*15);  % Allocation variable, time axis
step = 2;

% STEP 5
while (inside(1) <= inside(2))
    % STEP 6
    for i = inside(1):inside(2)  % Select all vehicles inside study area
        % STEP 7
        va = vv(i) + 2.5.*a(i).*T.*(1 - vv(i)./V(i)).*(0.025 + ... vv(i)./V(i)).^(1/2);
```
if (i==inside(1))  % If first vehicle in area
    vv(i) = va;  % then speed is set to v_a
else
    vb = b(i).*T + ((b(i).*T).^2 - b(i).*((2.*((xx(i-1) - s(i-1)) ...
    - xx(i)) - vv(i).*T - vv(i-1).^2./bs(i-1))).^(1/2));  % Calculate v_b in Gipps' model
    vv(i) = min(va, vb);  % Set speed to smallest
    of v_a and v_b
end

xx(i) = xx(i) + vv(i)*T;  % Update position of vehicle

if (xx(inside(1)) > l)  % If vehicle has left study area
    inside(1) = inside(1) + 1;  % Set next vehicle to first vehicle in study area
end

if (xaxis(step-1) >= next) && (inside(2) < n)  % If time for next vehicle entry and there are still vehicles waiting
    inside(2) = inside(2) + 1;  % Add new vehicle
    hwy = exprnd(1/lambda) + tm;  % Find random headway from exponential distribution
    next = xaxis(step-1) + round(hwy/T)*T;  % Calculate time for next vehicle entry
end

v(:,step) = vv;  % Store speeds
x(:,step) = xx;  % Store positions
xaxis(step) = xaxis(step-1)+T;  % Add timestep to simulation clock
step = step + 1;  % Add timestep

q(1:floor(xaxis(step-1)/900+1)) = 0;  % Allocate variables for counting flow rates at 15 minute intervals
detectortimefront = zeros(1,n); detectortimerear = zeros(1,n); gap = ...
zeros(1,n); v_detector = zeros(1,n); headway = zeros(1,n);  % Allocate variables
for i = 2:n
    j = find(x(i,:) > detector,1);  % Find first recorded pos. which exceeds detector
    detectortimefront(i) = T * (detector - x(i, j-1))/(x(i, j) - x(i, ...
    j-1)) + xaxis(j-1);  % Detector time front of veh.
    headway(i) = ((detectortimefront(i) - xaxis(j-1))*(x(i-1, j) - ...
x(i-1, j-1)/T + x(i-1, j-1)) - detector;  % Headway

v_detector(i) = v(i, j - 1);  % Speed at detector

j = find(x(i,:), > (detector + lv(i)), 1);  % Find first recorded pos.
% exceeding detector + length

detectortimerear(i) = T * ((detector + lv(i)) - x(i, j))/((x(i, ...)
  j) - x(i, j - 1)) + xaxis(j - 1);  % Detector time rear of veh.

if (i > 2)
  gap(i) = (detectortimefront(i) - detectortimerear(i - 1));  % Calculate time gap

end

timecode = floor((detectortimefront(i)-detectortimefront(2))/900+1);  % Find number of 15 min. int.
q(timecode) = q(timecode) + 1;  % Count vehicle

q(1:end-1) = q(1:end-1).*4;  % Estimate hourly flow rate

lasttimeint = ... 3600/(detectortimefront(n)-((length(q)-1)*900+detectortimefront(2))));  % Check length of last int.

if (lasttimeint > 5)  % If last interval should be rejected.
  red = q(end);  % Number of vehicles not to be included in results.
else  % If not to be rejected
  q(end) = ... q(end)*3600/(detectortimefront(n)-((length(q)-1)*900+detectortimefront(2))));  % No vehicles excluded
end

qall = [qall q];  % Store flow rates

for i = 3:(n-{red})
  timecode = floor((detectortimefront(i)-detectortimefront(2))/900+1);
  gapsm(end + 1,:) = [gap(i)*1000 1 q(timecode) lv(i)*100 ... v_detector(i)*3.6 weight(i) headway(i)*100];  % Save relevant variables to array to be available for analysis
end

% STEP 13
end

% STEP 14
end
D.3 Modified model

```matlab
%% MODIFIED GIPPS CAR FOLLOWING MODEL

% STEP 1
qs = [200;50;950]; % Entry flows to be simulated
% Vehicle weight distribution is not defined
% in the script, but derived from field data

gapsn = []; qall = []; % Allocate variables

%STEP 2
for replication = 1:20 % Number of replications
  % Replications loop starts here

%STEP 3
for m = 1:length(qs) % Entry flow loop starts here

  clearvars ~except m replication qs weight_cum weight_bins gaps gapsm ...
  gapsn qall gapsv % Clear all variables, except those that
  % need to be remembered between each
  % replication

%STEP 4
n = 800; % Number of vehicles in each simulation
n = n + 1; % First vehicle is not simulated

weight = zeros(n,1);a = zeros(n,1);b = zeros(n,1);lv = zeros(n,1);s = ...
zeros(n,1);V = zeros(n,1);bs = zeros(n,1); % Allocate space for variables
for i = 1:n
  weight(i) = weight_bins(find(weight_cum > rand(1),1));
  % Pick vehicle weight from weight
  % distribution
  a(i) = normrnd(3.0, 0.2); % Acceleration
  b(i) = -normrnd(3.0, 1.2); % Deceleration
  lv(i) = -5.8758e-9*weight(i)^2 + 0.00057928*weight(i) + 4.5758;
  % Length of vehicle
  s(i) = lv(i) + 1.0; % Length + safety distance
  V(i) = normrnd(20.5,1.4); % Desired speed
  bs(i) = -normrnd(6.7,1.2); % Estimated deceleration
  c1 = 0.78; c2 = -9e-7; c3 = -2.5e-4;
  b(i) = b(i)*(c1*exp(c2*weight(i))+(1-c1)*exp(c3*weight(i)));
  bs(i) = bs(i)*(c1*exp(c2*weight(i))+(1-c1)*exp(c3*weight(i)));
  % Adjust b and b' by weight
  % factor
  if (V(i) > 25) && (weight(i)>3500) % Maximum value for desired
    V(i) = 25; % speed of heavy veh.
  end
  ers = find(b>-0.5); % Deceleration parameter
  b(ers) = -0.5; % restraint
```
APPENDIX D. MATLAB CODE

ers = find(bs(1:n-1)>b(2:n)); % Estimated deceleration
bs(ers) = b(ers+1); % restraint

T = 0.8; % Reaction time / sim.step
l = 5600; % Length of study area [m]
including start position

detector = 5100; % Detector location

%Vehicle arrivals

Tm = 2; % Minimum headway [s]
lambda = (qs(m)/3600)/(1-tm*qs(m)/3600); % Lambda in exponential distribution
next = 0; % Time for next vehicle entry

v = zeros(n,n*15); % Allocate variable, save all recorded speed
x = zeros(n,n*15); % Allocate variable, save all recorded positions
v(1:n,1) = 15; % Initial speed
x(1:n,1) = 100; % Entering position set to 100 m to avoid potential problems with values below zero

vv = v(:,1); % Last recorded speed
xx = x(:,1); % Last recorded position

inside = [2 2]; % ID of first and last vehicle in study area
xaxis = zeros(1,n*15); % Allocation variable, time axis
step = 2; % Time step counter

while (inside(1) <= inside(2)) % Select all vehicles inside study area

    % STEP 7
    va = vv(i) + 2.5.*a(i).*T.*(1 - vv(i)./V(i)).*(0.025 + ... vv(i)./V(i)).^(1/2); % Calculate v_a in Gipps’ model

    if (i==inside(1)) % If first vehicle in area
        vv(i) = va; % then speed is set to v_a
    else
        vb = b(i).*T + ((b(i).*T).^2 - b(i).*(2.*(xx(i-1) - s(i-1) ... - xx(i)) - vv(i).*T - vv(i-1).*T.^2./bs(i-1))).^(1/2); % Calculate v_b in Gipps’ model
        vv(i) = real(min(va, vb)); % Set speed to smallest of v_a and v_b
    end

    xx(i) = xx(i) + vv(i).*T; % Update position of vehicle

end
D.3. MODIFIED MODEL

end

% STEP 9
if (xx(inside(1)) > l) % If vehicle has left
    inside(1) = inside(1) + 1; % Set next vehicle to first
end

% STEP 10
if (xaxis(step−1) >= next) && (inside(2) < n) % If time for next vehicle
    inside(2) = inside(2) + 1; % Add new vehicle
    hwy = exprnd(1/lambda) + tm; % Find random headway from
    next = xaxis(step−1) + round(hwy/T)*T; % Calculate time for next
end

v(:,step) = vv; % Store speeds
x(:,step) = xx; % Store positions
xaxis(step) = xaxis(step−1)+T; % Add timestep to simulation
step = step + 1; % Add timestep

% STEP 11

% STEP 12
q(1:floor(xaxis(step−1)/900+1)) = 0; % Allocate variables for
% counting flow rates at
% 15 minute intervals
detectortimefront = zeros(1,n); detectortimerear = zeros(1,n); gap = ...
zeros(1,n); v_detector = zeros(1,n); % Allocate variables
for i = 2:n
    j = find(x(i,:) > detector,1); % Find first recorded pos.
    detectortimefront(i) = T * (detector − x(i, j−1))/(x(i, ...
    j − 1)) + xaxis(j−1); % Detector time front of veh.
    headway(i) = ((detectortimefront(i) − xaxis(j−1))*x(i−1, j) − ...
    x(i−1, j−1))/T + x(i−1, j−1)) − detector; % Headway
    v_detector(i) = v(i, j−1); % Speed at detector
    j = find(x(i,:) > (detector + lv(i)),1); % Find first recorded pos.
    detectortimerear(i) = T * ((detector + lv(i)) − x(i, ...
    j) − x(i, j−1)) + xaxis(j−1); % Detector time rear of veh.
if (i > 2)
gap(i) = (detectortimefront(i) - detectortimerear(i - 1));  % Calculate time gap
end
timecode = floor((detectortimefront(i) - detectortimefront(2))/900 + 1);  % Find number of 15 min. int.
q(timecode) = q(timecode) + 1;  % Count vehicle
end
q(1:end-1) = q(1:end-1).*4;  % Estimate hourly flow rate
lasttimeint = ... 3600/(detectortimefront(n) - ((length(q)-1)*900 + detectortimefront(2)));  % Check length of last int.
if (lasttimeint > 5)  % If last interval should be rejected.
    red = q(end);  % Number of vehicles not to be included in results.
else  % If not to be rejected
    q(end) = ... q(end)*3600/(detectortimefront(n) - ((length(q)-1)*900 + detectortimefront(2)));  % No vehicles excluded
end
qall = [qall q];  % Store flow rates
for i = 3:(n-(red))
timecode = floor((detectortimefront(i) - detectortimefront(2))/900 + 1);
gapsn(end + 1,:) = [gap(i)*1000 1 q(timecode) lv(i)*100 ... v_detector(i)*3.6 weight(i) headway(i)*100];  % Save relevant variables to array to be available for analysis
end  % STEP 13
end  % STEP 14
end
D.4 Model calibration

```matlab
%% CALIBRATION OF FLOW−SPEED FOR GIPPS' MODEL
% APPLIES THE GOLDEN SELECTION SEARCH ALGORITHM

c_range = [20 0 0 22]; % Range of input parameter
% here: Desired speed of cars.
c_rmsp = [0 0]; % Allocate variable for errors
rmspx = []; % Allocate variables used for saving
rmspy = []; % RMSP graphs.

for c_reps = 1:5 % Loop for number of repetitions
    c_range(2:3) = c_range(1) + [0.382 0.618].*(c_range(4)−c_range(1)); % Find interior points
    if (c_reps==1) % If first repetition, both interior
        interval = 2:3;
    else % If repetition > 2, then
        interval = find(c_rmsp == 0) + 1;
    end

    for c_i = interval % Loop for interior points
        Gipps_weight5_c % Run simulation
        % It runs a script which is
        % adapted for calibration purposes.
        % This means that one input param.
        % is determined by a variable.
        gaps0 = gaps(find(gaps(:,6)<3500),:);
        gapsm0 = gapsm(find(gapsm(:,6)<3500),:);
        % Narrow result variables for field
        % and model to include cars only.
        groups = [100:100:900]; % Define flow rate intervals
        avrf = []; avrs = []; stdf = []; stdf = [];
        % Allocate variables
        for i = 1:length(groups)
            a = find(groups(i)<gaps0(:,3) & gaps0(:,3)<groups(i+1));
            b = find(groups(i)<gapsm0(:,3) & gapsm0(:,3)<groups(i+1));
            % Find all vehicle observations
            % recorded in the current interval
            avrf = [avrf harmmean(gaps0(a,5))];
            avrs = [avrs harmmean(gapsm0(b,5))];
            % Calculate average mean section speed
            % for field and model results
        end
        n = length(groups) − 1; % Calculate number of intervals
        c_rmsp(c_i−1) = sqrt(1/n*sum(((avrs − avrf)./avrf).^2)) % Calculate RMSP
        rmspx = [rmspx c_range(c_i)];
```
APPENDIX D. MATLAB CODE

```matlab
rmepy = [rmepy c_rmsp(c_i-1)];
% Store RMSP to variables for later
% analysis.
end
t1 = find(c_rmsp==min(c_rmsp));
% Determine interior point with lowest
% RMSP.
c_range = [c_range(t1) 0 0 c_range(t1+2)];
% Define new search range.
if (t1==1) % Reuse already calculated RMSP values
c_rmsp = [0 c_rmsp(1)];
elseif (t1==2)
c_rmsp = [c_rmsp(2) 0];
end
end
```
D.5 \( \chi^2 \)-VALUES

\[ \chi^2 \]-VALUES

D.5 \( \chi^2 \)-values

\[
\begin{align*}
\text{1} & \quad \% \text{ CHI-SQUARE TEST FOR TIME-GAP DISTRIBUTIONS} \\
\text{2} & \quad \% \text{ Extract time-gaps} \\
\text{3} & \quad \text{gaps1 = gaps(find(gaps(:,1)<6000,:),:);} \\
\text{4} & \quad \% \text{ Extract gaps smaller than 6 sec} \\
\text{5} & \quad \text{gapsn1 = gapsn(find(gapsn(:,1)<6000,:),:);} \\
\text{6} & \quad \% \text{ gaps=field results} \\
\text{7} & \quad \text{gapsm1 = gapsm(find(gapsm(:,1)<6000,:),:);} \\
\text{8} & \quad \% \text{ gapsm=org. model} \\
\text{9} & \quad \% \text{ gapsn=mod. model} \\
\text{10} & \quad \text{x_tg = 500:500:6000;} \\
\text{11} & \quad \% \text{ Bins} \\
\text{12} & \quad \text{groups = [3500 10000:10000:60000];} \\
\text{13} & \quad \% \text{ Weight groups} \\
\text{14} & \quad \text{results = zeros(length(groups)−1,4);} \\
\text{15} & \quad \% \text{ Results variable} \\
\text{16} & \quad \text{for j = 1:length(groups)−1} \\
\text{17} & \quad \quad \% \text{ Weight group loop} \\
\text{18} & \quad \quad \text{thm = []; devm = []; thn = []; devn = [];} \\
\text{19} & \quad \quad \% \text{ Lower weight limit} \\
\text{20} & \quad \quad \text{lower = groups(j);} \\
\text{21} & \quad \quad \% \text{ Upper weight limit} \\
\text{22} & \quad \quad \text{upper = groups(j+1);} \\
\text{23} & \quad \text{a = find(gaps1(:,6) < upper & gaps1(:,6)>lower);} \\
\text{24} & \quad \text{b = find(gapsm1(:,6) > 3500);} \\
\text{25} & \quad \text{c = find(gapsn1(:,6) < upper & gapsn1(:,6)>lower);} \\
\text{26} & \quad \% \text{ Extract data from weight group} \\
\text{27} & \quad \text{f = histc(gaps1(a,1),x_tg); f = f(1:end−1);} \\
\text{28} & \quad \text{fm = histc(gapsm1(b,1),x_tg); fm = fm(1:end−1);} \\
\text{29} & \quad \text{fn = histc(gapsn1(c,1),x_tg); fn = fn(1:end−1);} \\
\text{30} & \quad \% \text{ Create distributions} \\
\text{31} & \quad \text{f = f./sum(f);} \\
\text{32} & \quad \% \text{ Find frequency for field distribution} \\
\text{33} & \quad \text{thm = f.*sum(fm);} \\
\text{34} & \quad \% \text{ Theoretical distribution for each model} \\
\text{35} & \quad \text{thn = f.*sum(fn);} \\
\text{36} & \quad \text{devm = fm − thm;} \\
\text{37} & \quad \text{devn = fn − thn;} \\
\text{38} & \quad \% \text{ Deviation from theoretical distr.} \\
\text{39} & \quad \text{x2m = sum(devm.^2./thm);} \\
\text{40} & \quad \text{x2n = sum(devn.^2./thn);} \\
\text{41} & \quad \text{results(j,:) = [groups(j) groups(j+1) x2m x2n];} \\
\text{42} & \quad \% \text{ Results} \\
\text{43} & \quad \text{end}
\end{align*}
\]
Appendix E

Derivations and Calculations

E.1 Time-gap at constant speed

This section shows the derivation of an expression for the time-gap, with which a vehicle follows the downstream vehicle, when they drive at the same constant speed. The desired speed of the vehicle is higher than that of the downstream vehicle, \( U_n > U_{n-1} \). The vehicle’s speed is in this situation regulated by the second equation in Gipps’ model:

\[
\begin{align*}
    u_n(t + \tau) = b_n \tau + \sqrt{b_n^2 \tau^2 - b_n \left[ 2[x_{n-1}(t) - s_{n-1} - x_n(t)] - u_n(t) \tau - \frac{u_{n-1}(t)^2}{b} \right]}
\end{align*}
\]  
(E.1)

When both vehicles keep the same constant speed, the following applies:

\[
    u = u_n(t + \tau) = u_n(t) = u_{n-1}(t)
\]  
(E.2)

The spatial gap between the two vehicles, \( x_g \), is:

\[
    x_g = x_{n-1} - l_{n-1} - x_n
\]  
(E.3)

By subtracting the safety zone, into which the following vehicle is not willing to intrude, \( s_{n-1} - l_{n-1} \), from both sides of the equations, the following is obtained:

\[
    x_g - s_{n-1} + l_{n-1} = x_{n-1} - s_{n-1} - x_n
\]  
(E.4)

Inserting Equations (E.2) and (E.4) into Equation (E.1):
\[ u = b_n \tau + \sqrt{(b_n \tau)^2 - b_n \left[ 2[x_g - s_{n-1} + l_{n-1}] - u\tau - \frac{u^2}{b} \right]} \] (E.5)

Extracting the spatial gap:

\[ (u - b_n \tau)^2 = (b_n \tau)^2 - b_n \left[ 2[x_g - s_{n-1} + l_{n-1}] - u\tau - \frac{u^2}{b} \right] \] (E.6)

\[ u^2 - 2ub_n \tau = -2b_n(x_g - s_{n-1} + l_{n-1}) + b_nu\tau + \frac{b_nu^2}{b} \] (E.7)

\[ u^2 + 2b_n(x_g - s_{n-1} + l_{n-1}) = 3ub_n\tau + \frac{b_nu^2}{b} \] (E.8)

\[ x_g - s_{n-1} + l_{n-1} = \frac{3ub_n\tau + \frac{b_nu^2}{b} - u^2}{2b_n} \] (E.9)

\[ x_g - s_{n-1} + l_{n-1} = \frac{3}{2} u\tau + \frac{u^2}{2b} - \frac{u^2}{2b_n} \] (E.10)

\[ x_g = \frac{3}{2} u\tau + \frac{u^2}{2} \left( \frac{1}{b} - \frac{1}{b_n} \right) + s_{n-1} - l_{n-1} \] (E.11)

At constant speed the time-gap can be found by dividing the spatial gap by the vehicle’s speed:

\[ t_g = \frac{x_g}{u} \] (E.12)

The time-gap can thus be found by dividing both sides of the equation by \( u \):

\[ \frac{x_g}{u} = \frac{1}{u} \left[ \frac{3}{2} u\tau + \frac{u^2}{2} \left( \frac{1}{b} - \frac{1}{b_n} \right) + s_{n-1} - l_{n-1} \right] \] (E.13)

\[ t_g = \frac{3}{2} \tau + \frac{u}{2} \left( \frac{1}{b} - \frac{1}{b_n} \right) + \frac{1}{u} (s_{n-1} - l_{n-1}) \] (E.14)
This section shows the calculation of the statistically largest appearing headway for every 250,000th vehicle, provided a flow rate of \( q = 200 \) veh/h and minimum headway \( h_{\min} = 2 \) s.

Calculating the \( \gamma \) value:

\[
q = \frac{\gamma}{\gamma h_{\min} + 1} \tag{E.15}
\]

\[
\gamma = \frac{q}{1 - q h_{\min}} \tag{E.16}
\]

\[
\gamma = \frac{200/3600 \text{ veh/s}}{1 - 200/3600 \text{ veh/s} \cdot 2 \text{ s}} \tag{E.17}
\]

\[
\gamma = 0.0625 \tag{E.18}
\]

By taking the derivative of the cumulative distribution function, one can find the probability of a headway, \( h \), is:

\[
P(X = h) = \frac{d}{dh} (1 - \exp(-\gamma(h - h_{\min}))) \tag{E.19}
\]

\[
P(X = h) = \gamma \exp(-\gamma(h - h_{\min})) \tag{E.20}
\]

Calculating the headway appearing with a probability of 1/250,000:

\[
\frac{1}{250,000} = 0.0625 \exp(-0.0625(h - 2)) \tag{E.21}
\]

\[
h = -\ln \left( \frac{1}{250,000 \cdot 0.0625} \right) + 2 \tag{E.22}
\]

\[
h = 156.5 \text{ s} \approx 157 \text{ s} \tag{E.23}
\]