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Discussion paper

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Abstract: Hong and Kao (2004) proposed a panel data test for serial correlation of unknown form. However, their test is computationally difficult to implement, and simulation studies show the test to have bad small-sample properties. We extend Gencay's (2011) time series test for serial correlation to the panel data case in the framework proposed by Hong and Kao (2004). Our new test maintains the advantages of the Hong and Kao (2004) test, and it is simpler and easier to implement. Furthermore, simulation results show that our test has quicker convergence and hence better small-sample properties.

JEL classification: C11, C12, C15

Key word: Energy Distribution, MODWT, Serial correlation, Static and dynamic panel models

1. Introduction

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Serially correlated errors in regression models have several implications for econometric modeling, such as making parameter estimation inefficient and invalidating the commonly used Student's t -test and F -tests. Moreover, for the panel data case, commonly used estimators of dynamic models such as system-generalized method of moments (GMM) and Arrelano-Bond (1991) are only valid as long as the errors in the models are serially uncorrelated. Testing for serially correlated errors is thus an essential part of econometric modeling.

Most panel data tests for serial correlation; for example, Breusch and Pagan (1980), Bhargava *et al.* (1982), Baltagi and Li (1995) and Bera *et al.* (1995) test for no serial correlation against the alternative of serial correlation of some known form. Extending a time series test by Lee and Hong (2001), Hong and Kao (2004) relax the assumption that the serial correlation form is known. Because the Hong and Kao (2004) test is more general than other tests, it also has higher power. Additional strengths of the Hong and Kao test are that it may be applied to residuals from a wide range of different panel data models: static models, dynamic models, one- or two-way error-component models, fixed effects or random effects models. Among the weaknesses of the Hong and Kao (2004) test are its complex structure, which causes the convergence rate to be slow and makes the test computationally time consuming.

We propose an alternative serial correlation test for panel data models that maintains the strengths of the Hong and Kao (2004) test and at the same time has a more simplified structure, higher convergence rate and better small-sample properties. Our test is constructed by combining the variance ratio-test proposed by Gencay (2011) for time series models with the Fisher-type test applied in Choi (2001). The small-sample properties of our test are evaluated in a simulation study and compare favorably to other commonly used tests.

The rest of this paper has the following structure: Section 2 introduces the wavelet transform and the Hong and Kao test, Section 3 introduces the panel data test, Section 4 contains the simulation study, and Section 5 concludes the paper.

2. Wavelet method and the Hong and Kao test

2.1 Introduction to the wavelet transform

Wavelet transform methods began to gain the attention of statisticians and econometricians after a series of articles in the field of economics and finance. Introductory texts for economists are given by Ramsey (1999), Schleicher (2002) and Crowley (2005), and more extensive descriptions have been provided by Vidakovic (1999), Percival and Walden (2000)

and Gençay *et al.* (2001). The wavelet methodology represents an arbitrary time series in both time and frequency domains by convolution of the time series with a series of small wavelike functions. Corresponding to the time-infinite sinusoidal waves in the Fourier transform, the time-located wavelet basis functions $\{\psi_{jk} : j, k \in \mathbb{Z}\}$ used in the wavelet transform are generated by translations and dilations of a basic mother wavelet $\psi \in L^2(\mathbb{R})$. The function basis is constructed through $\psi_{jk}(t) = 2^{j/2} \psi(2^j t - k)$, where k is the location index and j is the scale index that corresponds to the information inside the frequency band $(\frac{1}{2^j}, \frac{1}{2^{j-1}})$. For a signal f , its wavelet transform is given by the wavelet coefficients $f^* = \{\gamma(j, k)\}_{k, j \in \mathbb{Z}}$ with $\gamma(j, k) = \langle f, \psi_{jk} \rangle = \int f(t) \psi_{jk}^*(t) dt$, which represent the resolution at time k and scale j . The resolutions in the time domain and the frequency domain are achieved by shifting the time index k and the scale index j , respectively. A lower level of j corresponds to higher frequency bands, and a higher level of j corresponds to lower frequency bands. Accordingly, the information at high frequency bands, such as noise, outliers or data spikes, is captured by $\gamma(j, k)$ at a lower level of j . By contrast, the long persistent information at low frequencies, *e.g.*, trends or structural breaks, are captured by $\gamma(j, k)$ at a higher level of j .

For a time series sampled at discrete time points, the coefficients of the time series for the wavelet basis are obtained via the discrete wavelet transform (DWT) and maximum overlap discrete wavelet transform (MODWT). The DWT transforms a dyadic time series to the form of $T-1$ wavelet coefficients structured as $J = \ln_2(T)$ scales and one scaling coefficient.

Scale $j = 1, \dots, J$ contains information in the frequency band $\left[\frac{1}{2^{j+1}}, \frac{1}{2^j} \right]$ and consists of $\frac{T}{2^j}$

coefficients that correspond to strictly adjacent wavelet functions. The DWT is implemented by applying a cascade of orthonormal high-pass and low-pass filters to a time series that separates its characteristics at different frequency bands (Mallat, 1989). The maximum overlap wavelet transform is a variation of the wavelet transform, projecting a time series X on $[\log_2 T] \times T$ wavelet functions. The T functions in each scale are translated by only one time period per iteration and thus overlap to a great extent, making a difference to the strictly adjacent wavelet functions of the DWT. The overlapping property allows considerably greater smoothness in the reconstruction of selected frequency bands at the cost of losing the

orthogonality property. For detailed illustration of DWT and MODWT, we refer to Vidakovic (1999), Percival and Walden (2000) and Gençay *et al.* (2001).

2.2 The Hong and Kao (2004) test

The panel data model in Hong and Kao (2004) is given by:

$$Y_{it} = \alpha + X'_{it}\beta + \mu_i + \lambda_t + u_{it}, \quad t = 1, \dots, T_i; i = 1, \dots, n \quad (1)$$

where X_{it} can be either static or dynamic in the form of including lag values of Y_{it} , μ_i is an individual effect, and λ_t a common time effect. In the Hong and Kao test, the null hypothesis is $H0: \text{cov}(u_{it}, u_{it-|h|}) = 0$ for all $h \neq 0$ and i vs. the alternative hypothesis $H1: \text{cov}(u_{it}, u_{it-|h|}) \neq 0$ for some $h \neq 0$ and some i . The test statistic is constructed using the spectral density function in which the assumption that h is known under the alternative may be relaxed.

The test is performed on the demeaned estimated residual $\hat{v}_{it} = \hat{u}_{it} - \bar{u}_i - \bar{u}_t + \bar{u}$, ($t = 1, \dots, T_i; i = 1, \dots, n$), where $\hat{u}_{it} = Y_{it} - X'_{it}\hat{\beta}$, $\bar{u}_i = T_i^{-1} \sum_{t=1}^{T_i} \hat{u}_{it}$, $\bar{u}_t = n^{-1} \sum_{i=1}^n \hat{u}_{it}$, $\bar{u} = (nT)^{-1} \sum_{i=1}^n \sum_{t=1}^{T_i} \hat{u}_{it}$ and $\hat{\beta}$ indicates the consistent estimators under $H0$. Instead of using the autocovariance function $R_i(h) = E(v_{it}v_{it-|h|})$, Hong and Kao (2004)

use the power spectrum $f_i(\omega) = (2\pi)^{-1} \sum_{h=-\infty}^{+\infty} R_i(h)e^{-ih\omega}$, $\omega \in [-\pi, \pi]$ to build the test statistic

because it can contain the information on serial correlation at all lags. Instead of Fourier representation of the spectral density, a wavelet-based spectral density $\Psi_{jk}(\omega)$ using the above-mentioned wavelet basis $\psi \in L^2(\square)$ is used, with $\Psi_{jk}(\omega)$ defined as:

$$\Psi_{jk}(\omega) \equiv (2\pi)^{(-1/2)} \sum_{m=-\infty}^{\infty} \psi_{jk}\left(\frac{\omega}{2\pi} + m\right), \quad \omega \in [-\pi, \pi].$$

$\Psi_{jk}(\omega)$ effectively captures the local peaks and spikes in spectral density by shifting the time effect index k . Based on the empirical wavelet coefficients $\hat{\alpha}_{ijk} = (2\pi)^{-1/2} \sum_h R_i(h)\Psi_{jk}^*(\omega)$, the heteroscedasticity-consistent test statistic \tilde{W}_1 and heteroscedasticity-corrected test statistic \tilde{W}_2 , as well as their distribution under $H0$ are defined separately as:

$$\tilde{W}_1 = \frac{\sum_{i=1}^n \left[2\pi T_i \sum_{j=0}^{J_i} \sum_{k=1}^{2^j} \hat{\alpha}_{ijk}^2 - \hat{R}_i^2(0)(2^{J_i+1} - 1) \right]}{2 \left[\sum_{i=1}^n \hat{R}_i^4(0)(2^{J_i+1} - 1) \right]^{1/2}} \xrightarrow{d} N(0,1),$$

$$\tilde{W}_2 = \frac{1}{\sqrt{n}} \sum_{i=1}^n \frac{2\pi T_i \sum_{j=0}^{J_i} \sum_{k=1}^{2^j} \hat{\alpha}_{ijk}^2 - (2^{J_i+1} - 1)}{2(2^{J_i+1} - 1)^{1/2}} \xrightarrow{d} N(0,1)$$

where “ \xrightarrow{d} ” convergence in distribution. The Hong and Kao (2004) test has three main disadvantages. First, both test-statistics are constructed using the hyperparameters J_i (resolution level in wavelet decomposition), which are determined in a computationally-intensive, data-driven method. Second, although Hong and Kao (2004) show $\tilde{W}_1 \xrightarrow{d} N(0,1)$ and $\tilde{W}_2 \xrightarrow{d} N(0,1)$, the slow convergence rates of both test statistics show a serious bias below the nominal size when using the asymptotic critical values directly. Empirical or bootstrapped critical values must be generated by simulations, further complicating the test. Third, because the test statistics are based on *DWT*, the data set is restricted as T_i should be a multiple power of 2. All of these disadvantages hinder the test’s popularity. We propose a much more simplified test and show that our test overcomes the shortcomings of \tilde{W}_1 and \tilde{W}_2 while still giving good results in testing a wide range of serial correlations in the generalized panel model.

3. A Panel data test based on wavelet variance ratio

The test in Hong and Kao (2004) is the panel version extension of the wavelet spectrum-based serial correlation test in single series proposed by Lee and Hong (2001). However, the Lee and Hong (2001) test has a slow convergence rate because of the estimation of the nonparametric spectrum density. An alternative time series test for serial correlation of unknown form is the Gencay (2011) variance ratio-test, which converges to normal distribution at a much faster parametric rate.

We extend the Gencay test to the panel data case by using a Fisher-type test combining the p -values from individual serial correlation tests based on Gencay (2011). This p -value combination strategy is inspired by Maddala and Wu (1999) and Choi (2001). Choi (2001) noted that the method to combine p -values can allow more general assumptions of the underlying panel models such as stochastic or nonstochastic models, balanced or non-balanced data and homogeneous or heterogeneous alternatives. This general assumption coincides with the aforementioned wide range assumption of the panel models in Hong and

Kao (2004), making our further comparisons possible. Choi (2001) also shows this p -value combination test generally has better size and power performance compared with the previous panel unit root test.

Our test procedure for serial correlation is straightforward. First, the errors in Equation (1) for each individual i are estimated. Second, the estimated errors are transformed to the wavelet domain using the MODWT. Unlike the DWT, the MODWT does not impose any restrictions on the sample size, whereby in the Hong and Kao restriction, the sample size is restricted to the power of 2. The MODWT on the estimated errors yields two sets of transform coefficients, W_i and V_i . For the discrete time series, which has the frequency band $(0, \frac{1}{2}]$, where W_i represents the higher half part of frequency component in the errors (frequency band is $(\frac{1}{4}, \frac{1}{2}]$), and V_i represents the lower half part of frequency component in the errors (frequency band is $(0, \frac{1}{4}]$). Third, if the errors are white noise, each frequency has the same

energy in which the following wavelet-ratio test statistic $G_i = \frac{\sum_{t=1}^{T_i} W_{it}^2}{\sum_{t=1}^{T_i} W_{it}^2 + \sum_{t=1}^{T_i} V_{it}^2}$ has the expected value $\frac{1}{2}$. Gencay shows that the test statistic

$S_i = \sqrt{4T}(\frac{1}{2} - G_i) = N(0,1) + O_p(T_i^{-1}) \xrightarrow{d} N(0,1)$ and it can be used to test for an unknown order of serial correlation in time series. Gencay further shows that the test based on S_i performs well in small samples. However, we cannot use S_i directly to obtain the p -values in the panel framework because it is a two-sided normal test and will lose power seriously when the panel data contains both positive and negative correlations. However, as $S_i^2 \sim \chi^2(1)$, the test based on S_i^2 is then a one-sided test and can be used to test the heterogeneous alternatives. Another advantage of using S_i^2 instead of S_i is that the convergence rate turns into $O_p(T_i^{-2})$ and will lead to even better small-sample performance. Among the three p -value

based test statistics P , Z , L defined as $P = -2 \sum_{i=1}^n \ln(p_i)$, $Z = \frac{1}{\sqrt{n}} \sum_{i=1}^n \Phi^{-1}(p_i)$,

$L = \sum_{i=1}^n \ln(\frac{p_i}{1-p_i})$, we choose the inverse normal test statistic $Z = \frac{1}{\sqrt{n}} \sum_{i=1}^n \Phi^{-1}(p_i)$ where

$p_i = 1 - \Gamma(S_i^2)$ and Φ and Γ are separately cumulative density functions for the normal and chi-square distributions. Stouffer *et al.* (1949) already proved that $Z \xrightarrow{d} N(0,1)$, and Choi (2001) showed that, compared to other Fisher-type of tests P and L , the inverse normal test statistic Z performs best and is recommended for empirical applications.

4. Simulation study

The small-sample properties of our proposed inverse normal test statistic Z are compared with the Hong and Kao test, \hat{W}_1 and \hat{W}_2 , in a simulation study. We also compare it with Hong's (1996) kernel-based tests, \hat{K}_1, \hat{K}_2 , Baltagi and Li's (1995) Lagrange multiplier (LM) test BL and Bera *et al.*'s (2001) modified LM test BSY. We use the same data generation process used in Hong and Kao (2004) for direct comparison. To evaluate the size of the test, we assume both a static panel model with a data generating process DGP_1 : $Y_{it} = 5 + 0.5X_{it} + \mu_i + u_{it}$, $X_{it} = 5 + 0.5X_{it-1} + \eta_{it}$ with $\eta_{it} \sim i.i.d.U[-0.5, 0.5]$ and a dynamic panel model DGP_2 : $Y_{it} = 5 + 0.5Y_{it-1} + \mu_i + u_{it}$ with $\mu_i \sim i.i.d.N(0, 0.4)$ and $u_{it} \sim i.i.d.N(0, 1)$ in both models. For the power part, the error process $\{u_{it}\}$ has two alternatives:

$$\text{AR(1) Alternatives: } \left\{ \begin{array}{l} \text{AR(1).a: } u_{it} = 0.2u_{it-1} + \varepsilon_{it}, i = 1, \dots, n \\ \text{AR(1).b: } u_{it} = -0.2u_{it-1} + \varepsilon_{it}, i = 1, \dots, n \\ \text{AR(1).c: } \begin{cases} u_{it} = 0.2u_{it-1} + \varepsilon_{it}, i = 1, \dots, n/2 \\ u_{it} = -0.2u_{it-1} + \varepsilon_{it}, i = 1, \dots, n/2 \end{cases} \end{array} \right. \quad (2)$$

$$\text{ARMA(12,4) Alternatives: } \left\{ \begin{array}{l} \text{ARMA(12,4).a: } u_{it} = -0.3u_{it-1} + \varepsilon_{it} + \varepsilon_{it-4}, i = 1, \dots, n \\ \text{ARMA(12,4).b: } u_{it} = 0.3u_{it-1} + \varepsilon_{it} - \varepsilon_{it-4}, i = 1, \dots, n \\ \text{ARMA(12,4).c: } \begin{cases} u_{it} = -0.3u_{it-1} + \varepsilon_{it} + \varepsilon_{it-4}, i = 1, \dots, n/2 \\ u_{it} = 0.3u_{it-1} + \varepsilon_{it} - \varepsilon_{it-4}, i = 1, \dots, n/2 \end{cases} \end{array} \right. \quad (3)$$

[Table 1]

[Table 2]

For the size and power of the Z test, we take the critical values at 10% and 5% significance levels directly from the $N(0,1)$ distribution and set the replication number for the simulation to

1000. We use Z_1 to represent our Z test constructed from the static data generation process DGP_1 and Z_2 to represent the Z test from the dynamic data generation process DGP_2 . Table 1 shows the size performance of our test statistic Z_1 , and Table 2 shows the size performance of our of our test statistic Z_2 . Table 1 may be compared with Hong and Kao's (2004) Table I for the static model, and Table 2 may be compared to Hong and Kao's (2004) Table II for the dynamic model.

In Tables 1 and 2, $\hat{K}_1, \hat{K}_2, BL, BSY$ are separately heteroscedasticity-consistent Daniell kernel-based tests; heteroscedasticity-corrected Daniell kernel-based tests; Baltagi-Li (Baltagi and Li, 1995) tests; Bera, Sosa-Escudero and Yoon tests (Bera *et al.*, 1995). The replication number is set to 1000, as in Hong and Kao, and the confidence intervals for a unbiased size at

the 10% and 5% significance levels are $0.10 \pm 1.96 * \sqrt{\frac{0.10(1-0.10)}{1000}} = (0.0814, 0.1186)$ and

$0.05 \pm 1.96 * \sqrt{\frac{0.05(1-0.05)}{1000}} = (0.0365, 0.0635)$, respectively. Tables 1 and 2 show that the

sizes of our test statistic Z are almost all unbiased for both the static panel model and the dynamic panel model in all cases, even for very small sample sizes such as when $(n, T) = (5, 8)$ or $(8, 5)$. However, Tables I and II in Hong and Kao (2004), show that when using the asymptotic critical values, the size is seriously under biased for \hat{W}_1 and \hat{W}_2 , \hat{K}_1 and \hat{K}_2 ; for BL , the size is seriously over-biased, and for BSY , the size is either under-biased or over-biased. The wide bootstrapped critical value is then used for Hong and Kao (2004) to adjust the size and there is still an under- or over-bias problem for all of the tests. On the contrary, our test uses critical values directly from the normal distribution, and the results are mostly unbiased.

To evaluate the power of the proposed tests, we let the error process follow an AR(1) and ARMA(12.4) process, and we get Table 3 and Table 4 corresponding to Table III and Table IV in Hong and Kao (2004). We use Z_1 and Z_2 to report separately the results for static and dynamic cases, whereas Hong and Kao (2004) did not show the results for the dynamic model.

[Table 3]

[Table 4]

In the power case for DGP_1 , because of poor performance for small-sample sizes for their tests, Hong and Kao (2004) use a simulated empirical critical value and bootstrap critical

value for power comparison. This is a challenging task because critical values must be simulated for all combinations of (n, T) . However, we use the $N(0,1)$ distribution for power tests, making it much more straightforward and easy to conduct.

For the AR(1) type of error, Table 3 shows that our test performance is modestly improved over the previous test types, its performance is much better than almost all of the tests for the AR(1) model for sample size $(5,8)$ and is better than all three models for the sample size $(50, 64)$. All of these examples demonstrate the much higher convergence rate of our test-statistic. However, for sample sizes $(10, 16)$ and $(25, 32)$, even though the power performance of our test is modest, it is still quite acceptable. Table 4 shows that for the ARMA(12, 4) type of error, the tests in Hong and Kao (2004) have almost no power when the sample size is $(10, 16)$, whereas our test performs much better. For sample size $(25, 32)$ and $(50, 64)$ in all three models, the only Hong and Kao (2004) test that compares well with our test is their $\tilde{W}_1(J_0)$ test. However, this test requires a computationally intensive, data-driven procedure for the choice of J_0 , making the already complex test even more of an obstacle. Moreover, compared with \tilde{W}_1 and \tilde{W}_2 , our test places no restrictions on T , whereas \tilde{W}_1 and \tilde{W}_2 require T to be a multiple of a power of two.

5. Conclusion

Compared with the tests in Hong and Kao (2004), our test statistic Z has a simplified construct, much faster convergence rate and much better performance in small samples. As \tilde{W}_1 , \tilde{W}_2 and Z are all constructed by the Lindeberg–Lévy central limit theorem and have the same convergence rate $O_p(n^{-1})$, the faster convergence rate of Z may be explained in two factors: first, the nonparametric spectral density estimation in \tilde{W}_1 and \tilde{W}_2 slows the convergence rate; second, the p -values in Z are derived from S_i^2 instead of S_i , which lead to a convergence rate in individuals being $O_p(T_i^{-2})$ instead of $O_p(T_i^{-1})$. Moreover, by using the inverse normal test, our test is easily extended to a cross-sectional dependence robust test by using a modified inverse normal test (Hartung, 1999) when combining the p -values. Generally speaking, just by using the $N(0,1)$ distribution, we obtain unbiased size and quite comparable power performance when compared with all previous tests.

References:

- Arellano, M. and S. Bond. (1991). Some tests of specification for panel data: Monte Carlo evidence and an application to employment equations, *The Review of Economic Studies*, 58, 277-297.
- Baltagi, B. H. and Li, Q. (1995). Testing AR(1) Against MA(1) Disturbances in an Error Component Model, *Journal of Econometrics*, 68, 133-151.
- Bera, A.K., Sosa-Escudero, W. and Yoon, M. (2001). Tests for the Error Component Model in the Presence of Local Misspecification, *Journal of Econometrics*, 101, 1-23.
- Bhargava, A., Franzini, L. and Narendranathan, W. (1982). Serial correlation and the fixed effects model, *Review of Economic Studies* 49(4), 533-549.
- Breusch, T. S. and A. Pagan. (1980): The Lagrange Multiplier Test and Its Applications to Model Specification in Econometrics, *Review of Economic Studies*, 47, 239-253.
- Choi, I. (2001). Unit Root Tests for Panel Data, *Journal of International Money and Finance*, 20, 249-272.
- Gençay, R. (2011). Serial correlation Tests with wavelets. Online on <http://web.uvic.ca/econ/research/seminars/Gencay.pdf>
- Gençay, R., Selçuk F. and Whitcher B. (2001). *An Introduction to Wavelets and Other Filtering Methods in Finance and Economic*, Academic Press, San Diego, CA, USA.
- Hartung, J. (1999) "A note on combining dependent tests of significance," *Biometrical Journal*, 41(7), 849-855.
- Hong, Y. (1996): Consistent Testing for Serial Correlation of Unknown Form, *Econometrica*, 64, 837-864.
- Hong, Y. and Kao, C. (2004) Wavelet-based testing for serial correlation of unknown form in panel models. *Econometrica*, 72, 1519-1563.
- Lee, J. and Hong, Y. (2001). Testing for Serial Correlation of Unknown Form Using Wavelet Methods, *Econometric Theory*, 17, 386-423.
- Percival, D.B. and Walden, A.T. (2000). *Wavelet Methods for Time Series Analysis*, Cambridge Univ. Press.
- Stouffer, S. A., Suchman, E. A., DeVinney, L. C. , Star, S. A. and Williams R. M. Jr. (1949) *The American Soldier*, vol. 1 of Adjustment during Army Life, Princeton University Press, Princeton, NJ, USA.

Tables

Table 1. Size table for static panel model (DGP₁)

| <i>(n,T)</i> | <i>(5,8)</i> | | <i>(10,16)</i> | | <i>(25,32)</i> | | <i>(50,64)</i> | | <i>(5,5)</i> | | <i>(10,10)</i> | | <i>(25,25)</i> | | <i>(50,50)</i> | | <i>(8,5)</i> | | <i>(16,10)</i> | | <i>(32,25)</i> | | <i>(64,50)</i> | |
|----------------------|--------------|------|----------------|------|----------------|------|----------------|------|--------------|------|----------------|------|----------------|------|----------------|------|--------------|------|----------------|------|----------------|------|----------------|------|
| | 10% | 5% | 10% | 5% | 10% | 5% | 10% | 5% | 10% | 5% | 10% | 5% | 10% | 5% | 10% | 5% | 10% | 5% | 10% | 5% | 10% | 5% | 10% | 5% |
| <i>Z</i> | 8.6 | 4.5 | 8.8 | 4.7 | 10.7 | 5.6 | 10.4 | 5.7 | 8.0 | 4.1 | 9.0 | 4.3 | 11.0 | 4.6 | 10.0 | 5.2 | 9.2 | 3.9 | 9.3 | 3.8 | 9.1 | 4.9 | 10.0 | 4.3 |
| <i>W₁</i> | 1.8 | 1.3 | 3.3 | 1.9 | 6.3 | 4.0 | 7.9 | 4.0 | 0.6 | 0.0 | 2.0 | 1.3 | 4.5 | 2.5 | 7.8 | 4.3 | 0.4 | 0.1 | 1.7 | 0.7 | 4.1 | 1.7 | 6.9 | 3.8 |
| <i>W₂</i> | 1.0 | 0.4 | 3.0 | 1.1 | 4.7 | 2.3 | 8.2 | 3.7 | 0.1 | 0.0 | 0.8 | 0.3 | 3.0 | 1.6 | 7.2 | 3.6 | 0.1 | 0.0 | 0.7 | 0.3 | 2.6 | 1.3 | 6.3 | 3.7 |
| <i>K₁</i> | 2.5 | 1.5 | 3.6 | 1.7 | 6.6 | 3.9 | 7.2 | 3.9 | 0.9 | 0.0 | 2.3 | 1.3 | 5.0 | 2.9 | 7.5 | 4.0 | 0.4 | 0.2 | 1.6 | 0.8 | 4.2 | 2.1 | 6.7 | 3.5 |
| <i>K₂</i> | 1.4 | 0.4 | 2.8 | 1.2 | 5.8 | 2.6 | 8.0 | 3.8 | 0.1 | 0.0 | 1.2 | 0.2 | 3.3 | 2.0 | 7.5 | 3.4 | 0.1 | 0.0 | 0.7 | 0.2 | 3.1 | 1.7 | 7.1 | 3.2 |
| <i>BL</i> | 30.5 | 19.9 | 22.6 | 14.9 | 23.2 | 14.6 | 22.5 | 14.5 | 45.3 | 32.4 | 35.2 | 23.9 | 30.2 | 20.8 | 28.2 | 19.2 | 57.8 | 46.3 | 48.0 | 35.4 | 34.7 | 24.5 | 35.3 | 23.8 |
| <i>BSY</i> | 9.0 | 3.0 | 5.7 | 2.1 | 6.7 | 1.8 | 7.7 | 2.2 | 18.3 | 9.3 | 13.5 | 5.1 | 9.9 | 3.4 | 7.3 | 2.8 | 31.9 | 20.7 | 22.0 | 10.3 | 14.2 | 5.9 | 11.4 | 4.8 |

Table 2. Size table for dynamic panel model (DGP₂)

| <i>(n,T)</i> | <i>(5,8)</i> | | <i>(10,16)</i> | | <i>(25,32)</i> | | <i>(50,64)</i> | | <i>(5,5)</i> | | <i>(10,10)</i> | | <i>(25,25)</i> | | <i>(50,50)</i> | | <i>(8,5)</i> | | <i>(16,10)</i> | | <i>(32,25)</i> | | <i>(64,50)</i> | |
|----------------------|--------------|-----|----------------|------|----------------|------|----------------|-----|--------------|------|----------------|-----|----------------|------|----------------|------|--------------|------|----------------|------|----------------|------|----------------|------|
| | 10% | 5% | 10% | 5% | 10% | 5% | 10% | 5% | 10% | 5% | 10% | 5% | 10% | 5% | 10% | 5% | 10% | 5% | 10% | 5% | 10% | 5% | 10% | 5% |
| <i>Z</i> | 8.4 | 4.3 | 8.7 | 3.9 | 8.2 | 4.3 | 9.2 | 5.6 | 10.0 | 3.3 | 8.8 | 3.4 | 8.8 | 6.1 | 10.8 | 4.2 | 8.1 | 5.3 | 10.2 | 4.0 | 9.0 | 5.8 | 9.7 | 5.1 |
| <i>W₁</i> | 0.5 | 0.2 | 1.1 | 0.4 | 2.8 | 1.1 | 6.3 | 3.3 | 0.0 | 0.0 | 0.4 | 0.3 | 2.3 | 1.5 | 3.5 | 1.8 | 0.4 | 0.1 | 1.7 | 0.7 | 4.1 | 1.7 | 6.9 | 3.8 |
| <i>W₂</i> | 0.1 | 0.0 | 0.4 | 0.2 | 3.2 | 1.1 | 5.9 | 2.7 | 0.0 | 0.0 | 0.0 | 0.0 | 1.2 | 0.7 | 2.8 | 0.9 | 0.1 | 0.0 | 0.7 | 0.3 | 2.6 | 1.3 | 6.3 | 3.7 |
| <i>K₁</i> | 0.6 | 0.3 | 1.6 | 0.7 | 3.7 | 2.2 | 7.0 | 3.8 | 0.0 | 0.0 | 0.3 | 0.2 | 2.8 | 1.4 | 3.2 | 1.5 | 0.4 | 0.2 | 1.6 | 0.8 | 4.2 | 2.1 | 6.7 | 3.5 |
| <i>K₂</i> | 0.1 | 0.0 | 0.8 | 0.2 | 3.4 | 2.1 | 6.7 | 2.8 | 0.0 | 0.0 | 0.1 | 0.0 | 1.8 | 0.9 | 3.3 | 1.6 | 0.1 | 0.0 | 0.7 | 0.2 | 3.1 | 1.7 | 7.1 | 3.2 |
| <i>BL</i> | 16.6 | 9.8 | 4.8 | 1.8 | 2.1 | 0.3 | 0.3 | 0.0 | 40.2 | 31.4 | 12.8 | 5.8 | 2.8 | 1.0 | 1.3 | 0.2 | 56.7 | 44.2 | 17.7 | 10.0 | 1.9 | 0.5 | 0.6 | 0.2 |
| <i>BSY</i> | 12.9 | 6.2 | 29.8 | 19.8 | 99.9 | 99.7 | 0.0 | 0.0 | 24.0 | 16.3 | 14.2 | 7.3 | 96.9 | 93.1 | 94.9 | 94.9 | 23.3 | 16.0 | 14.5 | 8.1 | 99.4 | 98.6 | 59.0 | 59.0 |

Table 3. Power for Static panel model (Z_1) and Dynamic panel model (Z_2) with AR(1) alternatives

| (n, T) | AR(1).a | | | | | | | | AR(1).b | | | | | | | | AR(1).c | | | | | | | |
|------------------|--------------|------|----------------|------|----------------|------|----------------|------|--------------|------|----------------|------|----------------|------|----------------|------|--------------|------|----------------|------|----------------|------|----------------|------|
| | <u>(5,8)</u> | | <u>(10,16)</u> | | <u>(25,32)</u> | | <u>(50,64)</u> | | <u>(5,8)</u> | | <u>(10,16)</u> | | <u>(25,32)</u> | | <u>(50,64)</u> | | <u>(5,8)</u> | | <u>(10,16)</u> | | <u>(25,32)</u> | | <u>(50,64)</u> | |
| | 10% | 5% | 10% | 5% | 10% | 5% | 10% | 5% | 10% | 5% | 10% | 5% | 10% | 5% | 10% | 5% | 10% | 5% | 10% | 5% | 10% | 5% | 10% | 5% |
| Z_1 | 8.2 | 4.2 | 13.5 | 6.1 | 55.6 | 42.9 | 100.0 | 99.9 | 18.5 | 8.0 | 39.0 | 27.9 | 95.4 | 92.4 | 100.0 | 99.9 | 13.5 | 5.4 | 16.1 | 11.5 | 79.3 | 71.5 | 99.9 | 99.9 |
| Z_2 | 9.7 | 6.2 | 11.2 | 5.4 | 29.0 | 19.9 | 93.0 | 99.9 | 14.4 | 7.9 | 30.7 | 23.4 | 87.3 | 78.8 | 100.0 | 99.9 | 10.1 | 5.0 | 16.2 | 9.0 | 80.3 | 68.3 | 100.0 | 99.9 |
| \bar{W}_1 | 4.2 | 1.4 | 11.5 | 6.9 | 56.4 | 41.7 | 99.9 | 99.8 | 24.6 | 16.0 | 36.9 | 28.3 | 90.2 | 82.8 | 99.9 | 99.9 | 16.1 | 10.5 | 25.7 | 15.9 | 80.0 | 67.8 | 99.9 | 99.9 |
| \bar{W}_2 | 5.6 | 1.4 | 10.2 | 4.9 | 47.7 | 37.3 | 99.9 | 99.8 | 24.5 | 14.0 | 36.6 | 25.7 | 88.3 | 84.0 | 99.9 | 99.9 | 15.5 | 7.2 | 22.1 | 12.7 | 75.8 | 66.6 | 99.9 | 99.9 |
| $\hat{\kappa}_1$ | 4.7 | 1.6 | 15.7 | 9.6 | 72.0 | 60.0 | 99.9 | 99.9 | 27.0 | 17.9 | 50.8 | 39.2 | 94.9 | 90.8 | 99.9 | 99.9 | 16.4 | 10.3 | 34.6 | 24.2 | 87.9 | 80.7 | 99.9 | 99.9 |
| $\hat{\kappa}_2$ | 3.8 | 1.5 | 11.6 | 5.8 | 65.3 | 51.1 | 99.9 | 99.9 | 26.5 | 16.3 | 47.6 | 35.4 | 94.6 | 90.5 | 99.9 | 99.9 | 14.9 | 8.1 | 28.7 | 6.9 | 86.6 | 77.5 | 99.9 | 99.9 |
| BL | 3.5 | 1.1 | 19.9 | 11.9 | 97.8 | 96.0 | 99.9 | 99.9 | 40.1 | 24.3 | 83.0 | 71.4 | 99.9 | 99.9 | 99.9 | 99.9 | 17.7 | 8.3 | 11.6 | 61.0 | 17.4 | 12.4 | 13.9 | 7.4 |
| BSY | 30.7 | 20.9 | 74.8 | 59.9 | 99.9 | 99.9 | 99.9 | 99.9 | 7.9 | 5.2 | 9.5 | 6.3 | 70.8 | 59.2 | 99.9 | 99.9 | 11.1 | 6.2 | 8.7 | 4.5 | 10.7 | 6.0 | 12.6 | 6.3 |

Table 4. Power for Static panel model (DGP_1) and Dynamic panel model (DGP_2) with ARMA(12,4) alternatives

| (n, T) | ARMA(12,4).a | | | | | | ARMA(12,4).b | | | | | | ARMA(12,4).c | | | | | |
|------------------|----------------|-----|----------------|------|----------------|------|----------------|-----|----------------|------|----------------|------|----------------|-----|----------------|------|----------------|------|
| | <u>(10,16)</u> | | <u>(25,32)</u> | | <u>(50,64)</u> | | <u>(10,16)</u> | | <u>(25,32)</u> | | <u>(50,64)</u> | | <u>(10,16)</u> | | <u>(25,32)</u> | | <u>(50,64)</u> | |
| | 10% | 5% | 10% | 5% | 10% | 5% | 10% | 5% | 10% | 5% | 10% | 5% | 10% | 5% | 10% | 5% | 10% | 5% |
| Z_1 | 16.0 | 7.6 | 34.0 | 27.0 | 66.5 | 57.5 | 11.8 | 6.0 | 34.3 | 26.0 | 67.1 | 58.0 | 11.0 | 8.1 | 39.0 | 18.0 | 65.2 | 55.2 |
| Z_2 | 13.9 | 7.4 | 31.6 | 22.6 | 68.9 | 53.1 | 11.7 | 6.0 | 31.8 | 21.2 | 63.3 | 56.1 | 11.3 | 6.1 | 31.5 | 17.3 | 61.1 | 49.6 |
| \bar{W}_1 | 1.0 | 0.0 | 43.2 | 29.9 | 96.8 | 95.2 | 0.1 | 0.0 | 34.4 | 22.8 | 95.5 | 95.5 | 0.0 | 0.0 | 37.1 | 25.8 | 94.3 | 93.6 |
| \bar{W}_2 | 0.0 | 0.0 | 34.8 | 26.2 | 96.8 | 95.6 | 0.0 | 0.0 | 16.5 | 28.9 | 95.5 | 94.4 | 0.0 | 0.0 | 28.9 | 19.6 | 94.4 | 93.9 |
| $\hat{\kappa}_1$ | 0.0 | 0.0 | 35.0 | 24.1 | 93.0 | 90.3 | 0.0 | 0.0 | 11.6 | 5.9 | 95.8 | 95.0 | 0.0 | 0.0 | 23.0 | 24.0 | 94.8 | 93.7 |
| $\hat{\kappa}_2$ | 0.0 | 0.0 | 27.7 | 19.1 | 94.1 | 91.5 | 0.0 | 0.0 | 7.3 | 3.2 | 96.1 | 94.4 | 0.0 | 0.0 | 16.3 | 10.2 | 94.4 | 93.9 |
| BL | 2.7 | 0.0 | 29.1 | 17.7 | 19.4 | 12.3 | 2.0 | 0.0 | 8.6 | 5.0 | 6.2 | 3.5 | 2.0 | 0.0 | 16.2 | 9.8 | 14.7 | 7.2 |
| BSY | 0.0 | 0.0 | 9.5 | 4.7 | 6.6 | 2.9 | 0.0 | 0.0 | 33.9 | 23.7 | 22.9 | 14.8 | 0.0 | 0.0 | 21.0 | 12.4 | 17.1 | 10.3 |