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Team Incentives in Relational Employment Contracts

by

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Team Incentives in Relational Employment Contracts*

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Abstract

The paper analyzes conditions for implementing incentive schemes based on, respectively joint, relative and independent performance, in a relational contract between a principal and a team of two agents. A main result is that the optimal incentive regime depends on the productivity of the agents, or more precisely on the returns from high effort. This occurs because agents’ productivities affect the principal’s temptation to renegotiate the relational contract. The analysis suggests that we will see a higher frequency of relative performance evaluation (RPE) - and schemes that lie close to independent performance evaluation - as we move from low-productive to high-productive environments. In particular, it is shown that if effort-productivity is sufficiently high, the optimal scheme for the principal is (for a range of discount factors) a collusion-proof RPE scheme, even if there is no common shock that affects the agents’ output.

1 Introduction

High-productive workers are more often than low-productive workers governed by high-powered incentive regimes (see e.g. Lazear, 1998). And while

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group incentives typically are enjoyed by low-wage ‘blue-collar’ workers, incentives based on individual or relative performance (including promotion-tournaments) are more common among ‘white collar’ workers at higher organizational levels (see Prendergast, 1999; and Appelbaum and Berg, 1999).

These observations can be explained by endogenous matching,¹ but in this paper we explore the relationship between productivity and incentive regime by taking a different approach: We show that when workers’ productivity affects the employer’s temptation to renege on contracts, then the optimal incentive regime (based on joint, relative or independent performance) will depend on the productivity of the workers.

We build our analysis on a model that capture quite common features of employment contracts: 1. They are long-term (or dynamic). The employer (principal) and her workers (agents) interact repeatedly, and the principal must deal with problems of dynamic moral hazard. 2. Employment contracts are (to some extent ) relational. A relational contract contains elements that cannot be verified by third parties. A worker’s performance, for instance, may be observable, but still difficult to verify by a court, since the assessment of performance may be complicated. In repeated interaction the parties are able to self-enforce contracts that are not court-enforceable, since contract deviation can ruin the ongoing relationship. 3. Employment contracts are multilateral. In organizations with more than one worker, the employer must not only control the decisions of a single agent; she must also take into account that her treatment of one worker may affect the behavior of other workers. In addition she must consider the strategic behavior of workers interacting with each other.

In the literature on employment contracts, each of these features have attained much attention, but usually they are not analyzed within the same model. We incorporate the above features in one model, and compare the efficiency of different incentive regimes. While this may seem ambitious, we stick to a quite simple model: We assume risk neutrality, and as shown by Levin (2003), optimal relational contracts between risk neutral parties have the nice feature of being stationary.² Hence, the potentially complex con-

¹ Observed correlation between contracts and other variables, such as riskiness and productivity, can be understood as heterogenous agents choosing different environments, see e.g. Ackerberg and Botticini (2002) and Chiappori and Salanie (2002).
² In a stationary contract, the principal promises the same contingent compensation in each period.
tracting problems that arise in dynamic relationships can be solved by simple self-enforcing stationary contracts, given some assumptions. So we analyze a multiagent moral hazard problem within a repeated game framework of self-enforcing relational contracts, and compare incentive schemes based on joint performance evaluation (JPE), relative performance evaluation (RPE) and independent performance evaluation (IPE).

Two papers come closest to ours: First, Levin (2002) who compares multilateral to bilateral relational employment contracts in a model with $n$ agents. An important insight is that multilateral contracts need only satisfy the sum of individual constraints. This favors relative performance evaluation since the principal can credibly commit to a limited total payment. Second, Che and Yoo (2001) who analyze a repeated game between one principal and two agents where the agents engage in implicit contracting with each other. This repeated agent-interaction favors joint performance evaluation since the agents have means for peer sanctions, which lowers the principal’s costs of providing incentives. While Levin does not consider repeated agent-agent interaction, Che and Yoo do not consider relational contracting between principal and agents. We complement Che and Yoo’s model by assuming that the agents’ output is not verifiable. In such we model a multilateral relational employment contract that includes repeated interaction between agents. And we can thus run a ‘horse race’ between the commitment advantage of RPE and the peer-monitoring advantage of JPE.

A main result is that the optimal incentive regime (JPE, RPE or IPE) depends on the productivity of the agents, or more precisely on the returns from high effort. This occurs because agents’ productivity affect the principal’s temptation to renege on the relational contract: The higher effort-productivity, the more there is to lose from breaking promises. One interpretation of the model is that we will see a higher frequency of RPE (and

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4Relational’ contracts and ‘implicit’ contracts are used synonymously in the literature. MacLeod and Malcomson (1989), Baker, Gibbons and Murphy (1994) and Schmidt and Schnitzer (1995) used ‘implicit’, while Bull (1987) used both ‘implicit’ and ‘relational’. Newer papers such as Baker, Gibbons and Murphy (2002) and Levin (2003) use ‘relational’, inspired by the legal literature, particularly MacNeil (1978). Since implicit contracts can be interpreted as vaguer than relational contracts (due to the antonym implicit versus explicit), we will in this paper use the term ‘implicit’ on the contract between the agents (like Che and Yoo), since it is most natural to think about this contract as a verbal informal agreement. But we will use ‘relational’ on the contract between the principal and his agents, since this most likely is a formally written wage contract.
schemes that lie close to IPE) as we move from low productive to high-productive environments. For low effort productivity, the cost of contract deviation is low, and the principal has therefore larger reneging temptations. Ceteris paribus, this calls for RPE due its commitment advantage. However, for low effort productivity, relational contracts can only be implemented for high discount factors. This favors JPE since implicit contracts between the agents are then easy to self-enforce. As we move towards high-productive environments, the cost of contract deviation becomes higher, and relational contracts can therefore be implemented for lower discount factors. We will then expect a larger fraction of RPE, since the repeated peer-monitoring device of JPE is vulnerable to low discount factors. This result is interesting since RPE seems more common in high-productive environments.

In the last section we open for the possibility of collusion between the agents. RPE is susceptible to collusion since both agents can jointly be better off by exerting low effort. We deduce an RPE scheme that is proof to collusion strategies, and show that our basic conclusions are not altered. In fact, in the setting where no common shock affects the agents’ outputs, we show that if effort-productivity is sufficiently high, the optimal scheme for the principal is (for a range of discount factors) a collusion-proof RPE scheme.\footnote{As is well known, a main advantage of RPE is that it helps the principal filter out common noise, see in particular Holmström (1982).} But since the collusion problem increases the cost of RPE, we also find that IPE in some cases may come out as the uniquely optimal contract. Moreover, if collusion is possible, IPE may be uniquely optimal \textit{even if output is verifiable}. For this to happen, however, there must be some common noise. With verifiable output and no common noise JPE dominates IPE, due to the advantage of peer monitoring. But if we introduce common noise, JPE does not dominate IPE for all parameter values since the cost of providing JPE incentives is high if the probability of positive common shock is high.

An interesting feature of the model is that optimal schemes do not always take the typical stark forms that are often found in the literature.\footnote{A notable exception is Carmichael (1983) who identify conditions where it is optimal to combine RPE with IPE.} For instance, we identify schemes that pay the worker both for relative and individual performance, or both for joint and individual performance. These less extreme schemes are not efficient when output is verifiable. But the
extreme schemes are seldom observed in practice, which indicates that the relational contract approach is fruitful. Moreover, we find that small changes in parameter values induce small changes in the form of the optimal contract. This is in contrast to models where small changes either make no changes on contract form, or major changes from one extreme to another extreme.

1.1 Related literature

There is a larger literature on multiagent moral hazard in static settings that compares the costs and benefits of RPE and JPE. As noted, a main advantage of RPE, stressed in particular by Holmström (1982), is that it can help the principal filter out common noise so that compensation to the largest possible extent is based on effort, not random shocks that are outside the agent’s control. With RPE’s special form, rank-order tournaments, the agents are also completely insulated from the risk of common negative shocks (see Lazear and Rosen, 1981; Stiglitz and Nalebuff, 1983). With risk averse agents, Green and Stokey (1983) show that piece rate schemes dominate tournaments when there is no common shocks (since RPE would only increase risk exposure without improving incentives), but that tournaments may dominate piece rates if there are common shocks. In a model with two sided moral hazard, (i.e. the principal is involved in production), Carmichael (1983) finds that RPE may dominate IPE even if there are no common shocks. Under sufficient level of risk aversion, however, IPE dominates RPE.

A problem with RPE, in addition to the collusion problem (analyzed in particular by Mookherjee; 1984), is that it may induce sabotage and discourage cooperation (see e.g. Lazear, 1989). JPE, on the other hand, can promote cooperation since an agent is rewarded if his peers perform well (see e.g. Holmström and Milgrom, 1990; Itoh 1993; and Macho-Stdler and Perez-Castrillo, 1993). JPE can also provide implicit incentives not to shirk (or exert low effort), since shirking may have social costs (as in Kandel and Lazear, 1992). But the classic problem with JPE is of course the free rider problem, analyzed in particular by Alchian and Demsetz (1972) and Holmström (1982). However, Che and Yoo (2001) elegantly show how repeated interaction between the agents can generate incentives and overcome the free-rider problem.\(^7\)

\(^7\)Radner (1986), Weitzman and Kruse, (1990), and FitzRoy and Kraft (1995) have all
In the literature on relational contracts, the majority of models have focused on environments where the parties have symmetric information (as in Klein and Leffler, 1981; Shapiro and Stiglitz, 1984; Bull 1987; MacLeod and Malcomson, 1989, Kreps 1990, Levin 2002). But as noted by Levin (2003), the assumption of symmetric information contrasts with the traditional incentive theory view that asymmetric information, rather than enforcement, is the central impediment to effective contracting (e.g. Holmström, 1979). Prior to Levin (2003), who makes a general treatment of the self-enforcing relational contract model with asymmetric information and risk neutral agents, only a few, such as Baker, Gibbons and Murphy (1994, 2002), have analyzed relational contract models with moral hazard in effort. We complement this literature. To our knowledge, the present paper is the first to analyze repeated agent-agent interaction within the framework of relational incentive contracts.

The rest of the paper is organized as follows: Section 2 presents the basic model. A comparative analysis of optimal contracts is presented in Section 3. Section 4 discusses the implications of collusion, while Section 5 concludes. Unless noted otherwise, all proofs are in the appendix.

2 The Model

Consider an economic environment consisting of one principal and two identical agents who each period produce either high, $Q_H$, or low, $Q_L$, values for the principal. Their effort level can be either high or low, where high effort has a disutility cost of $c$ and low effort is costless. The principal can only observe the realization of the agents’ output, not the level of effort they choose. But the agents can observe each other’s effort decisions. The agents’ output depend on effort. The probability for agent $i$ of realizing $Q_H$ is $q_H$ if the agent’s effort is high and $q_L$ if the agent’s effort is low, where $1 > q_H > q_L > 0$.

Agent $i$ receives a bonus vector $\beta = (\beta^i_{HH}, \beta^i_{HL}, \beta^i_{LH}, \beta^i_{LL})$ where the subscripts denote respectively agent $i$ and agent $j$’s realization of $Q_i$, ($i = H, L$). It is assumed that all parties are risk neutral, except that the pointed out that the folk theorem of repeated games provides a possible answer to the free rider critique of group incentives. But Che and Yoo is the first to demonstrate this in a repeated game between the agents.
agents are subject to limited liability: The principal cannot impose negative wages.\(^8\)

Let agent \(i\) and \(j\) choose efforts \(k \in \{H, L\}\) and \(l \in \{H, L\}\) respectively. Agent \(i\)’s expected wage is then

\[
\pi(k, l, \beta^i) \equiv q_k q_l \beta^i_{HH} + q_k (1 - q_l) \beta^i_{HL} + (1 - q_k) q_l \beta^i_{LH} + (1 - q_k) (1 - q_l) \beta^i_{LL}. \quad (1)
\]

For each agent, a wage scheme exhibits joint performance evaluation if \((\beta^i_{HH}, \beta^i_{LH}) > (\beta^i_{HL}, \beta^i_{LL})\).\(^9\) (For the most part we suppress agent-notation in superscript since the agents are identical.) In this case \(\pi(k, H, \beta) > \pi(k, L, \beta)\), so an agent’s work yields positive externalities to his partner. A wage scheme exhibits relative performance evaluation if \((\beta^i_{HH}, \beta^i_{LH}) < (\beta^i_{HL}, \beta^i_{LL})\). In this case \(\pi(k, H, \beta) < \pi(k, L, \beta)\), so an agent’s work generates a negative externality on his partner. A wage scheme exhibits independent performance evaluation if \((\beta^i_{HH}, \beta^i_{LH}) = (\beta^i_{HL}, \beta^i_{LL})\), which implies \(\pi(k, H, \beta) = \pi(k, L, \beta)\), so an agent’s work has no impact on his partner.

2.1 Verifiable output

Assume first that output is verifiable and that high effort is sufficiently valuable to the principal so that she always prefers to induce the agents to exert high effort. The principal’s problem is then to minimize the wage payments subject to the constraints that the agents must be induced to yield high effort. In a static setting, a contract \(\beta\) induces high effort from both agents as a unique equilibrium if

\[
\pi(H, H, \beta) - c \geq \pi(L, H, \beta), \quad (2)
\]

\[
\pi(H, L, \beta) - c \geq \pi(L, L, \beta). \quad (3)
\]

The left hand sides (LHS) show the expected wage from exerting high effort, while the right hand sides (RHS) show the expected wage from exerting low effort. We will solve the principal’s problem regarding (2), and then discuss

---

\(^8\)Limited liability may arise from liquidity constraints or from laws that prohibit firms from extracting payments from workers.

\(^9\)The inequality means weak inequality of each component and strict inequality for at least one component.
optimum against (3). The principal solves
\[
\min_{\beta \geq 0} \pi(H, H, \beta), \text{ subject to (2).} \tag{4}
\]
The incentive constraint (2) can be written
\[
q_H \beta_{HH} + (1 - q_H) \beta_{HL} - q_H \beta_{LH} - (1 - q_H) \beta_{LL} \geq \frac{c}{\Delta q},
\]
where \(\Delta q = q_H - q_L\). Since the LHS of the constraint is decreasing in \(\beta_{LH}\) and \(\beta_{LL}\), while the objective is increasing in \(\beta_{LH}\) and \(\beta_{LL}\), it is optimal to set \(\beta_{LH} = \beta_{LL} = 0\). With \(\beta_{LH} = \beta_{LL} = 0\), then any combination of \(\beta_{HH}\) and \(\beta_{HL}\) that satisfies the IC-constraint with equality is optimal, and yields expected wage \(\pi = q_H \frac{c}{\Delta q}\). We have the following lemma:

**Lemma 1** The optimal static wage scheme when output is verifiable is any scheme \(\beta^* = (\beta_{HH}, \beta_{HL}, 0, 0)\) that satisfies (2) with equality. The expected wage per agent is then \(\pi(H, H, \beta^*) = q_H \frac{c}{\Delta q}\).

The scheme \(\beta^*\) in the lemma can be RPE, IPE or JPE. It can be shown that any RPE scheme and the IPE scheme in \(\beta^*\) also satisfy (3) \(\pi(H, L, \beta) - c \geq \pi(L, L, \beta)\), while JPE schemes in \(\beta^*\) do not (see appendix). Hence, with incentive scheme \(\beta^*\) and \(\beta_{HH} > \beta_{HL}\), low effort from both agents is also an equilibrium. However, the agents are indifferent between the two equilibria (H,H) and (L,L) since the JPE scheme satisfying (3) has wage \(\frac{q_HC}{\Delta q}\), and we have \(\frac{q_HC}{\Delta q} - c = \frac{q_HC}{\Delta q}\).

Let us now proceed to the repeated setting, but still assume that output is verifiable. In a repeated setting, the agents can exploit the fact that they are able to observe each other’s effort decisions. In particular, they can play a repeated game where they both play high effort if the other agent played high effort in the previous period. In order for such a strategy to constitute a subgame perfect equilibrium, we must have:
\[
\frac{1}{1 - \delta} (\pi(H, H; \beta) - c) \geq \pi(L, H; \beta) + \frac{\delta}{1 - \delta} \min \{\pi(L, L; \beta), \pi(L, H; \beta)\}, \tag{5}
\]
where \(\delta\) is the discount factor. The LHS shows the expected present value of playing high effort, while the RHS shows the expected present value from unilaterally playing low effort in one period and being subsequently punished by the worst possible equilibrium payoff. Hence (5) says that, given
the strategy to play high effort if the other agent played high effort in the
previous period, an agent will play high effort as long as the present value
from playing high effort is greater than the present value from playing low
effort. Note that (5) is a necessary but not sufficient condition. For (5) to
be sufficient, the punishment path specified on the right hand side must also
be self-enforcing.

Observe that in a JPE scheme \( \pi(L, H; \beta) > \pi(L, L; \beta) \). Thus the right
hand side of (5) becomes \( \pi(L, H, \beta) + \frac{\delta}{1-\delta} \pi(L, L; \beta) \). In an RPE scheme,
however, where \( \pi(L, L; \beta) \geq \pi(L, H; \beta) \), the right hand side of (5) is re-
duced to \( \frac{1}{1-\delta} \pi(L, H; \beta) \). This makes (5) coincide with the static incentive
constraint, equivalent to \( \delta = 0 \). Hence, we see that repeated interaction
between the agents can increase the punishment of playing low effort in a
JPE scheme, but not in an RPE scheme. We have:

**Lemma 2** (Che and Yoo 2001) The optimal repeated wage contract when
output is verifiable is the JPE scheme \( \beta^I \equiv (\beta_{HH}, 0, 0, 0) \) where \( \beta_{HH} =
\frac{c}{(q_H + \delta q_L) \Delta q} \).

The intuition is straightforward: In the JPE scheme, low effort from
agent \( i \) does not only imply a reduced chance for him to realize high values,
it also implies that his peer plays low effort and thus lowers the chance of
realizing high values in the future. This is costly for the agent since a JPE
scheme promises highest wages if both realize high values. This is the peer-
monitoring advantage of JPE: the repeated interaction between the agents
yields both direct and implicit incentives to exert high effort. This lowers
the cost of providing incentives.

As shown in the appendix any JPE contract \( (\beta_{HH}, \beta_{HL}, 0, 0) \) (including
\( \beta^I \)) for which (5) is binding has the property that the worst sustainable
punishment - low effort from both workers \( (L, L) \) - is self-enforcing. This
makes high effort from both agents \( (H, H) \) a subgame perfect equilibrium,
and this equilibrium moreover yields each worker a higher payoff than \( (L, L) \).
(Che and Yoo call this a ‘team equilibrium’). Hence, the incentive constraint
given by (5) is sufficient.

**2.2 Relational contract between principal and agents**

Unlike Che and Yoo, we will now assume that the value realizations are
not verifiable to a third party. Hence, the contract between the agents
and the principal must therefore be self-enforcing, and thus ‘relational’ by definition. We consider a multilateral relational contract, which implies that any deviation by the principal triggers low effort from both agents. The principal honors the contract only if both agents honored the contract in the previous period. The agents honor the contract only if the principal honored the contract with both agents in the previous period. A natural explanation for this multilateral feature is that the agents interpret a unilateral contract breach (i.e. the principal deviates from the contract with only one of the agents) as evidence that the principal is not trustworthy (see Bewley, 1999; and Levin, 2002).

The contract is self-enforcing if the present value of honoring is greater than the present value of reneging. Ex post realizations of values, the principal can renege on the contract by refusing to pay the promised wage, while the agents can renege by refusing to accept the promised wage. The parties play trigger strategies. Like Baker, Gibbons and Murphy (2002), we assume that if one of the parties renege on the relational employment contract, the other insists on spot employment forever after. Spot employment implies that the agents exert low effort, but receive zero wage.\footnote{As opposed to here, where it is (implicitly) assumed that the principal owns the goods once they are produced, Kvaløy and Olsen (2005) analyze a multilateral relational contract where the agents can hold-up values ex post.}

We will now deduce the condition for the relational contract to be self-enforcing. Note first that the agents will always honor the relational contract as long as the incentive constraints hold. Hence, when deducing the enforceability constraint (or ‘implementability condition’), it is the condition that makes the principal honor the contract that is relevant. For each output realization \(Q_i, Q_j, i, j = L, H\), the principal must then find it better to pay the agents the specified bonuses \(\beta_{ij} + \beta_{ji}\) rather than renege on these payments. This is expressed by the following condition:

\[
\max\{2\beta_{HH}, \beta_{HL} + \beta_{LH}, 2\beta_{LL}\} \leq \frac{2\delta}{1-\delta} [\Delta q \Delta Q - \pi(H, H; \beta)].
\]

The left hand side is today’s loss from honoring the contract, while the right hand side is the future gain from honoring, namely the present value of the expected gain from high rather than low effort minus the wage cost. The constraint will clearly bind at the outcome where the contract specifies the
highest total bonus payments.

As we will see, an optimal contract will never bind at outcome \((Q_L, Q_L)\). Note then that the binding constraint for the principal to honor the contract depends on whether \(\beta_{HL} + \beta_{LH} > 2\beta_{HH}\) or not. For \(\beta_{HL} + \beta_{LH} > 2\beta_{HH}\) the constraint is binding at outcome \((Q_H, Q_L)\), but for \(\beta_{HL} + \beta_{LH} < 2\beta_{HH}\) the constraint is binding at outcome \((Q_H, Q_H)\) (paying each agent at \((Q_H, Q_L)\) costs more than paying, say, only the best at \((Q_H, Q_L)\)). In a JPE and IPE scheme the constraint is thus binding at outcome \((Q_H, Q_H)\).

We clearly see the 'commitment advantage' of RPE. While the principal 'risks' paying both agents high bonuses in the JPE scheme, she only risks paying one of the agents the highest bonus in the RPE scheme (at least in its extreme form with only \(\beta_{HL}\) positive). But, due to the 'peer monitoring advantage' of JPE, the necessary JPE wage is for most parameter values lower-powered and thus easier to implement. Hence, there is a trade off between enforcing a double set of 'medium size' JPE bonuses and a single, but (in most cases) larger 'winner prize' in RPE.

### 3 Comparative analysis of optimal contracts

We will now seek to determine the optimal contract for a given discount factor \(\delta\). Consider first the constraints for the principal's wage minimization program. Note that the implementability condition discussed above can be written as

\[
\frac{1 - \delta}{\delta} \max\{\beta_{HH}, \frac{\beta_{HL} + \beta_{LH}}{2}, \beta_{LL}\} + q_H^2 \beta_{HH} + q_H(1 - q_H)(\beta_{HL} + \beta_{LH}) + (1 - q_H)^2 \beta_{LL} \leq \Delta q \Delta Q. \quad \text{(GE)}
\]

As pointed out in the discussion following (5), the agents’ IC condition for an RPE contract takes the following form: \(^{11}\)

\[
q_H \beta_{HH} + (1 - q_H) \beta_{HL} - q_H \beta_{LH} - (1 - q_H) \beta_{LL} \geq \frac{c}{\Delta q}. \quad \text{(IR)}
\]

From that discussion it also follows that for a JPE contract the IC condition (5) is of the form \(\frac{1}{1 - \delta} \pi(H, H; \beta) \geq \pi(L, H, \beta) + \frac{\delta}{1 - \delta} \pi(L, L; \beta)\), and hence

\(^{11}\)Note that IR is not collusion proof. We analyze collusion in the next section.
can be written as
\[
(q_H + \delta q_L)\beta_{HH} + (1 - q_H - \delta q_L)\beta_{HL} \\
+ (\delta - q_H - \delta q_L)\beta_{LH} - (1 - q_H + \delta(1 - q_L))\beta_{LL} \geq \frac{c}{\Delta q}. \quad \text{(IJ)}
\]

Consider then the wage minimization problem for the principal;
\[
\min \pi = [q_H^2\beta_{HH} + q_H(1 - q_H)(\beta_{HL} + \beta_{LH}) + (1 - q_H)^2\beta_{LL}] \\
\text{subject to the implementability condition (GE), and the agents’ IC conditions (IR) and (IJ), respectively. We first note that we can restrict attention to contracts with } \beta_{LH} = \beta_{LL} = 0:
\]

**Lemma 3** If the set of implementable contracts is non-empty, the least-cost contract has \( \beta_{LH} = \beta_{LL} = 0 \).

**Proof.** Note that a reduction of \( \beta_{LL} \) will relax all constraints and hence ease implementation. It will of course also reduce wage costs. Next note that for an RPE contract a reduction of \( \beta_{LH} \) will relax the relevant constraints IR and GE, and it will reduce costs. For a JPE contract we see that a unit reduction of \( \beta_{LH} \) combined with a unit increase of \( \beta_{HL} \) will not affect GE but will strictly relax IJ. Hence we can increase \( \beta_{HL} \) less (so that \( \Delta(\beta_{HL} + \beta_{LH}) < 0 \)) and still satisfy both constraints (GE and IJ). Reducing \( \beta_{LH} \) this way will thus ease implementation and reduce wage costs. This proves the lemma.

The lemma allows us to consider only contracts of the form \((\beta_{HH}, \beta_{HL}, 0, 0)\). We will refer to contracts with two positive elements as JPE2 when \( \beta_{HH} > \beta_{HL} \) and RPE2 (when \( \beta_{HH} < \beta_{HL} \)), respectively. The border case (when \( \beta_{HH} = \beta_{HL} \)) is still referred to as an IPE contract. We will also refer to contracts with a single positive element as JPE1 (when \( \beta_{HH} > 0 \)) and RPE1 (when \( \beta_{HL} > 0 \)), respectively.

For these contracts the constraints take the form
\[
\frac{1 - \delta}{\delta} \max\{\beta_{HH}, \frac{\beta_{HL}}{2}\} + q_H^2\beta_{HH} + q_H(1 - q_H)\beta_{HL} \leq \Delta q \Delta Q, \quad \text{(GE)}
\]
\[
q_H\beta_{HH} + (1 - q_H)\beta_{HL} \geq \frac{c}{\Delta q}, \quad \text{(IR)}
\]

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\[(q_H + \delta q_L)\beta_{HH} + (1 - q_H - \delta q_L)\beta_{HL} \geq \frac{c}{\Delta q}\]  \hspace{1cm} (IJ)

The constraints and the set of feasible contracts can be represented graphically, as illustrated in Figure 1.

Points above and below the diagonal (where \(\beta_{HH} = \beta_{HL}\)) represent JPE and RPE contracts, respectively. The shaded area shows the set of implementable contracts. Note that the implementability constraint GE has a kink at \(\beta_{HH} = \beta_{HL}\). The figure illustrates a case where a limited set of JPE and RPE contracts are feasible, but no JPE1 contract can be implemented. The least costly contract is thus a JPE2 contract; more specifically the contract at the intersection point between GE and IJ. This contract is optimal because iso-cost lines (represented by \(q_H^2\beta_{HH} + q_H(1 - q_H)\beta_{HL} = \text{const}\)) are parallel to the line representing IR, and this line is always steeper than IJ. The best RPE2 and IPE contracts are equally costly (the common cost is \(q_H \frac{c}{\Delta q}\), see Lemma 1), but they cost more than the best JPE2 contract.

Let \(\delta_J\) be the minimal discount factor for which a JPE1 contract can be implemented. The figure illustrates a case where \(\delta < \delta_J\), so no JPE1 con-
tracts are feasible. The factor $\delta_J$ can be found by solving the two constraints (GE and IJ) for the minimal $\delta$ when $\beta_{HL} = 0$. This yields the following equation for $\delta_J$:

$$\frac{1 - \delta_J}{\delta_J} - (\Delta q)^2 \frac{\Delta Q}{c} q_L \delta_J = \left[ (\Delta q)^2 \frac{\Delta Q}{c} - q_H \right] q_H. \quad (6)$$

The figure also makes it clear that for $\delta < \delta_J$ there are implementable JPE2 contracts if and only if some part of the diagonal is within the shaded area, meaning that there is a range of contracts with $\beta_{HH} = \beta_{HL}$—i.e. IPE contracts—that satisfy both constraints (GE and IJ). Let $\delta_I$ be the minimal discount factor for which an IPE contract can be implemented. This factor can be found by solving the two constraints (GE and IJ) for the minimal $\delta$ when $\beta_{HH} = \beta_{HL}$. This yields the following equation for $\delta_I$:

$$\frac{1 - \delta_I}{\delta_I} = \left[ (\Delta q)^2 \frac{\Delta Q}{c} - q_H \right]. \quad (7)$$

Note that an IPE contract will be implementable (i.e. $\delta_I < 1$) only if the expression in the square bracket is positive. Recall that the minimal cost (per agent) for an IPE contract is $q_H \frac{c}{\Delta q}$ (the cost for any contract on IR). In order for the principal to find it profitable to induce high effort by the agents via such a contract, the expected gain in output must be higher than the cost, i.e. we must have $\Delta q \Delta Q \geq q_H \frac{c}{\Delta q}$. This is precisely the condition that makes $\delta_I$ well defined in (7).

As noted, the graphs in Figure 1 correspond to a case where $\delta_I < \delta < \delta_J$, and here a unique JPE2 contract is optimal. As the discount factor moves from $\delta_J$ to $\delta_I$, the intersection point between GE and IJ moves southeast towards the diagonal. Hence, the closer the discount factor gets to $\delta_I$, the more the principal must reduce $\beta_{HH}$ and increase $\beta_{HL}$. The intuition for this is as follows: As the discount factor decreases, exploiting peer monitoring becomes more costly. A JPE contract where most of the bonus payments are concentrated in $\beta_{HH}$ (such as the stark JPE1 contract) has a lower wage cost than the least costly IPE, but the maximum total pay that the principal risks paying is also larger compared to IPE (or to a JPE2 scheme that is close to IPE). Hence, when $\delta = \delta_I + \varepsilon$, ($\varepsilon$ close to zero), the optimal scheme is a unique JPE2 contract that is quite close to IPE.

For yet a lower discount factor; $\delta < \min\{\delta_I, \delta_J\}$, neither JPE1 nor JPE2
contracts can be implemented. For such a $\delta$ it is possible that an RPE2 contract will be feasible, and if so is the case it follows that such a contract will be optimal.

Let $\delta_{R2}$ be the minimal discount factor for which an RPE2 contract can be implemented. From the geometry of the constraints it follows that the set of implementable RPE2 contracts is non-empty if and only if some RPE2 contract with $\beta_{HH} = \frac{\beta_{HL}}{2}$ can be implemented (see Figure 1 and note that the implementability constraint GE is always steeper than IR for $\beta_{HH} < \frac{\beta_{HL}}{2}$). The critical discount factor $\delta_{R2}$ can thus be found by solving the two constraints (GE and IR) for the minimal $\delta$ when $\beta_{HH} = \frac{\beta_{HL}}{2}$. This yields

$$1 - \frac{\delta_{R2}}{\delta_{R2}} = [(\Delta q) \frac{2\Delta Q}{c} - q_H] (2 - q_H).$$

Comparing with the condition (7) defining $\delta_I$ we see that $\delta_{R2} < \delta_I$. Thus, whenever it is possible to implement an IPE contract (i.e. when $\delta_I < 1$), it is also possible to implement some RPE2 contract. This illustrates the commitment advantage of RPE.

But the fact that $\delta_{R2} < \delta_I$ does not mean that any RPE contract is implementable when IPE is implementable. For instance, if $\delta = \delta_I > \delta_{R2}$, the RPE1 contract $(0, \beta_{HL}, 0, 0)$ is not necessarily implementable. When $q_H > \frac{1}{2}$ a single RPE bonus $\beta_{HL} = \frac{c}{\Delta q(1-q_H)}$ is larger than a double set of IPE bonuses $2\frac{c}{\Delta q}$. Hence the incentive for the principal to deviate is larger under this RPE1 contract than under the IPE contract. Thus, if $\delta_{R2} < \delta \leq \min\{\delta_I, \delta_J\}$ and $q_H > \frac{1}{2}$, then the stark RPE1 contract $(0, \beta_{HL}, 0, 0)$ is not part of the set of feasible RPE2 contracts.

The discussion so far can be summarized in the following Proposition:

**Proposition 1** (i) When $\delta \geq \delta_J$ a unique JPE1 contract is optimal. For $\delta < \delta_J$ we have:

(ii) If $\delta_I < \delta < \delta_J$ a unique JPE2 contract is optimal.

(iii) If $\delta_{R2} \leq \delta \leq \min\{\delta_I, \delta_J\}$, all RPE2 contracts that satisfy (IR) with equality and (GE) are optimal.

(iv) For $\delta < \min\{\delta_{R2}, \delta_I, \delta_J\}$ no incentive contracts are implementable.

Recall that $\delta_{R2} < \delta_I$ holds as long as the principal finds it profitable to induce high effort; see the discussion following (7). In light of this, Proposition 1 elucidates the trade-off between RPE’s commitment advantage and
JPE’s peer monitoring advantage. Since there is no common noise in our model, a JPE scheme is always optimal if it can be implemented (Proposition 1, parts (i) and (ii)). However, RPE schemes are in general easier to implement, and if no JPE contracts can be implemented there may exist discount factors where RPE is implementable and thus optimal (part (iii)).

In order to draw more economic intuition out of Proposition 1, we now compare further the critical discount factors necessary to implement the defined contracts. In particular, we want to determine under which conditions the various cases delineated in the proposition will arise. We find the following:

**Proposition 2** There are bounds $b_i = b_i(q_H, q_L)$, such that

$$\delta_i < \delta_J \iff (\Delta q)^2 \frac{\Delta Q}{c} - q_H > b_i, \ i \in \{I, R2\}.$$  
The bounds $b_i$ are given by

$$b_i(q_H, q_L) = \sqrt{\left(\frac{a_i - q_H - q_L}{2(a_i - q_H)a_i}\right)^2 + \frac{q_H q_L}{2(a_i - q_H)a_i}} - \frac{a_i - q_H - q_L}{2(a_i - q_H)a_i},$$  
where $a_I = 1$, $a_{R2} = 2 - q_H$.

We are here particularly interested in when it is the case that no JPE1 contract is implementable, but some ‘second best’ contract (JPE2, IPE or RPE2) is implementable, i.e. when it is the case that $\delta_I < \delta_J$ and/or $\delta_{R2} < \delta_J$. From the proposition we see that this can occur only if the expression $A = (\Delta q)^2 \frac{\Delta Q}{c} - q_H$ is sufficiently large. Recall that in RPE or IPE the principal will induce high effort only if $A > 0$. We see that for $A$ small ($A \approx 0$) we will clearly have $\delta_I < \delta_J, \delta_{R2}$, and a JPE1 contract can then be implemented whenever an RPE2 or JPE2 (or IPE) contract can be implemented. But if $A$ becomes sufficiently high, we will conversely have $\delta_i < \delta_J$. Hence, a necessary condition for RPE2 or JPE2 (or IPE) to be optimal is that the effort-productivity, $\Delta q \frac{\Delta Q}{c}$, is sufficiently high.\(^{12}\)

If $q_l$ and $Q_L$ are constants (e.g. equal to zero) variations in $\Delta q \frac{\Delta Q}{c}$ will reflect variations in skills. We can then say that the agents’ skills must be sufficiently high in order for JPE2 or RPE2 to be implementable.

Propositions 1 and 2 are illustrated in figure 2.\(^{13}\)

\(^{12}\)Effort-productivity $\Delta q \frac{\Delta Q}{c}$ measures the expected gain in output per unit cost of effort when effort increases. This differs from labor-productivity (or ‘agent productivity’), since a high-productive agent may have low effort-productivity (i.e both $Q_L$ and $Q_H$ high, but $\Delta Q$ low). However, it is reasonable to assume that a high-productive agent has higher effort-productivity than a low-productive agent.

\(^{13}\)The figure is generated with $q_H = 0.6$ and $q_L = 0.3$. 

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Figure 2. Critical discount factors as function of productivity

The figure shows the relationship between $\delta$, $\frac{\Delta Q}{c}$, and optimal incentive regimes. The curves show critical discount factors as functions of $\frac{\Delta Q}{c}$, where the thick solid line is $\delta_J$, the thin solid line is $\delta_{R2}$, and the dotted line is $\delta_I$. A change in other parameters would shift all three curves. In particular, note that an increase in $q_H - q_L = \Delta q$, will shift the curves leftward, and thus ease implementation.

Like in Che and Yoo, we see that RPE tends to dominate for low discount factors, while JPE dominates for high discount factors. However, in Che and Yoo the optimality of RPE depends on a positive common shock. Since we assume no common noise, RPE is always more costly than JPE in our model. The expected JPE wage is always lower than the expected RPE wage due to the peer-monitoring advantage of JPE. But the relational contract constraint that we add to the analysis makes the starkest JPE contract unfeasible on lower discount factors. If the JPE bonus $\beta_{HH}$ becomes sufficiently high (as it does with low discount factors), the maximum wage that the principal ‘risks’ paying is higher in JPE than in RPE, and relational JPE contracts are therefore relatively harder to implement when the discount factor is low.

Let us now discuss variations in productivity generated by variations in
the ratio $\frac{\Delta Q}{c}$. In Che and Yoo such variations are not relevant, except that productivity must be sufficiently large in order that the principal should have incentives to induce high effort. In our model $\frac{\Delta Q}{c}$ is decisive for the determination of incentive regime. We see that when $\frac{\Delta Q}{c}$ is very high, any scheme can be implemented (since the principal’s loss from reneging is high).

For a given discount factor, say $\delta = 0.6$, we see that as $\frac{\Delta Q}{c}$ decreases, the principal cannot commit to JPE1, and she must must offer a JPE2 contract. With further decreases in $\frac{\Delta Q}{c}$, the principal cannot commit to JPE2, and she must offer an RPE2 contract. This shows the commitment advantage of RPE, which here makes such a contract optimal for relatively low values of $\frac{\Delta Q}{c}$.

The latter observation seems at odds with our statements that we will see a higher frequency of RPE as we move from low productivity to high productivity environments. However, it is not natural to think of all transactions/relationships/industries as governed by the same discount factor. We can interpret the discount factor as a measure for the dependency, or trust level, between the transacting parties (see e.g. Hart, 2001, on interpreting $\delta$ as trust; and James Jr., 2002, for a nice survey on the economic concept of trust). We can say that the vertical axis represents dependency levels, or trust levels, where $\delta = 0$ is a spot market, and $\delta = 1$ is a fully dependent high-trust relationship. We see that as $\frac{\Delta Q}{c}$ increases, the fraction of $\delta$’s where RPE2 is optimal increases more than the fraction of $\delta$’s where JPE2 is optimal, which again increases more than the fraction of $\delta$’s where JPE1 is optimal. Hence, our interpretation of the analysis is that we will see a higher frequency of RPE (and JPE2 schemes that lies close to IPE) as we move from low-productive to high-productive environments. On low effort-productivity, relational contracts can only be implemented on high trust-levels, i.e. high discount factors. This favors JPE since implicit contracts between the agents are then easy to self-enforce (in Figure 2 we see that when $\frac{\Delta Q}{c} < 9.3$, only JPE1 can be implemented, but this requires $\delta > 0.76$). As we move towards high-productive environments, the cost of contract deviation becomes higher and relational contracts can therefore be implemented on lower discount factors. We will then expect a larger fraction of RPE, since JPE is vulnerable to low discount factors.

However, irrespective of how one interprets Figure 2, it clearly suggests a relationship between productivity, measured as return from effort, and
incentive regime. This a quite different approach than the standard moral hazard models that stress the trade-off between incentives and insurance. Since we assume risk neutrality, insurance is not as issue in our model. As indicated in the introduction, a technical difficulty with risk aversion is that it does not ensure the optimality of stationary contracts (see Levin 2003). Introducing risk aversion would thus considerably complicate the analysis. At the outset, this does not justify the neutrality-assumption. So one may ask whether risk aversion would alter our results. In a more general model with verifiable output, Green and Stokey (1983) show that IPE dominates RPE when agents are risk averse and there are no common shocks. Our Lemma 1 would thus be altered if we introduce risk aversion. Risk aversion would also imply relatively higher critical discount factors for JPE and RPE than for IPE, since social surplus from these schemes are lowered when agents are exposed to their peer’s risk. In Figure 2 this should imply that the solid lines take a larger shift to the right than the dotted line. But the basic trade-off between the peer-monitoring advantage of JPE and the commitment-advantage of RPE should still apply.

4 Collusion

In contrast to IPE and JPE, RPE is vulnerable to collusion. If the principal offers an RPE contract, then the agents can be better off if they both play low effort (L,L) than if they both play high effort (H,H). To see this note that a least-cost RPE contract (which satisfies IR with equality) yields expected wages equal to $q_H \frac{c_H}{\Delta q_H}$ per agent if efforts are (H,H). If efforts are (L,L), the expected wage is straightforwardly seen to exceed $q_L \frac{c_L}{\Delta q_L}$ iff $\beta_{HL} > \beta_{HH}$. The difference in wage payments is then smaller than the cost $c$ of high effort, hence in such a contract the agents are better off if they can coordinate on both exerting low rather than high effort.

As we shall see, the agents can sustain such a low-effort ‘collusive’ outcome as a subgame perfect equilibrium, but only if the discount factor is sufficiently high. The agents can, however, for the given RPE contract always sustain a coordinated randomized strategy, and in particular a correlated randomization where they play (H,L) and (L,H) with equal probabilities (for parameters such that $\frac{q_L}{q_H} \leq \frac{1}{2}$ the latter is in fact optimal for the agents). To prevent such an outcome, the principal must then modify the
RPE contract such that the collusive strategy is no longer an equilibrium for the agents. This will typically increase expected wage costs, and hence make RPE contracts more expensive than the least-cost IPE contract (which is not vulnerable to collusion) for the principal. If the IPE contract is feasible, it may therefore be uniquely optimal. On the other hand, if neither this IPE contract nor any JPE contract is implementable, it can still be possible to implement a set of 'collusion proof' RPE contracts, and the least-cost contract among those will be the uniquely optimal one for the principal.

We show in this section the following results:

1. The least-cost collusion proof RPE scheme entails the lowest possible $\beta_{HL}$, i.e. the best RPE contract lies as close as possible to IPE. As a result IPE can be uniquely optimal.

2. If we introduce common noise when agents can collude, IPE can be uniquely optimal even if output is verifiable.

3. The main results in the previous section regarding variations in effort productivity still apply.

Turning to the analysis, consider first a correlated randomization where the agents play (H,L) and (L,H) with equal probabilities. Can the agents sustain this as a subgame perfect equilibrium (SPE) outcome in the repeated game? Yes, but for this to be the case it is necessary that an agent is not tempted to deviate when he is to play low effort as a part of (L,H). This means that we must have:

$$\pi(L;H;\beta) + \frac{\delta}{1 - \delta} \left( \frac{1}{2} \pi(L;H;\beta) + \pi(H;L;\beta) - c \right) \geq \pi(H;H;\beta) - c + \frac{\delta}{1 - \delta} (\pi(H;H;\beta) - c).$$

The LHS is the payoff associated with adhering to the collusive strategy. The RHS is the payoff obtained by deviating to high effort and then being punished in the future by the worst SPE, namely (H,H) forever.

To prevent such a 'collusive' equilibrium, the principal must choose bonuses such that the above inequality is reversed. This entails bonuses that fulfill the following condition:

$$q_H \beta_{HH} + (1 - q_H - \frac{\delta}{2}) \beta_{HL} > (1 - \frac{\delta}{2}) \frac{c}{\Delta q}.$$  \hspace{1cm} (IRC)

An RPE contract that satisfies IRC is thus immune to collusion that involves playing (H,L) and (L,H) with equal probabilities. As we shall see it is immune to a much wider class of collusive strategies.
Consider next the collusive strategy where the agents coordinate on both exerting low effort. To sustain this as a SPE outcome in the repeated game, an agent must not be tempted to deviate when he is to play low effort as a part of (L,L). By a similar reasoning as above we find that the principal can upset this equilibrium by choosing bonuses that satisfy the following condition (see the appendix):

\[(\delta q_H + q_L)\beta_{HH} + (1 - q_L - \delta q_H)\beta_{HL} > \frac{c}{\Delta q}.\] (IRL)

It turns out that, depending on the parameters, one of the two collusive strategies we have considered here will be optimal for the agents. Moreover, this implies that the two conditions IRC and IRL completely characterizes the set of collusion proof RPE contracts. We have the following result:

**Lemma 4** For any RPE contract that satisfies IR we have:

(i) For \(\frac{q_L}{q_H} \leq \frac{1}{2}\) the correlated strategy where the agents play (H,L) and (L,H) with equal probabilities is optimal for the agents in the sense of maximizing their per period payoff. For \(\frac{q_L}{q_H} > \frac{1}{2}\) the pure (L,L) strategy is optimal.

(ii) The contract is collusion proof if and only if it satisfies IRC and IRL.

(iii) For a set of parameters including \(\frac{q_L}{q_H} \leq 2(\sqrt{2} - 1) = 0.828\) the contract is collusion proof if and only if it satisfies IRC.

An RPE contract satisfying IR is thus immune to any form of collusion if and only if it satisfies both IRC and IRL. Collusion on (L,L) is feasible if the discount factor is sufficiently high, and for such discount factors contracts may have to be constrained more than what is implied by IRC in order to prevent collusion. But for a wide set of parameters, including \(\frac{q_L}{q_H} \leq 0.828\), it is the case that IRL is satisfied whenever IRC is, and hence IRC alone is then necessary and sufficient to prevent collusion. To make the exposition cleaner we consider only the latter case in the following.

Given the parameter restriction, condition IRC defines the set of collusion proof RPE2 contracts, and it is geometrically represented by a line delineating this set such as illustrated in Figure 3.
The line representing IRC intersects IR at the point $\beta_{HH} = \beta_{HL} = \frac{c}{\delta q}$, and we see that the condition is binding (reduces the set of feasible contracts) since IRC is flatter than IR. An RPE contract is vulnerable to collusion, and we see that the principal must increase the bonus $\beta_{HH}$ for good performance by both agents relative to the bonus $\beta_{HL}$ for good performance by only one agent (making the constraint flatter) in order to prevent the agents from colluding on low efforts.

Recall that iso-cost lines are parallel to IR. All RPE contracts that satisfy IRC are therefore more costly than, and thus dominated by the IPE contract $\beta_{HH} = \beta_{HL} = \frac{c}{\delta q}$. Hence, if the IPE contract is feasible, i.e. if $\delta > \delta_I$, it will be preferred by the principal to any feasible RPE contract.

Figure 3 illustrates a case where neither the IPE contract nor any JPE contracts are feasible, and where the collusion constraint binds, i.e. where $\delta < \min\{\delta_I, \delta_J\}$. In this case there is a set of feasible collusion proof RPE contracts (the shaded area), and we see that the uniquely optimal contract is represented by the point in this set that is on IRC and is closest to IPE (the northwest intersection between GE and IRC).

From the figure it is clear that there is a non-empty set of feasible col-
lusion proof RPE contracts if IRC intersects the line $\beta_{HH} = \frac{\beta_L}{2}$ to the left of where GE intersects this line. Algebraically this condition takes the following form (see the appendix for the derivation):

$$\frac{1 - \delta}{\delta} - \left[ (\Delta q)^2 \frac{\Delta Q}{c} - q_H \right] (2 - q_H) \leq \frac{1 - \delta}{2} - \delta \left[ (\Delta q)^2 \frac{\Delta Q}{c} - q_H \right] \frac{\delta q_H}{c}. \quad (9)$$

The condition defines the set of discount factors for which such a set of feasible and collusion proof RPE contracts exists. The terms on the RHS makes this condition different from the one defining the critical $\delta_{R2}$ in the absence of collusion (8). The condition will hold for $\delta$ in an interval $(\delta_{RC}, \delta)$, where $\delta \leq 1$. (Written as $g(\delta) \leq 0$ condition (9) is represented by a second-order polynomial.) The smaller root $\delta_{RC}$ is the critical lowest discount factor for which there will be an (economically interesting) collusion proof RPE contract that can be implemented. Comparing (9) and (8), we see that (9) does not hold for $\delta = \delta_{R2}$, hence we must have $\delta_{RC} > \delta_{R2}$.15 The collusion proofness constraint naturally increases the critical discount factor for implementation. Comparing with the critical discount factor $\delta_I$ for IPE contracts defined in (7), we see that $\delta_I$ will satisfy (9) and hence will exceed $\delta_{RC}$ ($\delta_{RC} < \delta_I$), only if the square bracket (or effort-productivity) is sufficiently large.

We can now state the following proposition:

**Proposition 3** For a set of parameters including $\frac{q_L}{q_H} \leq 2\sqrt{2} - 1$ the following holds. When contracts must be proof to collusion by the agents we have:

(i) If $\delta_I < \delta < \delta_J$ a unique JPE2 contract is optimal.

(ii) If $\delta = \delta_I < \delta_J$, then a pure IPE contract is uniquely optimal.

(iii) If $\delta \in (\delta_{RC}, \delta)$ and $\delta < \min\{\delta_I, \delta_J\}$, the optimal scheme is a unique RPE2 contract (the one that lies as close as possible to IPE).

(iv) If $\delta < \min\{\delta_{RC}, \delta_I, \delta_J\}$ no collusion proof incentive contracts are implementable.

Proposition 3 is similar to Proposition 1 (where collusion is not possible),

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14In the present setting the condition is sufficient but strictly speaking not necessary, since IRC may be flatter than GE (for large $\delta$) and hence may intersect GE to the left of the line $\beta_{HH} = \beta_{LH}/2$. But such a case is not economically interesting, since any feasible RPE contract would then be dominated by the IPE contract.

15For $\delta = \delta_{R2}$ defined in (8) the left hand side of (9) is zero while the right hand side is negative.
except for one important aspect. Contrary to the former case, the optimal RPE2 contract is now not any RPE2 contract that lies on IR and satisfies the enforceability constraint. Instead there is a unique optimal RPE2 contract, and this lies as close as possible to IPE. This also implies that IPE may be uniquely optimal. The intuition behind this is straightforward: By increasing $\beta_{HH}$ and reducing $\beta_{HL}$, the agent’s future gain from sustaining collusion is reduced. Hence the wage costs incurred by the principal to prevent collusion are reduced.

It is easy to see that a sufficiently high $\frac{\Delta Q}{c}$ is still a necessary condition for an RPE2 contract to be optimal. The easiest way to verify this is to note that when $\frac{\Delta Q}{c}$ is sufficiently small, then the relational contract can only be implemented for $\delta$ close to one. But for $\delta$ close to one, RPE schemes are impossible to implement because collusion is then easy to sustain. As $\frac{\Delta Q}{c}$ increases, it is possible to implement relational contracts on lower discount factors. And with lower discount factors collusion is harder to sustain (and thus easier to prevent). Hence, the result that relative performance evaluation can only be optimal when effort productivity is sufficiently high is clearer under the threat of collusion. Figure 4 is similar to Figure 2 and illustrates how the critical discount factors, including $\delta_{RC}$, varies with $\frac{\Delta Q}{c}$.

Figure 4. Critical discount factors under collusion proofness.
**Verifiable output.** Before we leave this section, we will show that under the threat of collusion IPE can be optimal even if the contract is verifiable. For this to occur, however, we must introduce common noise. As we have seen, JPE is less costly than IPE when output is verifiable (Lemma 2). However, if we introduce common noise JPE becomes less attractive since the cost of providing JPE is high if the probability of a positive common shock is high. Hence, common noise makes IPE less costly than JPE for some parameter values. What about IPE vs RPE? As we have seen, collusion makes IPE less costly than RPE. If we introduce common noise, though, RPE becomes less costly than IPE for some parameter values (since RPE filters out common noise). But to sum up: Collusion weakens RPE, while common noise weakens JPE. This makes IPE potentially uniquely optimal even if the contract is verifiable:

We follow Che and Yoo, assuming that a favorable shock occurs with probability $\sigma \in (0, 1)$, in which both agents produce high values for the principal. If the shock is unfavorable, the probability for agent $i$ of realizing $Q_H$ is still $q_H$ if the agent’s effort is high and $q_L$ if the agent’s effort is low. Observe now that a marginal increase in $\beta_{HH}$, while holding the IC constraint (IRC) with equality, will reduce the principal’s surplus (increase the wage) when $\sigma$ is sufficiently high:

$$\frac{\partial \pi}{\partial \beta_{HH}} = \sigma + (1 - \sigma)q_H^2(1 - \frac{1-q_H}{1-q_H-\delta}).$$

Hence the optimal collusion proof incentive scheme with $\beta_{HH} \leq \beta_{HL}$ has either $\beta_{HH} = 0$ or $\beta_{HH} = \beta_{HL}$. When $\beta_{HH} = 0$, the collusion proof RPE scheme has wage cost $q_H(1 - q_H)\frac{2-2q_H-\delta}{q_H(1-q_H)}$ per agent, which exceeds the per agent wage cost under IPE $(\sigma + (1 - \sigma)q_H)\frac{\sigma}{\Delta q(1-\sigma)}$ when $\delta > \frac{2\sigma(1-q_H)}{q_H(1-\sigma)+\sigma}$. The expected cost per agent under the JPE1 contract is $(\sigma + (1 - \sigma)q_Hq_H)\frac{\sigma}{\Delta q(1-\sigma)}\sigma_q + q_L$. Hence, the expected cost per agent is lower under IPE than under JPE if $\delta < \frac{\sigma(1-q_H)}{q_L(1-q_H)}\frac{\sigma}{\Delta q(1-\sigma)}$, which is true for $1 - \sigma > \frac{2q_Hq_L-q_H^2}{2q_Hq_L-2q_Hq_L+1}$, which can only hold for $q_L \leq \frac{1}{2}$. We can thus state:

**Proposition 4** When output is verifiable, and collusion is possible, then if parameters are such that $\delta > \delta$, there exist discount factors $\delta > \delta > \delta$ where independent performance evaluation is optimal.
5 Concluding remarks

A large literature has explored the merits of RPE and JPE in static settings. It is, however, often natural to think of managers and workers interacting repeatedly and basing pay on non-contractible performance measures. We know that joint performance evaluation can be optimal in dynamic settings since peer monitoring works as an incentive device. On the other hand, non-contractable performance measures gives relative performance evaluation a commitment advantage since the principal need only commit to a limited total payment. In this paper we have compared these competing effects. Our main result is that the agents’ productivity is decisive for the feasibility of different incentive regimes. Ceteris paribus, low productivity calls for RPE, since low productivity increases the principal’s incentives to deviate from the relational contract, and RPE limits total payment. However, low productivity makes high discount factors necessary for implementing relational contracts, and high discount factors support JPE’s peer-monitoring device. We have thus argued that our model predicts a higher frequency of RPE as we move from low to high-productive environments, since high productivity makes it possible to implement relational contracts on low discount factors.

Interestingly, our result seems consistent with the established hypothesis that high productive workers are more common under high-powered incentive regimes. However, we do not develop a "sorting argument", in which high productive workers seek high-powered incentive contracts. Instead, it is the principal’s opportunity to renege from promises that makes productivity matter, since the cost from reneging is positively related to the productivity of the agents. Our analysis thus indicates that the causality puzzle between contracts and other relevant variables may still occur even if one controls for adverse selection.

6 Appendix

Appendix to Lemma 1:

We first show that any RPE scheme and the IPE scheme in $\beta^*$ also satisfy (3). Note that for any contract $(\beta_{HH}, \beta_{HL}, 0, 0)$ we have

$$\pi(H, H, \beta) + \pi(L, L, \beta) - \pi(H, L, \beta) - \pi(L, H, \beta) = (\Delta q)^2(\beta_{HH} - \beta_{HL}). \quad (10)$$
Hence, \( \pi(H, H, \beta^s) - c + \pi(L, L, \beta^s) \leq \pi(H, L, \beta^s) - c + \pi(L, H, \beta^s) \) when \( \beta_{HL} \geq \beta_{HH} \). This implies that (3) is satisfied, since \( \pi(H, H, \beta^s) - c \geq \pi(L, H, \beta^s) \).

Note that JPE schemes in \( \beta^s \) may not satisfy (3). When \( \beta_{HL} < \beta_{HH} \), then \( \pi(H, H, \beta^s) + \pi(L, L, \beta^s) - \pi(H, L, \beta^s) - \pi(L, H, \beta^s) > 0 \), hence \( \pi(H, H, \beta^s) - c + \pi(L, L, \beta^s) > \pi(H, L, \beta^s) - c + \pi(L, H, \beta^s) \), which does not ensure that \( \pi(L, L, \beta^s) \leq \pi(H, L, \beta^s) - c \).

We finally find the optimal JPE scheme satisfying \( \pi(H, L, \beta) - c \geq \pi(L, L, \beta) \). This constraint can be written

\[
q_L \beta_{HH} + (1 - q_L) \beta_{HL} - q_L \beta_{LH} - (1 - q_L) \beta_{LL} \geq \frac{c}{\Delta q}, \tag{11}
\]

Now any combination of \( \beta_{HH} \) and \( \beta_{HL} \) that satisfies the IC-constraint (11) with equality and has \( \beta_{LL} = \beta_{HL} = 0 \) is optimal. Solving the constraint for \( \beta_{HH} \) or \( \beta_{HL} \) and inserting in the objective function yields expected wage \( \pi = \frac{q_L c}{\Delta q} \) always.

**Proof of Lemma 2:**

When \( \pi(L, H; \beta) > \pi(L, L; \beta) \), we can write the constraint (5) as

\[
(q_H + \delta q_L) \beta_{HH} + (1 - q_H - \delta q_L) \beta_{HL} - (\delta - q_H - \delta q_L) \beta_{LH} - (1 - q_H + \delta (1 - q_H)) \beta_{LL} \geq \frac{c}{\Delta q}.
\]

The left hand side of the constraint is decreasing in \( \beta_{LL} \), while the objective function \( \pi(H, H; \beta) = (q_H^2 \beta_{HH} + q_H (1 - q_H) (\beta_{HL} + \beta_{LL}) + (1 - q_H)^2 \beta_{LL}) \) is increasing in \( \beta_{LL} \). Hence, it is optimal to set \( \beta_{LL} = 0 \). Observe that the coefficient of \( \beta_{HL} \) is weakly greater than that of \( \beta_{HL} \) in the left hand side of the contraint, but that their coefficients are the same in the objective function. It is thus optimal to set \( \beta_{HL} = 0 \). Moreover, observe that a marginal increase in \( \beta_{HL} \), while holding the constraint with equality, will reduce the principal’s surplus (increase the wage):

\[
\frac{\partial \pi(H, H; \beta)}{\partial \beta_{HL}} = q_H^2 \frac{\partial \beta_{HH}}{\partial \beta_{HL}} + q_H (1 - q_H) (\frac{\partial \beta_{HL}}{\partial \beta_{HL}} + (1 - q_H)^2 \frac{\partial \beta_{LL}}{\partial \beta_{HL}}) > 0 \quad \text{for } \delta > 0.
\]

Hence, it is optimal to set \( \beta_{HL} = 0 \). The optimal JPE scheme is thus \( \beta^J \equiv (\beta_{HH}, 0, 0) \) where \( \beta_{HH} = \frac{c}{(q_H + \delta q_L) \Delta q} \). For JPE to dominate IPE and RPE we must have

\[
\frac{q^J_H c}{(q_H + \delta q_L) \Delta q} < \frac{q_L c}{\Delta q}, \tag{12}
\]

which holds for \( \delta > 0 \).

By an argument similar to Che and Yoo, we can show that any JPE contract \( \beta^J = (\beta_{HH}, \beta_{HL}, 0, 0) \) where (5) is binding and \( \beta_{HH} > \beta_{HL} \) (including \( \beta^J \)) makes the worst sustainable punishment -low effort from both workers.
(L,L)- self-enforcing. Given that (5) is binding we have \(\frac{1}{1-\delta}(\pi(H, H; \beta^i) - c) = \pi(L, H; \beta^i) + \frac{\delta}{1-\delta} \pi(L, L; \beta^i).\) Since \(\pi(L, H; \beta^i) > \pi(L, L; \beta^i),\) this implies \(\pi(H, H; \beta^i) - c < \pi(L, H; \beta^i).\) Then, since \(\pi(H, H; \beta^i) + \pi(L, L; \beta^i) - \pi(H, L; \beta^i) - \pi(L, H; \beta^i) > 0\) by (10), we have \(\pi(L, L; \beta^i) > \pi(H, L; \beta^i) - c\) (Comment: Note that in the static case \(\pi(H, H; \beta^i) - c < \pi(L, H; \beta^i)\) and \(\pi(L, L; \beta^i) > \pi(H, L; \beta^i) - c\) would break the IC contraint. The virtue of JPE is that the IC contraint is slack in the repeated setting).

**Proof of Proposition 2:**

Here we compare the critical \(\delta\)'s; \(\delta_f, \delta_f, \delta_{R2}.\) They are given by (6),(7) and (8), respectively. The equation (6) for \(\delta_f\) has the form \(F(\delta_f) = 0,\) where \(F' < 0.\) Hence \(\delta_i < \delta_f\) iff \(F(\delta_i) > 0.\) Each \(\delta_i\) is defined by an equation of the form
\[
\frac{1 - \delta_i}{\delta_i} = A a_i, \quad A = [(\Delta q)^2 \Delta Q_c - q_H] > 0.
\]
Substituting into the expression for \(F(\delta)\) we thus have
\[
\delta_i < \delta_f \iff 1 - \frac{\delta_i}{\delta_i} = A a_i > A q_H \iff A(a_i - q_H) > \frac{[A + q_H] q_L}{A a_i + 1}.
\]

For \(a_i - q_H > 0\) the last inequality holds iff \(A\) exceeds the unique positive root of the equation \((a_i - q_H)A(A a_i + 1) - [A + q_H] q_L = 0,\) i.e. iff
\[
A > b_i = \sqrt{\frac{(a_i - q_H - q_L)^2}{2(a_i - q_H)a_i} + \frac{q_H q_L}{2(a_i - q_H)a_i} - \frac{a_i - q_H - q_L}{2(a_i - q_H)a_i}}.
\]

**Proof of Lemma 4.**

Let \(\Pi(a, b; \beta)\) denote the expected per period payoff per agent when they play a correlated (symmetric) strategy with probabilities \((a, b, b, 1 - a - 2b)\) on effort combinations \((H, H), (H, L), (L, H), (L, L)\) respectively. We prove statement (i) below, i.e. that \(b = \frac{1}{2}\) is optimal when \(\frac{q_L}{q_H} \leq \frac{1}{2}\), and that \(a = b = 0\) (the pure low-effort strategy) is optimal otherwise.

A strategy with \(0 < b < \frac{1}{2}\) is feasible (sustainable as a SPE) iff an agent is not tempted to deviate when he is to play L as part of (L,H) nor as part of (L,L), i.e. iff the following two conditions hold
\[
\pi(L, H; \beta) + \frac{\delta}{1 - \delta} \Pi(a, b; \beta) \geq \pi(H, H; \beta) - c + \frac{\delta}{1 - \delta} (\pi(H, H; \beta) - c), \quad (12)
\]
\[ \pi(L, L; \beta) + \frac{\delta}{1 - \delta} \Pi(a, b; \beta) \geq \pi(H, L; \beta) - c + \frac{\delta}{1 - \delta} (\pi(H, H; \beta) - c). \]  

(The agent will never be tempted to deviate from H as a part of (H,L), since \( \pi(H, L; \beta) - c > \pi(L, L; \beta) \)). The pure (L,L) strategy is sustainable iff (13) holds for \( a = b = 0 \), and since the payoff then is \( \Pi(0, 0; \beta) = \pi(L, L; \beta) \), this condition amounts to

\[ \frac{1}{1 - \delta} \pi(L, L; \beta) \geq \pi(H, L; \beta) - c + \frac{\delta}{1 - \delta} (\pi(H, H; \beta) - c). \]

Substituting for the \( \pi \)-terms, we see that this condition is not satisfied precisely when IRL holds.

It follows immediately that conditions IRL and IRC are necessary for collusion proofness. For if IRL is not satisfied, then the collusive strategy (L,L) is feasible (at least for sufficiently large \( \delta \)). And if IRC is not satisfied, the collusive strategy \( b = \frac{1}{2} \) is feasible.

To prove sufficiency suppose IRC and IRL are satisfied. Since IRC holds, the strategy \( b = \frac{1}{2} \) is not feasible. By construction, IRL makes the pure strategy (L,L) \((a = b = 0)\) infeasible. We will show that no other collusive strategy \((0 < b < \frac{1}{2})\) can be feasible either.

Consider first \( \frac{q_H}{q_H} \leq \frac{1}{2} \). Since \( b = \frac{1}{2} \) is optimal in this case (statement (i)), we have \( \Pi(a, b; \beta) \leq \Pi(0, \frac{1}{2}; \beta) \). But then (12) cannot hold, since this condition by construction of IRC does not hold for \( b = \frac{1}{2} \).

Consider next \( \frac{q_L}{q_H} > \frac{1}{2} \), in which case the pure strategy (L,L) \((a = b = 0)\) is optimal, i.e. \( \Pi(a, b; \beta) \leq \Pi(0, 0; \beta) \). But then (13) cannot hold, since this condition by construction of IRL does not hold for the pure (L,L) strategy \((a = b = 0)\). This proves that no collusive strategy is feasible, and hence that statement (ii) holds.

To prove statement (iii), we compare the slopes of the lines defined by equalities in IRC and IRL, respectively. The claim holds if the line for IRC is flatter than that for IRL, which occurs if \( \frac{1 - q_H - \frac{1}{2}}{q_H} \leq \frac{1 - q_L - \delta q_H}{q_L + \delta q_H} \). This condition is equivalent to \( q_L(1 - \frac{1}{2} \delta) - q_H + \delta q_H - \frac{1}{2} \delta^2 q_H \leq 0 \). The latter inequality can be written as \( (\delta^2 - (1 + Q) \delta + 2Q) \frac{1}{2} q_H \geq 0 \) where \( Q = 1 - \frac{q_L}{q_H} \). The polynomial is nonnegative for all \( \delta \in (0, 1) \) if \( Q \geq 3 - 2\sqrt{2} \), i.e. if \( \frac{q_L}{q_H} \leq 2(\sqrt{2} - 1) = 0.828 \). This proves statement (iii) in the lemma.

It remains to prove statement (i). The expected payoff \( \Pi(a, b; \beta) \) per agent is given by \( 2 \Pi(a, b; \beta) = a(2\pi(H, H) - 2c) + 2b(\pi(H, L) + \pi(L, H) - \)

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\[ c + (1 - a - 2b)2\pi(L, L). \] Hence

\[ \Pi(a, b; \beta) = a(\pi(H, H) - \pi(L, L) - c) + b(\pi(H, L) + \pi(L, H) - 2\pi(L, L) - c) + \pi(L, L). \]

This is to be maximized, subject to \( a + 2b \leq 1 \). For an RPE scheme we always have, from (10) \( \pi(H, L) + \pi(L, H) \geq \pi(H, H) + \pi(L, L) \), and hence \( \pi(H, L) + \pi(L, H) - 2\pi(L, L) \geq \pi(H, H) - \pi(L, L) \).

This shows that the payoff multiplying \( a \) in \( \Pi(a, b; \beta) \) is smaller than that multiplying \( b \), hence \( a = 0 \) is optimal. Then \( b = \frac{1}{2} \) is optimal iff \( \pi(H, L) + \pi(L, H) - 2\pi(L, L) \geq c \). As shown below, this is equivalent to \( \left[ \beta_{HL} - 2qL(\beta_{HL} - \beta_{HH}) \right] - \frac{qH}{qL} \Delta q \geq 0 \). This inequality is satisfied for all RPE contracts \( \beta \) that satisfy IR iff (comparing slopes) \( \frac{1 - 2qL}{2qL} \geq \frac{1 - qH}{qH} \). This is equivalent to \( \frac{qH}{qL} \leq \frac{1}{2} \), and hence proves that \( b = \frac{1}{2} \) is optimal. It follows that for \( \frac{qH}{qL} > \frac{1}{2} \) it is optimal to set \( b = 0 \).

It then only remains to verify the condition for \( \pi(H, L) + \pi(L, H) - 2\pi(L, L) \geq c \). We have

\[ \pi(H, L) + \pi(L, H) = (qHqL\beta_{HH} + qH(1 - qL)\beta_{HL}) \]

\[ + (qLqH\beta_{HH} + qL(1 - qH)\beta_{HL}) \]

\[ = 2qHqL(\beta_{HH} - \beta_{HL}) + (qh + qL)\beta_{HL}. \]

Thus

\[ \pi(H, L) + \pi(L, H) - 2\pi(L, L) = (2qHqL(\beta_{HH} - \beta_{HL}) + (qh + qL)\beta_{HL}) \]

\[ - 2(qL^2(\beta_{HH} - \beta_{HL}) + qL\beta_{HL}) \]

\[ = 2(qH - qL)qL(\beta_{HH} - \beta_{HL}) + (qh - qL)\beta_{HL} \]

\[ = (\beta_{HL} - 2qL(\beta_{HL} - \beta_{HH})) \Delta q. \]

This completes the proof.

**Derivation of (9)**

Consider the minimal \( \delta \) for which IRC intersects \( \beta_{HL} = 2\beta_{HH} \) to the left of where GE intersects this line. The two points are given by, respectively

\[ qH\beta_{HH} + (1 - qH - \frac{\delta}{2})2\beta_{HH} = (1 - \frac{\delta}{2}) \frac{c}{2\Delta q}, \]

\[ (\frac{1 - \delta}{2} + qH^2))\beta_{HH} + qH(1 - qH)2\beta_{HH} = \Delta q \Delta Q. \]

So the condition is

\[ \frac{(1 - \frac{\delta}{2}) \frac{c}{2\Delta q}}{(2 - qH - \delta)} \leq \frac{\Delta q \Delta Q}{\frac{1 - \delta}{2} + qH(2 - qH)}. \]

This is equivalent to (9).

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References


