Production, safety, exchange, and risk

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Abstract: Two agents convert resources into safety investment and production while exchanging goods voluntarily. Safety investment ensures reduction of costly risk. High unit cost of safety effort reduces both productive effort and safety effort, which reduces income.

Keywords: production; safety; exchange; risk; trade; price.

Classical exchange theory was developed by Smith (1776) and Ricardo (1817). More recent accounts are Allen (2000), Arrow et al. (1961), Hausken and Moxnes (2005a, 2005b) and Taylor (1993). Recently, exchange theory and conflict have been merged, accounting for production and fighting (see Anderton, 1999; Anderton et al., 1999; Bowles and Gintis, 1993; Hausken, 2004; Rider, 1999; Skaperdas and Syropoulos, 2001). This article makes one step further accounting for safety investment in an exchange model.

Safety risk has not received much attention in the economics literature. Safety concerns are often considered as constraints imposed by law and regulations. Firms face risks due to internal factors related to production, equipment failure, human failure, due to interaction with other firms within the industry, or external factors. The latter can be societal changes in general, or targeted action such as crime, theft, espionage, hacking, blackmail, terrorism. Asche and Aven (2004) argue “that safety measures have a value in an economic sense”, and consider for one firm “the business incentives for investing into safety”. Similarly, Viscusi (1986) considers market incentives for safety.
Recent changes in US accounting laws have made CEOs liable to legal malpractice if accounting information is found to be fraudulent. This has caused a certain panic among firms as to whether they should invest more in information assurance technologies, given that an increase in such investments could lead to a decrease in firms’ productivity. Firms, most of which have finite resource constraints, are thus naturally led to determine optimal investments in information assurance technologies versus production technologies. The former can be perceived as investment to reduce the risk of legal malpractice. This article intends to understand the factors that influence the trade-off between safety and productive investment during exchange.

Each agent $i$ can produce one good $i$, but also attaches utility to another good $j$, $i, j = 1, 2, i \neq j$. Agent $i$ has a resource $R_i$ (e.g., a capital good, or labour) which can be converted with unit conversion cost $a_i$ into productive effort $E_i$, and with unit cost $b_i$ into safety effort $S_i$, where

$$R_i = a_i E_i + b_i S_i \Rightarrow S_i = \left( R_i - a_i E_i \right) / b_i \tag{1}$$

The production cost coefficient $a_i$, where $1/a_i$ is the productive efficiency, measures the resources required to maintain the agent and machinery he uses in production. Analogously, $1/b_i$ is the safety efficiency. As a practical aid, it may be convenient to think of good $i$ as a consumption good such as oil, and the resource $R_i$ as a capital good such as oil drilling equipment. Alternatively, the product may be a consumption good such as fish, and the resource $R_i$ a capital good such as fishing nets. The productive effort $E_i$ is designed to generate good $i$, i.e., extract income from resources currently employed. Without risk, the production function for good $i$ or income $Y_i$ takes the simple form

$$h_i Y_i = \left[ E_i \left( 1 - f \left( r_i, S_i \right) \right) \right]^h \tag{2}$$

where $c_i$ is a parameter that scales the safety effort relative to the risk $r_i$. A large $c_i$ reduces the risk more efficiently. The risk function $f(r_i, S_i)$ increases in the risk $r_i$, $\partial f / \partial r_i > 0$, which reduces income, and decreases in the safety effort $S_i$, $\partial f / \partial S_i < 0$, which constrains risk. The functional form is chosen for convenient analytical solutions. The agent can invest heavily in safety effort, which reduces the risk considerably, but that also reduces the production due to the budget constraint in (1). Hence, the agent faces a trade-off between $E_i$ and $S_i$.

Agent 1 exports an amount $X_1$ of good 1 to agent 2 in exchange for an amount $X_2$ in return. The agents have equivalent Cobb-Douglas preferences for the two goods, with utilities

$$U_1 = (Y_1 - X_1)^{\alpha} X_2^{1-\alpha}, \quad U_2 = X_1^{\alpha} (Y_2 - X_2)^{1-\alpha}, \quad X_1 = P_2 X_2, \quad \alpha \in [0, 1] \tag{3}$$

where $\alpha$ is the relative preference parameter for good 1 for both agents, and $P_2$ is an interior terms-of-exchange price denoting the price of good 2 in terms of good 1. To determine the first order conditions, we let agent 1 choose $E_1$ and $X_1$, and agent 2 choose $E_2$ and $X_2$, simultaneously and independently, to maximise utility. This gives
\[
\frac{\partial U_i}{\partial E_i} = 0 \Rightarrow E_i = \frac{R_i - \sqrt{R_i b_i r_i} / c_i}{a_i}, \quad S_i = \frac{\sqrt{R_i r_i}}{\sqrt{b_i c_i}}.
\]

\[
Y_i = \left(\frac{\sqrt{R_i c_i} - \sqrt{b_i r_i}}{a_i c_i}\right)^2, \quad \sqrt{R_i c_i} \geq \sqrt{b_i r_i}
\]

Proposition 1: The productive effort \(E_i\) increases in the resource \(R_i\) and in the risk reduction efficiency \(c_i\), and decreases in both unit costs \(a_i\) and \(b_i\), and in the risk \(r_i\).

Proposition 2: The safety effort \(S_i\) increases in the resource \(R_i\) and risk \(r_i\), and decreases in the unit cost \(b_i\) of safety effort, and in \(c_i\).

Proposition 3: The income \(Y_i\) increases in the resource \(R_i\) and in \(c_i\), and decreases in both unit costs \(a_i\) and \(b_i\), and in the risk \(r_i\).

Especially interesting among these results is that high unit cost of safety effort reduces both productive effort and safety effort, and thus of course reduces income. Focusing on reducing \(b_i\) is thus beneficial. We next insert \(X_1 = P_2 \alpha X_1\) into the first equation in (3) and differentiate \(U_1\) with respect to \(X_1\), and thereafter insert \(X_1 = P_2 X_2\) into the second equation in (3) and differentiate \(U_2\) with respect to \(X_2\). This gives

\[
\frac{\partial U_1}{\partial X_1} = 0 \Rightarrow X_1 = Y_1(1-\alpha), \quad \frac{\partial U_2}{\partial X_2} = 0 \Rightarrow X_2 = Y_2 \alpha
\]

To determine the market equilibrium condition, inserting (5) into (3) gives the price equation

\[
P_2 = \frac{X_1}{X_2} = \frac{Y_1(1-\alpha)}{Y_2 \alpha}
\]

The price \(P_2\) of good 2 in terms of good 1 is determined endogenously on a supply-demand basis. When agent 1 acquires more resources (\(R_i\) increases), he produces more (\(Y_1\) increases), exports more (\(X_1\) increases), and the price \(P_2 = X_1/X_2\) increases. Conversely, when the relative preference parameter \(\alpha\) for good 1 increases so that both agents attach higher utility to good 1 than to good 2, the demand for good 1 increases, causing a lower price \(P_2\) of the less valuable good 2 in terms of the more valuable good 1.

Inserting (5) into (3) gives the utilities

\[
U_1 = Y_1^\alpha Y_2^{1-\alpha} \alpha, \quad U_2 = Y_1^\alpha Y_2^{-\alpha} (1-\alpha)
\]

Essential for the utilities is the agents’ preference \(\alpha\) for goods. Agent 1 does better if good 1 is more preferred, and conversely if good 2 is preferred.
References


Notes