
The Population Externality

by

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THE POPULATION EXTERNA LITY

This paper addresses the important question: Are we in danger of over-populating the earth? The issues raised by the question are quite complex, involving in particular (1) structural features of aggregate production possibilities, (2) assumptions concerning substitution possibilities, (3) currently unknown resource availabilities/technological improvements and (4) value judgements concerning the social rate of discount and social worth of an extra life. We seek to sort these issues out from an economist's perspective and see what light (if any) economic theory can throw on the subject.

We take as our point of departure the theory of externalities. In particular, we ask whether or not the private decision to have an extra child imposes identifiable externalities on society at large. If we could show that such externalities were significant and negative then we could argue that a world in which the decision to have children was left to decentralized private decisionmaking would lead to overpopulation relative to the first best.

A number of obvious externalities have been identified in the recent literature.\(^1\) Obviously, if direct costs of childrearing (childcare, education, etc.) are shared publicly, then we have an associated externality. Also, if capital markets are imperfect, there may be distorted incentives to have children for social security purposes. Here, we will assume perfect markets and ignore direct effects so as to concentrate on a broad class of crowding effects that are harder to identify and somewhat more controversial.

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\(^1\)See for example Horn (1991) and the references therein.
The literature identifies one crowding externality in particular as being important to the population question: the "global commons." To the extent that access to such common resources is essential to well being an extra person imposes external costs by crowding other people out. By way of introduction, we will provide a general setup for measuring this externality. It has been suggested recently that if the commons can be effectively "enclosed" so that all fixed factors are privately owned, the externality can be effectively internalized. This argument has been made in the context of the "extended family" (or dynasty) model and it was not clear whether it can be extended to an overlapping generations (OLG) setup.

Even in the absence of a pure crowding externality, population choices may introduce incentives to mismanage intertemporal assets (such as physical capital, renewable resources and the like). Again, there have been recent papers showing that these distortions disappear in the dynasty model and some variants of it.\(^2\) Further, it has been argued recently by Måler (1989) that if all such assets are sold on complete markets with perfect foresight and the social rate of discount is appropriately chosen then there will be no intertemporal distortions even in the OLG model. However, these models did not consider population choice explicitly, and leave open the possibility of an interaction effect.\(^3\)

We will start by ignoring the second set of issues. Namely, we will assume that

\(^2\)See Löfgren (1991) and Hultbrantz (1991) for the case of forest management.

\(^3\)Indeed, with population fixed and markets complete, the dynasty model must lead to an efficient outcome due to the first welfare theorem. And while this theorem does not always hold in the OLG model it is not very surprising that results along the same lines can be found there.
there are no intertemporal assets that are subject to distortion. In particular, the only such asset will be a single indestructible exogenously fixed factor (which we may want to think of as land and/or the global commons. Even in this quite restricted context, we will argue that full private ownership eliminates the population externality only in versions of the dynasty model that appear to be contradicted by simple stylized facts.

I The Social Welfare Function

One very important value judgement concerning population involves how an extra (hypothetical) person should be counted in the social welfare function. There is a long literature on this subject and we will merely summarize some if its salient features here. The strict utilitarian point of view would have it that all persons, present and future should count equally and additively in the social welfare function. Even if one is sympathetic to the utilitarian view, there are conceptual difficulties with implementation since it is impossible to find a mathematical objective function reflecting these features that will lead to well defined optimal policies over an indefinite time horizon. At the very least, some discounting of the future seems 'required."

A welfare function at the opposite extreme would be one that takes the standard of living (or the average utility) as a social objective. Obviously, total utility and average utility lead to the same objective as long as population is fixed but will have markedly different implications for policy with respect to changing population; in particular, an

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4 For classic treatments of this subject, see Meade (1955) and Dasgupta (1969). For a discussion of the issues in a resource planning context, see Koopmans (1974).

5 The 'mathematical necessity' of discounting was demonstrated by Koopmans et al (1964) and generalized by Koopmans (1972).
average utility objective will make it more difficult to justify population increases (ceteris
paribus) than will a utilitarian objective.

The conflict between these objectives can be seen quite starkly in the context
discussed here. In a world with positive marginal products but diminishing returns to
fixed factors, additional people will always increase aggregate output (and thus, aggregate
utilitarian welfare in the absence of non-market externalities). But at the same time,
adding people will eventually drive the standard of living to zero unless the fixed factor is
inessential to production. Here we will focus on the standard of living, believing it to be
the more relevant measure. However, we will also reinterpret results from the utilitarian
perspective.

As might also be expected, the social rate of discount plays a key role in evaluating
the population externality. Indeed, we will show that in the pure overlapping generations
model, there is a precise connection between the private rate of discount (market rate of
interest) and the 'importance' of the fixed factor in production; as the fixed factor be-
comes more indespensible, the discount rate goes up. Thus, unless the social rate of
discount goes up with the private rate, a divergence will develop and private sector
decisions (including population policy) will underrepresent future generations.

II. Fixed factors and the global commons

The main argument for a population externality (on the standard of living) relies on
diminishing returns to fixed factors. (We will formalize this argument later.) This argument

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^There is a long literature on determination of the correct social rate of discount and its relevance in the
measurement of various externalities. See, for instance, Arrow/Fisher (1974) or Krutilla/Fisher (1975).
is in a state of disrepute among some economists due to its abuse at the hands of the "club of Rome," who invoked it with a vengeance by assuming away substitution possibilities. However, we want to point out that the theoretical foundations of the argument are perfectly valid in a general production framework; legitimate controversy is confined to the issue of how quickly diminishing returns set in and measurements of its importance.

The logical argument for elements of diminishing returns is based on the view that any fixed technology "must" exhibit constant returns to scale at sufficiently large scales. The argument is that at sufficiently large scale the best way to expand scale further is through replication and that this option is always available. For our purposes, it is enough that the world economy be a sufficiently large scale in this sense; that is, if we had a second "earth" exactly like the first, we could do no better than replicate (specialization at that scale has no benefits).

Thinking now of the earth as a fixed factor, it follows logically under maintained assumptions that we will observe diminishing returns (from a fixed technology) to adding variable population. Consequently, increased population can only be offset by improvements in the technology. The current debate over population issues focusses on the extent to which the benefits of technical change can continue to overcome the detrimental effects of population increase. Although this debate is obviously quite important, its resolution is not crucial to the question of concern here. As long as technological progress is independent of population size (a proposition that seems reasonable in a

\footnote{See, for example, Meadows et al (1972).}
world of our size where R&D effort is widely duplicated), the presence of externality will
not depend on whether or not technical progress occurs. With or without such progress,
output per person will be lower with extra population. On the other hand, the size of
externality (especially in the long run) may depend on the rate of technical innovation, and
we will return to this matter in a later section.

III. A two factor static model

In this section we study a simple example that serves to isolate the effect of fixed
factors on welfare. Later we will show that a relatively general dynamic formulation
exhibits qualitatively the same general features under certain conditions. The framework
here is one in which a single consumption good (Y) is produced from labor (L) and the
fixed factor (T) only. Drawing on the argument made earlier we assume without loss of
generality that the production function F(L,T) exhibits constant returns to scale.

As might be expected, the nature of population externality will depend on the
institutions associated with ownership of the fixed factor. We consider two extremes here:
(1) the fixed factor is identified with the "global commons" and treated as a nonexcludable
(but partially rivalrous) good; (2) the fixed factor is identified with privately owned land.

A. The global commons as a fixed factor

When the fixed factor is treated as a nonexcludable good, all people are assumed
to have equal access to it. Ignoring for now differences in ability, we take this to mean
that each person receives the average product of labor (output per person). Given
constant returns to scale overall, and a fixed common size (T) each additional person will
lower the average product of labor and consequently, the decision to have an extra child
confers negative externalities on everyone else. This externality can be thought of as representing added congestion on the common and as such is well understood.\(^8\)

For comparison purposes, we develop an expression for the size of this externality. When one extra person is added to an initial labor supply of \(L^o\), output per person falls by

\[
F(L^o, T)/L^o = F(L^o + 1, T)/(L^o + 1).
\]

Since this cost is incurred by \(L^o\) people the global external cost (GEC) is

\[
GEC = F(L^o, T) - [L^o/L^o + 1]F(L^o + 1, T)
= [F(L^o, T) - L^oF_L(L^o, T)]/[L^o + 1]
= [TF_T(L^o, T)]/[L^o + 1],
\]

where the approximate equality is derived from a first order Taylor’s expansion and the last equality follows from Euler’s equation for functions homogeneous of degree one.

Thus, we see that the externality imposed is equal to the “value” per person of the global commons. Is this a large or small number? Ultimately, of course this question is empirical and cannot be answered on the basis of theory alone. However, it is worth pointing out a certain fallacy of composition that can lead intuition astray. There is little doubt that the addition of a single person will have a very small affect on the average product of labor. Consequently if there is allocation symmetry, there will be a practically negligible effect on any other single individual (and the impact would surely go to zero as the number of people grows without bound). But the aggregate impact certainly need

\(^8\)For a general discussion of common property externalities, see Baumol and Oates (1975)
not be negligible in this sense. In particular, for the Cobb-Douglas production function, 
GEC is a constant fraction of per capita output (measured by the ’land’ parameter),
independent of population size.

Only a small fraction of this cost is borne by a particular family deciding on an extra 
child. Indeed, let us look at the family incentive to have a child; here we treat the net 
benefits of a child as measured by its consumption and ignore any noneconomic satisfac-
tion that might be derived. Net family benefit (NFB) is measured as

\[ NFB = [\varepsilon^o + 1][F(L^o, T)/L^o] - \varepsilon^o F(L^o, T)/L^o \]

\[ = F(L^o + 1, T)/[L^o + 1] - \varepsilon^o TF_T(L^o, T)/[L^o (L^o + 1)], \]

where \( \varepsilon^o \) represents the initial family size. The first term in NFB will be counted as a 
social benefit in the utilitarian framework (though not in the standard of living framework) 
but the second is only a fraction of the global external cost. And this will be a negligible 
fraction as long as the family is small relative to the population at large.

As mentioned before, the net social benefit will always be positive in this context 
under the utilitarian criteria but always negative under the standard of living criteria. 
Although a value judgement clearly is involved here, we think that the second measure 
is more appropriate as a measure of the external effect from the decision to have an extra 
child. Unless society at large feels better off knowing that an extra person exists, it is 
hard to see why that person’s output (utility) should be added to external welfare. 
Assuming we do take the standard of living view, note that if \( T \) is taken as a proxy for all 
fixed factors, the external cost surely would be a significant fraction of income; if this cost 
were imposed as a tax, we would expect a substantial effect on incentives to have
children.

B. Privately owned land as the fixed factor.

On the other hand, if the fixed factor is excludable and privately owned we might expect the "congestion" externality to be internalized much as land enclosures internalize the problem of the common. This result is quite obvious in a world where all incomes are earned on the two markets and the marginal child has no influence on equilibrium prices. However, we argued earlier that there is danger in equating "a very small effect" with "no effect" so we give a more general demonstration.

We assume competitive pricing so that all factors are paid their marginal products and let \( F^0_x \) and \( F^1_x \) stand for the marginal products with respect to \( x \) before and after the new people respectively. Further, let \( T_n \) and \( L_n \) stand for the land and labor owned by nonparents. To avoid possible distributional effects, we assume that nonparents are representative of the ex ante population in that they own a fixed fraction (\( \alpha \)) of total resources: \( T_n = \alpha T^0 \), \( L_n = \alpha L^0 \). Then, income of nonparents before \( (I^0_n) \) and after \( (I^1_n) \) the arrival of new population will be

\[
I^0_n = \alpha [F^0_T T^0 + F^0_L L^0] \\
I^1_n = \alpha [F^1_T T^0 + F^1_L L^0]
\]

Computing the difference, we see that

\[
I^1_n - I^0_n = \alpha [F^0_L T^0 + F^0_L L^0] [L^1 - L^0] = 0,
\]

where the approximation is obtained using a first order Taylor's expansion for the change in marginal products and the last equality follows from the fact that \( F_L \) is homogeneous of degree zero.
Thus, as long as the population change is small enough so that a second order approximation is valid for induced changes in production, then private ownership of the fixed factor internalizes the population externality. And while a single child surely is small in this sense, we might point out for future reference that even for large changes, there is no presumption one way or the other concerning the sign of the externality. The intuition for this result is that with private ownership, the child adds to family 'wealth' only its marginal labor product; consequently, the family absorbs the difference between marginal and average product as a cost of increasing family size, and per capita family consumption falls accordingly.

Summarizing the message from static models, only the commonly owned part of fixed factors counts toward measuring the crowding externality and while the global commons is not unimportant we may still find that the associated cost is relatively small. We ask next whether this same view will hold up in models that are explicitly intertemporal (and intergenerational).

IV. A typology of Intertemporal models

There seem to be three "pure" models to consider and a number of hybrids. We refer to the pure models as (1) the dynasty (or extended family) model, (2) the transfer model (with bequests) and (3) the life cycle model (without bequests).9

The dynasty model treats parents and all their descendents as a single optimizing unit faced with a resources constraint that aggregates all resources controlled by the

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9See Bernheim (1988) for a general discussion of these models and a survey of attempts to distinguish empirically among them.
"family." When markets are complete, the constraint on resources collapses to a single wealth constraint and the dynamic model reduces formally to one that is equivalent to a purely static one. Nerlove et al (1989) studied a model of this sort and were among the first to show formally that there was no population externality in the dynasty model with private ownership of all fixed factors. We will not reproduce this model explicitly here but will comment on its relevance shortly. Of course, it remains true in the dynasty model that population generates a crowding externality on the global commons.

The other two models are both overlapping generations variants in which members of each generation are separate maximizing agents. They differ in the way in which wealth is passed from one generation to the next. In the pure transfer model, all wealth that passes between them is transferred directly whereas in the pure life cycle model all wealth is sold from older generations to younger ones. Clearly, reality is likely to be some hybrid of these extremes, but we will study them separately for convenience.

V. A Pure Transfer Model

Consider first the case in which there are no asset transactions between generations (so that all saving is effectively bequest saving). The only way this outcome could be fully consistent with a maintained assumption of complete markets is if the older generation finishes consuming before the younger one has earned enough to purchase assets. Although it is relatively simple to formulate a market equilibrium system for such a model, its essential features are sufficiently intuitive that we will confine ourselves to a

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10Many attempts have been made to attribute wealth accumulation to these two motives. See Kotlikoff (1986) and Modigliani (1988) for alternative views on their relative importance.
mostly verbal discussion.

Assuming symmetry among members of each cohort (both in terms of land holdings and status quo number of children), the outcome of market equilibrium must be autarchy (no trade). Each person will consume the output per person of her cohort and will bequeath whatever land she inherited to her offspring. Whether or not there is a population externality in this context depends on the nature of the bequest motive. The "family" with the extra child collectively pays the cost in that each child inherits less land. The associated loss is the full cost since other members of the now larger cohort are indifferent to the change for the same reason nonparents were indifferent in the static model (they gain on their landholdings but lose a commensurate amount on labor income).

If the bequest motive is "optimal" in that bequests are valued only for the future family welfare they enable, then (as is well known) the transfer model is equivalent to the dynasty model and all costs are fully internalized. Presumably the parents would choose to "share" some of the burden with their children by purchasing land from nonparents in their cohort and using it to enhance the bequest per offspring.

However, with any other kind of bequest motive, it is appropriate to think of parents as imposing a "population externality" on their children. And under plausible assumptions about this motive, the size of this externality will be similar to what it was when the fixed factor was common property. Suppose that parents care about the size of their bequest independent of the number of children who share it. Then the act of having an extra child
does not change their utility possibility set\textsuperscript{11} so they will not seek to change their bequest, and the full cost of crowding falls upon their offspring.

We now measure this cost in the same terms as we did for the global commons. Letting $T^i$ and $L^i$ stand for the ex ante levels of land and labor (children) in the family having the extra child. Then, the bequest per child falls by

\[
\frac{T^i}{L^i} - \frac{T^i}{L^{i+1}} = \frac{T^i}{(L^i)^2}
\]

Thus, the family external cost (FEC) is measured by

\[
FEC = qL^i \frac{T^i}{(L^i)^2} = \frac{qT^i}{L^i},
\]

where $q$ stands for the value of the land as a bequest asset. Obviously, if the family is representative this measure is closely related to the one we found for the global common although we would have to explore the stock flow price relationship a little more closely to be more precise. Nevertheless, this analysis suggests that in the transfer model, at least, the population externality should be measured by the share of all fixed factors (commonly shared and privately owned) in the value of output.

These results fit quite naturally into the general theory of externalities. Once bequests are not optimal in the sense of the dynasty model, bequests represent decision variables of one agent that enter the utility function of another agent and as such can be

\textsuperscript{11}Recall that we are ignoring private costs and benefits of having children and focussing on incentive biases.
expected to confer externalities. Obviously, it matters a great deal for this reason whether or not bequest behavior is consistent with the dynasty model. In particular, it is critical whether bequests are used to undo perceived changes in equity between generations.\textsuperscript{12} Although there are disagreements on this matter, evidence seems to be mounting against the "neutrality" position.\textsuperscript{13} For example, the very large runup of foreign debt in the United States in the 80's should, according the the dynasty model, have lead to a commensurate increase in private (bequest) saving, whereas there has been no such increase.

Also, it is worth pointing out that the dynasty model requires not only provisions for the next generation, but also for all future generations. In the context above, this means that current parents must be consciously planning for all the descendents their current extra child will have when deciding on current bequests.

VI. A Single Asset Life Cycle Model

Let us turn, then, to a model in which there are no obvious nonmarket externalities between generations, namely the life cycle model with no bequests. In this situation, land becomes a store of value that people acquire and hold as part of their life-cycle saving.

The simplest equilibrium model that can incorporate these features was first studied by Calvo (1978). People live two periods, working only in the first. They save during that period and spend the proceeds during the second (retirement) period. The fixed factor

\textsuperscript{12}The dynasty model of Barro (1997) and even the weaker version of Bernheim (1987) imply full neutrality so that any attempt (say) to transfer money from future to present will be undone by increases in family bequests.

\textsuperscript{13}See Bernheim (1988) for a discussion of the evidence as of approximately 1987.
(Land) is the only available asset, so the young people save by purchasing it from the old. Calvo used this model to study indeterminacies of competitive equilibrium in infinite horizon models. We will make enough assumptions here to eliminate potential indeterminacy and see how the resulting equilibria vary with population choices.

A. Production

As before, there are two factors of production Labor (L) and Land (T). A single consumption good is produced from a time-invariant constant returns to scale production function \( F(L, T) \). Competitive conditions are assumed so that each factor is paid the value of its marginal product. Letting output be the current value numeraire, \( W_t, R_t \) be the current value returns to labor and land respectively, and writing the factor ratio as \( \ell_t = L_t/T \), we can summarize the equilibrium production relationships with a pair of functions: \( W_t = W(\ell_t) \), \( R_t = R(\ell_t) \), where \( W' < 0 \) and \( R' > 0 \).

B. Households

There is no labor-leisure margin so (normalizing individual labor supply to one) each household born in date \( t \) will earn income \( W_t \) and must decide how to divide it between early consumption and saving. Thus, she faces a problem of the form

\[
\text{Max } U(c^1, c^2)
\]

subject to

\[
c^1 = W_t - s,
\]

\[
c^2 = s[1 + r_t],
\]

where \( r_t \) stand for the rate of return on savings. This problem determines a savings function of the form \( s_t = S(W_t, 1 + r_t) \). All households are assumed alike and clearly there
must be \( L_t \) of them born at date \( t \).

C. Equilibrium

Since land is the only asset, the rate of return on saving will be determined from its flow payout plus realized capital gains.\(^{14}\) Letting \( q_t \) stand for the current value land price at date \( t \), we have

\[
   r_t = \frac{q_{t+1} - q_t}{q_t - R_t} + \frac{R_t}{q_t - R_t}, \quad \text{and} \quad p_t = 1 + r_t = \frac{q_{t+1}}{q_t - R_t}.
\]

Since all land must be held by the young generation, the equilibrium condition on that market at date \( t \) takes the form

\[
   L_t S(W_t, \frac{q_{t+1}}{q_t - R_t}) = (q_t - R_t) T.
\]

Using our production equilibrium relationships, we rewrite this relationship as a function of \( \varepsilon_t \) (which can be thought of as the state of the system at date \( t \)):

\[
   (1) \quad \varepsilon_t S(W(\varepsilon_t), \frac{q_{t+1}}{q_t - R(\varepsilon_t)}) = q_t - R(\varepsilon_t).
\]

Given Walras law, this equation characterizes the equilibrium price relationships at date \( t \). Note that it is a nonlinear difference equation in land prices as a function of \( \varepsilon_t \). Assuming perfect foresight, it defines equilibrium price path(s) into the future.

Calvo treated \( \varepsilon_t \) as constant and showed that this difference equation need not

\(^{14}\) We use the convention here that all flow transactions take place at the beginning of the period.
have a unique solution even when an appropriate transversality condition is imposed. Nonuniqueness occurs when savings functions are sufficiently backward bending, and can have profound implications for a number of intertemporal issues. We start here by assuming the 'normal' case in which savings responds positively to the rate of interest, in which case nonuniqueness is not an issue. Since there are no stock variables in the system, this means that (with $\varepsilon$ constant through time) the land price would immediately adjust to its steady state level defined by the equation:

$$
\varepsilon S(W(\varepsilon),\frac{q(\varepsilon)}{q(\varepsilon)-R(\varepsilon)}) = q(\varepsilon)-R(\varepsilon).
$$

Even if $\varepsilon$ varies initially, there will be a unique perfect foresight path converging to the eventual steady state.

We explore how the 'perfect foresight' solution will vary as a function of population policy.

D. Fixed factors and discounting

In this (admittedly quite stark) model, the productivity of the fixed factor accrues to people only as a rate of return to saving (since land is not collectively owned, nor is it passed by bequest, it can only be acquired in an act of saving). It follows that an increased productivity of land must ultimately translate into increased interest rates and (consequently) market discount rates. Each generation is induced to discount the future, since it is relatively cheap to provide for it. And the market effectively discounts net benefits to the offspring generation more than it does those of the parent generation. We will explore the implications of this relationship below.
E. Comparative dynamics

Suppose that there is only a single date t at which population is increased; specifically, someone in cohort t-1 has an extra child. From then on, \( \ell \) will be unaffected by this population policy change.\(^{15}\) The reader might think that future land prices could be affected by the policy; however, we will show in a moment that (under appropriate assumptions) this cannot happen so that the only relevant transfers will be between the parent and child generation.

We ask how the price of land in date t will change as \( \ell \) increases. This change will determine the transfer between the child generation (who purchase the land during their work period) and parent generation (who sell). Differentiating (1) (holding \( q_{t+1} \) fixed) yields

\[
[S(\cdot) + \varepsilon_tS_w(\cdot)W'(\ell_t) + (1 + \varepsilon_tS_p(\cdot)\frac{q_{t+1}}{(q_t-R(\ell_t))^2})R'(\ell_t)]d\ell_t
\]

(3)

\[
= [1 + \varepsilon_tS_p(\cdot)\frac{q_{t+1}}{(q_t-R(\ell_t))^2}]dq_t.
\]

We assume here that \( S_p \) is positive so that the coefficient of \( dq_t \) is unambiguously positive. And under the additional assumption that consumption in the first period is a normal good, we can show that the first square bracketed term is positive as well. To see this, use the zero degree homogeneity of \( F_L \) to rewrite (3) as

\(^{15}\)We can string together a series of such changes with no difficulties. See later remarks.
\[
[S(.) - \varepsilon_t C_w(.) W'(\varepsilon_t) + \varepsilon_t S_p(.) \frac{q_{t+1}}{(q_t - R(\varepsilon_t))^2} R'(\varepsilon_t)]d\varepsilon_t = [1 + \varepsilon_t S_p(.) \frac{q_{t+1}}{(q_t - R(\varepsilon_t))^2}]dq_t.
\]

Now, all terms multiplying \(d\varepsilon_t\) are positive under maintained assumptions so the sale price of land to the offspring generation will rise with the population increase. This would certainly accord with our intuition since land ought to have increased scarcity value in this circumstance.\(^{16}\)

We now argue that under the same conditions that guarantee uniqueness of the equilibrium path, there will be no price changes in subsequent years. To see this, we examine the price dynamics during adjacent periods (say \(s\) and \(s+1\)) when there is no change in the state variable. The relevant calculations yield

\[[\frac{\varepsilon_s S_p(.)}{q_s - R(\varepsilon_s)}]dq_{s+1} = [1 + \varepsilon_s S_p(.) \frac{q_{s+1}}{(q_s - R(\varepsilon_s))^2}]dq_s.\]

Letting \(\alpha_s\) stand for the square bracketed term on the left side of (4), and recalling the definition of \(\rho\), we can write

\(^{16}\)Of course, this calculation is done under the assumption that selling price of land does not change in later periods. We comment on that possibility later.
\[ \frac{dq_{s+1}}{dq_s} = \frac{1}{\alpha_s} + \rho_s. \]

Note that, under maintained assumptions, the right hand side of (5) is positive and larger than one as long as the equilibrium interest rate is positive. Consequently, this equation is unstable in the forward direction (and stable in the backward direction). Instability in the forward direction is the main condition needed to guarantee uniqueness of the equilibrium path. \(^{17}\) And once we have uniqueness, we can justify our claim that future land prices are unaffected by changes in date \(t\); given the OLG formulation, the state of the system after date \(t\) reverts to its status quo ante value so that the price path must as well. \(^{18}\)

V. Lessons from the theory of welfare measurement

There are propositions in the theory of welfare measurement that suggest externalities should be absent from the model just introduced (and generalizations of it). In any complete markets model, effects that work themselves out entirely through changes in prices represent pure transfers: agents on one side of the market gain by exactly the

\(^{17}\)If there is a long run steady state in the model, transversality generally will require convergence to it and only one price path will be consistent with this requirement.

\(^{18}\)The reader might ask why the land price does not change in period \(t-1\) given the equilibrium conditions (3) and (5). The answer must be that this price is predetermined at the time of decision so that the price change at \(t\) is an unanticipated capital gain.
same amount as agents on the other side lose.\footnote{This proposition has many incarnations. For a recent exposition of the subject, see Starrett (1988), Chapter 9.} Since population choice affects only prices, this proposition would appear to apply to the Calvo model.

Actually, the proposition does not apply exactly although the required modifications are relatively small. The proposition applies only in a closed economy in which the agents are the same both before and after change occurs. But, when population changes new players are introduced into the economy, and this introduces a small distortion as we shall see.

Suppose we consider a one time population increase at date $t$ (undertaken by cohort $t-1$).\footnote{Since everything is being measured to a first order, later small population increases will have an additive effect so the same results would apply.} This increase will affect rental rates and wages in period $t$ when the labor force is increased (with possible consequent welfare effects on cohort $t$) and will change land prices (thereby generating transfers among cohorts). We examine the welfare effect of these two types of changes separately.

Recall the problem (slightly rewritten) for a member of the now larger generation:

$$\text{Max } U(c^1,c^2)$$

subject to

$$c^1 + \frac{q_t^* R(e_t)}{q_{t+1}} c^2 = w(e_t).$$

The solution defines an indirect utility function of the form
\[ V^t(q_t, q_{t+1}, R(e_t), w(e_t)). \] Evaluating the first order change holding the q's constant and normalizing as usual, we find

\[ \frac{dV^t/d\lambda}{\lambda} \bigg|_q = \frac{V^t_R}{\lambda} R'(e_t) + \frac{V^t_w}{\lambda} W'(e_t). \]

From duality theory, the normalized indirect utility derivatives are net supplies on associated markets and from the market equilibrium conditions, net demand for second period consumption divided by the price of land must be equal to the supply of land per person.

Using these facts together with marginal productivity conditions yields

\[ \frac{dV^t/d\lambda}{\lambda} \bigg|_q = \frac{T_t}{L_t+1} F^t_{TL} + F^t_L. \]

Multiplying through by the numbers affected and using the fact that \( F_L \) is homogeneous of degree zero yields for global external cost

\[ GEC = \frac{1}{L_t} T_t F^t_{TL}. \]

The remaining term reflects externality due to the presence of new population. Although not zero, it is first order small given a large population. Consequently, the only significant welfare effects from increased population involve a pure transfer between cohorts due to an increase in the price of land.

It is interesting to look more closely the way in which the markets ‘eliminate’

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21 For example, in the Cobb-Douglas case, \( GEC/W = (1-\alpha)/L \)
externality here. How can it be that no one loses on balance given that output per person must fall just as it does when the fixed factor is common property? Part of the answer derives from the new dynamic aspects of the problem. Since old people do not work, the ratio of work force to population actually goes up during the first period of extra population. Consequently, output per population may go up in this period even though it must go down in the subsequent period. And notice that these same statements will be true whether or not the fixed factor is common property. Therefore, a sufficiently high discount rate could make people indifferent to the corresponding changes in consumption per person per period.

Unfortunately, this cannot be the whole story since it is quite possible that output per population may go down in both periods. This will happen if the labor land ratio gets high enough so that $F_L$ falls below $F/N$, where $N$ stands for the total population. As long as $F_L \rightarrow 0$ when $L/T \rightarrow \infty$, we would eventually find ourselves in that range with a sufficiently large labor supply. In this circumstance, everyone would be worse off in the global commons world yet somehow no one is worse off on balance in the private property world.

The resolution of this puzzle again involves discounting. Given the price changes that we found above, the child generation's consumption per person is sure to fall in second period of life (there are more of them selling the same amount of land at the same price) while the parent generation's consumption must rise in the second period of life. The only way this can be consistent with a neutral welfare effect is if child generation's consumption falls in the first period by no more than parent generation's rises in
that period (so that after a corresponding transfer from parent to child, the child would not be unambiguously worse off.

To see how these conditions could be met let us compute the change in consumption per person of the young holding constant the consumption per person of the old, during the period when the new population works. The material balance constraints imply

\[ L^p c^p + L^c c^{co} + F^1_L = L^p c^p + (L^c + 1)c^{c1}, \]

where \( p \) and \( c \) superscripts index parent and child generation respectively while \( oh \) and one index before and after population change. Solving for \( c^{c1} \) yields

\[ c^{c1} = (L^c c^{co} + F^1_L) / (L^c + 1). \]

Observe that new consumption is a convex combination of old consumption and the new marginal product of labor. Consequently, consumption for the young can rise only if old consumption was less than the new marginal product of labor. Since we are now in a world where the marginal product of labor is below the level of output per person, this will in turn imply that young consumption is below old consumption in that period.

In the market equilibrium, the interest rate must rise sufficiently far to induce people to postpone their consumption this way. We see now the importance of our assumption that \( S_p \) is positive. Indeed, it begins to be difficult to believe that the interest rate is capable of doing that much work.

**VI. Transfers across generations**

Even when net welfare effects cancel, if a decisionmaker gains at the expense of others, we ought to say the cost imposed on others is an externality. Here, we saw that parent generation gains (from an increased sale price of land) at the expense of offspring
generation.

Is the externality here of any consequence? Usually we would argue 'no' on the grounds that the price change induced by any particular parent's decision will be quite small so that the private increase in welfare will be first order negligible; this will be true here as long as the economy is well behaved. The effect of a particular individual on prices would be computed as

\[
\frac{dq_t}{dl_t} = \frac{1}{T} \frac{dq_t}{dl_t}
\]

so as long as the derivative on the right side is properly bounded, and the individual is a small part of the economy, the effect is small. Of course, the aggregate effect (on all members of cohort t-1) need not be small but as long as we are still cancelling gainers against losers (in cohort t), the externality will be negligible.

VII The social rate of discount and equity considerations

It is well known that the price-neutrality proposition ignores equity considerations. If the losers on one side of a market are more socially deserving (at the margin) than the gainers on the other, then there is a consequent net social loss. Such a situation applies here if the social rate of discount is lower than the private rate. As we just saw, it is the offspring generation that loses at the expense of the parent generation. Effects through the sale price of land do cancel, but they still matter more to the children (whom they affect in the first period of life) than to the parents (whom they affect in the second, discounted, period of life). Consequently, if we should choose to count children and
parents equally in the social accounting, the parent's decision to have a child generates a negative externality if we take all social objectives into account.

Indeed, if we count people equally, a social cost reemerges which looks surprisingly similar in magnitude to those found before. Now, the global external cost would be measured as the cost to young from price increase \((\Delta qT)\) minus the discounted benefit to old \((\Delta qT/(1+r))\). In steady state, \(r = R/(q-R)\) so we have

\[
GEC = RT \frac{\Delta q}{q} = \epsilon \frac{RT}{L},
\]

where \(\epsilon\) stands for the elasticity of land price with respect to population increase.

Thus, we see that only if we socially discount the future at the private interest rate (which is ultimately determined by the importance of fixed factors in generating crowding) is the population externality internalized here. We think it is difficult to justify social discounting in the context of these sorts of policy issues. Our model "justifies" new population on the grounds that the extra workers can generate a small short term gain during their productive time even though a relatively heavy cost is paid in the future. As long as the future is sufficiently discounted, there is no net social cost. However, the overall quality of life definitely declines over time and we think this constitutes a legitimate external cost.\(^{22}\)

The issues are even more dramatically illustrated if we consider a permanent long

\(^{22}\)Of course, there will be no actual decline if technological improvement is sufficiently rapid, but as discussed earlier, there will still be an external cost in that people will be worse off than they would have been without the extra population.
run change in population size. That is, suppose someone in cohort t-1 has an extra child who is expected to also have an extra child, and so forth. If we were initially in a steady state, the price system will adjust in one period to the new steady state. The steady state price of land will go up, and in fact will rise by more than the short run price increase. To see this, differentiate (2) and rearrange to yield

\[ [S(.) - \epsilon_t C_\omega(.)W'(\epsilon_t) + \epsilon_t S_\rho(.) \frac{q_{t+1}}{(q_t-R(\epsilon_t))^2}R'(\epsilon_t)]d\epsilon_t \]

\[ = [1 + \epsilon_t (\rho-1) \frac{S_\rho(.)}{q_t-R(\epsilon_t)}]dq_t. \]

The term multiplying dq is smaller than before but still positive as long as the interest rate is positive. Consequently, the long run price change is positive and larger than the short one. Exactly the same calculations as we did before, now imply that each generation following cohort t-1 suffers a welfare loss whose size has the same general form as (6). (Indeed, it should be obvious that welfare per person must fall by this amount in the new steady state.)

Consequently, if all cohorts were really counted equally, no finite benefit could justify extra population. Of course, these types of calculations force us to confront paradoxes associated with zero discounting (whereby we are forced to count the future as "infinitely" more important than the present); nonetheless, it would seem difficult to defend the position that the appropriate size of external cost is "zero."

VIII Conclusions and directions for further research

We have examined the global externality attributable to population crowding and
argued that it is significant and similar in form for a number of models in which the property ownership arrangements and intergenerational structure take various forms. In particular, only for the case in which all fixed factors are privately owned and the family fully internalizes all its decisions into the indefinite future do we fail to see such an external cost.

The models we have considered are restrictive in at least two important respects. First, we consider only two period OLG models and it is well known that saving behavior in such models is not representative of what can happen in multi-period generalizations. Also, in this connection, we did not allow for substantial income effects in saving, and consequently have yet to address the dynamic indeterminacy issues first raised by Calvo. Second, we have not allowed for the presence for intertemporal capital and resource assets that surely ought to play a role in savings decisions and the intergenerational allocation of resources. We hope to take up these, and other matters, in later work.
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