Discussion paper

Transfer Pricing and Debt Shifting in Multinationals

BY
Dirk Schindler AND Guttorm Schjelderup
Transfer Pricing and Debt Shifting in Multinationals

Dirk Schindler, Guttorm Schjelderup*
Norwegian School of Economics, NoCeT, and CESifo
May 25, 2014

Abstract

There is a growing concern that governments lose substantial corporate tax revenue due to transfer pricing and debt shifting strategies. Existing literature studies debt shifting and transfer pricing separately. In practice, however, the choice of debt-to-asset ratios in affiliates and the transfer price of internal debt are interrelated management decisions that are also mutually affected by government regulation. This paper models these strategies as intertwined. We find that the tax sensitivity of the corporate tax base depends on whether debt shifting and transfer pricing are cost complements or substitutes. A second result is that stricter regulation of debt shifting and transfer pricing may have the effect of fostering such activities.

Keywords: Multinational corporations, profit shifting, debt shifting, concealment costs

JEL classification: H25, F23, D21
1 Introduction

Worldwide, there is a growing concern that governments are losing substantial corporate tax revenue because of tax planning by multinational companies aimed at shifting profits in ways that erode the taxable base to locations where they are subject to a more favourable tax treatment. The case of Starbucks in the UK is one example of why such worries are not unfounded (Bergin, 2012). In their final declaration in June 2012 in Mexico, the G20 leaders, in response to such worries explicitly referred to “the need to prevent base erosion and profit shifting.” From a corporate point of view, however, it is often claimed that business leaders have a responsibility towards their shareholders to legally reduce their company’s tax bill.

The OECD (2013) report on “Base erosion and profit shifting” identifies transfer pricing and debt shifting (thin capitalization) as major reasons for the tax-revenue drain in high-tax countries. Both strategies are regulated by the OECD’s ‘arm’s length’ standard, which states that transfer prices should reflect market prices chosen by unrelated parties engaged in similar trades under similar circumstances (Eden, 1998; OECD, 2010, art. 9). As pointed out in several studies, ‘arm’s-length’ pricing may be difficult to enforce because of the lack of market parallels, multinationals’ use of tax havens, and lack of disclosure of either earnings worldwide or pricing methods.1

In the literature, the choice of transfer prices in affiliates is analyzed separately from the choice of firms’ tax efficient financing structure.2 In this paper we argue that these two choices are interrelated. There are several reasons for why this is so. First, consider an agreement for an intercompany loan. In this type of transaction the interest rate is a transfer price that is decided jointly with the loan amount. In such cases the two decisions are interrelated. On a more general level, public regulation that pertains to leverage, say, may affect the scope for transfer pricing (and vice versa). Thin capitalization rules, for instance, may make it relatively “cheaper” for the management to manipulate the interest rate on intercompany loans. There may also be economies of scale and scope related to tax planning that intertwines these decisions. For example, skills in concealing abusive transfer-pricing practices may have positive spillover effects on the firm’s ability to disguise its real debt-to-asset ratio. Our study, therefore, responds to calls for more research on how regulation and tax law affect managerial decisions (see, e.g., Shackelford and Shevlin, 2001; Hanlon and Heitzman, 2010).3

Some of our core results provide a theoretical foundation for empirical findings related

---

1 These issues are discussed in Taylor and Richardson (2013), Dyreng and Lindsey (2009), Hope et al. (2013), and Lo and Wong (2011).
2 The literature on debt shifting is surveyed by Mintz and Weichenrieder (2010), whilst Gresik (2001) and Gox and Schiller (2007) provide surveys of the transfer-pricing literature. Shackelford and Shevlin (2001) and Hanlon and Heitzman (2010) review (empirical) tax research in the accounting literature.
3 Weichenrieder and Windischbauer (2008), Büttner et al. (2012), and Overesch and Wamser (2013) study how firms respond to thin capitalization rules.
to debt shifting and transfer pricing. Pak and Zdanowicz (2001) find that the volume of profit shifting in U.S. multinationals was equal to 18% of total reported corporate profits in 2000. Bartelsman and Beetsma (2003) study OECD data and point out that 65% to 87% of the (potential) additional tax revenue, stemming from a unilateral tax increase, is lost due to profit shifting by transfer pricing. Despite the widespread use of tax havens to streamline tax efficient financing structures, debt shifting seems to be less tax sensitive. Studies on debt shifting and its response to changes in taxes show that the semi-elasticity of internal debt lies between 0.69 and 1.3, which indicates small behavioral changes following a tax change. A typical example is Büttner and Wamser (2007, p. 25), who state:

“...our findings suggest that the implied magnitude of tax-revenue losses is rather modest even for wholly-owned firms. To conclude, our findings are indicative for substantial costs of adjusting the capital structure for means of profit-shifting.”

Our analysis ties in with the above literature and the conclusion by Büttner and Wamser (2007), in that we show that when management takes into account all costs related to tax minimization at the global level, including the restructuring of the business (cf. Scholes et al., 2009, ch. 1.2), the effects of tax-rate differentials on debt shifting are modest under reasonable assumptions. Our analysis, then, shows that transfer pricing often is the more attractive profit shifting instrument.

A second set of results pertains to how the rental rate of capital is affected by debt shifting and transfer pricing. We find that debt shifting reduces the rental rate of capital and therefore increases investments. In contrast, manipulation of interest rates (the transfer price) does not affect the rental rate of capital or investments.

In a third step of our analysis, we investigate how management responds when the corporate tax rate is changed. We show that a rise in the corporate tax rate makes it more attractive to increase the debt-to-asset ratio and the volume of profits shifted, if we have concealment cost complementarity. Complementarity here means that a higher debt-to-asset ratio reduces marginal concealment costs of transfer pricing (and vice versa). Concealment cost substitutability exists when marginal concealment costs related to profit shifting rise when debt shifting increases (and vice versa). When we have cost substitutability, we find that management behaves in such a way that the corporate tax base becomes less tax sensitive.

Evidence for transfer pricing in the U.S. is given in Clausing (2003) and Bernard et al. (2006); for Norway in Langli and Saudagran (2004); for Germany in Weichenrieder (2008). Oyelere and Emmanuel (1998); Emmanuel and Oyelere (2002) show that foreign-owned affiliates in the UK are characterized by lower profits but higher dividend distributions (than UK-controlled firms). Evidence for transfer pricing in European multinationals is given in Dharmapala and Riedel (2013).

Taylor and Richardson (2013) find that multinationals that use tax havens as part of their strategy to shift debt are more thinly capitalized compared to firms that do not use tax havens.

Our findings with respect to government regulation are surprising. If a government introduces thin-capitalization rules (or tightens existing rules), then, under concealment cost substitutability, we show that management may respond by increasing the interest premium, or by increasing leverage. This result is counterintuitive, because it suggests that such rules could have unintended effects (both on tax revenue and on the capital structure). On the other hand, if we have concealment cost complementarity, the response to stricter regulation is to lower both the debt-to-asset ratio and the volume of abusive interest expenses.

Our analysis is undertaken in a setting where the central management of a multinational firm decides on capital investments, leverage and the price of internal debt across affiliates in different countries in order to save taxes globally. Thus, our analysis is sharply focused on the tax saving role of transfer prices and debt. One could object that our approach misses out the role of transfer prices for internal management in multinationals. However, only rarely do tax savings enter these contributions. Furthermore, there is a strand of literature that presents evidence that transfer pricing and debt shifting in multinationals are largely engineered to shift corporate income to low-tax jurisdictions.7 This literature suggests that the transfer prices that shift income are different from those that provide managerial incentives. In addition, Göx and Schiller (2007; p 692) point out that profits are usually higher in firms that avoid overburdening the transfer price with dual roles.

If the multinational is restricted in its use of transfer prices so that a conflict arises between profit shifting and managerial incentives, the multinational could carry more than one set of books. There is controversy in the literature over whether multinationals employ two sets of books rather than one. Göx and Schiller (2007) in their survey of the transfer pricing literature suggests that a non-negligible number of firms uses only one set of books.8 Whether transfer prices are overburdened by conflicting roles or not, and whether firms use one or two sets of books is a discussion we shall leave out here. Our focal point is to examine how firms set leverage and the transfer price on leverage, since these two decisions are interrelated.

The sections of the paper are organized as follows. In section 2, we describe the basic model and introduce the concealment cost functions. We derive the optimal use of debt policy and of interest-rate manipulation, and analyze the implications of tax engineering on real investment of the multinational firm in section 3, while in section 4, we examine the tax sensitivity of debt shifting and of profit shifting. The effectiveness and spill-over effects of regulation for the protection of tax bases are analyzed in section 5. In section 6, we offer some concluding remarks.

---


8 The dual role of transfer prices and the use of two books are also discussed in Smith (1992) and Nielsen and Raimondos-Møller (2012).
2 The Model

We set up a model of a multinational firm (henceforth MNC) that has its headquarters (henceforth HQ) located in any country \( p \in \{1, n\} \). The MNC can invest in affiliates in \( n \) countries. These affiliates are assumed for simplicity to be price takers and they are wholly owned. Each affiliate \( i \) employs \( K_i \) units of real capital that is used to produce \( x_i = F(K_i) \) units of a homogenous good whose output price is normalized to unity. The production function \( F(K_i) \) exhibits positive and decreasing returns to capital (i.e., \( F'_K > 0 \) and \( F''_K < 0 \)). We shall further assume that world markets for real and financial capital are integrated and that capital is perfectly mobile. Each country is small and cannot influence interest rates and the market interest rate is exogenously given by \( r > 0 \).

To finance its investments in an affiliate in country \( i \), the HQ can use equity \( E_i \) and debt \( D_i \). Debt can be further broken down into external debt \( (D^E_i) \) and internal debt \( (D^I_i) \), where internal debt is obtained by borrowing from related affiliates. We define \( K_i \) as the total (real) capital employed by affiliate \( i \) and let \( b^E_i = D^E_i / K_i \) be the external debt-to-asset ratio. In a similar fashion, \( b^I_i = D^I_i / K_i \) is the internal debt-to-asset ratio, and we define the overall leverage ratio \( (b_i) \) of the MNC by \( b_i = b^E_i + b^I_i = (D^E_i + D^I_i) / K_i \).

Within the MNC, it must be the case that the sum of market interest payments on internal borrowing and lending is zero across all affiliates, that is,

\[
\sum_i r \cdot D^I_i = \sum_i b^I_i \cdot r \cdot K_i = 0. \tag{1}
\]

The MNC can shift income to affiliates in other countries by under- or overinvoicing intra-firm transactions. We model this by allowing the firm to deviate from the market interest rate by levying a surcharge \( \tilde{r}_i \) on the market interest rate in affiliate \( i \). The total interest costs of internal debt are then \( r + \tilde{r}_i \), and the amount of profit shifted away from affiliate \( i \) is given by

\[
P_i = \tilde{r}_i \cdot b^I_i \cdot K_i. \tag{2}
\]

The sum of shifted profits across all affiliates can now be written as

\[
\sum_i \tilde{r}_i \cdot b^I_i \cdot K_i = 0. \tag{3}
\]

Theories of optimal capital structure assume that there are convex costs per unit of capital associated with the use of external and internal debt.

\footnote{Alternative ways of conducting transfer pricing would be to overinvoice the use of an intermediate or fixed factor. These methods are traditionally analyzed in the profit-shifting literature. It can be shown that our findings hold in such settings as well. However, implementing additional inputs would further increase notation and complexity.}

External debt is seen as useful in order to discipline local managers from lax man-
agement and “empire-building” strategies. However, as the leverage ratio goes up, the risk of bankruptcy increases and may cause bankruptcy costs, or induce a debt-overhang situation, in which profitable investment is not undertaken. Too much external debt may also be associated with a higher risk premium due to informational asymmetries. As is usual in the literature, we define costs of external debt by a U-shaped function $C_E(b_E)$, where the optimal external leverage in absence of taxation (i.e., the cost-minimizing level of external debt) is denoted by $\hat{b}$.\textsuperscript{10}

**Internal debt** also carries costs. In the literature, these are related to various tax-engineering expenses incurred in order to avoid or relax regulations such as thin-capitalization rules and/or controlled-foreign-company (CFC) rules (see, e.g., Fuest and Hemmelgarn, 2005).\textsuperscript{11} We add to the cost structure of internal debt by allowing for the possibility that low profits caused by either profit shifting ($P_i$) or/and high leverage may arouse suspicion by the tax authorities and lead to a costly audit. Hence, low profits due to transfer pricing, say, makes it more costly to use internal debt. In line with this, we define the cost function for internal debt as $C_I(b_I, P_i)$.

The costs and benefits of internal and external debt differ as is clear from the definitions of the cost functions above. Internal debt could be seen as tax-favored equity, since it does neither affect the risk of bankruptcy nor reduce any informational asymmetry.\textsuperscript{12} It is therefore not unreasonable to assume that the total cost function for debt is additively separable in external and internal leverage, that is, $C_D(b_E, b_I, P_i) = C_E(b_E) + C_I(b_I, P_i)$, if external credit markets are perfect (with the exception for costs related to financial distress and bankruptcy).

In line with the standard trade-off literature, we assume that agency costs of debt are convex in leverage, but proportional in real capital employed. For internal debt, designing strategies to avoid anti-avoidance regulation (particularly, working around thin-capitalization rules), and asking for experts’ advice imply higher costs.

In terms of imposing structure on the cost function, we assume that there are no debt-related concealment costs when $b_I \leq 0$; $C_I(0, P_i) = 0$, even if the firm engages in

\textsuperscript{10}See, e.g., Jensen (1986) on free cash-flows, Meckling and Jensen (1976) on moral hazard, Kraus and Litzenberger (1973) on bankruptcy costs, and Myers (1977) on debt-overhang problems. Hovakimian et al. (2004) and Aggrawal and Kyaw (2010) provide recent overviews on the full set of costs and benefits of external debt. To focus on the interplay of internal debt and profit shifting and to keep the model simple, we neglect overall bankruptcy costs on the parent level. The latter would set an incentive to shift external debt internationally; see Huizinga et al. (2008).

\textsuperscript{11}See for example Mintz and Smart (2004), Fuest and Hemmelgarn (2005), and Schindler and Schjelderup (2012). Thin-capitalization rules are in place in many countries such as Germany, the U.K, and the U.S., and also apply to foreign subsidiaries. See, e.g., Gouthière (2005) for a description of several EU and non-EU countries’ rules. Controlled-foreign-company rules are in place, e.g., in the US and Germany and they deny tax-exemption of passive income in the home country of the MNC, provided that tax avoidance is suspected (see Ruf and Weichenrieder, 2012).

\textsuperscript{12}Indeed, Gertner et al. (1994) point out that internal debt does not show the properties of external debt and that it should rather be seen as equity. Stonehill and Stitzel (1969) and Chowdhry and Coval (1998, pp. 87) qualify internal debt as “tax-preferred equity”, supporting this view.
abusive transfer pricing. In all other cases, costs of internal debt are affected positively by the total amount of profit shifting so that $\partial C_I/\partial P_i > 0$.

Formally, the properties applied to the cost function of debt can be summarized as:

**Assumption 1** External credit markets are assumed to be perfect except for the debt tax shield and financial distress costs. The debt cost function is additively separable, $C_D(b^E_i, b^I_i, P_i) = C_E(b^E_i) + C_I(b^I_i, P_i)$, and exhibits the properties:

\[
C_E(b^E_i) > 0 \quad \text{with} \quad C'_E(b^E_i) > 0, \quad C''_E(b^E_i) > 0 \quad \text{if} \quad b^E_i > b^E_i^\star, \\
C'_E(b^E_i) \leq 0, \quad C''_E(b^E_i) > 0 \quad \text{if} \quad b^E_i \leq b^E_i^\star,
\]

\[
C_I(b^I_i, P_i) > 0 \quad \text{with} \quad \frac{\partial C_I(b^I_i, P_i)}{\partial b^I_i} > 0, \quad \frac{\partial^2 C_I(b^I_i, P_i)}{\partial (b^I_i)^2} > 0 \quad \text{if} \quad b^I_i > 0, \\
\frac{\partial C_I(b^I_i, P_i)}{\partial P_i} > 0, \quad \frac{\partial^2 C_I(b^I_i, P_i)}{\partial P_i^2} > 0 \quad \text{if} \quad b^I_i > 0,
\]

\[
C_I(b^I_i, P_i) = 0 \quad \text{with} \quad \frac{\partial C_I(b^I_i, P_i)}{\partial b^I_i} = 0 \quad \forall P_i \quad \text{if} \quad b^I_i \leq 0.
\]

Not only do MNCs face costs related to the use of debt, but shifting profit by transfer prices also entails costs. Inspired by the literature on tax evasion (cf. Allingham and Sandmo, 1972; Yitzhaki, 1974), these costs can be interpreted either as costs due to the use of lawyers and accountants, and/or as expected penalties imposed if illegal interest-rate manipulation is detected and fined by the tax authorities. In the latter case, the cost function would imply that the detection probability as well as the fines increase in the amount of shifted profits. Furthermore, we shall assume that the concealment costs of profit shifting depend on the level of internal debt. Accordingly, we define the concealment cost function related to transfer pricing by $C_P(P_i, b^I_i)$, which is a convex function in the level of income shifted ($P_i$). The convexity in leverage $b^I_i$ is due to that it is more costly to hide (illegal) profit shifting if the debt-to-asset ratio is very high and taxable profits low due to excessive interest deductions. It follows from this that $\frac{\partial^2 C_P}{\partial b^I_i \partial P_i} > 0$. If $P_i \leq 0$, we assume that no costs occur because enlarging the tax base and increasing tax payments...
in such an affiliate should not induce local tax authorities to investigate and audit the affiliate more closely. Formally, our assumptions are summarized below by

**Assumption 2** The cost function of profit shifting exhibits

\[
C_P(P_i, b_i^l) > 0 \quad \text{with} \quad \frac{\partial C_P(P_i, b_i^l)}{\partial P_i} > 0, \quad \frac{\partial^2 C_P(P_i, b_i^l)}{\partial P_i^2} > 0 \quad \text{if} \quad P_i > 0, \\
\frac{\partial C_P(P_i, b_i^l)}{\partial b_i^l} > 0, \quad \frac{\partial^2 C_P(P_i, b_i^l)}{\partial (b_i^l)^2} > 0 \quad \text{if} \quad P_i > 0, \\
C_P(P_i, b_i^l) = 0 \quad \text{with} \quad \frac{\partial C_P(P_i, b_i^l)}{\partial P_i} = \frac{\partial C_P(P_i, b_i^l)}{\partial b_i^l} = 0 \quad \text{if} \quad P_i \leq 0.
\]

To summarize, the timeline of the model is as follows. First, the government sets its tax and regulatory instruments and the ultimate owners of the MNC equip the HQ in country \(p\) with the necessary equity. Then, the HQ decides for all affiliates \(i\), the amount of equity, leverage (external and internal), and the price on internal debt in affiliate \(i\). After that, the financial capital assigned to each affiliate is used to hire real capital for producing the final output good. The latter is sold at a given world-market price at unity.

In the next section, we first solve for the tax-efficient capital structure and the optimal transfer price on internal debt. In a second step, we analyze optimal investment of real capital.\(^\text{17}\) Finally, we investigate how HQ’s decisions are affected by changes in government’s tax and regulation policies.

## 3 Profit Shifting and Debt Shifting

The HQ maximizes global profits after corporate taxation. Net global profits of the MNC are given by

\[
\Pi = \sum_i \left[ \pi_i^e - t_i \cdot \pi_i^t \right],
\]

where \(\pi_i^e\) is economic profit in subsidiary \(i\), \(\pi_i^t\) is taxable profit, and \(t_i\) is the corporate tax rate in country \(i\). Economic profit is given by revenue minus user costs of capital and profit shifting,

\[
\pi_i^e = F(K_i) - [r + C_E(b_i^E) + C_I(b_i^I, P_i)] \cdot K_i - \tilde{r}_i b_i^l K_i - C_P(P_i, b_i^l),
\]

The tax code in most countries do not allow costs of equity to be deducted against tax whilst interest expenses are deductible. As a consequence, taxable profit differs from true economic profit. In defining taxable profit, we assume that costs per unit of capital associated with both external and internal borrowing are tax deductible. Some of these

\(^{17}\)Note that it does not matter in our setting which stage of the maximization problem is solved first, choosing the tax-avoidance devices or determining real capital investment.
costs may be associated with informational asymmetries between investors and managers of the firm, or illegitimate action from the point of view of the tax authority. One could argue that these costs should not be tax deductible. It is straightforward to show by examination of the equations to follow that even if they were not deductible, it would not affect our results.

Taxable profit income can, after some manipulations, be written as

$$\pi_i = F(K_i) - [rb_i^E + (r + \tilde{r}_i)b_i^I + CE(b_i^E) + CI(b_i^I, P_i)] \cdot K_i - CP(P_i, b_i^I),$$  \hspace{1cm} (6)$$

where capital invested in country $i$ is financed either by debt $D_i = D_i^I + D_i^E$ or by equity $E_i$, so that $K_i = D_i^I + D_i^E + E_i$.

The HQ maximizes the value of the MNC after corporate taxes. Personal taxes do not matter, since MNCs often either are owned by many institutional investors, or shareholders located in different countries.\footnote{It can be shown that from the viewpoint of a shareholder in a MNC, maximizing profits of the MNC after global corporate taxation and maximizing the net pay-off on equity investment after opportunity costs and personal (income) taxes, yield identical results under mild assumptions. For example, if corporate taxes cannot be deducted against personal income tax and if the personal tax rate on dividends and interest income is the same, it is straightforward to show that maximizing the value of the firm to the owner and maximizing corporate profits coincide. These restrictions are fulfilled for a wide range of real world tax codes: the classical corporate taxation system (e.g., in the U.S.), the German system since 2009 (“Abgeltungssteuer”), where interest income, dividends and capital gains are taxed at 25% and deductions for corporate taxes are not possible, and the Norwegian shareholder tax, introduced in 2006.} The optimization problem of the firm can be seen as a two-tier process: First, it chooses its optimal debt-to-asset ratio and the optimal interest rate on internal debt for any given value of real investment $K_i$. Second, the firm decides on how much real capital to use and therefore how much of the final good to produce in each country. Taking real investment $K_i$ as fixed initially, the firm’s optimal tax-planning behavior is found by maximizing equation (4). Inserting for equations (5) and (6), collecting terms, and taking into account the constraints on internal lending and on profit shifting, that is, equations (1) and (3), the maximization problem can be written as

$$\max_{b_i^E, b_i^I, \tilde{r}_i} \Pi = \sum_i \left\{ (1 - t_i) \left[ F(K_i) - CP(P_i, b_i^I) \right] - K_i \left[ r - t_i r(b_i^E + b_i^I) + (1 - t_i) \left(C_E(b_i^E) + CI(b_i^I, P_i)\right) + (1 - t_i) \tilde{r}_i b_i^I \right] \right\}$$

$$\text{s.t. } \sum_i r \cdot b_i^I \cdot K_i = 0 \quad (\lambda) \quad \text{s.t. } \sum_i \tilde{r}_i \cdot b_i^I \cdot K_i = 0 \quad (\eta),$$

where $\lambda$ and $\eta$ are the associated Lagrangian parameters for internal debt and transfer pricing, respectively.
Optimal manipulation of interest rates. Maximizing (7) with respect to $\tilde{r}_i$, we obtain

$$\eta - (1 - t_i) \leq (1 - t_i) \left( \frac{\partial C_I}{\partial P_i} + \frac{\partial C_P}{\partial P_i} K_i \right) \quad \forall i. \quad (8)$$

The left hand side is the net marginal benefit of profit shifting. It should be equal to or less than the after-tax marginal concealment cost of interest-rate manipulation (right-hand side). The Lagrangian parameter $\eta$ gives the shadow value of an additional unit of profit income shifted and can be shown to be equal to $\eta = \max_i (1 - t_i)$. We shall for convenience let country 1 be the country with the lowest tax rate so that by definition $\eta \equiv (1 - t_i)$. The first-order conditions in (8), then, imply that, for internal debt, each affiliate $i > 1$ pays a (positive) surcharge on the market interest rate in order to shift profits into affiliate 1 located in the lowest-tax country. Structuring transactions in this way maximizes the gain from transfer pricing.

Tax efficient financing structure. The first-order condition for external debt ($b^{E}_{i}$) is given by

$$C_{E}^r(b^{E}_{i}) = \frac{t_i}{1 - t_i} \cdot r > 0 \quad \forall i. \quad (9)$$

Equation (9) states that the value of the debt tax shield should be exploited up until the point where the associated costs of using external debt equals the marginal value of the tax shield. The positive value of the debt tax shield implies that the optimal leverage ratio of external debt in the presence of taxation ($b^{E*}_{i}$) is higher than the optimal leverage ratio in absence of taxation ($\tilde{b}^{E}_{i}$), that is, $b^{E*}_{i} > \tilde{b}^{E}_{i}$.

Deriving and rearranging the first-order condition for internal leverage $b^{I}_{i}$, we obtain

$$(t_i - \lambda) r = (1 - t_i) \left( \frac{\partial C_I}{\partial b^{I}_{i}} + \frac{\partial C_P}{\partial b^{I}_{i}} \frac{1}{K_i} \right), \quad (10)$$

where we have used that either equation (8) holds with equality, or that $\tilde{r}_i = 0$.

The left hand side of equation (10) is the net marginal benefit of debt shifting. It should be equal to the tax-adjusted marginal cost of concealing debt and profit shifting. The bracket on the left hand side of (10) consists of the marginal value of interest deductions, $t_i$, minus the the shadow cost of lending given by the Lagrangian multiplier $\lambda$. It is straightforward to show that $\lambda = \min_i t_i = t_1$, since we have defined country 1 as the lowest-tax country. The implication of this is that, in order to maximize its value after tax, a MNC will minimize tax payments by conducting lending activities from the affiliate located in the country with the lowest rate of tax (i.e., affiliate 1 in our model). Consequently, the value of the debt tax shield related to internal debt is given by $t_i - t_1$.

Optimal Real Investment. After determining the optimal degree of leverage and the interest rate on internal debt, the HQ derives the effective cost of capital (evaluated at
a tax-efficient financial structure with optimal $b_{t}^{E*}$ and $b_{t}^{f*}$ and for the optimal transfer price $\tilde{r}_{i}^*$. The effective rental rate of capital can be shown to be equal to

\[
  r_{i}^{eff} = r - t_{i}b_{t}^{E*}r + (1 - t_{i})C_{E}(b_{t}^{E*}) - (t_{i} - t_{1})b_{t}^{f*}r + (1 - t_{i})C_{I}(b_{t}^{f*}, P_{t}^{*}) - (t_{i} - t_{1})b_{t}^{f*}\tilde{r}_{i}^* + (1 - t_{i})C_{P}(P_{t}^{*}, b_{t}^{f*}) \frac{1}{K_{i}}. \tag{11}
\]

In what follows, we use (11) to derive the following conditions\footnote{In deriving these results, we have used equation (8) twice.}

\[
  \frac{\partial r_{i}^{eff}}{\partial \tilde{r}_{i}^*} = -(t_{1} - t_{i})b_{t}^{f*} + (1 - t_{i})b_{t}^{f*}\left(\frac{\partial C_{I}}{\partial P_{i}} K_{i} + \frac{\partial C_{P}}{\partial P_{i}} K_{i}\right) = 0, \tag{12}
\]

\[
  \frac{\partial r_{i}^{eff}}{\partial K_{i}} = -\frac{1}{K_{i}} \left[(1 - t_{i})C_{P}(P_{t}^{*}, b_{t}^{f*}) \frac{1}{K_{i}} - (t_{i} - t_{1})b_{t}^{f*}\tilde{r}_{i}^* \right]. \tag{13}
\]

Inserting for the optimal values of debt and the rental rate of capital into the maximization problem (7), we can express the MNC’s maximization problem with respect to its use of capital by

\[
  \max_{K_{i}} \sum_{i} \left((1 - t_{i})F(K_{i}) - r_{i}^{eff}(K_{i}) \cdot K_{i}\right),
\]

where, after applying equations (12) and (13), the first order condition for capital can be written as

\[
  F_{K_{i}}^{i} = \frac{r}{1 - t_{i}} - \frac{t_{i}}{1 - t_{i}}r b_{t}^{E*} + C_{E}(b_{t}^{E*}) - \left(\frac{t_{i} - t_{1}}{1 - t_{i}}\right) b_{t}^{f*} + C_{I}(b_{t}^{f*}, P_{t}^{*}). \tag{14}
\]

Equation (14) shows that since debt is tax deductible the use of external and internal debt to save taxes lowers the user cost of capital and leads to higher investment. In contrast, interest-rate manipulation has no direct effect on the user cost of capital. We summarize this as

**Lemma 1** Thin capitalization reduces effective capital costs and increases real investment. Manipulating the interest rate on internal debt affects the investment decision only indirectly via the interplay with internal debt in the concealment cost functions.

It follows from Lemma 1 that excessive interest premiums do not affect the real activity of firms as long as the use of internal debt does not affect concealment costs related to transfer pricing (and vice versa). However, as seen from equations (11) and (10), manipulating interest rates affects the user cost of capital as well as the tax sensitivity of internal debt if concealment costs of debt shifting and profit shifting also depend on the level of abusive internal interest expenses and internal debt, respectively. The topic of the next section is to explore what the consequences are of such a relationship.
4 The Tax Sensitivity of Debt and of Profit Shifting

In this section, we examine how transfer pricing and leverage decisions are affected by a change in the corporate tax rate. In order to assess how a change in the corporate tax rate affects the use of internal debt, we totally differentiate the first-order condition (9). This yields

\[ \frac{db_i^E}{dt_i} = \frac{r}{(1 - t_i)^2 \cdot C_{ii}(b_i^E)} > 0. \]  

Equation (15) shows that an increase in the tax rate of country \( i \) will induce the MNC to use more external debt, since the value of the debt tax shield has risen. Note that the higher tax sensitivity of external debt is independent of how much profit is shifted through interest manipulation or the use of internal debt.

To facilitate a discussion on how the transfer price \( \tilde{r}_i \) and the internal debt-to-asset ratio \( b_i^I \) are affected by a tax increase, we must make assumptions on how the marginal cost of internal leverage is affected by profit shifting, that is, on the sign of \( \partial^2 C_I / (\partial b_i^I \partial P_i) \).

We assume that the effects of one activity on concealment costs of the other activity are qualitatively symmetric, that is, \( \text{sign}\{\partial^2 C_I / (\partial b_i^I \partial P_i)\} = \text{sign}\{\partial^2 C_P / (\partial b_i^I \partial P_i)\} \). The sign of this cross-derivative is ambiguous and depends on how debt shifting and transfer pricing affect total concealment costs.

We define concealment cost substitutability the following way:

**Definition 1** Concealment cost substitutability exists when the marginal concealment costs related to profit shifting \( P_i \) rise when debt shifting \( b_i^I \) increases (and vice versa), that is;

\[ \frac{\partial^2 C_I}{\partial b_i^I \partial P_i}, \frac{\partial^2 C_P}{\partial b_i^I \partial P_i} > 0. \]

To see why marginal concealment costs may rise due to an increase in either transfer pricing or debt shifting, one can perceive that tax authorities compare profits of MNCs’ affiliates to profits of their peer group in order to decide on an audit. If an affiliate is having a high internal debt-to-asset ratio, then, if the firm also uses the transfer price to shift profit this reduces profit further and increases the likelihood of a costly audit. Another example relates to thin-capitalization rules. Such rules are meant to prevent a too high debt-to-asset ratio. If the firm shifts too much profit by manipulating the interest rate, profits will be low and this has a negative effect on book equity. Consequently, profit shifting may lead to that thin-capitalization rules come into force and might even induce tax authorities to audit the firm. In order to avoid an audit, the firm must make more use of accountants and lawyers to reduce the probability of an audit.

In line with the definition above, we define concealment cost complementarity as:

**Definition 2** Concealment cost complementarity exists, when the marginal concealment costs related to profit shifting \( P_i \) fall when debt shifting \( b_i^I \) increases (and vice versa), that is;

\[ \frac{\partial^2 C_I}{\partial b_i^I \partial P_i}, \frac{\partial^2 C_P}{\partial b_i^I \partial P_i} < 0. \]
Definition 2 indicates that the cross derivatives may be negative as well. This could happen if there are pure economies of scale. For example, a MNC has acquired special skills in concealing profit-shifting activities due to the sheer volume of such transactions and can use these skills for debt shifting as well (and vice versa).

In order to examine the management response with respect to excessive interest deductions and to internal leverage following a change in the corporate tax rate \( t_i \), we differentiate the first-order conditions (8) and (10) with respect to \( t_i \).

The change in internal debt is given by

\[
\frac{db^i_t}{dt_i} = \frac{(1 - t_1) \left[ A \cdot rb^i_t K_i - B \cdot b^i_t \right]}{(1 - t_i)^2 SOC} \begin{cases} > 0 & \text{if } \frac{\partial^2 C_i}{\partial b^i_t \partial P_i} < 0, \\ \geq 0 & \text{if } \frac{\partial^2 C_i}{\partial b^i_t \partial P_i} > 0, \end{cases}
\]

where \( A = \frac{\partial^2 C_P}{\partial P_i^2} + (\partial^2 C_I/\partial P_i^2) K_i > 0 \) is the direct effect related to increased profit shifting \( (P_i) \). It measures the change in marginal concealment costs of profit shifting following a change in the amount of profit shifted (i.e., the curvature of the concealment cost function related to profit shifting). The term \( B = (\partial^2 C_I/\partial b^i_t \partial P_i) K_i + \partial^2 C_P/(\partial b^i_t \partial P_i) \) is the indirect cost interaction effect. It shows how transfer pricing affects the cost of shifting debt (and vice versa), i.e., whether the concealment cost function inhibits cost substitutability or complementarity. The second order condition is given by the term \( SOC > 0 \).

If we have concealment cost complementarity \((\partial^2 C_I/\partial b^i_t \partial P_i < 0)\), the squared bracket in the numerator in equation (16) is unambiguously positive. In this case internal debt will rise following a tax increase since the direct effect as well the cost interaction effect go in the same direction.

Under concealment cost substitutability the numerator cannot be signed, since the direct effect goes against the cost interaction (indirect) effect. Internal debt may fall or rise following a tax increase depending on the relative magnitudes of the two terms in the squared bracket.

The change in firm behavior when it comes to the amount of profit shifted is given by

\[
\frac{dP_i}{dt_i} = (d\tilde{r}_i/dt_i) b^i_t K_i + (db^i_t/dt_i) \tilde{r}_i K_i, \quad \text{which can be written out in full as}
\]

\[
\frac{dP_i}{dt_i} = \frac{(1 - t_1) \left[ D \cdot b^i_t - B \cdot rb^i_t K_i \right]}{(1 - t_i)^2 SOC} \begin{cases} > 0 & \text{if } \frac{\partial^2 C_i}{\partial b^i_t \partial P_i} < 0, \\ \geq 0 & \text{if } \frac{\partial^2 C_i}{\partial b^i_t \partial P_i} > 0, \end{cases}
\]

where \( D = (\partial^2 C_I/\partial (b^i_t)^2) K_i + \partial^2 C_P/\partial (b^i_t)^2 > 0 \) is the direct effect on the cost function of increasing internal debt (i.e., the curvature of debt-shifting-related concealment costs).

We may now state the following results:

\[\text{For a full derivation see the Appendix}\]
Proposition 1 The tax sensitivity of internal debt and profit shifting \((P_i)\) is affected by the concealment cost function in the following way:

(a) Concealment cost complementarity \((\frac{\partial^2 C_i}{\partial b_i \partial P_i} < 0)\) increases the tax sensitivity of both debt and profit shifting; that is \(db_i^d/dt_i > 0\) and \(dP_i/dt_i > 0\).

(b) Concealment cost substitutability \((\frac{\partial^2 C_i}{\partial b_i \partial P_i} > 0)\) reduces the tax sensitivity of both debt and profit shifting and \(db_i^d/dt_i \leq 0\) and \(dP_i/dt_i \leq 0\).

All else equal, a rise in the corporate tax rate \(t_i\) makes it more attractive to increase the debt-to-asset ratio and the interest rate. The exact response from management, however, depends on the properties of the concealment cost functions and in particular the interaction between the direct cost effect and indirect cost interaction effect.

Proposition 1 states that under concealment cost complementarity the direct and indirect cost effect go in the same direction so that a rise in the corporate tax rate induces the MNC to shift more debt and increase the transfer price. As a result, more profits are shifted and the corporate tax base is more tax sensitive. The reason is that under concealment cost complementarity the indirect cost effect mitigates the increase in marginal concealment costs of debt shifting and transfer pricing.

Under concealment cost substitutability, one profit shifting activity, say, manipulating the interest rate, makes it more costly to shift debt. Hence, the direct effect which indicates that it has become more profitable to shift profits is offset by the indirect cost interaction effect. The end outcome, then, depends on the relative magnitudes of the direct and the indirect effect. As a consequence, a higher corporate tax rate may under certain circumstances induce the MNC to shift less profit/or reduce the debt-to-asset ratio.\(^{21}\) What is certain is that the tax sensitivity of the corporate tax base is lower than if the two tax-engineering efforts did not interact.

Proposition 2 Irrespective of the properties of the concealment cost function, the tax sensitivity of interest-rate manipulation \((d\tilde{r}_i/dt_i)\) cannot be signed, even if a higher tax rate increases the amount of profit shifted \((dP_i/dt_i > 0)\).

The effect on the optimal interest-rate manipulation \(\tilde{r}_i\) is ambiguous for any specification of concealment costs. In general, a higher tax rate leads to more profit shifting (rise in \(P_i = \tilde{r}_i \cdot b_i^f \cdot K_i\)) and induces the MNC to use more internal leverage \(b_i^f\). Since a higher leverage ratio \(b_i^f\) also shifts more profit, the interest rate \(\tilde{r}_i\) may have to fall to ensure that the optimal amount of profit is shifted.

\(^{21}\)In any case, the tax base \((TB)\) of the MNC will shrink, however, because it can be shown that \(\frac{dTB}{dt_i} = -\frac{(r - t_i)K_i}{(1 - t_i)\text{SOC}} '\left[(rK_i)^2 A + 2rK_iB + D\right] < 0\) even for fixed capital investment. The result follows from applying the fact that \(AD - B^2 > 0\) from the \(SOC > 0\) to hold.
Our results above should be contrasted to the findings in the empirical literature where a main insight is that the management of a MNC is more likely to respond to a tax change by manipulating transfer prices than debt. In particular, evidence suggests that internal debt is not very sensitive to changes in the corporate tax rate (see e.g., Büttner and Wamser, 2007; Møen et al., 2011). Based on our results in equations (16) and (17), the findings in the empirical literature could be explained by the availability of multiple profit shifting instruments, where debt is an instrument that is more expensive to manipulate. In other words, transfer pricing and debt shifting are cost substitutes and the profile of the concealment cost curve differs for the two, with a high concealment cost curvature for debt shifting and a low for transfer pricing.\footnote{In technical terms, the tax-rate sensitivity for each instrument - confer equations (16) and (17) – is determined by the gradients of the marginal concealment costs (all else equal) not the absolute level of the marginal concealment costs.}

Cost substitutability will decrease the tax-rate sensitivities of both instruments, all else equal. If under cost substitutability, we have a large increase in marginal concealment costs related to internal debt whereas costs related to transfer pricing are low, we obtain magnitudes of tax sensitivities in line with the empirical literature. Such differences in costs may be explained, for example, by binding thin-capitalization rules, whereas regulation of transfer prices provides the MNC with a larger degree of discretion.

5 Government Regulation

In this section, we study how political measures to protect the tax base affect management decisions. In particular, we examine how thin-capitalization rules, and rules that place restrictions on the amount of profit shifted affect management decisions.

In order to facilitate the analysis, we rewrite the concealment cost function of internal debt as $C_I = C_I(b^i_I, P_i, \sigma_i)$, where $\sigma_i$ is a parameter that measures the tightness of thin-capitalization rules in country $i$. A higher $\sigma_i$ (i.e., tighter thin-capitalization rules) is taken to imply that it becomes more costly to circumvent such rules. We shall also invoke the reasonable assumption that tighter thin-capitalization rules make it more costly to shift debt and profit, that is, $\partial^2 C_I / (\partial b^i_I \partial \sigma_i) > 0$ and $\partial^2 C_I / (\partial P_i \partial \sigma_i) > 0$, but we shall not allow these effects to go to infinity. The latter implies that MNCs may still find ways to circumvent thin-capitalization rules as this seems to be in line with empirical research on thin capitalization rules (see, e.g., Weichenrieder and Windischbauer, 2008; Büttner et al., 2012).

We denote the concealment costs of profit shifting as $C_P = C_P(b^i_P, P_i, \alpha_i)$, where $\alpha_i$ is a parameter that indicates the strictness of arm’s-length pricing regulation in country $i$. An increase in $\alpha_i$ implies higher concealment costs or higher fines if profit shifting is detected. Similar to the case of thin-capitalization rules, stricter transfer-pricing regu-
lation increases marginal concealment costs of manipulating interest expenses, that is, \( \partial^2 C_P / (\partial b_i^l \partial \alpha_i) > 0 \), \( \partial^2 C_P / (\partial P_i \partial \alpha_i) > 0 \).

Differentiating the first-order conditions (8) and (10) and doing comparative statics on tighter thin-capitalization rules \((\sigma_i)\), we find that

\[
\frac{db_i^l}{d\sigma_i} = \frac{b_i^l K_i}{SOC} \left[ \frac{\partial^2 C_l}{\partial P_i \partial \sigma_i} \cdot B - \frac{\partial^2 C_l}{\partial b_i^l \partial \sigma_i} \cdot A \right] \begin{cases} < 0 & \text{if } \frac{\partial^2 C_l}{\partial b_i^l \partial P_i} < 0, \\ \geq 0 & \text{if } \frac{\partial^2 C_l}{\partial b_i^l \partial P_i} > 0, \\ \end{cases}
\]

(18)

\[
\frac{dP_i}{d\sigma_i} = \frac{b_i^l K_i}{SOC} \left[ \frac{\partial^2 C_l}{\partial b_i^l \partial \sigma_i} \cdot B - \frac{\partial^2 C_l}{\partial P_i \partial \sigma_i} \cdot D \right] \begin{cases} < 0 & \text{if } \frac{\partial^2 C_l}{\partial \beta_i \partial P_i} < 0, \\ \geq 0 & \text{if } \frac{\partial^2 C_l}{\partial \beta_i \partial P_i} > 0. \\ \end{cases}
\]

(19)

Comparative statics on the profit-shifting regulation parameter \((\alpha_i)\) yields

\[
\frac{db_i^l}{d\alpha_i} = \frac{b_i^l K_i}{SOC} \left[ \frac{\partial^2 C_P}{\partial P_i \partial \alpha_i} \cdot B - \frac{\partial^2 C_P}{\partial b_i^l \partial \alpha_i} \cdot A \right] \begin{cases} < 0 & \text{if } \frac{\partial^2 C_P}{\partial b_i^l \partial P_i} < 0, \\ \geq 0 & \text{if } \frac{\partial^2 C_P}{\partial b_i^l \partial P_i} > 0, \\ \end{cases}
\]

(20)

\[
\frac{dP_i}{d\alpha_i} = \frac{b_i^l K_i}{SOC} \left[ \frac{\partial^2 C_P}{\partial b_i^l \partial \alpha_i} \cdot B - \frac{\partial^2 C_P}{\partial P_i \partial \alpha_i} \cdot D \right] \begin{cases} < 0 & \text{if } \frac{\partial^2 C_P}{\partial \beta_i \partial P_i} < 0, \\ \geq 0 & \text{if } \frac{\partial^2 C_P}{\partial \beta_i \partial P_i} > 0. \\ \end{cases}
\]

(21)

Based on equations (18) and (19), we may state:

**Proposition 3** Tighter thin-capitalization regulation decrease both debt shifting and profit shifting under concealment cost complementarity. With concealment cost substitutability, tighter thin-capitalization regulation may foster more debt shifting (thin capitalization) or transfer pricing.

The mechanisms that lead to these results are similar to those explained in the previous section. For concealment cost complementarity, there is a win-win situation from the point of view of the government. Stricter thin-capitalization rules will increase marginal concealment costs and reduce debt shifting and profit shifting. Reduced debt shifting increases marginal costs of profit shifting further and the indirect cost interaction effects induce an even stronger reduction in both kinds of tax engineering. In contrast, when one activity increases concealment costs related to other tax-engineering efforts (i.e., for concealment cost substitutability), the outcome is in general ambiguous and depends on the specific form of the concealment cost functions. In this case, paradoxical outcomes may result. One example is that rules intended to reduce thin capitalization could relax

\(^{23}\) A full derivation for both \(\alpha_i\) and \(\sigma_i\) is given in the Appendix.
the costs of transfer pricing ($\frac{\partial^2 C_I}{\partial F_I^2}, \frac{\partial^2 C_P}{\partial F_I^2} > 0$) and thus increase profit shifting. Such an outcome would be particularly inauspicious, because profit shifting appears to be more tax aggressive and does not result in any higher investment (contrary to debt shifting; cf. Lemma 1).

When it comes to regulation that affects the firm’s ability to shift profit, we summarize the insights from equations (20) and (21) as follows:

Proposition 4 Under concealment cost complementarity, stricter regulation to prevent profit shifting decreases both debt shifting and profit shifting. Under concealment cost substitutability, regulation may lead to more debt shifting or more profit shifting.

The unintended effects of regulation under concealment cost substitutability is clearly seen from equation (19), where tougher regulation of profit shifting may actually foster more profit shifting and reduce the costs of working around thin-capitalization rules so that the debt-to-asset ratio rises. We stress, however, that regulation in this case may also result in less leverage and profits shifted, and that in general, the outcome depends on the relative magnitudes of the interplay between concealment costs related to the use of internal leverage and profit shifting.

Propositions (3) and (4) show that it is of crucial importance to have knowledge about functional forms of the concealment function when enacting policy to protect the corporate tax base. If concealment cost functions exhibit substitutability, government action may lead to management responses that go in the opposite direction of what the policy aims at achieving.

6 Conclusions

In this paper we have departed from the traditional view in the literature where the choice of transfer prices on internal leverage in affiliates is decided separately from the choice of firms’ tax efficient financing structure. Rather we have argued that these decisions are interrelated. The analysis has answered two key questions. The first pertains to how concealment costs related to debt shifting and profit shifting affect management responses to changes in corporate taxes. The second concerns how government regulation intended to curb profit shifting affects tax planning and profit shifting by multinationals. We show that policies intended to protect national tax bases may have unintended effects under concealment cost substitutability. Our findings point to that it is of crucial importance to have more knowledge about costs related to activities that often are labelled tax avoidance or tax evasion in order to understand how management in multinationals behaves. These costs may differ depending on the type of activity the firm engages in, and they may affect other tax-engineering efforts as well. It can be shown that our study carries over to a more general setting where the transfer price is not related to debt.
A Appendix

The first-order condition of external debt (9) is fully separable from the other decisions. Hence, we can neglect it and focus on the other two conditions (8) and (10). Denote the tightness of anti-profit-shifting regulation in country \(i\) by parameter \(\alpha_i\) and the tightness of thin-capitalization rules in country \(i\) by parameter \(\sigma_i\). The first-order conditions for internal debt (10) and for interest-rate manipulation pricing (8) can be transformed into

\[
\frac{(t_i - t_1)r}{1 - t_i} - \frac{\partial C_I}{\partial b_i} - \frac{\partial C_P}{\partial b_i} \frac{1}{K_i} = 0, \quad (22)
\]

\[
\frac{t_i - t_1}{1 - t_i} - \frac{\partial C_P}{\partial P_i} - K_i \frac{\partial C_I}{\partial P_i} = 0, \quad (23)
\]

where we made use of \(\lambda = t_1\) and \(\eta = 1 - t_i\).

Totally differentiating these expressions leads to

\[
\begin{align*}
\left[ \frac{\partial^2 C_I}{\partial (b_i^t)^2} + \frac{\partial^2 C_I}{\partial b_i^t \partial P_i} \tilde{r}_i K_i + \frac{\partial^2 C_P}{\partial (b_i^t)^2} K_i \right] \frac{\partial b_i^t}{\partial P_i} + \left[ \frac{\partial C_I}{\partial b_i^t} b_i^t K_i + \frac{\partial C_P}{\partial b_i^t} b_i^t K_i \right] \frac{\partial \tilde{r}_i}{\partial P_i} & = \frac{1 - t_i}{(1 - t_i)^2} r dt_i - \frac{\partial^2 C_P}{\partial b_i^t \partial \alpha_i} K_i \frac{d\alpha_i}{\partial P_i} - \frac{\partial^2 C_I}{\partial b_i^t \partial \sigma_i} \frac{d\sigma_i}{\partial P_i} + \frac{\partial^2 C_I}{\partial P_i^2} \frac{d\tilde{r}_i}{\partial P_i} + \frac{\partial^2 C_P}{\partial P_i^2} \frac{d\tilde{r}_i}{\partial P_i}, \\
\left[ \frac{\partial^2 C_P}{\partial P_i^2} \tilde{r}_i K_i + \frac{\partial^2 C_P}{\partial \alpha_i \partial P_i} \frac{d\alpha_i}{\partial P_i} + \frac{\partial^2 C_I}{\partial \alpha_i \partial P_i} \frac{d\alpha_i}{\partial P_i} K_i \right] \frac{\partial b_i^t}{\partial P_i} + \left[ \frac{\partial^2 C_P}{\partial P_i^2} b_i^t K_i + \frac{\partial^2 C_I}{\partial P_i^2} b_i^t K_i^2 \right] \frac{\partial \tilde{r}_i}{\partial P_i} & = \frac{1 - t_i}{(1 - t_i)^2} d t_i - \frac{\partial^2 C_P}{\partial P_i \partial \alpha_i} \frac{d\alpha_i}{\partial P_i} - \frac{\partial^2 C_I}{\partial P_i \partial \sigma_i} \frac{d\sigma_i}{\partial P_i} + \frac{\partial^2 C_P}{\partial P_i^2} \frac{d\tilde{r}_i}{\partial P_i} + \frac{\partial^2 C_I}{\partial P_i^2} \frac{d\tilde{r}_i}{\partial P_i},
\end{align*}
\]

and collecting terms results in

\[
\begin{pmatrix}
\frac{\partial^2 C_I}{\partial (b_i^t)^2} + \frac{\partial^2 C_I}{\partial b_i^t \partial P_i} \tilde{r}_i K_i + \frac{\partial^2 C_P}{\partial (b_i^t)^2} K_i \\
\frac{\partial^2 C_P}{\partial P_i^2} \tilde{r}_i K_i + \frac{\partial^2 C_P}{\partial \alpha_i \partial P_i} \frac{d\alpha_i}{\partial P_i} + \frac{\partial^2 C_I}{\partial \alpha_i \partial P_i} \frac{d\alpha_i}{\partial P_i} K_i
\end{pmatrix}
\begin{pmatrix}
\frac{\partial b_i^t}{\partial P_i} \\
\frac{\partial b_i^t}{\partial P_i}
\end{pmatrix}
+ \left( \begin{pmatrix}
\frac{\partial^2 C_I}{\partial (b_i^t)^2} K_i \\
\frac{\partial^2 C_P}{\partial P_i^2} b_i^t K_i + \frac{\partial^2 C_I}{\partial P_i^2} b_i^t K_i^2
\end{pmatrix}
\begin{pmatrix}
\frac{\partial \tilde{r}_i}{\partial P_i} \\
\frac{\partial \tilde{r}_i}{\partial P_i}
\end{pmatrix}
\right)
= \left( \frac{1 - t_i}{(1 - t_i)^2} \right) dt_i - \left( \begin{pmatrix}
\frac{\partial^2 C_P}{\partial b_i^t \partial \alpha_i} \\
\frac{\partial^2 C_P}{\partial \alpha_i \partial \alpha_i}
\end{pmatrix}
\begin{pmatrix}
\frac{d\alpha_i}{\partial P_i} \\
\frac{d\alpha_i}{\partial P_i}
\end{pmatrix}
\right) d t_i - \left( \begin{pmatrix}
\frac{\partial^2 C_I}{\partial b_i^t \partial \sigma_i} \\
\frac{\partial^2 C_I}{\partial \sigma_i \partial \alpha_i}
\end{pmatrix}
\begin{pmatrix}
\frac{d\sigma_i}{\partial P_i} \\
\frac{d\sigma_i}{\partial P_i}
\end{pmatrix}
\right) d \sigma_i.
\]
The second-order condition implies

\[
SOC = \left[ \frac{\partial^2 C_I}{\partial (b_i')^2} + \frac{\partial^2 C_I}{\partial b_i' \partial P_i} \tilde{r}_i K_i + \frac{\partial^2 C_P}{\partial b_i' \partial P_i} \tilde{r}_i K_i + 1 \right] \left[ \frac{\partial^2 C_P}{\partial P_i^2} b_i^l K_i + \frac{\partial^2 C_I}{\partial P_i^2} b_i^l K_i^2 \right] > 0.
\]  

(25)

Utilizing the definitions \( A = \frac{\partial^2 C_P}{\partial P_i^2} + \frac{\partial^2 C_I}{\partial P_i^2} K_i \), \( D = \frac{\partial^2 C_I}{\partial (b_i')^2} K_i + \frac{\partial^2 C_P}{\partial (b_i')^2} \) and \( B = \frac{\partial^2 C_I}{\partial b_i' \partial P_i} K_i + \frac{\partial^2 C_P}{\partial b_i' \partial P_i} \), the SOC can be simplified to

\[
SOC = (AD - B^2) b_i^l > 0.
\]  

(26)

Applying Cramer’s rule, we finally receive for internal debt

\[
\frac{db_i^l}{dt_i} = \frac{(1 - t_i) \left[ \frac{\partial^2 C_P}{\partial P_i^2} r b_i^l K_i + \frac{\partial^2 C_I}{\partial P_i^2} r b_i^l K_i^2 - \frac{\partial^2 C_I}{\partial b_i' \partial P_i} b_i^l K_i - \frac{\partial^2 C_P}{\partial b_i' \partial P_i} b_i^l \right]}{(1 - t_i)^2 SOC} \begin{cases} > 0, & \text{if } \frac{\partial^2 C_I}{\partial b_i' \partial P_i} < 0, \\ \geq 0, & \text{if } \frac{\partial^2 C_I}{\partial b_i' \partial P_i} > 0, \end{cases}
\]

(27)

\[
\frac{db_i^l}{d\sigma_i} = -\frac{\frac{\partial^2 C_I}{\partial b_i' \partial \sigma_i} \left[ \frac{\partial^2 C_P}{\partial P_i^2} b_i^l K_i + \frac{\partial^2 C_I}{\partial P_i^2} b_i^l K_i^2 \right] + \frac{\partial^2 C_I}{\partial b_i' \partial P_i} K_i \left[ \frac{\partial^2 C_P}{\partial b_i' \partial P_i} b_i^l K_i + \frac{\partial^2 C_P}{\partial b_i' \partial P_i} b_i^l \right]}{SOC} \begin{cases} < 0, & \text{if } \frac{\partial^2 C_I}{\partial b_i' \partial P_i} < 0 \text{ or } \frac{\partial^2 C_I}{\partial P_i \partial \sigma_i} = 0, \\ \geq 0, & \text{if } \frac{\partial^2 C_I}{\partial b_i' \partial P_i} > 0, \end{cases}
\]

(28)

\[
\frac{db_i^l}{d\alpha_i} = -\frac{\frac{\partial^2 C_P}{\partial \alpha_i} \left[ \frac{\partial^2 C_P}{\partial P_i^2} b_i^l K_i \right] + \frac{\partial^2 C_I}{\partial P_i \partial \alpha_i} \left[ \frac{\partial^2 C_I}{\partial \alpha_i} b_i^l K_i + \frac{\partial^2 C_P}{\partial \alpha_i} b_i^l \right]}{SOC} \begin{cases} < 0, & \text{if } \frac{\partial^2 C_I}{\partial b_i' \partial P_i} < 0, \\ \geq 0, & \text{if } \frac{\partial^2 C_I}{\partial b_i' \partial P_i} > 0. \end{cases}
\]

(29)
The effects on manipulating the interest rates for transfer pricing are

\[
\frac{d\tilde{r}_i}{dt_i} = \frac{(1 - t_i)}{(1 - t_i)^2 SOC} \left[ \frac{\partial^2 C_I}{\partial (b'_i)^2} + \frac{\partial^2 C_P}{\partial (b'_i)^2} \frac{1}{K_i} - \left( \frac{\partial^2 C_P}{\partial P_i^2} + \frac{\partial^2 C_I}{\partial P_i^2} \right) \tilde{r}_i r K_i - \left( \frac{\partial^2 C_I}{\partial b'_i \partial P_i} K_i + \frac{\partial^2 C_P}{\partial b'_i \partial P_i} \right) (r - \tilde{r}_i) \right] \geq 0,
\]

(30)

\[
\frac{d\tilde{r}_i}{d\sigma_i} = -\frac{1}{SOC} \left[ \frac{\partial^2 C_I}{\partial P_i \partial \sigma_i} K_i \left( \frac{\partial^2 C_I}{\partial (b'_i)^2} + \frac{\partial^2 C_P}{\partial (b'_i)^2} \frac{1}{K_i} \right) - \frac{\partial^2 C_P}{\partial b'_i \partial \sigma_i} \left( \frac{\partial^2 C_I}{\partial b'_i \partial P_i} K_i + \frac{\partial^2 C_P}{\partial b'_i \partial P_i} \right) \tilde{r}_i K_i \right] \geq 0,
\]

(31)

\[
\frac{d\tilde{r}_i}{d\alpha_i} = -\frac{1}{SOC} \left[ \frac{\partial^2 C_P}{\partial P_i \partial \alpha_i} \left( \frac{\partial^2 C_I}{\partial (b'_i)^2} + \frac{\partial^2 C_P}{\partial (b'_i)^2} \frac{1}{K_i} \right) - \frac{\partial^2 C_P}{\partial b'_i \partial \alpha_i} \left( \frac{\partial^2 C_I}{\partial b'_i \partial P_i} K_i + \frac{\partial^2 C_P}{\partial b'_i \partial P_i} \right) \tilde{r}_i \right] \geq 0.
\]

(32)

The effects on interest-rate manipulation are ambiguous in all cases, since the level of shifted profits also depends on internal leverage \( b'_i \). For example, a higher tax rate induces more profit shifting (first terms in squared bracket in equation (30)). However, since a higher internal leverage also increases profit shifting \( P_i = \tilde{r}_i \cdot b'_i \cdot K_i \), there is also a negative effect on the interest rate \( \tilde{r}_i \) (the first term in brackets in the squared bracket in (30)). Finally, we have the cross effects on marginal costs, which can enforce or reduce the former effects (see last term in squared bracket).

Focusing on total profit shifting \( P_i \), the effects simplify and become as expected. Since total profit shifting is given by \( P_i = \tilde{r}_i \cdot b'_i \cdot K_i \),
we find:

\[
\frac{dP_i}{dt_i} = \frac{d\tilde{r}_i}{dt_i} b'_i K_i + \frac{db'_i}{dt_i} \tilde{r}_i K_i = \frac{1 - t_i}{(1 - t_i)^2 SOC} \left[ \left( \frac{\partial^2 C_l}{\partial (b'_i)^2} K_i + \frac{\partial^2 C_P}{\partial (b'_i)^2} \right) b'_i - \left( \frac{\partial^2 C_l}{\partial b'_i \partial P_i} K_i + \frac{\partial^2 C_P}{\partial b'_i \partial P_i} \right) r b'_i K_i \right] \begin{cases} > 0, & \text{if } \frac{\partial^2 C_l}{\partial b'_i \partial P_i} < 0, \\ \leq 0, & \text{if } \frac{\partial^2 C_l}{\partial b'_i \partial P_i} > 0, \end{cases} (33)
\]

\[
\frac{dP_i}{d\sigma_i} = \frac{d\tilde{r}_i}{d\sigma_i} b'_i K_i + \frac{db'_i}{d\sigma_i} \tilde{r}_i K_i = -\frac{1}{SOC} \left[ \frac{\partial^2 C_l}{\partial P_i \partial \sigma_i} \left( \frac{\partial^2 C_l}{\partial (b'_i)^2} K_i + \frac{\partial^2 C_P}{\partial (b'_i)^2} \right) b'_i K_i - \frac{\partial^2 C_l}{\partial b'_i \partial \sigma_i} \left( \frac{\partial^2 C_l}{\partial b'_i \partial P_i} K_i + \frac{\partial^2 C_P}{\partial b'_i \partial P_i} \right) b'_i K_i \right] \begin{cases} < 0, & \text{if } \frac{\partial^2 C_l}{\partial b'_i \partial P_i} < 0, \\ \geq 0, & \text{if } \frac{\partial^2 C_l}{\partial b'_i \partial P_i} > 0, \end{cases} (34)
\]

\[
\frac{dP_i}{d\alpha_i} = \frac{d\tilde{r}_i}{d\alpha_i} b'_i K_i + \frac{db'_i}{d\alpha_i} \tilde{r}_i K_i = -\frac{1}{SOC} \left[ \frac{\partial^2 C_P}{\partial P_i \partial \alpha_i} \left( \frac{\partial^2 C_l}{\partial (b'_i)^2} K_i + \frac{\partial^2 C_P}{\partial (b'_i)^2} \right) b'_i - \frac{\partial^2 C_P}{\partial b'_i \partial \alpha_i} \left( \frac{\partial^2 C_l}{\partial b'_i \partial P_i} K_i + \frac{\partial^2 C_P}{\partial b'_i \partial P_i} \right) b'_i \right] \begin{cases} < 0, & \text{if } \frac{\partial^2 C_l}{\partial b'_i \partial P_i} < 0, \\ \geq 0, & \text{if } \frac{\partial^2 C_l}{\partial b'_i \partial P_i} > 0. \end{cases} (35)
\]
References


