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Abstract (184 words)

Alpine ski racing is a popular sport in many countries and a lot of research has gone into optimizing athlete performance. Two factors influence athlete performance in a ski race: speed and the chosen path between the gates. However, to date there is no objective, quantitative method to determine instantaneous skiing performance that takes both of these factors into account. The purpose of this short communication was to define a variable quantifying instantaneous skiing performance and to study how this variable depended on the skiers’ speed and on their chosen path. Instantaneous skiing performance was defined as time loss per elevation difference $\Delta t/\Delta z$, which depends on the skier’s speed $v(z)$, and the distance travelled per elevation difference $\Delta s/\Delta z$. Using kinematic data collected in an earlier study, it was evaluated how these variables can be used to assess the individual performance of 6 ski racers in two slalom turns. The performance analysis conducted in this study might be a useful tool not only for athletes and coaches preparing for competition, but also for sports scientists investigating skiing techniques or engineers developing and testing skiing equipment.
Introduction

Alpine ski racing is among the most popular winter sport in several countries. Many sports scientists and engineers have strived to improve the performance of alpine ski racers. Some studies have directly investigated how skiing technique might affect performance (Watanabe & Ohtsuki, 1977; Barelle & Tavernier, 2000; Müller & Schwameder, 2003; Federolf et al., 2008; Supej, 2010, Reid, 2010) while others have studied the interdependencies of mechanical and biomechanical variables during skiing (Federolf et al., 2008; Supej, 2008; Federolf, Lüthi, Roos, & Dual, 2010; Supej & Holmberg, 2010; Supej, Kipp, & Holmberg, 2010) or have focused particularly on skiing equipment (Glenne, DeRocco, & Vandergrift, 1997; Nordt, Springer, & Kollár, 1999; Colbeck & Perovich, 2004; Federolf et al., 2008; Schiestl, Kaps, Mössner, & Nachbauer, 2006; Bäurle, Kaempfer, Szabo & Spencer, 2007; Federolf, Roos, Lüthi, & Dual, 2010; Federolf et al., 2010; Heinrich, Mössner, Kaps, & Nachbauer, 2010).

An important limitation of research into biomechanical and physical factors influencing the success in ski races is that the existing methods to quantify skiing performance are often inadequate for a detailed analysis (Kirby, 2009; Supej et al., 2010). Section time, which is probably the most frequently used variable to quantify racing performance, has several important limitations (Supej et al., 2010). But all other variables that have been used for a performance analysis, e.g. speed, acceleration, differential specific mechanical energy (Supej, 2008; Supej et al., 2010), difference in mechanical energy divided by initial speed (Supej et al., 2010), or lateral skidding of the skis (Kirby, 2009), also suffer from an important limitation: They only quantify variables that relate to the skiers’ speed or energy state. The actual performance of a ski racer, however, depends not only on speed, but also on the path chosen by the skier. These two aspects are interlinked. A longer path often enables skiers to maintain a higher speed but takes...
more time to negotiate while a direct approach to the gates reduces the distance skied but may lead to tighter turns and reduced speeds. In many competitions the skiers’ “strategy”, i.e. what trajectory a skier chooses and how this trajectory allows increasing or maintaining speed, has become as important for success as the skiing technique itself (Le Master, 2010).

In a competition the performance variable deciding over victory or defeat is the overall run time. When coaches evaluate the performance of their athletes in sections of the run, they often use the expression “the athlete lost time” or “athlete A gained time as compared to athlete B.” Video analysis software such as Dartfish® (Dartfish video software solutions, Fribourg, Switzerland) is often used by coaches for a qualitative comparison of the performance of two selected skiers. However, this method requires time consuming post-processing and its precision depends on the camera perspective. To date, a quantitative method that provides sufficient accuracy for scientific investigations has, to the best knowledge of the author, not been described or used.

The purposes of this study were therefore to a) develop a variable quantifying instantaneous performance by developing a mathematical concept for what practitioners describe as “loss of time” and b) to determine if the “loss of time” occurred due to a decline in speed or due to a longer trajectory. The method outlined in this paper was evaluated using the kinematic data of 6 junior ski racers in two slalom turns (Reid, 2010), which was generously provided by Reid and colleagues.

**Methods**

**Participants and Data Collection**

The data used in this study to demonstrate the calculation and evaluation of a variable quantifying instantaneous skiing performance was recorded and analysed by Reid and
colleagues (Reid, 2010). In summary, 6 Norwegian junior Eurocup skiers (male, age 17-20, height $1.81 \pm 0.08$ m, weight $83.82 \pm 8.75$ kg, FIS points $22.25 \pm 8.24$ (mean ± SD), world rank in their age classes between 1 and 6) performed a slalom simulation of which two consecutive turns were recorded with a camera-based motion analysis system. All participants gave informed written consent and the study was approved by the appropriate institutional review board. The skiers’ movements were characterized by 25 reference points which allowed calculation of the centre of mass position (CM). The measurement frequency of the motion analysis system was 50Hz and the point reconstruction error of the measurement system was calculated to be between 6 and 17 mm RMSE (Reid, 2010). The current analysis requires a reference trajectory that characterizes a skier’s position on the slope. The CM trajectories were therefore projected onto the plane of the snow surface and expressed in global coordinates (Figure 1).

**Calculation of Instantaneous Skiing Performance**

As pointed out earlier, the performance variable deciding over victory or defeat in a competition is the overall run time. The instantaneous performance was therefore quantified by calculating the time difference $\Delta t$ between two points of a skier’s trajectory. To compare the performance of different skiers this time difference had to be expressed as a function of a variable that is common to all skiers. It has been suggested in previous studies that the elevation $z$ could be such a common variable (Supej, 2008). Hence, an instantaneous performance $p$ at each elevation $z$ was defined as

$$p^{-1}(z) = \frac{\Delta t}{\Delta z}$$

(Eq. 1)
The variables \( \Delta t \) and \( \Delta z \) were determined at each known point \( i \) of the trajectory: 
\[
\Delta t_i = t_i - t_{i-1} \quad \text{and} \quad \Delta z_i = z_i - z_{i-1}.
\]
For the elevation difference the later value, \( z_i \), was subtracted from the earlier value, \( z_{i-1} \), since the participants skied from higher elevation to lower elevation. **Hence, both differences \( \Delta t \) and \( \Delta z \), and consequently the performance variable** \( p \) **were positive.**

Equation 1 allowed a direct comparison of the instantaneous performance between competitors (Figure 2). However, it could not answer the question *why* an athlete “lost” or “gained” time. As pointed out in the introduction, a skier may lose time due to a decline in speed, or due to a longer path. Using the definition of speed, \( v = \Delta s / \Delta t \), these variables were introduced into Eq. 1:

\[
p^{-1}(z) = \frac{\Delta t}{\Delta z} = \frac{1}{\Delta z} \left( \frac{\Delta s}{v} \right)
\]

or

\[
p^{-1}(z) = \frac{1}{v} \frac{\Delta s}{\Delta z}
\]

(Eq. 2)

where \( \Delta s \) is the distance between two adjacent points of the trajectory and \( v \) the speed:

\[
\Delta s_i = \sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2 + (z_i - z_{i-1})^2}
\]

(Eq. 3)

\[
v_i = \frac{\Delta s_i}{t_i - t_{i-1}}
\]

(Eq. 4)
Eq. 2 is only well defined if a) the distance between two points i and i-1 of the trajectory is reasonably small such that the rules of infinitesimal calculus apply, and b) if the skier’s speed is not zero. However, if both sides of the equation are inverted then the equation implies that a skier who is not moving (v=0) will have a Zero performance \( p(z) = 0 \). Hence, even in this situation the definition is consistent with the aim of defining an instantaneous performance variable.

The results of equations Eq. 1 or 2 were validated by integrating the time between two selected elevations \( z_{initial} \) and \( z_{final} \). This integral should be equal to the section time \( T \) between the two elevations:

\[
T = \int_{z_{initial}}^{z_{final}} p^{-1}(z) \, dz
\]

(Eq. 5)

All calculations discussed here were implemented in MATLAB (The MathWorks Inc., Natick, MA, USA).

Results

For all participants the loss of time spent per meter of elevation difference is shown in Figure 2. The graph shows a saw tooth shape with loss of time decreasing between the gates and increasing near the gates. A similar shape is found when plotting the skiers’ speed as a function of elevation (Figure 3, top). The skiers increased their speed continuously and almost linearly when they traversed between gates. As the skiers initiated the turn, their speed declined sharply.Shortly after crossing the fall line, which is the direction of the steepest decent on the slope, the speed started to increase again. The distance travelled per meter of elevation difference (Figure 3 bottom) depends on the skier’s speed, the inclination of the slope and the angle between the skier’s velocity
vector and the fall line. The latter influencing variable caused local minima in the $\Delta s/\Delta z$-graph when the skier crossed the fall line and maxima when the skier traversed. The result of the validation calculation (Table 1) showed that time loss was systematically slightly overestimated by 0.01-0.03 s (1%).

The comparison of the individual skiers’ loss of time (Figure 2) showed that two skiers (Subjects 3 and 5) performed worse than their peers since they lost more time in large sections of the analysed turns. However, the examination of the two factors contributing to performance (Figure 3) revealed that the reason for their inferior performance was different. Subject 3 was substantially slower than his peers throughout large parts of the analysed turns. Interestingly, his initial speed (between 9 m and 8.5 m elevation) did not differ markedly from his competitors. This suggests that this skier made a “mistake” at an elevation of approximately 7 m reducing his speed, which did not recover in the following two turns. In contrast, subject 5 was not notably slower than his peers, however, when comparing the travelled distance per meter of elevation difference (Figure 3, bottom) it becomes obvious that subject 3 chose a substantially longer path compared to the other skiers. In fact, a detailed analysis of the performance factors presented in Figure 3 would allow to pinpoint for every skier how he could have improved his performance. For example, subject 1 skied a comparatively short path at a high speed, but in the last turn he lost more speed than participants 2 and 6. Subject 2 had the best overall performance, however, at the beginning of the analysed section he choose a path that was longer than necessary and in the last turn he lost more speed than others.

A general feature of all three graphs is that the skier’s relative performance differed substantially from turn to turn: The skier with the best performance in the beginning of the analysed section (skier 1) showed only the fourth best instantaneous performance
at the end. In the two turns analysed in this study, fluctuations in the skiers’ speeds had a larger impact on the fluctuations in performance than distance travelled. The performance depends linearly on both factors, however, the range of typical fluctuations in the speed was around 9%, whereas the fluctuations observed in distance travelled amounted to only 3 to 7%. If analysing the performance differences between skiers within a selected turn (Figure 4, top) it was found that the rate at which the speed decreased (indicated by grey bars in Figure 4, top) differed already at the turn initiation and persisted till the completion of the turn. This may suggest that the turning technique (carving or skidding) may have differed between these skiers. Differences in path length between skiers occurred predominantly at the completion of the turn (arrows in Figure 4, bottom). Several causes might be responsible for these differences, e.g. that some skiers were “early” or late” in their turn, that they did not approach the gate as closely as others, or that they were not able to “hold their line” to use coaching terminology and therefore lost more time than their peers.

Discussion

The instantaneous skiing performance of six participants was evaluated in this study by calculating the three variables time loss per elevation difference $\Delta t/\Delta z$, speed $v$ and distance travelled per elevation difference $\Delta s/\Delta z$ and expressing them as a function of the elevation $z$. The main advantages of this approach compared to existing methods of analysing skiing performance (Supej, 2008; Supej et al., 2010; Supej & Holmberg, 2010) are that a) it allows a continuous evaluation, while several previous methods relied on the analysis of sections of the run; b) causes for decline of performance due to a loss of speed or due to skiing a longer trajectory can be distinguished; and c) it is an intuitive method that is close to how coaches qualitatively analyse the performance of their athletes.
The proposed approach is therefore well suited for studying the trade-off between maintaining a high velocity and skiing a short trajectory. In theory, a shorter trajectory requires tighter turns and may therefore lead to slower speeds, while a longer trajectory would allow rounder turns which might allow the skier to maintain a higher speed. It is also important to note, that a loss of speed will continue to influence the performance until the speed is regained while a longer trajectory will only instantaneously reduce the performance. However, in the actual situation of a race there are several influencing factors that can potentially change how speed or path length affect the ultimate performance. Of particular importance are the slope inclination, changes in the slope inclination, or differences in the snow surface properties.

Accurate reference trajectories are needed that quantify the skiers’ positions as a function of time. As demonstrated in the current paper, camera-based motion analysis systems can provide such trajectories, however, the validation calculation (Table 1) showed that time loss was slightly overestimated. The main cause for this deviation was an underestimation of the velocity in Eq. 4 due to a linearization of an actually curvilinear trajectory. This overestimation depends on the measurement frequency. At a frame rate of 50 Hz the distance between measurement points was approximately 25-30 cm. Considering the fast changes of direction occurring in slalom and giant slalom, this distance may constitute a lower limit for an accurate analysis. In super g and downhill, changes of direction do not occur as rapidly, however, in these disciplines the speed is considerably higher. This suggests that the measurement frequency has a critical impact on the accuracy of the results and that frequencies below 50 Hz may not be suitable.
A limitation of the camera-based motion analysis systems is that this technology requires time consuming post processing of the data. However, high-end global navigation satellite systems (GNSS) already provide accurate position data at similar measurement frequencies (Brodie, Walmsley, & Page, 2008; Waegli A. 2009; Supej, 2010; Supej & Holmberg, 2011). GNSS might therefore become a promising alternative for data recording in the near future.

While an obvious application of a GNSS combined with the performance analysis method described here would be in coaching, several other fields might also benefit from such a tool. One application could be testing of skiing equipment. There are many studies evaluating the mechanical properties of alpine skis (Nordt et al., 1999; Glenne et al., 1997; Schiestl et al., 2006; Federolf et al., 2010; Federolf et al., 2010; Heinrich et al., 2010). However, the optimum mechanical properties of skis remain unknown. The method presented here would allow testing of how skis with different mechanical properties perform in different parts of a turn.

Moreover, in recent years alpine ski racing has suffered from a decline of spectator interest. One reason for this decline might be that even for trained observers such as TV commentators of ski races, it has become very difficult to comprehend why one athlete finishes with a better time than another one. Displaying the speed and the distance travelled per elevation difference would enable any observer to directly evaluate the effectiveness of the skiing technique and the strategy of the ski racers and might make this sport more interesting to watch.

**Conclusions and Perspective**

The definition of the variables time loss per elevation difference $\frac{\Delta t}{\Delta z}$, speed $v(z)$, and distance travelled per elevation difference $\frac{\Delta s}{\Delta z}$ offers an intuitive formalism that not
only quantifies instantaneous performance in alpine ski racing but also allows an assessment of the reason for performance differences between athletes or between different trials. The performance analysis conducted in this study would be a useful tool for athletes or coaches, but might also be useful for sports scientists investigating skiing techniques or engineers developing and testing skiing equipment. Implementation of the method outlined in this manuscript using a high-end GNSS offers a realistic prospect of a real-time analysis system to quantify instantaneous skiing performance on-slope.

References


Figure Captions

Figure 1 Coordinate system and trajectories of the skiers analysed in this study.

Figure 2 Instantaneous performance of the skiers: time loss per elevation difference.

Figure 3 Factors contributing to instantaneous skiing performance: speed (top) and distance travelled per elevation difference (bottom).

Figure 4 Factors contributing to instantaneous skiing performance displayed for one turn for the 4 fastest skiers. The black arrows highlight points in the turn where the skiers won or lost time as compared to their peers. These points are of particular interest for an analysis how the skiers could improve their individual performance. Note: Participants 3 and 5 were omitted for better clarity.
Table 1: section times $T$ determined between elevation levels of 9 m and 1 m

<table>
<thead>
<tr>
<th>participant number</th>
<th>time $T$ 9 m to 1 m elevation [s]</th>
<th>$\int_{9m}^{1m} p^{-1}(z)dz$ [s]</th>
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<tbody>
<tr>
<td>1</td>
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<td>$2.14$</td>
</tr>
<tr>
<td>2</td>
<td>$2.09 \pm 0.01$</td>
<td>$2.12$</td>
</tr>
<tr>
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<td>$2.17 \pm 0.01$</td>
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<td>5</td>
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</tr>
<tr>
<td>6</td>
<td>$2.11 \pm 0.01$</td>
<td>$2.12$</td>
</tr>
</tbody>
</table>
Figure 1  Coordinate system and trajectories of the skiers analysed in this study.
Figure 2  Instantaneous performance of the skiers: time loss per elevation difference.
Figure 3  Factors contributing to instantaneous skiing performance: speed (top) and distance travelled per elevation difference (bottom).
Figure 4  Factors contributing to instantaneous skiing performance displayed for one turn for the 4 fastest skiers. The black arrows highlight points in the turn where the skiers won or lost time as compared to their peers. These points are of particular interest for an analysis how the skiers could improve their individual performance. Note: Participants 3 and 5 were omitted for better clarity.