Abstract:
The paper analyses how equilibrium adjustments of the wage rate affect the scope for tax rate reductions when the government experiences an exogenous increase in non-tax revenues. It shows within a stylized model that increased revenue in the form of a tradable will increase the wage rate, which diminishes the scope for tax rate reduction, provided that the initial wage dependent government net expenditures are positive. In this case the wage rate adjustment represents an automatic channel for redistributing increased non-tax government revenues. When the revenue increases in the form a non-tradable, the wage rate adjustment reinforces the scope for tax rate reduction. Simulations on a CGE model of the Norwegian economy confirm the theoretical results, and demonstrate that the fiscal wage effect can be strikingly large.

Keywords: Tax incidence, fiscal policy, general equilibrium effects

JEL classification: D58, H22, H61

Acknowledgement: Thanks to Ådne Cappelen for comments on an earlier version. The usual disclaimer applies.

Address: Erling Holmøy, Statistics Norway, Research Department. E-mail: erl@ssb.no
**Discussion Papers**  
comprise research papers intended for international journals or books. A preprint of a Discussion Paper may be longer and more elaborate than a standard journal article, as it may include intermediate calculations and background material etc.

Abstracts with downloadable Discussion Papers in PDF are available on the Internet:  
http://www.ssb.no  
http://ideas.repec.org/s/ssb/dispap.html

For printed Discussion Papers contact:  
Statistics Norway  
Sales- and subscription service  
NO-2225 Kongsvinger  
Telephone: +47 62 88 55 00  
Telefax: +47 62 88 55 95  
E-mail: Salg-abonnement@ssb.no
1 Introduction

What is the scope for tax rate reductions if the government receives a given amount of non-tax revenues or cut in expenditures? In more technical terms: What is the trade-off between a specific tax rate and a given change in non-tax net revenues implied by the government budget constraint, when one takes general equilibrium effects into account?

The purpose of this paper is to shed light on this question. Not surprisingly, it’s motivating claim is that a naive estimate, obtained by dividing the additional revenue by the base of the tax that is to be reduced, is likely to be quite misleading, and for some reasons that to my knowledge has not been given due attention in the literature.

It is easy to motivate the introductory question for economies in which the government collects the revenue from natural resources. A good example is Norway where the government collects most of the petroleum rent through taxes and stakes in petroleum companies. The fiscal policy rule adopted in 2001 imposes a precise constraint on the non-petroleum government budget deficit. This constraint has, however, been violated in most years, and the high oil price after 2004 has reinforced the pressure from various interest groups for tax cuts and higher government spending. The response to this pressure should rest on a thorough analysis of the introductory question. However, the relevance of this analysis should be quite general. For example, most governments in the OECD are indebted, and a permanent fall in the expected interest rate or revaluations of government assets will be equivalent to an increase in resource rents. Other examples include changes in world prices of imports used for government consumption, productivity improvements in the production of public services and demographic changes affecting government pension expenditures and tax financed health and social care.

This paper will focus on one particular general equilibrium mechanism: Changes in government non-tax net revenues affect the real exchange rate, through e.g. wage adjustment, which influences tax bases and prices of government spending. I examine this mechanism analytically within a stylized model, and its empirical significance by simulations on an applied general equilibrium model. Some of the analytical results are perhaps surprising. For example, it is possible that an increase in non-tax revenue warrants an increase in the tax rates. This result is not due to labour supply responses. Instead, I emphasize that wage adjustment automatically distributes a significant portion of the additional non-tax revenues from the government to households, reducing the scope for discretionary redistribution through lower tax rates and/or increased government spending. To my knowledge this mechanism has not been given the attention it deserves in the literature on tax incidence and fiscal sustainability.
The fiscal effect of wage adjustments can be summarized in the case where the government receives more of a traded good, e.g. oil revenues. Additional oil revenue allows an increase in net imports of other traded goods, restoring balanced trade. As is well known\textsuperscript{1}, resources are reallocated from production of traded goods to the non-traded goods sector, through real appreciation brought about by a higher wage rate and/or exchange rate appreciation. The contribution in this paper is to deduce the fiscal consequences of the wage adjustment, especially the price effects on government expenditures and tax bases. A higher wage rate raises the current value of tax bases and government expenditures. Depending on the relative wage content, expenditures may increase more than the tax bases, thereby reducing the scope for cuts in the tax rates. This possibility is more likely the higher is the wage dependency of government expenditures. The wage share in government consumption is typically very large, and public pensions and other transfers are indexed to the wage rate in several countries.

Another result in this paper is that the direction of the fiscal effect of the wage adjustment depends on whether the government receives more of a traded good or a non-traded good. Assuming labour to be internationally immobile, one example of the latter case is productivity improvement in public service production, reducing the government employment required to meet political targets. In this case the wage rate adjustment will contribute to reinforce the revenue neutral tax rate reduction if the increase in non-tax government revenues comes in form of a non-traded good.

The scope for wage rate adjustments depends on the technologies in the industries producing tradeables. The standard textbook model of the small open economy (SOE), assumes constant returns to scale. Then the wage rate and other prices of non-traded factors are determined by the exogenous world prices and exogenous productivity parameters in the most profitable traded goods industries. When these variables are fixed, the scope for wage rate adjustments is limited to incidence of changes in the taxes on labour paid by the producers, such as the payroll tax. However, if technologies exhibit decreasing returns the scope for wage rate adjustments is greater. Decreasing returns at the industry level is consistent with empirical findings in several economies. This is particularly true for economies that have specialized their composition of the T-sector to comparative advantages based on natural resources. Specifically, the main T-sectors in Norway have been based on natural resources, such as crude oil, natural gas, hydro power, fish stocks, forests and certain minerals. Expanding the use of natural resources typically runs into decreasing returns. Section 3.1 provides further justification of assuming decreasing returns to scale in Norwegian industries.

\textsuperscript{1}See e.g. Corden and Neary (1982), van Wijnbergen (1984) and Krugman (1987).
The literature on the basic determinants of the tax burden has grown rapidly as a result of the strong concern for the problems of fiscal sustainability (FS) caused by ageing facing most of the industrial countries. However, the fiscal effect of wage rate adjustments has rarely been discussed in this literature. Exceptions are the projections of the fiscal position in Denmark and Norway discussed in, respectvively, Andersen, Jensen and Pedersen (2004) and Heide, Holmøy, Solli and Strøm (2006) (HHSS). Both papers find that policy makers cannot rely on productivity growth in the private sector as a source to finance the increase in government expenditure, since the resulting wage growth raises expenditures more than the tax bases. HHSS also emphasize how the wage effect modifies the scope for tax rate reductions when government petroleum revenues increases. However, neither of the papers offer the precise formal analysis provided in the present paper.

The situation where the government receives less of a traded good is given a normative analysis in Steigum and Thøgersen (2003) (ST). ST consider the optimal dynamic fiscal policy response to an unanticipated drop in the price of a natural resource, when the government itself loses revenue from this adverse shock. As pointed out by ST, relatively few studies take into account that the government itself has stakes in the sectors hit by external shocks. ST include non-traded goods in their model, which in principle allows for a wage effect on the government budget through endogenous real exchange rate adjustments. However, the specifications of government revenues and expenditures in the ST-model are too simple to capture fiscal effects of wage adjustments. Buiter (1988) includes an analysis of how a lump-sum tax must adjust to exogenous changes in the world interest rate and internal or external government debt in order to balance the government budget. Aggregating all taxes and transfers into a lump-sum tax removes a priori the wage effect on these budget components, but Buiter’s results are influenced by an endogenous wage effect on government spending on non-traded goods. However, this wage effect is not pointed out explicitly. Macklem (1993) uses a dynamic CGE model of the Canadian economy to analyse macroeconomic affects and tax rate adjustments to an exogenous increase in government debt. However, his model implies that any changes in the real exchange rate are transitory. Consequently, his model is not designed to capture the wage effect discussed in the present paper. All papers referred to above have used intertemporal models, but the nature of the wage effect adressed in the present paper is most efficiently explained within

---

a static model. The simulation experiments take dynamics into account, but dynamic aspects play no noteworthy role for the empirical importance of the wage effect.

In analyses of the marginal cost of public funds (MCF) a key question is the increase in a specific tax rate necessary to bring about a given increase in government revenues. The MCF literature has focused on endogenous responses in the tax bases to such a policy shift, in particular the labour supply responses. However, this literature has to my knowledge not paid attention to the wage effect analysed in the present paper, despite it’s obvious relevance for the magnitude of the necessary increase in a distortionary tax rate.

The rest of the paper is organized as follows. Section 2 sets up a theoretical model, which is stripped down to the minimum ingredients necessary to explain the fiscal effect of endogenous wage adjustment. Comparative statics results show how the wage rate diminishes or reinforces the revenue neutral tax rate reduction depending on the nature of the increase in non-tax government revenue. Section 3 shows that the wage effect is empirically significant by running relevant simulation experiments on a realistic general equilibrium model of a particular economy. Specifically, I employ a large scale CGE model of the Norwegian economy to estimate the revenue neutral reduction of the payroll tax rate when the government non-tax revenue increases by an exogenous amount. I consider an increase in the world prices of oil and gas as an example of an increase in the form of a traded good, and reduced government employment as an example of an increase in form of a non-traded good. Among the conclusions drawn in Section 4, I emphasize the relevance of relatively detailed CGE models for assessing the fiscal trade-offs between non-tax revenues and revenue neutral tax rate adjustments, since the fiscal effects of general equilibrium repercussions are likely to be highly country specific.

2 Analytical results

2.1 A stylized model

We consider a model of a small open economy producing one Traded (T) and one Non-traded (N) good. The model is static, so there are no savings. Households are represented by a consumer who consumes the N-good and the T-good. The world price is exogenous and normalised to unity. So is the fixed nominal exchange rate, as it has no real effects. The government produces a given level of services by using labour. It finances the wage costs by collecting taxing on labour income, profits and consumption. In addition, the

\[3\text{It is straightforward to let the good produced by the T-sector differ from the traded good purchased by the consumer.} \]
government owns an exogenous endowment of a tradable, possibly different from the other goods.

A key property of the model is decreasing returns to scale, giving rise to a continuous negative relationship between the supply of the T-good and the wage rate, which is equivalent to the real exchange rate in this model. In order to show how endogenous wage rate adjustments affects the tax rate required to balance the government budget, it suffices to have decreasing returns in the T-sector, whereas the N-sector produces at constant returns. Also, assuming exogenous labour supply makes the model much simpler without losing the main points.

The model consists of the following 9 equations:

\[ X_T = L_T^s \]  
\[ C_N = L_N \]  
\[ L_T = \left( \frac{s}{W} \right) \frac{1}{1-s} \]  
\[ \pi_T = (1 - s) X_T \]  
\[ P_N = W \]  
\[ (1 + t_C) (C_I + P_N C_N) = (1 - t_L) WL + (1 - t_{\pi}) \pi_T \]  
\[ t_L = \frac{WL_G - t_C (C_I + P_N C_N) - t_{\pi} \pi_T - O}{WL} \]  
\[ \frac{C_I}{C_N} = P_N^\sigma \]  
\[ L_T + L_N + L_G = L \]

\( X_T \) is output of the traded good, which equals exports of the T-good. \( C_N \) denotes output and consumption of the non-traded good. \( L \) is total employment, \( L_G, L_T \) and \( L_N \) is employment in, respectively, the government sector, the T- and the N-sector. \( P_T \) is the world price facing the exporting T-sector, and \( P_N \) is the price of the N-good. \( 0 < s < 1 \) is the elasticity of scale in the T-sector. \( W \) is the wage rate paid by producers, i.e. the pre-tax consumer wage rate. \( \pi_T \) is profits in the T-sector. \( C_I \) is consumption of the traded good, which equals imports. \( \sigma \) is the elasticity of substitution between imports and N-goods in the CES utility function of the representative consumer. \( O \) is the value of the exogenous amount of another tradable owned by the government, denominated in terms of the imported consumer good. \( t_L, t_C \) and \( t_{\pi} \) are, respectively, the tax rates on labour income, consumption and profits.
(1) is the decreasing returns to scale production function of the T-sector. (2) is a constant returns to scale production function of the N-sector, assuming equilibrium in the market for the N-good. Productivity parameters are normalised to unity. (3) is the labour demand function in the T-sector derived from profit maximization of price taking firms. (4) is the maximum profit in the T-sector. (5) follows from perfect competition in the N-sector. (6) is the budget constraint of the representative consumer, and (7) is the government budget constraint. We shall let $t_L$ be the fiscal instrument that endogenously adjusts to meet this constraint. (8) is the first order condition for utility maximization, reflecting a uniform consumption tax rate levied both on imports and the N-good, and that the share parameters in the CES utility function are normalised to unity. The (fixed) level of services produced by the government sector has no effect on the demand functions for imports and the N-good. (9) is the labour market equilibrium condition.

$O$, $L$, $L_G$, $t_\pi$, $t_C$ and $P_T = 1$ are exogenous. The model determines the 9 endogenous variables $X_T$, $L_T$, $\pi_T$, $L_N$, $P_N$, $W$, $C_N$, $C_I$ and $t_L$. We shall examine how changes in $O$ and $L_G$ affect $t_L$, with particular focus on the influence from equilibrium adjustments in $W$. As will be shown below, $W$ is independent of the tax rates. It is therefore convenient to proceed in two steps. First, we establish the equilibrium relationships $\frac{\partial W}{\partial O}$ and $\frac{\partial W}{\partial L_G}$. Thereafter we examine $\frac{\partial t_L}{\partial O}$ and $\frac{\partial t_L}{\partial L_G}$.

Although the model is very simple, flexible interpretation of the variables extends the relevance of the conclusions. First, $O$ may be interpreted as the value of property rights of natural resources such as oil and/or predetermined financial claims on the rest of the world. $O$ may be negative, e.g. if the government sector uses imports to produce services or is indebted to foreigners. Second, it is easy to include wage indexed transfers, e.g. public pension benefits, from the government to the consumer. To see this, let $N$ be the exogenous number of transfer recipients and let $b$ the exogenous individual benefit ex ante wage indexation. $WbN$ should be added to consumer income and to government expenditure. Government expenditures then equals $W (L_G + bN)$, which plays exactly the same role as $WL_G$ in the model above.

The equilibrium wage rate is implicitly found by substitutions into the labour market equilibrium condition, using (3), $L_N = C_N = W^{-\sigma} C_I$, that the two budget constraints implies the the external balance constraint $C_I = X_T + O$, and $X_T = (\frac{s}{W})^{\frac{1}{1-s}}$:

$$
\left(\frac{s}{W}\right)^{\frac{1}{1-s}} + W^{-\sigma} \left[\left(\frac{s}{W}\right)^{\frac{1}{1-s}} + O\right] = L - L_G,
$$

which is independent of the tax rates since labour supply is exogenous. Define the initial share variables $\lambda_L = \frac{L}{L - L_G}$, $\lambda_i = \frac{L_i}{L - L_G}$, $i = T, N, G$, $\delta_O = \frac{O}{C_I}$ and $\delta_T = \frac{X_T}{C_I}$.
Logarithmic differentiation of (10) yields the equilibrium adjustment in the wage rate, see Appendix 1:

$$\dot{W} = \frac{\lambda_G \dot{L}_G + \lambda_N \delta_O \dot{O}}{\lambda_N \sigma + \frac{\lambda_T + \lambda_N \delta_T s}{1-s}},$$

(11)

where $\dot{W} \equiv \frac{dW}{dt}$ etc. The denominator, henceforth denoted $l_W > 0$, is the absolute value of the aggregate labour demand elasticity with respect to $W$. $W$ is proportional to simultaneous equal relative changes in $P_T$ and $O$ (reflecting that the model does not determine the nominal price level). $\dot{W}_O = \frac{\lambda_N \sigma}{l_W} > 0$, since a higher $O$ is equivalent to improved terms of trade, which allows an expansion of consumption and a contraction of the T-sector. $\dot{W}_{LG} = \frac{\lambda_G}{l_W} > 0$. The intuition is that the supply of labour to the private sector increases. Higher private sector employment raises privat consumption of both the N-good and imports. Balanced trade is restored by a fall in $W$, expanding exports and reducing the import share. When $s \to 1$ or $\sigma \to \infty$, then $l_W \to \infty$, and $\dot{W}_O$ and $\dot{W}_{LG} \to 0$, since the model then converges to the textbook model of the small open economy in which there are constant returns and imports are perfect substitutes for home goods. Given $W$, the solution for the other endogenous variables are recursively found.

2.2 The government receives more tradeables

The revenue neutral adjustment of $t_L$ to a marginal change in $O$, which takes into account the dependency between $W$ and $O$, follows from derivation of (7) (see Appendix 1):

$$\frac{\partial t_L}{\partial O} = -1 + \frac{\dot{W}_O}{WL} \left[ L_G - t_L L - t_C \left( (1 + \sigma W^{-\sigma-1}) C_N + \left( \frac{W^{-\sigma-1} + 1}{1-s} \right) L_T + t_\pi L_T \right) \right].$$

(12)

The bracket contains the government budget effects of a change in the wage rate: $L_G$ captures the change in the government wage bill, $t_L L$ captures the change in the base of labour income taxation, $t_\pi L_T$ captures the change in the profit tax base, and the terms multiplied by $t_C$ are different sources to changes in the consumption tax base. Here $t_C C_N$ captures the consumption tax base effect due to the isolated change in the price of the N-good, $t_C \sigma W^{-\sigma-1} C_N = \frac{t_C \sigma C_I}{W}$ captures the change in the taxes collected from consumption of imports attributable to the change in the import share, and $t_C \left( W^{-\sigma-1} + 1 \right) \left( \frac{1}{1-s} \right) L_T = - t_C (W^\sigma + W) \frac{\partial L_T}{\partial W} = t_C (W^\sigma + W) \frac{\partial C_N}{\partial W}$ captures the change in the consumption tax base caused by a proportional change in $C_N$ and $C_I$. An alternative expression follows by inserting the government budget constraint into (12) (see Appendix 1):

$$\frac{\partial t_L}{\partial O} = -1 + \frac{\dot{W}_O}{WL} \left[ t_C \left( 1 - \sigma \right) C_I + \left( t_\pi - \frac{t_C \sigma}{\theta_N} \right) s X_T + O \right].$$

(13)
where \( \theta_N \equiv \frac{P_N C_N}{P_N C_N + P_C I} \) is the budget share of the home good in private consumption, and
\[
\hat{W}_O \equiv \frac{\partial W}{\partial O} \bigg|_O = \frac{\lambda_N \delta_O}{\lambda_N} > 0.
\]

In order to build up an understanding of the model in general, and of (13) in particular, it is instructive to examine some simple special cases.

### 2.2.1 Special cases

**\( O = t_C = t_\pi = 0 \) initially.** Now \( \frac{\partial t_L}{\partial O} \) degenerates to
\[
\frac{\partial t_L}{\partial O} \bigg|_{t_C = t_\pi = 0} = -\frac{1}{WL}.
\]

Note that since the general equilibrium effects captured by \( \frac{\partial W}{\partial O} \) are irrelevant in this case, the correct estimate of \( \frac{\partial t_L}{\partial O} \) coincides with a naive estimate ignoring all equilibrium effects. In this case the initial government budget constraint simplifies to \( t_L = \frac{W L G}{W L} = \frac{L G}{L} \). Since both government revenues and expenditures are proportional to \( W \) the wage adjustment induced by a marginal change in \( O \) from 0 has no effect on the necessary change in \( t_L \).

**\( O > 0, t_C = t_\pi = 0 \) initially.** Now a marginal increase in \( O \) implies that \( t_L \) must be changed by
\[
\frac{\partial t_L}{\partial O} \bigg|_{t_C = t_\pi = 0} = -1 + \hat{W}_O \frac{W L}{W L} > -\frac{1}{WL}
\]

since \( \hat{W}_O > 0 \). To get the intuition behind this modification of the scope for tax rate reduction, recall that the government budget constraint in this special case is \( O = W (L_G - t_L L) > 0 \), i.e. the wage dependent expenditures exceed the wage dependent revenues. Consequently, the real appreciation through the rise in \( W \) magnifies the non-oil budget government deficit.

This intuition suggests that \( \frac{\partial t_L}{\partial O} \bigg|_{t_C = t_\pi = 0} \) is monotonically increasing from \(-\frac{1}{WL} \) when \( O \) increases from 0. Inspection of \( \hat{W}_O = \frac{\lambda_N \delta_O}{\lambda_N} = \frac{\lambda_N \delta_O}{\sigma (1-s) + \frac{1-s}{\lambda_N}} \) confirms this. Intuitively, the higher is \( O \) the smaller is the T-sector necessary to keep trade balanced. Thus, a higher \( O \) implies a higher \( \delta_O \) and a smaller \( \delta_T \). In addition \( \frac{l_N}{\lambda N} \) would be smaller. Both effects make \( \hat{W}_O \) an increasing function in \( O \), cet. par.

Is it possible that \( \frac{\partial t_L}{\partial O} \bigg|_{t_C = t_\pi = 0} > 0 \)? This requires \( \hat{W}_O > 1 \Leftrightarrow \delta_O > \frac{l_N}{\lambda N} \). Recall that \( \hat{W}_O \to 0 \) when \( \sigma \to \infty \) and/or \( s \to 1 \), because \( l_N \to \infty \). Thus, for \( \hat{W}_O \) to be non-negligible, imports must be rather different from the N-good, and the marginal labour productivity in the T-sector must be significantly reduced when \( L_T \) increases. A precise characterization
is possible in the extreme case when the oil revenue is large enough to pay for all imports, i.e. when \( \delta_O = 1, \delta_T = 0 \) and \( \lambda_N = 1 \). Then \( \frac{L_T}{N} = \sigma + \frac{LT + \delta_T s}{1-s} \) degenerates to \( \sigma \). When there is no T-sector, the wage rate affects net exports through import substitution. Thus, in the case where \( t_C = t_\pi = 0 \), if \( \sigma < 1 \) and \( \delta_O = \lambda_N = 1 \), then \( \hat{W}_O > 1 \), and additional government oil revenues would require an increase in the labour income tax rate in order to restore the government budget constraint!

Reducing the initial \( O \) from the extreme situation when \( \delta_O = \lambda_N = 1 \) would increase the initial values of \( \delta_T \) and \( L_T \) from zero. Cet. par. this reduces the difference \( \delta_O - \frac{L_T}{N} = \delta_O - \left( \sigma + \frac{LT + \delta_T s}{1-s} \right) \). To keep the difference - and thereby \( \hat{W}_O \) - constant, would require a reduction in \( \sigma \) and/or an increase in \( s \) towards unity.

The general insights from the discussion above can be summarized in the following points:

1. For a given size of the T-sector, the more price elastic is import demand and T-good supply, i.e. the higher are \( \sigma \) and \( s \).

2. For given \( \sigma \) and \( s \), the scope for reducing the labour income tax as a revenue neutral response to higher government oil revenues, is greater the higher is the share of imports financed by exports of T-goods rather than oil exports.

3. It is the adverse budget effect of a higher wage rate that becomes more significant the smaller are \( \sigma \), \( s \) and the initial size of the T-sector (relative to import demand).

**Accounting for other taxes.** When \( t_C = 0 \) but \( t_\pi \neq 0 \), the government budget constraint takes the form \( W (L_G - t_L L) = t_\pi \pi_T + O > 0 \). Thus, if the revenue from profit taxation has been used to finance a higher wage dependent expenditures (here represented by \( L_G \)) or a lower \( t_L \) initially, for given \( L \) and \( O \), the scope for tax rate reduction is further reduced when \( O \) increases. Alternatively, if profit taxes is used to finance a reduction in \( O \), for example by lower taxes on the resource rent, \( \frac{\partial t_L}{\partial O} \bigg|_{t_C=0} = \frac{\partial t_L}{\partial O} \bigg|_{t_C=t_\pi=0} \), since the fiscal effect of the wage rate adjustment is the same.

Like the other taxes, \( t_C \) does not affect \( \hat{W}_O \). But the counteracting effect of higher wage rate on the scope for reducing \( t_L \) is diminishing in the \( t_C \). Note that the influence of \( W \) on the consumption tax base must be read from (12), not from (13), which is obtained after using the government budget constraint to eliminate wage dependent budget components. The change in the consumption tax base equals \( \frac{\partial (C_I + P_N C_N)}{\partial W} = C_N + \sigma W^{\sigma-1} C_N + W^{\sigma} \frac{\partial C_N}{\partial W} + W \frac{\partial C_N}{\partial W} \). Recall that the terms here, successively, capture the price effect on the consumption tax base, the increase in taxed consumption of imports for a given \( C_N \), the increase in taxed
consumption of imports for a given import share, and the increase in taxed consumption of the N-good for a given N-good price. All these effects are positive. (Specifically, \( \frac{\partial C_N}{\partial W} = \frac{L_T}{(1-s)W} > 0 \).) Consequently, when the increase in \( O \) induces real appreciation through an increase in \( W \), the government collects more revenue from the consumption tax. This effect contributes to magnify the scope for reducing the labour income tax rate, and is is stronger the higher is \( t_C \). The magnification also increases with \( \sigma \) and \( s \). The higher is \( \sigma \), the greater is the increase in taxed imports for a increase in the wage rate. The closer \( s \) is to unity, the stronger is the export contraction of a given increase in \( W \), and the assumption of full employment implies that the lost exports is replaced by taxed consumption of N-goods. For sufficiently large values of \( t_C \), \( \sigma \) and/or \( s \) close to unity, the bracketed factor multiplied with \( \hat{W}_O \) in (12) or (13) may even be negative, in which case the positive general equilibrium effect on \( W \) exacerbates the scope for reducing \( t_L \).

2.3 The government receives more non-tradeables

We consider the case where the government gets more of some resources that can replace labour in the production of public services, i.e. \( L_G \) can be reduced without reducing the supply of (the unspecified) public services. This shift may result from for example increased productivity in the government sector or improved health in the population, reducing the need for resources on public health services. \( L_G \) may also be interpreted as including the recipients of wage indexed transfers, which may decrease due to changes in norms etc. Anyhow, the supply of labour to the private sector increases, at the same time as government expenditure falls.

Appendix 1 shows that the revenue neutral adjustment of \( t_L \) to a marginal change in \( L_G \), taking into account the dependency between \( W \) and \( L_G \), can be written

\[
\frac{\partial t_L}{\partial L_G} = \frac{W + \hat{W}_{LG}}{WL} \left[ t_C (1 - \sigma) C_I + \left( t_L - \frac{L_T}{NL} \frac{s}{1-s} \right) X_T + O \right],
\]

(16)

where \( \hat{W}_{LG} = \frac{\dot{L}_G}{\dot{W}} > 0 \). The bracketed term is identical to the one (13). As above, the naive estimate, ignoring all general equilibrium effects, will be correct if \( t_C = t_L = 0 \Leftrightarrow L_G = t_L L \) initially. When the bracketed term is positive, the wage rate adjustment now reinforces the direct budget effect of changing \( L_G \). This is contrary to the case in which the government gets more of the traded good, i.e. an increase in \( O \). The similarity between (16) and (13) makes further analysis of \( \frac{\partial t_L}{\partial L_G} \) an unnecessary repetition of the results above.
3 Empirical importance

3.1 A CGE model of the Norwegian economy

Realistic estimates of the empirical significance of the influence of wage rate adjustments on the relationship between the tax rate and government endowments require simulations on a CGE model of a specific economy. My conjecture is that the relevance of an empirical model may be even greater for the question analysed in this paper than for many other problems in the applied literature on taxation. Two points support this view: 1) The analysis above has verified that the initial composition of the government revenues and expenditures is crucial for the magnitude of the fiscal effect of the wage rate adjustment. This composition is highly country specific. Norway is probably an extreme case due to the large government petroleum revenues. 2) The magnitude of the wage rate adjustment is also likely to differ substantially between economies, due to e.g. different institutional settings for the wage formation. The same is likely to be true for relevant industry characteristics, for example the scale elasticities of the production functions emphasized in the previous section.

This section presents results from simulations on a large dynamic CGE model, MSG6, of the Norwegian economy. The structure and empirical properties are described and explained in Heide, Holmøy, Lerskau and Solli (2004). Here we confine the description to the characteristics most important for our analysis.

MSG6 shares some basic characteristics with the stylized model above, which makes the analysis in the previous section relevant for understanding the simulation results presented below: The nominal exchange rate and all world prices of exports and imports are exogenous; labour is internationally immobile; all goods, services and production factors are perfectly mobile between industries within the economy; supply equals demand in all markets in all periods; a representative consumer maximizes utility, and producers maximize the value of firms; agents consider imports of manufactures and services as close but imperfect substitutes for the corresponding deliveries from Norwegian firms, which implies that import shares are decreasing functions of the import price relative to the price index of corresponding domestic deliveries; the balanced trade constraint is met by the consumption possibilities are constrained by endogenous wage adjustment ensuring profitability in a sufficiently large production of traded goods. The model is closed by letting endogenous payroll tax rate adjustment balance the government budget.

In order to serve as a realistic description of how the Norwegian economy works in the long run, MSG6 is much more complex than the stylized model in several respects:
1. MSG6 provides a rather disaggregated description of the Norwegian economy, specifying 60 commodity groups.

2. MSG6 is dynamic; the consumer maximizes an intertemporal utility function, and producers maximize the present value of the net-of-tax cash flow to the owners. Agents have access to the international credit market, and the world interest rate is exogenous to the Norwegian economy. Consumers and producers have perfect foresight. The balanced trade constraint takes the form of a non-Ponzi game condition on net foreign debt.

3. MSG6 takes into account that firms employ goods as intermediaries and capital goods in addition to labour.

4. Individual labour supply is endogenous, with the uncompensated wage elasticity equal to 0.1.

5. Producers of manufactures and tradable services allocate their output between the domestic and the foreign market. It is costly to change the output composition between these markets. Whereas domestic firms are price takers in all factor markets and the export markets, they participate in Large Group Monopolistic Competition with free entry/exit in the domestic markets.

6. Whereas only the production function in the T-sector exhibits decreasing returns to scale in the stylized model, all private industry production functions have this property in MSG6. The scale elasticities range from 0.85 - 1.00.

Since the scale elasticity is a key determinant of the scope for wage rate adjustments, it is important to support it with empirical evidence. Klette (1999) estimates scale elasticities close to 0.9 for Norwegian manufacturing firms. On the other hand, Biorn, Lindquist and Skjerpen (2002) find constant or moderately increasing returns to scale for the average plant in Norwegian manufacturing. However, increasing returns at the plant level may be compatible with decreasing returns at the aggregate industry level when there are capacity constraints for each plant and significant cost heterogeneity among the plants within the same industry. This logic was explored in the pioneering work on the correspondence between micro and macro production functions by Houthakker (1955-56), Johansen (1959, 1972) and Salter (1960). Bye, Holmøy and Heide (2006) shows that decreasing returns to scale at the industry level for the the most export oriented Norwegian manufacturing industries results from productivity heterogeneity, combined with profit based exit of firms. Large and persistent productivity differences even within narrowly defined industries have been well documented, see e.g. Sutton (1997), Baily, Hulten and Campbell (1992) and
Klette (1999). MSG6 takes into account that output and input in an industry can change both because of changes at the firm level and as a result of entry or exit of firms, which differ with respect to productivity within the same industry.

Decreasing returns to scale also represents an answer to the challenge to applied model builders advanced by Kehoe (2003). Kehoe evaluates the performances of three prominent multisectoral CGE models in estimating the impact of NAFTA on the economies of Canada, Mexico and the US. He concludes that these models drastically underestimated the impact on North American trade, and that they failed to capture most of the relative impacts on different sectors. Partly, he attributes the poor performance to the modelling approach based on the Dixit-Stiglitz specification of preferences over home goods and imports. Instead, he suggests that a Ricardian model specification, emphasizing productivity heterogeneity within the specified industries, will do a better job.

Three growth scenarios are compared:

1. A baseline scenario assuming prolongation of the present fiscal and welfare policy, including present tax rates in real terms, the present public pension system and other welfare schemes, and wage indexation of most cash transfers from the government to households. See Heide, Holmøy, Solli and Strøm (2006) for more details.

2. A high petroleum price scenario resting on the baseline assumptions except that the real world prices of crude oil and natural gas are twice as high as in the baseline, i.e. the real oil price increases from 25 to 50 $/barrel, and the gas price follows the oil price. This is a highly relevant illustration of the case where the government gets more tradeables, since the Norwegian government collects most of the petroleum revenues through taxation and direct state ownership. Petroleum revenues amounted to 34.5 percent of total government revenues in 2005.\(^4\) We disregard any repercussions on other world markets and increases in the oil and gas production.

3. A reduction of government employment caused by higher productivity growth without changing the output of services from the baseline. This illustrates the case where the government gets more non-tradeables.

In all scenarios the fiscal policy rule adopted in 2001 is followed. Strictly, this rule implies that the annual government petroleum revenue is transferred to the Government Pension Fund - Global (GPFG), and that only an estimated real return of 4 percent of the fund can be used to finance the non-petroleum budget deficit. In order to meet the

\(^4\)According to the Revised National Budget 2006, St.meld. nr. 2, Norwegian Ministry of Finance.
meet the resulting time path of the government budget surplus, the payroll tax rate adjusts annually. The payroll tax rate, rather than the tax rate on personal labour income is chosen as the endogenous fiscal instrument, because it is a very broad tax and flat tax on labour income, whereas the personal income tax system is much more complex. However, since the payroll tax is levied on producers rather than workers, it has some effects not captured in the formal analysis above. These effects are identified formally in Appendix 2. They are not qualitatively important in the case of higher petroleum prices. Their importance for the results of reducing government employment will be commented below.

3.2 Higher petroleum prices

Accounting for unit costs of petroleum production, doubling the petroleum prices implies a 140 percent increase in the government petroleum wealth, computed as the present value of the government net cash flow from the petroleum sector over the period 2004-2050. This increase corresponds to 72 percent of the present value over the same period of government old-age pension expenditures. Whereas the payroll tax rate increases from the present 13 to 31 percent along the baseline scenario, doubling of petroleum prices allows for a reduction in this tax rate by 13-14 percentage points, so that it passes 17 percent in 2050.

The empirical significance of the influence of general equilibrium effects on the payroll tax rate reduction is best demonstrated by comparing it with a naive direct estimate. In 2050 the GPFG capital would be 160 percent higher with a real oil price of 50 $/barrel than in the baseline scenario. The fiscal policy rule allows the non-petroleum budget deficit to equal 4 percent of the GPFG capital. The effective base of the payroll tax is the payroll in the private sector. One obtains a naive estimate of the reduction of the payroll tax rate by dividing the 4 percent of the increase in the GPFG capital by the payroll in the baseline scenario. In 2050 such a naive estimate would imply a 26 percentage points reduction from the baseline level of 31 percent. In contrast the corresponding model based estimate is a reduction of 14 percentage points, roughly half of the naive estimate. Thus, general equilibrium effects do not generate only slight modifications, they change the order of magnitude of the fiscal effects.

The by far most important general equilibrium effect is the effect of wage rate adjustments discussed in Section 2. The terms-of-trade gain allows a real appreciation corresponding to a 16 percent increase in the wage cost per hour in 2050 relative to the non-petroleum world prices. The corresponding increase in the wage rate equals 28.9 percent as most of the cut in the payroll tax rate is passed over to labour. The increased wage rate is the main reason why the revenue from direct taxes increases by 4.1 percent in 2050 despite the reduction of employment and the payroll tax rate. The increase in the
labour cost also raises the tax bases of indirect taxes through mark-up pricing of domestic deliveries. About 14 points of the 24.9 percent increase in the indirect tax revenue can be attributed to higher prices.

Table 1. Long run macroeconomic effects in Norway of 1) a permanent doubling of the real petroleum prices and 2) reduced real spending on government Health and Social Care (Labour productivity). Deviations in 2050 from the baseline scenario. Percent unless otherwise stated.

<table>
<thead>
<tr>
<th></th>
<th>Petroleum prices</th>
<th>Labour productivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pay roll tax, percentage points</td>
<td>-14.0</td>
<td>-4.5</td>
</tr>
<tr>
<td>Total revenues, of which</td>
<td>28.6</td>
<td>-3.0</td>
</tr>
<tr>
<td>Indirect taxes</td>
<td>24.9</td>
<td>1.9</td>
</tr>
<tr>
<td>Direct taxes, excl. petroleum revenues</td>
<td>4.1</td>
<td>-5.2</td>
</tr>
<tr>
<td>Total expenditures, of which</td>
<td>21.0</td>
<td>-3.2</td>
</tr>
<tr>
<td>Government consumption</td>
<td>27.9</td>
<td>3.4</td>
</tr>
<tr>
<td>Cash transfers to households</td>
<td>15.4</td>
<td>-10.4</td>
</tr>
<tr>
<td>Private consumption</td>
<td>10.2</td>
<td>2.7</td>
</tr>
<tr>
<td>Employment</td>
<td>-1.3</td>
<td>-0.3</td>
</tr>
<tr>
<td>Government employment</td>
<td>0</td>
<td>-8.4</td>
</tr>
<tr>
<td>Wage cost per hour</td>
<td>16.0</td>
<td>-2.0</td>
</tr>
<tr>
<td>Wage rate</td>
<td>28.9</td>
<td>2.6</td>
</tr>
<tr>
<td>Consumer real wage rate</td>
<td>13.6</td>
<td>3.1</td>
</tr>
</tbody>
</table>

However, due to the current spending of the petroleum wealth allowed by the fiscal policy rule, government expenditures increase more than government revenues. Wage indexed government pension expenditures account for more than 60 percent of the increase in expenditures. Moreover, the government wage bill increases, given the assumption that competition in the labour market spreads the wage growth throughout the economy, including the government sectors. The main reason why the percentage increase in the government consumption is only slightly more than half the increase in transfers, is the reduction in the payroll tax rate. However, this obviously reflects an equal drop in the payroll tax revenue from government employment.

The results demonstrate that wage indexation, as well as the spread of wage growth from the main producers of traded goods to other sectors, represent two important channels through which increased higher government wealth automatically is rebated to households. This automatic rebatement significantly reduces the room for tax rate reductions.
The terms-of-trade gain allows for an expansion of private consumption, which significantly reinforces the tax rate reduction as the indirect consumption tax rate is close to 20 percent in Norway. In 2050 private consumption in fixed prices would be 10.2 percent above the baseline scenario. The formal analysis in Section 2 ruled out by assumption fiscal effects of endogenous labour supply responses. The simulation shows that employment falls, which reduces most tax bases. However, the employment reduction is rather small, 1.3 percent in 2050. The result reflects that households experience an increase in non-labour income, which weakly dominates the positive labour supply effect of the large increase in the consumer real wage rate.

3.3 Reduced government employment

Accelerating labour productivity in the government sector Health and social care from 0.5 to 1.0 percent per year from 2006 to 2050 accumulates to an 8.4 percent reduction of total government employment in 2050. Ignoring any other adjustments, this will also equal the percentage decrease in the government labour cost. Even a naive estimate would take into account that the labour cost include the payroll tax rate, which should be netted out as a payment from the government to itself. In 2050 the baseline average payroll tax rate has increased from 13 percent in 2005 to 31 percent. In addition, we subtract the 35 percent average tax on personal labour income in order to obtain the direct effective budget effect of reduced government employment. This implies a direct budget effect in 2050 which is about 14 percent of the direct budget effect of doubling the prices of oil and gas. Consequently, since the payroll tax is a flat tax, a naive estimate on the scope for reducing the payroll tax rate is $0.14 \times 26 = 3.6$ percentage points.

The corresponding model based estimate is a 4.5 percent reduction. Thus, the simulation result is consistent with the analytical result derived in the previous section: The general equilibrium fiscal effect of the wage rate adjustment will magnify the scope for tax rate reductions when the government gets more revenue in terms of a non-traded good. However, this conclusion needs qualifications since other equilibrium effects also influence the payroll tax rate reduction, especially as the difference between the model based and the naive estimates is only 0.9 percentage points.

First, one may question that the wage effect magnifies the tax rate reduction since the wage rate increases (by 2.6 percent) in the simulation, whereas it falls in the theoretical analysis. Recall, however, that the theoretical analysis considered adjustments of the tax rate levied on personal labour income - not the payroll tax rate. The simulated reduction of the unit labour cost (2 percent) is consistent with the theoretical analysis, and the payroll tax rate reduction makes it possible to combine lower labour cost with a higher
consumer wage rate. Appendix 2 shows that the sign of the wage effect on the payroll tax rate reduction is determined by the sign of the change in the labour cost. It also shows that the magnitude - but not the sign - of the necessary adjustment of the payroll tax rate will be somewhat magnified compared to the corresponding adjustment of the labour income tax rate.

In addition to the price effects of the wage rate adjustment, two other general equilibrium effects are worth mentioning: 1) the use of other inputs in the government sector Health and Social Care is reduced by the same proportion as employment, but the spending effect is only 5 percent of the reduced labour costs. 2) The increase in private consumption raises the indirect tax revenue, as well as government income from market sales of services. This revenue effect is modified by the decrease in consumer prices entailed by the fall in unit labour costs. These effects dominate the negative tax base effect caused by the slight reduction of total employment. The small labour supply reduction reflects that the effect of the increase in real non-labour income weakly dominates the substitution effect of the 3.1 percent increase in the consumer real wage rate. Note that the reason why the decrease in government consumption exceeds the 8.4 percent drop in government employment is basically due to the decrease in the unit labour cost driven by the payroll tax rate reduction. As pointed out above, this is a gross effect, which also explains most of the reduction of 5.2 percent in the revenues from direct taxes.

4 Conclusions

This paper has tried to answer the question: What is the scope for tax rate reductions when the government experiences an exogenous increase in non-tax revenues or cut in expenditures? To this end it has clarified the trade-off between a tax rate levied on labour income and non-tax net revenues implied by the government budget constraint, when one takes into account general equilibrium effects of a given change in non-tax revenues. The analysis has paid particular attention to the fiscal effects of the general equilibrium adjustment of the wage rate, which represents the real exchange rate. The main effects were highlighted in a stylized general equilibrium model of a small open economy, which deviates from the textbook model by allowing decreasing returns to scale in the sector producing tradeables. Simulations on a large scale empirical CGE model of the Norwegian economy confirm the qualitative theoretical results, and estimates the empirical fiscal significance of the wage rate adjustment and other general equilibrium effects. The analysis justifies the following conclusions:

1. The scope for a revenue neutral tax rate reduction as a response to an increase in
non-tax revenues may be significantly affected by the endogenous wage rate adjustment. This is strikingly demonstrated by the CGE estimate on the payroll tax rate reduction in Norway of doubling the world prices of oil and gas. Whereas a naive estimate, ignoring all equilibrium effects, is a 26 percentage points reduction in 2050, the CGE estimate is 14 points.

2. A general insight is that wage rate adjustments represent an automatic channel for redistributing changes in non-tax government net revenues. The potential importance of this redistribution mechanism is especially high in economies similar to the so-called "Scandinavian Model", characterised by relatively strong egalitarian norms. Specifically, the impact of a change in the wage rate on government expenditures is relatively strong in such economies for two reasons. First, a relatively high degree of centralised wage formation ensures that wage changes are to a large extent and relatively rapidly spread to the government sectors. Second, the relatively generous public pension benefits and most other government transfers to households are indexed to the average wage level. Depending on the wage formation and the composition of the government budget with respect to wage dependent revenue and expenditure components, this automatic redistribution mechanism may diminish or magnify the revenue neutral tax rate reduction.

3. The fiscal effect of the wage rate adjustment depends on the nature of non-tax revenue that changes. If the government gets more revenue in the form of a tradable, the wage rate will increase, which diminishes the revenue neutral tax rate reduction, provided that the initial wage dependent government expenditures exceed wage dependent government revenues. On the other hand, the wage rate adjustment reinforces the revenue neutral tax rate reduction when the government gets more non-tax net revenue in the form a non-tradable, e.g. decrease in the government employment required to produce a given output of services. Such a shift induces a drop in the wage rate, which, _cet. par_, increases the budget surplus when the initial wage dependent government expenditures exceed wage dependent government revenues.

4. Since the strength of the fiscal effect of wage rate adjustments is likely to differ significantly between countries and over time, because it depends on both the magnitude of the change in the wage rate, and the initial wage dependent government net revenues. Norway is a special case, particularly because of large government petroleum revenues. Given the present fiscal policy rule for the intertemporal consumption of the petroleum wealth, the petroleum revenues allow a substantial deficit of wage dependent government net revenues. Moreover, the wage dispersion is relatively low and constant, the government employed as much as 27 percent of total man-hours in
2005, and government welfare transfers are generous and indexed to wages. In coun-
tries where both the government and the economy as a whole must serve substantial
debt, wage dependent government net revenues may be positive. In addition to the
direct positive budget effect of a drop in the interest rate, the increase in the wage
rate compatible with the external balance constraint will in this case magnify the
direct effect. Hence, the present analysis is relevant for other countries as well.

The magnitude of the changes in the wage rate depends crucially on the degree of
decreasing returns to scale in the traded goods sector. In my opinion empirical evidence
suggest that this is a realistic assumption at the aggregate industry level. It also meets
the criticism raised in Kehoe (2003) against alternative applied modelling approaches of
exports and imports.

Since the fiscal effects of general equilibrium repercussions are likely to be highly coun-
try specific, relevant CGE models seem to be the appropriate tool for assessing the fiscal
trade-offs between non-tax revenues and revenue neutral tax rate adjustments. Probably,
model based estimates will also be more respected by the public than more simple es-
timates, despite some loss of transparency. In light of both the possible magnitudes of
the general equilibrium effects, and the unpopular fiscal policy decisions facing ageing
economies, it seems important to provide such a common understanding of the country
specific fiscal trade-offs.

References

towards fiscal sustainability in Denmark. Mimeo.

Baily, N. M., C. Hulten and D. Campbell (1992): Productivity Dynamics in Manufac-

Beetsma, R., L. Bettendorf and P. Broer (2003): The Budgeting and Economic Con-

Biørn, E., Lindquist, K.-G. and T. Skjerpen: Heterogeneity in Returns to Scale: A Ran-
dom Coefficient Analysis with Unbalanced Panel Data. Journal of Productivity Analysis
18, 2002, 39-57

Bovenberg, A. L. and T. Knaap (2005): Ageing, funded pensions and the Dutch econ-
omy. CESIFo Working Paper 1403.


Appendix 1: Calculating the partial derivatives of $t_L$

Recall the reduced form of the labour market equilibrium condition determining $W$, (10):

$\left(\frac{s}{W}\right)^{\frac{a}{1-a}} + W^{-a}\left(\frac{s}{W}\right)^{\frac{a}{1-a}} + O = L - L_G,$

which is independent of the tax rates since labour supply is exogenous. Let $\lambda_L = \frac{L_L}{L - L_G}$ and $\lambda_i = \frac{L_i}{L - L_G}$, $i = T, N, G$, i.e. the shares of labour in the private sector absorbed by the T-, N- and the the government sector, respectively. $\delta_O = \frac{O}{C_T}$ and $\delta_T = \frac{X_T}{C_T}$ are the shares of, respectively, the oil revenue and the value of the traded goods output in the consumer imports. Logarithmic differentiation of (10) (in the initial equilibrium) yields the equilibrium adjustment in the wage rate

$\hat{W} = \frac{\lambda_T + \lambda_N \delta_T}{\lambda_N \sigma} \hat{P_T} - \lambda_L \hat{L} + \lambda_G \hat{L}_G + \lambda_N \delta_O \hat{O} \lambda_N \sigma + \frac{\lambda_T + \lambda_N \delta_T}{1-\delta_T}$

(17)
where \( \hat{W} \equiv \frac{dW}{d\lambda} \) etc. The partial elasticities become

\[
\hat{W}_O = \frac{\lambda_N \delta_O}{\lambda_N \sigma + \frac{\delta_t + \lambda_N \delta_s}{1-s}}, \tag{18}
\]

\[
\hat{W}_{LG} = \frac{\lambda_G}{\lambda_N \sigma + \frac{\delta_t + \lambda_N \delta_s}{1-s}}. \tag{19}
\]

\( t_L \) follows from one of the budget constraints:

\[
t_L = \frac{WL_G - O - t_C (C_I + P_N C_N) - t_\pi \pi_T}{WL} \tag{20}
\]

When calculating the partial derivatives of \( t_L \) we take into account that the change in \( W \) affects the tax bases as follows:

\( C_I = W^\sigma C_N \Rightarrow \frac{\partial C_I}{\partial W} = \sigma W^{\sigma-1} C_N + W^\sigma \frac{\partial C_N}{\partial W} \). It follows from (9) that \( \frac{\partial C_N}{\partial W} = \frac{\partial L_N}{\partial W} = -\frac{\partial L_T}{\partial W} \). (3) implies \( \frac{\partial L_T}{\partial W} = -\left( \frac{1}{1-s} \right) \frac{L_T}{W} \). Consequently, \( \frac{\partial C_N}{\partial W} = \left( \frac{1}{1-s} \right) \frac{L_T}{W} \). Moreover, \( \frac{\partial \pi_T}{\partial W} = -L_T \).

Derivation of (20) with respect to \( O \) yields:

\[
\frac{\partial t_L}{\partial O} = \frac{WL \left[ \frac{\partial W}{\partial \sigma} L_G - 1 - t_\pi \frac{\partial \pi_T}{\partial O} - t_C \left( \frac{\partial (C_I + WC_N)}{\partial W} \right) \right]}{(WL)^2} - t_L WL \frac{\partial W}{\partial \sigma} L
\]

\[
= \frac{-1 + \frac{\partial W}{\partial \sigma} \left[ L_G - t_L L - t_\pi \frac{\partial \pi_T}{\partial O} - t_C \left( \frac{\partial C_I + WC_N}{\partial W} \right) \right]}{WL} \]

\[
= \frac{-1 + \frac{\partial W}{\partial \sigma} \left[ L_G - t_L L + t_\pi L_T - t_C C_N - t_C \left( \sigma W^{\sigma-1} C_N + \left( \frac{W^{\sigma-1} + 1}{1-s} \right) L_T \right) \right]}{WL}
\]

The bracket contains the government budget effects of a change in the wage rate: \( L_G \) captures the change in the government wage bill, \( t_L L \) captures the change in the base of labour income taxation, \( t_\pi L_T \) captures the change in the profit tax base, \( t_C C_N \) captures the consumption tax base effect due to the isolated change in the price of the N-good, \( t_C \sigma W^{\sigma-1} C_N = t_C \frac{\sigma C_I}{W} \) captures the change in the taxes collected from consumption of imports attributable to the change in the import share, and \( t_C \left( W^{\sigma-1} + 1 \right) \left( \frac{1}{1-s} \right) L_T = -\left( \frac{1}{1-s} \right) \frac{L_T}{W} \). \( t_C \left( W^{\sigma-1} + 1 \right) \frac{\partial W}{\partial \sigma} L_T = t_C \left( W^{\sigma-1} + 1 \right) \frac{\partial W}{\partial \sigma} L_T = t_C \left( W^{\sigma-1} + 1 \right) \frac{\partial W}{\partial \sigma} L_T \) captures the change in the consumption tax base caused by a proportional change in \( C_N \) and \( C_I \).

We obtain an alternative expression for \( \frac{\partial \pi_T}{\partial W} \) by inserting \( \hat{W}_O \equiv \frac{\partial W}{\partial \sigma} \frac{O}{W} \), using that the government budget constraint implies that \( W (L_G - t_L L - t_C C_N) = t_C C_I + t_\pi \pi_T + O \), \( \pi_T = X_T - WL_T \), and that \( WL_T = sX_T \) when profit is maximized:

\[
24
\]
\[
\frac{\partial t_L}{\partial O} = -1 + \frac{\partial W}{\partial O} \left[ L_G - t_L L - t_C C_N + t_\pi L_T - t_C \left( \sigma W^{\sigma - 1} C_N + \left( \frac{W^{\sigma-1}+1}{1-s} \right) L_T \right) \right] \frac{W L}{WL} -1 + \frac{\partial W}{\partial O} \left[ t_C C_I + t_\pi \pi_T + O + t_\pi W L T - t_C \left( \sigma W^{\sigma} C_N + \left( \frac{W^{\sigma}+W}{1-s} \right) L_T \right) \right] \frac{W L}{WL} -1 + \frac{\hat{W}_L}{O} \left[ t_C C_I + t_\pi X_T + O - t_C \left[ \sigma C_I + \left( \frac{C_I}{P_N} + 1 \right) \left( \frac{W L_T}{1-s} \right) \right] \right] \frac{W L}{WL} -1 + \frac{\hat{W}_L}{O} \left[ t_C C_I + t_\pi X_T + O - t_C \left[ \sigma C_I + \left( \frac{C_I+P_N C_N}{P_N} \right) \left( \frac{s X_T}{1-s} \right) \right] \right] \frac{W L}{WL} -1 + \frac{\hat{W}_L}{O} \left[ t_C (1 - \sigma) C_I + \left( t_\pi - \frac{t_C}{\theta_N \frac{s}{1-s}} \right) X_T + O \right] \frac{W L}{WL}. \tag{21}
\]

where \( \theta_N \equiv \frac{P_N C_N}{P_N C_N + \tau} \) is the budget share of the home good in private consumption.

After the similar manipulations we obtain the partial derivation of \( t_L \) with respect to \( L_G \), recognizing the dependency between \( W \) and \( L_G \):

\[
\frac{\partial t_L}{\partial L_G} = \frac{W L \left[ \frac{\partial W}{\partial L_G} L_G + W - t_\pi \frac{\partial W}{\partial L_G} \frac{\partial W}{\partial L_G} - t_C \frac{\partial (C_I + W C_N)}{\partial L_G} \frac{\partial W}{\partial L_G} \right] - t_1 W L \frac{\partial W}{\partial L_G} L}{(WL)^2} -1 + W \left[ L_G - t_1 L - t_\pi \frac{\partial W}{\partial W} - t_C \left( \frac{\partial C_I}{\partial W} + \frac{\partial (W C_N)}{\partial W} \right) \right] \frac{\partial W}{\partial L_G} W + \frac{\hat{W}_L}{L_G} \left[ t_C (1 - \sigma) C_I + \left( t_\pi - \frac{t_C}{\theta_N \frac{s}{1-s}} \right) X_T + O \right] \frac{W L}{WL}. \tag{22}
\]

**Appendix 2: Changes in the payroll tax rate**

Let \( a \) be the payroll tax rate, and define the unit labour cost as \( Z = (1 + a) W \). \( Z \) plays the same role as \( W \) in the model in Section 2. Thus, the solutions for all endogenous variables but \( t_L \) are identical to the solutions of that model and independent of the tax rates. The solution for \( Z \) follows from appropriate substitutions into the labour market equilibrium condition:

\[
\left( \frac{s}{Z} \right)^{\frac{1}{1-s}} + Z^{-\sigma} \left[ \left( \frac{s}{Z} \right)^{\frac{s}{1-s}} + O \right] = L - L_G. \tag{23}
\]
Logarithmic differentiation with respect to $O$ and $L_G$ and $Z$ yields:

$$
\dot{Z} = \frac{\lambda_G \dot{L}_G + \lambda_N \delta_O \dot{O}}{\lambda_N \sigma + \frac{\lambda_N \delta_O}{1-\sigma}}.
$$

(24)

So the partial elasticities become

$$
\dot{Z}_O = \frac{\lambda_N \delta_O}{\lambda_N \sigma + \frac{\lambda_N \delta_O}{1-\sigma}} > 0,
$$

(25)

$$
\dot{Z}_{L_G} = \frac{\lambda_G}{\lambda_N \sigma + \frac{\lambda_N \delta_O}{1-\sigma}} > 0.
$$

(26)

The government budget constraint can be written as

$$
(1 - t_L)WL_G = t_C (C_I + P_N C_N) + t_L \pi_T + (t_L + a) W (L_T + L_N) + O \Leftrightarrow
$$

$$
WL_G - t_L WL_G = t_C (C_I + ZC_N) + t_L \pi_T + (t_L + a) W L - t_L WL_G - aWL_G + O \Leftrightarrow
$$

$$
ZL_G = t_C (C_I + ZC_N) + t_L \pi_T + \left(\frac{t_L + a}{1 + a}\right) ZL + O \Leftrightarrow
$$

$$
t_L + a \frac{1}{1 + a} = Y \equiv \frac{ZL_G - O - t_C (C_I + ZC_N) - t_L \pi_T}{ZL}.
$$

(27)

The effects on $a$ or $t_L$ of changes in $O$ and $L_G$ goes through the auxiliary variable $Y$, which is independent of the choice of the endogenous fiscal instrument.

We differentiate the expression for $Y$ with respect to $Y$, $Z$, $L_G$ and $O$, using that the government budget constraint may be written as $t_C (C_I + ZC_N) + t_L \pi_T + O = Z (L_G - YL) \Leftrightarrow Z (L_G - YL - t_C C_N) = t_C C_I + t_L \pi_T + O$:

$$
dY = \frac{dL_G}{L} ZL \left[ t_C \left( \frac{\partial C_I}{\partial Z} + \frac{\partial (ZC_N)}{\partial Z} \right) dZ + t_L \frac{\partial \pi_T}{\partial Z} dZ + dO \right] - ZL (L_G - YL) dZ
$$

$$
= \frac{Z \dot{d}L_G - dO}{ZL} + \frac{L_G - YL - t_C \left( \frac{\partial C_I}{\partial Z} + \frac{\partial (ZC_N)}{\partial Z} \right) - t_L \frac{\partial \pi_T}{\partial Z}}{ZL} dZ.
$$

$$
= \frac{Z \dot{d}L_G - dO}{ZL} + \frac{L_G - YL + t_L \pi_T - t_C C_N - t_C \left( \sigma Z^{\sigma - 1} C_N + \left( \frac{Z^{\sigma - 1}}{1-\sigma} \right) \right)}{ZL} dZ.
$$

$$
= \frac{Z \dot{d}L_G - dO}{ZL} + \frac{Z (L_G - YL + t_L \pi_T - t_C C_N) - t_C \left( \sigma Z^{\sigma} C_N + \left( \frac{Z^{\sigma + 1}}{1-\sigma} \right) \right)}{ZL} dZ.
$$

$$
= \frac{Z \dot{d}L_G - dO}{ZL} + \frac{t_C C_I + t_L \pi_T + t_C (\pi_T + ZL_T) + O - t_C \left( \sigma (C_I + \frac{C_N Z_C}{ZC_N}) \right)}{ZL} dZ.
$$

$$
= \frac{Z \dot{d}L_G - dO}{ZL} + \frac{t_C (1 - \sigma) C_I + (\pi_T - \frac{t_L \pi_T}{1-\sigma}) X_T + O}{ZL} dZ.
$$

26
Thus, $\frac{\partial Y}{\partial \theta}$ and $\frac{\partial Y}{\partial L_G}$ become

$$
\frac{\partial Y}{\partial \theta} = -1 + \frac{\partial \theta}{\partial \theta} \left[ t_C (1 - \sigma) C_I + O + \left( t_x - \frac{t_C}{\theta N} s \right) X_T \right], \quad (28)
$$

$$
\frac{\partial Y}{\partial L_G} = Z + \frac{\partial L_G}{\partial L_G} \left[ t_C (1 - \sigma) C_I + O + \left( t_x - \frac{t_C}{\theta N} s \right) X_T \right], \quad (29)
$$

which are analogous to $\frac{\partial L}{\partial \theta}$ and $\frac{\partial L}{\partial L_G}$ above. Having found the impact on $Y$, the adjustment of $a$ follows from

$$
t_L + a = Y \iff a = \frac{Y - t_L}{1 - Y},
$$

which implies

$$
\frac{\partial a}{\partial Y} = \frac{(1 + a)^2}{1 - t_L}.
$$

(30)

Alternatively, when $t_L$ is the endogenous fiscal instrument:

$$
t_L + a = Y \iff t_L = Y (1 + a) - a \Rightarrow \frac{\partial t_L}{\partial Y} = 1 + a
$$

Thus, the necessary adjustment of $a$ magnifies the adjustment in $Y$ by the factor $\frac{(1+a)^2}{1-t_L}$, which is $\frac{1+a}{1-t_L} > 1$ times the corresponding effect on $t_L$.

**Appendix 3: The model determines the real exchange rate**

This appendix demonstrates that all real variables, including the endogenous tax rate, is invariant to proportional changes in the wage rate and the nominal exchange rate. For completeness, we take into account imports used by the government, $I_G$, and we allow the imported good to differ from the exported one. The world prices $P_I$ and $P_T$ are now measured in foreign currency. For expositional simplicity we consider the case where $t_C = t_x = 0$. Thus, it verifies the conventional insight that the general equilibrium model determines relative prices and real variables, leaving one degree of freedom for nominal prices.

The model consists of the following 8 equations:

$$
X_T = L_T^* \quad (31)
$$

$$
L_T = \left( \frac{seP_T}{W} \right)^{\frac{1}{1-x}} \quad (32)
$$

$$
P_N = W \quad (33)
$$

$$
\frac{C_I}{C_N} = \left( \frac{eP_I}{P_N} \right)^{-\sigma} \quad (34)
$$

$$
eP_I C_I + P_N C_N = (1 - t_L) (eP_T X_T + P_N X_N + W L_G) \quad (35)
$$
\[ WL_G + eP_I I_G = t_L (eP_T X_T + P_N X_N + WL_G) + eO \]  
(36)

\[ X_N = C_N \]  
(37)

\[ L_T + L_N + L_G = L \]  
(38)

Adding the two budget constraints, and using \( X_N = C_N \), yields the national budget constraint:

\[
eP_I C_I + eP_I I_G = eO + eP_T X_T \iff P_I (C_I + I_G) = O + P_T X_T,
\]

which is independent of \( e \).

A reduced form of the model follows from substitutions into the labour market equilibrium condition:

\[
L - L_G = \left( \frac{seP_T}{W} \right)^\frac{1}{x} + C_N \quad \text{(39)}
\]

\[
= \left( \frac{seP_T}{W} \right)^\frac{1}{x} + \left( \frac{eP_I}{P_N} \right)^\sigma C_I \quad \text{(40)}
\]

\[
= \left( \frac{seP_T}{W} \right)^\frac{1}{x} + \left( \frac{eP_I}{W} \right)^\sigma \left[ P_T \left( \frac{seP_T}{W} \right)^\frac{x}{x} + O \right] - I_G. \quad \text{(41)}
\]

This equilibrium condition demonstrates that the model only determines \( \frac{t_L}{P_T} \), which represents the real exchange rate. The tax rate will also be invariant to proportional changes in \( e \) and \( W \), since \( P_N = W \) implies

\[
t_L = \frac{WL_G + eP_I I_G - eO}{eP_T X_T + WX_N + WL_G}.
\]
### Recent publications in the series Discussion Papers

<table>
<thead>
<tr>
<th>Page</th>
<th>Title</th>
<th>Authors</th>
<th>Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>380</td>
<td>A. Langørgen and D. Rønningen (2004): Local government preferences, individual needs, and the allocation of social assistance</td>
<td>A. Langørgen and D. Rønningen</td>
<td>2004</td>
</tr>
<tr>
<td>382</td>
<td>T. A. Galloway (2004): To What Extent Is a Transition into Employment Associated with an Exit from Poverty</td>
<td>T. A. Galloway</td>
<td>2004</td>
</tr>
<tr>
<td>384</td>
<td>A. Bruvoll and T. Fæhn (2004): Transboundary environmental policy effects: Markets and emission leakages</td>
<td>A. Bruvoll and T. Fæhn</td>
<td>2004</td>
</tr>
<tr>
<td>389</td>
<td>B. Halvorsen (2004): Effects of norms, warm-glow and time use on household recycling</td>
<td>B. Halvorsen</td>
<td>2004</td>
</tr>
<tr>
<td>400</td>
<td>B. Halvorsen and Runa Nesbakken (2004): Accounting for differences in choice opportunities in analyses of energy expenditure data</td>
<td>B. Halvorsen and Runa Nesbakken</td>
<td>2004</td>
</tr>
<tr>
<td>407</td>
<td>P. Frenger (2005): The elasticity of substitution of superlative price indices</td>
<td>P. Frenger</td>
<td>2005</td>
</tr>
<tr>
<td>411</td>
<td>J. Larsson and K. Telle (2005): Consequences of the IPPC-directive’s BAT requirements for abatement costs and emissions</td>
<td>J. Larsson and K. Telle</td>
<td>2005</td>
</tr>
<tr>
<td>418</td>
<td>D. Fredriksen and N.M. Stolen (2005): Effects of demographic development, labour supply and pension reforms on the future pension burden</td>
<td>D. Fredriksen and N.M. Stolen</td>
<td>2005</td>
</tr>
<tr>
<td>421</td>
<td>J. Bjørnstad (2005): Non-Bayesian Multiple Imputation</td>
<td>J. Bjørnstad</td>
<td>2005</td>
</tr>
</tbody>
</table>
447 G. Liu (2006): A causality analysis on GDP and air emissions in Norway


451 P. Frenger (2006): The substitution bias of the consumer price index

452 B. Halvorsen (2006): When can micro properties be used to predict aggregate demand?


454 G. Liu (2006): On Nash equilibrium in prices in an oligopolistic market with demand characterized by a nested multinomial logit model and multiproduct firm as nest


456 L-C Zhang (2006): On some common practices of systematic sampling

457 Â. Cappelen (2006): Differences in Learning and Inequality


459 G.H. Bjertnaes (2006): Income Taxation, Tuition Rates and Change Lower Electricity Prices and Increasing Demand: An Application to the Nordic Countries


461 T. Ericson (2006): Time-differentiated pricing and direct load control of residential electricity consumption


464 K.M.Heide, E. Holmøy, I. F. Solli and B. Strom (2006): A welfare state funded by nature and OPEC. A guided tour on Norway's path from an exceptionally impressive to an exceptionally strained fiscal position


466 S. Hol (2006): The influence of the business cycle on bankruptcy probability


468 Suzan Hol and Nico van der Wijst (2006): The financing structure of non-listed firms

