PRA 5.7
Mixing of a jet into a stratified environment

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FOREWORD

This program system is the result of a cooperation between Norsk Institutt for Vannforskning, Oslo (NIVA) and A. S Computas, Oslo. The physical and mathematical fundament has been worked out by NIVA, while A. S Computas has been responsible for the development of the program system. There has been a high degree of cooperation in the lay-out and testing of the system.

A.S Computas, January 1973
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1. Introduction.

The NIVA * JET. MIX is a program complex designed to compute the behaviour of a buoyant jet into a stratified ambient environment. It has the ability to take a various number of data and to save data for later use. The program can also be used to compute a density profile from given temperature and salinity at different depths.


This program analyses the problem of an inclined buoyant jet, mixing by entrainment into a stagnant environment of vertically varying density with stable stratification. The jet may be of two types:

a) Three-dimensional, with circular symmetry.

b) Two-dimensional, that is with practically infinite extension in one horizontal direction normal to jet velocities.

A system of differential equations has been derived based on the theoretical description of turbulent buoyant jet presented by Loh-Nien Fan and Normann H. Brooks, 1969 (1), and by John D. Ditmars 1969 (2).

2.1. Assumptions.

A 1. The fluid is incompressible.

A 2. Density-variations over the flow field are small compared to the reference density.

A 3. The vertical density gradient approximates a constant over the extension of a jet cross-section.
A 4. Density has linear mixing properties.

A 5. The flow is fully turbulent.

A 6. The flow is stationary.

A 7. The radius of curvature of the jet trajectory is much greater than the jet width.

A 8. Similarity over all cross-sections of the jet is assumed for the profiles of velocity, buoyancy and concentration of tracer. Gaussian profiles are good approximations.

A 9. Longitudinal dispersion may be neglected relative to longitudinal convection.

2.2. Definitions.

Fixed space coordinates are \((x, y, z)\), with \(z\) in the vertical, downwards direction. The middle velocities of the jet over all cross-sections must lie in the same vertical plane, defined as the \(x, z\)-plane.

A jet trajectory (centerline) is defined in the \(xz\)-plane, through the centers of the assumed Gaussian profiles. The cross-sections, planes normal to the centerline, are defined by the distance \(s\) along the centerline from a chosen reference point. The points in such a cross-section are defined by polar coordinates \((r, \phi)\) for a circular jet, and by the distance \(n\) in the \(xz\)-plane from the centerline to the point for a two-dimensional jet. The coordinates \((s, r, \phi), (s, n, y)\) then define a point in space. Figures A1, A2 (Appendix 1) shows the coordinates for both situations.

The following quantities are defined:
a. **Scalar fields in space:**

**Jet geometry**

<table>
<thead>
<tr>
<th>Circular</th>
<th>Two dimensional</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u^*(s,r) )</td>
<td>( u^*(s,n) )</td>
</tr>
<tr>
<td>( \rho_a^*(s,r,\phi) )</td>
<td>( \rho_a^*(s,n) )</td>
</tr>
<tr>
<td>( \rho_s^*(s,r,\phi) )</td>
<td>( \rho_s^*(s,n) )</td>
</tr>
<tr>
<td>( c^*(s,r,\phi) )</td>
<td>( c^*(s,n) )</td>
</tr>
</tbody>
</table>

b. **Functions of \( s \):**

- \( u(s) \): centerline velocity in \( s \)-direction
- \( \rho_a(s) \): centerline value of ambient density
- \( \rho(s) \): centerline jet density
- \( c(s) \): centerline tracer concentration
- \( b(s) \): characteristic jet width
- \( \Theta(s) \): angle between \( x \)-axis and jet trajectory

c. **Constants.**

- \( g \): acceleration of gravity

\( \lambda^2 \): turbulent Schmidt number, expressing the difference in dispersion coefficient for momentum and for tracers. Equation A(1) - A(3) implicitly define \( \lambda \).

\( \alpha \): Entrainment coefficient, expressing the rate of mixing of the jet with the environment. This mixing is due to the jet turbulence, which disperses into the stagnant environment and absorbs it into the jet.
The volume flux $Q$ through a cross-section thereby increases along the jet trajectory, and this increase of $Q$ is given by:

$$\frac{dQ}{ds} = 2\pi \rho_{\text{ub}} \text{ in the circular case,}$$
and

$$\frac{dQ}{ds} = 2\rho u \text{ in the two-dimensional case,}$$

where $Q$ = volume flux per length in $y$-direction.

$dQ/ds$ may be understood as the inflow to the jet through the side boundary.

$\rho_a$ = ambient density at the discharge point, used as reference density.

Gaussian profiles are assumed as follows:

For circular jet:

$$u(s,r) = u(s) e^{-r^2/b^2} \quad \text{(A1)}$$

$$\rho_a(s,r,\phi) - \rho(s,r,\phi) = \left[\rho_a(s) - \rho(s)\right] e^{-r^2/\lambda b^2} \quad \text{(A2)}$$

$$c(s,r) = c(s) e^{-r^2/\lambda b^2} \quad \text{(A3)}$$

Nominal jet width: $W = \sqrt{n} b \quad \text{(A4)}$

For a two-dimensional jet, the identical equations, with $r$ replaced by $n$, are used.
2.3. Equations.

a) **Circular symmetry.**

Continuity relations for volume, x-momentum, z-momentum, buoyancy and tracer, together with two obvious geometric relations, give the equations:

\[
\frac{du}{ds} = -2\alpha \frac{u}{b} - \frac{2g\lambda^2}{b} \cdot \sin \theta (p_a - p) \tag{A5}
\]

\[
\frac{db}{ds} = 2\alpha - \frac{g\lambda^2}{b} \cdot \sin \theta (p_a - p) \tag{A6}
\]

\[
\frac{d(p_a - p)}{ds} = -\frac{1 + \lambda^2}{\lambda^2} \frac{dpa}{dz} \cdot \sin \theta - 2\alpha \frac{pa - p}{b} \tag{A7}
\]

\[
\frac{d\theta}{ds} = \frac{2g\lambda^2}{\rho_o} \cdot \cos \theta \left( p_a - p \right) \tag{A8}
\]

\[
\frac{dx}{ds} = \cos \theta \tag{A9}
\]

\[
\frac{dz}{ds} = -\sin \theta \tag{A10}
\]

\[
\frac{d (\text{cub}^2)}{ds} = 0 \tag{A11}
\]

Equations A(5) - (10) constitute a system of simultaneous differential equations in the six dependent variables \( u, b, \theta, (p_a - p), x, z \), as functions of \( s \).
Eq. A(11) integrates into an algebraic equation for the tracer concentration:

$$c = c_0 \cdot \frac{u_0 b^2}{ub}$$  

A(12)

or for the centerline dilution $$S = \frac{c_0}{c}$$:

$$S = \frac{u b}{u_0 b}$$  

A(13)

Equations A(5)-(13) apply only to the zone of established flow, with gaussian profiles. A zone of flow establishment (zone 1) is therefore incorporated. The jet is assumed discharged from a circular hole of diameter $$D$$, and with uniform velocity $$U$$, density $$\rho_d$$, dilution $$1$$, and at initial angle $$\Theta_0$$. Deflection is neglected in zone 1, which is taken of length

$$s_1 = 6.2 \cdot D$$  

A(14)

and at the end of which a gaussian velocity profile with

$$u_1 = U$$  

A(15)

are assumed to be established. Continuity and geometric relations then give:

$$b_1 = DJ \sqrt{2}$$  

A(16)

$$\left(\rho_a - \rho\right)_1 = \left(\rho_a (s_1) - \rho_d\right) \cdot \frac{1+\lambda^2}{2\lambda}$$  

A(17)

$$\Theta_1 = \Theta_0$$  

A(18)

$$x_1 = 6.2 \cdot D \cdot \cos \Theta \ (\Theta_0 \text{ is set to zero})$$  

A(19)

$$Z_1 = 6.2 \cdot D \sin \Theta + Z_0$$  

A(20)

$$S_1 = \frac{2\lambda^2}{1+\lambda^2}$$  

A(21)
Eq. A(15)-(20) are used as initial values for the system A(5)-(10), and A(21) is used with A(15), (16) for correction of A(13) into:

\[ S = \frac{4\lambda^2}{1+\lambda} \cdot \frac{ub^2}{UD} \]  

A(22)

b) Two-dimensional jet.

The different geometry leads to the following changes of eq. A(5)-A(22):

\[ \frac{du}{ds} = -2a \frac{u + \sqrt{2g\lambda}}{\frac{u + \sqrt{2g\lambda}}{b} \rho_0} \]  

A(5')

\[ \frac{db}{ds} = \frac{ua - \sqrt{2g\lambda}}{\sqrt{\pi} \rho_0} \cdot b \frac{\sin \theta (\rho_0 - \rho)}{u} \]  

A(6')

\[ \frac{d(\rho_a - \rho)}{ds} = -\sqrt{\frac{1+\lambda}{\lambda}} \frac{\rho_a - \rho}{\Sigma} \cdot \frac{\rho_a - \rho}{b} \]  

A(7')

\[ \frac{d\theta}{ds} = \frac{\sqrt{2g\lambda}}{\rho_0} \cdot \cos \theta \frac{\rho_0 - \rho}{u} \]  

A(8')

Eq. A(9) and A(10) unchanged.

\[ \frac{d}{ds} \ (\text{cub}) = 0 \]  

A(11')

\[ c = c_0 \frac{u_0 b_0}{u_0 b_0} \]  

A(12')

\[ S = \frac{ub}{u_0 b_0} \]  

A(13')

With these initial values eq. A(5)-(10) are used up to the point of neutral density, that is the point where \( q_a - q \) passes zero. From here the program shifts to a simple potential energy calculation for the vertical movement, where entrainment and friction are neglected, and gravitation is considered the only active force. Such calculation is also used directly for special initial conditions. The details are given in section 2.6.
The jet is now discharged from an infinite slot of width $B$, with velocity, density, dilution and initial angle as before, and the endpoint of zone 1 is given by:

$$s_1 = 5.2 \cdot B$$  \hspace{1cm} A(14')

$$b_1 = \sqrt{\frac{2}{\pi}} \cdot B$$  \hspace{1cm} A(16')

$$(\rho_a - \rho)_1 = (\rho_a - \rho_0) \cdot \sqrt{\frac{1 + \lambda^2}{2\lambda^2}}$$  \hspace{1cm} A(17')

$$x_1 = 5.2 \cdot B \cos \theta$$  \hspace{1cm} A(19')

$$z_1 = z_0 - 5.2 \cdot B \sin \theta$$  \hspace{1cm} A(20')

$$s_1 = \sqrt{\frac{2 \cdot \lambda^2}{1 + \lambda^2}}$$  \hspace{1cm} A(21')

with A(15) and A(18) unchanged.

The correction of A(13') becomes:

$$S = \sqrt{\frac{2}{\pi}} \cdot \frac{u_b}{UB}$$  \hspace{1cm} A(22')

From the point of neutral density, or for special initial conditions, the equations in section 2.6 are used.

2.4. Numerical values of constants.

Suggested values for the constants $\alpha$, $\lambda$ (from lit. ref.[1]) are:

<table>
<thead>
<tr>
<th></th>
<th>$\alpha$</th>
<th>$\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Round, buoyant jet:</td>
<td>0.082</td>
<td>1.16</td>
</tr>
<tr>
<td>Round, simple momentum jet:</td>
<td>0.057</td>
<td>1.16</td>
</tr>
<tr>
<td>Two-dimensional, buoyant jet:</td>
<td>0.16</td>
<td>0.89</td>
</tr>
</tbody>
</table>
For the last case, eq. A(21') gives $S_1 < 1$ which is obviously unphysical, and follows from the fact that we assume a gaussian distribution of tracer before it is fully established. A similar remark applies to eq. A(17'). This should give a relatively small error in $(\rho_a - \rho)$ over a short $s$-interval after zone 1, increasing calculated values for terminal rising heights or sinking depths a bit.

2.5. Density calculations.

Ambient and jet density are calculated from salinity and temperature as given in Maintenance Manual, Appendix C2 and C3, and are then assumed to mix linearly.

However, density as a function of temperature is convex upwards, with a positive apex at approximately $T = 4^\circ C$. If molecular diffusion is effective in smoothing temperature profiles resulting from turbulent mixing, density no longer mixes linearly.

For mixing of two freshwater volumes of different temperatures, the actual resulting density will then be higher than the linearly mixed density occurring in the differential equations. (Cfr. mixing of two equal volumes of water with temperatures $0^\circ C$ and $8^\circ C$ respectively).

For waters of different salinities (or other tracer concentrations affecting density) the density is approximately a function of $T$ as before, linearly displaced in the vertical direction by salinity. If the mixing process is considered in two stages, the first stage being mixing of salt and the second mixing of temperature, it becomes clear that we have the same effect as for fresh water.

The result of the error in density will be that the calculated trajectory will rise higher, or sink less, than the actual trajectory, and the results will be on the safe side in terms of upwelling.
2.6 Calculation of max/min depth.

When the jet passes the point of neutral density, and goes towards its terminal rizing height or sinking depth, the equations presented will not be valid physically, and mathematically may give convergence trouble because of low velocities.

For the part of the jet path after the point of neutral density we therefore neglect entrainment and friction, and assume the vertical acceleration to be

$$\frac{dv}{dt} = g \cdot \frac{\rho_a - \rho}{\rho}$$  \hspace{1cm} \text{A(23)}$$

with \(v\) = velocity in neg. z-direction (upwards).

\(v_0 = u \cdot \sin \theta\) as the initial value.

This is transformed into:

$$\frac{dv^2}{dz} = 2g \cdot \frac{\rho_a - \rho}{\rho}$$ \hspace{1cm} \text{A(24)}$$

which is solved in steps by the given piecewise linear density profile up to a z value where \(v = 0\) or the limits in z is reached. Method is given in Maintenance Manual, section 3.6. Equation A(24) is also used directly for special initial conditions:

- For \(\theta = 90^\circ\) if \(\rho_a - \rho_d < 0\).
- For \(\theta = -90^\circ\) if \(\rho_a - \rho_d > 0\).
- For \(\theta = -90^\circ\) or \(90^\circ\), if \(\frac{d\rho_a}{dz} = 0\) and \(\frac{d\rho_d}{dz} > 0\).

The program is therefore hardly of any use for those conditions: for a heavy jet in vertical upwards direction, and for a light jet in vertical downwards direction.

For most normal initial conditions, however, the procedure chosen will give reasonable results, eq. (A5) - (13) will be used up to the point of neutral density, and eq. A(24) will then give an upper limit for max. depth, or a lower limit for min. depth.
3. **Accuracy.**


The differential equations A(5) - (13) are automatically solved to a preset accuracy, regulated by the program, and independent of DELTAS. The distance between output points is given by DELTAS.

3.2. Density profile intervals.

The density gradient is taken by program to be linear between given points. It is, therefore, important to use short intervals in areas where the density changes rapidly.

4. **User interface.**

This program is specially designed to be run from an interactive terminal. The run is divided into different kinds of modi: a) Control mode. b) Zero mode. c) Directive mode. Having got connection with the computer by calling up and typing site identification, the following control statement must be written, assuming running against the Univac 1100 Series operating system:

"RUN run-id, account-no, project"

Initiates run

"ASG, option filename 1., F"
"ASG, option filename 2., F"
"USE 7., filename 1.
"USE 8., filename 2."
Assigns files for saving of data. The filenames are chosen by user, but the numbers 7 and 8 are obligatory. The same pair of files must always be used together. For choice of option, see operating systems manual. (UP - 4144).

"XQT NIVA\*JET.MIX

This starts program execution and sets run in zero mode. This means system is ready to take one of the following directives. Each directive sets run in one of 19 directive modi. On return from a directive mode, run is back in zero mode, except END mode. when run goes back to control mode.

4.1. HELP.

Prints a question and waits for one of the letters I, O, D or F as answer. Prints out information of input, output, directives or file handling, respectively.

4.2. CIN.

Prints a heading and waits for input of five constants:
DELTA S   : Step length along jet curve in m.
XLIM      : Max. distance from discharge in m.
GRAV      : Acceleration of gravity in m/s².
ALPHA     : Entrainment coefficient.
LAMBDA    : Square root of Schmidt number.

4.3. CCOR.

Prints out heading and old values and waits for corrected values of one or more constants. As many constants are corrected as the number of fields in the input line. An asterisk (*) leaves the constant unchanged.

4.4. CSAV.

Saves current constants (if any) on file unit 7 for later use.

*) For use from cards:
"XQT NIVA\*JET.MIX/CARD
4.5. COUT.

Prints out current constants (if any) or constants from file.

4.6. CGET.

Reads constants from file 7.

4.7. JIN.

Prints heading and wait for 1 or 2 as answer. 1 means two-dimensional jet, 2 means round.

Then prints new heading and waits for input of
OUTF ALL SITE: Any 1-6 characters.
MANIFOLD NO: Integer number, not more than six digits.
Then prints third heading and waits for input of one integer number and
five real numbers:
N: Discharge hole no.
DEPTH(N): Discharge depth in m.
DIST(N): Discharge distance from shore in m.
DEFF(N): Effective diameter of hole in m.
THETA(N): Elevation angle of discharge in degrees.
U(N): Initial velocity of jet in m/s.

When one line is inputted, the same line is returned as a control, and it
then waits for a new line of the same kind. A maximum of ten lines of
this kind may be inputted. The input may be stopped before by writing
-1 as the first item of the line.

4.8. JCOR.

Correction of values inputted by JIN. Asterisk (*) means unchanged value.
As many values in one line are corrected as the number of items in the
correction line. If the whole line is to be unchanged, one asterisk is
typed in. Correction is stopped by typing -1 as first item in the line.
Old values are always printed out before corrected values are expected.
4.9. PIN P

Input of density profile with number $P(0 \leq N \leq 100)$. Prints heading and waits for input of:

STATION: Any 1-6 characters.
REG. TIME: Two items, each any 1-6 characters.
DENS WW: Real number, density of waste water in kg/l.

Prints new heading and waits for input of minimum 2, maximum 40 sets of:

DEPTH: Depth in m.
TEMP: Temperature in degrees centigrade, (or density).
SAL: Salinity in 0/00, (or an arbitrary negative number).

These data has to be given in sequence from the uppermost depth (closest to surface) and down. Zero salinity gives fresh water. The corresponding density is computed for each inputted line. Input is stopped by typing DEPTH= -1 or when 40 lines have been inputted.

The density can be given directly by typing in density instead of temperature. This is indicated by giving a negative salinity.

4.10. PCOR.

Same data as PIN, same procedure as JCOR.

4.11. POUT P1 P2 .......... PN

Prints out specified density profiles (N$\leq$6). Prints out all data given by PIN or PCOR. In addition, the corresponding density is printed out. After execution, the last printed density profile is the current one.

4.12. PSAV.

Saves current density profile on file unit 8. Should always be used after PIN or PCOR if the density profile is to be used later.

4.13. PGET P.

Makes density profile no. P the current one. The earlier current profile is lost if it has not been saved. PGET is automatically called by POUT and CALC.
4.14. PLIB.

Prints out list of saved profiles where:
PROFILE: Profile number.
STATION: Name of station.
REGTIME: Registration time.
NO OF DEPTHS: Number of depths given in profile.

4.15. INIT.

Initiates new file No. 8.
Must always be used when a new file No. 8 has been introduced. If this directive is used when the file contains data, these data are all lost.

4.16. PDEL P1 P2...PN.

Deletes the specified density profiles from file 8.

4.17. PPAC.

Packs file No. 8. Should be used when density profiles has been deleted or corrected or if a new profile with an earlier used profile No. has been saved. This prevents the file from growing too much.

4.18. CALC P1 P2...PN.

Does the calculation with the discharge data given by JIN or JCOR, the constants given by CIN or C COR or earlier CSAV and the density profiles specified.
The normal output gives:
MANIFOLD NO.
OUTFALL SITE
DENSITY PROFILE NO.
DISCHARGE N.
DEPTH (N)
DIST (N)
DEFF (N)
THETA (N)
U (N)
DENS WW Given by PIN or PCOR.

And the computed values for each step of length DELTAS:
DEPTH (J): "y-coordinate" of curve
DIST(J): "x-coordinate" of curve (from discharge)
WIDTH (J): Width of jet at point (x, y) in meters.
DILUT (J): Relative dilution of jet at point (x, y).
AMBDENS: Ambient density at point (x, y).
VEL (J): Jet velocity at point (x, y) in m/s.
The following special lines may be printed:
y x DEPTH/MIN
y x DEPTH/MAX
y x DEPTH/NEUTRAL
y x XLIMIT REACHED
y DEPTHLIM/MIN REACHED
y DEPTHLIM/MAX REACHED

The meaning is:
DEPTH/MIN or DEPTH/MAX: The tangent of the curve is horizontal, i.e. zero derivative. Note that MIN denotes minimal depth which is a top of the curve, and vice versa.
DEPTH/NEUTRAL: Zero density difference between jet and ambient fluid.
XLIMIT REACHED: Given value of XLIM (one of the constants) has been reached during calculation.
DEPTHLIM/MIN REACHED or DEPTHLIM/MAX REACHED:
Calculation has come outside the defined depth interval of the density profile.
Diagnostics and error messages:
YOU MUST INPUT JET PARAMETERS.

Program is going back to zero mode and input must be done by JIN. Then a new CALC Pl P2.... PN must be written to get back to CALC mode.

DISCHARGE IS OUTSIDE DEFINED DENSITY AREA.

Selfexplanatory. The density profile must be extended or a new discharge depth must be given.

UNSTABLE STRATIFICATION. DENSITY GRADIENT IS SET TO ZERO.

The density is found to decrease going downwards. The density gradient is then set to zero and calculation continued.

NOT CONVERGING.

Calculation goes on without finding anything of interest.

4.19. END.

Terminates program and brings one back to control mode. Must always be the last directive in a run.

4.20. General remarks on input.

All written by user in program mode is input. All input is in free format; that means that the situation on the line is indifferent. Each input item can have from 1 to six characters and must be separated from the others by at least one space. The number of input items is important, except in correction lines where the number of items may be from one to maximum. Writing directives, the first item must be the directive. Six other items may be written.
For directives CALC, PDEL, POUT they must be integer numbers denoting density profiles. For directives PIN and PGET the first item after the directive must be a profile number. The other items are ignored. For all the other directives, only the directive itself is taken into consideration, all the other items are ignored.

Back in control mode, the last statements must be:
"FREE, B 7.
"FREE, B 8.
"FIN

5. Limitations.

Only one set of constants can be saved. It is, however, possible to have one current and one saved set of constants.

Discharge data for one manifold at one outfall site with a maximum of ten data sets can be run simultaneously.

Up to one hundred density profiles can be saved on one file. Data must be given for minimum 2, maximum 40 different depths.

A maximum of six density profiles can be specified in a CALC, POUT or PDEL directive.


a) The program is written in FORTRAN IV. A few special UNIVAC FORTRAN features has been used. One subroutine is written in UNIVAC 1100-series Assembler.

b) Core requirements of program: 18000 words decimal. Word-length 36 bits.
c) There are two external files:
Unit 7: One logical record of 6 words.
Unit 8: DATRAN file. Max 16 700 words.
  Minimum 400 words with no data saved.

d) Source program: Close to 2000 cards.

e) Typical run-time (CPU-time) per density profile per discharge data set:
  0.5 to 1.0 sec.

f) The program has been tested on A/S Computas' UNIVAC 1108 both from
cards and from teletype. On machine compatibility, see a).

7. Examples.

A typical run is shown in Appendix 2.

8. Literature references.

   Report No. KH-R-18, January 69, California Institute of technology

2. Computer program for round buoyant jets into stratified ambient envi-ron-
   California Institute of technology.

3. Mixing of a jet into a stratified ambient environment, by Bjerkeng
Fig A.1 Circular symmetric jet.

Fig A.2 Two-dimensional jet.
Appendix 2.

NB! All input is underlined.

A/S FJERNDATA TIME/SHARING EXEC-2 VERS 2670158*014A
R-BOOT: FEB 02, 73 AT 08:31:15 T-BOOT: JAN 30, 73 AT 02:45:51
RUN JET MIXING
DATE: 020273 TIME: 131012
@ASG,A NIVA*KONST.
READY
@USE 7., NIVA*KONST.
READY
@USE 8., NIVA*PROF.
READY
@ASG,A 8.
READY
*QOT NIVA*JET,MIX

JET MIXING PROGRAM READY
GIVE FIRST DIRECTIVE:
INIT
NEW FILE READY FOR USE.

GIVE NEW DIRECTIVE:
CIN
WRITE:
DELTAS XLI M GRAV ALPHA LAMBDA
5 1000 9.81 0.082 1.16

GIVE NEW DIRECTIVE:
CSAV

GIVE NEW DIRECTIVE:
PIN 1
WRITE STATION, REG. TIME AND DENS.WW:
DST 11/1 71 1.0
WRITE:
DEPTH TEMP SAL
0 3.3 28.4
1 3.6 28.5
2 3.3 30.4
10 6.3 30.7
12.7 31.6
14 7.1 31.6
16 8.6 32.8
20 8.8 32.89
25 8.7 32.2
30 3.4 33.02
40 7.9 33.11 *DEL*
40 7.9 33.1
55 7.3 33.11
80 6.9 33.18
-1

GIVE NEW DIRECTIVE:
PNAV
GIVE NEW DIRECTIVE:
PLIB

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<th>NO. OF DEPTHS</th>
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GIVE NEW DIRECTIVE:
PIN 2

WRITE STATION, REG.TME AND DEPTH:
DKI 13/8 1971

WRITE STATION, REG.TME AND DEPTH:
DKI 13/8 71 1

WRITE:

DEPT H TEMP SAL

**TAPE START**

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END OF TAPE

ERROR IN INPUT.
THE INTERPRETATION OF MEANINGLESS INPUT WAS ATTEMPTED.

THE FOLLOWING RECORD IS ERRONEOUS OR DOES NOT CORRESPOND TO FORMAT SPECIFICATIONS:
9 14.9 25.4

1/0 CALLED AT SEQUENCE NUMBER 000163 OF PIN

ERROR IN INPUT.
THE INTERPRETATION OF MEANINGLESS INPUT WAS ATTEMPTED.

THE FOLLOWING RECORD IS ERRONEOUS OR DOES NOT CORRESPOND TO FORMAT SPECIFICATIONS:
10 17.7 23.7

1/0 CALLED AT SEQUENCE NUMBER 000163 OF PIN
### Depth and Temperature Table

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### Initial Jet Mixing (All Data in m, Sec and deg)

**Manifold No:** 1  
**Outlet Site No:** VESTFJ  
**Density Profile No.:** 3

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<th>Dist (m)</th>
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<th>THETA</th>
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<th>Density</th>
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### Depth and Width Table

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<th>AMBODENS</th>
<th>VEL</th>
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**Extremal Depth Estimated by Potential Calculation**

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