Discussion Papers

Taxation, Unemployment and Growth: Dynamic Welfare Effects of "Green" Policies

Brita Bye

Statistics Norway
Research Department

\[ \hat{\beta} = \bar{y} - \bar{x} \bar{\beta} \]
Brita Bye

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Abstract:
This paper analyses the effects of so-called "green" tax reforms on a small, open economy producing an imperfect substitute for foreign goods, using an intertemporal general equilibrium model. The labour market is characterised by union wage setting, and a fixed exchange rate implies wage rigidity and involuntary unemployment. The long run effects on instantaneous utility, employment and the stock of real and financial capital of a revenue neutral increase in the tax on fossil fuels combined with a) lump sum rebating or b) change in the labour income tax rate, are discussed. Due to the changes in instantaneous utility during the time path following the implementation of the tax reform, the total welfare effect may be positive even with a reduction in long run consumption. The total welfare effect is in general more positive (or less negative) with wage tax reduction than lump-sum rebating.

Keywords: Dynamic equilibrium analysis, unemployment, environmental tax reforms

JEL classification: D50, D60, D90, H2, J51, Q43

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Address: Brita Bye, Statistics Norway, Research Department, P.O.Box 8131 Dep., N-0033 Oslo, Norway. E-mail: bby@ssb.no
1 Introduction

Environmental taxes seem to be an attractive instrument to enhance environmental quality without seriously damaging the growth possibilities. By increasing environmental taxes to curb pollution and using the revenues to cut distortionary taxes on income, it may be possible to obtain a "double dividend", i.e. not only a better environmental standard, but also a less distortionary tax system, thereby improving economic welfare. However, some authors interpret "double dividend" differently. Goulder (1994) and Christiansen (1996) discuss different interpretations and definitions of the term "double dividend", and give an overview of the literature.

The recent literature on environmental taxation and the double dividend, shows that in a second best economy with other distortionary taxes and a revenue requirement, the optimal tax will in general deviate from the pigouvian tax\(^2\), see e.g. Bovenberg and de Mooij (1994a) and Bovenberg and van der Ploeg (1994a,b). Bovenberg and van der Ploeg (1994a,b) and Bovenberg and de Mooij (1994a) find it optimal to cut the labour tax in response to increased concern for environmental quality, using static general equilibrium models with endogenous labour supply and a clearing labour market. In their analyses employment and welfare decline, rejecting the double dividend hypothesis. This conclusion may be modified when introducing wage rigidity causing involuntary unemployment, see Bovenberg and van der Ploeg (1993, 1994c, 1996).

In a static model framework with involuntary unemployment, Bovenberg and van der Ploeg (1993,1994c) find that a shift in the tax burden away from labour towards other resources has a positive effect on employment, obtaining an employment double dividend. Bovenberg and de Mooij (1994b) explores the effects of an environmental tax reform on pollution, economic growth and welfare in an intertemporal general equilibrium model with economy-environment-economy linkages for a closed economy with no labour input in the production. An environmental tax reform may raise economic growth by reducing the environmental production externality, which has a positive effect on the productivity of capital. Sørensen et al. (1994) use an intertemporal general equilibrium model (with economy-environment-economy linkages) for a closed economy with wage rigidity caused by union wage setting, when analysing the effects of environmental taxation. Higher emission taxes combined with a lower tax burden on labour income, support both an employment and a welfare double dividend, they claim, since the effects on consumption and growth may also be positive.\(^3\)

In this paper both the dynamics of endogenous capital accumulation and involuntary unemployment due to wage rigidity, are implemented in the model framework of a small  

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1 Bovenberg and van der Ploeg (1993, 1994a, b) and Bovenberg and de Mooij (1994a) use the expression "employment double dividend" when an environmental tax reform has a positive effect on employment and the environment, but not necessarily on total welfare.

2 The pigouvian tax is discussed in e.g. Sandmo (1975).

open economy, when analysing the total welfare effects of a unilateral increase in the tax on fossil fuels. Combustion of fossil fuels is an important source of environmental pollution. The tax revenue is rebated through reductions in other taxes. There is no economy-environment-economy feedbacks in the model, and hence no optimisation of the fossil fuel tax rate. This implies that only the non-environmental welfare costs are analysed. The intertemporal model pictures a small open economy producing a product which is an imperfect substitute for foreign goods as in Bye and Holmøy (1992) and Bye (1995). This formulation deviates from the traditional model of a small open economy where the prices are fixed at the world market, see e.g. Sen and Turnovsky (1989) for an intertemporal model with such a characteristic. The alternative assumption adopted in this paper can be justified for several reasons. Econometric work at an aggregate industry level often end up with surprisingly small trade elasticities, see e.g. Lindquist (1993) and Naug (1995). Norman (1990) criticises the Armington approach as an approximation for oligopolistic structures and product differentiation which are essentially supply-side effects, by showing that the Armington approach may give quite different results compared to a model where imperfect competition is modelled directly on the firm level. On the other hand, Branson (1990) in a comment to Norman (1990) points out that more competition on the supply side and higher substitutability on the demand side are themselves substitutes in explaining adjustments to trade liberalisation.

In the model used in this paper, the introduction of environmental taxation combined with changes in other taxes, will through their effects on prices and costs, have long term welfare effects through changing the rate of capital accumulation, employment and economic growth. The model differs in a substantial way from the models in Bye and Holmøy (1992) and Bye (1995) by including labour market imperfections as originally suggested by Blanchard and Kiyotaki (1987). Monopolistic trade unions set the wage level and a fixed exchange rate implies real wage rigidity and involuntary unemployment, which change both the properties and the solution of the model, compared to the model in Bye (1995). By including involuntary unemployment, the model is made relevant for a discussion of a possible employment dividend.

The paper analyses tax reforms from an arbitrary initial situation where the initial tax rates on labour and fossil fuels are not optimised. The existence of a welfare gain of an environmental tax reform depends on whether the reform reduces other imperfections in the economy. The imperfections present in the model are related to imperfect competition in the labour market giving involuntary unemployment, and an unexploited terms of trade gain due to a declining export demand curve. In addition the taxes are creating distortions in the use of production factors, and in the distribution of consumption on different commodities.

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4 Hoel (1996) gives a survey over the literature on the effects of tax changes on wage formation and unemployment. There are some support in the literature for a hypothesis that a wage tax influences the producers wage costs and hence also the level of unemployment.

5 Andersen (1995) presents an intertemporal model with structural unemployment caused by efficiency wage setting. He claims that other labour market imperfections such as union wage formation would lead to approximately the same qualitative results when analysing the effects of fiscal policy.
The long run effects on instantaneous utility, employment and the stock of real and financial capital of a revenue neutral increase in the tax on fossil fuels combined with a) lump sum rebating or b) change in the wage tax, are discussed. In general the total welfare effect is ambiguous, depending on substitution effects which in the case of a non-optimal initial tax structure create efficiency losses, income effects from changes in terms of trade, employment, real capital and net foreign wealth. Due to the changes in instantaneous utility during the time path following the implementation of the tax reform, the total welfare effect may be positive even with a reduction in long run consumption. The total welfare effect is in general more positive (or less negative) with tax reform b) than tax reform a). Hence, there may exist a welfare double dividend by increasing the tax on fossil fuels, and using the revenue to reduce the wage tax, which is causing involuntary unemployment. The effect on the long run level of employment is also more positive with tax reform b).

The paper is organised as follows: Section 2 describes the analytical framework. Section 3 describes some of the long run effects of the tax reforms, while section 4 conducts the corresponding welfare analyses. Section 5 concludes, while some of the technical details are relegated to the appendices.

2 The model framework

2.1 Technology and preferences

The economy is using two goods which are both used as capital inputs by the firms and consumed by the households. Good 1 is a composite good of a domestic and a foreign variety which are imperfect substitutes for each other. The domestic and the foreign varieties compete both on the domestic and on the world market. Good 2, fossil fuels, is supplied by imports only. All firms have identical linearly homogeneous technology and all households have identical homothetic preferences. The structure of technology and preferences is assumed to be separable so that the two goods enter both the production function and the utility function through a macro commodity. The household preferences and the production technology are assumed to be similar with respect to the composition of the macro commodity. Due to constant returns to scale and competitive behaviour, the part of the demand structure that determines the composition of the macro commodity can be represented in dual terms by the following linearly homogeneous price functions:

\begin{align}
(1) & \quad P_1 = p_1(P_1^H, P_1^I) \\
(2) & \quad P = p(P_1, P_2)
\end{align}

where \( P_2 = P_2^*(1 + \tau) \). \( P_2^* \) is the world market price of fossil fuels and \( \tau \) denotes the carbon tax corresponding to the aggregate commodity fossil fuels. The domestic price \( P_1 \)

\footnote{Taxation of fossil fuels will change the composition of the macro commodity.}
of the competing commodity is the unit cost of acquiring good 1. It is an ideal price index of the domestic producer price \( P_1 \) and the price of the competing imports \( P_1^l \). The price of the macro commodity \( P \) is formed in an analogous way and represents an ideal price index of the price of this competing commodity \( P_1 \) and the price of fossil fuels \( P_2 \). The import prices \( P_1^l \) and \( P_2^* \) are both determined exogenously on the world market. The exchange rate is fixed to unity and all values are measured in domestic currency. Good 1 consists both of non-fossil energy commodities such as hydro based electricity and other non-energy commodities, but the energy commodities constitute only a small fraction of the aggregated good 1. Hence, the substitution possibilities between fossil fuels (good 2) and good 1 represented by the elasticity of substitution \( \sigma_{12} \), are likely to be smaller than the substitution possibilities between the domestic and foreign variety of good 1, represented by the elasticity of substitution \( \sigma_{1H} \).

2.2 Producer behaviour

Due to competition among a sufficiently large number of domestic firms, the price of the domestic product is forced down to equal unit cost. The individual firms are price takers in all markets and face given time paths of wages \( w \), a world market interest rate \( r \), and product prices. They rent labour \( L \), but own the capital stock \( K \). Investment, \( J \), is financed by retained profits. The supply of labour is assumed to be fixed. The labour market is unionised consisting of \( n \) different labour unions representing \( n \) different labour skills. This model of the labour market was originally suggested by Blanchard and Kiyotaki (1987), and is also used by Sørensen et al (1994). The total input of labour consists of the following CES-aggregate of \( n \) different labour skills.

\[
L = \left( \sum_{i=1}^{n} L_i \sigma_{L}^{\frac{\sigma_{L}-1}{\sigma_{L}}-1} \right)^{\frac{1}{\sigma_{L}}-1}
\]

\( L_i \) is the input (in hours) of labour skill \( i \) and \( \sigma_{L} \) is the elasticity of substitution between labour of any two skills. We assume that \( \sigma_{L} > 1 \). The dual unit cost function of the labour input aggregate is given by

\[
w = \left( \sum_{i=1}^{n} w_i^{\frac{1}{1-\sigma_{L}}} \right)^{\frac{1}{1-\sigma_{L}}}
\]

Using Shepards lemma on the corresponding labour cost function gives the demand for each labour skill.

\[
L_i = \left( \frac{w_i}{w} \right)^{-\sigma_{L}} L
\]

The production function is linearly homogeneous. The decision problem of the representative firm at time 0 is to choose the time paths of the control variable gross investment so that the present value of the cash flow, \( V \), is maximised:
\[ V_0 = \int_{t=0}^{\infty} \left[ P_1^H f(k) - PJ - wL \right] e^{-rt} dt \]

subject to
\[ \dot{K} = J \]

\[ K(0) = K_0 \]

where \( K_0 \) is predetermined so \( \dot{K} \) is backward looking. The transversality condition is given by
\[ \lim_{t \to \infty} e^{-rt} PK = 0 \]

The necessary f.o.c. are
\[ P_1^H f'(k) = (r - \frac{\dot{P}}{P})P \]

\[ P_1^H [f(k) - kf'(k)] = w \]

where \( \dot{P} \) is forward looking.

We may interpret (10) as determining \( k \), while (11) determines \( P_1^H \) given \( k \) and the wage rate \( w \).

### 2.3 Union wage setting

The labour market is unionized, with \( n \) monopolistic unions of \( n \) different labour skills. All workers are organized in unions according to their skill type and are perfectly mobile across firms, and all unions have the same number of members, \( M \). The assumption of monopolistic unions implies that the union in the market for skill \( i \) sets the nominal wage rate to the employers, and the labour demand curve for each skill determines the level of employment. This level of employment is equally shared among all members of the union. Each union sets its wage rate with the aim of maximizing the lifetime utility of its representative member.

We assume the number of skills (unions) to be large, hence each union has a negligible influence on the general wage \( w \) and price level, the aggregate level of employment and capital accumulation. Following these assumptions and given that there is no disutility from work, the union may maximise the lifetime utility of its representative member by simply maximising the individual utility during each time period. Since the stock of members is fixed, this is equivalent to maximising the union's utility. It is assumed that it is only the total disposable income for the worker that matters (see e.g. Rødseth (1995)). The union for workers with skill \( i \) must thus solve the problem

\[ \max_{w_i} w_i^* L_i + s^*(\bar{L}_i - L_i) \]

subject to
(13.a) \[ L_i(w_i) = \left( \frac{w_i}{w} \right)^{-\sigma_L} L \]

(13.b) \[ L_i < \bar{L}_i \]

where \[ w_i^* = \frac{w_i(1 - t)}{P} \]

\( w^* \) is the net of tax real wage rate. \( L_i \) is total employment (in hours) of skill \( i \) and \( \bar{L}_i \) is union \( i \)'s fixed supply of labour (in hours). \( s^* = s/P \) is the real value of the unemployment benefit and \( t \) is the wage tax. From the first order condition for the union’s utility maximisation we get the following simple wage equation.

(14) \[ w_i = \frac{m s}{(1 - t)} = w \]

where \( m = \sigma_L / (\sigma_L - 1) \). The union will set the nominal wage rate as a mark-up over the unemployment benefit corrected for taxes. Due to the assumptions of symmetric unions and a constant elasticity of substitution between different labour skills, the equilibrium wage rate depends only on exogenous variables and is independent of labour skill. The unemployment benefit is assumed to be high enough to cause involuntary unemployment \( \bar{L}_i - L_i > 0 \), in the model.

2.4 Consumer behaviour

The household sector is described by a large number of atomistic consumers with an infinite horizon. They are all endowed with labour skill of type \( i \). We assume that there is no disutility from work and that all consumers are rationed in the labour market. Since all consumers are rationed in the labour market, total labour supply in the number of work hours \( \bar{L}_i \) is given. Hence, the representative consumer’s objective is to maximise total discounted utility with respect to total consumption \( C \) of the macro commodity. Tax revenues are rebated to the consumer as lump-sum transfers and unemployment benefit. The consumer’s income consists of wage, interest on financial wealth, net cash flow from the firms and transfers from the government. The wage rate is independent of skill and there is no aggregation problem, hence the household sector can be described by a representative consumer with an infinite horizon. The time path of aggregate consumption is determined by solving the following maximisation problem:

(15) \[ \max_{(C_t)} U_0 = \int_0^\infty u(C_t) e^{-\rho t} dt \]

subject to the budget constraint:

(16) \[ \dot{b} = rb + P^H L f(k) - PC - PJ - twL + s(\bar{L} - L) + \Omega \]
let $b(0) = b_0$

$b$ is net financial wealth and $b_0$ is predetermined. $\Omega$ is lump sum transfer from the government. It is standard in the representative agent framework to assume that the agent, being one of a large number of agents, is unable to infer his share of the total tax revenue. Hence (16) with $\Omega$ treated as exogenous, is the budget constraint considered. $\rho$ is the subjective rate of time preference. Recall that $P$ is determined by (1) and (2) because of the assumptions of separability in the preference structure. Equation (17) is the transversality condition.

From the utility maximisation we have the following first order condition:

$$\frac{u'_C}{P} = \mu$$

where $\mu$ is the costate variable associated with the financial wealth accumulation equation (17). The costate variable evolves according to

$$\dot{\mu} = \frac{u'_C}{P}(r - \rho)$$

A necessary condition for obtaining a steady state solution is $r = \rho$. From the single country's perspective this is a "razor's edge" requirement as both the interest rate and the rate of time preference are exogenous in the model of a small open economy. For convenience it is assumed that the condition holds at all points in time, which implies that $\mu$ is constant over a dynamic path.

We assume the following well known specification of the utility function:

(21a)  $$u(C_t) = \frac{\sigma_C}{\sigma_C - 1} C_t^{\sigma_C - 1}$$  ,  when $\sigma_C \neq 1$

(21b)  $$u(C_t) = \ln C_t$$  ,  when $\sigma_C = 1$

$\sigma_C$ is the intertemporal elasticity of substitution.

From the first order conditions of intertemporal utility maximisation we get the following simple relationship between consumption and the costate variable $\mu$:

$$C = (\mu P)^{-\sigma_C}$$

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\(^7\) The dynamic stability properties of an intertemporal model of a small open economy is discussed in numerous papers, see e.g. Sen and Turnovsky (1989).
2.5 Equilibrium

The condition for equilibrium in the domestic product market is given by

\[ Lf(k) = \theta H \frac{P}{P_1^H} (C + kL + \dot{L}k) + A \left( \frac{P_1^f}{P_1^H} \right) \]

where \( \theta H \equiv \theta_1, \theta_1^H \) is the total home share of domestic absorption. \( \theta_i, i = 1, 2 \), are the budget shares of good 1 and fossil fuels respectively. \( \theta_1^j, j = H, I \), are the budget shares of the domestic and imported variety of good 1 respectively. Export demand \( A \) is an increasing function of the ratio of the world market price to the domestic product price. The elasticity of substitution between domestic and foreign varieties of good 1 is assumed to be the same on the domestic market as on the export market. This simplifies the analysis without losing any main points.

The description of the macroeconomic equilibrium is completed by introducing the government budget constraint. In accordance with standard practice in the theory of domestic distortions, it is assumed that lump-sum subsidies are available to redistribute tax revenues. The government’s budget is assumed to balance in each period, i.e.

\[ P_2^* \tau P_2^f (C + J) + twL = s(\dot{L} - L) + \Omega \]

Inserting the government budget constraint into the consumer’s budget constraint given in equation (16) gives

\[ \dot{b} = rb + P_1^H Lf(k) - GC - GJ \]

where \( G \equiv p_1'(p_1^H P_1^H + p_1^l P_1^l) + \mu_2 P_2^* \). \( G \) can be interpreted as the price of the domestic macro commodity exclusive of the tax on fossil fuels. Equation (25) is the current account. Hence, it is sufficient to impose the transversality condition on the consumer to ensure that the stock of net foreign wealth does not explode.

The complete macroeconomic equilibrium is thus described by the equations (1), (2), (10), (11), (14), (17), (22), (23) and (25). The model has a block recursive structure: equation (14) determines \( w \), then equations (1), (2), (10) and (11) determine \( P, P_1, P_1^H \) and \( k \), given import prices and the net wage rate. Given the prices consumption follows from equation (22), and labour demand is determined in (23) by total demand for the domestic product. (25) determines the accumulation of net foreign wealth. The transversality condition (17) adjusts \( \mu \) such that the net foreign wealth does not explode.

A reduced form of the dynamic equilibrium system is obtained by eliminating \( P \) from equations (1) and (2). \( P \) can be replaced by the function \( P = g(P_1^H, P_1^f, P_2) = p(p_1(P_1^H, P_1^f), P_2) \). We assume for simplicity that the exogenous variables \( P_1^f, P_2^*, \tau \) and \( w \) are constant through time.

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8 The partial derivatives of the price function are defined as \( \frac{\partial p(p_1, P_2)}{\partial P_1} = p_1'(P_1, P_2) \) and \( \frac{\partial p(p_1^f, P_1^H)}{\partial P_1^f} = p_1^f(P_1^f, P_1^H) \) a.s.o.
Equation (11) give \( P_i^H \) as a function of \( k \)

\[
(11') \quad P_i^H = \frac{w}{f(k) - kf'(k)},
\]

which holds at all points of time. Thus the dynamics of \( P_i^H \) and \( k \) obey

\[
\dot{P}_i^H = -\frac{\theta^K P_i^H}{\sigma_K \dot{L}}.
\]

\(\sigma_K\) is the substitution elasticity between labour and capital in the production function, and \(\theta^K\) and \(\theta^L\) are the cost shares of capital and labour, respectively.

Inserting the expression for \(\dot{P}_i^H\) into equation (10), the dynamic system derived from (10), (23) and (25) then takes the following form:

\[
\begin{align*}
(26.a) \quad \dot{k} &= h_1(k) = \frac{\sigma_K k}{\theta^K g(P_i^H)} \left[ P_i^H f'(k) - r g(P_i^H) \right] \\
(26.b) \quad \dot{L} &= h_2(k, L; \mu) = \frac{P_i^H}{\theta^K g(P_i^H)} \left[ Lf(k) - A \right] - \frac{1}{k} \left[ (\mu g(P_i^H))^{-\sigma_c} + h_1(k)L \right] \\
(26.c) \quad \dot{b} &= rb + P_i^H Lf(k) - G \left[ (\mu g(P_i^H))^{-\sigma_c} + Lh_1(k) + kh_2(k, L; \mu) \right]
\end{align*}
\]

where the functions \(h_1(k)\) and \(h_2(k, L; \mu)\) are defined after substitution of \(P_i^H\) given by (11').

### 2.6 Equilibrium dynamics

The time path of \( k \) can be solved in terms of exogenous variables only from equation (26.a), when (11') and (14) are taken into account. Inserting the solution into (26.b), yields the time path for \( L \), conditioned on a constant value of \( \mu \). Finally \( b \) follows from equation (26.c). However, for an arbitrary value of \( \mu \), the resulting accumulation of \( b \) will in general be inconsistent with the transversality condition imposed by (17) and will eventually explode. The complete equilibrium therefore requires a solution for \( \mu \) that satisfies (17), see e.g. Sen and Turnovsky (1989) and Brock and Turnovsky (1993) for the same kind dynamic structure. A special feature of this structure is the path dependency of the stationary solution.\(^9\) A closed form solution of the model can therefore only be found in approximate terms by linearising around the steady state \((\bar{k}, \bar{L})\) solution.

The long run equilibrium is a saddle-point since \( h_{1k} = \frac{\partial h_1(k)}{\partial k} < 0 \) and \( h_{2L} = \frac{\partial h_2(k, L)}{\partial L} > 0 \), evaluated at steady state, see appendix A. The stable solution for \( k \) in the neighbourhood of steady state is

\[
(27) \quad k(t) = \bar{k} + (k_0 - \bar{k})e^{h_{1kt}}
\]

\(^9\) This complicates the analysis substantially compared to Bovenberg and de Mooij (1994b) and Sorensen et al (1994) who use closed economy models. In Bovenberg and de Mooij (1994b) and Sorensen et al (1994) transitional dynamics are absent. After a policy shock the economy moves immediately to a new steady state.
Inserting this into the linearised form of (26.b) (when $\dot{k} = 0$ and $\dot{L} = 0$) we obtain the stable solution for $L$

\[ L(t) = \tilde{L} + \left( \frac{-h_{2k}}{h_{1k} - h_{2L}} \right)(k_0 - \tilde{k})e^{h_{1k}t}, \]

where $h_{2k}$, given in appendix A, is assumed to be negative. To determine the dynamics of the current account we linearise equation (26.c) around steady state

\[ \dot{b} = r(b - \bar{b}) + P_1^H f(k)(L - \bar{L}) + \left[ \bar{P}_1^H \bar{L} f'(\bar{k}) - \frac{\theta K P_1^H}{\sigma K k} \left( \bar{A} + \frac{\theta H}{P_1^H} \left( \tau \sigma_{12} \left( \frac{\theta_2}{P_1^H P_2^G} \right)^2 + G \sigma_C \right) \right) \right] (k - \bar{k}). \]

Substituting the solution of $L$ from (28) and $k$ from (27) we obtain the following differential equation in $b$

\[ \dot{b} - \tau b = \alpha(k_0 - \bar{k})e^{h_{1k}t} - \tau \bar{b}, \]

where

\[ \alpha \equiv \bar{L} \bar{P}_1^H f'(\bar{k}) + \bar{P}_1^H f'(\bar{k}) \left( \frac{h_{2k}}{h_{2L} - h_{1k}} \right) - \frac{\theta K P_1^H}{\sigma K k} \left[ \bar{A} + \frac{\theta H}{P_1^H} \left( \tau \sigma_{12} \left( \frac{\theta_2}{P_1^H P_2^G} \right)^2 + G \sigma_C \right) \right]. \]

Assuming that the economy starts out with an initial stock of net financial wealth, $b(0) = b_0$, the solution to equation (30) is

\[ b = \bar{b} + \frac{\alpha(k_0 - \bar{k})e^{h_{1k}t}}{h_{1k} - r} + \left[ (b_0 - \bar{b}) - \frac{\alpha(k_0 - \bar{k})}{h_{1k} - r} \right] e^{rt}. \]

In order for the transversality condition (17) to be satisfied, the last term must vanish, which implies

\[ b_0 - \bar{b} = \frac{\alpha(k_0 - \bar{k})}{h_{1k} - r}. \]

The solution which is consistent with long run solvency is then given by

\[ b = \bar{b} + \frac{\alpha(k_0 - \bar{k})e^{h_{1k}t}}{h_{1k} - r}. \]

Equation (33) describes the equilibrium relationship between the change in the equilibrium capital-labour ratio and the change in the equilibrium net wealth of the economy, depending crucially on the sign of $\alpha$. Consider the expression for $\alpha$. The term $P_1^H L f'(k)$ which measures the increase in the net product of an increase in $k$, is positive. The second term in the expression for $\alpha$, $\bar{P}_1^H f'(\bar{k}) \frac{h_{2k}}{h_{2L} - h_{1k}}$, represents the effect of changes in employment of an increase in the capital-labour ratio. The term in the last square brackets represents the terms of trade effect and the direct intra- and intertemporal substitution effects, following
an increase in the capital-labour ratio. The increase in the capital-labour ratio induces a fall in the domestic product price, and intratemporal substitution over to domestic production, in addition to the positive effect on consumption through the intertemporal consumption function. $h_{2k}$ is negative due to these substitution effects. By inserting into the expression $\frac{b_{2k}}{h_{2k} - h_{1k}}$ from Appendix A, it can be shown that $\alpha < 0$, if the substitution effects, which contribute negatively to $\alpha$, outweighs the positive effect on the net product of an increase in $k$. The substitution elasticities are assumed to be large enough to fulfil this requirement. $\alpha < 0$ has the following interpretation: An increase in the capital-labour ratio induces a fall in the domestic product price and the price fall favours demand for the domestic product, but implies a loss in terms of trade. Provided that the substitution effects dominate the terms of trade effect, i.e. the Marshall-Lerner condition is satisfied, the increase in $k$ has a positive effect on net foreign wealth. In addition the higher capital-labour ratio has a positive effect on the demand for labour, see equation (28), even though $\dot{L}$ is falling as the economy approaches steady state.

The property of the positive relationship between net foreign wealth and the capital-labour ratio is the opposite of what is the case in a traditional small open economy model. With exogenous employment, higher investment in real capital leads to a corresponding reduction in financial investment and reduces net foreign wealth. Sen and Turnovsky (1989) modifies this model by assuming endogenous labour supply, but there is still a negative relationship between the accumulation of real and financial capital. In both Bye and Holmøy (1992) and Bye (1995) total employment is exogenous, but the small open economy model is modified by the assumption of the domestic product being an imperfect substitute for the world market product. The substitution possibility over to domestic production when the price of the domestic product is reduced following an increase in the capital-labour ratio, in addition to the terms of trade effect, modify the negative effect on net foreign wealth. But, there is still an inverse relationship between the accumulation of real and financial capital. The assumption of involuntary unemployment as modelled in this paper, represents an additional modification compared to the model presented in Bye and Holmøy (1992) and Bye (1995), implying a positive relationship between accumulation of real and financial capital.

The stationary solution of the linearised system is given by the following equations (for convenience the bar's over the steady state values are omitted in the rest of the paper):

\begin{align}
(34.a) & \quad P_1^H = \frac{w}{f(k) - kf'(k)} \\
(34.b) & \quad rg(P_1^H) = P_1^H f'(k) \\
(34.c) & \quad \frac{P_1^H}{g(P_1^H)}(Lf(k) - A(P_1^I/P_1^H)) = (\mu g(P_1^H))^{-\sigma_c} \\
(34.d) & \quad rb = G(P_1^H)(\mu g(P_1^H))^{-\sigma_c} - P_1^H Lf(k)
\end{align}
which determine the steady state values $P_1^H$, $k$, $L$, $b$ and $\mu$.

3 Long run effects of tax reforms

In order to calculate the welfare effects of a tax reform, both short and long run effects must be found. To derive the long run effects of a domestic tax reform, it is necessary to find the total logarithmic differentials of the steady state solution (34.a) - (34.e) w.r.t. $w$ and $P_2$. The domestic tax reforms are assumed to have no influence on international commodity markets, implying that the foreign prices $r$, $P_1^I$ and $P_2^I$ are constant in the analyses. The expression for consumption $C = (\mu g(P_1^H))^{-\sigma_C}$ is substituted in the steady state solution (34.a) - (34.e). $\dot{P}_2$, $\dot{P}_1^H$, $\dot{k}$, a.s.o. represent the logarithmic derivatives.

$$
\begin{align*}
(35.a) \quad \dot{P}_1^H &= \dot{w} - \frac{\theta^k}{\sigma_K} \dot{k} \\
(35.b) \quad \dot{P}_1^H &= \frac{1}{\theta^\ell}(\theta_2 \dot{P}_2 + \frac{\theta^L}{\sigma_K} \dot{k}) \\
(35.c) \quad \dot{L} &= s_{1L} \dot{P}_1^H + s_{12} (\dot{P}_2 - \theta^H \dot{P}_1^H) + \frac{\theta^H PC}{P_1^H X} \dot{C} - \theta^k \dot{k} \\
(35.d) \quad GC\dot{C} &= P_1^H X (\dot{P}_1^H + \dot{L} + \theta^k \dot{k}) + rbb - GC\dot{G} \\
(35.e) \quad \ddot{b} &= - \frac{\alpha k}{b(r - h_{1k})} \ddot{k}
\end{align*}
$$

We have the following definitions;

$$
s_{12} \equiv (1 - \frac{A}{X})\theta_2 \sigma_{12} > 0, \quad s_{1L} \equiv -(\theta^H \frac{A}{X} + \theta^\ell_1) \sigma_{1L} < 0
$$

$$
\theta^\ell \equiv (1 - \theta^H), \quad X \equiv Lf(k),
$$

and the following expressions;

$$
\dot{G} = \frac{P}{G}[\theta_1 \dot{P}_1 + \theta_1 \theta_2 \tau_{1+\tau} \sigma_{12} (\dot{P}_2 - \theta^H \dot{P}_1^H)], \quad \dot{P}_2 = \frac{\tau}{1+\tau} \ddot{r},
$$

$$
\dot{C} = -\sigma_C (\ddot{\mu} + \ddot{P}), \quad \ddot{w} = m\ddot{\delta} + \frac{t}{1-t} \dddot{t}
$$
is the economy’s total import share. $s_{lH}$ summarises the substitution effects between the domestic and foreign variety of good 1, and $s_{12}$ is the effect on the demand for the domestic product induced by substitution between fossil fuels and good 1.

The effects of a revenue neutral increase in the domestic tax on fossil fuels $\tau > 0$, combined with either a) a lump sum transfer or b) a revenue neutral change in the labour income tax, are analysed.

3.1 Taxation of fossil fuels with lump sum rebating

With tax reform a) we have

$$\hat{P}_2 = \frac{\tau}{1+\tau} \hat{r}, \quad \hat{w} = 0$$

The reduced form equations of the differentiated model are given in appendix C. The total effect on the level of consumption in steady state can be expressed by the following equation

$$\dot{C} = \frac{1}{GC} \left[ \frac{1}{1-\theta^H \theta^K} \left[ \theta^K \theta_2 P^H_1 A - s_{12} P^H_1 X \left( \frac{1-\theta^H \theta^K}{\theta^K_1} \right) \left( \frac{\tau}{1+\tau} \right) \right. \right. \right.$$

$$+ \left( PL - \frac{\alpha}{\tau - h_{1K}} \right) \tau \sigma_K \theta_2 \left. \hat{P}_2 + P^H_1 X \hat{L} \right]. \quad (36.a)$$

The effects on steady state consumption of an increase in the tax on fossil fuels $\tau$ can be divided into income effects due to changes in terms of trade, employment, real capital and net foreign wealth, and finally substitution effects. Since the tax income is redistributed lump sum, all other income effects disappear.

Following the increase in the tax on fossil fuels, terms of trade for good 1 are improved through higher export prices, and the first term within the brackets, $\theta^K \theta_2 P^H_1 A$, is a positive income effect from the change in these terms of trade. The second effect present in equation (36.a), $s_{12} P^H_1 X \left( \frac{1-\theta^H \theta^K}{\theta^K_1} \right) \left( \frac{\tau}{1+\tau} \right)$, which we denote the initial tax wedge effect, is the direct substitution effect on consumption from an increase in $P_2$ relative to the endogenous adjustment of $P_1$, due to the increase in $\tau$. This effect is larger the larger the substitution between good 1 and fossil fuels. An increase in the fossil fuel tax brings about substitution from fossil fuels to good 1. By further reducing the demand for fossil fuels, the efficiency loss generated by the discrepancy between the marginal willingness to pay for fossil fuels and the marginal production costs, represented by the world market price for fossil fuel, increases. This effect is modified through the term $\theta^K \theta^H$, which captures that the price of good 1 also depends on the price of fossil fuels through the domestic production of capital. This modifies the initial tax wedge effect of an increase in the carbon tax.

The effect represented by the term $(PL - \frac{\alpha}{\tau - h_{1K}}) \tau \sigma_K \theta_2 k$, which is denoted the capital accumulation term, consists of; (i) the value of the marginal product of capital measured by $PLkr$, and (ii) the interest from the capitalised value of the trade surplus generated.

\[10\] The calculations of the reduced form equations are available from the author.
by an increase in the capital-labour ratio, measured by \( \frac{r_{ok}}{r-h_{ik}} \), where \( h_{ik} \) takes into account the adjustment speed of the capital-labour ratio. This gives the total effect on net foreign wealth. These two effects summarise the net impact of \( k \) on the national income induced by substitution following the rise in \( P_2 \) which enters the user cost of capital. An increase in the carbon tax implies a reduction in the capital-labour ratio, see equation (C.3) in appendix C, implying that the capital accumulation term must be represented in the reduced form equation for consumption with a negative sign. This has the following interpretation; since the capital-labour ratio is lower, the returns from both real capital (measured as \( k \)) and financial wealth are reduced, contributing negatively to the stationary level of consumption.

The terms of trade effect, the initial tax wedge effect and the productivity effect from capital are all present in a situation with exogenous employment, see Bye (1995). With involuntary unemployment there is in addition a positive income effect on consumption from the increase in employment. The overall effect on consumption of tax reform a) is larger the larger the total home share in demand and the share of capital in production. The effects of the relative price changes are modified since domestic production uses domestic produced capital as input.

The total effect on employment is given by the following equation

\[
\hat{L} = \frac{1}{1 - \theta_H \frac{P}{G}} \left[ (S\theta^K + \gamma_K \sigma_K) \left( \frac{\theta_2}{1 - \theta_H \theta_K} \right) + s_{12} \left( \frac{1}{1 + \tau\theta_1} \right) \right] \hat{P}_2
\]

where

\[
S \equiv s_{1H} - s_{12} \left( \frac{\theta_1}{1 + \tau\theta_1} \right) + \theta_H \frac{P A}{G X} < 0^{11}
\]

and

\[
\gamma_K \equiv \theta^K (1 - \theta_H \frac{P}{G}) + \frac{\theta_H}{XG} (\frac{r_{ok}}{r-h_{ik}}) > 0
\]

Consider first the last term in the brackets in equation (36.b), \( \frac{s_{12}}{1+\tau\theta_1} \). This expresses the direct positive substitution effect over to good 1 when the price on fossil fuel increases, which implies higher demand for the domestic product and labour for a given home share of good 1. The effect is modified by the initial fossil fuel tax and the budget share of good 1 in demand. The initial tax rate induces an efficiency loss, modifying the positive effect on the demand for labour.

The effect of the fossil fuel tax on the capital-labour ratio is negative. A reduction in the productivity of labour induces an increase in the domestic product price, reinforcing the rise in the price of the macro good and the user cost of capital. These effects are accounted for by the first two terms in (36.b). The first term in \( S \), \( s_{1H} \), is the substitution effect between the domestic and the foreign variety of good 1, which is negative. An increase in the domestic product price following a tax on fossil fuels implies substitution away from the

\[
S \equiv s_{1H} - s_{12} \left( \frac{\theta_1}{1+\tau\theta_1} \right) + \theta_H \frac{A_P}{G X G} = -\theta_1^t \sigma_{1H} - \theta_1^H \frac{A}{G} (\sigma_{1H} - \frac{P}{G} \theta_1) - s_{12} \left( \frac{\theta_1}{1+\tau\theta_1} \right) < 0
\]
domestic variety both on the domestic and export market. This contributes to a reduction in labour demand. The second term in $S$, $-s_{12}(\frac{\theta^H}{1+\theta^H_1})$, reflects that an increase in the domestic product price, following a higher fossil fuel tax, induces an increase in the price of good 1 and brings about substitution to good 2. This effect modifies the direct positive substitution effect on good 1 and contributes negatively to the level of employment. The last term in $S$, $\theta^H \frac{P_1}{G_2}$, represents the terms of trade effect. The positive income effect following an increase in terms of trade implies higher demand for the macro commodity for consumption and investment purposes, and hence has a positive effect on the demand for the domestic product and labour.

The term $\gamma_K \sigma_K$ represents the effect of a change in the capital-labour ratio on labour demand. The last term in $\gamma_K$ represents the effect on the demand for the domestic product and labour of the change in net foreign wealth generated by a change in the capital-labour ratio. Since $\alpha < 0$, this effect is negative and increases with the total home share. $\theta_K$ represents the direct contribution of the reduced capital-labour ratio on the demand for labour. The second term in $\gamma_K$ is divided by total production, and it is assumed that the direct effect from the change in the capital-labour ratio dominates, such that $\gamma_K$ is positive. This implies that the reduction in the capital-labour ratio following a carbon tax increase, has a positive effect on employment. The same effects are present in the reduced form equation for the stock of real capital, see equation (C.4) in appendix C. The stock of real capital is reduced by the tax reform, implying lower savings in both real- and financial capital. The total effect on employment is increasing in the total home share of domestic absorption. The term $\frac{1}{1-\theta^H \frac{P_1}{G}}$ is then interpreted as an activity multiplier.

It may be optimal to impose an export tariff in a model where there are possibilities of obtaining terms of trade gains, see Dixit (1985) and Vennemo (1992) for both a theoretical and empirical exposition. In the model presented in this paper there are substitution possibilities both between good 1 and fossil fuels and between the domestic and foreign varieties of good 1. The existence of these substitution possibilities implies that it is not equivalent to impose an export tariff on the domestic product instead of the carbon tax. If there is no carbon tax initially, $\tau = 0$, $P = G$ and $P_2 = d\tau$, the initial tax wedge effect in equation (36.a) disappears, and the total effect on consumption is more likely to be positive. The trade off between a possible terms of trade gain and the efficiency loss from the initial tax wedge is discussed in Bye (1995).

To summarise, the long run effects on both consumption and employment are uncertain. The existence of unexplored market power on the export market, implies that it is possible to obtain a positive effect on long run consumption of an increase in the carbon tax combined with lump sum rebating of the tax revenue through a positive terms of trade effect. The intratemporal substitution effects following the increase in the domestic product price reduce the demand for the domestic product and labour. The initial carbon tax rate contributes negatively to consumption through the initial tax wedge term. The income effects from lower capital-labour ratio and net foreign wealth, contribute negatively to the steady state.
level of consumption, but lower capital-labour ratio has a positive effect on employment. The use of a dynamic model with endogenous accumulation of capital and the possibility of a terms of trade gain modify the results in Bovenberg and van der Ploeg (1994b) where an environmental tax reform in a small open economy framework with no possibilities of obtaining terms of trade gains, had a negative effect on employment.

3.2 Taxation of fossil fuels combined with changes in the labour income tax

Consider now the effects on employment and welfare of tax reform b) where

\[ \dot{P}_2 = \frac{\tau}{1 + \tau} \bar{r}, \quad \bar{w} = \frac{t - \bar{t}}{1 - \bar{t}} \]

For simplicity \( \dot{P}_2 \) and \( \bar{w} \) are used in the reduced form expressions. The logarithmic differentiated government’s budget balance (equation (24)) is given by (when \( \bar{s} = 0, \bar{\Omega} = 0, \bar{t} = \frac{1 - t}{\bar{t} - \bar{t}} \bar{w}, \bar{r} = \frac{1 + \tau}{\bar{r}} \dot{P}_2 )

\[
(37) \quad \bar{w} = -\frac{1}{\kappa_t} \left[ (s + tw)LL + \frac{\tau}{1 + \tau} \theta_2 PC \dot{C} + \kappa_r \dot{P}_2 \right] < 0,
\]

where

\[
\kappa_t \equiv 1 + \left( \frac{\tau}{1 + \tau} \right) \left( \frac{\theta_2 s_{12}}{1 - \theta H \theta K} \right) wL > 0,
\]

\[
\kappa_r \equiv \theta_2 PC \left[ 1 - \frac{\tau}{1 + \tau} \sigma_{12} \left( \frac{1 - \theta H \theta K}{1 - \theta H \theta K} \right) \right] > 0.
\]

Both \( \kappa_t \) and \( \kappa_r \) are positive. They represent the direct effects on the governmental income of a partial increase in \( t \) and \( \tau \) respectively. \( \kappa_r \) is increasing in the value of consumption, which is the only base for this tax in the steady state. The second term in \( \kappa_r \) captures the effect of adjustment of the tax base; increased carbon tax makes consumers substitute good 1 for fossil fuels.

The overall effect on the wage tax includes the adjustment in the tax bases of employment and consumption. It is assumed that the initial equilibrium in the economy is on the upward-sloping parts of the Laffer curves for both the labour tax and the fossil fuel tax. The additional public revenue from carbon taxation thus allows for a reduction in the wage tax rate. The reduction in the wage tax rate is smaller the larger the initial wage tax base, reducing the effects of the tax reform. To find the overall effects on employment and consumption of such a revenue neutral tax reform, equation (37) is inserted into the reduced form equations (C.1) and (C.2). The effect on the capital-labour ratio follows then from equation (C.3). Consider first the effects on consumption, which are given in equation (38.a).
\[ \hat{C} = \frac{1}{CG + \frac{\tau}{1+\tau} \theta_2 P \gamma_w} \left[ (P_1^H X - L(s + tw)\gamma_w)\bar{L} + \Delta C \bar{P}_2 \right] \]

where

\[ \gamma_w \equiv \frac{1}{\kappa_t(1 - \theta^H \theta^K)} \left[ \theta^L \left( P_1^H A + P_1^H X s_{12}(\frac{\tau}{1 + \tau}) \right) + (PL - \frac{\alpha}{\tau - h_{t1k}})r k \sigma K \theta^l \right] > 0 \]

\[ \Delta C \equiv \frac{1}{1 - \theta^H \theta^K} \left[ -\left( \frac{1 - \theta^K \theta^H}{\theta^H_1} + \theta^L \frac{K_T}{\kappa_t} \right) \frac{\tau}{1 + \tau} P_1^H X s_{12} + \left( \theta^K \theta_2 - \theta^L \frac{K_T}{\kappa_t} \right) P_1^H A \right] \]

\[ - (PL - \frac{\alpha}{\tau - h_{t1k}})r k \sigma K \left( \theta_2 + \frac{\kappa_T}{\kappa_t} \theta^l \right) \].

The term \( \Delta C \) represents the total effect on consumption of changes in both \( P_2 \) and \( w \), see equation (C.2) in appendix C. Compared to reform a), all the same effects are present, but there are additional effects from the term \( \frac{\kappa_T}{\kappa_t} \) which represents the relation between the carbon tax’s and the wage tax’s direct revenue raising device, i.e. the direct effect on the wage tax rate. The larger \( \frac{\kappa_T}{\kappa_t} \), the larger the fall in the wage tax rate, partially. This term contributes to reduce the positive terms of trade effect\(^{12}\), and increase the effect of the initial tax wedge and the income effect from the capital-labour ratio. The reduction in the wage tax rate contribute to a reduction in the domestic product price and the price of good 1, compared to tax reform a). This induces additional substitution over to good 1, and strengthens the efficiency loss of the initial carbon tax. An increase in the carbon tax implies a terms of trade gain through the increase in the domestic product price, but a reduction in the wage tax contributes negatively to such a terms of trade gain by lowering the domestic product price. From equation (C.3) it follows that a reduction in the wage rate implies further reduction in the capital-labour ratio, compared to tax reform a), contributing negatively to the net income effect of the capital-labour ratio (the capital accumulation term). \( \Delta C \) is negative if the initial tax wedge effect and the effect of the capital-labour ratio outweighs the terms of trade effect.

Whether the long run effect of the tax reform on consumption is positive or negative, depends in addition on the general equilibrium effect from employment which is also modified compared to reform a) by the term \( \gamma_w(s + tw) \), following from the assumption of revenue neutrality of the tax reform. \( \gamma_w \) represents the direct effect on consumption of an increase in the wage rate, see equation (C.2). This term consists also of the three effects; the terms of trade effect, the initial tax wedge effect of the fossil fuel tax which is positive with an increase in the wage rate since the domestic product price increases, giving substitution from good 1 to fossil fuels, and the total effect of an increase in the capital-labour ratio on the net national income. This last effect comes into the reduced form equation (C.2) with a positive

\(^{12}\)It can be shown that \( \theta^K \theta_2 - \theta^L \frac{K_T}{\kappa_t} \) is positive.
sign since a wage increase has a positive effect on the capital-labour ratio (see equation (C.3)). It follows that the term $\gamma_w$ contributes negatively to the level of consumption since the wage rate is reduced.

The effects on the level of employment is given by

$$L = \frac{1}{\gamma_{Lt}} \left[ \left( S \left( \theta_2 \theta^K - \frac{\kappa_r}{\kappa_2} \theta^L \right) + \gamma_K \sigma_K \left( \theta_2 + \frac{\kappa_r}{\kappa_2} \theta^I \right) \right) \frac{1}{1 - \theta^H \theta^K} + s_{12} \left( \frac{1}{1 + \tau \theta_1} \right) \right] P_2$$

(38.b) $$- \frac{1}{\kappa_t} \left( S \theta^L - \gamma_K \sigma_K \theta^I \right) \left( \frac{\tau_2 \theta^H P}{(1 + \tau)G + \gamma_2 \gamma_w \Delta C} \right) \frac{1}{1 - \theta^H \theta^K} \right] P_2$$

where

$$\gamma_{Lt} \equiv 1 - \frac{\theta^H P}{G} + \frac{S \theta^L - \gamma_K \sigma_K \theta^I}{\kappa_t} \left( \frac{\tau_2 \theta^H X + L(s + tw)(1 + \tau)G}{(1 + \tau)G + \gamma_2 \gamma_w \Delta C} \right) > 0.$$ 

Compared to reform a), all the same effects are present, but there are additional effects from the term $\frac{\theta^L}{\kappa_2}$ which contribute to reduce the negative intratemporal substitution effects, represented by $S$, since the reduction in the wage tax contribute negatively to the domestic product price. The positive effect of a reduction of the capital-labour ratio represented by $\gamma_K \sigma_K$, is reinforced through the additional negative wage rate adjustment. The direct positive substitution effect of the carbon tax on the domestic product and employment, $\frac{\theta^L}{\kappa_2}$, is unchanged. The term in the denominator, $\gamma_{Lt}$, is assumed to be positive (but is less than 1), contributing to reinforcing the effects on employment. Compared to tax reform a), the total effect on employment of these three first terms in equation (38.b), is more likely to be positive.

In the last term in equation (38.b), the term $(S \theta^L - \gamma_K \sigma_K \theta^I) < 0$, represents the direct effect on employment of a change in the wage rate, see equation (C.1) in appendix C. The effect is multiplied by the total effect on consumption of changes in both $P_2$ and $w$, represented by the term $\Delta C$, following from the assumption of revenue neutrality of the tax reform. The positive effect on employment is modified if $\Delta C$ is negative. This points to the possible tax base erosion effect of the carbon tax. If the increase in the carbon tax actually leads to a reduction in consumption, the possible reduction in the wage tax is smaller, having a negative effect on employment. It is not possible to have multiple increases in the tax rate on fossil fuels which are neutralised with reductions in the wage tax rate, since the effect on the tax revenue may become negative, i.e. the tax-base erosion effect of the fossil fuel tax will dominate.

Compared to tax reform a) with lump sum rebating, the positive effect on employment is larger with tax reform b), but the lower capital-labour ratio contributes negatively to the long run level of consumption through the capital accumulation term. The stock of real capital is further reduced, see equation (C.4).\(^\text{13}\) Bovenberg and van der Ploeg (1994c) using a static model framework with involuntary unemployment due to wage rigidity, obtain a positive effect on both employment and consumption of a similar kind of tax reform, if

\(^\text{13}\)It is assumed that the positive substitution effects over to good 1 do not outweigh the negative substitution effects, both due to substitution over to the imported variety of good 1 and to labour.
the resource tax as they call it, is not too high initially. By using a dynamic model with endogenous capital accumulation, it is shown that an environmental tax reform will have long term effects on the level of consumption through the change in the capital-labour ratio and the stock of net foreign wealth, which modifies the positive long run effect on consumption. This effect is absent in Bovenberg and van der Ploeg (1994c). Sørensen et al (1994) find support for a double dividend hypothesis (positive effects on both employment, growth and consumption) in an intertemporal model with involuntary unemployment for a closed economy. This conclusion is modified by using an intertemporal model for an open economy where both savings in real and financial capital are reduced. This highlights the need to take into consideration the dynamic effects from changes in capital accumulation and long run production possibilities, when evaluating environmental tax reforms.

4 Welfare effects of tax reforms

To analyse the total welfare effects of a tax reform in a dynamic model implies computation of the entire time path of instantaneous utility of the representative agent, see e.g. Brock and Turnovsky (1993). The representative consumer’s instantaneous utility at time \( t \), \( Z(t) \), is defined as

\[
Z(t) = u(C_t)
\]

and the overall level of welfare

\[
W = \int_0^\infty u(C_t) e^{-rt} dt = \int_0^\infty Z(t) e^{-rt} dt
\]

To be able to analyse the welfare effects of the tax reforms, it is necessary to linearise equation (39.a) around its steady state level, substitute the linearised expression into equation (39.b) and integrating, to find an approximate measure of the welfare of the representative consumer. The utility function is specified in equation (21.a) and the corresponding consumption function is given in equation (22). The marginal utility of wealth, \( \mu \), adjusts instantaneously to a new constant level following changes in the other variables. Equations (11) and (14) give \( P_t^H \) as a function of \( k \) when the exogenous variables are constant through time. Hence, \( P \) is a function of \( k \), and from equation (22) it follows that the adjustment in \( k \) around steady state determines the adjustment in consumption and then \( Z(t) \) around steady state. Using equation (27), the transitional path of \( Z(t) \) may be linearly approximated by

\[
Z(t) \cong u(\tilde{C}) + \omega (k_0 - \tilde{k}) e^{kt}
\]

where

\[
\omega \equiv u'_C C_P g_{PH} P'_{Hk} = \sigma_C \tilde{C} \frac{\sigma_{C}}{\sigma_{H}} g_{H} g_{K} > 0
\]

\( \omega \) is constant since it is only a small area around the steady state solution that is considered.
Inserting equation (40.a) into equation (39.b) and integrating, gives the linear approximation for $W$.

\[
W = \frac{u(\ddot{C})}{r} + \omega \left( \frac{k_0 - \bar{k}}{r - h_{ik}} \right)
\]

(40.b)

The first term in equation (40.b) represents the consumer’s utility if the steady state level of consumption was attained instantaneously. The second term reflects the adjustment towards this level due to the fact that the steady state is reached only gradually along the transitional path.

**Taxation of fossil fuels**

To derive the total welfare effects of a domestic tax reform, it is necessary to find the total logarithmic differentials of equations (40.a) and (40.1D). The expressions are given by

\[
\ddot{Z}(t) = \frac{1}{Z} \left[ \ddot{C} \left( \frac{r_{cc}-1}{r_{cc}} \right) \ddot{C} - \omega e^{h_{it} r_{kk}} \right]
\]

(41.a)

\[
\ddot{W} = \frac{1}{W} \left[ \ddot{C} \left( \frac{r_{cc}-1}{r_{cc}} \right) \ddot{C} - \omega \left( \frac{\bar{k}}{r - h_{ik}} \right) \ddot{k} \right]
\]

(41.b)

By using the reduced form expressions for $\dot{C}$ and $\bar{k}$ given in section 3.1. with lump sum rebating, or in section 3.2. with a change in the wage tax, the total welfare effects can be analysed. Tax reform a) is used as an example, but the interpretations are easily extended to incorporate the effects of tax reform b).

Equation (41.a) gives the effect of a tax on fossil fuels on the level of instantaneous welfare $Z(t)$. Using equation (C.3) it is easily seen that an increase in the tax on fossil fuels $\ddot{\tau} > 0$ reduces the steady state level of capital per labour unit, i.e. $\bar{k} < 0$. An increase in the tax on fossil fuels will have the following effects on the short run ($t = 0$) and steady-state ($t \to \infty$) levels of instantaneous utility.

\[
\ddot{Z}(0) = \frac{1}{Z} \left[ \ddot{C} \left( \frac{r_{cc}-1}{r_{cc}} \right) \ddot{C} - \omega \ddot{k} \bar{k} \right]
\]

(42.a)

\[
\ddot{Z}(\infty) = \frac{1}{Z} \ddot{C} \left( \frac{r_{cc}-1}{r_{cc}} \right) \ddot{C}
\]

(42.b)

From equations (42.a) and (42.b) it is easily seen that

\[
\ddot{Z}(0) > \ddot{Z}(t) > \ddot{Z}(\infty),
\]

independent of whether the steady state effect on consumption is positive or negative. This implies that instantaneous utility follows a downward sloping path as illustrated in figure 4.1. Following an initial jump in instantaneous utility initiated by the tax reform, it will gradually adjust to the new steady state level. The shaded areas are the change in instantaneous utility discounted, i.e. the total welfare effect. Along the path the capital-labour ratio is reduced,
lowering the share of investment in total demand. This gives more room for consumption. On the other hand a lower capital-labour ratio has a negative effect on production, which gives less room for consumption in steady state.

The total welfare effect given by equation (41.b) shows that the welfare effect may be positive even with a negative steady state effect on consumption. The second term within the brackets represents the effect of the reduction in the capital-labour ratio along the path towards steady state. The reduction in the capital-labour ratio is accompanied by a reduction in net foreign wealth, which changes proportionally to the capital-labour ratio in the neighbourhood of the steady state. This give room for a slowdown in the fall in consumption along the path. Discounting a transitional path with a higher level of consumption and instantaneous utility along the path may give a positive welfare effect and supports the double dividend hypothesis even if the steady state effect on instantaneous utility is negative, as in figure 4.1.

**Figure 4.1. Welfare effects of an environmental tax reform**

The total welfare effect of tax reforms a) and b), depends on the long run effects on both the capital-labour ratio and the level of consumption. The capital-labour ratio is reduced under both reforms, with the largest reduction under tax reform b). This has a positive impact on both instantaneous utility and total welfare, which is largest with reform b). The long run effect on consumption is more uncertain. As the discussion in section 3.1 of tax reform a) states; if the effect on employment is negative, the effect on the long run level of consumption is also negative, contributing negatively to both instantaneous utility and
total welfare. But even with a reduction in the long run level of consumption, the total welfare effect may be positive due to the positive effects on instantaneous utility along the transitional path. With tax reform b) the effect on employment is also likely to be positive. By using the terminology introduced by Bovenberg and van der Ploeg (1993), both the employment double dividend and the welfare double dividend may be obtained with tax reform b).

5 Concluding remarks

This paper has analysed the effects of green tax reforms on a small, open economy producing an imperfect substitute for foreign goods. The model is based on intertemporal optimisation, and union wage setting and a fixed exchange rate imply wage rigidity and involuntary unemployment. The long run effects on instantaneous utility, employment and the stock of real and financial capital, of a revenue neutral increase in the tax on fossil fuels combined with a) lump sum rebating or b) change in the wage tax, are discussed. In general the total welfare effect is ambiguous, depending on substitution effects which in the case of a non-optimal initial tax structure create efficiency losses, and income effects from changes in terms of trade, employment, real capital and net foreign wealth. Due to the changes in instantaneous utility during the time path following the implementation of the tax reform, the total welfare effect may be positive even with a reduction in long run consumption. The total welfare effect is in general more positive (or less negative) with tax reform b) than tax reform a). Hence, there may exist a welfare double dividend from increasing the tax on fossil fuels, and using the revenue to reduce the wage tax, which is causing involuntary unemployment. The effect on the long run level of employment is also more positive with tax reform b).

The model presented in this paper represents the most stylised version of a model incorporating; i) intertemporal dynamics, ii) involuntary unemployment, iii) open economy and iv) imperfect substitution between products from different countries. The assumption of imperfect substitutes is a commonly used specification of many empirical short, medium and also long term economic analyses. An intertemporal analysis will also depend on the short term effects, implying that it is necessary to incorporate the assumption of imperfect substitutes in an intertemporal model if it is a part of the short term analysis. This paper illustrates that such an extension of the small open economy framework complicates the effects and interpretation of an environmental tax reform substantially.

Compared to other analytical studies of environmental tax reforms and the double dividend issue, this paper's major contribution in addition to the assumption of imperfect substitutes between the domestic and foreign variety, is the dynamic model framework, which makes it possible to analyse the time path of instantaneous utility. The short and long term effects may be different for the two tax reforms considered, implying that it is important to evaluate both the short and long run implications of a suggested environmental
tax reform. The literature on gradual reforms of taxation, see e.g. Dixit (1985), gives no general rules for the effects of tax reforms when there are existing distortionary taxes. As quoted from Dixit (1985); "The welfare effect of any primary reform, will depend crucially on the other elements of the whole reform package. In practice each application must be examined on its own." The analysis presented in this paper points to many important effects which clearly will affect the economy if an environmental tax reform is implemented, even if the conclusions are somewhat unclear. There is then a need for numerical simulations to be able to give more strong conclusions on the welfare effects of environmental tax reforms.
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Appendix A

The derivatives of the dynamic system (26.a)-(26.c) in steady state are given by the following expressions:

(A.1) \[ h_{1k} = -\frac{r}{\theta^H \theta^K} (1 - \theta^H \theta^K) < 0 \]

(A.2) \[ h_{1L} = 0 \]

\[ h_{2k} = -\frac{\theta^K P^H_1}{\sigma_K k^2 P} \left[ \frac{P_1 P^H_1}{(P^H_1)^2} (\sigma_{12} \theta^H + \sigma_{1H} \theta^I) + \frac{\sigma_{1H}}{\theta^H} \theta^I + \sigma_C \theta^H \frac{P}{P^H_1} C \right] \]

(A.3) \[ + \frac{L r}{k \theta^H} \left[ \theta^I + \frac{1}{\theta^K} \right] \]

\( h_{2k} \) can be positive/negative.

(A.4) \[ h_{2L} = \frac{r}{\theta^H \theta^K} > 0 \]
Appendix B

Due to the recursive structure of the model, the price expectations are substituted with changes in the capital stock, which is backward looking. Hence, the level of employment is also backward looking. The dynamic adjustments of $k$ and $L$ can be described by a phase diagram of the dynamic system (26.a) and (26.b). In section 2.6 it is shown that this dynamic system has the property of saddle-point stability, irrespective of the real value of the marginal utility $\mu$. We denote the loci generated by setting $\dot{k} = 0$ in (26.a) and $\dot{L} = 0$ in (26.b) as locus A and B respectively. The slopes of these two loci are based on the steady state values of the derivatives which enter the dynamic system.

Inserting equation (35.a) and the expression for $\hat{G}$ into equation (35.b) gives the following equation for the $\dot{k} = 0$ locus

\[
\frac{1}{\sigma_K} (\theta^K + \frac{\theta^L}{\theta^L})\dot{k} = \ddot{w} - \frac{\theta_2}{\theta^L} \dot{p}_2
\]

It is easily seen from equation (35.b) that the $\dot{k} = 0$ locus is vertical in the point $rg = P^H f'(k)$ on the $k$-axis in the $L, k$-diagram.

Inserting equations (35.a), (35.d) and (35.e) into equation (35.c) gives the slope of locus B.

\[
(1 - \frac{\theta^H P}{G}) \ddot{L} = - (s_{12} \frac{\theta^H}{1 + \theta_1}) + S \ddot{w}
\]

where

\[
\gamma_K \equiv \theta^K (1 - \frac{\theta^H P}{G}) + \frac{\theta^H}{XG} \frac{r_1 k}{r - h_k} > 0
\]

\[
S \equiv s_{1H} - s_{12} (\frac{\theta^H}{1 + \theta_1}) + \frac{\theta^H P A}{G X} < 0
\]

Locus B is upward sloping in the $L, k$-diagram if the negative term $S \frac{\theta^K}{\sigma_K}$ outweighs the positive term $\gamma_K$. The absolute value of the elasticity of $L$ w.r.t. $k$ is larger the larger is the marginal elasticity of capital measured by the cost share of capital (in the neighbourhood of steady state), and the larger are the intratemporal substitution possibilities represented by the terms $s_{1H}$ and $s_{12}$. If the intratemporal substitution effects are not large enough to outweigh the positive term $\gamma_K$, the slope of locus B is negative.

The two loci are presented in figures B.1 and B.2. The saddle path is denoted SS.

\[\text{\textsuperscript{14}SS \equiv s_{1H} - s_{12} (\frac{\theta^H}{1 + \theta_1}) + \theta^H \frac{\Delta K}{XG} = - \theta_1' s_{1H} - \theta_1' \frac{\Delta K}{XG} (1 - \frac{P A}{G}) \theta_1 - s_{12} (\frac{\theta^H}{1 + \theta_1}) < 0}\]
Figure B.1. Phase diagram

"Small" intratemporal substitution effects.
Figure B.2. Phase diagram

"Large" intratemporal substitution effects.
Appendix C

The reduced form equations for $\bar{L}$, $\bar{k}$ and $\bar{C}$ can be derived from (35.a) - (35.e) and the expression for $\bar{G}$. The reduced form equations of $\bar{L}$, $\bar{C}$ and $\bar{k}$ are given by

$$
\bar{L} = \frac{1}{1-\theta^H P/G} \left[ (S\theta^K + \gamma_K \sigma_K) \left( \frac{\theta_2}{1-\theta^H \theta^K} \right) \bar{P}_2 + s_{12} \left( \frac{1}{1 + \tau \theta_1} \right) \bar{P}_2 \right.
\right.
$$
\begin{align*}
&+ \left. \left( S\theta^L - \gamma_K \sigma_K \theta^f \right) \left( \frac{1}{1-\theta^H \theta^K} \right) \bar{w} \right],
\end{align*}
(C.1)

$$
\bar{C} = \frac{1}{GC} \left[ \frac{1}{1-\theta^H \theta^K} \left[ \theta^K \theta_2 P^H A - s_{12} P^H X \left( 1 - \theta^H \theta^K \right) \left( \frac{\tau}{1+\tau} \right) \right.ight.
\right.
$$
\begin{align*}
&- \left. \left[ (PL - \frac{\alpha}{r-h_{1k}}) r_k \sigma_K \theta_2 \right] \bar{P}_2 + \frac{1}{1-\theta^H \theta^K} \left[ \theta^L (P^H A + P^H X \left( \frac{\tau}{1+\tau} \right) s_{12}) \right.ight.
\right.
\end{align*}
$$
\begin{align*}
&+ \left. \left[ (PL - \frac{\alpha}{r-h_{1k}}) r_k \sigma_K \theta^f \right] \bar{w} + P^H X \bar{L} \right]
\right.
\end{align*}
(C.2)

(\text{where (C.1) can be substituted for $\bar{L}$), and}

$$
\bar{k} = \frac{\sigma_K}{1-\theta^H \theta^K} \left[ \theta^f \bar{w} - \theta_2 \bar{P}_2 \right]
$$
(C.3)

where

$$
\gamma_K \equiv \theta^K (1 - \theta^H P/G) + \frac{\theta^H}{XG} \left( \frac{r \alpha k}{r - h_{1k}} \right) > 0,
$$

$$
S \equiv s_{1H} - s_{12} \left( \frac{\theta^H}{1+\tau \theta_1} \right) + \theta^H \frac{P A}{G X} < 0.
$$

The reduced form equation for the stock of real capital, $\bar{K}$, is then given by

$$
\bar{K} = \frac{1}{1-\theta^H P/G} \left[ (S\theta^K + \sigma_K \left( \frac{\theta^H (r \alpha k)}{XG (r - h_{1k})} - \theta^L (1 - \theta^H P/G) \right) \left( \frac{\theta_2}{1-\theta^H \theta^K} \right) \bar{P}_2 + \right.ight.
\right.
$$
\begin{align*}
&\left. + s_{12} \left( \frac{1}{1 + \tau \theta_1} \right) \bar{P}_2 \right]
\end{align*}
$$
\begin{align*}
&+ \left. \left[ (S\theta^L - \sigma_K \theta^f \left( \frac{\theta^H (r \alpha k)}{XG (r - h_{1k})} - \theta^L (1 - \theta^H P/G) \right) \left( \frac{1}{1-\theta^H \theta^K} \right) \bar{w} \right] \right]
\end{align*}
(C.4)

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