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On the meaning and measurement of redistribution in cross-country comparisons

Abstract:
Empirical findings on the relationship between income inequality and redistribution from a cross-country perspective are not conclusive. One reason may be that observers have in mind different concepts of redistribution. A major factor is that comparator countries’ pre-fisc distributions typically differ markedly, and account is taken of this differently (if at all) by different measures of redistribution. The ambiguities can be resolved by applying the “transplant-and-compare” approach, rendering fiscal regimes into a common base by adjusting for differences in pre-fisc income inequality, and then measuring the “pure” effect of tax-and-transfer policies using this benchmark. We illustrate both what is possible, and what remains problematic, using this technique, by conducting an exploratory international comparison, based on microdata from the Luxembourg Income Study database in combination with more aggregated information from the OECD, for 15 countries.

Keywords: Redistributive effect; Personal income tax; Cross-country comparison

JEL classification: H11; H23; H53

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Discussion Papers  comprise research papers intended for international journals or books. A preprint of a Discussion Paper may be longer and more elaborate than a standard journal article, as it may include intermediate calculations and background material etc.
Sammendrag

Er det slik at land med stor inntektsulikhet omfordeler mer enn land med mindre ulikhet? Dette er en svært mye diskutert sammenheng i litteraturen, men de mange internasjonale analysene som diskuterer dette temaet gir ikke noe klart svar på hvilken sammenheng som gjelder. I denne studien vises det til at ulike målemetoder i de forskjellige analysene bidrar til uklarheten. Blant annet er det anvendt ulike operasjonaliseringer av omfordeling. I denne studien foreslås det å anvende en metode for vurderinger av omfordeling i ulike land basert på etablering av en felles referanse for sammenlikninger. Det argumenteres for at denne metoden etablerer et sammenlikningsgrunnlag som identifiserer skattepolitikkens bidrag i landene. Videre hevdes det at dette omfordelingskonseptet er relevant for mange analyser av omfordeling i et internasjonalt perspektiv. Metodens fordeler og begrensninger diskuteres og dens anvendelse illustreres ved hjelp av mikrodata fra Luxembourg Income Study i kombinasjon med tall fra OECD for 15 land, Norge inkludert.
1. Introduction

What is the relationship between income inequality and redistribution from a cross-country perspective? It depends – not least because observers have in mind different concepts of redistribution and apply different methods. We argue that the ambiguous results found in the literature may stem at least in part from the use of different definitions of redistribution, with regard to both the way that pre-tax inequality is taken account of by the measure employed, and to which part of public policies (i.e., taxes or expenditure side characteristics of budgets) are analyzed, and that these ambiguities can be reduced by applying the “transplant-and-compare” approach, rendering fiscal regimes into a common base by adjusting for differences in pre-fisc (pre-tax/transfer) income inequality. The objective of the procedure is to make the comparator countries’ pre-fisc distributions as similar as possible in order to tease out the “pure” effect of tax-and-transfer policies (leaving out the effect which may be due to the fact that the starting points, namely pre-fisc distributions between the countries, differ).

We illustrate both what is possible using this technique, and what remains problematic, by conducting an exploratory international comparison based on microdata from the Luxembourg Income Study database for 15 countries around the end of the last decade.

Results are consistent with a conclusion that the relationship between countries’ common-base redistributional efforts for the personal income tax and their pre-tax income inequality levels is weaker than has been supposed in studies using standard measures of redistribution which do not correct for pre-tax inequality differences. Issues of significance, from both economic and statistical points of view, remain, and are discussed from a pragmatic point of view in the concluding section of the paper.

The relationship between redistribution and original inequality is undefined from a theoretical standpoint, but the literature (briefly described in Section 2) offers simplified “stories” to two possibilities: the median voter hypothesis predicts that pre-fiscally more unequal countries redistribute more whereas the “Robin Hood paradox” is an often used characteristic and starting point of theories aiming at explaining the opposite (i.e., that they redistribute less). In Section 2, we also briefly review the different measures of redistribution typically used in empirical work. In Section 3, we discuss our data sources.

A significant degree of ambiguity in empirical findings undoubtedly stems from the use of different measurement methodologies. In Section 4, by means of the transplant-and-compare procedure, which is designed to filter out influences stemming from or reflecting diverse original inequality experiences,
we lessen or remove these ambiguities. For example, we find that a “familiar” cross-country finding, that greater pre-fisc inequality goes with greater redistribution (as suggested by the median voter hypothesis), is considerably weakened when we use the transplant and compare procedure.

An ever-present problem, of course, is that we do not have access to truly original, or counterfactual, no-tax-no-benefit distributions. Fiscal systems evolve in response to the existing underlying distributions of assets and hence to existing pre-fisc distributions. The “transplant and compare” methodology offers a benchmark scenario for international (and indeed intertemporal) comparisons, but to be fully effective, it requires that one country’s distribution of income can be “imaged” onto another’s, by a transformation which is linear in logarithms. If the log-linear transformation fails to be effective, we can still examine “net” redistributive impacts after both countries’ pre-fisc distributions are made to look the same, although there may be a base-dependency issue – which can be tested for and possibly rejected. In the final section of the paper we discuss this and other remaining problems.

2. The relationship between redistribution and original inequality in existing literature

2.1 Empirical and theoretical investigations
Wagstaff et al. (1999) is a comprehensive analysis of redistributional effects and tax progressivity effects of the personal income tax for twelve OECD countries. They find that there is no link between pre-tax income inequality and the degree of redistribution brought about by the personal income tax. For example, Ireland, Spain, the UK and the US are the countries with the most unequal pre-tax income distributions, and are found to be more redistributive than many other countries, but less redistributive than Sweden and Finland, which have very low levels of pre-tax income inequality.

Intuitively one might expect that the countries with the highest pre-tax income inequality are the most redistributive, simply because the potential for redistribution is highest. The median-voter model (Romer, 1975; Roberts, 1977; Meltzer and Richard, 1981) provides a theoretical framework within which such a relationship could be manifested. For subsequent work in this line, see Alesina and Rodrik (1994), Persson and Tabellini (1994), Perotti (1996) and Milanovic (2000). However, it is often claimed that redistribution from rich to poor is least present when and where it seems most needed; a finding that Lindert (2004) calls “the Robin Hood paradox”. Bassett et al. (1999) find some empirical support for this. See also Bénabou (2000) and Moene and Wallerstein (2001). In Milanovic
(2010), econometric issues and problems are highlighted which may have contributed to the mixed results in the literature.

Other investigations include: that of Alesina and Angeletos (2005), who suggest that differences in redistributional efforts reflect differences between societies’ social perceptions regarding the fairness of market outcomes; that of Lambert et al. (2003), according to which different attitudes to what are socially acceptable levels of inequality in different countries can account for differing redistributional policies; and that of Slemrod and Kopczuk (2002), who suggest that low income countries will restrain both the size of government and the progressivity with which the size is financed.

2.2 Approaches to the measurement of redistribution

Links between outcome measures of inequality and of redistribution differ for different definitions of redistribution. A central strand of the measurement literature, initiated by Kakwani (1977), defines Gini-based summary progressivity measures which contain a contribution from the pre-tax income distribution and a contribution from either the distribution of the tax burden or the distribution of post-tax income. In either case, overall progressivity results from the interrelationship between taxes and the pre-tax income distribution, a characteristic which from a public policy point of view makes tax progressivity measurement often more interesting than pure inequality measurement: it establishes relationships between initial conditions (pre-tax income distributions), policy tools (taxes) and outcomes (post-tax income distributions).

Progressivity is measured by *redistributive effect* if there is a contribution from the distribution of post-tax income, and by *disproportionality* if there is a contribution from the distribution of the tax burden. Kakwani (1977), who showed that the link between his disproportionality measure and Reynolds and Smolensky’s (1977) redistributive effect measure is conditioned by tax level. Bourguignon and Morrison (1980) proved that the Reynolds-Smolensky index is the only conventional measure of redistributive effect which is determined jointly by tax level and the inequality impact of an identically-distributed one dollar tax. Aronson et al.’s (1994) decomposition of the Reynolds-Smolensky index shows how to isolate vertical, horizontal and reranking contributions from this measure, and has recently been refined (Urban and Lambert, 2008). Going back in time, Musgrave and Thin (1948) and Pechman and Okner (1974) offered precursor measures to the Reynolds-Smolensky index which can (now) be seen as normalizations of the achieved inequality reduction, in which the normalization is by the equality or inequality level prevailing before the tax was applied. Fellman et al.’s (1999) “optimal yardstick” approach measures the redistributive property of an in-place income
tax relative to the redistribution that could have been achieved had the tax been designed to have maximal inequality impact given the tax level. This approach uses the extended Gini coefficient of Yitzhaki (1983), providing the analyst with an opportunity to input a distributional judgment parameter, which conditions the results.

An important non-Gini strand of the progressivity literature uses the welfare-function-based Atkinson (1970) index of inequality, for a chosen degree of inequality aversion, to quantify redistributive effect. The Blackorby and Donaldson (1984) and Kiefer (1985) progressivity indices each adopt this approach, which, like that of Fellman et al., requires an assumption about the degree of inequality aversion of the social observer. The Blackorby and Donaldson index measures the proportion of after-tax income the social observer would hypothetically pay to convert a flat tax system with the same yield into the given one. When pre-tax inequality is low, the observer will pay less in order to avoid a proportional tax. Therefore, when such a measure is used as an indicator of the degree of tax progressivity, we can expect smaller redistributive effects in countries which are more equal before tax.¹

All of these approaches result in progressivity measures which conflate tax system differences with pre-tax distributional differences. When used to make cross-country comparisons of redistributive effect, this tangled, twofold informational content prevents pre-tax inequality and what we might call “redistributinal effort” from being explored as distinct phenomena. In Dardanoni and Lambert (2002) a methodology is proposed which fixes this problem, rendering common-base estimators for cross-country comparisons of redistribution. Post-tax income distributions are adjusted on the basis of differences between the pre-tax distributions they are derived from by controlling for location and spread differences. By eliminating all other pre-tax income inequality differences, for whatever reason they happened (behavioral incentive differences or demographic variations, for instance), we get closer to identifying the effect of tax policies or “redistributional ambitions” across countries. Redistributional properties are measured and compared (using any or all of the conventional methodologies) for the “transplanted” income distributions, now for a common baseline. Comparisons after such transplantation are guaranteed invariant to the choice of baseline if candidate distributions are isoelastic transformations of one another (i.e. if the pre-tax income distributions only differ in

¹ Lambert and Thoresen (2009) found, when applying the Blackorby-Donaldson index, that the Norwegian tax system is not very redistributive. Duclos and Lambert (2000) contains evidence of much higher values for Canada.
logarithms by location and scale, thus not exhibiting major shape dissimilarities). This is the approach we shall follow in the remainder of the paper.²

Progressivity measurement can be extended to the benefits flowing from government expenditure programs, if these are attributed to individuals or households in cash-equivalent terms, and to a combined tax and benefit system. See Kim and Lambert (2009) and also Fellman et al. (1999) for extensions of Gini-based measures to a combined tax and benefit system, and Duclos and Lambert (2000) in respect of the Atkinson-index-based measures. Horizontal and reranking effects are also considered in these papers. In an Appendix, we provide some technical details about the measures of redistribution used in this study, and about implementation of the transplant and compare procedure.

3. Data and definitions

Information about redistributional effects of the personal income tax across countries are computed from the LIS database (http://www.lisproject.org), which provides a high-quality source of internationally comparable data on household incomes. The selection of countries has been restricted by availability of information about pre-tax income distributions and the need for information across countries from approximately the same time period. This leaves us with data from 14 countries in the time period 1999-2001 (Australia (AUS), Canada (CAN), Denmark (DNK), Finland (FIN), Germany (DEU), Israel (ISR), Netherlands (NLD), Norway (NOR), Poland (POL), Sweden (SWE), Switzerland (CHE), Taiwan (TWN), United Kingdom (UK) and the United States (US)) and information about 1 country from 1997, Belgium (BEL). We have utilized estimates of summary measures across countries provided by LIS, but mainly we have employed information from the LIS data by submitting SAS-programmes to the micro data bases of the respective countries, obtaining tailor-made measures.³

The sizes of the datasets vary, from observations on more than 80,000 Danish households to about 3,600 households for Switzerland. For all datasets, income and tax variables are measured in “equivalent values”, which means that the nominal values have been weighted by an equivalence scale (the square root of number of household members). The representation of each household when obtaining summary measures of inequality is in terms of the number of household members; this is often characterized as employing the individual as the unit of analysis.

² In a series of transplant-and-compare exercises undertaken for the Norwegian income tax system over the period 1992-2004, Lambert and Thoresen (2009) reported that the transplant-and-compare procedure yielded “cogent and seemingly robust results, which are also intuitively satisfying”.
³ In particular for the “transplant and compare” method it has been crucial to have access to micro data for each country.
Following from our focus on effects of both tax and transfer policies, we discuss constellations for both the tax and expenditure side of governments’ interventions with respect to one pre-tax/transfer definition of income, corresponding to what Milanovic (2000) characterizes as *factor P income*: income (including pensions) before tax and social transfers, where the latter includes social insurance transfers and social assistance transfers (see [http://www.lisproject.org/techdoc.htm](http://www.lisproject.org/techdoc.htm) for further details). Countries’ initial conditions are determined by pre-tax/transfer (pre-fisc) income inequality, and the different measures of redistribution are obtained by use of the relevant income variables. For instance, estimates of tax redistributional effects are derived by inputting the inequality of gross income (including taxed and non-taxed transfers) and the inequality of post-tax income. In the scatterplots that are used to describe links, we use measures of pre-tax/transfer income inequality to define countries’ places in the international pre-intervention hierarchy.

In some of the constellations information from LIS is combined with information from OECD, who for instance have produced measures of the “social inclination” of the expenditure side of the budgets; such measures are presented and utilized in OECD (2004, 2005, 2008).

### 4. Relationships between pre-fisc income inequality, redistribution and government size

#### 4.1 Findings using standard approaches

As a first step in our explorations, we refrained from using the transplant and compare procedure, in order to derive unadjusted findings in respect of the six (Gini and Atkinson index based) redistribution measures defined for income taxes in the Appendix, as well as two measures of vertical effect, and four that characterize the expenditure side of the budget. We also explored relationships between redistribution and some appropriate measures of the size of government. Detailed graphical analysis for these scenarios, using scattergrams and fitted regression lines to facilitate identifying relationships, can be found in Lambert et al. (2010), the main features of which we describe here before applying the transplant and compare procedure which takes into account, rather than neglecting, the differing pre-tax/transfer inequality experiences in the different countries (namely, by eliminating them).

We found that redistribution increases with pre-tax/transfer income inequality across countries for all six measures of income tax redistribution. Each measure produced a surprisingly similar “story”, given their different designs, although the Pechman-Okner measure, which normalizes redistribution by pre-tax inequality behaves slightly differently in some respects than indices embodying other forms of
normalization, and displays a weaker relationship with pre-tax/transfer inequality. For instance, the three Scandinavian countries (Denmark, Norway and Sweden) are less redistributive than the US according to the latter group of measures, whereas results for the Pechman-Okner measure indicate that redistribution is higher in the Nordic countries than in the US. We also examined two measures of vertical redistribution, and again saw clear positive links with pre-tax income inequality, similar in form and substance to those for overall redistribution (suggesting that differences between countries according to horizontal effects are limited). As for links between redistribution and what we might call the size of the government, we tried three measures of size: the share of total income taken in personal income tax, the share of GDP accounted for by total tax revenue, (including corporate taxes, indirect taxation, etc), and total government outlays (consolidations of accounts for the central, state, and local governments plus social security) as a share in GDP. The two latter measures are based on data from OECD (2004) for the years 2000 and 2001. The relationships we found were weak, coming close to exhibiting negative correlation.

Many authors associate redistribution with features of the expenditure side of the budget. We also examined relationships between pre-tax/transfer income inequality and redistribution characteristics of the expenditure using our unadjusted data (i.e. without applying the transplant and compare technique). The “social inclination” of a country’s government might be measured by transfer redistributional effects and the progressivity of cash benefits, along with “size” or “scale” measures such as the share in GDP of social expenditures (cash benefits and benefits in-kind with one or more social goals) and public cash benefits as a share of disposable income (the former for 2001 in terms of available data, and the latter for the mid-1990s). Our findings indicate that in countries with high pre-tax/transfer inequality, transfers have larger offsetting effects, but the relationship for the progressivity of cash benefits is very weak across the board. In contrast, the scale measures are negatively related to pre-fisc income inequality. These findings using unadjusted data indicate that there is no relationship between how much is used for social support and the progressivity of what is used (which is independent of size). We found, indeed, that the UK and Australia are at the low end in terms of shares of social expenditures and cash benefits, but both countries are among the most transfer redistributional and have high, large and negative transfer progressivities. This may well illustrate the trade-off between low-income targeting and the size of redistributive budgets (universalism), as emphasized by e.g. Korpi and Palme (1998).

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4 However, from a tax policy perspective, it can of course be very important to distinguish between horizontal and vertical effects.
When interpreting these results we need to keep in mind that differences between countries’ pre-tax/transfer inequality experiences have not been allowed for, and other cross-country differences may also be at work. A high value of social expenditures as share of GDP may reflect a high level of the social welfare system, but if the beneficiaries have to pay taxes on these benefits, social expenditures net of taxes as a share of GDP may be at the same level as in countries with lower social expenditures relative to GDP.

4.2 Findings using the transplant and compare procedure

It was evident from using unadjusted data, as in the previous section, that conclusions flowing from empirical work vary somewhat according to different measures of redistribution adopted. Thus there is no clear answer to a question such as “which country is most redistributive?” According to our reading of the redistribution hypothesis and other conjectures of the link between pre-tax inequality and redistribution, the most relevant measure(s) of redistribution would come from using a methodology that provides result in terms of a common baseline: one would like to see how countries’ redistributional efforts would compare if computed for (hypothetically) the same level of pre-tax income inequality. Therefore we redid all of our analyses using the “transplant and compare” method.5

The transplant and compare method necessitates adjusting post-tax income values by a fitted deformation function, whose parameter estimates are obtained by OLS regression, as explained in the Appendix. A useful precursor to the comparisons is to validate the base independence of results. Do results depend on the choice of reference distribution, i.e. on whether the pre-tax income distribution of (for instance) the US or Norway is employed as the base? In Figure 1, we show how one measure of redistribution compares for five selected countries, using each of those five countries as the reference pre-tax income distribution in turn. The figure shows strong similarities between descriptions of redistributitional effects for different bases.

5 The conflation of pre-tax income distributions with tax redistributional effects has led authors in the past to search for evidence according to a common base to come closer to identifying policy-makers’ efforts to redistribute income; see particularly Kasten et al. (1994) and Thoresen (2004) for discussions of over time developments in tax progressivity and redistribution, employing tax simulation models to simulate effects of tax schedules for different years on the same pre-tax income distribution.
Figure 1. Redistributional effects (Reynolds-Smolensky index) of the personal income tax according to the transplant and compare procedure for different reference distributions. Results for Germany, Canada, Norway, the UK, and the US.

Clearly there is not a serious issue of base dependence, and we elect to work using Norway as reference distribution. Is there a relationship between common-base redistribution and pre-tax/transfer income inequality across countries? See Figure 2, in which redistribution for a reference level of pre-tax income inequality is measured on the vertical axis and pre-tax/transfer income inequality is measured on the horizontal axis (by the Gini coefficient in two cases, and the Atkinson index in one case). In this scenario, results would not be expected to depend on whether a redistribution index with or without normalization with respect to pre-tax inequality were used, since the effect of differences in pre-tax income inequality levels are now controlled for directly. We show specimen results for three measures of overall redistribution and two measures of vertical redistribution. The overall redistribution measures are those of Reynolds-Smolensky and Kiefer (which are not normalized), and the Fellman et al. measure, which is normalized with respect to the socially optimal inequality-reducing yardstick tax. See the Appendix for formal definitions of these. For vertical effects, we use the measures devised by Kakwani and Urban-Lambert.

In Figure 2, common-base tax redistribution and pre-tax income inequality are plotted together. The figure includes fitted linear relations with upward slopes, but as reflected by the $t$-value (in parentheses), the identification of these relationships is weak in three of the charts – those for the
Reynolds-Smolensky index and the two measures of vertical effects - whereas the links for the Kiefer index and the Fellman et al. measure are clearly positive. Of course, it is somewhat “heroic” to carry out regressions with such a limited number of data points, but it provides a helpful way to summarize the empirical links.6

These results signify that the choice of index for redistribution is important. Also, relationships between countries’ common-base redistributional efforts for the personal income tax and their pre-tax income inequality levels are weaker now than were perceived when using standard approaches. The reason is that the deformations reduce the redistributional effects in countries with high pre-tax income inequality, as the non-equiproportionate compression reduces pre-tax income differentials more than the post-tax income differentials. Thus, the standard procedure may suggest that high pre-tax income inequality countries, such as the US, deliver more redistribution than is actually true for an empirical strategy that seeks to identify cross-country “redistributional efforts”. The positive relationship for the Fellman et al. measure is particularly interesting because this measure captures redistributional effort relative to an optimal design of taxes, suggesting perhaps that countries with high inequality succeed in taxing people more closely to the relevant optimum. According to the two measures of vertical redistribution, the US, Sweden and Norway are about equally redistributive, which means that the vertical “redistributive efforts” are about on the same level in these three countries.

We also tested the transplant and compare procedure for the combined effect of taxes and transfers, which means that deformations are carried out for an income concept before taxes and transfers (which is similar to the income concept of the horizontal axes in Figure 2). The reliability of results of the common base measurement method depends on being able to characterize pre-policy income distributions (across countries) by a two-parameter deformation function (see the Appendix). As it turns out, the income distributions before both taxes and transfers are more difficult to align, and so we refrain from presenting results of these comparisons here. We have no ground, therefore, for confirming or negating the indications we found using unadjusted data.

6 Another complicating factor is that measures for each country are prone to statistical uncertainty. Standard errors can be approximated by bootstrapping procedures, along the lines described by Lambert and Thoresen (2009, p.231).
Figure 2. Common-base tax redistributional and tax vertical effects vs pre-fisc income inequality

Reynolds-Smolensky index

Common-base tax redistribution

Kiefer index

Common-base tax redistribution

Fellman et al. index

Common-base tax redistribution

Urban-Lambert vertical effect

Kakwani vertical effect

Common-base vertical tax effect

\[ y = 0.12x - 0.01 \]  

\[ (1.40) \]

\[ y = 0.07x - 0.001 \]  

\[ (2.18) \]

\[ y = 2.03x - 0.53 \]  

\[ (2.44) \]

\[ y = 0.11x - 0.002 \]  

\[ (1.20) \]

\[ y = 0.11x - 0.01 \]  

\[ (1.25) \]
We end this section by examining if there is a trade-off between common-base redistribution and size of government on the financing side of public budgets. Figure 3 describes the relationship between common base redistribution and total tax revenues as share of GDP; for the latter, data are obtained from OECD (2004). We see no indication of countries trading off redistribution; rather the contrary: countries with a large public sector appear to put in more redistributive efforts too. Of course, such trade-offs may still pertain for the expenditure side of budgets, as suggested by Korpi and Palme (1998).

Figure 3. Relationship between (common base) tax redistribution and tax revenue share

5. Overview and ways forward

The understanding of the development of fiscal policies in response to pre-fiscal distributions of income is an intriguing but very challenging question. One major problem is that policies and income distributions evolve over time in close interrelation to each other, suggesting that they are endogenous in a cross-country perspective. Nevertheless, the relationship attracts substantial attention, not the least from a political economy perspective: does inequality create a political demand for redistribution (redistribution hypothesis) or do we observe a “Robin Hood paradox”?

The literature offers mixed results. Given the data we used for this study, for 15 countries from the LIS database, we see a rather clear picture concerning the conflicting results on the relationship between pre-tax/transfer income inequality and redistribution. When each income tax system is evaluated

\[ y = 0.13x - 0.02 \]

\[ (2.32) \]

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7 We see a similar pattern when letting the average personal income tax be plotted against common base redistribution.
according to its own pre-tax/transfer inequality baseline, personal taxes appear to be more redistributive in more unequal countries, even though results to some extent depend on the choice of index. Similarly, we also see indications that transfers are more targeted in countries with high pre-tax/transfer income inequality. However, relationships between measures of size of government and redistribution, although very weak, come closer to exhibiting negative correlation.

Our results, indicating that the relationship between redistributional effort and pre-tax inequality is weaker than suggested when using standard measures, have been obtained despite some open questions about issues of significance, from both economic and statistical points of view. When the log-linear transformation required for the transplant and compare procedure fails to be effective, we can still examine redistributive impacts after both countries’ pre-fisc distributions are made to look the same, either reporting failure if nothing is transparent (as we did here for expenditure progressivity), or, where results are suggestive, reporting as a rider that there may be a base-dependency issue. Ours was a “pilot study” only. The issue of robustness, i.e. of the “reach” of the transplant-and-compare technique, clearly will bear further examination.

The contrasting evidence presented here and from past studies clearly invites confusion around such questions as “which country is the most redistributive?”. We believe that what many authors actually have in mind, when discussing the relationship between income inequality and redistribution across countries, is the concept of redistributional effort for (hypothetically) the same level of pre-tax income inequality. For the redistribution hypothesis and other conjectures of the link between pre-tax inequality and redistribution, certainly, the most relevant measure(s) of redistribution would be common-base measures – those of the Dardanoni and Lambert (2002) variety, which come from applying the “transplant and compare” methodology to raw tax and income data.

The cross-country tax redistributive effects of personal income taxes are functions of the measurement system adopted, which has until now been the traditional one, in which the existing pre-tax inequality values for the various countries inevitably condition the results and confound comparative work. The results of using the “transplant and compare” procedure, as in this study, show weaker relationships for tax redistributional and vertical effects than results according to traditional methods. Thus, the common-base evidence presented here suggests that there may well be no link between pre-tax/transfer income inequality and redistributional effects of the personal income tax, in a cross-country perspective. In that sense the present approach supports neither the redistribution hypothesis nor the “Robin Hood paradox” depiction of the relationship.
References


Technical appendix

1. Indices of progressivity used in Section 4

Let \( x \) be individual or household pre-tax income, let \( n \) be post-tax income and let the aggregates of these across the population be \( X \) and \( N \). Denote the Gini coefficient for an attribute \( u = u(x) \) by \( G_U \), and denote the Atkinson inequality index with parameter \( e \) by \( I_U(e) \). Principally, we shall encounter \( G_X \) and \( G_N \) and \( I_X(e) \) and \( I_N(e) \) in what follows. The concentration coefficient for \( u \) when income units are ranked by \( x \) will be denoted \( C_U \). We shall encounter \( C_N \) and \( C_T \) here, where \( t = x - n \) is the individual tax payment and \( T \) is the total tax. Let the total tax ratio (or overall average tax rate) be \( g = T/X \). The factor \( \frac{g}{(1-g)} \) is often called tax level: it measures total taxes relative to the total remaining (post-tax) income. This notation, along with all of what follows, is exactly as in Kim and Lambert (2009).

We first enumerate the Gini-based measures that will feature in our empirical investigations. The Reynolds-Smolensky (1977) index of redistributive effect is \( RE = G_X - G_N \) and the Kakwani (1977) index of disproportionality is \( PK = C_T - G_X = \frac{G_X - C_N}{G_X} \) where \( V^K = [G_X - C_N] \) quantifies the “vertical” redistributive characteristic of the income tax, namely, its inequality-reducing impact on average, from which additional terms need netting out in order to obtain \( RE \). A negative “horizontal” contribution arises in case there is unequal tax treatment of pre-tax equals (introducing inequality where there was none before). Kakwani did not allow for this feature of a typical tax system in his 1977 paper, but in his 1984 paper he introduced the decomposition \( RE = V^K - R^K \), where \( R^K = G_N - C_N \) is an index of the reranking which occurs when income units which were better-off than others before tax end up worse-off, because of differential tax treatments. In Urban and Lambert (2008) it is shown how to properly adapt this decomposition when, as is common, there is a sparsity or complete lack of exact pre-tax equals in one’s data. The final decomposition is of the form \( RE = \frac{V^K}{G_X} - H - R^K \), where \( H \) is a term measuring the effect of the unequal treatment of exact equals, which is effectively netted out of Kakwani’s \( V^K \); his reranking measure stands. An algorithm for computing \( V^K \) (or \( H \)) is given in Urban and Lambert (2008).\(^8\)

The earlier Gini-based indices of redistributive effect are those of Musgrave and Thin,

\[
RE^{MT} = \frac{1 - G_N}{1 - G_X}, \quad \text{and Pechman and Okner, } \quad RE^{PO} = \frac{G_X - G_N}{G_X}. \quad \text{The “optimal yardstick” version, due}
\]

\(^8\) H may be negative if there are few or no exact pre-tax equals in the sample: \( H \) is an estimate only, and can be an underestimate.
to Fellman et al., takes the form \( RE^F (\nu) = \frac{G_N(\nu) - G_0(\nu)}{G_X(\nu) - G_0(\nu)} \) where \( G_X(\nu) \) and \( G_N(\nu) \) are extended Gini coefficients with parameter \( \nu > 1 \), and \( G_0(\nu) \) is the (counterfactual) post-tax Gini which would pertain if the same tax revenue were raised by a 100% tax on all income above a certain pre-tax threshold, with no tax lower down the distribution.\(^9\)

The Blackorby-Donaldson (1984) index of redistributive effect is

\[
RE^{BD} = \frac{I_X(e) - I_N(e)}{1 - I_X(e)} \quad \text{and the Kiefer index is} \quad RE^{KI} = I_X(e) - I_N(e).
\]

The last of these (the Kiefer index), as well as the first (Reynolds-Smolensky) measures redistributive effect as inequality reduction, without normalization. The others normalize either by pre-tax inequality \( RE^{PO} \) or by pre-tax equality \( RE^{MT}, RE^{BD} \). All can be applied to a benefit system or net tax and benefit system, simply by re-interpreting \( N \) as respectively income including benefits or income including benefits and net of tax.

2. The transplant and compare procedure

Let \( F(x) \) be the distribution function for pre-tax income in a given country, and, as before, let \( u = u(x) \) be some attribute of a person or household having \( x \) before tax. If \( g(x) \) is a mapping of pre-tax incomes into \( \mathbb{R}^+ \), the conjugate mapping \( u^g(x) = g(u(g^{-1}(x))) \), i.e. \( u^g = g \circ u \circ g^{-1} \), operates on the distribution \( F \circ g^{-1} \). If an isoelastic function \( g(x) \) can be found such that \( F \circ g^{-1} \) equals a chosen reference distribution – e.g. the standard lognormal distribution, call this \( LN(0,1) \),\(^{10}\) although for our empirics we chose to put the income distribution of Norway in this role, see on - then as Dardanoni and Lambert (2002) have shown, the conjugate of the pre-tax/post-tax income mapping \( x \to n \) can be regarded as the transplant of the tax system into \( LN(0,1) \). This can be done with the data of each country, to enable a set of comparisons, of the actions of transplanted tax and benefit systems upon \( LN(0,1) \), in which actual schedules have all been adjusted for pre-tax distributional differences.

The lognormal is not required for this “transplant and compare” procedure. A key result of Dardanoni and Lambert (2002) is that base-independent results are guaranteed, i.e. it does not influence results whether the pre-tax income distribution of Norway or some other country is used as base instead of \( LN(0,1) \), provided that the deformation function is isoelastic. Thus whenever within-country pre-tax

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\(^9\) In practice, we used an iteration procedure to find the pre-tax threshold which divides the population into two: those who pay no tax and those who are taxed 100 percent on income above the threshold; see also footnote 6 in Fellman et al. (1999).

\(^{10}\) That is to say, \( y \sim LN(0,1) \iff \ln(y) \sim N(0,1) \), where \( N(0,1) \) is the standard normal distribution.
income distributions differ in logarithms only by location and scale, any appropriate “reference”
distribution can be selected, and the comparisons made with tax systems that have been adjusted for
international differences in pre-tax location and scale.

Empirically, one wants to find that, for each country \( i \), there exist \( a_i \) and \( b_i > 0 \) such that the
distribution of \( a_i + b_i \ln(x) \) is sufficiently close to the chosen reference distribution, where \( x \) is pre-tax
income. Thus, the method implies finding estimates of \( a_i \) and \( b_i \) that minimize the differences between
the two distributions in terms of location and scale. This corresponds to finding the intercept and slope
in a traditional OLS regression, and the \( R^2 \) statistic becomes the relevant measure of goodness-of-fit.
The post-tax income values are then adjusted by the fitted deformation function \( g_i(x) = e^{a_i x^{b_i}} \) before
making comparisons of progressivity and redistributive effect.

Before carrying out the regressions some data trimming was done. First of all, to remove effects of
outliers, all records with negative or zero income were dropped. Then the datasets were bottom-coded
at 1 percent of mean pre-tax income and top-coded at 50 times the median; see Brandolini and
Smeeding (2007) for similar (but not identical) data manipulations of LIS data. Next, the datasets were
adjusted to the same sample size;\(^{11}\) we used sizes of 5,000 observations and 10,000 observations,
dependent on the samples in question. In the procedures to derive Norway-adjusted estimates of
redistribution, we added in an error term, which was drawn from a normal distribution. This and
additional practical issues concerning implementation of the “transplant and compare” method are
discussed in Lambert and Thoresen (2009).

The results of the 14 OLS regressions that were used to establish the common-base results can be
found in Lambert et al. (2010). The slope estimates are above or below 1 dependent on whether the
respective pre-tax income distributions are more equal \( (b > 1) \) or more unequal \( (b < 1) \) than the pre-tax
income distribution of Norway, involving either expansion \( (b > 1) \) or contraction \( (b < 1) \) in the
transformation to replicate the Norwegian distribution. The last column shows how well the regression
estimates in combination with data from the respective countries were able to reproduce the pre-tax
income distribution of Norway, in terms of pre-tax inequality estimates. The largest deviation was
observed for the regression against Switzerland (CHE), which overshoots pre-tax income inequality by
6.4 percent. In the calculation of the common-base redistributinal effects, see Figure 2, we employed

\(^{11}\) See footnote 23 in Dardanoni and Lambert (2002), where the necessity to equalize sample sizes is discussed.
the deformation functions for both the pre-tax and post-tax income distributions; we could have instead used the actual value for pre-tax income inequality for Norway.\textsuperscript{12}

\textsuperscript{12} This issue is extensively discussed in Appendix C in Lambert and Thoresen (2009).